STATS 509 HOMEWORK 7

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Question 1 (page 357 exercise 7, (a)&(b) is written by hand) (c)

```
x = solve(matrix(c(1,0.4,0.4,1), nrow = 2))
phi = x %*% c(0.4,0.2)
options(digits = 3); phi
```

```
##        [,1]
## [1,] 0.3810
## [2,] 0.0476
```

```
rho3 = phi[1]*0.2 + phi[2]*0.4; rho3
```

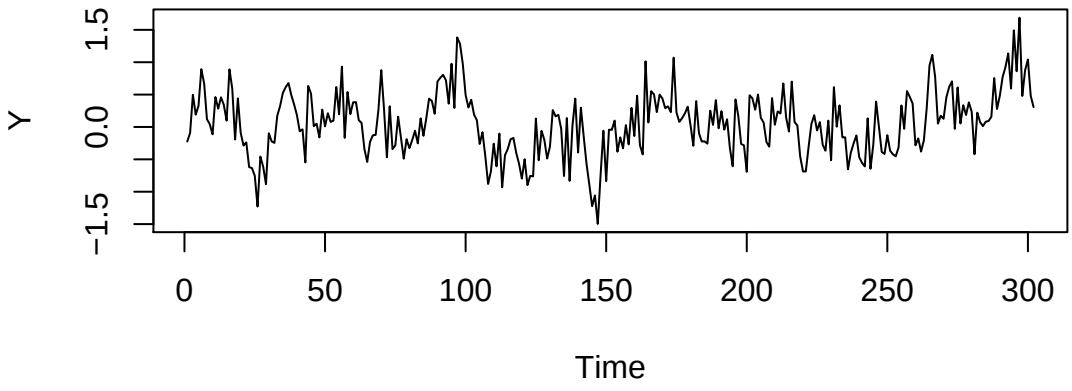
```
## [1] 0.0952
```

- The result is that $\phi_1 = 0.3810, \phi_2 = 0.0476, \rho(3) = 0.0952$.

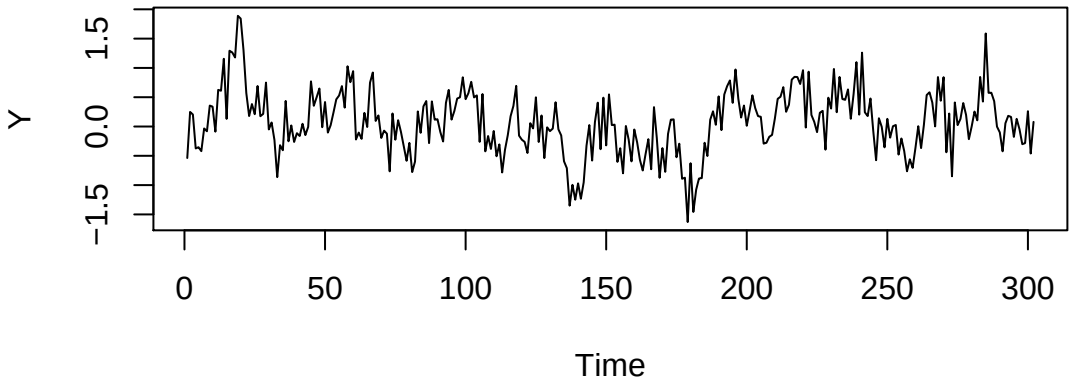
Question 2

(a)

