Quiz 3.3 – Gibbs Energy and T

Name: Key

The Equilibrium Temperature

$4 \cdot \frac{7}{2}R$ $\quad 2 \cdot 4R \quad \Delta C_p = -6R = -49.88 \frac{kJ}{mol \cdot K}$

Consider the Haber-Bosch process: $\underbrace{3\,H_2(g) + N_2(g)}_{}$ $\rightleftharpoons$ $2\,NH_3(g)$

At 298 K this reaction has $\Delta G^{\ominus} = -32.90\frac{kJ}{mol}$, $\Delta H^{\ominus} = -92.22\frac{kJ}{mol}$, and $\Delta S^{\ominus} = -198.76\frac{J}{mol\ K}$

In CHEM 1220 you learned that $\Delta G = \Delta H - T\Delta S$, and used this formula to calculate the equilibrium temperature: $T = \dfrac{\Delta H}{\Delta S}$. Use this formula to find the equilibrium temperature.

$$T = \frac{-92.22 \frac{kJ}{mol}}{-0.19876 \frac{kJ}{mol \cdot K}} = 464\ K$$

That formula relies on the assumption that ΔH and ΔS are both independent of temperature, which is only a good approximation over very small temperature ranges. Use Kirchoff's law to find ΔH and use $\Delta S(T_2) = \Delta S(T_1) + \displaystyle\int_{T_1}^{T_2} \frac{\Delta C_p}{T} \mathrm{d}T$ to find ΔS at the equilibrium temperature you calculated above

$\Delta T = 166\ K$

$$\Delta H(464) = -92.22 \frac{kJ}{mol} - 0.04988 \frac{kJ}{mol \cdot K} \cdot 166\ K = -100.5 \frac{kJ}{mol}$$

$$\Delta S(464) = -198.76 \frac{J}{mol \cdot K} - 49.88 \frac{J}{mol \cdot K} \cdot \ln\left(\frac{464}{298}\right) = -220.8 \frac{J}{mol \cdot K}$$

Use these values of ΔH and ΔS to find ΔG at the equilibrium temperature you calculated above. Is the system really at equilibrium?

$$\Delta G = -100.5 \frac{kJ}{mol} - 464\ K \cdot -0.2208 \frac{kJ}{mol \cdot K} = +1.95 \frac{kJ}{mol}$$

No, $\Delta G \neq 0$

Find the true equilibrium temperature using the Gibbs-Helmholtz equation: $\left(\dfrac{\mathrm{d}\,(\Delta G/T)}{\mathrm{d}T}\right)_p = -\dfrac{\Delta H}{T^2}$

The answer is 454.4 K, but this is a bad question so please don't feel like you need to solve it.

Find ΔH and ΔS at that temperature, and confirm that the system is at equilibrium at that temperature