Quiz 7.4 – Translational Motion

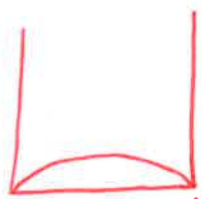
Name: Key

Particle in a Box

$E = \frac{n^2 h^2}{8 m L^2}$ $\quad m = 1.00794\ AMU = 1.67 \cdot 10^{-27}\ kg$

Consider a H atom confined in a box with $L = 1.5\ nm$. Model this system as a particle in a box. For each of the first three energy levels, draw the wavefunction and give the energy. Point out any nodes on your drawn wavefunctions.

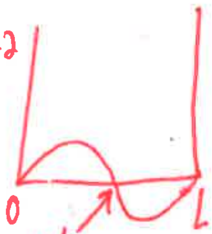
n=1



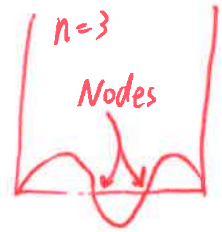
$E_1 = \frac{1^2 h^2}{8 \cdot m \cdot L^2} = 1.46 \cdot 10^{-23}\ J$

$E_2 = \frac{2^2 h^2}{8 m L^2} = 5.84 \cdot 10^{-23}\ J$

n=2



n=3



$E_3 = \frac{3^2 h^2}{8 m L^2} = 1.31 \cdot 10^{-22}\ J$

Find an expression for the spacing between energy levels for a particle in a box ($E_{n+1} - E_n$), and describe its trend, if any.

$E = n^2 E_1$ $\quad \Delta E = \left[(n+1)^2 - n^2\right] E_1 = \left[n^2 + 2n + 1 - n^2\right] E_1$

$\Delta E = (2n+1) E_1$ $\quad$ Energy gaps increase linearly with n

Quantum Well

Consider a particle confined to a 2-dimensional box. This system is commonly called a *quantum well*. If the two sides are equal in length, give the energies and degeneracies to the first four energy levels of this quantum system

If $L_1 = L_2$, $\quad E_{n_1, n_2} = (n_1^2 + n_2^2) \frac{h^2}{8 m L^2} = (n_1^2 + n_2^2) E_0$

↑ E

Level	Degeneracy	Energy	(n_1, n_2)
4	2	$10\ E_0$	(1,3) or (3,1)
3	1	$8\ E_0$	(2,2)
2	2	$5\ E_0$	(1,2) or (2,1)
1	1	$2\ E_0$	(1,1)

Tunneling

Consider an electron approaching a potential energy barrier. The barrier is $5.0\ nm$ wide, and $4.0 \times 10^{-23}\ J$ high, while the electron has kinetic energy of $1.0 \times 10^{-23}\ J$ $V-E = 3.0 \cdot 10^{-23} J$ $\varepsilon = \frac{E}{V} = \frac{1}{4}$

What will be the probability that the electron is transmitted through the barrier?

$k = \sqrt{2m(V-E)}/\hbar = \sqrt{2 \cdot 9.109 \cdot 10^{-31} kg \cdot 3.0 \cdot 10^{-23} J}/1.05 \cdot 10^{-34}\ J \cdot s = 7.01 \cdot 10^{7}\ m^{-1}$

$w = 5 \cdot 10^{-9} m \rightarrow kW = 0.3505$

$$T = \left[1 + \frac{(e^{kw} - e^{-kw})^2}{16\varepsilon(1-\varepsilon)}\right]^{-1} = \left[1 + \frac{(e^{0.3505} - e^{-0.3505})^2}{16 \cdot \frac{1}{4}(1-\frac{1}{4})}\right]^{-1} = 0.854 \rightarrow 85\%$$

Sketch this system below, showing both the potential energy and the electron wavefunction in qualitative terms.