Quiz 7.3 – Fundamentals of Quantum Mechanics

Name: ____

Wavefunction Normalization

The wavefunction for a 1s electron orbital is:

$$\psi_{1s}(r, \theta, \phi) = e^{-r/a_0}$$

Note that this is a function in spherical polar coordinates, and that a_0 is the Bohr radius. Find the normalization constant, and give the complete normalized wavefunction $\psi_{1s}(r, \theta, \phi)$

Expectation Values

For electronic orbitals, we can define an orbital angular momentum operator: $\hat{l}^2$

Some eigenvalues are:

$$\hat{l}^2 \psi_{3s} = 0$$

$$\hat{l}^2 \psi_{3p} = 2\hbar^2 \psi_{3p}$$

$$\hat{l}^2 \psi_{3d} = 6\hbar^2 \psi_{3d}$$

If an electron is in the superposition state $\Psi = \left(\frac{1}{\sqrt{2}} \psi_{3s} + \frac{1}{\sqrt{3}} \psi_{3p} + \frac{1}{\sqrt{6}} \psi_{3d} \right)$, what will be the expectation value $\left\langle \hat{l}^2 \right\rangle$?

Schrödinger Equation and Wavefunctions

For a particle confined in the region $0 \leq x \leq L$, the appropriate wavefunctions are:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

∘ Another function, $\phi(x) = -4x^2 + 4x$ has a similar shape and obeys the same boundary conditions. Prove whether or not this function is also a solution to the Schrödinger equation.

∘ Find the average position $\langle x \rangle$ for the states $n = 1$ and $n = 2$

∘ Assume $L = 1$, and give the probability that the system is observed with $0.4 < x < 0.6$ for the states $n = 1$ and $n = 2$