$$\Psi = \sum_i c_i \phi_i$$

$c_i \phi_i \Psi \Psi$

$$c_i = \langle \phi_i | \ \Psi \rangle = \int \phi_i^* \Psi d\tau$$

$[-1, 1]$

$$\phi_0(x) = \frac{1}{\sqrt{2}}, \phi_1(x) = \sqrt{\frac{3}{2}} x, \phi_2(x) = \frac{\sqrt{15}}{4} \left(x^2 - \frac{1}{3} \right), \phi_3(x) = \sqrt{\frac{175}{8}} \left(x^3 - \frac{3}{5} x \right)$$

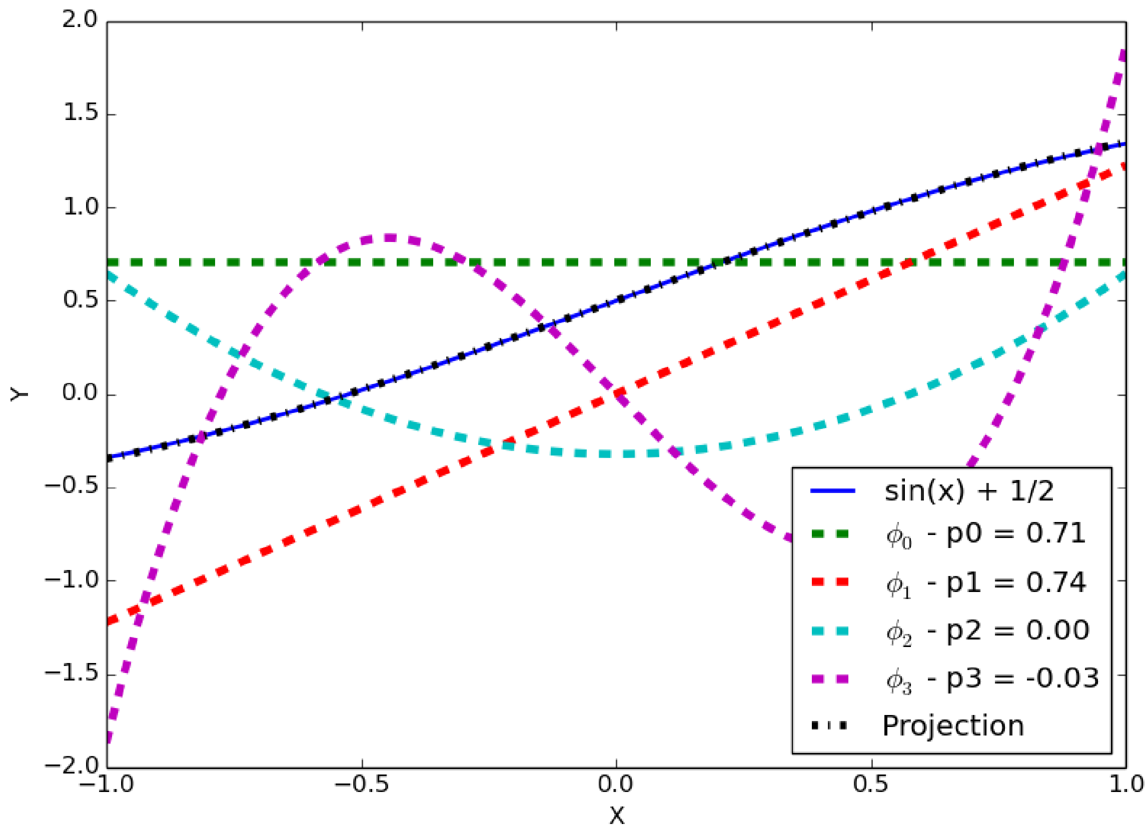
$$F(x) = \sin x + \frac{1}{2}$$

$$c_0 = \int_{-1}^{1} \frac{1}{\sqrt{2}} \left(\sin x + \frac{1}{2} \right) dx = \frac{1}{\sqrt{2}} \left[-\cos x + \frac{1}{2} x \right]_{-1}^{1} = \frac{1}{\sqrt{2}}$$

$$c_1 = \int_{-1}^{1} \sqrt{\frac{3}{2}} x \left(\sin x + \frac{1}{2} \right) dx = \sqrt{\frac{3}{2}} \left[\sin x - x \cos x + \frac{1}{4} x^2 \right]_{-1}^{1} = \sqrt{6} \left(\sin(1) - \cos(1) \right)$$

$$c_2 = \int_{-1}^{1} \frac{\sqrt{15}}{4} \left(x^2 - \frac{1}{3} \right) \left(\sin x + \frac{1}{2} \right) dx = 0(exactly)$$

$$c_3 = \int_{-1}^{1} \sqrt{\frac{175}{8}} \left(x^3 - \frac{3}{5}x \right) \left(\sin x + \frac{1}{2} \right) dx = \sqrt{14}\left(14\cos(1) - 9\sin(1)\right)$$



$$f(y) = y^2$$

$$f(y) = d_0 + \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi y}{a}\right) + d_n \cos\left(\frac{n\pi y}{a}\right)$$

$$d_0 \frac{1}{\sqrt{2a}} c_n \frac{1}{\sqrt{a}} \sin\left(\frac{n\pi y}{a}\right) d_n \frac{1}{\sqrt{a}} \cos\left(\frac{n\pi y}{a}\right)$$

$$c_i = \langle \phi_i | \ \Psi \rangle = \int \phi_i^* \Psi d\tau$$

$$f(x) = y^2 \sin\left(\frac{n\pi y}{a}\right) c_n$$

$$d_0 = \int_{-a}^{a} \frac{1}{\sqrt{2a}} y^2 dy = \frac{1}{\sqrt{2a}} \left[\frac{1}{3} y^3\right]_{-a}^{a} = \frac{2\sqrt{2}}{6} a^{5/2}$$

