Basic Quantum Chemistry Reference Compiled By: Matthew Rowley

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Particle in a Box

$$\psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}, \quad n = 1,2,3,...$$

$$\widehat{H} = \frac{\widehat{p}^2}{2m} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$

$$E_n = \frac{h^2 n^2}{8m l^2} \qquad \Delta E_n = \frac{(2n+1)h^2}{8m l^2}$$

3-Dimensional Particle in a Box

$$\psi_{n} = \left(\sqrt{\frac{2}{l}}\right)^{3} \sin \frac{n_{x}\pi x}{l} \sin \frac{n_{y}\pi y}{l} \sin \frac{n_{z}\pi z}{l}, \quad n_{x}, n_{y}, n_{z} = 1,2,3,...$$

$$\widehat{H} = \frac{\widehat{p}^2}{2m} = \frac{-\hbar^2}{2m} \nabla^2$$

$$E_{n} = \frac{h^{2}}{8m} \left(\frac{n_{x}^{2}}{l_{x}^{2}} + \frac{n_{y}^{2}}{l_{y}^{2}} + \frac{n_{z}^{2}}{l_{z}^{2}} \right)$$

Harmonic Oscillator

$$\psi_{\rm n} = \sqrt{2^{\rm n} n!} \left(\frac{\alpha}{\pi}\right)^{1/4} H_{\rm n} \left(\sqrt{\alpha} x\right) e^{-\alpha x^2/2} \quad n = 0,1,2,... \quad \alpha \equiv \frac{2\pi \nu m}{\hbar}$$

$$\widehat{H} = \frac{\widehat{p}^2}{2m} + \frac{1}{2}kx^2 = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}kx^2$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) = \hbar\nu\left(n + \frac{1}{2}\right)$$

Hermite Polynomials

$$H_n(x) = \sum_{m=0}^{m=n} c_m^{(n)} x^m$$

$$c_m^{(n)} = \frac{1}{\sqrt{m\hbar\omega}} e^{-i\pi/2}$$

$$H_{n}(x) = (-1)^{n} e^{x^{2}} \frac{\partial^{n}}{\partial x^{n}} e^{-x^{2}}$$

Ladder Operators

$$a^{+} = (2m)^{-1/2}(\hat{p} + im\omega x);$$

$$a^{-}=(2m)^{-1/2}(\hat{p}-im\omega x);$$

$$\psi_n = \frac{1}{\sqrt{n!}} \left(\frac{a^+}{\sqrt{\hbar \omega}} \right)^n \psi_0(-i)^n = \frac{1}{\sqrt{n!} (2\hbar \omega m)^n} \left(-\hbar \frac{\partial}{\partial x} + \omega mx \right)^n \psi_0$$

The First Few Normalized Wave Functions

$$\overline{\psi_0 = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}}$$

$$\psi_1 = \left(\frac{4\alpha^3}{\pi}\right)^{1/4} x e^{-\alpha x^2/2}$$

$$\psi_2 = \left(\frac{\alpha}{4\pi}\right)^{1/4} (2\alpha x^2 - 1)e^{-\alpha x^2/2}$$

Alternate General Form of the Wave Function

$$\psi_{n} = e^{-\alpha x^{2}/2} \sum_{m=0 \text{ or } 1}^{n} c_{m}^{(n)} x^{m}$$

Note that only even or only odd terms are included in the sum

For n = even,
$$c_0^{(n)} = 1$$

For
$$n = odd$$
, $c_1^{(n)} = 1$

$$c_{m+2}^{(n)} = \frac{2\alpha(m-n)}{(n+1)(n+2)} c_{m}$$

The entire wave function must be normalized after the summation

Rigid Rotor and Spherical Harmonics

$$\psi = Y_i^m(\theta, \phi)$$

$$\widehat{H} = \frac{\widehat{J}^2}{2I}$$

$$E_j = \frac{j(j+1)\hbar^2}{2I}$$

Spherical Harmonics
$$\overline{Y_j^m(\theta,\phi) = S_{j,m}(\theta)T_m(\phi)}$$

$$T_m(\phi) = \frac{1}{\sqrt{2\pi}}e^{im\phi}$$

$$S_{j,m}(\theta) = \left[\frac{(2j+1)[(j-|m|)!]}{2[(j+|m|)!]} \right]^{1/2} P_j^{|m|}(\cos\theta)$$

