SAT-Based Sudoku Solving

CSC 320 Project

By Jamie Kihira, Roxanne Dewing, Justin Bao and Zirui Li

August 5, 2018

Contents

1.0 Introduction

The boolean satisfiability problem (SAT), is the problem of determining whether or not given a boolean formula, there can be a truth value assigned to the variables, such that the formula evaluates to true . A boolean formula is in conjunctive normal form, (CNF) if it is expressed as conjunctions of clauses where each clause consists of one or more literals, joined by OR. Partially solved Sudoku puzzles can be translated into a list of clauses that can be interpreted by a miniSAT solver. For the basic task, our group found that the miniSat solver took, on average, 0.0025 seconds and used 14.27 MB of memory for each puzzle. However, for extended task one (hard inputs) our group found that the miniSat solver took, on average, 0.0056 seconds and used 14.40 MB of memory for each puzzle. Therefore, the hard inputs experienced a 124% increase in time and roughly a 1% increase in memory. For extended task two, the alternate minimal encoding, we found the miniSat solver took, on average, 0.107 seconds and used 14.75 MB of memory. Therefore, our alternate encoding experienced a 4180% increase in time and a 3.4% increase in memory used. For extended task three, we generalized the version of the minimal encoding for Sudoku boards of size $n \times n$.

We found upon solving one puzzle for each size of n:

When $n = 6$, the miniSat solver took 0.0 seconds and used 14 MB of memory.
When $n = 8$, the miniSat solver took 0.0156 seconds and used 14.13 MB of memory.
When $n = 9$, the miniSat solver took 0.0 seconds and used 14.14 MB of memory.
When $n = 12,$ the miniSat solver took 0.015625 seconds used 15.06 MB of memory.
When $n = 16,$ the miniSat solver took 0.078125 seconds and used 17.66 MB of memory.

2.0 Basic Task

2.1 Sat2Sut

Sud2sat reads a Sudoku puzzle (in some specified text format) and converts it to a CNF formula suitable for input to the miniSAT SAT solver (described below.) For the basic task, you only need to consider the “minimal” encoding of puzzles as CNF formulas (described in class)

2.2 Sud2Sat

Sat2sud reads the output produced by miniSAT for a given puzzle instance and converts it back into a solved Sudoku puzzle (suitable for printing)

2.3 Results

2.3.1 CPU Results

Average Time for all puzzles: 0.0025
Median for all puzzles: 0.0
Mode for all puzzles: 0

2.3.1 Memory Used

Average Memory: 14.265399999999985 MB
Mode for Memory: 14.28 MB
Median for Memory: 14.28 MB

Appendix A contains all of the 50 solved Sudoku puzzles.
Appendix B contains each problem summarized as follows:

```
Grid #
============================[ Problem Statistics ]=============================
|  Number of variables:         x                                |
|  Number of clauses:           x                                |
|  Parse time:                  x                                |
|  Simplification time:         x                          |
restarts          : x
conflicts         : x             (-nan /sec)
decisions                 : x             (0.00 % random) (inf /sec)
propagations              : x             (inf /sec)
conflict literals         : x             (-nan % deleted)
Memory used               : x MB
CPU time                  : x s

[SATISFIABLE] or [UNSATISFIABLE]
[pass] or [fail]
```

*Note: pass or fail is indicated by our own verification program, that tests whether or not each solution is a valid Sudoku solution.

3. 0 Extended Task 1

Appendix C contains the list of hard puzzles and their corresponding solutions

3.1 CPU Results

Average Time for all puzzles: 0.005756578947368421
Median for all puzzles: 0.0
Mode for all puzzles: 0.0

3.2 Memory Used

Average Memory: 14.403894736842112 MB
Mode for Memory: 14.28 MB
Median for Memory: 14.43 MB

4. 0 Extended Task 2

4.1 Alternate Minimal Encoding

Our alternate to the minimal encoding is as follows:

Every cell has at most one number

$$\bigwedge_{i=1}^{9} \bigwedge_{j=1}^{9} \bigwedge_{k=1}^{8} \bigwedge_{l=k+1}^{9} (-S_{ijk} \vee -S_{ilk})$$

Every number is in every row

$$\bigwedge_{k=1}^{9} \bigwedge_{i=1}^{9} \bigvee_{j=1}^{9} (S_{ijk})$$

Every number is in every column

$$\bigwedge_{k=1}^{9} \bigwedge_{j=1}^{9} \bigvee_{i=1}^{9} (S_{ijk})$$

Every number is in every subgrid

$$\bigwedge_{k=1}^{9} \bigwedge_{u=0}^{2} \bigwedge_{v=0}^{2} \bigvee_{i=3*u+1}^{3*u+3} \bigvee_{j=3*v+1}^{3*v+3} (S_{ijk})$$

Appendix A contains all of the solutions to this alternate minimal encoding (they are the same solutions as what the basic task produced).

4.2 CPU Results

Average Time for all puzzles: 0.1071875
Median for all puzzles: 0.0625
Mode for all puzzles: 0.03125

4.3 Memory Used

Average Memory: 14.753800000000002 MB
Median for Memory: 14.684999999999999 MB
Mode: There were 5 equally occurring modes (occured 3 times each)
- 14.27 MB
- 14.47 MB
- 14.55 MB
- 14.62 MB
- 14.70 MB

5.0 Extended Task 3

5.1 Generalized version of minimal encoding for size $n \times n$.

For our third extended task, we generalized the Sudoku solving problem (9×9) to solving n by n Sudoku puzzles. We generalized the minimal encoding for 9 by 9 sudoku puzzles as follows:

Every cell contains at least one number

$$\bigwedge_{i=1}^{n} \bigwedge_{j=1}^{n} \bigvee_{k=1}^{n} (S_{ijk})$$

Each number appears at most once in every row

$$\bigwedge_{i=1}^{n} \bigwedge_{k=1}^{n} \bigwedge_{j=1}^{n-1} \bigwedge_{l=j+1}^{n} (-S_{ijk} \vee -S_{ilk})$$

Each number appears at most once in every column

$$\bigwedge_{j=1}^{n} \bigwedge_{k=1}^{n} \bigwedge_{i=1}^{n-1} \bigwedge_{l=i+1}^{n} (-S_{ijk} \vee -S_{ljk})$$

Each number appears at most once in every r by c subgrid

$$\bigwedge_{k=1}^{n} \bigwedge_{a=1}^{c-1} \bigwedge_{b=1}^{r-1} \bigwedge_{u=1}^{r} \bigwedge_{v=1}^{c-1} \bigwedge_{w=v+1}^{c} (-S_{(r*a+u)(c*b+v)k} \vee -S_{(r*a+u)(c*b+w)k})$$

$$\bigwedge_{k=1}^{n} \bigwedge_{a=1}^{c-1} \bigwedge_{b=1}^{r-1} \bigwedge_{u=1}^{r-1} \bigwedge_{v=1}^{r} \bigwedge_{w=u+1}^{c} \bigwedge_{t=1}^{r} (-S_{(r*a+u)(c*b+v)k} \vee -S_{(r*a+w)(c*b+t)k})$$

* Note: r is the number of rows in a subgrid and c is the number of columns in a subgrid.

5.2 Results

We tested our solution on boards of size $n \times n$, where $n = 6,\ 8,\ 9,\ 12\ and\ 16$. The puzzles and the corresponding solutions can be viewed in Appendix D.

6.0 Conclusion

The increasing time and memory for an increasing Sudoku board size is as our group expected. However, our group found that the alternate minimal encoding contained 3159 base clauses. This is less than the 8829 base clauses for the basic tasks minimal encoding. Also, we found that our general encoding for boards of size $n \times n$ has $3/2n^4 - 3/2n^3 + n^2$ base clauses. Our results for the basic task, and the extended tasks can be summarized as follows:

For the basic task, our group found that the miniSat solver took, on average, 0.0025 seconds and used 14.27 MB of memory for each puzzle. However, for extended task one (hard inputs) our group found that the miniSat solver took, on average, 0.0056 seconds and used 14.40 MB of memory for each puzzle. Therefore, the hard inputs experienced a 124% increase in time and roughly a 1% increase in memory. For extended task two, the alternate minimal encoding, we found the miniSat solver took, on average, 0.107 seconds and used 14.75 MB of memory. Therefore, our alternate encoding experienced a 4180% increase in time and a 3.4% increase in memory used. For extended task three, we generalized the version of the minimal encoding for Sudoku boards of size $n \times n$.

