

The Gram-Schmidt process

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Example:

ORTHOGONALIZE: $a_1' = (2, 1, 2)$ $a_2' = (3, -1, 5)$ $a_3' = (0, 1, -1)$

$\rightarrow (a_1')$ or first reference vector (arbitrarily chosen)

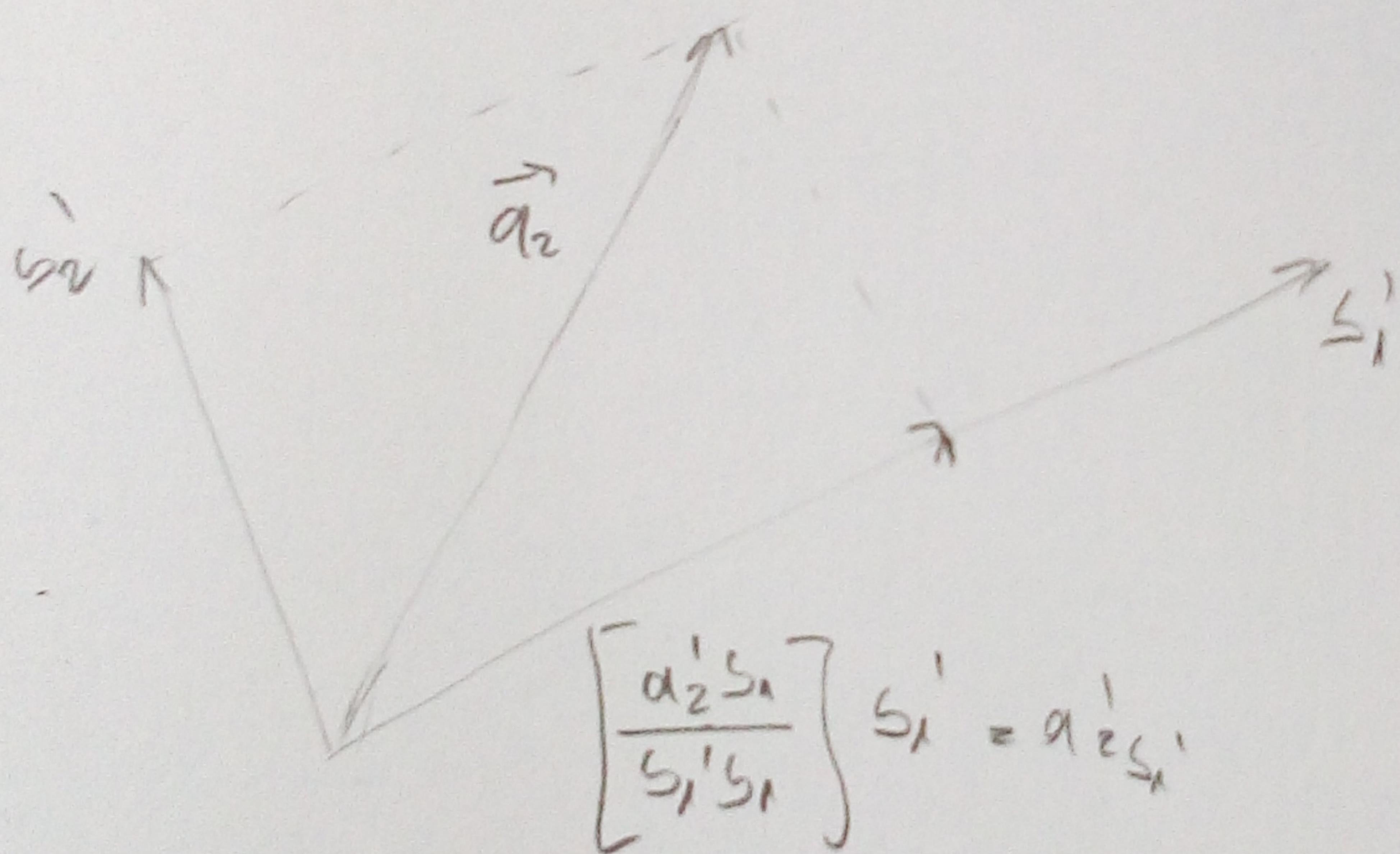
$$\boxed{s_1' = a_1'}$$

\rightarrow

$$s_2' = a_2' - \left[\frac{a_2' \cdot s_1'}{s_1' \cdot s_1'} \right] s_1'$$

Orthogonal projection
of a_2' onto s_1'

$$s_2' = (-0.33, -2.67, 1.67)$$



$\rightarrow s_3' = a_3' - \left[\frac{a_3' \cdot s_2'}{s_2' \cdot s_2'} \right] s_2' - \left[\frac{a_3' \cdot s_1'}{s_1' \cdot s_1'} \right] s_1'$

Orthogonal proj.
of a_3' onto s_2'

Orthogonal proj.
of a_3' onto s_1'

$$s_3' = (0.08, -0.04, -0.06)$$

\rightarrow for r vectors we have

$$\boxed{s_r' = a_r' - \left[\frac{a_r' \cdot s_{r-1}'}{s_{r-1}' \cdot s_{r-1}'} \right] s_{r-1}' - \dots - \left[\frac{a_r' \cdot s_1'}{s_1' \cdot s_1'} \right] s_1'}$$

Grau-Schmidt process

2) NORMALIZE:

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25/09/17

→ divide all the new basis vectors by their norm:

$$s_1' = (2, 1, 2) \rightarrow \|s_1'\| = 3$$

$$b_1^{(x)} = \frac{1}{3} (2, 1, 2) = (0.67, 0.33, 0.67)$$

$$s_2' = (-0.33, -2.67, 1.67) \rightarrow \|s_2'\| = 3.17$$

$$b_2^{(x)} = \frac{1}{3.17} (-0.33, -2.67, 1.67) = (-0.10, -0.84, 0.53)$$

$$s_3' = (0.08, -0.04, -0.06) \rightarrow \|s_3'\| = 0.108$$

$$b_3^{(x)} = \frac{1}{0.108} (0.08, -0.04, -0.06) = (0.74, -0.32, -0.56)$$

Orthogonal transformation of vectors (solution)

Page
25/29

- The analogous matrix is a square matrix that satisfies the property:

$$(A^T A = AA^T = I)$$

i.e. any two column vectors or any two row vectors
in the matrix A are mutually orthogonal, and,
furthermore, each vector is of unit length.

