The Pivotal Method of Evaluating Determinants

(Row Reduction)

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$$A = \begin{bmatrix} \boxed{2} & 3 & 1 & 2 \\ 4 & 2 & 3 & 4 \\ 1 & 4 & 2 & 2 \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 4 & 2 & 3 & 4 \\ 1 & 4 & 2 & 2 \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 - 4R_1}$$

$$\begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & \boxed{-4} & -9 & 0 \\ 1 & 4 & 2 & 2 \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_2} \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 1 & 4 & 2 & 2 \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - R_1}$$

$$\rightarrow \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 0 & 2.5 & 1.5 & 1 \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - 2.5R_2} \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 0 & 0 & \boxed{-4.125} & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{R_3}{-4.125}}$$

$$\rightarrow \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 0 & 0 & 1 & +0.(24) \\ 3 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - 3R_1} \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 0 & 0 & 1 & +0.(24) \\ 0 & -3.5 & -1.5 & -2 \end{bmatrix} \xrightarrow{R_4 + 3.5R_2}$$

$$\rightarrow \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 0 & 0 & 1 & +0.(24) \\ 0 & 0 & 6.375 & -2 \end{bmatrix} \xrightarrow{R_4 - 6.375R_3} \begin{bmatrix} 1 & 1.5 & 0.5 & 1 \\ 0 & 1 & 2.25 & 0 \\ 0 & 0 & 1 & -0.(24) \\ 0 & 0 & 0 & \boxed{+0.4545} \end{bmatrix}$$

$$\Longrightarrow \boxed{|A|} = (2)(-4)(-4.125)(0.4545) = \boxed{15.00}$$

SSCP matrix

from: Carroll & Green

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$$S = A'A - \frac{1}{m}(A'\cdot\vec{1})(\vec{1}'A)$$

$$\begin{bmatrix} y & \cdots \\ x_1 & \cdots \\ x_2 & \cdots \end{bmatrix}\begin{bmatrix} y & x_1 & x_2 \\ \vdots & \vdots & \vdots \end{bmatrix} - \frac{1}{m}\begin{bmatrix} y & \cdots \\ x_1 & \cdots \\ x_2 & \cdots \end{bmatrix}\begin{bmatrix} 1 \\ \vdots \\ \vdots \end{bmatrix}\begin{bmatrix} 1 & \cdots \end{bmatrix}\begin{bmatrix} y & x_1 & x_2 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

3×1 $\frac{\Sigma}{m}$ 1×3 $\frac{\Sigma}{m}$

RAW CROSS-PROD. MATRIX

$$\boxed{B = \begin{bmatrix} \Sigma Y^2 & \Sigma Y x_1 & \Sigma Y x_2 \\ \Sigma x_1 Y & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 Y & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix}}$$

$$\frac{1}{m}\begin{bmatrix} \Sigma Y \\ \Sigma x_1 \\ \Sigma x_2 \end{bmatrix}\cdot\begin{bmatrix} \Sigma Y & \Sigma x_1 & \Sigma x_2 \end{bmatrix} = \frac{1}{m}\begin{bmatrix} (\Sigma Y)^2 & \Sigma Y \Sigma x_1 & \Sigma Y \Sigma x_2 \\ \Sigma x_1 \Sigma Y & (\Sigma x_1)^2 & \Sigma x_1 \Sigma x_2 \\ \Sigma x_2 \Sigma Y & \Sigma x_1 \Sigma x_2 & (\Sigma x_2)^2 \end{bmatrix}$$

SSCP MATRIX

$$S = B - \begin{bmatrix} \frac{(\Sigma Y)^2}{m} & \frac{\Sigma Y \Sigma x_1}{m} & \frac{\Sigma Y \Sigma x_2}{m} \\ \frac{\Sigma x_1 \Sigma Y}{m} & \frac{(\Sigma x_1)^2}{m} & \frac{\Sigma x_1 \Sigma x_2}{m} \\ \frac{\Sigma x_2 \Sigma Y}{m} & \frac{\Sigma x_1 \Sigma x_2}{m} & \frac{(\Sigma x_2)^2}{m} \end{bmatrix}$$

Alternatively, if the column means are subtracted out of A to begin with, yielding the MEAN-CORRECTED matrix A_d, then:

$$\boxed{S = A_d' A_d.}$$

Covariance Matrix (C)

$$C = \frac{1}{(m-1)} S$$

- off-diagonal elements of C are covariances
- diagonal elements of C are variances

$$cov(Y x_1) = \frac{\Sigma x y_1}{(m-1)}$$

$$S_Y^2 = \frac{\Sigma y^2}{(m-1)}$$

from Carroll & Green 21/09/17

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Correlation matrix (R)

- correlation betw. two variables, say Y and x_1 is:

$$r_{yx} = \frac{\sum Y x_1}{\sqrt{\sum Y^2}\,\sqrt{\sum x_1^2}}$$

sum of coproducts

sum of squares

→ correlated to SSCP matrix (sums of coproducts and sums of squares)

→ its standard error $s_r = \sqrt{\frac{(1-r^2)}{(n-2)}}$

→ Y and x_1 are mean-centered

→ the main entries of the SSCP matrix diagonal are the mean-corrected sum of squares of the three variables Y, x_1 and x_2.

