Problem 22.2

Code:

```
m = 60;
b = 2:-1:-57; b(m) = -58; b = b';
x0 = ones(m,1);
A = tril(-ones(m),-1) + diag(ones(m,1));
A(:,m) = 1;
[L,U,P] = lu(A);

d = P * b; y = zeros(m,1); x = zeros(m,1);
y(1)=d(1);
for i=2:m
    for j=1:i-1
        d(i)=d(i)-L(i,j)*y(j);
    end
    y(i)=d(i);
end

x(m)=y(m)/U(m,m);
for i=(m-1):-1:1
    for j=m:-1:i+1
        y(i)=y(i)-U(i,j)*x(j);
    end
    x(i)=y(i)/U(i,i);
end

max(abs(U(:)))
norm(x - x0, inf)

B = A;
B(:,m) = 1.1;
[L1,U1,P1] = lu(B);

d1 = P1 * b; y1 = zeros(m,1); x1 = zeros(m,1);
y1(1)=d1(1);
for i=2:m
    for j=1:i-1
        d1(i)=d1(i)-L1(i,j)*y1(j);
    end
    y1(i)=d1(i);
end

x1(m)=y1(m)/U1(m,m);
```

```
for i=(m-1):-1:1
    for j=m:-1:i+1
        y1(i)=y1(i)-U1(i,j)*x1(j);
    end
    x1(i)=y1(i)/U1(i,i);
end

x2 = B \ b;
max(abs(U1(:)))
norm(x1 – x2, inf)
```

Answer:

I choose real solution of x_0 as (1, 1, …, 1), and compute its corresponding b. And the growth factor of A is 5.7646e+17. The computed solution x with Gaussian Elimination is almost all 1 but few of elements are 0. Thus, the rounding number (inf-norm of the true x_0 and x) is 1, which is not "very" big. However, if I modify matrix A slightly, i.e. I make the last column of A as 1.1 rather than 1. And I compute the real value as x_2 above (compute the inverse straightly). The growth factor is 6.3411e+17, which is nearly the same as before, but the solution of x_1 is much different with before, some of the elements are far more from 1. And the rounding error is 13.6727, which is larger than before.

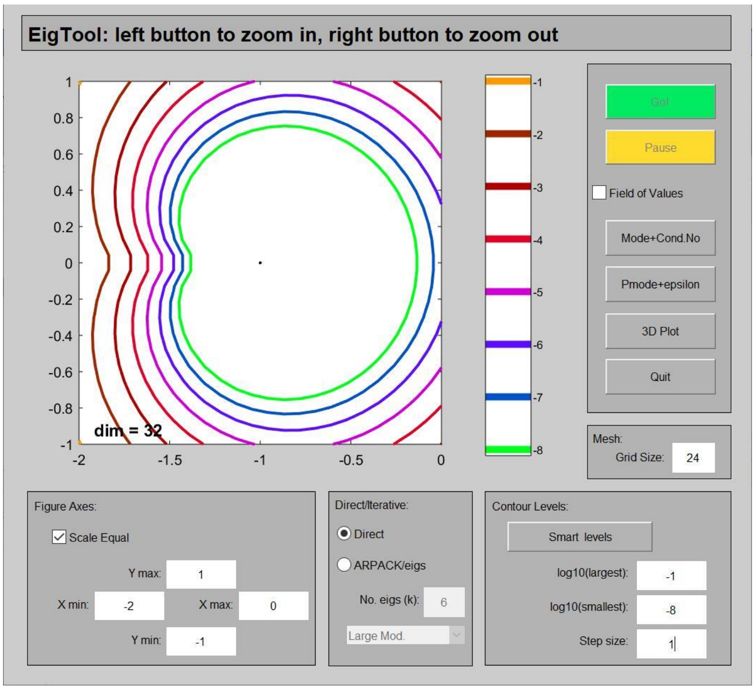
Problem 26.2

Code:

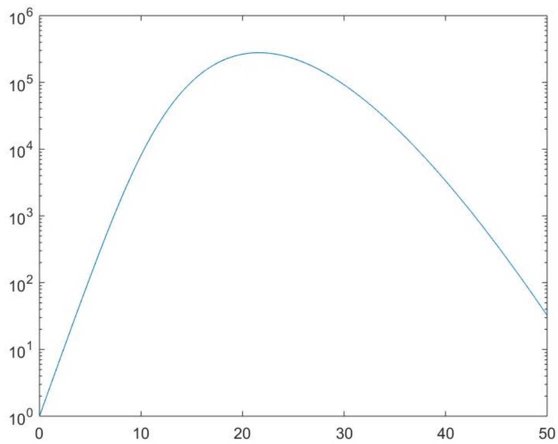
(a) & (b)

```
m = 32;
supdia1 = diag(ones(m-1,1));supdia2 = diag(ones(m-2,1));
appd1 = zeros(m-1,1); appd2 = zeros(1,m);
appd3 = zeros(m-2,1); appd4 = zeros(1,m-1);
A = diag(-ones(m,1)) + [[appd1 supdia1]; appd2] + [[appd1 [[appd3 supdia2]; appd4]];
appd2];

eigtool(A)

y = [];

for i = 0:0.5:50
    y = [y norm(expm(i .* A),2)];
end

t = 0:0.5:50;

figure
semilogy(t,y)
```

Answer:
I use eigtool to plot the boundaries of 2-norm epsilon-pseudospectra of A for given epsilon. And the plot is as follows:



We can see that the boundary converges to the black point as epsilon decreases.
Then I plot the semiology as follows:



Notice the initial growth rate is nearly linear increasing. Since it's semiology plot, which means the value is increasing exponentially. Notice that the 2-norm is related to the largest singular value of A, and according the conclusion of 26.1. pseudospectra is related to the smallest singular value of A.