A Brief Introduction to the Proton Spin Puzzle

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I. Introduction

Over the past summer and throughout this semester I have had the opportunity to work on a particle physics research project. My project concerns the validity of TMD factorization in the Semi-Inclusive deep inelastic scattering (SIDIS) at CLAS12, a detector at Jefferson Lab. Specifically, I have worked on calculating SIDIS kinematics from CLAS12 data (and simulation) that can be used to calculate a quantity called affinity that estimates how well certain factorization schemes describe the QCD physics occuring in SIDIS. My project is tiny part of the giant global effort to describe the dynamics and structure of nucleons: the proton and neutron. This effort was jump started in 1988 when the European Muon Collaboration published the first results of g_1^p, a quantity that describes how much of the proton's spin is made up from valence quark spin. The simplest quark model of the proton describes the nucleon as a collection of three quarks bound together by nuclear gluon, where the proton's 1/2 spin is the sum of the spins of the 1/2 spin quarks. SIDIS experiments have forced physicists to create more complex nucleon models where the three valence quarks orbit through a sea of quark-antiquark pairs. The proton spin can thus originate in parts from valence quark spin and angular momentum, sea quark spin and angular momentum, and even gluon spin and angular momentum. This paper aims to introduce the experiments that began and continued these efforts in proton spin studies, explain the impact of SIDIS studies on the field, and connect the physics back to the basics of spin taught in Modern Physics courses

First the paper will provide an overview of spin for elementary particles and how the quantity can be used to describe (word for not elementary) particles. I will then discuss the Bjorken and Ellis-Jaffe sum rules and their relevence to nucleon spin, as well as the experiments that tested their validity. These experiments lead into the current advances concerning the proton spin puzzle where the paper concludes.

II. What is spin? From electrons to protons

Before the discovery of electron spin, physicists knew about the orbital angular momentum of the electron that results from it's orbits around the nucleus. Furthermore, the introduction of quantum mechanics brought on new experiments to verify the quantization of electron angular momentum. Perhaps the most famous experiment on the matter was thought up by Otto Stern and performed by Walther Gerlach, and has since been referred to as the Stern-Gerlach experiment. In this experiment, a beam of silver atoms are sent through a non-uniform magnetic field and deposited onto a glass plate for observation. The physicists used a non-uniform field to exert a force on any particle with magnetic moment. This deflection depends on the alignment of the particle's magnetic moment, and thus the physicists intended to show the quantization of space due to discrete levels of alignment creating discrete collections on the observation plate. Under the assumption that the electron's magnetic moment depends solely on the orbital angular momentum, they expected to see $(2l+1)$ deposits on the plate. From a classical perspective, there would be no constraint on the number of deposits as the angular momentum is not quantized in classical physics, and thus the experiment was created as an attempt to show space quantization. However, the result did not follow either theory, and instead two distinct components showed on the plate. Furthermore, the silver atoms used in the experiment had no angular momentum in their ground state where the valence electron resides, corresponding to $l = 0$. The present theory of quantized orbital angular momentum had no explanation for the two components, suggesting the electron may have another contribution to its magnetic moment (__Physics textbook citation).

T. E. Phipps and J. B. Taylor performed the Stern-Gerlach experiment in 1927, this time using hydrogen atoms to remove any sources of uncertainty in using a complex silver atom. The physicists' data showed the same result as the original experiment, confirming that there was another source of the electron's magnetic moment. Graduate students at the University of Lieden developed a theory to explain the phenomena and described the magnetic moment as a result of the electron spinning on its axis. Although they were incorrect about the electron spinning, the name stuck and a theory of spin angular momentum was created. The students conjectured that the moment obeyed the same space quantization rules as orbital angular momentum, producing a formula for the number of components of spin angular momentum for a particle: $(2s + 1)$. In both the previously mentioned experiments, the results showed two discrete components which indicates the value for the spin quantum number: $s = \frac{1}{2}$. Furthermore, they gave formulas for the z-component and magnitude of spin angular momentum:

$$S_z = m_s \hbar$$

$$|S| = \sqrt{s(s+1)\hbar} = \frac{\sqrt{3}}{2}\hbar$$

Here, $m_s = \pm\frac{1}{2}$. Because the magnitude of the spin angular momentum depends only only constants, the spin magnetic moment must be an intrinsic property of the electron.

