Quiz 16.2 – Ion Transport

Name: Key

Conductive Materials

A wire sample made of an unknown metal has a length of $2.5m$ and a cross-sectional area of $0.080cm^2$. The resistance of this wire sample is measured to be 12Ω. What is the conductivity (κ) of the unknown metal?

$0.080cm^2 \rightarrow 0.08 \cdot 10^{-4} m^2$

$$\kappa = G\frac{l}{A} = \frac{1}{R}\frac{l}{A} = \frac{1}{12\Omega}\frac{2.5\,m}{0.08\cdot10^{-4}\,m^2} = 26{,}000\ \frac{1}{\Omega\cdot m}$$

An outdoor grounding wire must be designed to safely carry a very high current, as from a lightning strike. Copper wire has conductivity of $5.8 \times 10^7 \frac{1}{\Omega\, m}$. Find the resistance of a wire made from pure copper, with $l = 25m$ and $A = 0.22cm^2$ $\rightarrow 0.22\cdot10^{-4}\,m^2$

$$R = \frac{l}{\kappa A} = \frac{25\,m}{5.8\cdot10^{7}\frac{1}{\Omega\,m}\cdot 0.22\cdot10^{-4}\,m^2} = 0.020\Omega$$

Electrolyte Conductivity

In water at $298K$, Ca^{2+} has an ion mobility of $u = 6.17 \times 10^{-8} \frac{m^2}{V\,s}$ and Br^- has an ion mobility of $u = 8.09 \times 10^{-8} \frac{m^2}{V\,s}$. Use the viscosity of water ($0.0089 \frac{kg}{m\,s}$) to find the Stokes radius of each ion

$$u = \frac{ze}{6\pi\eta a} \rightarrow a = \frac{ze}{6\pi\eta u}$$

$$Ca^{2+}: a = \frac{2\cdot 1.602\cdot10^{-19}\,C}{6\cdot\pi\cdot 0.00089\,kg/m\cdot s\cdot 6.17\cdot10^{-8}\frac{m^2}{Vs}} = 3.10\cdot10^{-10}\,m$$

$$Br^-: a = \frac{1\cdot 1.602\cdot10^{-19}\,C}{6\cdot\pi\cdot 0.00089\,kg/m\cdot s\cdot 8.09\cdot10^{-8}\,m^2/Vs} = 1.18\cdot10^{-10}\,m$$

Find the molar conductivity of a solution of $CaBr_2$

$$\Lambda_M = (z_+u_+\nu_+ + z^-u^-\nu^-)\cdot F = \left(2\cdot 6.17\cdot10^{-8}\frac{m^2}{Vs}\cdot 1 + 1\cdot 8.09\cdot10^{-8}\frac{m^2}{Vs}\cdot 2\right)\cdot 96485\frac{C}{mol} = 0.0275\ \frac{m^2}{\Omega\,mol}$$

Two electrodes with $A = 6.0cm^2$ are placed $1.5cm$ apart in a $0.75\ M$ $CaBr_2$ solution. Find the resistance across the electrolyte solution

$$\kappa = \Lambda_m \cdot c = 0.0275\frac{m^2}{\Omega\cdot mol}\cdot 0.75\frac{mol}{L}\cdot\frac{1000L}{1m^3} = 20.6\ \frac{1}{\Omega\cdot m}$$

$$G = \kappa\frac{A}{l} \rightarrow R = \frac{l}{\kappa A} = \frac{0.015\,m}{20.6\,\frac{1}{\Omega\cdot m}\cdot 6\cdot10^{-4}\,m^2} = 1.21\Omega$$

If $120V$ are placed across the electrodes, how long does it take a Ca^{2+} ion to traverse from one electrode to the other?

$$s = u\,E = u\,\frac{\Delta V}{d} = 6.17\cdot10^{-8}\frac{m^2}{V\cdot s}\,\frac{120V}{0.015m} = 0.000494\ m/s$$

$$t = \frac{d}{v} = \frac{0.015m}{0.000494\ m/s} = 30.4\,s$$