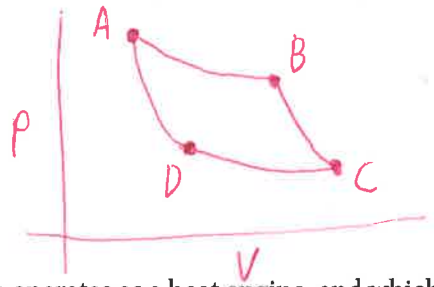
Quiz 3.1 – Entropy

Name: Key

Carnot cycle

Consider a heat engine based around the Carnot cycle. Sketch this cycle on a p/V diagram, labeling the states in the process as A, B, C, and D



Tell which direction around this cycle operates as a heat engine, and which direction operates as a heat pump

- A-B-C-D-A: Heat Engine
- A-D-C-B-A: Heat Pump

Fill in the table below for the cycle when operating as a heat engine. Use generic variables (C_V, T_H, T_C, V_A, V_B, etc.)

Step	w	q	ΔU	ΔS
A-B	$-nRT_H \ln\left(\frac{V_B}{V_A}\right)$	$+nRT_H \ln\left(\frac{V_B}{V_A}\right)$	$\varnothing$	$nR \ln\left(\frac{V_B}{V_A}\right)$
B-C	$C_V (T_C - T_H)$	$\varnothing$	$C_V (T_C - T_H)$	$\varnothing$
C-D	$-nRT_C \ln\left(\frac{V_D}{V_C}\right)$	$+nRT_C \ln\left(\frac{V_D}{V_C}\right)$	$\varnothing$	$nR \ln\left(\frac{V_D}{V_C}\right)$
D-A	$C_V (T_H - T_C)$	$\varnothing$	$C_V (T_H - T_C)$	$\varnothing$
net (A-B-C-D-A)	$-nR\left[T_H \ln\left(\frac{B}{A}\right) + T_C \ln\left(\frac{D}{C}\right)\right]$	$+nR\left[T_H \ln\left(\frac{B}{A}\right) + T_C \ln\left(\frac{D}{C}\right)\right]$	$\varnothing$	$nR\left(\ln\left(\frac{V_B}{V_A}\right) + \ln\left(\frac{V_D}{V_C}\right)\right)$

$-nR \ln\left(\frac{V_B}{V_A}\right)(T_H - T_C)$ $\quad +nR \ln\left(\frac{V_B}{V_A}\right)(T_H - T_C)$ $\quad = \varnothing$, so $\ln\left(\frac{V_D}{V_C}\right) = -\ln\left(\frac{V_B}{V_A}\right)$

A car engine is a type of heat engine, and burns gasoline burns at about 600 $°C$. If the ambient temperature is 25 $°C$, what is the thermodynamic maximum efficiency a car engine can achieve?

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{\cancel{8}\ 298\ K}{873\ K} = 0.659 \rightarrow 65.9\%\ \text{efficient}$$

Measuring molar entropy

He has $T_{boil} = 4.25\ K$ and $\Delta H_{vap} = 83\frac{J}{mol}$. The isobaric heat capacity for liquid helium is very complex, but can be approximated as $C_p(l) \approx 7.4 \times 10^{-3} T^3 \frac{J}{mol\ K^4}$. The isobaric heat capacity for gaseous He is simply $C_p(g) = \frac{5}{2}R$. Use these data to calculate the molar entropy for He gas at room temperature, and compare it to the value given in our textbook appendix.

$$S(298\,K) = \int_0^{4.25} 7.4 \cdot 10^{-3} \frac{J}{mol \cdot K^4} T^3 \cdot \frac{1}{T} dT + \frac{83 \frac{J}{mol}}{4.25\,K} + \int_{4.25}^{298} \frac{5}{2} \cdot 8.314\ J/mol \cdot K \cdot \frac{1}{T}\, dT$$

$$S(298\,K) = 7.4 \cdot 10^{-3} \frac{J}{mol\,K^4} \cdot \frac{1}{3} T^3 \Big|_0^{4.25} + 19.53 \frac{J}{mol \cdot K} + 20.785 \frac{J}{mol \cdot K} \cdot \ln T \Big|_{4.25}^{298} = 108\ J/mol \cdot K$$

From textbook: $126.15 \frac{J}{mol \cdot K}$

Irreversibility in Mechanical Systems

Consider a spring which obeys Hook's law: $F = -kx$ where x is the displacement away from equilibrium and $k = 650\frac{N}{m}$. The acceleration due to gravity is $9.80665\ \frac{m}{s^2}$.

- Calculate the equilibrium displacement if a $10\ kg$ weight is placed on the spring

$F_{spring} = F_{gravity}$ $\quad -650 \frac{N}{m} X = 10\,kg \cdot 9.80665 \frac{m}{s^2}$

$$X = -0.151\ m$$

Considering the same weight-on-a-spring in Problem 1:

- Calculate the work done by the falling weight.

$$w = F \cdot d = 10\,kg \cdot 9.80665 \frac{m}{s^2} \cdot 0.151\,m = 14.8\ J$$

How much work would be done if instead the spring was stretched reversibly to the same equilibrium displacement. Bonus – Explain the discrepancy!

Spring force = applied force

$\int dw = \int F dx$

$$w = \int_0^{-0.151\,m} -650 \frac{N}{m} x\, dx = -650 \frac{N}{m} \cdot \frac{1}{2} x^2 \Big|_{0\,m}^{-0.151\,m} = 7.4\ J$$

The free-falling weight ~~start~~ ends with 7.4 J of kinetic energy + the energy stored in the spring. Reversible extension has no kinetic energy

The spring-weight system will lose kinetic energy through friction with the air until it rests at its equilibrium position. What is $\Delta S_{universe}$ for both the reversible and irreversible processes if they are done at room temperature ($25°C$)?

reversible: $q_{rev} = \emptyset$ so $\Delta S_{universe} = \emptyset$, $\Delta S_{sys} = \emptyset$, and $\Delta S_{surr} = \emptyset$

irreversible: $\Delta S_{sys} = \int \frac{q_{rev}}{T} = \emptyset$ $\quad \Delta S_{surr} = \frac{-q_{sys}}{T} = \frac{7.4\,J}{298\,K} = 0.025 \frac{J}{K}$

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{surr} = 0.025 \frac{J}{K}$$