Decades after the breakthrough of the Stern-Gerlach experiment, the European Muon Collaboration (EMC) created an experiment to investigate the spin makeup of the proton. The naive quark models of the time predicted that all of the spin angular momentum came from the intrinsic spin of the valence quark constituents of the proton. However, the EMC results showed that only a small portion of proton spin can be attributed to the valence quarks. Just as in the Stern-Gerlach experiment physicists found that the current models did not account for the total angular momentum of the electron, the simple quark model of the proton did not account for the total spin. In a sense, the proton spin puzzle can be thought of as a repeat of the Stern-Gerlach result at the next level lower on the scale of the fundamental building blocks of the universe. Hence, this paper serves as a brief introduction to the more recent expansion of spin studies performed after the discovery of spin.

Section III. Beginnings of the Proton Spin Crisis

Before we go into the theory and experiments that broke apart the naive quark model of the nucleon, we must develop a basic understanding of key concepts in scattering experiments. The 1987 EMC experiment mentioned previously used deep inelastic scattering to probe the proton and calculate the spin contribution of its constituents. Inelastic scattering refers to experiments where a particle (typically in a particle accelerator) collides with another particle and through the transfer of momentum "breaks" the particle into different particles. In experiments such as (**give electron proton scattering experiment examples**), leptons (electrons and muons) are accelerated to high energies before being shot at target nucleons. When the two particles collide, the lepton transfers part of its momentum to a virtual photon which the proton absorbs (the interaction is explained by quantum electrodynamics, or QED). At sufficiently high energies the virtual photon is able to hit an individual quark, and can knock the quark out of the nucleon. Because quarks must be bound to other quarks in hadrons, sets of two or three quarks (protons and neutrons are both examples of hadrons), the scattered quark hadronizes into a new particle (this effect is described by quantum chromodynamics, or QCD). Hence, we refer to these experiments as depp inelastic scattering if the square of the momentum transferred from the lepton to the hadron, Q^2, is large. When only the incident and scattered lepton are measured in the experiment, the process is called inclusive, while if at least one high energy hadron is measured in the final state (such as the hadron created by the struck quark), then the experiment is called semi-inclusive. Bringing this all together, we can describe the experiments performed by the EMC and SLAC (**double check this**) as inclusive deep inelastic scattering, or DIS, and those performed by JLab, (**Add sidis experiments**), as semi-inclusive deep inelastic scattering, or SIDIS. All of the kinematics of DIS experiments are contained in the momentum transfer Q^2 as well as the energy transferred between the lepton and the target ν. SIDIS experiments have utilized more complex kinematics, some of which will be detailed further in the paper. Another important concept to scattering experiments are cross sections (and differential cross sections). In this context, a cross section refers to the probability of a certain event happening. When one quantum particle approaches another, there is a non-zero probability that the two do not interact. Hence, the total cross section for a particle collision describes the probability of measuring a specific endstate given an interaction. For example, we could find a cross section that describes the probability of measuring four leptons in the final state of a proton-proton collision. The differential cross section describes the probability of measuring an endstate within a range of a kinematic variable (Dotson, 2020 **Add citation for youtube video: https://www.youtube.com/watch?v=ojPQN86BW9o&ab_channel=AndrewDotson**)

In 1966, James Bjorken theorized that the operators mediating the electromagnetic interaction may obey the same rules as free-field operators leading him to create a sum rule for DIS of polarized leptons off polarized protons and neutrons. This rule, called the Bjorken sum rule, describes the relationship between the structure functions g_1^p (proton) and g_1^n (neutron) and DIS kinematics:

$$\int_0^\infty \frac{Q^2}{M\nu^2} d\nu \Big(g_1^p(Q^2,\nu) - g_1^n(Q^2,\nu) \Big) = \frac{g_A}{3}$$

There are four total structure functions that together summarize the composition of the proton. Here, g_1 is proportional to the polarization asymmetry, which is defined as the difference of the cross sections where nucleon and lepton spins are parallel and antiparallel. Bjorken's sum rule predicts that the cross sections contained g_1 do not depend on Q^2, shown by the lack of Q^2 on the right side of the equation (Jaffe 26). This prediction suggests that the size of the probed object doesn't affect the cross section, meaning that the quarks and gluons must be point-like objects. The cross section thus depends only on the scaling variable x_{Bj} where:

$$x_{Bj} = \frac{Q^2}{2M\nu}$$

In electron proton DIS experiments, x_{Bj} can be thought of as the fraction of the target proton's momentum attributed to the quark about to be struck by the incident electron (viewed in a high proton momentum frame). This allows us to think of the structure functions as probability distributions in x for the quarks inside the nucleon (Jaffe 26). When labeled by flavor, we find:

