FACULTY OF DIGITAL MEDIA AND CREATIVE INDUSTRIES
HBO – Information and Communication Technologies

Assignment # 6

Quantum Computing Introduction

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Figure 1: An arbitrary quantum circuit.

Figure 2: An arbitrary quantum circuit.

Welcome to the sixth assignment! During this assignment, you will practice your math skills in an environment more similar to the one that you will face during the course assessment. Although the questions will not be the same, they will follow the same structure and you will have to solve the exercises using the Dirac notation or matrix–vector multiplication.

Together with this assignment, you will find the LaTeX template for you to use when solving the exercises and writing down your answers. Remember to upload a single pdf file with the full development of the assignment and the answers.

Question 1 *Let $|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. What will you always obtain when measuring two of the three qubits (any of them)?*

`Write down your solution here:`

$|00\rangle + |11\rangle$, Because, you can't measure $|01\rangle$ and $|10\rangle$

Question 2 *The quantum teleportation process, depicted in Figure 1, transmits quantum information from one location to another using two previously entangled qubits. Knowing that when measuring these entangled qubits, the system immediately collapses to the resulting state. Does the quantum teleportation process also enable faster–than–light communication between the two locations? Explain your answer.*

`Write down your solution here:`

Question 3 *Consider the quantum circuit presented in Figure 2. What is the state of the qubits $|q_1q_0\rangle$ after measuring $|q_2\rangle$ with result 1 ($M(|q_2\rangle) = 1$)?*

`Write down your solution here:`

$$\begin{aligned}
\mathsf{H}q_0 &\longrightarrow |000\rangle + |001\rangle \\
\mathsf{H}q_1 &\longrightarrow |000\rangle + |010\rangle + |001\rangle + |011\rangle \\
\mathsf{C}NOT_{0,2} &\longrightarrow |000\rangle + |010\rangle + |101\rangle + |111\rangle \\
\mathsf{C}NOT_{1,2} &\longrightarrow |000\rangle + |110\rangle + |101\rangle + |011\rangle \\
\mathsf{M}(|q_2\rangle) = 1 &= \frac{1}{2}|110\rangle \bigwedge |101\rangle \\
|1\rangle &\otimes \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle) \\
|q_1q_0\rangle &= \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)
\end{aligned}$$

Figure 3: An arbitrary quantum circuit.

Figure 4: An arbitrary quantum circuit.

Question 4 *Consider the quantum circuit presented in Figure 3 and assume* $|q_3q_2q_1q_0\rangle = |0000\rangle$*. Determine, by using the Dirac notation, what is the state vector of the quantum circuit just before the measurement?*

`Write down your solution here:`

$$
\begin{aligned}
\mathsf{H}q_0 &\longrightarrow \frac{1}{\sqrt{2}}\,|0000\rangle + |0001\rangle \\
\mathsf{H}q_1 &\longrightarrow \frac{1}{2}\,|0000\rangle\,|0010\rangle\,|0001\rangle\,|0011\rangle \\
\mathsf{C}NOT_{0,2} &\longrightarrow |0000\rangle\,|0010\rangle\,|0101\rangle\,|0111\rangle \\
\mathsf{C}NOT_{1,2} &\longrightarrow |0000\rangle\,|0110\rangle\,|0101\rangle\,|0011\rangle \\
\mathsf{C}NOT_{01,3} &\longrightarrow |0000\rangle\,|0110\rangle\,|0101\rangle\,|1011\rangle
\end{aligned}
$$

$$
\begin{aligned}
\text{Final state before measurement} = |\psi\rangle\,\frac{1}{2}\,|0000\rangle\,|0110\rangle\,|0101\rangle\,|1011\rangle \\
|0000\rangle = 0, |0001\rangle = 1, |0010\rangle = 2, |0011\rangle = 3, |0100\rangle = 4, |0101\rangle \\
= 5, |0110\rangle = 6, |0111\rangle = 7, |1000\rangle = 8, |1001\rangle = 9, |1010\rangle \\
= 10, |1011\rangle = 11, |1100\rangle = 12, |1101\rangle = 13, |1110\rangle = 14, |1111\rangle = 15 \\
\text{The amplitude is } \frac{1}{2} \text{ en goes on the spots 0, 5, 6, 11} \\
\text{State vector} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{2} & 0 \end{bmatrix}
\end{aligned}
$$

Question 5 *Considering the state vector obtained in Question 4. What is the probability* $\mathbf{Pr}\left\{M(|q_3\rangle) = 0\right\}$*?*

`Write down your solution here:`

$$
\begin{aligned}
\mathsf{Q4} = |\psi\rangle = \frac{1}{2}(|0000\rangle + |0110\rangle + |0101\rangle + |1011\rangle \\
q_3 = 0 \longrightarrow |0000\rangle + |0110\rangle + |0101\rangle = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} = 75\%
\end{aligned}
$$

