Quiz 7.6 – Rotational Motion

Name: Key

New Coordinate Systems

For rotations (and other systems, later) we will use non-cartesian coordinate systems. For cylindrical and spherical polar coordinates give:

- The Laplacian operator (∇^2)
  - Cylindrical: $\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$ (with $z = \emptyset$)
  - Spherical Polar: $\frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta} \sin\theta \frac{\partial}{\partial \theta}$
- The Jacobian (infinitesimal volume element)
  - Cylindrical: $d\tau = r\,dr\,d\phi\,dz$ -or- $d\tau = r\,dr\,d\phi$ (with $z = \emptyset$)
  - Spherical Polar: $d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$
- An integral of function $F(\tau)$ over all space, with the correct limits of integration and Jacobian
  - Cylindrical: $\int_0^{2\pi}\int_0^{\infty} f(r,\phi)\, r\, dr\, d\phi$ (for $z = \emptyset$)
  - Spherical Polar: $\int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} f(r,\theta,\phi)\, r^2 \sin\theta\, dr\, d\theta\, d\phi$

Rotation and Quantum Numbers

Quantum mechanical states are labeled by their *quantum numbers*. Give the symbol, name, and relation to observable properties for the quantum numbers in the following systems:

- Particle on a Ring

Angular Momentum Q.N., m_ℓ, $\ell_z = \hbar m_\ell$

- Rigid Rotor

Orbital Angular Momentum Q.N., ℓ, $|\ell| = \hbar\sqrt{\ell(\ell+1)}$ -or- $\ell^2 = \hbar^2 \ell(\ell+1)$

Magnetic Q.N., m_ℓ, $\ell_z = \hbar m_\ell$ and $E = \frac{\hbar^2 \ell(\ell+1)}{2I}$

Rigid Rotor

Consider a 3-dimensional rigid rotor with a moment of inertia $I = 7.4 \times 10^{-47}\ kgm^2$

- Give the energy (in J) and total angular momentum of the $l = 2$ energy level

$$E = \frac{\hbar^2 l(l+1)}{2I} = \frac{\hbar^2 \cdot 2 \cdot (2+1)}{2 \cdot 7.4 \cdot 10^{-47}\ kg\,m^2} = 4.51 \cdot 10^{-22}\ J$$

- List all of the allowed values for the z-component of the angular momentum

$m_l = (-2, -1, 0, 1, 2)$ $\quad l_z = \hbar m_l$

m_l	l_z
2	$2\hbar$
1	$\hbar$
0	0
-1	$-\hbar$
-2	$-2\hbar$

- List all the observables of a rigid rotor which we can know simultaneously

$E,\ l^2,\ (l_z \text{ -or- } l_x \text{ -or- } l_y)$

- List all pairs of observables for which there exists an uncertainty relationship

$[\hat{l}_x, \hat{l}_y]$ $\quad [\hat{l}_y, \hat{l}_z]$ $\quad [\hat{l}_z, \hat{l}_x]$