$$g_1^p(x) = \frac{4}{9}\Delta u(x) + \frac{4}{9}\Delta \bar{u}(x) + \frac{1}{9}\Delta d(x) + \frac{1}{9}\Delta \bar{d}(x) + \frac{1}{9}\Delta s(x) + \frac{1}{9}\Delta \bar{s}(x)$$

where $\Delta u(x)$ represents the difference between up quark x distributions for quarks with spin parallel and antiparallel to the spin of the proton. We get the fractional coefficients in front of each term from the square of the quark's charge due to DIS utilizing the QED interaction. While we do not find Q^2 dependence for the structure function here, it is important to note that QCD radiative effects introduce Q^2 dependence at high momentum transfer. Radiative effects are when the target proton emit(s) gluon(s) that are unmeasured, altering the energy and momentum of the system. By integrating the structure function over all x_{Bj}, we find:

$$\int_0^1 g_1^p(x)dx = \frac{1}{2}\Big(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\Big)$$

QCD flavor symmetry shows that matrix elements F and D can be related to $\Delta u - \Delta s$ and $\Delta u - \Delta d$, allowing substitutions leading to:

$$18\int_0^1 g_1^p(x)dx = 3F + D + 2\Sigma \quad (eq4)$$

(Jaffe 26). Here, Σ corresponds to the sum $\Delta u + \Delta d + \Delta s$, or the contribution of all the light quarks to the proton's spin.

As mentioned earlier, the simplest quark model of the proton consists of two up quarks and one down quark, while more sophisticated models describe a sea of virtual quark-antiquark pairs residing in the proton. In 1973, Arthur Jaffe and John Ellis theorized that unpolarized strange quarks may inhabit the nucleon. They thought that these quarks, which are much more massive than up and down quarks, may not contribute a significant amount to the proton's total spin, leading them to approximate the strange contribution as 0 in equation 4. They then calculated a value for the structure function g_1^p using current experimental values for F and D as well as QCD radiative corrections:

$$\int_0^\infty g_1^p(x, Q^2)dx = 0.176 \pm 0.006 \quad (eq\ 5)$$

(Jaffe 27). This sum rule became known as the Ellis-Jaffe sum rule, and was tested by DIS experiments conducted by the EMC and SLAC.

The first DIS experiments began in the 1970s, starting at SLAC. However, the earliest polarized proton scattering was carried out at too low x_{Bj} values to investigate the Bjorken sum rule and Ellis-Jaffe sum rule. By the end of the decade Vernon Hughes had begun an experiment capable of evaluating the sum rules. His group measured:

$$\int_0^1 g_1^p(x)dx = 0.17 \pm 0.05$$

While this result agreed with the Ellis-Jaffe sum rule, the scattering events occurred at low Q^2, leaving ambiguity about phase space close to x_{Bj} = 0. Following this experiment at SLAC, CERN produced the next results at the super Proton Synchrotron. This experiment was conducted by CERN's European Muon Collaboration (EMC). The EMC experiment scattering high energy muons off of unpolarized proton targets. Through decaying pions they achieved 80% polarization and energy levels as high as 100 GeV, much higher than previous experiments. Their results published in 1987 found the structure function to be much less than that measured by SLAC and the prediction from the Ellis-Jaffe sum rule:

$$\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$$

(Jaffe 27). This measurement corresponds to $\Sigma = 0.120 \pm 0.16$, suggesting that, within error, none of the proton's spin comes from the spin of the light quarks' inside it. Additionally, the result suggested that a significant portion of the proton's spin comes from strange quark-antiquark pairs with a value of:

$$\Delta s = -0.190 \pm 0.056$$

(Jaffe 27). Ellis and Jaffe had conjectured this value to be zero and thus the EMC data shows a violation of their sum rule.

The spin muon collaboration succeeded the EMC and set out to further their goals of describing the nucleon spin structure through measurements of the neutron structure function. SLAC followed suit and worked to measure the structure function as well, and both groups announced their first results in 1993. Assuming no contribution from the strange quark, the theoretical value is:

$$\int_0^1 g_1^n(x)dx = -0.002 \pm 0.005$$

The SMC reported an inconsistent value of:

$$\int_0^1 g_1^n(x)dx = -0.08 \pm 0.04$$

And lastly SLAC reported a closer to theoretical value:

$$\int_0^1 g_1^n(x)dx = -0.022 \pm 0.011$$

(Jaffe 28). Recalling the Bjorken sum rule, we can write a new integral which is equal to $1/2$ the left side of equation 1:

$$\int_0^1 (g(x)_1^p - g(x)_1^n)dx$$

The new SLAC and SMC data produced results for this quantity, allowing for evaluation of the Bjorken sum rule. The results from both experiments proved consistent with expected values, acting as a confirmation of the sum rule.

