Problem 1

Function:

```
function u = halton(p,n)
b = zeros(ceil(log(n)/log(p)),1);
for j = 1:n
    i =1;
    b(1) = b(1) + 1;
    while b(i) > p - 1 + eps
        b(i) = 0;
        i = i + 1;
        b(i) = b(i) + 1;
    end
    u(j) = 0;
    for k = 1: length(b(:))
        u(j) = u(j) + b(k) * p ^ (-k);
    end
end
end
```

Main:

```
Method 1
n = 10^5; x = rand(4,n); b = sum(x.*x); s = 0;
for i = 1:n
    if b(i) <= 1
        s = s+1;
    end
end
s/n * 16

Method 2
p1 = 2; p2 = 3; p3 = 5; p4 = 7; n = 10 ^ 5;
w = halton(p1,n);
x = halton(p2,n);
y = halton(p3,n);
z = halton(p4,n);
s = 0;
for i = 1:10 ^5
    if (2 * w(i) - 1)^2 + (2 * x(i) - 1)^2 + (2 * y(i) - 1)^2 + (2 * z(i) - 1)^2 < 1
        s = s + 1;
    end
end
s / 10^5 * 16
pi ^ 2 / 2
```

Result:

The approximation of the volume of the ball with Monte Carlo is 4.8922

The approximation of the volume of the ball with quasi - Monte Carlo is 4.9293

The exact volume is $\pi^2/2 \approx 4.9348$. It shows that our results are very close to the exact number. Specially, quasi - Monte Carlo approximation is better than the original one.

Problem 2

Main:
```
m = 0; n = 10000;
for j = 1:n
     s = 0; b = 0; k = 1000; sqdelt = sqrt(0.01);
    for i = 1:k
        a = b;
        b = b + sqdelt * randn;
        if i >= 300 & i <= 500 % change interval for different questions
            if a * b <= 0
                s = 1;
            end
        end
    end
    if s == 0;
        m = m + 1;
    end
end
(2 / pi) * asin(sqrt(3/5))
m/10000
```

(a) For interval [3, 5], the result we get is 0.5788, and the exact probability is $\arcsin\sqrt{t_1/t_2} \approx 0.5641$. It shows that they are very close to each other.

(b) For interval [2, 10], the result we get is 0.3111, and the exact probability is $\arcsin\sqrt{t_1/t_2} \approx 0.2952$. It show that they are very close to each other.

(c) For interval [8, 10], the result we get is 0.7189, and the exact probability is $\arcsin\sqrt{t_1/t_2} \approx 0.7048$. It show that they are very close to each other.

Problem 3 (on the previous page)

Problem 4

Function:
```
function E = european_option_price(g, S0, r, T, sigma, zmin, zmax, m)
h = (zmax - zmin)/m;
z = zmin : h : zmax; % m+1 points
f = @(z)exp(-r * T) * g(S0 * exp(sigma * sqrt(T) * z + (r - sigma ^ 2 / 2) * T)) * normpdf(z);
```

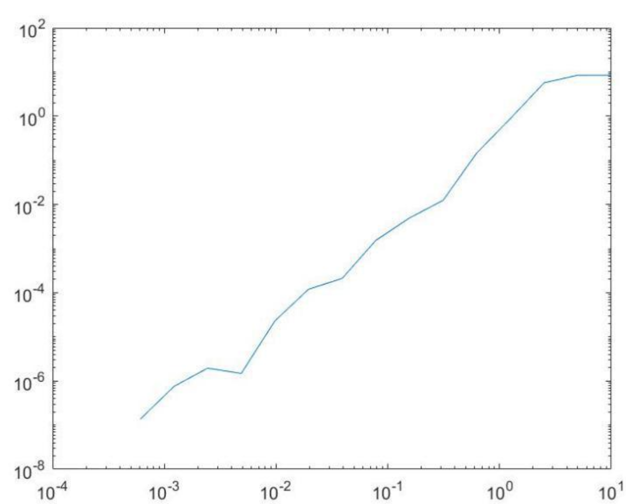
```
y0 = f(zmin); ym = f(zmax);
sumyi = 0;
for i = 2:m
    sumyi = sumyi + f(z(i));
end
E = h / 2 * (y0 + ym + 2 * sumyi);
end

function price = Cbs(T,S,K,r,sigma)
d1 = (log(S/K) + (r + sigma^2/2) * T)/(sigma * sqrt(T));
d2 = d1 - sigma * sqrt(T);
price = S * normcdf(d1) - K*exp(-r * T) * normcdf(d2);
end
```

Main:

```
S0 = 100; r = 0.01; sigma = 0.2;
K = 100; T = 1;
g = @(x)max(x - K, 0);
zmin = -10; zmax = 10; h = []; e = [];
for i = 1:15
    m = 2^i;
    E = european_option_price(g, S0, r, T, sigma, zmin, zmax, m);
    price = Cbs(T,S0,K,r,sigma);
    error = abs(E - price);
    e = [e, error];
    h = [h, (zmax - zmin)/m];
end
loglog(h, e)
```

The plot is as follows:



Finding that absolute error approach zero as h tends to zero

Problem 5

(b)

```
T =1; K = 100; B = 120; r = 0.1; sigma = 0.25; S0 = 100;
M = 1000; delt = T/M;
t = 0:delt:T; % M + 1 points
N = 1000;
k = M;
S = [];
for i = 1:N
    b = 0; sqdelt = sqrt(delt); St = [];
    for j = 1:k+1
        tk = t(j);
        b = b + sqdelt * randn;
        Stk = S0 * exp((r - 1/2 * sigma^2) * tk + sigma * b);
        St = [St, Stk];
    end
    S = [S; St];
end
```

(c):

```
T =1; K = 100; B = 120; r = 0.1; sigma = 0.25; S0 = 100;
M = 1000; delt = T/M;
t = 0:delt:T; % M + 1 points
N = 10000;
k = M; VT = 0;
for i = 1:N
    b = 0; sqdelt = sqrt(delt); l = 0;
    for j = 1:k+1
        tk = t(j);
        b = b + sqdelt * randn;
        Stk = S0 * exp((r - 1/2 * sigma^2) * tk + sigma * b);
        if Stk >= B
            l = 1;
        end
    end
    if l == 0;
        VT = VT + max(Stk - K, 0);
    end
end
E = VT/N * exp(-r * T)
```

When N = 10000, the result of our estimation is 0.7450;

When N = 100000, the result of our estimation is 0.7381.