Question 6 *Consider the quantum circuit presented in Figure 4 and assume* $|q_1q_0\rangle = |10\rangle$*. Determine, by using the matrix–vector multiplication, what is the state vector of the quantum circuit just before the measurement?*

Write down your solution here:

Apply H **on** q_0 :

$$\mathrm{I} \otimes H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$|\psi_1\rangle = (I \otimes H)\, |\psi_0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$$

Apply H **on** q_1 :

$$\mathrm{H} \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$|\psi_2\rangle = (H \otimes I)\, |\psi_1\rangle = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$$

Apply X **on** q_1 :

$$\mathrm{X} \otimes I = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$|\psi_3\rangle = (X \otimes I)\, |\psi_2\rangle = \frac{1}{2} \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{2}(-|00\rangle - |01\rangle + |10\rangle + |11\rangle)$$

Apply final H **on** q_0 :

$$|\psi\rangle = (I \otimes H)\, |\psi_3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Final state just before measurement:

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}\,(|10\rangle - |00\rangle)}$$

Question 7 *Confirm, by using the Dirac notation, the state vector obtained in Question 6.*

Figure 5: An arbitrary quantum circuit.

Figure 6: An arbitrary quantum circuit.

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Write down your solution here:
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$$H\,|q_0\rangle \longrightarrow |10\rangle + |11\rangle$$
$$H\,|q_1\rangle \longrightarrow |00\rangle - |10\rangle + |01\rangle - |11\rangle$$
$$X\,|q_1\rangle \longrightarrow |10\rangle - |00\rangle + |11\rangle - |01\rangle$$
$$H\,|q_0\rangle \longrightarrow |10\rangle + |11\rangle - |00\rangle + |01\rangle + |10\rangle - |11\rangle - |00\rangle - |01\rangle$$
$$|\psi\rangle \longrightarrow |10\rangle + -|00\rangle + |10\rangle - |00\rangle \longrightarrow \frac{1}{\sqrt{2}}(|10\rangle - |00\rangle)$$
$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |00\rangle)$$

Question 8 *At this point, I assume that you noticed that Figure 4 represents the Deutsch circuit for a constant function. Likewise, the quantum circuit presented in Figure 5 represents the Deutsch circuit for a balanced function. Assume $|q_1 q_0\rangle = |10\rangle$ and corroborate, by using the Dirac notation, that $M(|q_0\rangle) = 1$ with 100% probability.*

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Write down your solution here:
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$$|q_1 q_0\rangle = |10\rangle$$
$$Hq_0 \longrightarrow \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$$
$$Hq_1 \longrightarrow \frac{1}{2}(|00\rangle - |10\rangle + |01\rangle - |11\rangle)$$
$$\longrightarrow \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$$
$$Xq_0 \longrightarrow \frac{1}{2}(|01\rangle + |00\rangle - |11\rangle - |10\rangle$$
$$CNOT_{0,1} \longrightarrow \frac{1}{2}(|11\rangle + |00\rangle - |01\rangle - |10\rangle$$
$$Xq_0 \longrightarrow \frac{1}{2}\,|10\rangle + |01\rangle - |00\rangle - |11\rangle$$
$$Hq_0 \longrightarrow \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle + |00\rangle - |01\rangle - |00\rangle + |01\rangle - |10\rangle + |11\rangle)$$
$$\longrightarrow \frac{1}{\sqrt{2}}(|11\rangle)$$

Question 9 *Consider the quantum circuit presented in Figure 6 and assume $|q_2 q_1 q_0\rangle = |100\rangle$. Can you determine, by using the Dirac notation, whether the implemented function is constant or balanced?*

`Write down your solution here:`

$$H|1\rangle = |-\rangle$$
$$H|0\rangle = |+\rangle$$
$$|\psi_1\rangle = |-\rangle_{q2}\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)_{q1q0}$$
$$2 \cdot CNOT \longrightarrow |\psi_2\rangle = |-\rangle_{q2}\frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$$
$$|\psi_2\rangle = |-\rangle_{q2}|-\rangle_{q1}|-\rangle_{q0}$$
$$H_{q1q0} \longrightarrow |\psi_3\rangle = |-\rangle_{q2}|+\rangle_{q1}|+\rangle_{q0}$$
$$\longrightarrow |\psi_3\rangle = |-\rangle_{q2}|11\rangle$$
$$|11\rangle \neq 0$$

So is balanced

Question 10 *Refering to Question 9. Are you 100% sure about the type of the implemented function? Why?*

`Write down your solution here:`

$$2 \cdot CNOT \longrightarrow q_2 \longrightarrow q_2 \otimes q_1 \otimes q_0$$
$$f(q_1q_0) = q_1 \otimes q_0$$
$$\begin{bmatrix} f = |00\rangle = 0 \\ f = |01\rangle = 1 \\ f = |10\rangle = 1 \\ f = |11\rangle = 0 \end{bmatrix} so, 2 \cdot 0 \bigwedge 2 \cdot 1$$

output question 9=$|11\rangle \longrightarrow |11\rangle \neq 0 = balanced$