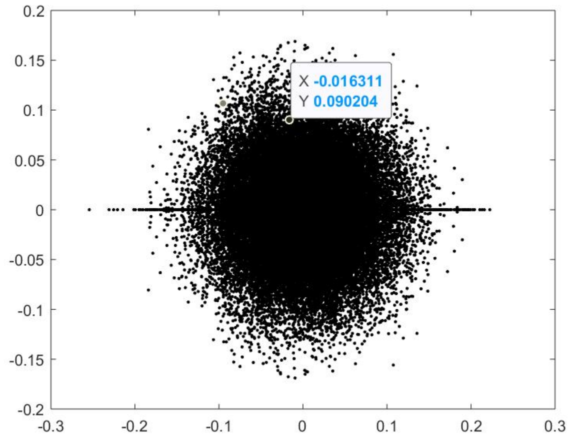
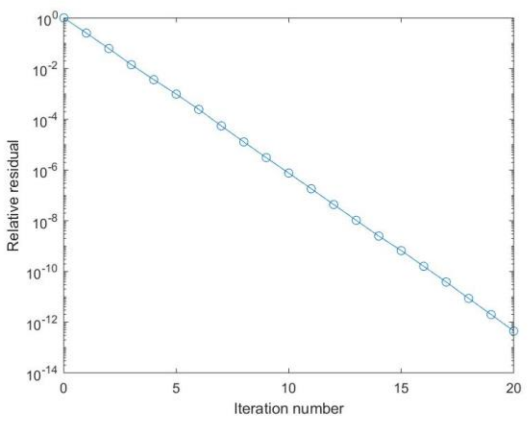
Question 1 (Repeat example 35.1)

```
m = 200; A = 2*eye(m) + 0.5*randn(m)/sqrt(m);
b = ones(m,1);

[V,~] = eig(A);
plot(V,'k.')

tol = 10^(-12);
maxit = 20;

[x, flag, relres, iter, resvec] = gmres(A,b,[],tol,maxit);

figure
semilogy(0:maxit, resvec/norm(b), '-o');
xlabel('Iteration number');
ylabel('Relative residual');
```





Question 2

(a)

```
A = [8 4 2 1; 4 8 4 2; 2 4 8 4; 1 2 4 8];
v = [1 1 1 1]'./2;
n = 10;
lambda1 = []; lambda2 = []; lambda3 = [];

format long
for k = 1:n
    w = A * v;
    v = w / norm(w);
    lambda1 = [lambda1 v'*A*v];
end

v = [1 1 1 1]'./2;

for k = 1:n
    w = A \ v;
    v = w / norm(w);
    lambda2 = [lambda2 v'*A*v];
end

v = [1 1 1 1]'./2;
lambda3 = [v'*A*v];

for k = 1:n
    w = (A - lambda3(k)*eye(4)) \ v;
    v = w / norm(w);
    lambda3 = [lambda3 v'*A*v];
end

lambda1
lambda2
lambda3
```

The answer is as follows:

Note that for both power method and Rayleigh quotient iteration, they compute the eigenvalue with the largest absolute value. Besides, Rayleigh quotient converges faster than power method. And for unshifted inverse iteration, it computes the eigenvalue with the smallest absolute value.

Power method	16.672131147540988	16.683819628647207	16.684602322430148	16.684654684513276	16.684658187307214
	16.684658421627784	16.684658437302733	16.684658438351320	16.684658438421469	16.684658438426155
Unshifted inverse iteration	14.399999999999999	7.135135135135133	4.554938956714762	4.331664534989951	4.316434838256360
	4.315414702695185	4.315346454398972	4.315341888880736	4.315341583468831	4.315341563038203
Rayleigh quotient	16.684615384615384	16.684658438426489	16.684658438426489	16.684658438426492	16.684658438426489
	16.684658438426492	16.684658438426489	16.684658438426492	16.684658438426489	16.684658438426492

(b)

```
A = [8 4 2 1; 4 8 4 2; 2 4 8 4; 1 2 4 8];
v = [1 1 1 1]'./2;
n = 10;
off_diag_max = [];

format long
max = 0;
for k = 1:n
    [Q,R] = qr(A);
    A = R*Q;
    for i = 1:4
        for j = 1:4
            if (i ~= j & A(i,j) > max)
                max = A(i,j);
            end
        end
    end
    off_diag_max = [off_diag_max max];
end

A
off_diag_max
```

For the answer below, note that the off diagonal elements at each step is truly decreasing to zero. And the matrix of A is convergent to somehow tridiagonal matrix.

A=

16.684650870697748	0.008106965374876	0.000018128314709	0.000000256400930
0.008106965374876	7.999920721769696	0.017888947809786	0.000253015404780
0.000018128314708	0.017888947809787	4.312593875744195	0.060995911929790
0.000000256400932	0.000253015404780	0.060995911929791	3.002834531788368

off_diag_max =

4.289906995109921	2.659558449077073	1.363329400421118	0.727713493424352
0.394169759053323	0.252862471417951	0.114259089109530	0.125365640161673
0.033176642774905	0.060995911929791		