$$P_j^{|m|}(w) \equiv \frac{1}{2^j i!} (1 - w^2)^{|m|/2} \frac{\partial^{j+|m|}}{\partial w^{j+|m|}} (w^2 - 1)^j$$

$$S_{j,m}(\theta)$$

$$j = 0 \quad m = 0 \quad \frac{1}{2}\sqrt{2}$$

<i>j</i> = 1	m = 0	$\frac{1}{2}\sqrt{6}\cos\theta$
	$m=\pm 1$	$\frac{1}{2}\sqrt{3}\sin\theta$
<i>j</i> = 2	m = 0	$\frac{1}{4}\sqrt{10}(3\cos^2\theta - 1)$
	$m=\pm 1$	$\frac{1}{2}\sqrt{15}\sin\theta\cos\theta$
	$m=\pm 2$	$\frac{1}{4}\sqrt{15}\sin^2\theta$
<i>j</i> = 3	m = 0	$\frac{3}{4}\sqrt{14}\left(\frac{5}{3}\cos^3\theta - \cos\theta\right)$
	$m=\pm 1$	В
	$m=\pm 2$	$\frac{1}{4}\sqrt{105}\sin^2\theta\cos\theta$
	$m = \pm 3$	$\frac{1}{8}\sqrt{70}\sin^3\theta$

Angular Momentum Operators

$$\widehat{J}_z = -i\hbar \frac{\partial}{\partial \theta}$$

$$J_z = m\hbar$$

$$\hat{J}^2 = -\hbar^2 \left(\frac{\partial^2}{\partial \theta^2} + \cot \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right)$$
$$J^2 = j(j+1)\hbar^2$$

$$\hat{J}_x = y\hat{p}_z - z\hat{p}_y$$

$$\hat{J}_y = z\hat{p}_x - x\hat{p}_z$$

$$\hat{J}_z = x\hat{p}_y - y\hat{p}_x$$

$$\begin{split} \left[\hat{J}_{x},\hat{J}_{y}\right] &= i\hbar\hat{J}_{z} \\ \left[\hat{J}_{y},\hat{J}_{z}\right] &= i\hbar\hat{J}_{x} \\ \left[\hat{J}_{z},\hat{J}_{x}\right] &= i\hbar\hat{J}_{y} \end{split}$$

$$\hat{J}_x = i\hbar \left(\sin \phi \, \frac{\partial}{\partial \theta} + \cot \theta \, \cos \phi \, \frac{\partial}{\partial \phi} \right)$$

$$\hat{J}_y = -i\hbar \left(\cos \phi \, \frac{\partial}{\partial \theta} - \cot \theta \, \sin \phi \, \frac{\partial}{\partial \phi} \right)$$

Ladder Operators

$$\overline{J_{\pm} = J_x \pm iJ_y}$$

$$J_{\pm}Y_j^m(\theta, \phi) = \hbar\sqrt{j(j+1) - m(m\pm 1)}Y_j^{m+1}(\theta, \phi)$$

