MATH 477/577. DUE SEPT. 23 **NAME**: ____________________

HOMEWORK/COMPUTER ASSIGNMENT **ID**: ____________________

HOMEWORK SET # 2

DATE: ____________________

Please read lecture 4, 5, 6, 7, and 8 of textbook and review your lecture notes. Graduate students do problems with a star ONLY; undergraduate students do problems without a star ONLY.

Question 1.

One more problem on Norm for graduate students ONLY.

Suppose M is an $n \times n$ matrix with induced norm $\|M\| < 1$, show that $(I - M)$ is nonsingular and $\|(I-M)^{-1}\| \leq \dfrac{1}{1 - \|M\|}$. *hint: consider* $\Sigma_{k=0}^{\infty} M^k$, *then use partial sum to establish its convergence.* .

Question 2.

Fundamental concepts on SVD.

(a): Problem 4.1 (c, d) and Problem 5.1

(b*): Problem 4.4 and Problem 5.3.

Question 3.

Application of SVD.

(a): Given the SVD of $A = U\Sigma V^*$, compute the SVD of $(A^*A)^{-1}, (A^*A)^{-1}A^*, A(A^*A)^{-1}A^*$

(b*): Suppose $A \in \mathbb{C}^{n\times n}$, show that there exist unitary matrix Q and semi-positive definite Hermitian matrices H_1 and H_2 such that $A = H_1Q = QH_2$, where $H_1^2 = AA^*, H_2^2 = A^*A$. This is known as the polar decomposition of matrix A and is often used in mechanics and materials science, i.e. a deformation can be decomposed into rotation (unitary matrix) and stretching (compression).

Question 4.

Fundamental concepts on projectors.

(a): Problem 6.1, 6.4.

(b*): Problem 6.4, 6.5.

Question 5.

QR factorization

(a): Problem 7.1(a)

(b*): Problem 7.1(a); and 7.3, just do the algebraic proof. *hint: consider* $A = QR$.

Computer Assignment #1 for ALL students: Note that if you are new to Matlab, please go through the Matlab Demos by first clicking the Matlab icon on the computer desktop, then clicking the "Demos" under the "Help" menu. (1): Problem 4.3, for example let

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$$

(2): Use function *svd* in Matlab to compute the SVD of matrix A in **Question 1**(c).

(3): I mentioned in class that SVD can be used in image processing. Here is an example. Get a picture of your favorite (decent, but not that complicated) and save it as a jpg file, then do the following in Matlab. Assuming I have a picture of rabbit and saved as *rabbit.jpg*.

```
A=rgb2gray(imread('rabbit.jpg')); %read in the picture here;

figure(1);% open a window here

imshow(A); %show the original picture of rabbit.

[U,S,V]=svd(double(A)); %do SVD here

r=10; %use only first 10 rank-one matrix to approximate A, and you can try other r values

A10=U(:,1:r)*S(1:r,1:r)*(V(:,1:r))';% this is for r=10;

figure(2);%

imshow(uint8(A10)); % show the approximation and compare with your original picture
```