Lab 1 Report: Speed of Light

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Background

Einstein's special theory of relativity suggests that the speed of light, c, is the speed limit of the universe. Maxwell's equations show that c is a fundamental quantity, and these qualities have given scientists motivation for measuring the speed of light. However, the speed of light inspired curiosity in physicists well before these discoveries, as many scientists attempted to measure the quantity without success before Maxwell or Einstein made their contributions. Fizeau accomplished this feat in the mid-eighteenth century using a rotating gear and mirrors, which allowed him to get circumvent the difficulty in measuring how long it takes light to travel in a linear path. Foucault improved Fizeau's design with a rotating mirror, which has inspired the experimental setup used for the present experiment.

Setup

Given that light travels at an incredibly fast speed, we cannot successfully measure the speed of light through timing how long time takes to travel a linear distance. However, we can learn from Foucault and Fizeau and incorporate mirrors and rotations to create our experimental design. Figure 1 shows how we use a polarized laser beam as our light source and reflect this light off of a spinning mirror. When correctly aligned, the laser reflects onto a second mirror that directs the laser onto a return mirror that reflects the light back to the second mirror and back to the return mirror. In the time that it has taken for the light to travel from the spinning mirror to the return mirror and back, the spinning mirror has rotated θ degrees from the position when it reflected the laser. The light reflects off of the spinning mirror at an angle of 2θ relative to the initial laser, and hits a beam splitter that reflects a fraction of the light through a polarizer and onto a camera sensor. We also placed a photodiode in the plane of the reflection of the laser so that we could measure the frequency of the spinning mirror. This frequency along with the positions captured on the camera allows us to measure the amount of time that has passed between the instants that the light reflected off the spinning mirror. Utilizing the distance that the light travels during this time, we can calculate a value for c.

Although the speed of light does not depend on the length of the path that the light takes between reflections on the spinning mirror, we can choose these values intentionally to help with data processing. In order to focus the laser on the second reflection of the spinning mirror, we placed a lens such that the focal length (2 meters) of the lens is the distance that the light travels from the spinning mirror to the lens. Using the thin lens equation (eq. 1) we can calculate S' and place the return mirror at that distance. This setup allows parallel light rays from the spinning mirror to converge onto the same point on the return mirror. The light then focuses on the spinning mirror, allowing for a tighter laser beam to focus on the camera sensor. Because we need accurate measurements of the position of the laser beam on the sensor to calculate θ and therefore c, we want a tighter beam that produces a more precise position measurement.

