Quiz 7.3 – Fundamentals of Quantum Mechanics

Name: _____

Wavefunction Normalization

The wavefunction for a 1s electron orbital is:

$$\psi_{1s}(r, \theta, \phi) = e^{-r/a_0}$$

Note that this is a function in spherical polar coordinates, and that a_0 is the Bohr radius. Find the normalization constant, and give the complete normalized wavefunction $\psi_{1s}(r, \theta, \phi)$

Expectation Values

For electronic orbitals, we can define an orbital angular momentum operator: $\hat{l}^2$

Some eigenvalues are:

$$\hat{l}^2\psi_{3s} = 0$$
$$\hat{l}^2\psi_{3p} = 2\hbar^2\psi_{3p}$$
$$\hat{l}^2\psi_{3d} = 6\hbar^2\psi_{3d}$$

If an electron is in the superposition state $\Psi = \left(\frac{1}{\sqrt{2}}\psi_{3s} + \frac{1}{\sqrt{3}}\psi_{3p} + \frac{1}{\sqrt{6}}\psi_{3d}\right)$, what will be the expectation value $\left\langle \hat{l}^2 \right\rangle$?

Schrödinger Equation and Wavefunctions

For a particle confined in the region $0 \leq x \leq L$, the appropriate wavefunctions are:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

∘ Another function, $\phi(x) = -4x^2 + 4x$ has a similar shape and obeys the same boundary conditions. Prove whether or not this function is also a solution to the Schrödinger equation.

∘ Find the average position $\langle x \rangle$ for the states $n = 1$ and $n = 2$

∘ Assume $L = 1$, and give the probability that the system is observed with $0.4 < x < 0.6$ for the states $n = 1$ and $n = 2$

Caged Bird

By Maya Angelou

A free bird leaps
on the back of the wind
and floats downstream
till the current ends
and dips his wing
in the orange sun rays
and dares to claim the sky.

But a bird that stalks
down his narrow cage
can seldom see through
his bars of rage
his wings are clipped and
his feet are tied
so he opens his throat to sing.

The caged bird sings
with a fearful trill
of things unknown
but longed for still
and his tune is heard
on the distant hill
for the caged bird
sings of freedom.

The free bird thinks of another breeze
and the trade winds soft through the sighing trees
and the fat worms waiting on a dawn bright lawn
and he names the sky his own

But a caged bird stands on the grave of dreams
his shadow shouts on a nightmare scream
his wings are clipped and his feet are tied
so he opens his throat to sing.
The caged bird sings
with a fearful trill
of things unknown
but longed for still
and his tune is heard
on the distant hill
for the caged bird
sings of freedom.