Hydrogenic Atom

Wave Functions

$$\overline{\psi = R_{n,l}(r)Y_l^m(\theta,\phi)}$$

$$\alpha_o = \frac{4\pi\varepsilon_o\hbar^2n^2}{\mu e^2Z}$$

	$R_{n,\prime}(r)$		
		$2\left(\frac{Z}{\alpha_o}\right)^{3/2}e^{-Zr/\alpha_o}$	
n = 2		$\frac{1}{\sqrt{2}} \left(\frac{Z}{\alpha_o}\right)^{3/2} \left(1 - \frac{Zr}{2\alpha_o}\right) e^{-Zr/2\alpha_o}$	
		$\frac{1}{2\sqrt{6}} \left(\frac{Z}{\alpha_o}\right)^{5/2} r e^{-Zr/2\alpha_o}$	
n = 3	l = 0	$\frac{2}{3\sqrt{3}} \left(\frac{Z}{\alpha_o}\right)^{3/2} \left(1 - \frac{2Zr}{3\alpha_o} + \frac{2}{27} \left(\frac{Zr}{\alpha_o}\right)^2\right) e^{-Zr/3\alpha_o}$	
	l = 1	$\frac{8}{27\sqrt{6}} \left(\frac{Z}{\alpha_o}\right)^{5/2} \left(1 - \frac{1}{6} \frac{Zr}{\alpha_o}\right) r e^{-Zr/3\alpha_o}$	
	<i>l</i> = 2	$\frac{4}{81\sqrt{30}} \left(\frac{Z}{\alpha_o}\right)^{7/2} r^2 e^{-Zr/3\alpha_o}$	

	Complex Hydrogenic Wave Functions		
1s	$\frac{1}{\sqrt{\pi}} \left(\frac{Z}{\alpha_o}\right)^{3/2} e^{-Zr/\alpha_o}$		
2 <i>s</i>	$\frac{1}{\sqrt{\pi}} \left(\frac{Z}{2\alpha_o}\right)^{3/2} \left(1 - \frac{Zr}{2\alpha_o}\right) e^{-Zr/2\alpha_o}$		
$2p_0$	$\left(\frac{1}{\sqrt{z}}\left(\frac{Z}{2\alpha}\right)^{3/2}re^{-Zr/2\alpha_0}\cos\theta\right)$		
$2p_{\pm 1}$	$\frac{1}{8\sqrt{\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} r e^{-Zr/2\alpha_o} \sin\theta \ e^{\pm i\phi}$		

$$3s \qquad \frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{\alpha_o}\right)^{3/2} \left(27 - 18\frac{Zr}{\alpha_o} + 2\left(\frac{Zr}{\alpha_o}\right)^2\right) e^{-Zr/3\alpha_o}$$

$$3p_0 \qquad \frac{1}{81} \sqrt{\frac{Z}{\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} \left(6 - \frac{Zr}{\alpha_o}\right) r e^{-Zr/3\alpha_o} \cos \theta$$

$$3p_{\pm 1} \qquad \frac{1}{81\sqrt{\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} \left(6 - \frac{Zr}{\alpha_o}\right) r e^{-Zr/3\alpha_o} \sin \theta e^{\pm i\phi}$$

$$3d_0 \qquad \frac{1}{81\sqrt{6\pi}} \left(\frac{Z}{\alpha_o}\right)^{7/2} r^2 e^{-Zr/3\alpha_o} (3\cos^2 \theta - 1)$$

$$3d_{\pm 1} \qquad \frac{1}{81\sqrt{\pi}} \left(\frac{Z}{\alpha_o}\right)^{7/2} r^2 e^{-Zr/3\alpha_o} \sin \theta \cos \theta e^{\pm i\phi}$$

$$3d_{\pm 2} \qquad \frac{1}{162\sqrt{\pi}} \left(\frac{Z}{\alpha_o}\right)^{7/2} r^2 e^{-Zr/3\alpha_o} \sin^2 \theta e^{\pm 2i\phi}$$