We found upon solving one puzzle for each size of n:

When $n = 6$, the miniSat solver took 0.0 seconds and used 14 MB of memory.
When $n = 8$, the miniSat solver took 0.0156 seconds and used 14.13 MB of memory.
When $n = 9$, the miniSat solver took 0.0 seconds and used 14.14 MB of memory.

When $n = 12$, the miniSat solver took 0.015625 seconds used 15.06 MB of memory.
When $n = 16$, the miniSat solver took 0.078125 seconds and used 17.66 MB of memory.

7.0 Calculations

*Note that for the following calculations, the base of an encoding shall refer to the clauses that must exist regardless of the puzzle given to sud2sat or variations of sud2sat.

7.1 Number of Clauses: Minimal Encoding (9×9)

Checking that each cell contains a number gives 81 different clauses since there are 81 different cells and we have 9 variables (for numbers) in each of the 81 clauses. For checking that each number appears at most once in every row, the calculation is $9 * 9 * {}_9C_2 = 2916$. 9 comes from each row, another 9 for each number, and ${}_9C_2$ comes from checking each pair of columns and making sure that both variable do not evaluate as true. Similarly, we have 2916 clauses for checking that each number appears at most once in every column and another 2916 for checking that each number appears at most once in each subgrid. This gives a total of $81 + 2916 + 2916 + 2916 = 8829$ clauses for the base of the minimal encoding.

7.2 Number of Clauses: Alternate Encoding (9×9)

Checking that each cell has at most one number entails going to each cell which there are 81 of and making sure that every pair of numbers, in total ${}_9C_2$, are not both simultaneously true. This gives $81 *_9 C_2 = 2916$ clauses. For each of checking that every number is in every row, column, or subgrid, for each combination of row/column/subgrid and number, $9 * 9 = 81$ in total, one of the 9 places must be true and is thus encoded with an OR. Hence, checking for row, columns, and subgrids each have 81 clauses. In total, the alternate encoding has $2916 + 3 * 81$ =3159 clauses in its base.

7.3 Number of Clauses: Generalized Encoding ($n \times n$)

The calculation here can be done by replacing the 9's with the minimal encoding with n's. The results is $n^2 + 3n^2{}_nC_2 = 3n^4/2 - 3n^3/2 + n^2$ clauses in the base of the generalized

minimal encoding. Notice that this has an $O(n^4)$ running time as the maximum number of singletons which are not part of the base is n^2.

8.0 Appendix A

Grid #	Unsolved	Solved
Grid 1	003020600 900305001 001806400 008102900 700000008 006708200 002609500 800203009 005010300	483921657 967345821 251876493 548132976 729564138 136798245 372689514 814253769 695417382
Grid 2	200080300 060070084 030500209 000105408 000000000 402706000 301007040 720040060 004010003	245981376 169273584 837564219 976125438 513498627 482736951 391657842 728349165 654812793
Grid 3	000000907 000420180 000705026 100904000 050000040 000507009 920108000 034059000 507000000	462831957 795426183 381795426 173984265 659312748 248567319 926178534 834259671 517643892
Grid 4	030050040 008010500 460000012 070502080 000603000 040109030 250000098 001020600	137256849 928314567 465897312 673542981 819673254 542189736 256731498 391428675

	080060020	784965123
Grid 5	020810740 700003100 090002805 009040087 400208003 160030200 302700060 005600008 076051090	523816749 784593126 691472835 239145687 457268913 168937254 342789561 915624378 876351492
Grid 6	100920000 524010000 000000070 050008102 000000000 402700090 060000000 000030945 000071006	176923584 524817639 893654271 957348162 638192457 412765398 265489713 781236945 349571826
Grid 7	043080250 600000000 000001094 900004070 000608000 010200003 820500000 000000005 034090710	143986257 679425381 285731694 962354178 357618942 418279563 821567439 796143825 534892716
Grid 8	480006902 002008001 900370060 840010200 003704100 001060049 020085007 700900600 609200018	487156932 362498751 915372864 846519273 593724186 271863549 124685397 738941625 659237418
Grid 9	000900002 050123400 030000160 908000000 070000090	814976532 659123478 732854169 948265317 275341896

	000000205 091000050 007439020 400007000	163798245 391682754 587439621 426517983
Grid 10	001900003 900700160 030005007 050000009 004302600 200000070 600100030 042007006 500006800	761928453 925743168 438615927 357461289 894372615 216589374 689154732 142837596 573296841
Grid 11	000125400 008400000 420800000 030000095 060902010 510000060 000003049 000007200 001298000	976125438 158436927 423879156 234761895 867952314 519384762 782513649 395647281 641298573
Grid 12	062340750 100005600 570000040 000094800 400000006 005830000 030000091 006400007 059083260	962341758 148975623 573268149 321694875 487512936 695837412 834726591 216459387 759183264
Grid 13	300000000 005009000 200504000 020000700 160000058 704310600 000890100 000067080 000005437	397681524 645279813 218534976 823956741 169742358 754318692 472893165 531467289 986125437
Grid 14	630000000 000500008	639218457 471539268

	005674000 000020000 003401020 000000345 000007004 080300902 947100080	825674139 564823791 793451826 218796345 352987614 186345972 947162583
Grid 15	000020040 008035000 000070602 031046970 200000000 000501203 049000730 000000010 800004000	697128345 428635197 315479682 531246978 286397451 974581263 149852736 752963814 863714529
Grid 16	361025900 080960010 400000057 008000471 000603000 259000800 740000005 020018060 005470329	361725948 587964213 492831657 638259471 174683592 259147836 746392185 923518764 815476329
Grid 17	050807020 600010090 702540006 070020301 504000908 103080070 900076205 060090003 080103040	359867124 648312597 712549836 876924351 524731968 193685472 931476285 465298713 287153649
Grid 18	080005000 000003457 000070809 060400903 007010500 408007020 901020000 842300000 000100080	786945312 219863457 534271869 165482973 327619548 498537126 951728634 842356791 673194285

Grid 19	003502900 000040000 106000305 900251008 070408030 800763001 308000104 000020000 005104800	743512986 589346217 126987345 934251768 671498532 852763491 398675124 417829653 265134879
Grid 20	000000000 009805100 051907420 290401065 000000000 140508093 026709580 005103600 000000000	782614359 439825176 651937428 293471865 568392714 147568293 326749581 975183642 814256937
Grid 21	020030090 000907000 900208005 004806500 607000208 003102900 800605007 000309000 030020050	428531796 365947182 971268435 214896573 697453218 583172964 849615327 752389641 136724859
Grid 22	005000006 070009020 000500107 804150000 000803000 000092805 907006000 030400010 200000600	425781936 178369524 369524187 894157362 652843791 713692845 987216453 536478219 241935678
Grid 23	040000050 001943600 009000300 600050002 103000506 800020007 005000200	348267951 571943628 269185374 697351482 123874596 854629137 415798263

	002436700 030000040	982436715 736512849
Grid 24	004000000 000030002 390700080 400009001 209801307 600200008 010008053 900040000 000000800	124986735 867435912 395712684 478359261 259861347 631274598 712698453 983547126 546123879
Grid 25	360020089 000361000 000000000 803000602 400603007 607000108 000000000 000418000 970030014	361524789 789361425 524879361 893157642 412683597 657942138 148796253 235418976 976235814
Grid 26	500400060 009000800 640020000 000001008 208000501 700500000 000090084 003000600 060003002	581479263 329156847 647328159 956731428 238964571 714582936 172695384 893247615 465813792
Grid 27	007256400 400000005 010030060 000508000 008060200 000107000 030070090 200000004 006312700	387256419 469781325 512439867 123548976 758963241 694127583 835674192 271895634 946312758
Grid 28	000000000 079050180 800000007 007306800	345871269 279653184 861429537 197346852

