The Gram-Schmidt process

25/09/17
Roi

Example:

1) ORTHOGONALIZE: $a_1' = (2, 1, 2)$ $\quad a_2' = (3, -1, 5)$ $\quad a_3' = (0, 1, -1)$

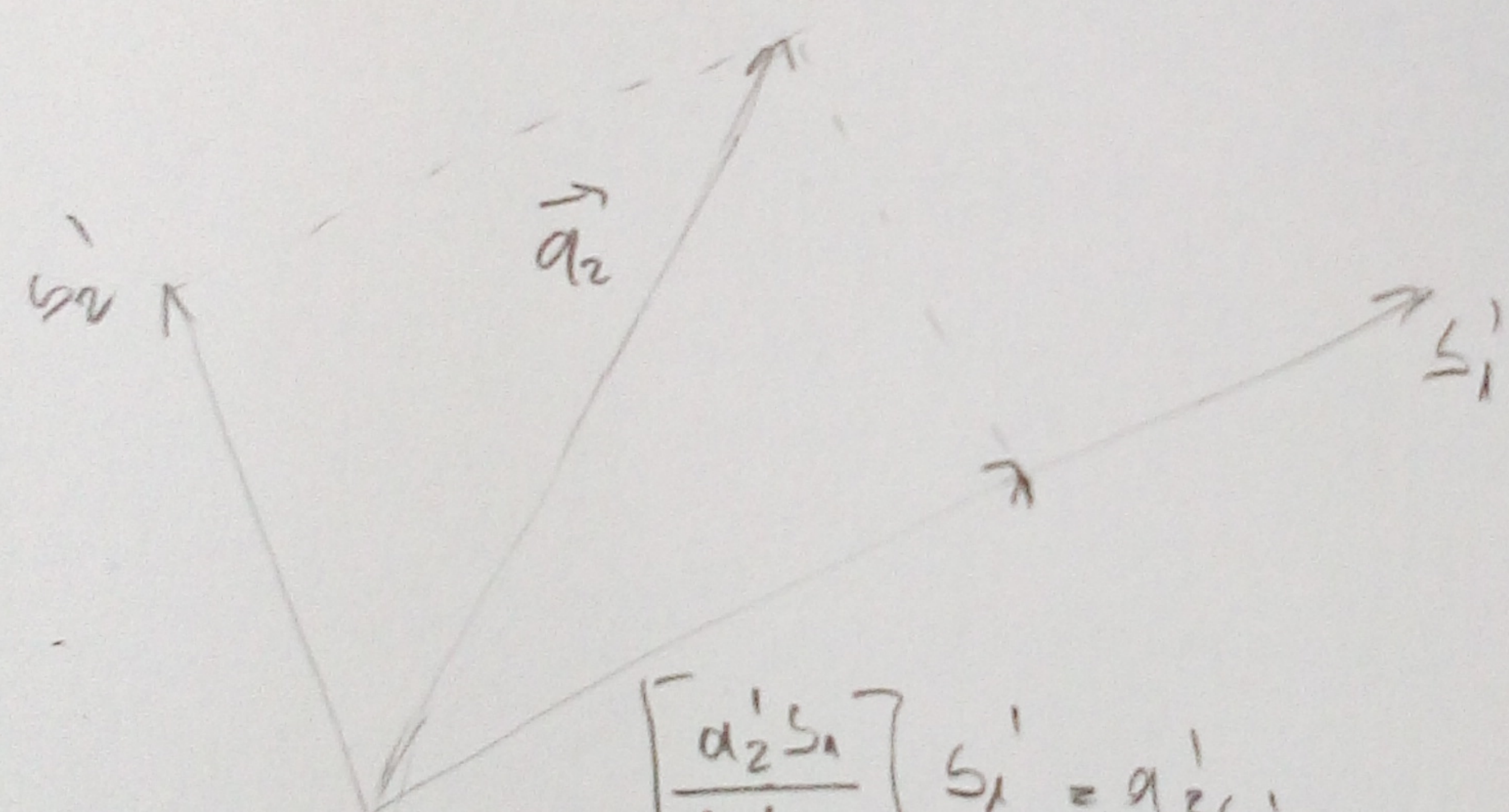
→ a_1' is first reference vector (arbitrarily chosen)

$$\boxed{s_1' = a_1'}$$

→ $$s_2' = a_2' - \left[\frac{a_2' s_1}{s_1' s_1}\right] s_1'$$

orthogonal projection of a_2' onto s_1'