Axis and point rotations

- we can rigidly rotate the axes for their original orientation, while leaving the point fixed.

OR

- we can leave the original axes fixed at rigidly rotate the vector to a new location.

Rotation (of axes):

1) cosine of angle between adjacent legs

- hypotenuse (opposite)
- adjacent leg (adj. adj.)
- Opposite by (adj. opp.)

θ_{ij} = angle betw. pairs of axes

i.e. betw. original axis i and the new axis j

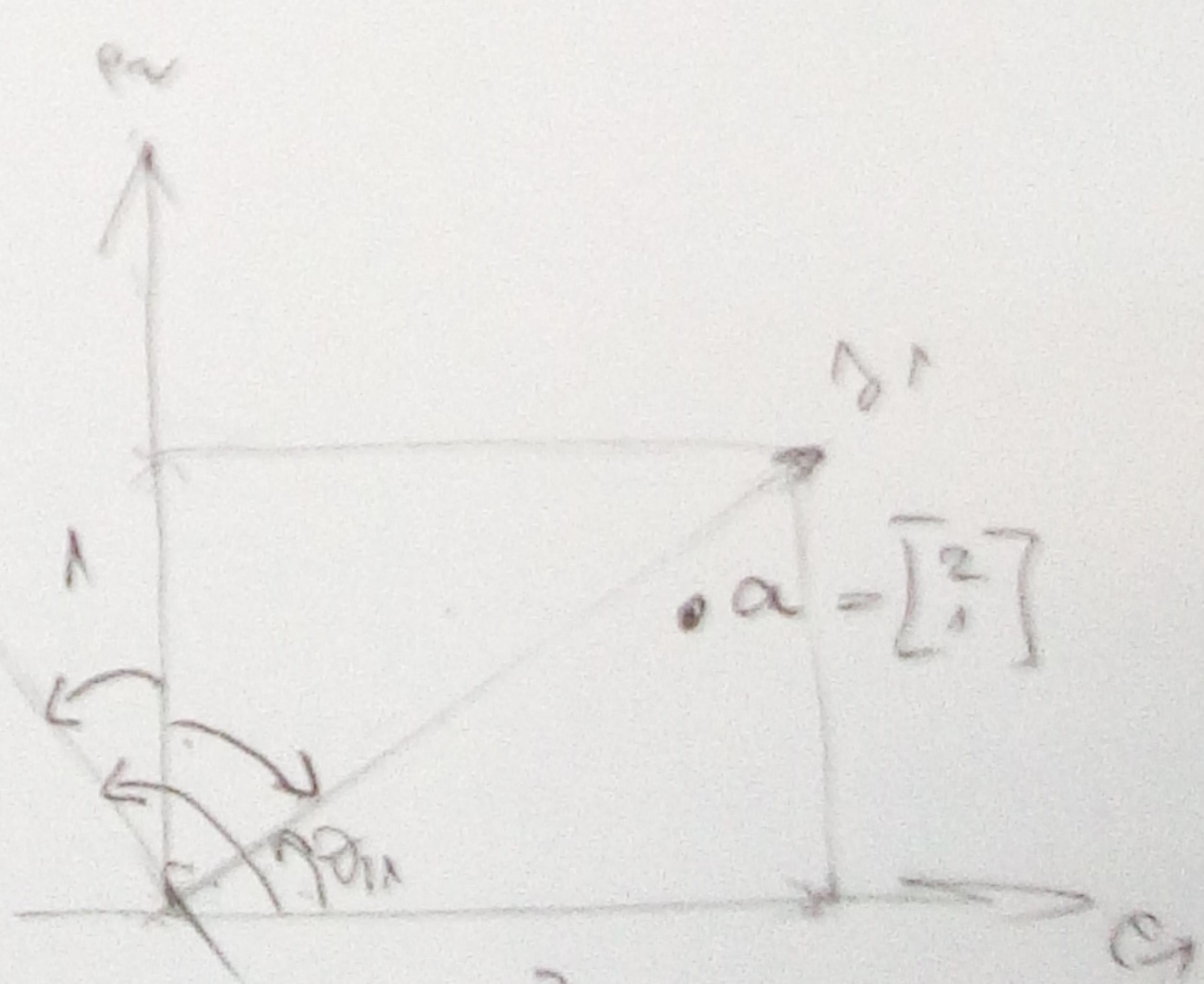
Example

$$\theta_{11} \approx 30^\circ = 0.862$$

$$\theta_{12} \approx 120^\circ = -0.5$$

$$\theta_{21} \approx 60^\circ = 0.5$$

$$\theta_{22} \approx 30^\circ = 0.862$$



$$J^1 = \begin{bmatrix} \cos \theta_{11} \cdot e_1 \\ \cos \theta_{21} \cdot e_2 \end{bmatrix} = \begin{bmatrix} 0.862 \\ 0.5 \end{bmatrix}$$

$$J^2 = \begin{bmatrix} \cos \theta_{12} \cdot e_1 \\ \cos \theta_{22} \cdot e_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.862 \end{bmatrix}$$

we have expressed J₁ and J₂ in terms of e₁ and e₂ ①