	450708096 003502700 700000005 016030420 000000000	452718396 683592741 738264915 516937428 924185673
Grid 29	030000080 009000500 007509200 700105008 020090030 900402001 004207100 002000800 070000090	235761489 419328576 867549213 746135928 521896734 983472651 394287165 652913847 178654392
Grid 30	200170603 050000100 000006079 000040700 000801000 009050000 310400000 005000060 906037002	298175643 657394128 134286579 821649735 573821496 469753281 312468957 785912364 946537812
Grid 31	000000080 800701040 040020030 374000900 000030000 005000321 010060050 050802006 080000000	761543289 832791645 549628137 374215968 128936574 695487321 417369852 953872416 286154793
Grid 32	000000085 000210009 960080100 500800016 000000000 890006007 009070052 300054000 480000000	132649785 758213649 964785123 543897216 276531894 891426537 619378452 327154968 485962371
Grid 33	608070502	698173542

	050608070 002000300 500090006 040302050 800050003 005000200 010704090 409060701	354628179 172549368 531897426 946312857 827456913 765931284 213784695 489265731
Grid 34	050010040 107000602 000905000 208030501 040070020 901080406 000401000 304000709 020060010	698173542 354628179 172549368 531897426 946312857 827456913 765931284 213784695 489265731
Grid 35	053000790 009753400 100000002 090080010 000907000 080030070 500000003 007641200 061000940	453218796 629753481 178496532 796582314 314967825 285134679 542879163 937641258 861325947
Grid 36	006080300 049070250 000405000 600317004 007000800 100826009 000702000 075040190 003090600	516289347 849173256 732465918 698317524 327954861 154826739 961732485 275648193 483591672
Grid 37	005080700 700204005 320000084 060105040 008000500 070803010 450000091 600508007	945681723 781234965 326759184 269175348 138942576 574863219 457326891 612598437

	003010600	893417652
Grid 38	000900800 128006400 070800060 800430007 500000009 600079008 090004010 003600284 001007000	365942871 128756493 974813562 819435627 537268149 642179358 296384715 753691284 481527936
Grid 39	000080000 270000054 095000810 009806400 020403060 006905100 017000620 460000038 000090000	134587296 278169354 695234817 359816472 821473569 746925183 917348625 462751938 583692741
Grid 40	000602000 400050001 085010620 038206710 000000000 019407350 026040530 900020007 000809000	193672485 462358971 785914623 538296714 674135298 219487356 826741539 941523867 357869142
Grid 41	000900002 050123400 030000160 908000000 070000090 000000205 091000050 007439020 400007000	814976532 659123478 732854169 948265317 275341896 163798245 391682754 587439621 426517983
Grid 42	380000000 000400785 009020300 060090000 800302009	384567921 126439785 759821346 563798214 847312659

	000040070 001070500 495006000 000000092	912645873 231974568 495286137 678153492
Grid 43	000158000 002060800 030000040 027030510 000000000 046080790 050000080 004070100 000325000	469158372 712463859 538297641 927634518 385719426 146582793 653941287 294876135 871325964
Grid 44	010500200 900001000 002008030 500030007 008000500 600080004 040100700 000700006 003004050	316549278 987321645 452678931 594236817 238417569 671985324 845162793 129753486 763894152
Grid 45	080000040 000469000 400000007 005904600 070608030 008502100 900000005 000781000 060000010	586127943 723469851 491853267 135974628 279618534 648532179 917246385 352781496 864395712
Grid 46	904200007 010000000 000706500 000800090 020904060 040002000 001607000 000000030 300005702	954213687 617548923 832796541 763851294 128974365 549362178 281637459 475129836 396485712
Grid 47	000700800 006000031	159743862 276589431

	040002000 024070000 010030080 000060290 000800070 860000500 002006000	348612759 624978315 917235684 583164297 435821976 861497523 792356148
Grid 48	001007090 590080001 030000080 000005800 050060020 004100000 080000030 100020079 020700400	861357294 597482361 432619785 916275843 358964127 274138956 789541632 143826579 625793418
Grid 49	000003017 015009008 060000000 100007000 009000200 000500004 000000020 500600340 340200000	294863517 715429638 863751492 152947863 479386251 638512974 986134725 521678349 347295186
Grid 50	300200000 000107000 706030500 070009080 900020004 010800050 009040301 000702000 000008006	351286497 492157638 786934512 275469183 938521764 614873259 829645371 163792845 547318926

9.0 Appendix B

Problem Statistics for each Grid

Grid 1

```
============================[ Problem Statistics ]=============================

| Number of variables:            729                                        |
| Number of clauses:              2968                                       |
| Parse time:               0.00 s                                           |
| Simplification time:            0.00 s                                     |

restarts              : 1
conflicts             : 0          (-nan /sec)
decisions             : 1          (0.00 % random) (inf /sec)
propagations          : 729                (inf /sec)
conflict literals     : 0          (-nan % deleted)
Memory used           : 14.14 MB
CPU time              : 0 s
SATISFIABLE
pass
```

Grid 2

```
============================[ Problem Statistics ]=============================
| Number of variables:            729                                        |
| Number of clauses:              3735                                       |
| Parse time:               0.00 s                                           |
| Eliminated clauses:             0.00 Mb                                    |
| Simplification time:            0.00 s                                     |

restarts              : 1
conflicts             : 4          (inf /sec)
decisions             : 9          (0.00 % random) (inf /sec)
propagations          : 699                (inf /sec)
conflict literals     : 10                 (0.00 % deleted)
Memory used           : 14.26 MB
CPU time              : 0 s

SATISFIABLE
pass
```

Grid 3

```
============================[ Problem Statistics ]=============================
```

| Number of variables: 729 |
| Number of clauses: 3843 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 10 (inf /sec)
decisions : 18 (0.00 % random) (inf /sec)
propagations : 1063 (inf /sec)
conflict literals : 47 (11.32 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 4

============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3479 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |

restarts : 1
conflicts : 2 (inf /sec)
decisions : 3 (0.00 % random) (inf /sec)
propagations : 656 (inf /sec)
conflict literals : 2 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 5
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 2662 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)

```
decisions     : 1       (0.00 % random) (inf /sec)
propagations            : 729             (inf /sec)
conflict literals  : 0       (-nan % deleted)
Memory used             : 14.14 MB
CPU time      : 0 s

SATISFIABLE
pass
```

Grid 6

```
============================[ Problem Statistics ]=============================
|  Number of variables:            729                                       |
|  Number of clauses:              4387                                      |
|  Parse time:              0.00 s                                           |
|  Eliminated clauses:             0.00 Mb                                   |
|  Simplification time:            0.00 s                                    |
restarts      : 1
conflicts     : 24              (inf /sec)
decisions     : 42              (0.00 % random) (inf /sec)
propagations            : 1348            (inf /sec)
conflict literals  : 136             (11.11 % deleted)
Memory used             : 14.28 MB
CPU time      : 0 s

SATISFIABLE
pass
```

Grid 7

```
============================[ Problem Statistics ]=============================
|  Number of variables:            729                                       |
|  Number of clauses:              3880                                      |
|  Parse time:              0.02 s                                           |
|  Eliminated clauses:             0.00 Mb                                   |
|  Simplification time:            0.00 s                                    |
restarts      : 1
conflicts     : 5       (160 /sec)
decisions     : 14              (0.00 % random) (448 /sec)
propagations            : 719             (23008 /sec)
conflict literals  : 21              (19.23 % deleted)
Memory used             : 14.28 MB
CPU time      : 0.03125 s

SATISFIABLE
pass
```

Grid 8

```
============================[ Problem Statistics ]=============================
| Number of variables:          729                                          |
| Number of clauses:            3028                                         |
| Parse time:           0.00 s                                               |
| Simplification time:          0.00 s                                       |
restarts        : 1
conflicts       : 0       (-nan /sec)
decisions       : 1       (0.00 % random) (inf /sec)
propagations            : 729           (inf /sec)
conflict literals  : 0       (-nan % deleted)
Memory used             : 14.13 MB
CPU time        : 0 s

SATISFIABLE
pass
```

