Is 8 °C Equal to 50 °F?

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"Good Enough for Chemistry": A Tragedy in Three Scenes

Scene 1.

A library study room, occupied by three freshman chemistry students comparing homework answers. It is the first week, so they are still eager to get things right. That is about to change:

Michelle [*reading*]: "Problem six. Convert 8 °C to the Fahrenheit scale."

Jeannette: Forty degrees.

Mike: Fifty degrees.

Michelle [*startled*]: What? How'd you get that?

Jeannette: That's to the right number of figures—you heard how important he thinks that is! Eight times 9/5 is 14.4, which to one sig fig is 10. Add 32, and since the 10 is only good to the 10's place, you get 40.

Mike: But you're not supposed to round off in the middle like that—you have to wait until the end. Add 32 to 14.4, which gets you 46.4. Then, since the 14.4 was only good to one figure, you round to 50. [To Michelle] What did you get?

Michelle: I used this method we learned in high school. Add 40 to the eight degrees—that gives 48. I got that by adding, so it's good to two figures. Multiply by 9/5—that gives 86.5. Now subtract 40, for 46.4. To two figs that's 46.

Mike: Sounds pretty roundabout to me. All those steps probably confuse the sig-fig thing.

Michelle: I don't think how you write a number should depend on how you calculate it, as long as you don't round off in the middle.

[*Jeannette has meanwhile been working her calculator.*]

Jeannette: I've never liked rounding off. It's like you should just throw away stuff you know. [*To Mike*] Your way, 5 °C is 41 °F; 6 and 7 °C are both 40; 8, 9, and 10 °C are all 50; and 11 °C is 52. That's weird!

Michelle: At home we have this window thermometer with both scales on it—the degrees are not at all that different in size. It just doesn't make sense to round to 10's.

Mike: [*Outnumbered*] Look, we've got 13 more problems to check. I'll take the ones we can't get to Professor Austreich.

Jeannette: And I'll see if some other text is better on sig figs. There's a bunch of them in the Help Room.

Michelle: So what's the next problem?

Mike [*Reading*]: "Problem 7. Write down your height to the nearest inch, and convert it to meters."

Jeannette and **Michelle** [In unison]: 1.7 m.

Mike: That can't be right—you guys aren't the same height. [*Michelle is 5 ft 5. Jeannette is 5 ft 8 and a guard on the basketball team.*]

Jeannette: Like I said, with these rules you're always throwing away stuff you know.

Mike: Maybe 3 or 4 in. doesn't matter in chemistry.

Michelle [*to Jeannette*]: I bet it matters to your coach.

Mike: OK, one more for Austreich.

[*They turn to Problem 8, and we fade to:*]

Scene Two

Later that day, in the office of Bernard Austreich, Professor of Chemistry, who is putting in his required office hours. He is listening, not too patiently, to Mike's account of the problem.

Mike: . . . so it looks like we ought to round the temperature to the 10's place.

Austreich: When you get an absurd answer, you're doing something wrong. Go back, and read the text more carefully.

Mike: I have, sir. I'm sure I'm doing it just as the rules say. Can you tell me where I went wrong?

Austreich: [*Doodles on his pad. He finally sees the difficulty.*]: Look—these sig fig rules are only an approximation, but they're good enough for chemistry.

Mike: So on a test, should we write 50 or 46? Can we ignore the sig figs?

Austreich: Certainly not! I don't intend to have you all writing down every digit your calculators will produce!

Mike: But just when do we use the rules, and when don't we? And what about this height thing?

Austreich [*Stands up.*] I'm sorry, you'll have to excuse me. I'm late for a Funding Committee meeting.

[*Abrupt cut, to:*]

Scene 3

The next day, in the study room. Our three freshmen have been joined by Ralph, a sophomore engineering major and new friend of Michelle.

Mike: . . . and he sure was in a hurry.

Jeannette: So where does that leave *us*? The rules are the same in all the books. Those authors can't all be wrong!

Ralph: Don't bet on it. They just copy each other. Chemists love these complicated rules. It's an initiation deal—you just have to get through it. Wait 'til you get oxidation numbers. It's good the sig-fig thing comes first. You learn right off not to take their stuff seriously.

Michelle: So what's right? I gather there's a reasonable answer to this problem. The 8 °C one, I mean.

Ralph: That's easy! Presumably you know the temperature to 1 °C. If that changes by one, how much does the Fahrenheit temperature change?

Jeannette [*immediately—she saw this yesterday*]: 1.8 degrees.

Ralph: So is that closer to one degree or to 10 degrees? That tells you the sensible place to round off.

