Math 584 (Math for Algo Trading), Spring 2020.

Homework 2

Due: Tue, Feb 25, 2020, NO LATER than 4:30pm.

For each ticker in the file 'TechTickers.scv', download its adjusted closing price for each business day between Jan 1, 2009, and Dec 31, 2019. This is your sample. All questions listed below must be answered using this sample. Assume the existence of riskless return 0.01. All exercises must be solved using only the modules listed in the file 'Template.ipynb'.

1. In this question, you construct dynamic utility-optimizing trading strategies. (Do not forget that the riskless return is a part of the universe of available assets.) Use the power utility with $\gamma = 1/2$, the trading time horizon $T = 100$ (measured in days), and the calibration window size $N = 250$.

   (a) 20 pts Using only the first asset and the riskless one, construct the optimal strategy. The model for the risky return is

   $$R^t = \mu + \varepsilon^t,$$

   with i.i.d. $\{\varepsilon^t\}_t$, s.t.

   $$\varepsilon^t = \begin{cases} \sigma, & \text{prob. } 1/2, \\ -\sigma, & \text{prob. } 1/2, \end{cases}$$

   where μ and σ should be estimated as the sample mean and the sample standard deviation.

   Produce and save in a 'csv' file the PnL process of this strategy. Output its annualized mean, variance, and the Sharpe ratio.

   Replace the first asset by the second one and repeat the same procedure. Repeat this for every remaining asset (note that, in each experiment you use only one risky asset and the riskless one).

   (b) 20 pts Using only the first asset, the riskless one, and the volume-exponentially-weighted average price, with $\rho = 1$, as the predictive factor, construct the optimal strategy. Use a linear regression model for the relationship between the risky asset and the predictive factor:

   $$R^t = a + cF^{t-1} + \varepsilon^t,$$

   with i.i.d. $\{\varepsilon^t\}_t$, s.t.

   $$\varepsilon^t = \begin{cases} \sigma, & \text{prob. } 1/2, \\ -\sigma, & \text{prob. } 1/2, \end{cases}$$

   where μ and σ should be estimated as the sample mean and the sample standard deviation. Use the following model for the daily traded volume η^t:

   $$\eta^t = \begin{cases} \hat{\mu} + \hat{\sigma}, & \text{prob. } 1/2, \\ \hat{\mu} - \hat{\sigma}, & \text{prob. } 1/2, \end{cases}$$

   where $\{\eta^t\}_t$ are i.i.d., and $\hat{\mu}$ and $\hat{\sigma}$ should be estimated as the sample mean and the sample standard deviation. To compute the value function via DPP, use an equidistant grid on $[\tilde{\mu} - 3\tilde{\sigma}, \tilde{\mu} + 3\tilde{\sigma}]$, consisting of 100 points, for the possible values of the factor. The values of $\tilde{\mu}$ and $\tilde{\sigma}$ are approximated as the sample mean and the sample standard deviation of the values of F in the calibration window.

To compute the value function and the optimal strategy outside of the grid, use linear interpolation/extrapolation.

Produce and save in a 'csv' file the PnL process of this strategy. Output its annualized mean, variance, and the Sharpe ratio.

Replace the first asset by the second one and repeat the same procedure. Repeat this for every remaining asset (note that, in each experiment you use only one risky asset and the riskless one).

(c) 20 pts Repeat part (a) with proportional transaction costs $\lambda = 0.01$. The model for the risky return is

$$R^t = \mu + \varepsilon^t,$$

with i.i.d. $\{\varepsilon^t\}_t$, s.t.

$$\varepsilon^t = \begin{cases} \sigma, & \text{prob. } 1/2, \\ -\sigma, & \text{prob. } 1/2, \end{cases}$$

where μ and σ should be estimated as the sample mean and the sample standard deviation. To compute the value function via DPP, use an equidistant grid on $[-2, 2]$, consisting of 100 points, for the possible values of the "weights before rebalancing". To compute the value function and the optimal strategy outside of the grid, use linear interpolation/extrapolation.

Produce and save in a 'csv' file the PnL process of this strategy. Output its annualized mean, variance, and the Sharpe ratio.

Replace the first asset by the second one and repeat the same procedure. Repeat this for every remaining asset (note that, in each experiment you use only one risky asset and the riskless one).