Grid 9

```
============================[ Problem Statistics ]=============================
| Number of variables:          729                                          |
| Number of clauses:            4175                                         |
| Parse time:           0.02 s                                               |
| Eliminated clauses:           0.00 Mb                                      |
| Simplification time:          0.00 s                                       |
restarts        : 1
conflicts       : 11      (704 /sec)
decisions       : 21            (0.00 % random) (1344 /sec)
propagations            : 878           (56192 /sec)
conflict literals  : 35            (2.78 % deleted)
Memory used             : 14.28 MB
CPU time        : 0.015625 s

SATISFIABLE
pass
```

Grid 10

```
============================[ Problem Statistics ]=============================
| Number of variables:          729                                          |
| Number of clauses:            3915                                         |
| Parse time:           0.00 s                                               |
| Eliminated clauses:           0.00 Mb                                      |
| Simplification time:          0.00 s                                       |
restarts        : 1
conflicts       : 7       (inf /sec)
decisions       : 15            (0.00 % random) (inf /sec)
```

propagations : 858 (inf /sec)
conflict literals : 37 (21.28 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 11
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3931 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 5 (inf /sec)
decisions : 15 (0.00 % random) (inf /sec)
propagations : 707 (inf /sec)
conflict literals : 15 (6.25 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 12
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3324 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)
propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 13
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3847 |

| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 1 (inf /sec)
decisions : 3 (0.00 % random) (inf /sec)
propagations : 677 (inf /sec)
conflict literals : 1 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 14
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4057 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 1 (inf /sec)
decisions : 8 (0.00 % random) (inf /sec)
propagations : 584 (inf /sec)
conflict literals : 6 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 15
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3611 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.02 s |
restarts : 1
conflicts : 0 (0 /sec)
decisions : 1 (0.00 % random) (64 /sec)
propagations : 665 (42560 /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.28 MB
CPU time : 0.015625 s

SATISFIABLE
pass

Grid 16
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 2586 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)
propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.13 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 17
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 2706 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)
propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.14 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 18
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3612 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 2 (inf /sec)
decisions : 13 (0.00 % random) (inf /sec)
propagations : 687 (inf /sec)

conflict literals : 4 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 19
==========[Problem Statistics]==========
| Number of variables: 729 |
| Number of clauses: 3252 |
| Parse time: 0.00 s |
| Simplification time: 0.02 s |
restarts : 1
conflicts : 0 (0 /sec)
decisions : 1 (0.00 % random) (64 /sec)
propagations : 729 (46656 /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.27 MB
CPU time : 0.015625 s

SATISFIABLE
pass

Grid 20
==========[Problem Statistics]==========
| Number of variables: 729 |
| Number of clauses: 3273 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)
propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.29 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 21
==========[Problem Statistics]==========
| Number of variables: 729 |
| Number of clauses: 3744 |
| Parse time: 0.00 s |

| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 2 (inf /sec)
decisions : 9 (0.00 % random) (inf /sec)
propagations : 637 (inf /sec)
conflict literals : 2 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 22
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3882 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 5 (inf /sec)
decisions : 13 (0.00 % random) (inf /sec)
propagations : 714 (inf /sec)
conflict literals : 8 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 23
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3771 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 1 (inf /sec)
decisions : 2 (0.00 % random) (inf /sec)
propagations : 660 (inf /sec)
conflict literals : 1 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 24
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3928 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.02 s |
restarts : 1
conflicts : 2 (128 /sec)
decisions : 11 (0.00 % random) (704 /sec)
propagations : 684 (43776 /sec)
conflict literals : 4 (20.00 % deleted)
Memory used : 14.28 MB
CPU time : 0.015625 s

SATISFIABLE
pass

Grid 25
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3990 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 6 (inf /sec)
decisions : 18 (0.00 % random) (inf /sec)
propagations : 797 (inf /sec)
conflict literals : 11 (8.33 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 26
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4372 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 26 (inf /sec)
decisions : 40 (0.00 % random) (inf /sec)
propagations : 1440 (inf /sec)

conflict literals : 135 (11.18 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 27
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3963 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 1 (inf /sec)
decisions : 3 (0.00 % random) (inf /sec)
propagations : 635 (inf /sec)
conflict literals : 1 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 28
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3771 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 4 (inf /sec)
decisions : 9 (0.00 % random) (inf /sec)
propagations : 725 (inf /sec)
conflict literals : 5 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 29
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4179 |

| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 4 (inf /sec)
decisions : 24 (0.00 % random) (inf /sec)
propagations : 724 (inf /sec)
conflict literals : 22 (8.33 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 30
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4044 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 8 (inf /sec)
decisions : 16 (0.00 % random) (inf /sec)
propagations : 880 (inf /sec)
conflict literals : 25 (3.85 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 31
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4165 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 5 (inf /sec)
decisions : 30 (0.00 % random) (inf /sec)
propagations : 776 (inf /sec)
conflict literals : 17 (5.56 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE

pass

Grid 32
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4102 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (0 /sec)
decisions : 8 (0.00 % random) (512 /sec)
propagations : 613 (39232 /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.28 MB
CPU time : 0.015625 s

SATISFIABLE
pass

Grid 33
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3477 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 2 (inf /sec)
decisions : 5 (0.00 % random) (inf /sec)
propagations : 657 (inf /sec)
conflict literals : 3 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 34
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3356 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)

propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 35
==========[Problem Statistics]==========
| Number of variables: 729 |
| Number of clauses: 3632 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 5 (0.00 % random) (inf /sec)
propagations : 599 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 36
==========[Problem Statistics]==========
| Number of variables: 729 |
| Number of clauses: 3089 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)
propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.14 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 37
==========[Problem Statistics]==========
| Number of variables: 729 |
| Number of clauses: 3580 |

| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 1 (inf /sec)
decisions : 4 (0.00 % random) (inf /sec)
propagations : 599 (inf /sec)
conflict literals : 1 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 38
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3529 |
| Parse time: 0.00 s |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 0 (-nan /sec)
decisions : 1 (0.00 % random) (inf /sec)
propagations : 729 (inf /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.27 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 39
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3411 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.02 s |
restarts : 1
conflicts : 0 (0 /sec)
decisions : 1 (0.00 % random) (64 /sec)
propagations : 623 (39872 /sec)
conflict literals : 0 (-nan % deleted)
Memory used : 14.28 MB
CPU time : 0.015625 s

SATISFIABLE
pass

Grid 40
```
============================[ Problem Statistics ]=============================
| Number of variables:            729                                        |
| Number of clauses:              3057                                       |
| Parse time:                     0.00 s                                     |
| Simplification time:            0.00 s                                     |
restarts              : 1
conflicts             : 0          (-nan /sec)
decisions             : 1          (0.00 % random) (inf /sec)
propagations          : 729        (inf /sec)
conflict literals     : 0          (-nan % deleted)
Memory used           : 14.13 MB
CPU time              : 0 s

SATISFIABLE
```
pass

Grid 41
```
============================[ Problem Statistics ]=============================
| Number of variables:            729                                        |
| Number of clauses:              4175                                       |
| Parse time:                     0.00 s                                     |
| Eliminated clauses:             0.00 Mb                                    |
| Simplification time:            0.00 s                                     |
restarts              : 1
conflicts             : 11         (inf /sec)
decisions             : 21         (0.00 % random) (inf /sec)
propagations          : 878        (inf /sec)
conflict literals     : 35         (2.78 % deleted)
Memory used           : 14.28 MB
CPU time              : 0 s

SATISFIABLE
```
pass

Grid 42
```
============================[ Problem Statistics ]=============================
| Number of variables:            729                                        |
| Number of clauses:              4067                                       |
| Parse time:                     0.00 s                                     |
| Eliminated clauses:             0.00 Mb                                    |
| Simplification time:            0.00 s                                     |
restarts              : 1
conflicts             : 7          (inf /sec)
decisions             : 22         (0.00 % random) (inf /sec)
propagations          : 839        (inf /sec)
conflict literals     : 25         (7.41 % deleted)
```

Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 43
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 3880 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 7 (inf /sec)
decisions : 21 (0.00 % random) (inf /sec)
propagations : 896 (inf /sec)
conflict literals : 36 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 44
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4314 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 26 (inf /sec)
decisions : 40 (0.00 % random) (inf /sec)
propagations : 1729 (inf /sec)
conflict literals : 175 (4.37 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 45
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4108 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |

| Simplification time: 0.00 s |
restarts : 1
conflicts : 15 (inf /sec)
decisions : 35 (0.00 % random) (inf /sec)
propagations : 1146 (inf /sec)
conflict literals : 78 (9.30 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 46
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4362 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 9 (inf /sec)
decisions : 29 (0.00 % random) (inf /sec)
propagations : 860 (inf /sec)
conflict literals : 75 (3.85 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 47
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4507 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 2
conflicts : 117 (inf /sec)
decisions : 155 (0.00 % random) (inf /sec)
propagations : 5316 (inf /sec)
conflict literals : 820 (22.20 % deleted)
Memory used : 14.42 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 48
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4216 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 6 (inf /sec)
decisions : 20 (0.00 % random) (inf /sec)
propagations : 870 (inf /sec)
conflict literals : 47 (11.32 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 49
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4563 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 81 (inf /sec)
decisions : 104 (0.00 % random) (inf /sec)
propagations : 4712 (inf /sec)
conflict literals : 644 (29.23 % deleted)
Memory used : 14.45 MB
CPU time : 0 s

SATISFIABLE
pass

Grid 50
============================[Problem Statistics]=============================
| Number of variables: 729 |
| Number of clauses: 4046 |
| Parse time: 0.00 s |
| Eliminated clauses: 0.00 Mb |
| Simplification time: 0.00 s |
restarts : 1
conflicts : 8 (inf /sec)
decisions : 19 (0.00 % random) (inf /sec)
propagations : 768 (inf /sec)

conflict literals : 23 (0.00 % deleted)
Memory used : 14.28 MB
CPU time : 0 s

SATISFIABLE
pass

10. Appendix C

Grid #	Unsolved	Solved
Grid 1 CPU time: 0s Memory used: 14.46MB	4.....8.5.3.....7.....2.....6.....8.4... ...1.....6.3.7.5..2.....1.4.....	417369825 632158947 958724316 825437169 791586432 346912758 289643571 573291684 164875293
Grid 2 CPU time: 0s Memory used: 14.46MB	52...6.....7.13.....4..8..6.....5.. .....418.....3..2...87.....	527316489 896542731 314987562 172453896 689271354 453698217 941825673 765134928 238769145
Grid 3 CPU time: 0.015625s Memory used: 14.46MB	6.....8.3.4.7.....5.4.7.3..2..... 1.6.....2.....5.....8.6.....1....	617459823 248736915 539128467 982564371 374291586 156873294 823647159 791385642 465912738

Grid 4 CPU time: 0.015625s Memory used: 14.46MB	48.3……71.2……7.5….6….2..8.. ……1.76…3…..4……5….	487312695 593684271 126597384 735849162 914265837 268731549 851476923 379128456 642953718
Grid 5 CPU time: 0s Memory used: 14.46MB	….14….3….2…7……9…3.6.1…… …..8.2…..1.4….5.6…..7.8…	962314857 134587269 578296413 847962531 651873942 329145786 285639174 793451628 416728395
Grid 6 CPU time: 0s Memory used: 14.46MB	……52..8.4……3…9…5.1…6..2..7….. …3…..6…1…….7.4…….3.	416837529 982465371 735129468 571298643 293746185 864351297 647913852 359682714 128574936
Grid 7 CPU time: 0.015625 Memory used: 14.46MB	6.2.5……..3.4…….43…8….1….2… …..7..5..27……..81…6…..	682154379 951763842 374892165 437528916 816937254 295416738 568271493 729345681 143689527
Grid 8 CPU time: 0s Memory used:	.524……..7.1……..8.2…3…..6… 9.5…..1.6.3……..897……..	652481937 834679152 971325864 467812593 315794628 298563471

14.42MB		186937245 523146789 749258316
Grid 9 CPU time: 0s Memory used: 14.46MB	6.2.5.....4.3.....43...8....1....2... .....7..5..27.....81...6.....	682153479 951764832 374892165 437528916 816947253 295316748 568271394 729435681 143689527
Grid 10 CPU time: 0.015625s Memory used: 14.41MB	.923.....8.1.....1.7.4.....65 8.....6.5.2...4.....7.....9.....	792351648 543786129 681429537 157648293 924137865 836295471 368572914 419863752 275914386
Grid 11 CPU time: 0s Memory used: 14.68MB	6..3.2....5.....1.....7.26.....54 3.....8.15.....4.2.....7..	614382579 953764812 827591436 742635198 168279354 395418627 286157943 579843261 431926785
Grid 12 CPU time: 0s Memory used: 14.28 MB	.6.5.1.9.1...9..539....7....4.8...7..... 5.8.817.5.3.....5.2.....76..8...	863521794 127496853 954387621 645839172 739142568 281765439 498653217 512974386 376218945

Grid 13 CPU time: 0s Memory used: 14.45MB	..5...987.4..5...1..7.....2...48....9.1... ..6..2.....3..6..2.....9.7.....5..	135426987 846957321 927381465 213748659 598163742 674295813 351674298 482539176 769812534
Grid 14 CPU time: 0.015625s Memory used: 14.46MB	3.6.7.....518.....1.4.5...7.....6... ..2.....2.....4.....8.3.....5.....	356871294 972643851 841952736 213465987 794318625 685297413 128736549 569184372 437529168
Grid 15 CPU time: 0s Memory used: 14.45MB	1.....3.8.7.4.....2.3.1.....95 8.....5.6...7.....8.2...4.....	129576348 376428519 584391627 293815764 417263895 865749132 958632471 731984256 642157983
Grid 16 CPU time: 0.015625s Memory used: 14.68MB	6..3.2....4.....1.....7.26.....54 3.....8.15.....4.2.....7..	615382479 943765812 827491536 752634198 168279354 394518627 286157943 579843261 431926785

Grid 17 CPU time: 0s Memory used: 14.41MB	....3..9....2....1.5.9.....1.2.8.4.6 .8.5...2..75.....4.1..6..3.....4.6.	718435692 963278541 254961378 547612839 192387456 386549127 675893214 421756983 839124765
Grid 18 CPU time: 0s Memory used: 14.41MB	45.....3....8.1....9.....5..9.2..7..... 8.....1..4.....7.2...6..8..	458276931 623891475 197534286 371452698 269783154 845169327 712948563 986315742 534627819
Grid 19 CPU time: 0s Memory used: 14.44MB	.237....68...6.59.9.....7......4.97.3.7. 96..2.....5..47.....2....8.....	123759486 874261593 965384721 216543978 357896142 498127365 532478619 641932857 789615234
Grid 20 CPU time: 0s Memory used: 14.28MB	..84...3....3.....9....157479...8.....7. .514.....2...9.6...2.5....4.....9..56	518476239 427359618 963821574 795248361 832617945 146935827 379564182 651782493 284193756

Grid 21 CPU time: 0s Memory used: 14.46MB	.98.1....2.....6.....3.2.5..84..... ...6.....4.8.93..5.....1..	498716523 257839461 136425987 971382654 684157392 523694718 765241839 319578246 842963175
Grid 22 CPU time: 0s Memory used: 14.28MB	..247..58.....1.4.....2...9528.9. 4....9...1.....3.3....75..685..2...	132479658 847563291 956281347 413725869 528196473 769348125 271854936 394617582 685932714
Grid 23 CPU time: 0s Memory used: 14.46MB	4.....8.5.3.....7.....2.....6.....5.4.... ..1.....6.3.7.5..2.....1.9.....	417369825 638125947 952748316 825437169 791856432 346912758 284693571 573281694 169574283
Grid 24 CPU time: 0.015625s Memory used: 14.28MB	.2.3.....63.....58.....15....9.3....7..... ...1....8.879..26.....6.7...6..7..4	925371486 163498725 874562391 542689137 618753942 739124658 487915263 351246879 296837514