Real Hydrogenic Wave Functions		
1s	$\frac{1}{\sqrt{\pi}} \left(\frac{Z}{\alpha_o}\right)^{3/2} e^{-Zr/\alpha_o}$	
2 <i>s</i>	$\frac{1}{\sqrt{\pi}} \left(\frac{Z}{2\alpha_o}\right)^{3/2} \left(1 - \frac{Zr}{2\alpha_o}\right) e^{-Zr/2\alpha_o}$	
$2p_z$	$\frac{1}{\sqrt{\pi}} \left(\frac{Z}{2\alpha_o}\right)^{5/2} r e^{-Zr/2\alpha_o} \cos \theta$	
$2p_x$	$\frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} r e^{-Zr/2\alpha_o} \sin\theta \cos\phi$	
$2p_y$	$\frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} r e^{-Zr/2\alpha_o} \sin\theta \sin\phi$	
3s	$\frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{\alpha_o}\right)^{3/2} \left(27 - 18\frac{Zr}{\alpha_o} + 2\left(\frac{Zr}{\alpha_o}\right)^2\right) e^{-Zr/3\alpha_o}$	
$3p_z$	$\frac{1}{81} \sqrt{\frac{2}{\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} \left(6 - \frac{Zr}{\alpha_o}\right) r e^{-Zr/3\alpha_o} \cos \theta$	
$3p_x$	$\frac{1}{81} \sqrt{\frac{2}{\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} \left(6 - \frac{Zr}{\alpha_o}\right) r e^{-Zr/3\alpha_o} \sin\theta \cos\phi$	
$3p_y$	$\frac{1}{81} \sqrt{\frac{2}{\pi}} \left(\frac{Z}{\alpha_o}\right)^{5/2} \left(6 - \frac{Zr}{\alpha_o}\right) r e^{-Zr/3\alpha_o} \sin\theta \sin\phi$	

Hamiltonian

$$\widehat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(r)$$

$$V(r) = \frac{-Ze^2}{4\pi\varepsilon_0 r} = \frac{-Ze'^2}{r}$$

$$\widehat{H} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial r^2} + \frac{2\partial}{r\partial r} \right) + \frac{1}{2mr^2} \widehat{L}^2 - \frac{Ze^2}{4\pi\varepsilon_o r}$$

Energies

Energies
$$E_n = -\frac{Z^2 e^2}{2n^2 (4\pi \varepsilon_o) \alpha_o}$$

$$E_n = -\frac{Z^2 \mu e^4}{2(4\pi \varepsilon_o)^2 \hbar^2 n^2}$$

$$E_n \approx -13.606 \frac{Z^2}{n^2} eV$$

Many Electron Atom

Wave Functions

Hamiltonian

$$\overline{\widehat{H} = \widehat{H}^o + \widehat{H}_{rep} + \widehat{H}_{S.O.}}$$

$$\widehat{H}^{o} = \sum_{i=1}^{n} \left(-\frac{\hbar^{2}}{2m} \nabla_{i}^{2} - \frac{Ze'^{2}}{r_{i}} \right)$$

$$\widehat{H}_{rep} = \sum_{i=1}^{n-1} \sum_{j>i}^{n} \frac{{e'}^2}{r_{ij}}$$

$$\widehat{H}_{S.O.} = \sum_{i=1}^{n} \xi_i \widehat{L}_i \cdot \widehat{S}_i$$

Energies

Term Symbols for Equivalent Electrons		
Configuration	Terms	
s^1	^{2}S	
s^2 ; p^6 ; d^{10}	¹ S	
$p^1; p^5$	² P	
$p^2; p^4$	³ P, ¹ D, ¹ S	
p^3	${}^{4}S$, ${}^{2}D$, ${}^{2}P$	
$d^{1}; d^{9}$	^{2}D	
d^{2} ; d^{8}	³ F, ³ P, ¹ G, ¹ D, ¹ S	
d^{3} ; d^{7}	${}^{4}F$, ${}^{4}P$, ${}^{2}H$, ${}^{2}G$, ${}^{2}F$, ${}^{2}D(2)$, ${}^{2}P$	
d^4 ; d^6	^{5}D , ^{3}H , ^{3}G , $^{3}F(2)$, ^{3}D , $^{3}P(2)$,	
$\begin{bmatrix} a^2; a^3 \end{bmatrix}$	^{1}I , $^{1}G(2)$, ^{1}F , $^{1}D(2)$, $^{1}S(2)$	
d^5	⁶ S, ⁴ G, ⁴ F, ⁴ D, ⁴ P, ² I, ² H,	
a	$^{2}G(2)$, $^{2}F(2)$, $^{2}D(3)$, ^{2}P , ^{2}S	
Term Symbols	for Nonequivalent Electrons	
Configuration	Terms	
s^1s^1	³ S, ¹ S	
s^1p^1	³ P, ¹ P	
s^1d^1	^{3}D , ^{1}D	
s^1f^1	${}^{3}F$, ${}^{1}F$	
p^1p^1	³ D, ³ P, ³ S, ¹ D, ¹ P, ¹ S	
p^1d^1	³ F, ³ D, ³ P, ¹ F, ¹ D, ¹ P	
p^1f^1	${}^{3}G, {}^{3}F, {}^{3}D, {}^{1}G, {}^{1}F, {}^{1}D$	
d^1d^1	${}^{3}G, {}^{3}F, {}^{3}D, {}^{3}P, {}^{3}S,$	
	${}^{1}G, {}^{1}F, {}^{1}D, {}^{1}P, {}^{1}S$	
d^1f^1	$^{3}H, ^{3}G, ^{3}F, ^{3}D, ^{3}P,$	

