Quiz 7.2 – Fundamentals of Quantum Mechanics

Name: ______________________________

Linear Algebra Review

○ Consider two even functions:

$$\phi_1(x) = \sqrt{\frac{3}{2}}x \qquad \phi_2(x) = \sqrt{\frac{175}{8}}\left(x^3 - \frac{3}{5}x\right)$$

Show that these two functions are orthogonal over the interval $[-1, 1]$

○ The wavefunction for a 1s electron orbital is:

$$\psi_{1s}(r, \theta, \phi) = e^{-r/a_0}$$

Note that this is a function in spherical polar coordinates, and that a_0 is the Bohr radius. Find the normalization constant, and give the complete normalized wavefunction $\psi_{1s}(r, \theta, \phi)$

○ For electronic orbitals, we can define an orbital angular momentum operator: $\hat{l}^2$

Some eigenvalues are:

$$\hat{l}^2\psi_{3s} = 0$$
$$\hat{l}^2\psi_{3p} = 2\hbar^2\psi_{3p}$$
$$\hat{l}^2\psi_{3d} = 6\hbar^2\psi_{3d}$$

If an electron is in the superposition state $\Psi = \left(\frac{1}{\sqrt{2}}\psi_{3s} + \frac{1}{\sqrt{3}}\psi_{3p} + \frac{1}{\sqrt{6}}\psi_{3d}\right)$, what will be the expectation value $\left\langle \hat{l}^2 \right\rangle$?

○ Consider two operators: $\hat{A} = \dfrac{\mathrm{d}^2}{\mathrm{d}x\mathrm{d}y}$ and $\hat{B} = x^2 + y$

Give the commutator $\left[\hat{A}, \hat{B}\right]$. You may assume that the operators will only be used with functions that are separable. i.e. You may use a trial function of $f(x)g(y)$

Schrödinger Equation and Wavefunctions

○ For a particle confined in the region $0 \leq x \leq L$, the appropriate wavefunctions are:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Another function, $X(x) = -4x^2 + 4x$ has a similar shape and obeys the same boundary conditions. Prove whether or not this function is also a solution to the Schrödinger equation.