EECS 545 Homework 3

Yuan Yin

October 15, 2018

Problem 2

(b) & (c)

The code and result is as follows:

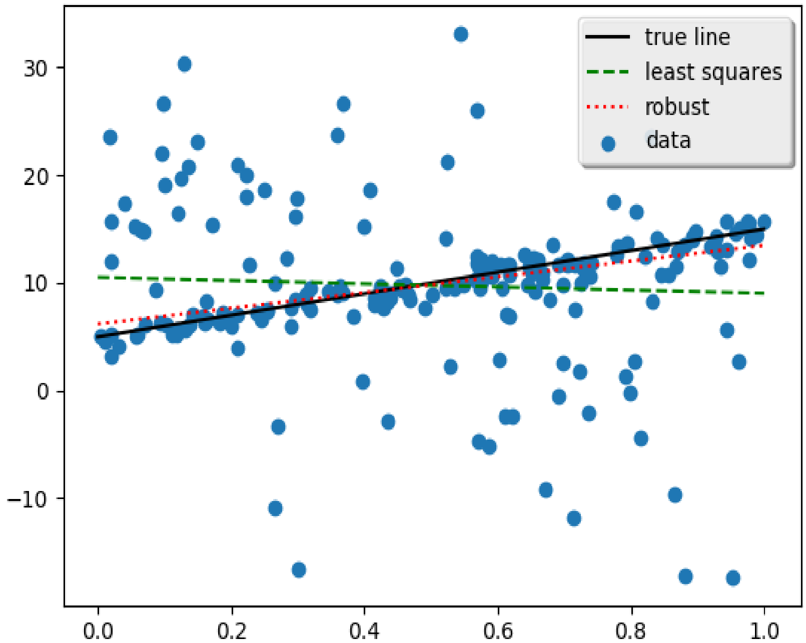
```
import numpy as np
import matplotlib.pyplot as plt
import pylab as pl

n = 200
np.random.seed(0) # Seed the random number generator
x = np.random.rand(n,1)
z = np.zeros([n,1])
k = n * 0.4
rp = np.random.permutation(n)
outlier_subset = rp[1:int(k)]
z[outlier_subset] = 1 # outliers
y = (1 - z) * (10 * x + 5 + np.random.randn(n,1)) + z * (20 - 20 * x + 10 *
    np.random.randn(n,1))

# Plot data and true line
plt.scatter(x, y, label = 'data')

t = pl.frange(0,1,0.01)
plt.plot(t, 10*t+5, 'k-', label = 'true line')

# Add your code for ordinary least squares below
x_ols = np.mat(x); y_ols = np.mat(y) # They are all 200*1 matrix
y_mdf = y_ols - np.mean(y_ols); x_mdf = x_ols - np.mean(x_ols)
A = 2 * (x_mdf.T * x_mdf); r = -2 * x_mdf.T * y_mdf; c = y_mdf.T * y_mdf
w_ols = -(A.I * r)[0,0]; b_ols = np.mean(y_ols) - w_ols * np.mean(x_ols)
print("Parameters of OLS are: w: ",w_ols, "b: ", b_ols)
```

```
plt.plot(t, w_ols*t+b_ols, 'g--', label = 'least squares')

#######################################################
# helper function to solve weighted least squares
    # add your code here
def wls(x,y,c):
    add = 1
    x_add = np.insert(x, 0, values=add, axis=1)
    theta = (x_add.T * c * x_add).I * x_add.T * c * y
    b = theta[0]; w = theta[1]
    return w, b

# Add your code for robust regression MM algorithm below
x_rob = np.mat(x); y_rob = np.mat(y)
w_rob = 0; b_rob = 0
error = 1
while error > 0.01:
    c = np.eye(len(x_rob))
    for i in range(len(x_rob)):
        r_t_i = y_rob[i] - w_rob * x_rob[i] - b_rob
        c[i,i] = 1/(2*np.sqrt(1+r_t_i**2))
    w_new, b_new = wls(x_rob, y_rob, c)
    error = np.sqrt((w_new - w_rob)**2 + (b_new - b_rob)**2)
    w_rob, b_rob = w_new[0,0], b_new[0,0]

print("Parameters of ROB are: w: ",w_rob, "b: ", b_rob)
plt.plot(t, w_rob*t+b_rob, 'r:', label = 'robust')
legend = plt.legend(loc='upper right', shadow=True)
plt.show()
```