	¹ H, ¹ G, ¹ F, ¹ D, ¹ P
f^1f^1	³ I, ³ H, ³ G, ³ F, ³ D, ³ P, ³ S, ¹ I, ¹ H, ¹ G, ¹ F, ¹ D, ¹ P, ¹ S
))	¹ I, ¹ H, ¹ G, ¹ F, ¹ D, ¹ P, ¹ S

(N)indicates that the term occurs N times

Variational Theory

$$\langle \widehat{H} \rangle \equiv \frac{\langle \phi | \widehat{H} | \phi \rangle}{\langle \phi | \phi \rangle} \ge E_0$$

Linear Variational Theory

For a set of *m* trial functions:

$$\psi_n = \sum_{i=1}^m c_i^{(n)} \phi_i$$

Energies

To find E_n solve for W in the secular equation:

$$\begin{vmatrix} \mathbf{H}_{11} - \mathbf{S}_{11} \mathbf{W} & \cdots & \mathbf{H}_{1m} - \mathbf{S}_{1m} \mathbf{W} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{m1} - \mathbf{S}_{m1} \mathbf{W} & \cdots & \mathbf{H}_{mm} - \mathbf{S}_{mm} \mathbf{W} \end{vmatrix} = 0$$

This gives m roots of W, corresponding to the upper bounds of the m lowest energy states.

Wave Functions

To find $c_i^{(n)}$ and ψ_n solve this system of equations:

$$\sum_{i=1}^{i=m} (H_{1i} - S_{1i}W_n)c_1^{(n)}$$

$$\sum_{i=m}^{i=m} (H_{2i} - S_{2i}W_n)c_2^{(n)}$$

$$\vdots$$

$$\sum_{i=1}^{i=m} (H_{mi} - S_{mi}W_n)c_m^{(n)}$$

Because the above system is not completely linearly independent (by one degree) we must finally normalize by:

$$\sum_{i=1}^m c_i^{(n)} = 0$$

Non-Degenerate Perturbation Theory

$$\psi_{n}^{(1)} = \sum_{i \neq n} \frac{\langle \psi_{i}^{(0)} | \widehat{H}' | \psi_{n}^{(0)} \rangle}{E_{n}^{(0)} - E_{i}^{(0)}} \psi_{i}^{(0)}$$

 $\widehat{H}=\widehat{H}^o+\widehat{H}'$ Where $\,\widehat{H}^o$ is exactly solvable for $\psi_n^{(0)}$ and \widehat{H}' is a perturbation

$$E_n = E_n^{(0)} + E_n^{(1)} + \cdots$$

$$E_n^{(0)} \psi_n^{(0)} = \widehat{H}^0 | \psi_n^{(m-1)} \rangle$$

The First Few Degrees of Energy Correction $\frac{E_n^{(1)} = \langle \psi_n^{(0)} | \widehat{H}' | \psi_n^{(0)} \rangle}{E_n^{(1)} = \langle \psi_n^{(0)} | \widehat{H}' | \psi_n^{(0)} \rangle}$

$$\overline{E_n^{(1)} = \langle \psi_n^{(0)} | \widehat{H}' | \psi_n^{(0)} \rangle}$$

$$E_{n}^{(2)} = \sum_{i \neq n} \frac{\left| \langle \psi_{i}^{(0)} | \widehat{H}' | \psi_{n}^{(0)} \rangle \right|^{2}}{E_{n}^{(0)} - E_{i}^{(0)}}$$

