MATH 477/577. DUE OCT. 2 **NAME**: _____

HOMEWORK/COMPUTER ASSIGNMENT **ID**: _____

HOMEWORK SET # 3

DATE: _____

Please read lecture 9, 10, 11, 12 and 13 in your textbook and review your lecture notes. Graduate students do problems with a star ONLY; undergraduate students do problems without a star ONLY. There are three computer problems in the assignment, please start to work on these problems **as soon as you can.**

Question 1.

One more practice on QR factorization

Let $A \in \mathbb{C}^{m\times n}$, $m > n$, rank $A = n$.

(a): Show that the operation count for modified GS is also $2mn^2$ (same as the classical GS).

(b*): In class, I showed an example of $A_{3\times 3}$ and the modified GS can be described in matrix multiplications $AR_1R_2R_3 = Q$ with $R = (R_1R_2R_3)^{-1}$, all Rs are upper triangular matrices with positive diagonal elements. Show that the classical GS can also be expressed in matrix multiplication form.

Question 2.

Fundamental concepts on Householder reflection

(a): Problem 10.1 and 10.4

(b*): Problem 10.1 and 10.4.

Question 3.

Perturbed by a rank-one matrix (for both undergraduates and graduates). let $\|x\|_2 = 1$ and $U = I - 2xx^*$,

(a) show that $U^2 = I$,

(b) if $x^*x = 2$, what is $(I - xx^*)^{-1}$?

(c) show that $I - xx^*$ is singular iff $x^*x = 1$, find the inverse in all nonsingular cases.

Question 4.

Fundamental concepts on Condition numbers

(a): Find the condition number of the addition of the two components of vector $x = (x_1, x_2)$: $f(x) = x_1 + x_2$ and the multiplication, $f(x) = x_1 * x_2$. explain your results.

(b*): Let $A \in \mathbb{R}^{100\times 100}$ be a lower bi-diagonal matrix that entries on the main diagonal are $\frac{1}{2}$ and the entries on the sub-diagonal is -1. Note that $det(A)$ is very small and this matrix is close to singular. Show that there is an element in A^{-1} is 2^{100} and the condition number of this matrix using ∞ norm $\kappa(A)_\infty > 2^{101}$. *hint: consider matrix of cofactors, i.e. an adjugate matrix.*

(c*): Problem 12.1.

Computer Assignment #3 for ALL students: If you are new to Matlab, please go through

the Matlab Demos by first clicking the Matlab icon on the computer desktop, then clicking the ”Demos” under the ”Help” menu.

(1): Do problem 8.2.

(2): Do problem 10.2.

(3): Do problem 11.3.