The result is:

```
Parameters of OLS are: w: -1.4749427077811519 b: 10.519708850514508
Parameters of robust regression are: w: 7.302235544182917 b: 6.186315755157274
```

The plot which shows the data, the true line, the OLS estimate and the robust estimate is as bellow:



Problem 4

(b)

The code and the result is as follows:

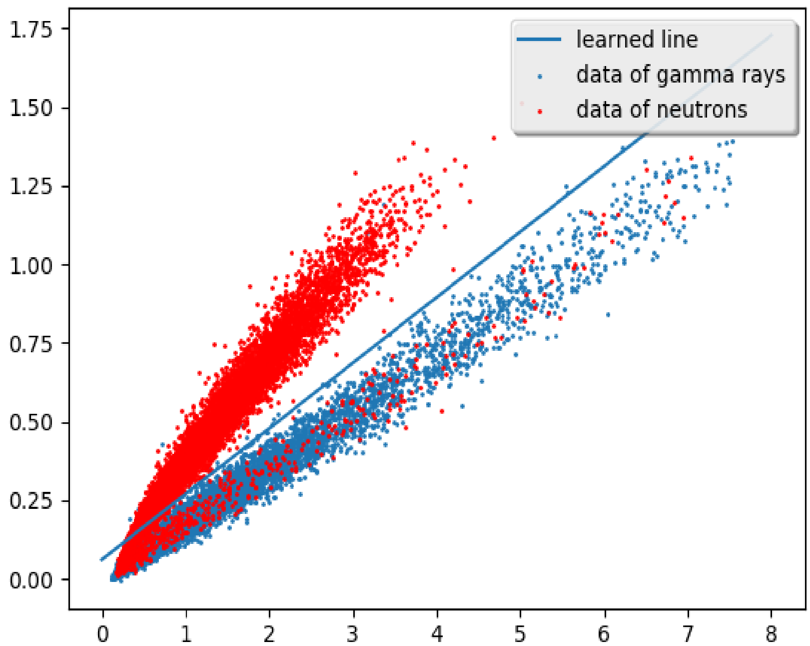
```
import numpy as np
import scipy.io as sio
import matplotlib.pyplot as plt
import pylab as pl

np.random.seed(0) # Seed the random number generator
# Load the data
nuclear = sio.loadmat('nuclear.mat')
x = nuclear['x']
y = nuclear['y'] # 1 represents neutrons and -1 represents gamma rays
d,n = x.shape

# Plot data
gamma_rays = []; neutrons = []
for i in range(n):
    if y[0,i] == 1:
```