Degenerate Perturbation Theory

$$\psi_{n}^{(1)} = \sum_{i \neq n} \frac{\langle \phi_{i}^{(0)} | \widehat{H}' | \phi_{n}^{(0)} \rangle}{E_{n}^{(0)} - E_{i}^{(0)}} \psi_{i}^{(0)}$$

$$\phi_n^{(0)} = \sum_{i=1}^d c_i \, \psi_i^{(0)}, \quad 1 \le n \le d, d \text{ is the degeneracy}$$

Finding Correct Zero-Order Wave Functions

To find c_i , solve the secular equation:

$$\begin{vmatrix} H'_{11} - E_n^{(1)} & H'_{12} & \dots & H'_{1d} \\ H'_{21} & H'_{22} - E_n^{(1)} & \dots & H'_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ H'_{d1} & H'_{d2} & \dots & H'_{dd} - E_n^{(1)} \end{vmatrix} = 0$$

The First Few Degrees of Energy Correction

$$\mathbf{E}_{\mathrm{n}}^{(1)} = \langle \phi_{\mathrm{n}}^{(0)} | \widehat{\mathbf{H}}' | \phi_{\mathrm{n}}^{(0)} \rangle$$

$$E_n^{(2)} = \sum_{i \neq n} \frac{\left| \langle \phi_i^{(0)} | \widehat{H}' | \phi_n^{(0)} \rangle \right|^2}{E_n^{(0)} - E_i^{(0)}}$$

Definite and Indefinite Integrals

Remember that

From the form
$$d\tau = r^2 \sin \theta \, dr d\theta d\phi$$

$$\nabla^2 = \left(\frac{\partial^2}{\partial \theta^2} + \cot \theta \, \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \, \frac{\partial^2}{\partial \phi^2}\right)$$

$$\int x \sin bx \, dx = \frac{1}{b^2} \sin bx - \frac{x}{b} \cos bx$$

$$\int \sin^2 bx \, dx = \frac{x}{2} - \frac{1}{4b} \sin(2bx)$$

$$\int x \sin^2 bx \, dx = \frac{x^2}{4} - \frac{x}{4b} \sin(2bx) - \frac{1}{8b^2} \cos(2bx)$$

$$\int x^2 \sin^2 bx \, dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right) \sin(2bx) - \frac{x}{4b^2} \cos(2bx)$$

$$\int \sin ax \sin bx \, dx = \frac{\sin[(a - b)x]}{2(a - b)} - \frac{\sin[(a + b)x]}{2(a + b)}, \quad a^2 \neq b^2$$

$$\int x e^{bx} dx = \left(\frac{x}{b} - \frac{1}{b^2}\right) e^{bx}$$

$$\int x^2 e^{bx} dx = \left(\frac{x^2}{b} - \frac{2x}{b^2} + \frac{2}{b^3}\right) e^{bx}$$

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}, \quad n > -1, b > 0$$

$$\int_0^\infty e^{-bx^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{b}}, \quad b > 0$$

$$\int_0^\infty x^{2n} e^{-bx^2} dx = \frac{(2n)!}{2^{2n+1} n!} \sqrt{\frac{\pi}{b^{2n+1}}}, \quad b > 0, n = 1, 2, 3, \dots$$

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}} e^{-bt} \left(1 + bt + \frac{b^2 t^2}{2!} + \dots + \frac{b^n t^n}{n!}\right), \quad b > 0, n = 0, 1, 2, \dots$$