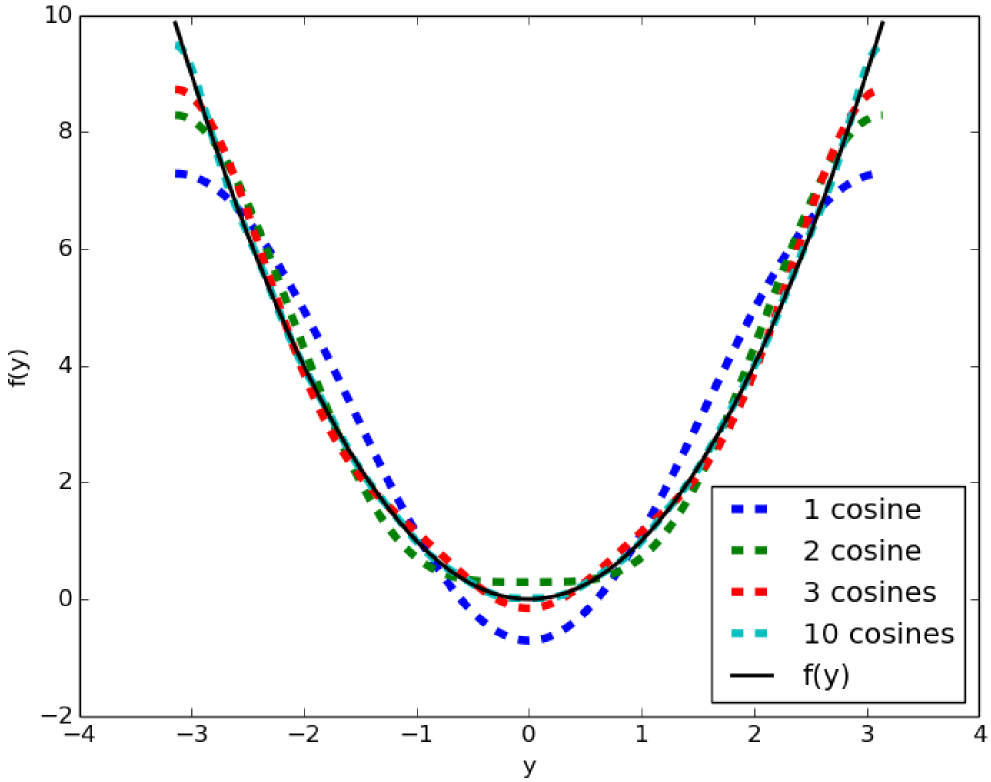
$$\begin{aligned}
d_{n>0} &= \frac{1}{\sqrt{a}} \int_{-a}^{a} y^2 \cos\left(\frac{n\pi y}{a}\right) dy \\
&= \frac{1}{\sqrt{a}} \left[\left(\frac{a}{n\pi}\right)^3 \left(\left(\frac{n\pi}{a}\right)^2 y^2 - 2\right) \sin\left(\frac{n\pi y}{a}\right) + \left(\frac{a}{n\pi}\right)^3 2y \cos\left(\frac{n\pi y}{a}\right)\right]_{-a}^{a} \\
&= \frac{1}{\sqrt{a}} \left[\frac{a}{n\pi} y^2 \sin\left(\frac{n\pi y}{a}\right) - \left(\frac{a}{n\pi}\right)^3 2 \sin\left(\frac{n\pi y}{a}\right) + \left(\frac{a}{n\pi}\right)^2 2y \cos\left(\frac{n\pi y}{a}\right)\right]_{-a}^{a}
\end{aligned}$$

$f(x) = \sin(x)[-\pi, \pi]$

$f(x) = \cos(x)[-\pi, \pi]-1 - (-1) = 0$

$$\sin\left(\frac{n\pi y}{a}\right)[-a, a]$$

$$\begin{aligned}
d_{n>0} &= \frac{1}{a} \left[\frac{a}{n\pi} y^2 \sin\left(\frac{n\pi y}{a}\right) - \left(\frac{a}{n\pi}\right)^3 2 \sin\left(\frac{n\pi y}{a}\right) + \left(\frac{a}{n\pi}\right)^2 2y \cos\left(\frac{n\pi y}{a}\right)\right]_{-a}^{a} \\
&= \frac{1}{a} \left[\left(\frac{a}{n\pi}\right)^2 2y \cos\left(\frac{n\pi y}{a}\right)\right]_{-a}^{a} \\
&= \frac{1}{\sqrt{a}} \left[\frac{2a^3}{n^2\pi^2}(-1) - \frac{-2a^3}{n^2\pi^2}(-1)\right] = -\frac{4a^{5/2}}{n^2\pi^2} foroddn \\
&= \frac{1}{\sqrt{a}} \left[\frac{2a^3}{n^2\pi^2}(1) - \frac{-2a^3}{n^2\pi^2}(1)\right] = \frac{4a^{5/2}}{n^2\pi^2} forevenn \\
&= (-1)^n \frac{4a^{5/2}}{n^2\pi^2}
\end{aligned}$$



10
8
6
4
2
0
−2
−4
−3
−2
−1
0
1
2
3
4
f(y)
y
1 cosine
2 cosine
3 cosines
10 cosines
f(y)

$y^2 y^2$

$a = \pi$