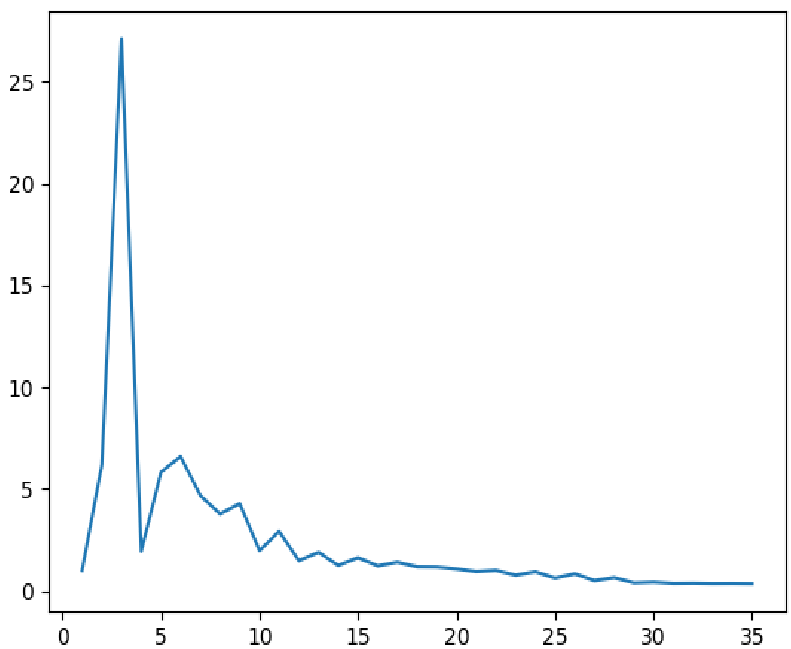
```
        neutrons.append(i)
    else:
        gamma_rays.append(i)
plt.figure()
plt.scatter(x[0,gamma_rays], x[1,gamma_rays], s = 1, label = 'data of gamma rays')
plt.scatter(x[0,neutrons], x[1,neutrons], s = 1, c = 'r', label = 'data of neutrons')

# Initialization and plot learned line
x = np.mat(x); y = np.mat(y)
add = 1; J = []
x_tilda = np.insert(x, 0, values=add, axis=0)
lamda = 0.001; omega = np.zeros(d); b = 0
Iteration = 35
for j in range(Iteration):
    u = 0; alpha = 100/(j+1); Loss = 0
    for i in range(n):
        h = 1 - y[0,i] * (np.dot(omega, x[:,i]) + b)
        if h[0,0] > 0:
            scale = 1
        elif h[0,0] < 0:
            scale = 0
        else:
            scale = np.random.rand()
        u += 1/n * (-y[0,i] * x_tilda[:,i]) * scale + lamda / n * np.mat([0, omega[0],
            omega[1]]).T
        Loss += max(0, h[0,0])
    J.append(Loss/n + lamda/2 * np.dot(omega, omega))
    b -= alpha * u[0]; omega[0] -= alpha * u[1]; omega[1] -= alpha * u[2]

print(b[0,0], omega[0], omega[1])
t = pl.frange(0,8,0.01)
n = pl.frange(1,Iteration)
plt.plot(t, -omega[0]/omega[1] * t - b[0,0]/omega[1], label = 'learned line')
legend = plt.legend(loc='upper right', shadow=True)
# Plot the objective function
plt.figure()
plt.plot(n, J, label = 'Objective Function')
plt.show()
```

The result is:

```
Parameters of estimated hyperplane are:
b = -1.1543275085302598 w_1 = -3.8809615376043824 w_2 = 18.631005972691405
the minimum achieved value of the objective function is: 0.3590574177462417
```

The plots showing the data and the learned line, and showing J as a function of iteration number is as bellow:





(c)

the code and the result is as follows:

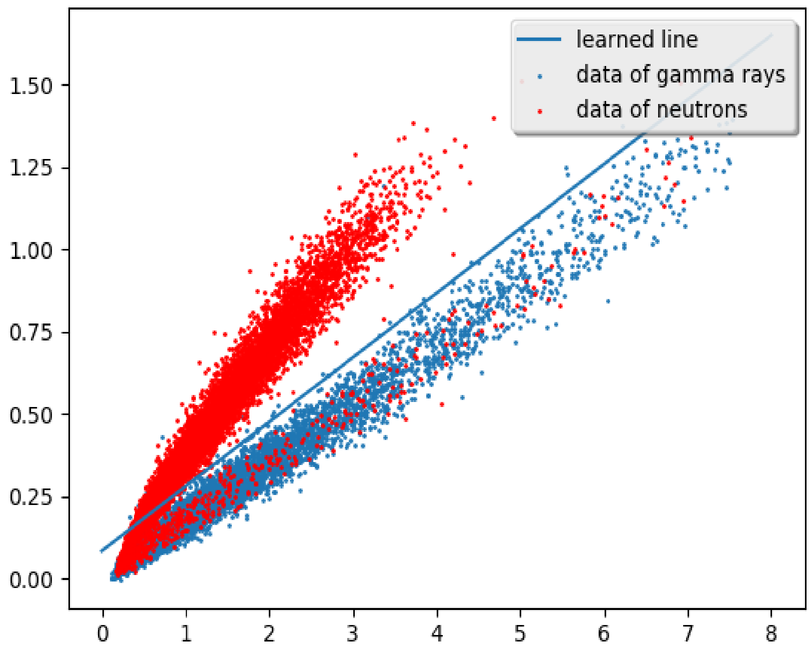
```
import numpy as np
import scipy.io as sio
import matplotlib.pyplot as plt
import pylab as pl

np.random.seed(0) # Seed the random number generator
# Load the data
nuclear = sio.loadmat('nuclear.mat')
x = nuclear['x']
y = nuclear['y'] # 1 represents neutrons and -1 represents gamma rays
d,n = x.shape

# Plot data
gamma_rays = []; neutrons = []
for i in range(n):
    if y[0,i] == 1:
        neutrons.append(i)
    else:
        gamma_rays.append(i)
plt.figure()
plt.scatter(x[0,gamma_rays], x[1,gamma_rays], s = 1, label = 'data of gamma rays')
plt.scatter(x[0,neutrons], x[1,neutrons], s = 1, c = 'r', label = 'data of neutrons')

# Initialization and plot learned line with stochastic sub-gradient method
x = np.mat(x); y = np.mat(y)
add = 1; J = []
x_tilda = np.insert(x, 0, values=add, axis=0)
lamda = 0.001; omega = np.zeros(d); b = 0
Iteration = 35
for j in range(Iteration):
    ui = 0; alpha = 100/(j+1); N = np.random.permutation(n); Loss = 0
    for i in N:
        h = 1 - y[0,i] * (np.dot(omega, x[:,i]) + b)
        if h[0,0] > 0:
            scale = 1
        elif h[0,0] < 0:
            scale = 0
        else:
            scale = np.random.rand()
        ui = 1/n * (-y[0,i] * x_tilda[:,i]) * scale + lamda / n * np.mat([0, omega[0],
            omega[1]]).T
```