IV. Current Solutions and Problems

This section is devoted to the current (as of publication of Aidala et. al. in 2013) understanding of the proton's spin structure. The singlet axial charge plays a large role in this puzzle, and we can attempt to explain its small experimental value through theoretical QCD. We can write the formula for the structure function:

$$g_A^{(0)} = \left(\sum_q \Delta q = 3\frac{\alpha_s}{2\pi}\Delta g\right)_{partons} + C_\infty$$

(for more on where this comes from, Aidala et al. (page 15) recommends reviewing Altarelli and Ross (1988), Efremov and Teryaev (1988), Carlitz et al. (1988), Bass et al. (1991) and Bass (2005)).

Here, $\Delta g_{partons}$ represents the spin carried by polarized gluons in the polarized proton, where $\alpha_s \Delta g$ is approximately constant as $Q^2 \to \infty$ (Aidala et al. 15). $\Delta q_{partons}$ represents the spin carried by the quarks (and anti-quarks). C_∞ relates to non-perturbative QCD processes adn is discussed in section VI of Aidala et al.

1. **Page 16 - maybe focus on the attempts to explain the small value of g by measuring strangeness and polarized glue?**

There are currently several attempts to explain the low value for $g_A^{(0)}$, including: positive gluon polarization screening processes, SU(3) breaking, negative strangeness polarization in the nucleon, and a potential topological contribution at $x_{Bj} = 0$. The remainder of the paper focuses on polarized strange contributions and polarized glue.

In SIDIS experiments, when a pion (made of an up anti-down pair or anti-up down par) or kaon (strange anti-strange pair) is reconstructed in the final state, the hadron most likely contains the struck quark. This is because the detected hadron is typically at high energy, and thus we expect that the struck quark gained its energy from the collision. We can use this hadron to tag the flavor of the struck quark, and hence calculate information relevant to that quark. One can calculate the virtual photon-proton double-spin asymmetry from SIDIS data, which is shown in Aidala et al. 16. The SMC and HERMES experiments have led this particular field of experimentation (Aidala et al. 16). Both COMPASS and HERMES have presented results regarding the sum of strange and anti-strange polarization which is plotted in figure Z (**Need to put figure from page 18 of aidala et al)**. The results suggest that there is no evidence for polarized strangeness in the nucleon across x_{Bj} values. However, these calculations depend on fragmentation functions which describe the processes that occur when a quark is knocked out of one hadron and creates a new hadron. Future improvements to these fragmentation functions could alter the calculations of Δs (Aidala et al. 18). A potential future approach for measuring the strange-quark polarization would involve elastic scattering neutrinos off of protons. These experiments could avoid SU(3) assumptions, and hence would provide an independent outlook on the strangeness of the proton. Pagliaroli et al. (2012) provides a suggestion for such an experiment.

One way to increase our understanding off the effect of polarized glue on the proton's spin involves polarized proton-proton scattering. This sort of scattering is sensitive to the ratio of polarized to unpolarized glue $\frac{\Delta g}{g}$, making proton-proton collisions an important part of proton spin studies. COMPASS, HERMES, and the SMC have made measurements of this fraction. These experiments reconstruct a charmed meson or high momentum hadron in the final state (see Adolph et al., 2012d and Alekseev et al., 2009c for charmed meson production and Ageev et al., 2006 for high momentum hadron studies) (Aidala et al. 19). These studies have shown no evidence for non-zero gluon polarization in the phase space they were conducted in. See figure (**delta g/g table on page 20**) for a table of the data. Despite these results failure to show gluon polarization, results from detectors PHENIX and StAR at CERN's RHIC provided the first non-zero measurements of polarized glue in the proton. The value of Δg measured at these detectors would not make up for all of the missing proton spin, but would contribute a significant portion:

$$\Delta g \approx 0.2 - 0.3$$

(Aidala et al. 20). This measurement concludes the experimental discussion in the present paper, and theoretical aspects of the discusses properties are discussed in Aidala et al. Section VI.

V. Conclusion

(**keep short**)Although physicists have not yet found where all of the proton's spin comes from, global studies on the matter have improved understanding significantly over the past four decades since the publishing of the EMC results. Current experiments are actively building off of the work described in this paper, and many future experiments are planned to continue the studies. The experiments discussed have evolved our understanding about spin and the structure of the proton and illustrates the process by which physics