Quiz 1.1 – An Ideal Gas

Name: Key

Question 1

A weather balloon starts in Cedar City with with $P = 0.82\ atm$, $T = 21.5\ °C$, and $V = 18.75\ L$

$\rightarrow 294.65\ K$

- Find the number of moles of gas inside the balloon

$$n = \frac{PV}{RT} = \frac{0.82\,atm \cdot 18.75\,L}{0.08206\,\frac{L\cdot atm}{mol\cdot K} \cdot 294.65\,K} = 0.636\ moles$$

- If the balloon is filled with He gas, find the mass of the gas inside the balloon

$$m = n \cdot M = 0.636\ moles \cdot 4.003\ g/mol = 2.55\ g$$

- Find the density of the He-filled balloon (assume the instruments and balloon itself have no mass)

$$d = \frac{m}{V} = \frac{2.55\ g}{18.75\ L} = 0.136\ g/L \quad \text{-or-} \quad d = \frac{PM}{RT} = \frac{0.82\ atm \cdot 4.003\ g/mol}{0.08206\ \frac{L\cdot atm}{mol\cdot K} \cdot 294.65\,K} = 0.136\ g/L$$

- Find the density of the surrounding air (assume it is 100% N_2 gas)

$$d = \frac{PM}{RT} = \frac{0.82\ atm \cdot 28.014\ g/mol}{0.08206\ \frac{L\cdot atm}{mol\cdot K} \cdot 294.65\,K} = 0.950\ g/L$$

Question 2

The weather balloon is released into the upper atmosphere and the instruments on-board indicate a pressure of $0.45\ atm$ and a temperature of $-32.4\ °C$

What will the new volume of the balloon be?

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad \frac{0.82\ atm \cdot 18.75\ L}{294.65\ K} = \frac{0.45\ atm \cdot V_2}{240.75\ K} \rightarrow V_2 = 27.9\ L$$

Question 3

A car engine burns about $0.1\ g$ of gasoline (C_8H_{18})for each engine cycle. A car engine may have a cylinder volume of $0.075\ L$ at the point of ignition, and operate at a temperature of $80.0\ °C$. If the gasoline combusts completely inside the $0.075\ L$ piston, what is the pressure of the combustion products?

$$2\ C_8H_{18}(g) + 25\ O_2(g) \rightarrow 16\ CO_2(g) + 18\ H_2O(g) \qquad M_{C_8H_{18}} = 114.23\ g/mol$$

$$\frac{0.1\ g\ C_8H_{18}}{} \cdot \frac{1\ mol}{114.23\ g} \cdot \frac{34\ mol\ (g)\ products}{2\ mol\ C_8H_{18}} = 0.0149\ moles$$

$$P = \frac{nRT}{V} = \frac{0.0149\ moles \cdot 0.08206\ \frac{L\cdot atm}{mol\cdot K} \cdot 353.15\ K}{0.075\ L} = 5.76\ atm$$

$\sigma = 0.21 nm^2 = 0.21 \cdot 10^{-18} m^2$ 81,060 Pa

Question 4

Find the following, regarding a sample of pure Helium at $25°C$ and $0.800\ atm$

- $v_{rms} = \left(\frac{3RT}{M}\right)^{1/2} = \left(\frac{3 \cdot 8.314\ J/mol \cdot K \cdot 298.15 K}{4.003\ g/mol \cdot 0.001\ kg/g}\right)^{1/2} = 1,360\ m/s$
- $v_{rel} = \sqrt{2}\, V_{mean} = \sqrt{2} \cdot \left(\frac{8RT}{\pi M}\right)^{1/2} = \sqrt{2}\left(\frac{8 \cdot 8.314\ J/mol \cdot K \cdot 298.15 K}{\pi \cdot 4.003\ g/mol \cdot 0.001\ kg/g}\right)^{1/2} = 1,780\ m/s$
- λ (mean free path)

$\lambda = \frac{V_{rel}}{Z} = \frac{1,780\ m/s}{7.36 \cdot 10^9 s^{-1}} = 2.42 \cdot 10^{-7} m$ 242 nm -or- $\lambda = \frac{k_B T}{\sigma p} = \frac{1.38 \cdot 10^{-23} J/K \cdot 298.15 K}{0.21 \cdot 10^{-18} m^2 \cdot 81,060\ Pa} = 2.42 \cdot 10^{-7} m$ 242 nm

- Collision frequency

$Z = \sigma \cdot V_{rel} \cdot N \longrightarrow Z = 0.21 \cdot 10^{-18} m^2 \cdot 1,780\ m/s \cdot \frac{0.800\ atm \cdot 6.022 \cdot 10^{23} mol^{-1}}{0.08206 \frac{L \cdot atm}{mol \cdot K} \cdot 298.15 K \cdot 0.001\ m^3/L}$

$N = \frac{p N_A}{RT}$ $Z = 7.36 \cdot 10^9 s^{-1}$

Question 5

What will be v_{rel} for collisions between O_2 and N_2 molecules in a sample of the atmosphere at $25°C$ and $0.800\ atm$ ($\chi_{N_2} = 0.80,\ \chi_{O_2} = 0.20$)

$V_{rel} = \left(\frac{8RT}{\pi \mu}\right)$ $\mu = \frac{M_{N_2} M_{O_2}}{M_{N_2} + M_{O_2}} = \frac{28.0134\ g/mol \cdot 31.9988\ g/mol}{28.0134\ g/mol + 31.9988\ g/mol} = 14.94\ g/mol$

$V_{rel} = \left(\frac{8 \cdot 8.314\ J/mol \cdot K \cdot 298.15 K}{\pi \cdot 14.94\ g/mol \cdot 0.001\ kg/g}\right)^{1/2} = 650\ m/s$

When I Heard the Learn'd Astronomer

By Walt Whitman

When I heard the learn'd astronomer,
When the proofs, the figures, were ranged in columns before me,
When I was shown the charts and diagrams, to add, divide, and measure them,
When I sitting heard the astronomer where he lectured with much applause in the lecture-room,
How soon unaccountable I became tired and sick,
Till rising and gliding out I wander'd off by myself,
In the mystical moist night-air, and from time to time,
Look'd up in perfect silence at the stars.