Michelle: Hey, that's neat. It ought to work for other calculations, too!

Ralph [*pleased by the response*]: It'll work for any calculation. Just one rule, and it works for adding, and multiplying, and logs, and trig functions—anything at all.

Mike: So why don't chemistry books explain it that way? [*Pauses.*] And why couldn't Austreich?

Ralph: I had Old Ostrich last year. I got points knocked off for sig figs because I reported a density to three digits,

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when the mass was only known to two. Both were about one part in a hundred, but the mass was just under 10 g and the density was just above one. I tried to tell Ostrich the sig-fig rules could be improved. He said they were...

Mike *[breaking in]*: "Good enough for chemistry!"

Ralph: That's right. He also said "If it's not broke, don't fix it."

Jeannette: That's silly! Imagine a thermometer that worked like these sig-fig rules. When you warmed it, it would go up by one- or two-degree steps to 41 °F, jog down a degree, stick at 40 until it jumped up to 50, and go by little steps again from there up. If I had a thermometer like that I'd sure figure it was broke.

Michelle: Or never was any good in the first place.

Mike: Don't throw it away. We'll give it to Old Ostrich. It's obviously...

All *[in chorus]*: "Good enough for chemistry!"

Commentary

Our play is a tragedy not because of what happens to the students—they are coping nicely. The tragedy is that they have us, the teachers and authors, exactly right. When I challenge a colleague for a rationale for our significant figure rules, I usually get some anecdote used in class to ridicule those who overreport digits. But funny stories often don't answer serious questions. Unless we do better, we are, as Ralph says, merely indulging in an initiation ritual. I have discovered no freshman text that gives a clean, complete objective for the rules for rounding off. Even those who write articles on the subject rarely discuss this most fundamental question.

The truth is: *there is no single adequate significant digit criterion!* As soon as we opt not to report an estimated uncertainty for a number, we are on shaky ground. The number of digits to be preserved is context-dependent. We need to know, for example:

(1) the use to which the answer is to be put. Is it to be published, and, if so, where? In a refereed journal? A freshman text? A newspaper? Or is it to be saved in a research notebook, for later use? In the last case no digit of even marginal significance should be discarded, any more than one would round in the middle of any multiple-stage calculation.
(2) the nature of the calculation.
(3) whether the data were themselves rounded (as is much information in textbooks). Such data has a smaller and better known uncertainty than, say, experimental results.

In practice, even the context of common experience may matter. We expect thermometer readings near room temperature to be good to a degree or so. We expect people to know their heights to the nearest inch, so rounding to 0.1 m (about 4 in.) clearly "throws away stuff you know".

The common text treatment of significant digits is simply indefensible. It gives ridiculous answers for about one multiplication or division out of four. It confuses students who have not developed a feeling of precision and distresses those who have. How can we justify forcing students to discard digits any careful researcher would preserve? We should stop it before another generation of teachers is inculcated with a trivalized view of an important subject. No rules at all would be better because nothing could be worse!

But we *can* do better and with only a little added effort.

A Modest Proposal

I propose a rationale we can explain to students, a general rule they can test out on their calculators, and specific rules for addition, multiplication, and division that are simple and consistent.

In what follows, the term *datum* will be used to describe a number that goes into a calculation, and *answer* will describe a calculated result.

A goal. There are two distinct, often conflicting, goals for a choice of the number of digits to report:

Goal A: to preserve in each answer all information implied by the data.

Goal B: not to imply too much or too little precision in an answer.

No simple set of rules will satisfy both goals in all circumstances. Take as an example the conversion of 68 in. to meters. To report this as 1.73 m might imply a precision of 1 cm, or less than half an inch, which could be an overstatement, compromising Goal B. To follow the text rule and round to two digits, to 1.7 m, throws away the distinction between 68 in. and 65 in., and clearly fails to meet Goal A.

The problems and compromises involved in attempting to satisfy both goals are beyond anything we can explain to beginning students. The standard text rules are slanted toward Goal B. I propose, instead, that we concentrate on Goal A, which is simpler to explain and justify. We will satisfy Goal B only as it can be done without violating Goal A. We can think of this as the "research notebook criterion". From it we get a general rule:

Data will be assumed to be precise to one in the least significant digit. If a change of one in the lsd of a datum produces a given change in an answer, the most significant digit of that change is the last digit to be preserved in that answer. ["Ralph's rule".]

Example 1: 8 °C is 46.4 °F, while 9 °C is 48.2 °F. The change is 1.8 °F, so the one's place should be carried in the answer: 8 °C is 46 °F.