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \text{ (eq. 1)}$$

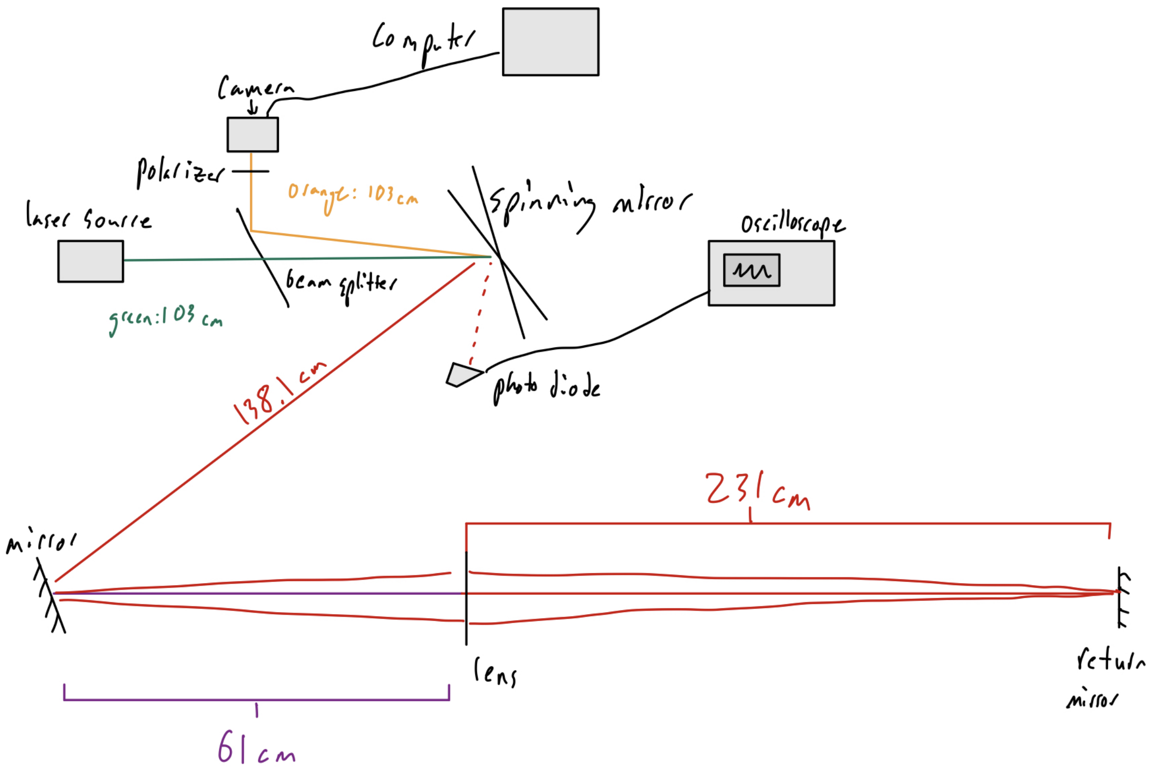
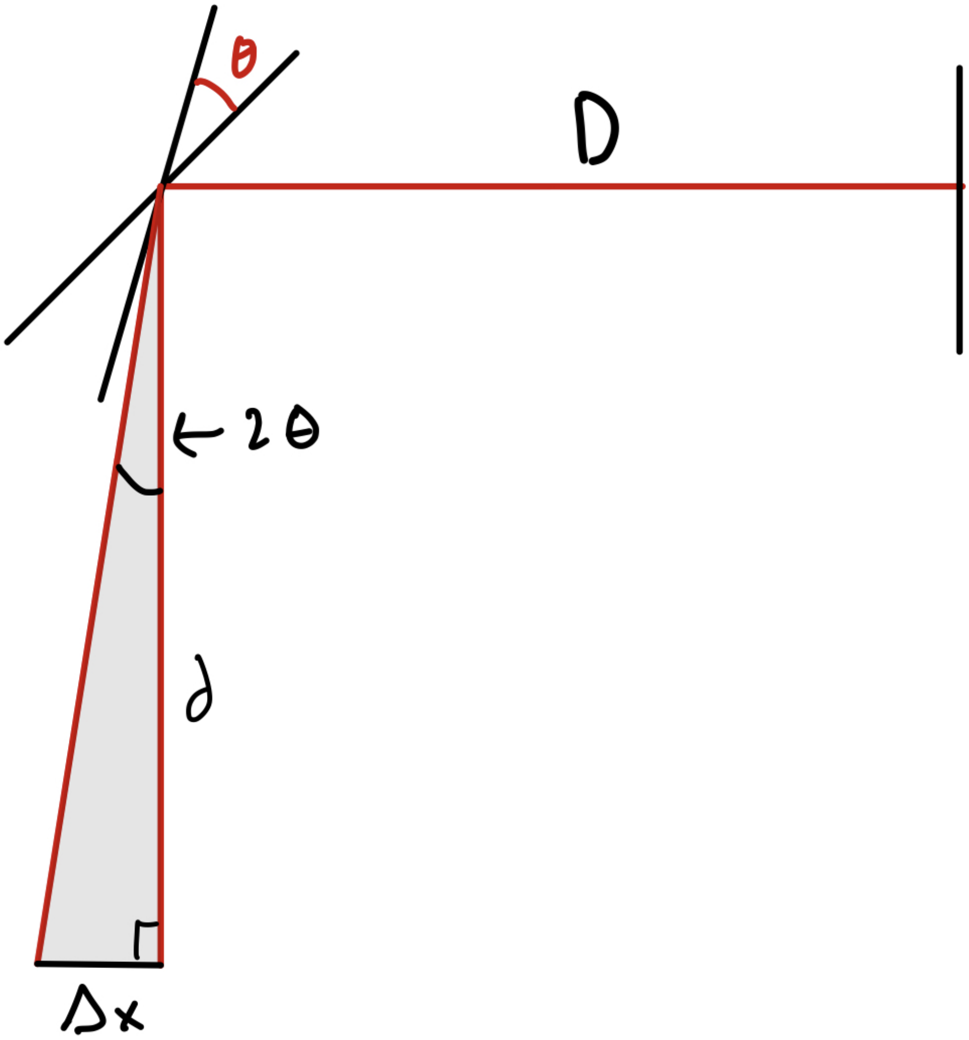