$s_2' = (-0.33, -2.67, 1.67)$



$$\left[\frac{a_2' s_1}{s_1' s_1}\right] s_1' = a_{2s_1}'$$

$$\boxed{s_2' = a_2' - a_{2s_1}'}$$

→ $$s_3' = a_3' - \left[\frac{a_3' s_2}{s_2' s_2}\right] s_2' - \left[\frac{a_3' s_1}{s_1' s_1}\right] s_1'$$

orthogonal proj. of a_3' onto s_2'

orthogonal proj. of a_3' onto s_1'

$s_3' = (0.08, -0.04, -0.06)$

→ for r vectors we have:

$$\boxed{s_r' = a_r' - \left[\frac{a_r' s_{r-1}}{s_{r-1}' s_{r-1}}\right] s_{r-1}' - \dots - \left[\frac{a_r' s_1}{s_1' s_1}\right] s_1'}$$

Gram-Schmidt process

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2) NORMALIZE :

→ divide all the new basis vectors by their norm :

$b_1' = (2,1,2) \rightarrow \|b_1'\| = 3$

$b_1^{*'} = \frac{1}{3}(2,1,2) = (0.67, 0.33, 0.67)$

$b_2' = (-0.33, -2.67, 1.67) \rightarrow \|b_2'\| = 3.17$

$b_2^{*'} = \frac{1}{3.17}(-0.33, -2.67, 1.67) = (-0.10, -0.84, 0.53)$

$b_3' = (0.08, -0.04, -0.06) \rightarrow \|b_3'\| = 0.108$

$b_3^{*'} = \frac{1}{0.108}(0.08, -0.04, -0.06) = (0.74, -0.32, -0.56)$

Orthogonal transformation of matrices (rotations)

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• an ORTHOGONAL MATRIX is a square matrix that exhibits the property:

$$\boxed{A'A = AA' = I}$$

i.e. any two column vectors or any two row vectors in the matrix A are mutually orthogonal, and, furthermore, each vector is of unit length.

Axis and point rotation

1) we can rigidly ROTATE THE AXES from their original e_i orientation, while leaving the point fixed

$\boxed{\text{OR}}$

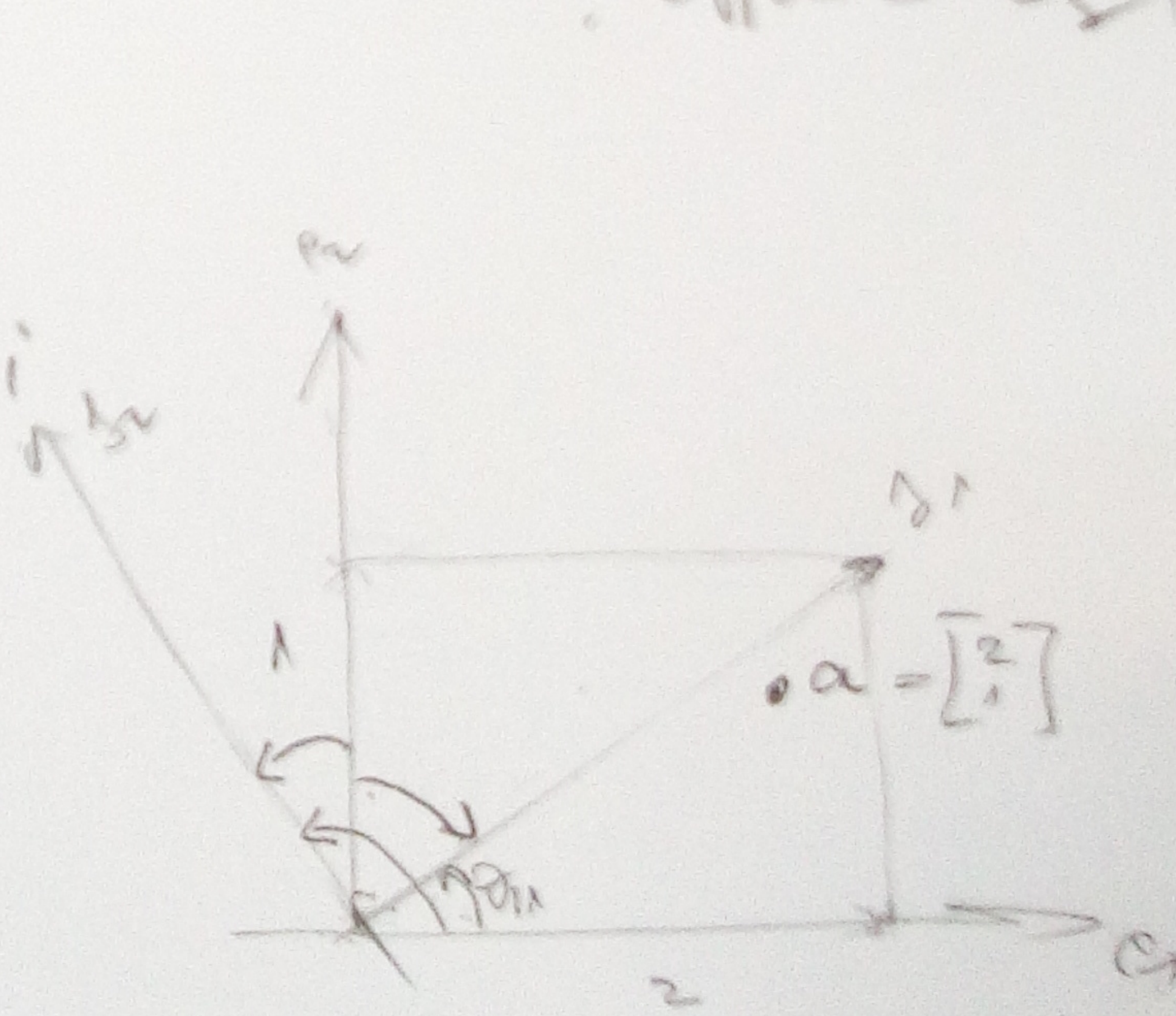
2) we can leave the original e_i axes fixed and rigidly rotate the vector to a new location

