Homework 7.1 – Introduction to Quantum Theory

Name: _____

Excercise 7A.5 (b) (10 points)

Calculate the energy of a photon and the energy per mole of photons for radiation of wavelength (i) $200\ nm$ (ultraviolet), (ii) $150\ pm$ (X-ray), (iii) $1.00\ cm$ (microwave)

Excercise 7A.11 (a) (5 points)

To what speed must an electron be accelerated for it to have a de Broglie wavelength of $3.0\ cm$?

Exercise 7A.12 (a) (5 points)

The 'fine-structure constant,' α, plays a special role in the structure of matter; its approximate value is $1/137$. What is the de Broglie wavelength of an electron travelling at αc, where c is the speed of light?

Exercise 7B.2 (b) (5 points)

Normalize (to 1) the wavefunction e^{-ax} in the range $0 \leq x \leq \infty$, with $a > 0$

Problem 7B.7 (10 points)

A normalized wavefunction for a particle confined between 0 and L in the x direction is $\Psi = (2/L)^{1/2} \sin(\pi x/L)$. Suppose that $L = 10.0\ nm$. Calculate the probability that the particle is (a) between $x = 4.95\ nm$ and $5.05\ nm$, (b) between $x = 1.95\ nm$ and $2.05\ nm$, (c) between $x = 9.90\ nm$ and $10.00\ nm$, (d) between $x = 5.00\ nm$ and $10.00\ nm$

Exercise 7C.2 (a) (10 points)

Identify which of the following functions are eigenfunctions of the operator $\mathrm{d}/\mathrm{d}x$: (i) $\cos(kx)$; (ii) e^{ikx}; (iii) kx, (iv) e^{-ax^2}. Give the corresponding eigenvalue where appropriate

Exercise 7C.3 (a) (5 points)

Functions of the form $\sin(n\pi x/L)$, where $n = 1, 2, 3, \ldots$ are wavefunctions in a region of length L (between $x = 0$ and $x = L$). Show that the wavefunctions with $n = 1$ and 2 are orthogonal; you will find the necessary integrals in the *Resource section*. (*Hint:* Recall that $\sin(n\pi) = 0$ for integer n) (*Even Better Hint:* Some integrals can be solved by symmetry arguments)

Exercise 7C.3 (b) (5 points)

For the same system as in Exercise E7C.3(a) show that the wavefunctions with $n = 2$ and $n = 4$ are orthogonal

Exercise 7C.9 (a) (5 points)

Calculate the minimum uncertainty in the speed of a ball of mass $500\ g$ that is known to be within $10\ \mu m$ of a certain point on a bat. What is the minimum uncertainty in the position of a bullet of mass $5.0\ g$ that is known to have a speed somewhere between $350.000\ 01\ ms^{-1}$ and $350.000\ 00\ ms^{-1}$?