Grid 25 CPU time: 0s Memory used: 14.41MB	1.....7.9.4...72..8.....7..1..6.3.....5 .6..4..2.....8..53...7.7.2....46	123456789 649837251 857291634 274518963 398672415 561943827 416725398 985364172 732189546
Grid 26 CPU time: 0s Memory used: 14.45MB	4.....3.....8.2.....7.....1...8734..... 6.....5...6.....1.4...82.....	475691328 961832745 823754196 259143687 347586219 618927534 534269871 796318452 182475963
Grid 27 CPU time: 0.015625s Memory used: 14.46MB	.....71.2.8.....4.3...7...6..5....2..3.. 9.....6...7.....8....4.....5....	349526871 521897643 876413529 718369254 465281397 932745186 654178932 187932465 293654718
Grid 28 CPU time: 0.015625s Memory used: 14.45MB	6..3.2....4.....8.....7.26.....54 3.....8.15.....8.2.....7..	618342579 943765182 527891436 752634891 861279354 394518627 286157943 179483265 435926718

Grid 29 CPU time: 0s Memory used: 14.43MB	.47.8...1.....6..7..6....357.....5... .1..6....28..4.....9.1...4.....2.69.	947582361 863471952 152639784 624813579 738295416 519764823 285946137 396157248 471328695
Grid 30 CPU time: 0.015625s Memory used: 14.45MB	......8.17..2.....5.6.....7...5..1....3... 8.....5.....2..4..8....6...3....	254379861 761248593 893516742 326791458 915824376 487653219 538167924 142985637 679432185
Grid 31 CPU time: 0.15625s Memory used: 14.42MB	38.6.....9.....2..3.51.....5....3..1..6 ....4.....17.5..8.....9.....7.32	385621497 179584326 426739518 762395841 534812769 891476253 917253684 243168975 658947132
Grid 32 CPU time: 0.015625s Memory used: 14.28MB	...5.....5.697.....2...48.2...25.1... 3..8..3.....4.7..13.5..9..2...31..	836521947 142379586 975648321 364892715 259167438 781435269 598214673 413756892 627983154

Grid 33 CPU time: 0s Memory used: 14.28MB	.2.....3.5.62..9.68...3...5.....64. 8.2..47..9....3.....1.....6...17.43....	427593186 315862479 968174325 659328714 731649852 284751963 593287641 842916537 176435298
Grid 34 CPU time: 0s Memory used: 14.46MB	.8..4....3.....1.....2...5...4.69..1..8.. 2.....3.9....6....5.....2.....	781942365 324576918 659831724 815723496 936154872 247698153 578369241 162487539 493215687
Grid 35 CPU time: 0s Memory used: 14.44MB	..8.9.1...6.5...2.....6....3.1.7.5..... 9..4...3...5....2...7...3.8.2..7....4	748392165 369514728 125876943 932147856 687235419 514689372 853461297 476923581 291758634
Grid 36 CPU time: Os Memory used: 14.45MB	4.....5.8.3.....7.....2.....6.....5.8... ...1.....6.3.7.5..2.....1.8.....	417369528 839125746 652748319 925837461 741956832 386412957 294683175 573291684 168574293

Grid 37 CPU time: Os Memory used: 14.45MB	1.....3.8.6.4.....2.3.1.....958.....5.6...7.....8.2...4.....	124597368 369428517 587361924 293815746 416273895 875946132 958632471 631784259 742159683
Grid 38 CPU time: 0s Memory used: 14.41MB	1....6.8..64.....4...7....9.6...7.4..5...5...7.1...5....32.3....8...4.....	137926485 964587231 825341967 241895673 673412598 589673142 758164329 396258714 412739856
Grid 39 CPU time: 0.015625 Memory used: 14.44MB	249.6...3.3....2..8.....5.....6.....2.....1..4.82..9.5..7....4.....1.7...3...	249865173 531974268 867132495 423786519 986251347 715349826 692518734 354627981 178493652
Grid 40 CPU time: 0s Memory used: 14.41MB	...8....9.873...4.6..7.....85..97.....43..75.....3....3...145.4....2..1	351846729 287319645 694725183 168534972 725198364 943267518 516483297 832971456 479652831

Grid 41 CPU time: Os Memory used: 14.46MB	...5.1....9....8...6.....4.1.....7..9... .....3.8.....1.5...2..4.....36....	748591326 195623847 263487519 421936758 356874291 987152634 832749165 679215483 514368972
Grid 42 CPU time: 0s Memory used: 14.46MB	......8.16..2.....7.5.....6...2..1....3... 8.....2.....7..3..8....5...4....	723469851 651238794 894715632 375691428 912874365 486523917 248356179 137982546 569147283
Grid 43 CPU time: 0s Memory used: 14.28MB	.476...5.8.3.....2.....9.....8.5..6...1.... .6.24.....78...51...6....4..9...4..7	947628351 863751492 125349678 734895126 589162734 612473985 478236519 256917843 391584267
Grid 44 CPU time: 0s Memory used: 14.41MB	.....7.95.....1...86..2.....2..73..85..... 6...3..49..3.5...41724.....	132467895 957381246 864529731 429673158 578912364 613854972 385296417 241735689 796148523

Grid 45 CPU time: 0s Memory used: 14.28MB	.4.5.....8...9..3..76.2.....146.....9.. 7.....36....1..4.5..6.....3..71..2..	143587962 852496731 976321584 214675398 635819427 789243615 321764859 468952173 597138246
Grid 46 CPU time: 0.015625s Memory used: 14.45MB	.834.....7..5.....4.1.8.....27. ..3.....2.6.5....5.....8.....1..	783465219 421973658 965281734 347128596 198546327 652397481 216854973 534719862 879632145
Grid 47 CPU time: 0s Memory used: 14.45MB	..9.....3.....9...7.....5.6..65..4.....3..... 28.....3..75.6..6.....12.3.8	219675843 865439721 743281596 936512487 157348962 428967135 382754619 671893254 594126378
Grid 48 CPU time: Os Memory used: 14.43MB	.26.39.....6....19.....7.....4..9.5....2. ...85.....3..2..9..4....762.....4	126739845 847625391 935481762 213864579 654973218 798512436 361248957 489157623 572396184

Grid 49 CPU time: 0.015625s Memory used: 14.45MB	2.3.8....8..7.....1...6.5.7...4.....3. ...1.....82.5....6...1.....	273681495 891754263 546392178 169537824 485269731 327148956 734916582 958423617 612875349
Grid 50 CPU time: 0s Memory used: 14.45MB	6..3.2....1.....5.....7.26.....84 3.....8.15.....8.2.....7..	654312879 913876452 827495136 742638591 165729384 398541627 286157943 471983265 539264718
Grid 51 CPU time: 0s Memory used: 14.41MB	1.....9...64..1.7..7..4.....3.....3.89..5 ....7....2.....6.7.9.....4.1....129.3.	152738946 864291375 973645281 216357498 348912567 597486123 421863759 639574812 785129634
Grid 52 CPU time: 0s Memory used: 14.44MB	.....9.....84.623...5....6...453...1... 6...9...7....1.....4.5..2....3.8....9	174589362 953261784 862347951 219673845 387415296 546928173 628194537 495732618 731856429

Grid 53 CPU time: 0s Memory used: 14.28MB	.2....5938..5..46.94..6...8..2.3.....6.. 8.73.7..2.....4.38..7....6.....5	126478593 837592461 945361278 412937856 569184732 783256914 251649387 374815629 698723145
Grid 54 CPU time: 0s Memory used: 14.28MB	9.4..5...25.6..1..31.....8.7...9...4..26 .....147....7.....2...3..8.6.4.....9.	964815237 258637149 317924658 872159364 495263781 631478925 783596412 529341876 146782593
Grid 55 CPU time: 0s Memory used: 14.40MB	...52.....9...3..4.....7...1.....4..8..453. .6...1...87.2.....8....32.4..8..1.	476529183 895173624 321864795 517398246 289645371 634712958 752431869 168957432 943286517
Grid 56 CPU time: 0s Memory used: 14.28MB	53..2.9...24.3..5...9.....1.827...7.. .....981.....64....91.2.5.43.	538127946 624839751 719645382 965314827 381762594 247598163 493281675 856473219 172956438

