FACULTY OF DIGITAL MEDIA AND CREATIVE INDUSTRIES
HBO – Information and Communication Technologies

Assignment # 3

Quantum Computing Introduction

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Figure 1: An arbitrary quantum circuit.

Welcome to the third assignment! During this assignment, you will practice your math skills in an environment more similar to the one that you will face during the course assessment. Although the questions will not be the same, they will follow the same structure and you will have to solve the exercises using the Dirac notation or matrix–vector multiplication.

Together with this assignment, you will find the LaTeX template for you to use when solving the exercises and writing down your answers. Remember to upload a single pdf file with the full development of the assignment and the answers.

Question 1 *Are the quantum states* $|\psi_1\rangle = |00\rangle$ *and* $|\psi_2\rangle = |{+}{+}\rangle$ *mutually orthogonal?*

`Write down your solution here:`

The two states are mutually orthogonal when the inner product is 0.

$$\rightarrow |{+}{+}\rangle = |+\rangle \otimes |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\rightarrow |{+}{+}\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle$$

$$\rightarrow \langle 00| \, |{+}{+}\rangle = |00\rangle \cdot \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle$$

$$\rightarrow \langle 00| \, |{+}{+}\rangle = \frac{1}{2}(\langle 00| \, |00\rangle + \langle 00| \, |01\rangle + \langle 00| \, |10\rangle + \langle 00| \, |11\rangle$$

$$\rightarrow \langle 00| \, |{+}{+}\rangle = \frac{1}{2}(1 + 0 + 0 + 0) = \frac{1}{2} \neq 0$$

The states are mutually orthogonal.

Question 2 *Let* $|\psi\rangle = \frac{4}{5}|00\rangle + \frac{2}{5}|01\rangle + \frac{2}{5}|10\rangle + \frac{1}{5}|11\rangle$. *What is the probability of obtaining* $M(q_1 = 0)$ *when measuring the system?*

`Write down your solution here:`

$$\mathbf{Pr}\{q_1 = 0\} \rightarrow (\frac{4}{5})^2 |00\rangle + (\frac{2}{5})^2 |01\rangle = \frac{16}{25} + \frac{4}{25} = \frac{20}{25} = \frac{4}{5} = 80\%$$

Question 3 *What is the resulting transformation matrix* T_1 *when applied the following operation* $T_1 = Y \otimes S$

`Write down your solution here:`

$$T_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 1 \\ i & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

Question 4 *Consider the quantum circuit presented in Figure 1 and assume* $|q_1\rangle = |1\rangle$ *and* $|q_0\rangle = |0\rangle$*; hence,* $|\psi_{in}\rangle = |10\rangle$. *Determine, by using the matrix–vector multiplication, what is the state vector of the quantum circuit just before the measurement?*

Figure 2: An arbitrary quantum circuit.

`Write down your solution here:`

$$I\,|Q_0\rangle = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$H\,|Q_1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}1\\-1\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}) = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} - \begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ -1 & 0\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$$

$$\mathsf{C}NOT_{1,0} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$\mathsf{X}Q_0 = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\bigwedge\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}\bigwedge\begin{bmatrix}0\\1\end{bmatrix} = (|01\rangle - |10\rangle)$$

$$\mathsf{Z}Q_1 = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}\bigwedge\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix}$$

Which results in:

$$|\psi\rangle = |01\rangle - |10\rangle$$

Question 5 *Consider the quantum circuit presented in Figure 1 and assume $|q_1\rangle = |0\rangle$ and $|q_0\rangle = |1\rangle$; hence, $|\psi_{in}\rangle = |01\rangle$. Determine, by using the Dirac notation, what is the state vector of the quantum circuit just before the measurement?*

`Write down your solution here:`

$$\mathsf{H}\,|q_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|q_1\rangle + |q_0\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$$

$$\mathsf{C}NOT_{1,0} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle$$

$$\mathsf{X}q_0 = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle$$

$$\mathsf{Z}q_1 = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle$$

$$\text{Final state} = |\psi_{out}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle$$

Question 6 *Consider the quantum circuit presented in Figure 2 and assume $|q_0\rangle = |0\rangle$, $|q_1\rangle = |0\rangle$, $|q_2\rangle = |0\rangle$ and $|q_3\rangle = |0\rangle$; hence, $|\psi_{in}\rangle = |0000\rangle$. Determine, by using the Dirac notation, what is the state vector of the quantum circuit just before the partial measurement?*