Example 2: 5 ft plus 5 in. is 65 in., which is 1.6510 m. 66 in. would be 1.6764 m, so the change is 0.0254 m. The answer will then be given to the hundredth's place: 65 in. is 1.65 m.

Ralph's rule can be applied to a variety of calculations. Ralph's claim that it will "work for any calculation" is only a little naive: related somewhat more sophisticated approaches (*1–3*) can be recommended to interested students.

Ralph's rule has the additional advantage that it does not have to be unlearned by students who go on to more detailed studies of numerical precision. Ralph's rule is a simplification of the "worse case" approach to uncertainty proposed by Gordon, Pickering, and Bisson (*4*). Many students (and teachers) never apply what they supposedly have learned about precision to the question of significant digits, simply because it has never occurred to them that the two are related!

The rule for addition and subtraction that follows from Ralph's rule is the familiar one: the least digit in the sum corresponds to the least digit in whichever datum is given to the least absolute precision.

The Product–Quotient Rule

The root of the troubles our student trio experienced, and of most of the "sig-fig" anomalies encounterd in general chemistry, is what Fields and Hawkes (*5*) call the simple rule, and I will call the oversimplified product–quotient rule (OPQR): "A product or quotient answer should be given to the same number of digits as the shortest datum used to calculate it." Fields and Hawkes estimate that the OPQR leads to "absurd" results about one time in four.

A simple alternative to the OPQR that satisfies Ralph's rule can be derived. For single or chained multiplications, and to a good approximation for divisions, the change ΔA in an answer A is related to a change ΔD in a datum D by:

$$\Delta A = \frac{\Delta D}{D} A$$

That is, the relative precision in A will be the relative precision in D. If ΔD is taken as one in the lsd of D, then $\Delta D/D$ is just one divided by the integer consisting of the digits of D without its decimal. Our rational product–quotient rule (RPQR) is then as follows:

(1) Identify the weakest datum—the datum given to the smallest number of digits, or, if two or more data are given to the same number of digits, the one of these that is the smallest number

when the decimal point is ignored. Write out these digits as an integer number, ignoring the decimal.

(2) divide this integer into the answer, and note the most significant digit in the result. The position of this digit is the position of the last digit that should be preserved in the answer.

Example [See our play, Scene 3]: What is the density of a solution if a 9.41-mL sample weighs 9.8 g?

$$9.8 \text{ g}/9.41 \text{ mL} = 1.0414 \text{ g/mL}$$

To find the lsd for the answer we perform the division

$$98\overline{)1.0414}\quad .010\ldots$$

Since the first digit is in the hundredth's place, the answer should be taken as 1.04 g/mL. [Austreich's exam key answer: 1.0 g/mL.]

It is not necessary to perform the division, only to visualize it, in order to identify the digits to be preserved. However, doing the division has value if one wishes to expand upon the treatment of precision, either now or later. What we have at this point is a noncalculus treatment of relative precision (*4*). The first digit of the quotient obtained indicates how important it may be to carry the answer to this position: a 7, 8, or 9 indicates that rounding one more digit will lose very little precision.

An alternative product–quotient rule, the "three rule" is proposed by Fields and Hawkes (*5*). The three rule is closely related to the RPQR: if the first digit obtained for ΔD is three or more, one would round off one more digit in D. The three rule is a compromise between Goals A and B above. Either the RPQR or the three rule is easy to apply once the result of any chain of multiplications and divisions is in a pocket calculator. Just key in the digits of the weakest datum and divide.

A Less Modest Proposal

I propose a new significant digit treatment, to encompass the arguments and observations above:

(1) A goal and general rule for significant digit treatment should be presented: that each answer should be given to enough digits that no information is lost if the weakest datum is known to ±1 in its lds.

(2) The usual addition–subtraction rule, and a revised multiplication rule, the RPQR above, should be given as applications of this rationale.

(3) Expression of precision in terms of an estimated uncertainty should be mentioned but not emphasized.

I have done this with several groups of freshman students, including a prenursing chemistry class. Students were required to master the RPQR in order to deal with extensive computer-supervised individualized homework assignments (*6*). After the usual initial resistance to anything at odds with their high school training, most students handled this approach well.

A number of related topics, including (1) cumulative uncertainty from multiple data and (2) the precision of averages, may arise in the freshman course and laboratory but need not be part of an initial treatment.

Text authors and publishers may give the negative response to this proposal that Guare (*7*) would predict. If so, it behooves those of us who care about numeracy to prepare the required alternative material.

Numerous discussions on this question with my colleagues Jimmie G. Edwards and E. Jean Jacob have helped clarify the ideas presented here.

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