Grid 57 CPU time: 0.015625s Memory used: 14.41MB	1....786...7..8.1.8..2....9.....24...1.. ....9..5...6.8.....5.9.....93.4	124597863 937648215 856231749 513786492 482913657 769425138 698374521 341852976 275169384
Grid 58 CPU time: 0.015625s Memory used: 14.43MB	....5...11.....7..6.....8.....4.....9.1.3.... .596.2..8..62..7..7.....3.5.7.2..	872459631 154683972 963721485 216834759 549217368 738596124 481362597 627945813 395178246
Grid 59 CPU time: 0s Memory used: 14.28MB	.47.2....8....1....3....9.2.....5...6..81.. 5.....4.....7....3.4...9...1.4..27.8..	947326581 852491673 136587942 284735169 693812457 715649238 579168324 328954716 461273895
Grid 60 CPU time: 0.015625s Memory used: 14.43MB	.....94.....9...53....5.7..8.4..1..463.... .....7.8.8..7.....7.....28.5.26....	215876943 678394215 349125876 587432169 463981752 192657384 826743591 734519628 951268437

Grid 61 CPU time: 0.03125s Memory used: 14.41MB	.2……6….41…..78….1……7….37…..6 ..412….1..74..5..8.5..7……39..	124397856 835641297 967825341 241538769 583769412 679412538 312974685 498256173 756183924
Grid 62 CPU time: 0.015625s Memory used: 14.46MB	1…..3.8.6.4……2.3.1……758 ……7.5…6…..8.2…4…….	125976348 369428517 784351926 253817694 416293875 897645132 978532461 631784259 542169783
Grid 63 CPU time: 0.015625s Memory used: 14.42MB	2….1.9..1..3.7..9..8…2…….85..6.4… ……7…3.2.3…6….5…..1.9…2.5	283741596 615239748 974865321 397126854 861453972 452978613 528394167 736512489 149687235
Grid 64 CPU time: 0s Memory used: 14.45MB	..7..8…..6.2.3…3……9.1..5..6…..1…. .7.9….2……..4.83..4…26….51.	957638421 146729385 832541679 419352768 628417953 375986142 791265834 583174296 264893517

Grid 65 CPU time: 0s Memory used: 14.41MB	...36....85.....9.4..8.....68.....1 7..9..45...1.5...6.4....9..2.....3...	127365489 853491276 964278351 231756894 548932617 679184523 312547968 485619732 796823145
Grid 66 CPU time: 0s Memory used: 14.45MB	34.6.....7.....2..8.57.....5....7..1..2 ....4.....36.2..1.....9.....7.82	345671298 987253146 621984573 264795831 573816429 198432657 836529714 712348965 459167382
Grid 67 CPU time 0.015625s Memory used: 14.45MB	.....4.18..2.....6.7.....8...6..4....3... 1.....6.....2..5..1....7...3....	265389471 874251693 193647852 327894165 946125387 518763249 631578924 452916738 789432516
Grid 68 CPU time: 0s Memory used: 14.46MB	.4..5..67...1...4....2.....1..8..3.....2.. .6.....4..5.3.....8..2.....	842359167 573186942 619274538 127865394 435791286 968423715 781942653 354617829 296538471

Grid 69 CPU time: 0s Memory used: 14.41MB	.....4...2..4..1.7..5..9...3..7....4..6.... 6..1..8...2....1..85.9...6.....8...3	538219746 962874531 174356298 283497615 741568329 695123874 329645187 857931462 416782953
Grid 70 CPU time: 0s Memory used: 14.43MB	8..7....4.5....6.....3.97...8....43.. 5....2.9....6.....2...6...7.71..83.2	863751294 957432681 124689573 532976148 619843725 748125936 386217459 295364817 471598362
Grid 71 CPU time: 0.03125s Memory used: 14.46MB	.8...4.5....7..3.....1..85...6.....2... ...4....3.26.....417.....	986324157 124759368 537861429 413285976 695173284 278946513 342617895 869532741 751498632
Grid 72 CPU time: 0.03125s Memory used: 14.28MB	....7..8...6...5...2...3.61.1...7..2..8..5 34.2..9.....2.....58...6.3.4...1....	945671283 136482597 827593461 614837952 798125346 253964178 362759814 581246739 479318625

Grid 73 CPU time: 0s Memory used: 14.46MB	……8.16..2……..7.5……6…2..1….3… 8…….2……7..4..8….5…3….	724369851 651248793 893715642 375691428 912874365 486523917 238456179 147982536 569137284
Grid 74 CPU time: 0s Memory used: 14.46MB	.2……….6….3.74.8……..3..2.8..4..1. 6..5……..1.78.5….9……….4.	126437958 895621473 374985126 457193862 983246517 612578394 269314785 548769231 731852649
Grid 75 CPU time: 0.03125s Memory used: 14.28MB	.52..68…….7.2…….6….48..9..2..41. …..1…..8..61..38…..9…63..6..1.9	152946837 963587421 847231695 574863912 289415763 631729548 796152384 415398276 328674159
Grid 76 CPU time: 0s Memory used: 14.46MB	….1.78.5….9……….4..2……….6….3. 74.8……..3..2.8..4..1.6..5…..	269314785 548769231 731852649 126437958 895621473 374985126 457193862 983246517 612578394

Grid 77 CPU time: 0s Memory used: 14.42MB	1.....3.6.3..7...7...5..121.7...9...7..... ...8.1..2....8.64....9.2..6....4.....	152678943 864391752 973245681 215763894 497582136 638914527 321856479 549127368 786439215
Grid 78 CPU time: 0s Memory used: 14.28MB	4...7.1....19.46.5.....1.....7....2..2.3... .847..6....14...8.6.2....3..6...9....	496573128 381924675 275861943 153789462 962435781 847216539 714352896 529648317 638197254
Grid 79 CPU time: 0s Memory used: 14.45MB	......8.17..2........5.6......7...5..1....3... 8.......5......2..3..8....6...4....	253479861 761238594 894516732 326791458 915824376 487653219 548167923 132985647 679342185
Grid 80 CPU time: 0s Memory used: 14.41MB	963......1....8......2.5....4.8......1....7.. ....3..257......3...9.2.4.7......9..	963741258 152398674 874265391 345872169 218956743 697134825 721489536 589623417 436517982

Grid 81 CPU time: 0s Memory used: 14.39MB	15.3……7..4.2….4.72…..8……..9..1. 8.1..8.79……38……..6….7423	152398647 973641285 864572931 598714362 247936158 316285794 725463819 431829576 689157423
Grid 82 CPU time: 0.015625s Memory used: 14.28MB	……..5724…98….947…9..3…5..9.. 12…3.1.9…6….25….56…..7……6	946731582 157248639 832659471 719423865 584976123 623815947 461397258 398562714 275184396
Grid 83 CPU time: 0s Memory used: 14.46MB	….75….1..2…..4…3…5…..3.2…8…1. ……6…..1..48.2……..7……..	932475861 617928534 845613279 568741392 429836715 173259648 356192487 294387156 781564923
Grid 84 CPU time: 0s Memory used: 14.45MB	6…..7.3.4.8……..5.4.8.7..2….. 1.3…….2…..5…..7.9……1….	618459723 342867519 579123468 296534187 784291635 153786294 927648351 861375942 435912876

Grid 85 CPU time: 0.015625s Memory used: 14.28MB	....6...4..6.3....1..4..5.77.....8.5...8.... .6.8....9...2.9....4....32....97..1..	957261384 846537921 123489567 734926815 295814736 618375492 572198643 481653279 369742158
Grid 86 CPU time: 0s Memory used: 14.28MB	.32.....58..3.....9.428...1...4...39...6. ..5.....1.....2...67.8.....4....95....6.	132749685 857361924 964285371 216457839 348692157 579813246 421536798 683974512 795128463
Grid 87 CPU time: 0s Memory used: 14.41MB	...5.3.......6.7..5.8....1636..2.......4.1. ......3...567....2.8..4.7.......2..5..	746513892 132869754 598742316 367925481 925481673 481637925 679154238 254378169 813296547
Grid 88 CPU time: 0.015625s Memory used: 14.28MB	.5.3.7.4.1.........3.......5.8.3.61....8..5 .9.6..1........4...6...6927....2...9..	956327841 127486395 834951267 548739612 271864539 369215478 793548126 415692783 682173954