Physical Constants and Conversions

Constant	Symbol	SI Value	Gaussian Value
Speed of Light in Vacuum	c	2.99792458 * 10 ^{8 m} / _S	2.99792458 * 10 ¹⁰ cm/ _S

Proton Charge	e	1.602177 * 10 ⁻¹⁹ C	
"	e'		4.803207 * 10 ⁻¹⁰ statC
Vacuum Permittivity	$\epsilon_{ m o}$	$8.8541878 * 10^{-12} ^{\text{C}^2} /_{\text{N-m}^2}$	
Avogadro Constant	N _A	$6.02214 * 10^{-12} \text{mol}^{-1}$	$6.02214 * 10^{-12} \text{mol}^{-1}$
Electron Rest Mass	m _e	$9.1093897 * 10^{-31}$ kg	$9.1093897 * 10^{-28}$ g
Proton Rest Mass	m _p	$1.672623 * 10^{-27}$ kg	$1.672623 * 10^{-24}$ g
Neutron Rest Mass	m _n	$1.674929 * 10^{-27}$ kg	$1.674929 * 10^{-24}$ g
Planck Constant	h	$6.6260755 * 10^{-34}$ J-s	$6.6260755 * 10^{-27}$ erg-s
Reduced Planck Constant	ħ	$1.0545727 * 10^{-34}$ J-s	$1.0545727 * 10^{-27}$ erg-s
Faraday Constant	F	96485.3 ^C / _{mol}	
Vacuum Permeability	μ_o	$4\pi * 10^{-7} \text{ N/}_{\text{C}^2-\text{S}^2}$	
Bohr Radius	α_o	5.291772 * 10 ⁻¹¹ m	5.291772 * 10 ⁻⁹ cm
Bohr Magneton	β_e	$9.27402 * 10^{-24} \text{ J/}_{\text{T}}$	
Nuclear Magneton	β_N	$5.05079 * 10^{-27} \text{ J}_{\text{T}}$	
Electron g Value	g_{e}	2.0023193044	2.0023193044
Proton g Value	g_p	5.585695	5.585695
Gas Constant	R	8.3145 ^J / _{mol-K}	$8.3145 * 10^{7} \frac{\text{erg}}{\text{mol-K}}$
Boltzmann Constant	k	1.38066 * 10 ^{-23 J} /K	1.38066 * 10 ⁻¹⁶ erg/K
Gravitational Constant	G	$6.673 * 10^{-11} \text{m}^3/\text{kg-s}^2$	$6.673 * 10^{-8} \text{ cm}^3/\text{g-s}^2$

Energy Conversion Factors
$1 \text{ erg} = 10^{-7} \text{J}$
1 cal ≅ 4.184 J
$1 \text{ eV} \cong 1.602177 * 10^{-19} \text{J} \cong 1.602177 * 10^{-12} \text{erg} \triangleq 23.0605 \text{ kcal/mol}$
1 hartree $\cong 4.35975 * 10^{-18} J \cong 27.2114 \text{ eV} \triangleq 627.510 \text{ kcal/mol}$

 $The \ symbol \triangleq means \ "corresponds \ to"$

Notes and Acknowledgements:

1. Spherical coordinates are represented in the way standard for chemists (which is different from the standard used by mathematicians). i.e. r = radius from origin, $\theta = angle$ with

- the positive z axis, ϕ = angle between the positive x axis and the projection onto the x/y plane.
- 2. While I have worked hard to ensure that this document is correct, I assume no responsibility for the accuracy of the information here.

The following were used in compiling this reference:

- 1. http://panda.unm.edu/Courses/Finley/P262/Hydrogen/WaveFcns.html
- 2. Levine, Ira N. *Quantum Chemistry* 6th ed. Pearson Prentice Hall (Upper Saddle River, NJ) 2009.
- 3. Hollas, J. Michael. *Modern Spectroscopy*. John Wiley & Sons, Ltd. (Chichester, West Sussex, England) 2004.