Quiz 2.4 – Exact Differentials

Name: Key

Natural Variables

Write the exact differentials for the four main thermodynamic potentials (U, H, G, and A) with respect to their natural variables in two forms: (The Wikipedia page on "Thermodynamic Potentials" can be very helpful here, but Helmholtz energy is F instead of A on that page)

- Showing the partial derivatives explicitly

$dH = \left(\frac{\partial H}{\partial S}\right)_p dS + \left(\frac{\partial H}{\partial p}\right)_S dp$

$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$ $\quad dG = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dT$ $\quad dA = \left(\frac{\partial A}{\partial T}\right)_V dT + \left(\frac{\partial A}{\partial V}\right)_T dV$

- Replacing the partial derivatives with the appropriate state variables

$dU = TdS - pdV$ $\quad dH = TdS + Vdp$

$dG = -SdT + Vdp$ $\quad dA = -SdT - pdV$

Properties of Ideal gases

Find the following properties in terms of state variables (i.e. solve the derivatives) for an ideal gas:

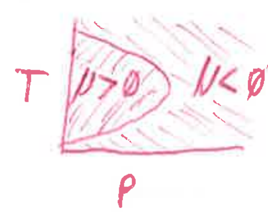
- $\pi_T = \left(\frac{\partial U}{\partial V}\right)_T$ for an ideal gas, $U = C_v T$ and C_v is independent of V so $\left(\frac{\partial U}{\partial V}\right)_T = \emptyset$
- $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{V}\frac{\partial}{\partial T}\frac{nRT}{p}\Big|_p = \frac{nR}{pV} = \frac{1}{T}$
- $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{V}\frac{\partial}{\partial p}\frac{nRT}{p}\Big|_T = -\frac{nRT}{V}\cdot -\frac{1}{p^2} = \frac{p}{p^2} = \frac{1}{p}$

Joule-Thompson Coefficients

The Joule-Thompson coefficient is positive for most gases under most conditions. State the following:

- What conditions will lead to a negative coefficient for most gases?

Extremely high or low T, and extremely high p



- Give an example of a gas with a negative coefficient at STP

He