Math 584 (Math for Algo Trading), Spring 2020.

Homework 3

Due: Thu, Mar 12, 2020, NO LATER than 4:30pm.

For each ticker in the file 'TechTickers.scv', download its adjusted closing price for each business day between Jan 1, 2009, and Dec 31, 2019. This is your sample (note that, in this hwk, the list of tickers is smaller). All questions listed below must be answered using this sample. Assume zero riskless return. All exercises must be solved using only the modules listed in the file 'Template.ipynb'.

A pair is considered cointegrated if the p-value of the Dickey-Fuller test (if you use the Augmented Dickey-Fuller test, you need to ensure that the order of the auto-regressive model tested is exactly one) does not exceed $p^* = 0.05$.

1. In this exercise, you test the performance of the pairs trading strategy, using the running window method with the estimation window size $N = 100$ days, and trading pairs whose **prices** are cointegrated. For the stop-loss mechanism, use the maximum holding time $\bar{T} = 40$ days and the maximum deviation from the mean $\bar{\lambda}\tilde{\sigma} = 3\tilde{\sigma}$, where $\tilde{\sigma}$ is the standard deviation of the pair (computed when a position in the pair is opened). You **do not** need to hedge your strategy with the market index.

   (a) 30 pts Implement the simplest version of the pairs trading, investing $\$\bar{C} = \$1,000,000$ on the long side of each cointegrated pair. If a position in the pair is opened when its centered price is below zero, then you need to choose the optimal exit threshold λ_2 by solving

   $$V = \max_{\lambda_2 \colon \lambda_2 \geq \lambda_1 + c} \frac{\lambda_2 - \lambda_1 - c}{P} (T - \mathbb{E}(\tau | X^0 = \lambda_1)), \tag{1}$$

   where

   $$\mathbb{E}(\tau | X^0 = \lambda_1) \approx \frac{\sqrt{\pi}}{\tilde{\sigma}\sqrt{\rho}} \int_{\lambda_1}^{\lambda_2} \exp(y^2 \rho / \tilde{\sigma}^2)(1 + \text{erf}(y\sqrt{\rho}/\tilde{\sigma}))dy,$$

   $$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, \quad c \approx 4P\lambda, \quad T = 30,$$

   λ_1 is the centered price of the pair at the time when the position is opened, and P is the price of the stock that is on the long side of the pair. If a position in the pair is opened when its centered price is above zero, you need to define V and λ_2 in a symmetric way (you need to figure out how exactly to do it).

   Test this strategy using the proportional T-costs $\lambda = 0$ and $\lambda = 0.005$. For each case, produce the plots of the absolute and relative PnL processes, save their values in csv files, and compute the associated Sharpe-ratios.

   (b) 10 pts Repeat part (a) using an allocation of investment size that is sensitive to the risk-return tradeoff. Namely, you need to invest $\$\bar{C} \cdot V = \$1,000,000 \cdot V$ on the long side of each cointegrated pair, where V is defined in (1).

2. 20 pts In this exercise, you test the performance of the pairs trading strategy, using the running window method with the estimation window size $N = 100$ days, and trading pairs whose **log-prices** are cointegrated. Construct and test the optimal mean-standard-deviation portfolio of cointegrated pairs (with daily rebalancing), using the

reciprocal risk tolerance $\gamma = 1$, the proportional T-costs $\lambda = 0$ and $\lambda = 0.005$, and the total size of investment on the long side of the pairs being $\$\hat{C} = \$50,000,000$. For each case, produce the plots of the absolute and relative PnL processes, save their values in csv files, and compute the associated Sharpe-ratios.