→ we need the reciprocal form of the SS for Y, x_1, x_2:

$$r_{yx} = \frac{1}{\sqrt{\sum Y^2}} \cdot \frac{1}{\sqrt{\sum x_1^2}} \cdot \sum Y x_1$$

→ we use Δ the DIAGONAL MATRIX with the entries: $\frac{1}{\sqrt{\sum Y^2}}, \frac{1}{\sqrt{\sum x_1^2}}, \frac{1}{\sqrt{\sum x_2^2}}$ to compute R the CORRELATION MATRIX

$$\Delta = \begin{bmatrix} \frac{1}{\sqrt{\sum Y^2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\sum x_1^2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\sum x_2^2}} \end{bmatrix} ; \quad \boxed{R = \Delta C \Delta}$$

mean-corrected form

$$\begin{bmatrix} \frac{1}{\sqrt{\sum Y^2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\sum x_1^2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\sum x_2^2}} \end{bmatrix} \cdot \begin{bmatrix} \sum Y^2 & \sum Y x_1 & \sum Y x_2 \\ \sum x_1 Y & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 Y & \sum x_1 x_2 & \sum x_2^2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{\sum Y^2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\sum x_1^2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\sum x_2^2}} \end{bmatrix}$$

→

$$\begin{bmatrix} \dfrac{\sum y^2}{\sqrt{\sum y^2}\sqrt{\sum y^2}} & \dfrac{\sum y x_1}{\sqrt{\sum y^2}\sqrt{\sum x_1^2}} & \dfrac{\sum y x_2}{\sqrt{\sum y^2}\sqrt{\sum x_2^2}} \\[2em] \dfrac{\sum y x_1}{\sqrt{\sum y^2}\sqrt{\sum x_1^2}} & \dfrac{\sum x_1^2}{\sqrt{\sum x_1^2}\sqrt{\sum x_1^2}} & \dfrac{\sum x_1 x_2}{\sqrt{\sum x_1^2}\sqrt{\sum x_2^2}} \\[2em] \dfrac{\sum y x_2}{\sqrt{\sum y^2}\sqrt{\sum x_2^2}} & \dfrac{\sum x_1 x_2}{\sqrt{\sum x_1^2}\sqrt{\sum x_2^2}} & \dfrac{\sum x_2^2}{\sqrt{\sum x_2^2}\sqrt{\sum x_2^2}} \end{bmatrix} = R$$

MEAN-CORRECTED SCORES!

column vector of the means

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A'

$j = 1$ to m

unit row vector $1 \times m$

$$A_d' = \begin{bmatrix} y & \cdots & \cdots \\ x_1 & \cdots & \cdots \\ x_2 & \cdots & \cdots \end{bmatrix} - \frac{1}{m}\begin{bmatrix} \Sigma y \\ \Sigma x_1 \\ \Sigma x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\boxed{A_d' = A' - \frac{1}{m}(\vec{a}\,\vec{1}')}$$

$$S = \left(A' - \underbrace{\frac{1}{m}(A'\vec{1}) \cdot \underbrace{[1 \cdots 1]}_{j=1 \text{ to } m}}_{M}\right) \cdot (A - M')$$

$$S = A'A$$

$$(A' - M)(A - M') = \underbrace{A'A}_{B} - A'M' - MA + MM'$$

$$A_d = \begin{bmatrix} y & x_1 & x_2 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix} - \begin{bmatrix} 1 \\ \vdots \\ \vdots \end{bmatrix}\begin{bmatrix} \Sigma y & \Sigma x_1 & \Sigma x_2 \end{bmatrix} \frac{1}{m}$$

A

unit column vector $m \times 1$

row vector of the means

$$\boxed{A_d = A - (\vec{1}\,\vec{a}')\frac{1}{m}}$$

from Carroll & Green

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$$
\begin{bmatrix}
\frac{\Sigma y^2}{\sqrt{\Sigma y^2}\sqrt{\Sigma y^2}} & \frac{\Sigma y x_1}{\sqrt{\Sigma y^2}\sqrt{\Sigma x_1^2}} & \frac{\Sigma y x_2}{\sqrt{\Sigma y^2}\sqrt{\Sigma x_2^2}} \\[2ex]
\frac{\Sigma y x_1}{\sqrt{\Sigma y^2}\sqrt{\Sigma x_1^2}} & \frac{\Sigma x_1^2}{\sqrt{\Sigma x_1^2}\sqrt{\Sigma x_1^2}} & \frac{\Sigma x_1 x_2}{\sqrt{\Sigma x_1^2}\sqrt{\Sigma x_2^2}} \\[2ex]
\frac{\Sigma y x_2}{\sqrt{\Sigma y^2}\sqrt{\Sigma x_2^2}} & \frac{\Sigma x_1 x_2}{\sqrt{\Sigma x_1^2}\sqrt{\Sigma x_2^2}} & \frac{\Sigma x_2^2}{\sqrt{\Sigma x_2^2}\sqrt{\Sigma x_2^2}}
\end{bmatrix} = R
$$

Statistical matrices : Summary

Rori 21/09/17

→ RAW SUMS-OF-SQUARES AND CROSS-PRODUCTS MATRIX (B):

$$\boxed{B = A'A}$$

→ MEAN-CORRECTED SSCP MATRIX (S):

$$\boxed{S = A_d'A_d}$$

A_d - matrix of mean-centred scores

→ COVARIANCE MATRIX (C):

$$\boxed{C = \frac{1}{m} A_d'A_d}$$

$$\boxed{\begin{aligned} A_d &= A - 1\bar{a}' \\ \bar{a}' &= 1'A/m \end{aligned}}$$

→ CORRELATION MATRIX (R):

$$\boxed{R = \frac{1}{m} A_s'A_s}$$

$$\boxed{A_s = A_d \Delta}$$

A_s - matrix of standardized scores, >