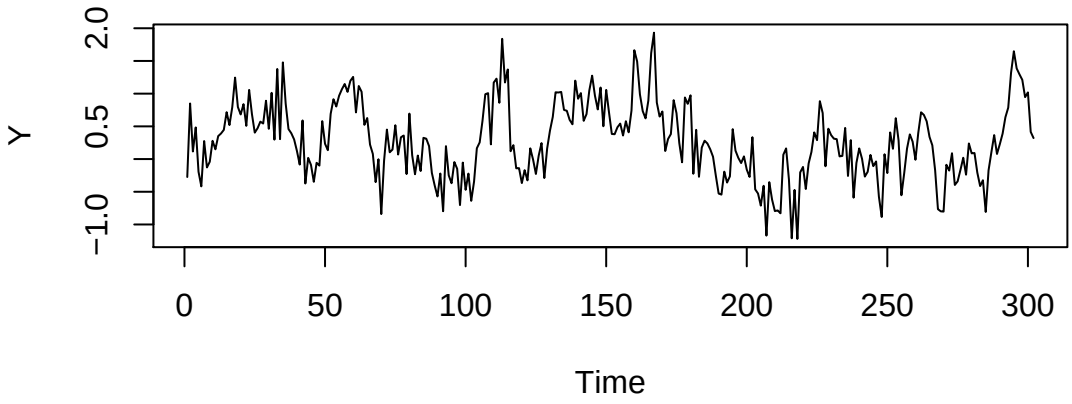
```
n = 302; alpha1 = 0.4; alpha2 = 0.4; sigma = 0.4
set.seed(123)
eps = rnorm(n,0,0.4)
y = eps
for (i in c(3:n)){
  y[i] = alpha1*y[i-1] + alpha2*y[i-2] + eps[i]
}
plot(y, type = "l", xlab = "Time", ylab = "Y")
```




```
set.seed(456)
eps = rnorm(n,0,0.4)
y = eps
for (i in c(3:n)){
  y[i] = alpha1*y[i-1] + alpha2*y[i-2] + eps[i]
}
plot(y, type = "l", xlab = "Time", ylab = "Y")
```



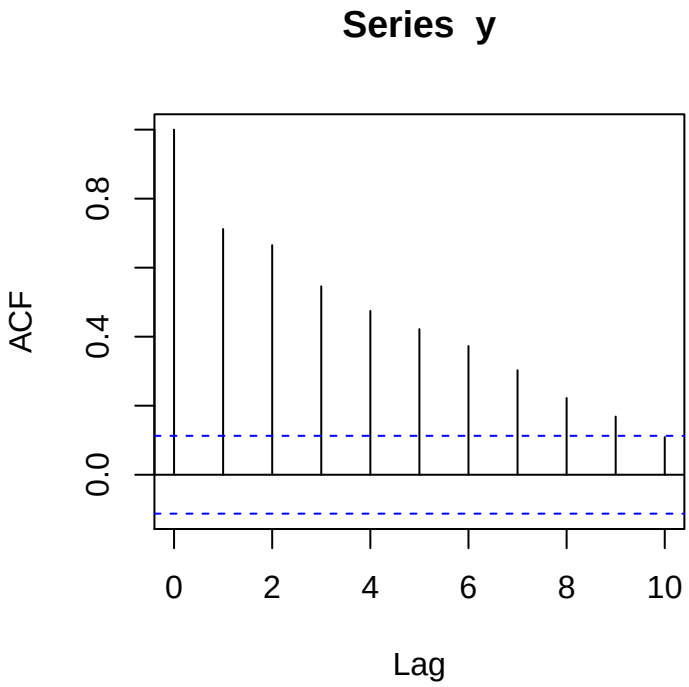

```
eps = rnorm(n,0,0.4)
y = eps
for (i in c(3:n)){
  y[i] = alpha1*y[i-1] + alpha2*y[i-2] + eps[i]
}
plot(y, type = "l", xlab = "Time", ylab = "Y")
```