Figure 1: Experimental Setup

Theory

As previously mentioned, we need to calculate how far the light travels in the time it takes the spinning mirror to rotate θ degrees as the quotient of these two quantities yields the speed of light: $c = \frac{\Delta x}{\Delta t}$. However, instead of explicitly solving for c, we can instead solve for the displacement of the laser beam after reflecting off of the spinning mirror, as shown in figure 2. This will allow use to write a linear function for displacement in terms of frequency, with c as a constant.



Using the formula for tangent $tan(2\theta) = \frac{\Delta x}{d}$ we can write displacement, Δx as a function of \theta, with constant d:

$$\Delta x = dtan(2\theta)\ (eq. 2)$$

Here, theta depends on the angular velocity of the spinning mirror where $\theta = \omega \cdot t$, and t is the amount of time it took the laser to travel D meters in figure 2. Because we can more easily and precisely measure frequency than angular velocity using the photodiode, we can rewrite our equation for θ in terms for frequency: $\theta = 2\pi \cdot f \cdot t$. Because the light travels at speed c over a distance of D meters while the spinning mirror turns θ degrees, we can substitute $t = \frac{2D}{c}$ as the light travels D meters to the return mirror, then D meters back to the spinning mirror:

$$\theta = \frac{4\pi \cdot f \cdot D}{c}\ (eq. 3)$$

We can now use this result inside of our formula for displacement:

$$\Delta x = dtan(\frac{8\pi \cdot f \cdot D}{c})\ (eq. 4)$$

Because light travels very fast relative to the angular velocity of the spinning mirror, the spinning mirror changes angle a small amount, allowing us to apply the small angle approximation where $tan(\theta) \approx \theta$:

$$\Delta x = \frac{8\pi \cdot d \cdot f \cdot D}{c} \ (eq. 5)$$

Now that we have this formula, we can measure the value of Δx at different frequencies, and use the results to determine a value for c.

Data

In order to measure the displacement of the laser beam we reflected the returning beam onto a camera sensor. Because beam displacement was on the scale of microns, we could not use a mirror to reflect the returning beam as the mirror would block the laser source. To circumvent this issue we places a beam splitter in the beams path. The beam splitter reflects half of the laser source, but half of the laser is unaffected, allowing the beam to travel through the apparatus. When the beam reaches the beam splitter on the way back, half of the laser is reflected onto the camera sensor. At each of 12 different frequencies we captured a photo from the camera and use the RGB values to find the photo's centroid. We chose to find the spot with the highest blue value as these centroids appeared closer to the center of the laser beam. We then took the horizontal value from the centroid as our displacement. See appendix I for a complete table of the collected data, and appendix II for the code used for processing the data.

Analysis

Now that we have data for the displacement of the laser in terms of frequency, we can fit the data to eq. 5 to calculate a slope of best fit. Because our model is a linear function, we can rewrite it for ease of computing the fit:

$$\Delta x = A \cdot f + y \ (eq. 6)$$

where y is the y intercept, and

$$A = \frac{8\pi \cdot d \cdot D}{c} (eq. 7)$$

Now we can fit our data to eq. 6 and find a value for A. From our fit, we calculated $A = -4.4 \cdot 10^{-7}$. We can use A to solve for c:

$$c = \frac{8\pi \cdot d \cdot D}{A} (eq. 8)$$

This formula gives:

$$c = 4.64 \cdot 10^{8} \ m/s.$$

When viewing a graph of the data (figure 3), we notice that there are outlier points at higher frequencies. Specifically the 8th and 9th data points appear to deviate from the pattern of the rest of the data. We can remove just those two points and plot the 10 remaining data points (figure 4), and hence calculate a new value for c with this refined data. This yields a value of:

$$c = 4.23 \cdot 10^{8} \ m/s$$

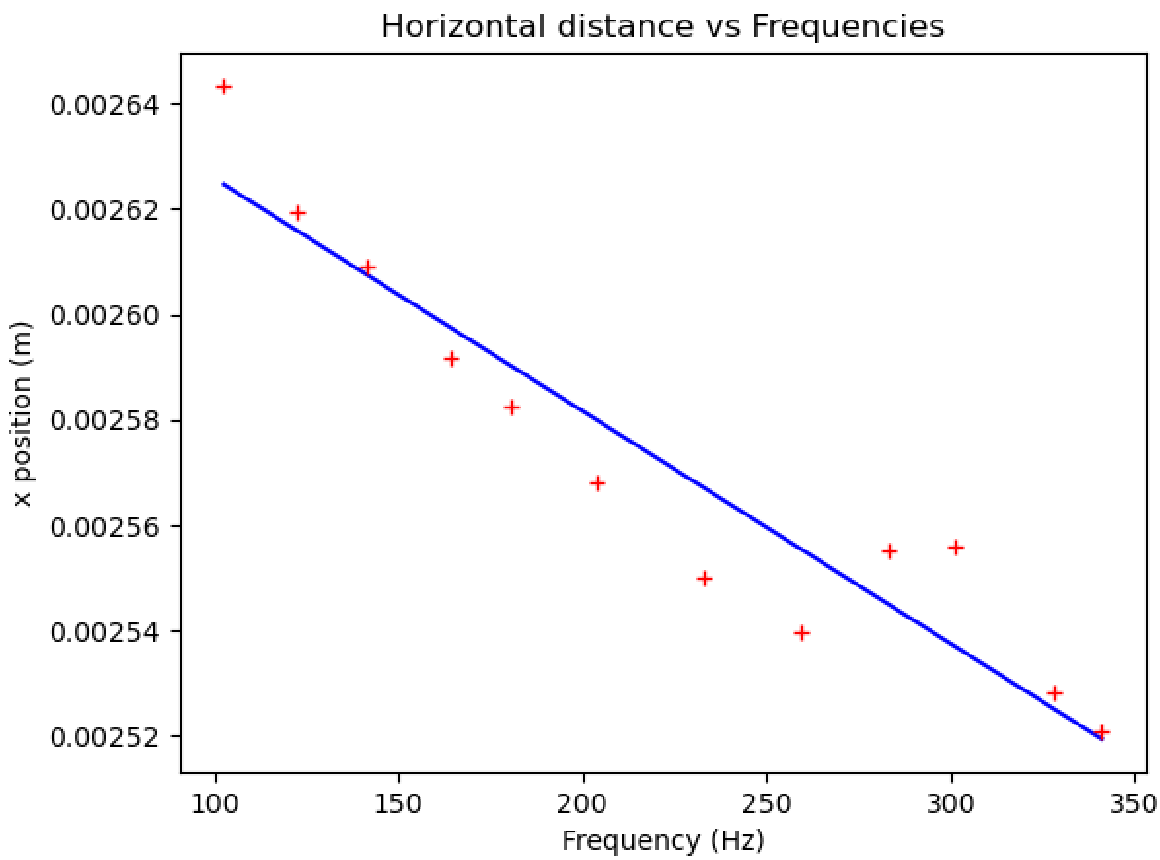


Figure 3: Displacement vs Frequency

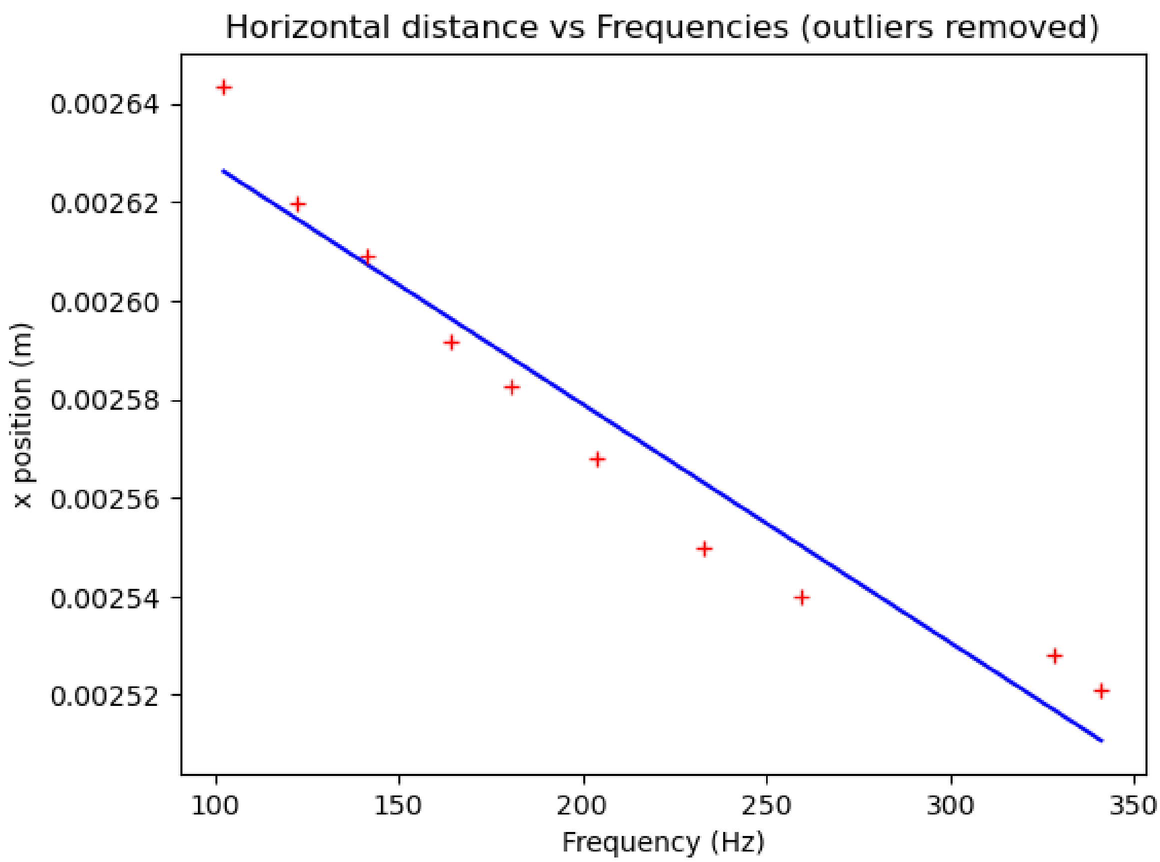


Figure 4: Reduced data plot

We find that the removal of those two outliers lowers the speed of light by about a tenth of its previous value. This new fit appears to match the data a bit better, the removal of the 9th and 10th point makes it clearer that the last two points, the 11th and 12th points do not match the trend of the first 8 points. To limit the influence of these points, we calculated a third value for c:

$$c = 3.16 \cdot 10^8\ m/s$$

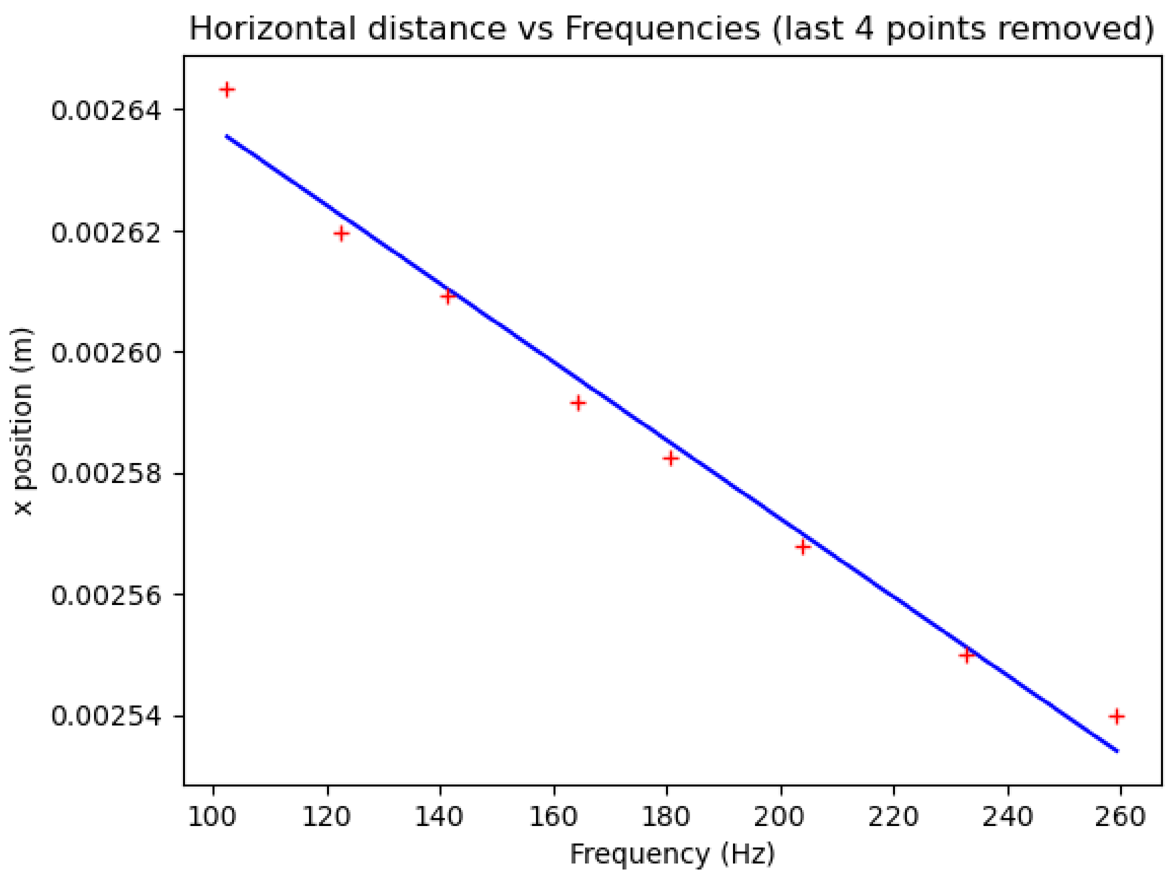


Figure 5: Final reduced data set

To help understand the meaning of our results, we calculated uncertainty in our value for c. Because c (eq. 8) is calculated from d, D, and A, we must propegate the uncertainty of c from these variables. Using the uncertainty propagation formula , we find:

$$(\frac{\delta c}{c})^2 = (\frac{\delta D}{D})^2 + (\frac{\delta d}{d})^2 + (\frac{\delta A}{A})^2\ (eq. 9)$$

with uncertainties and values:

	Value	Uncertainty
d	1.03 meters	0.01 meters
D	7.87 meters	0.05 meters
F	2.00 meters	0.01 meters
A	4.4 * 10^-7	0.3 * 10^-7

Substituting these values into eq. 9 and solving for c, we find our uncertainty in c:

$$\delta c = 0.2 \cdot 10^8\ m/s$$

Together, we have the final experimental result for the reduced data set:

$$c = (3.2 \pm 0.2) \cdot 10^8\ m/s$$

Discussion

From more precise experiments the speed of light is already known as $c = 2.998 \cdot 10^8\ m/s$. Knowing this, our experimental result appears to deviate only slightly from what we would expect. This value, from the most reduced dataset, also matches the expected value for c more closely than the original value, and the value calculated using 10 of the 12 data points. Given that this reduced data set fit better may suggest that something with the experimental setup went awry when collecting the last four data points, or could potentially signify a divergence from our model at higher frequencies. If we were to perform this experiment again, being more careful about the stability of the camera could eliminate the need to reduce the data set, and using equipment that can handle higher frequencies could allow for a study on the diversions we noticed for the high frequency data points. Furthermore, using a laser with a tighter beam, or alternatively a more precise beam focusing arrangement could produce more precise photos, allowing for a more accurate study.

Appendix I: Data

Frequency (Hz)	Displacement (meters)
102.6	0.002643
122.7	0.002620
141.5	0.002609
164.3	0.002592
180.5	0.002582
203.9	0.002568
232.9	0.002550
259.3	0.002540
283.1	0.002555
301.4	0.002556
328.1	0.002528
341.0	0.002521

Appendix II: Code

See https://github.com/RowanKell/PhysicsLab