Grid 89 CPU time: 0.015625s Memory used: 14.44MB	..5..8..18.....9.....78....4.....64....9. .....53..2.6.....138..5....9.714.	935748621 876231594 124695783 512469378 643872915 789153462 267514839 491386257 358927146
Grid 90 CPU time: 0.015625s Memory used: 14.28MB	.....72.6.1....51...82.8...13..4..... .37.9..1.....238..5.4..9.....79.	143258679 872964153 695137482 986541327 451372968 237896514 719623845 564789231 328415796
Grid 91 CPU time: 0.015625s Memory used: 14.28MB	...658.....4.....12.....96.7...3..5.. ..2.8...3..19..8..3.6.....4....473..	937658241 864291735 125734986 583419627 649372518 712586493 471963852 396825174 258147369
Grid 92 CPU time: 0s Memory used: 14.28MB	.2.3.....6..8.9.83.5.....2...8.7.9..5 .....6..4.....1...1...4.22..7..8.9	924361758 156478293 837592641 613247985 749185326 582936174 498623517 371859462 265714839

Grid 93 CPU time: 0s Memory used: 14.44MB	.5..9....1.....6.....3.8.....8.4...9514..... ..3....2.....4.8...6..77..15..6.	856491372 143572698 927368451 278645139 514923786 639817245 361789524 485236917 792154863
Grid 94 CPU time: 0s Memory used: 14.42MB	.....2.....7...17..3...9.8..7.....2.89.6. ..13..6....9..5.824.....891.....	659412378 238679451 741385296 865723149 427891635 913546782 396157824 574268913 182934567
Grid 95 CPU time: 0s Memory used: 14.41MB	3...8.....7....51.....36...2..4.... 7.....6.13..452.....8..	354186927 298743615 167952483 481527369 932614578 576398241 729865134 845231796 613479852

11. Appendix D

Grid #	Unsolved Sudoku	Solved Sudoku
Grid 1 (6 * 6) CPU time: 0 s Memory used: 14.00 MB	6 2 3 3 0 0 0 5 0 0 0 1 0 0 0 6 0 0 4 0 3 1 0 0 0 0 2 0 0 0 3 0 0 0 5 0 0 2 1	3 4 2 1 5 6 5 6 1 2 3 4 6 2 5 4 1 3 1 3 4 5 6 2 2 1 6 3 4 5 4 5 3 6 2 1

Grid 2 (8 * 8) CPU time: 0.015625 s Memory used: 14.13 MB	8 4 2 0 3 0 0 0 7 0 0 0 0 0 1 0 0 0 2 6 0 0 0 8 0 0 1 4 0 5 0 0 0 3 0 0 1 0 0 2 0 0 3 0 8 7 0 0 0 0 5 0 4 0 0 0 5 0 0 0 0 0 3 0 0 4 0	1 3 8 2 5 7 6 4 8 7 6 1 4 3 5 2 6 5 3 4 8 2 7 1 4 2 5 7 6 1 3 8 7 1 4 5 2 6 8 3 2 8 7 6 3 4 1 5 3 4 1 8 7 5 2 6 5 6 2 3 1 8 4 7
Grid 3 (9 * 9) CPU time: 0 s Memory used: 14.14 MB	9 3 3 0 0 3 0 2 0 6 0 0 9 0 0 3 0 5 0 0 1 0 0 1 8 0 6 4 0 0 0 0 8 1 0 2 9 0 0 7 0 0 0 0 0 0 0 8 0 0 6 7 0 8 2 0 0 0 0 2 6 0 9 5 0 0 8 0 0 2 0 3 0 0 9 0 0 5 0 1 0 3 0 0	4 8 3 9 2 1 6 5 7 9 6 7 3 4 5 8 2 1 2 5 1 8 7 6 4 9 3 5 4 8 1 3 2 9 7 6 7 2 9 5 6 4 1 3 8 1 3 6 7 9 8 2 4 5 3 7 2 6 8 9 5 1 4 8 1 4 2 5 3 7 6 9 6 9 5 4 1 7 3 8 2
Grid 4 (12* 12) CPU time: 0.015625 s Memory used: 15.06 MB	12 4 3 1 0 0 0 0 0 0 0 0 9 11 3 0 0 0 4 0 0 7 0 1 0 0 0 2 9 0 0 0 0 3 0 0 0 0 0 0 0 0 0 12 0 0 11 0 2 0 0 0 0 0 3 0 0 0 0 0 0 0 8 8 0 0 0 0 10 0 0 0 0 0 0 0 0 6 0 0 0 0 0 0 12 0 0 0 7 0 0 0 9 0 0 0 0 0 11 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 5 0 0 0 4 0 0 2 0 0 0 0 11 0 0 0 0 7 0 0 0 2 0 3 0 0 8 0 0 0	1 5 7 6 10 8 2 4 12 9 11 3 12 10 11 4 3 2 7 9 1 5 8 6 2 9 8 1 11 7 3 5 6 4 10 12 6 4 3 9 12 5 8 11 10 2 7 1 10 11 9 3 4 6 12 2 5 7 1 8 8 12 5 7 1 10 9 3 11 6 2 4 4 1 6 5 2 11 10 8 7 12 3 9 3 7 2 12 8 9 6 1 4 10 5 11 5 3 12 8 7 4 1 6 2 11 9 10 7 8 1 11 6 12 5 10 9 3 4 2 9 2 4 10 5 1 11 12 3 8 6 7 11 6 10 2 9 3 4 7 8 1 12 5

Grid 5 (16* 16) CPU time: 0.078125 s Memory used: 17.66 MB	16 4 4 2 0 0 0 9 0 0 0 0 0 0 11 0 0 0 3 0 0 0 4 0 0 0 0 8 12 0 0 0 9 0 0 0 1 0 15 0 0 0 0 0 0 0 9 0 0 0 0 0 0 0 0 0 0 0 16 0 0 0 0 7 0 0 0 0 0 3 0 0 0 0 0 0 0 0 0 0 0 0 14 5 0 0 0 0 0 6 0 0 0 0 13 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 11 0 0 6 0 0 0 0 0 0 7 0 0 0 12 0 0 0 0 0 8 0 0 0 0 0 9 0 0 0 0 0 0 0 15 0 0 16 0 0 0 10 0 0 0 0 0 0 0 2 0 0 0 0 0 11 0 0 0 0 0 2 0 0 0 3 0 4 0 0 0 0 7 0 0 0 0 0 0 10 0 0 0 11 0 0 0 0 0 0 4 0 0 0 0 0 0 8 0 0 0 0 0 0 0 0 0 0 0 15 0 6 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 8 0 16 0 0 0 0 0 10 0 0 0 0 0 0 0 0 2 0 0 0	2 16 8 7 9 10 14 13 1 4 15 11 12 6 5 3 10 3 5 4 7 1 2 11 8 12 6 16 15 9 14 13 14 1 13 15 8 12 3 6 10 5 7 9 11 2 16 4 12 11 9 6 5 4 15 16 13 2 14 3 7 1 10 8 9 7 3 12 2 11 4 15 6 8 10 1 16 5 13 14 5 2 4 8 10 16 6 14 7 11 3 13 1 15 9 12 1 10 15 16 12 13 9 3 2 14 4 5 8 11 6 7 6 13 11 14 1 5 8 7 15 16 9 12 3 10 4 2 7 8 2 1 4 3 5 9 11 13 16 10 6 14 12 15 3 5 16 11 15 14 10 1 12 6 8 7 13 4 2 9 15 6 10 13 11 8 16 12 4 9 2 14 5 7 3 1 4 14 12 9 6 7 13 2 5 3 1 15 10 8 11 16 11 9 7 5 3 15 1 4 16 10 12 2 14 13 8 6 8 4 1 3 16 2 11 10 14 15 13 6 9 12 7 5 13 12 6 2 14 9 7 5 3 1 11 8 4 16 15 10 16 15 14 10 13 6 12 8 9 7 5 4 2 3 1 11