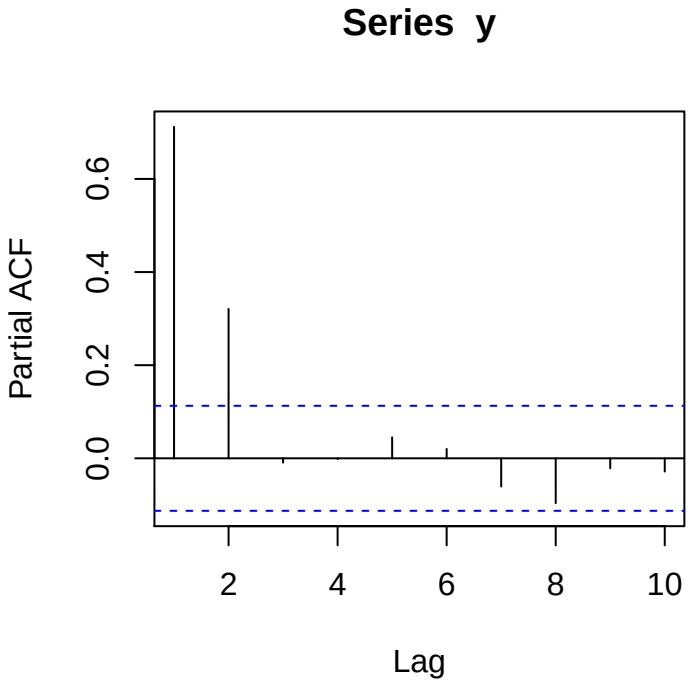
(b)

- Seeing that there are obvious differences among three plots. Now let's plot ACF and PACF.

```
acf(y, lag = 10)
```



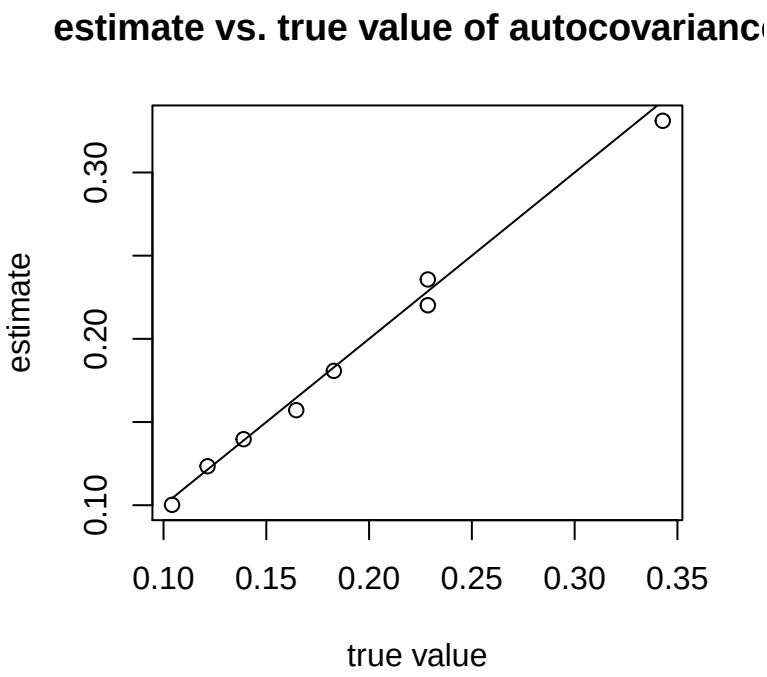

```
acf(y, lag = 10, type = "partial")
```



- Seeing that for ACF plot, AR(2) model shows significant autocorrelation with small lags, and this autocorrelation decreases gradually with lag increases.
- For PACF plot, it shows only lag 2 has significant autocorrelation, indicating that for lag>2, there is no significant autocorrelation when taking everthing between them in consideration. This indicates that this is an AR(2) model.

(c)

```
gamma_0 = (0.4)^2/(1-alpha1^2-alpha2^2)
x = solve(t(matrix(c(1,-alpha1,-alpha2,-alpha1,1-alpha2,0,alpha2,alpha1,-1), nrow = 3)))
gamma02 = x %*% c(sigma^2,0,0)
options(digits = 3);
gamma = rep(0,8)
for (i in 1:8){
  if (i <= 3){
    gamma[i] = gamma02[i]
  }
  else{
    gamma[i] = alpha1*gamma[i-1] + alpha2*gamma[i-2]
  }
}
gamma_est = acf(y, type = c("covariance"), plot = FALSE, lag = 7)
plot(gamma, gamma_est$acf,  xlab = "true value",
     ylab = "estimate", main = "estimate vs. true value of autocovariance")
lines(gamma,gamma)
```



- Seeing that the real line is true value itself and the circles are plot of **estimate vs. true value**.