Write down your solution here:

$$\mathsf{H}q_0 = \frac{1}{\sqrt{2}}(|0000\rangle + |0001\rangle)$$
$$\mathsf{H}q_2 = \frac{1}{2}(|0000\rangle + |0100\rangle + |0001\rangle + |0101\rangle)$$
$$\mathsf{C}NOT_{0,1} = \frac{1}{2}(|0000\rangle + |0100\rangle + |0011\rangle + |0111\rangle)$$
$$\mathsf{C}NOT_{2,3} = \frac{1}{2}(|0000\rangle + |1100\rangle + |0011\rangle + |1111\rangle)$$
$$\mathsf{C}NOT_{1,2} = \frac{1}{2}(|0000\rangle + |1100\rangle + |0111\rangle + |1011\rangle)$$
$$\mathsf{H}q_1 = \frac{1}{\sqrt{2}} \cdot \frac{1}{2}(|0000\rangle + |0010\rangle + |1100\rangle + |1110\rangle + |0101\rangle - |0111\rangle + |1001\rangle - |1011\rangle)$$
$$|\psi\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle + |0010\rangle + |1100\rangle + |1110\rangle + |0101\rangle - |0111\rangle + |1001\rangle - |1011\rangle)$$

Question 7 *Considering the state vector obtained in Question 6. What is the probability* $\mathbf{Pr}\{q_3 = 1\}$*?*

Write down your solution here:

$$\mathbf{Pr}\{q_3 = 1\} = |1001\rangle, |1011\rangle, |1100\rangle, |1110\rangle$$
$$\mathbf{Pr}\{q_3 = 1\} = \frac{4 \cdot 1}{8} = \frac{4}{8} = \frac{1}{2} = 50\%$$

Question 8 *Considering the state vector obtained in Question 6. Assume that the measuring process returned the following values:* $M(|q_2\rangle) = 1$ *and* $M(|q_1\rangle) = 0$*. What is the state vector of the quantum circuit after the described partial measurement? (Note: Remember to renormalize the vector).*

Write down your solution here:

$$|\psi\rangle = \frac{1}{2\sqrt{2}}(|1100\rangle + |0101\rangle)$$
$$\mathsf{n}ormalising \rightarrow (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4} \rightarrow \sqrt{\frac{1}{4}} = \frac{1}{2}$$
$$2 \cdot \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$\text{normalised state} \rightarrow |\psi\rangle = \frac{1}{\sqrt{2}}(|1100\rangle + |0101\rangle)$$

Question 9 *Considering the state vector and the measurements indicated in Question 8. What is the final state vector of the quantum circuit after applying the corrections?*

Write down your solution here:

$$\mathsf{X}q_3 = \frac{1}{\sqrt{2}}(|0100\rangle + |1101\rangle$$
$$\text{if } q_2 = 1 \implies \text{apply } X \text{ on } q_3$$
$$\text{if } q_1 = 1 \implies \text{apply } Z \text{ on } q_3$$
$$\mathsf{q}_1 \neq 1 \implies \text{Z is not being used}$$
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0100\rangle + |1101\rangle)$$

Question 10 *Considering the state vector obtained in Question 6. Assume that the measuring process returned the following values: $M(|q_2\rangle) = 1$ and $M(|q_1\rangle) = 1$. What is the final state vector of the quantum circuit after applying the corrections? (Note: Remember to renormalize the vector).*

Write down your solution here:

$$|\psi\rangle = \frac{1}{2\sqrt{2}}\,|1110\rangle - |0111\rangle$$
$$\mathrm{n}ormalising \to (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4} \to \sqrt{\frac{1}{4}} = \frac{1}{2}$$
$$\mathsf{X}q_3 \to \frac{1}{\sqrt{2}}(|0110\rangle - |1111\rangle$$
$$\mathsf{Z}q_3 \to \frac{1}{\sqrt{2}}(|0110\rangle + |1111\rangle$$
$$|\psi_{final}\rangle = \frac{1}{\sqrt{2}}(|0110\rangle + |1111\rangle$$