1) $\boxed{\text{ROTATION}}$ of axes:

$$\cos\theta_{ij} = \frac{\text{hypotenuse}}{\text{adjacent leg}}$$

· hypotenuse (ipotenusa)
· adjacent leg (cat. adiacente)
· opposite leg (cat. opposto)

θ_{ij} → angle betw. pairs of axes i.e. betw. original axis i and the new axis j



Examples:

$\theta_{11} = 30° = 0.862$

$\theta_{12} = 120° = -0.5$

$\theta_{21} = 60° = 0.5$

$\theta_{22} = 30° = 0.867$

$$f_1 = \begin{bmatrix} \cos\theta_{11} \cdot e_1 \\ \cos\theta_{21}\, e_2 \end{bmatrix} = \begin{bmatrix} 0.862 \\ 0.5 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} \cos\theta_{12} \cdot e_1 \\ \cos\theta_{22} \cdot e_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.867 \end{bmatrix}$$

→ we have expressed f_1 and f_2 in terms of e_1 and e_2

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→ Firstly, we express f_1 and f_2 in terms of e_1 and e_2

→ the assumed fixed point a is expressed in terms of e_1 and e_2 (i.e. the original basis vectors):

$$a = 2\begin{bmatrix}1\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$

→ Lastly, we find the coordinates of the same point, which we call now a^* in terms of the new basis vectors f_j.

$$a^* = \underbrace{2}_{a_1} \cdot \begin{bmatrix}\cos\theta_{11} e_1\\ \cos\theta_{12} e_1\end{bmatrix} + \underbrace{1}_{a_2}\begin{bmatrix}\cos\theta_{21} e_2\\ \cos\theta_{22} e_2\end{bmatrix}$$

for all points of the old system [illegible]

$$a^* = \begin{bmatrix}\cos\theta_{11} & \cos\theta_{21}\\ \cos\theta_{12} & \cos\theta_{22}\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}2\cos\theta_{11} + \cos\theta_{21}\\ 2\cos\theta_{12} + \cos\theta_{22}\end{bmatrix} = \begin{bmatrix}2.2\ldots\\ 0.3\ldots\end{bmatrix}$$

- the coordinates of the point with respect to the new basis f_i!

• If we denote θ_{11} as ψ:

$$\cos\theta_{11} \to \cos\psi$$
$$\cos\theta_{21} \to \sin\psi$$
$$\cos\theta_{12} \to -\sin\psi$$
$$\cos\theta_{22} \to \cos\theta_{11} \to \cos\psi$$

$$\Rightarrow \boxed{a^* = \begin{bmatrix}\cos\psi & \sin\psi\\ -\sin\psi & \cos\psi\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix}}$$