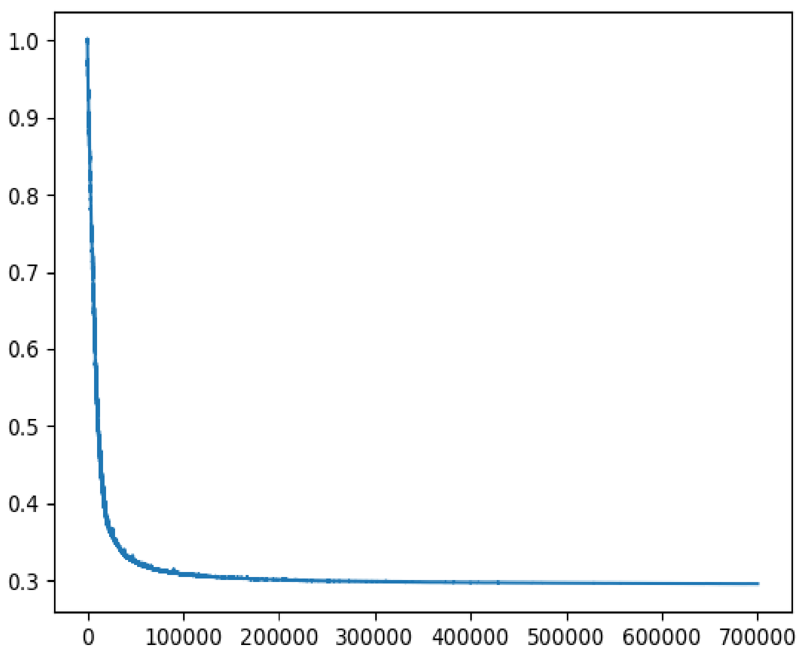
```
        b -= alpha * ui[0]; omega[0] -= alpha * ui[1]; omega[1] -= alpha * ui[2]
        t = (omega * x + b)
        Loss = np.array(1 - np.multiply(y, t))
        Loss *= (Loss > 0)
        # Loss += max(0, h[0, 0])
        J.append(np.sum(Loss)/n + lamda/2 * np.dot(omega, omega))

print("Parameters of estimated hyperplane are: b = ", b[0,0], "w_1 = ", omega[0], "w_2 = ",
    omega[1])
print("the minimum achieved value of the objective function is: ", min(J))
t = pl.frange(0,8,0.01)
n = pl.frange(1,Iteration*n)
plt.plot(t, -omega[0]/omega[1] * t - b[0,0]/omega[1], label = 'learned line')
legend = plt.legend(loc='upper right', shadow=True)

# Plot the objective function
plt.figure()
plt.plot(n, J, label = 'Objective Function')
plt.show()
```

The result is:

```
Parameters of estimated hyperplane are: b = -0.9463755603126593 w_1 = -2.200214343271486 w_2
    = 11.238670609411892
the minimum achieved value of the objective function is: 0.29596596817675697
```

The plots showing the data and the learned line, and showing J as a function of iteration number is as bellow:


learned line
data of gamma rays
data of neutrons
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
5
6
7
8


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
100000
200000
300000
400000
500000
600000
700000