

Quiz 7.4 - Translational Motion

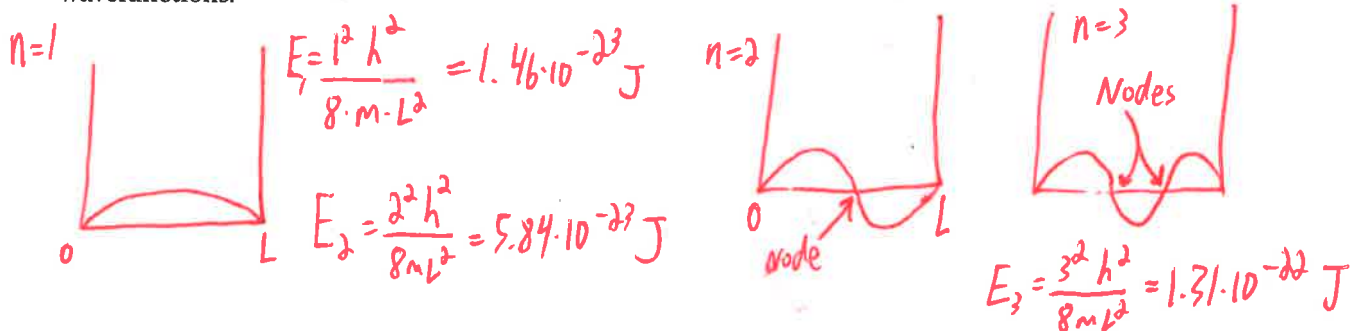
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Particle in a Box

$$E = \frac{n^2 h^2}{8mL^2}$$

$$m = 1.00794 \text{ AMU} = 1.67 \cdot 10^{-27} \text{ kg}$$

Consider a H atom confined in a box with $L = 1.5 \text{ nm}$. Model this system as a particle in a box. For each of the first three energy levels, draw the wavefunction and give the energy. Point out any nodes on your drawn wavefunctions.



Find an expression for the spacing between energy levels for a particle in a box ($E_{n+1} - E_n$), and describe its trend, if any.

$$E = n^2 E_1 \quad \Delta E = [(n+1)^2 - n^2] E_1 = [n^2 + 2n + 1 - n^2] E_1$$

$$\Delta E = (2n+1) E_1 \quad \text{Energy gaps increase linearly with } n$$

Quantum Well

Consider a particle confined to a 2-dimensional box. This system is commonly called a *quantum well*. If the two sides are equal in length, give the energies and degeneracies to the first four energy levels of this quantum system

$$\text{If } L_1 = L_2, \quad E_{n_1, n_2} = (n_1^2 + n_2^2) \frac{h^2}{8mL^2} = (n_1^2 + n_2^2) E_1$$

Level	Degeneracy	Energy	(n_1, n_2)
4	2	$10 E_1$	$(1, 3) \text{ or } (3, 1)$
3	1	$8 E_1$	$(2, 2)$
2	2	$5 E_1$	$(1, 2) \text{ or } (2, 1)$
1	1	E_1	$(1, 1)$

Tunneling

Consider an electron approaching a potential energy barrier. The barrier is 5.0 nm wide, and $4.0 \times 10^{-23} \text{ J}$ high, while the electron has kinetic energy of $1.0 \times 10^{-23} \text{ J}$

What will be the probability that the electron is transmitted through the barrier?

$$T = \left[1 + \frac{(e^{kW} - e^{-kW})^2}{16E(1-E)} \right]^{-1} \quad k = \sqrt{2mE}/\hbar$$

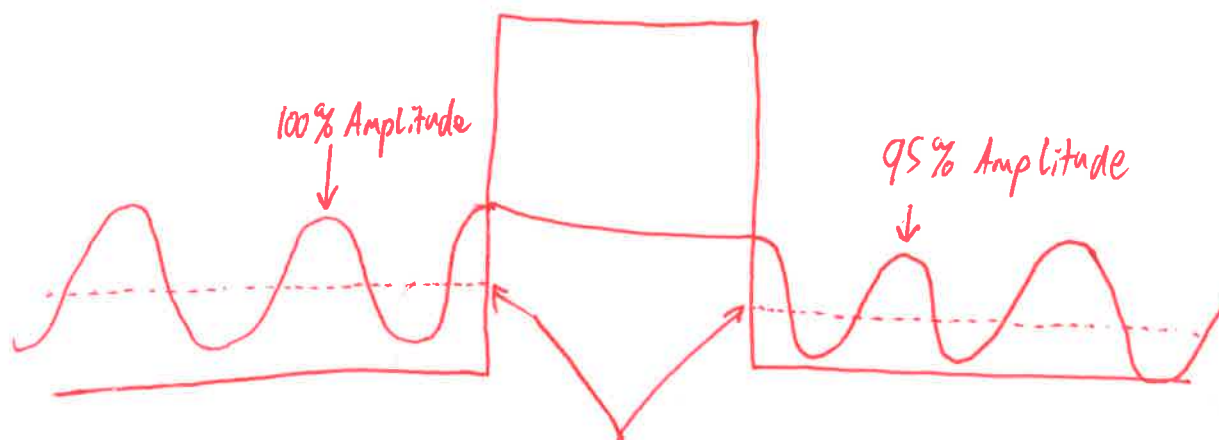
$$= \sqrt{2 \cdot 9.109 \cdot 10^{-31} \text{ kg} \cdot 1.0 \cdot 10^{-23} \text{ J}} / 1.05 \cdot 10^{-34} \text{ J}\cdot\text{s} =$$

$$W = 5 \cdot 10^{-9} \text{ m} \rightarrow kW = 0.203$$

$$k = 4.07 \cdot 10^7 \frac{1}{\text{m}}$$

$$E = \frac{E}{V_0} = \frac{1.0 \cdot 10^{-23} \text{ J}}{4.0 \cdot 10^{-23} \text{ J}} = \frac{1}{4} \rightarrow T = \left[1 + \frac{(e^{0.203} - e^{-0.203})^2}{16 \cdot \frac{1}{4} \cdot (1 - \frac{1}{4})} \right]^{-1} = 0.947 = 95\%$$

Sketch this system below, showing both the potential energy and the electron wavefunction in qualitative terms.



These should be equal height, $\frac{1}{4}$ the height of the barrier