Questions

1 Calculate the kinetic energy of N electrons in 3D system zero temperature $U_0 = \frac{3}{5} NE_F$.

2 Calculate the Fermi energy of a metal. What is the relationship between the electron density and the Fermi energy in the free electron model? Explain how this relationship would be derived. Hint: The volume in k-space corresponding to one allowed k-state is $((2\pi)/L)^3$, where L is the length of the crystal. The total volume of the crystal is L^3. There are twice as many electrons states as k-states due to spin.

3 Derive the relationship connecting pressure and volume of an electron gas at 0 K. The result may be written as $p = 2(U_0/V)/3$. Show that the bulk modulus $B = -V(dp/dV)$ of an electron gas at $T = 0$ is $B = 5p/3 = 10U_0/9V$.

4 Average electron energy in 2-D. For a metal at a temperature of T = 0 K, the conduction electrons at the bottom of the band have an energy E = 0 and the conduction electrons with the highest energy have an energy $E = E_F$. For free electrons in two-dimensions, what is the average energy of the conduction electrons?

5 Find the density of states as a function of energy for a non-interacting free electron gas in two dimensions. For this system it is possible to find an analytic expression for the temperature dependence of the chemical potential. Show that

$$\mu(T) = kT \ln\left[\exp\left(\frac{\pi n\hbar^2}{mkT}\right) - 1\right] = kT \ln\left[\exp\left(\frac{E_F}{kT}\right) - 1\right],$$

here n is the number of electrons per unit area.

6 Thermodynamic properties of a metal. A monovalent metal has a simple cubic Bravais lattice and a lattice constant of $a = 0.15$ nm. Calculate the chemical potential, the specific heat, the entropy, and the Helmholtz free energy of the electrons at temperatures of 10 K and 300 K assuming that the free electron model can be used.

7 Derive the density of states D(k) in two dimensions. The density of states for a free electron gas in two dimensions is, $D(E) = m/(\hbar^2\pi)$. What is the heat capacity for a free electron gas in two dimensions?