Problem 1

Function:

```
function price = Cbs(T,S,K,r,sigma)
d1 = (log(S/K) + (r + sigma^2/2) * T)/(sigma * sqrt(T));
d2 = d1 - sigma * sqrt(T);
price = S * normcdf(d1) - K*exp(-r * T) * normcdf(d2);
end
```

Problem 2

Function:

```
function y = bisection(f,a,b,TOL)
if sign(f(a)) * sign(f(b)) >= 0
   error('f(a)f(b)<0 not satisfied');
end
while (b-a)/2 >TOL
   c = (a + b)/2;
   if f(c) == 0
      break
   end
   if sign(f(a)) * sign(f(c)) < 0
      b = c;
   else a = c;
   end
end
y = (a + b)/2;
end

function xc = secant(f,x0,x1,TOL)
x = [x0, x1];
while abs(x(2) - x(1))>TOL
xc = x(2) - (f(x(2)) * (x(2) - x(1)))/(f(x(2)) - f(x(1)));
x(1) = x(2);
x(2) = xc;
end
end
```

Main:

```
K = [500 550 600 650 700 750 800 850 900 950];
C = [210.3400  166.3140  126.5249  89.0857  59.5878  43.5040  31.3392  25.2330  20.1734  15.7494];
S = 700;
T = 1/4;
r = 0.03;
sigmac1 = [];
for i = 1:10
   fi = @(sigma)Cbs(T,S,K(i),r,sigma) - C(i);
   a = 0.0001; b = 1; TOL = 10^(-6);
```

```
    sigmac1(i) = bisection(fi,a,b,TOL);
end
vpa(sigmac1, 6)

sigmac2 = [];
for i = 1:10
    fi = @(sigma)Cbs(T,S,K(i),r,sigma) - C(i);
    x0 = 0.3; x1 = 0.35; TOL = 10^(-8);
    sigmac2(i) = secant(fi,x0,x1,TOL);
end
vpa(sigmac2, 6)
```

Result :

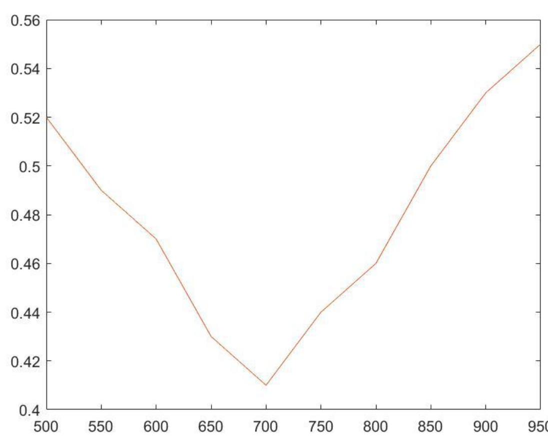
(1) $\sigma_{implied}(K)$ = [0.520000, 0.490000, 0.470000, 0.429999, 0.410001, 0.440000, 0.459999, 0.499999, 0.530001, 0.549999]

(2) $\sigma_{implied}(K)$ = [0.520000, 0.489999, 0.470000, 0.430000, 0.410000, 0.440000, 0.460000, 0.500000, 0.530000, 0.550000]

Problem 3

Main:

```
plot (K, sigmac1)
hold on
plot (K, sigmac2)
```



Problem 5 section 3.4 CP 3

Function:

Same as textbook of **splinecoeff.m** and **splineplot.m**

Main:

```
x = [0 1 2 3 4];
y = [1 3 3 4 2];
splinecoeff(x,y)
splineplot(x,y,10)
```

The coefficient of function:

2.660714285714286 0 -0.660714285714286

0.678571428571429 -1.982142857142857 1.303571428571429
0.625000000000000 1.928571428571428 -1.553571428571428
-0.178571428571429 -2.732142857142857 0.910714285714286

Thus, the function we get is:

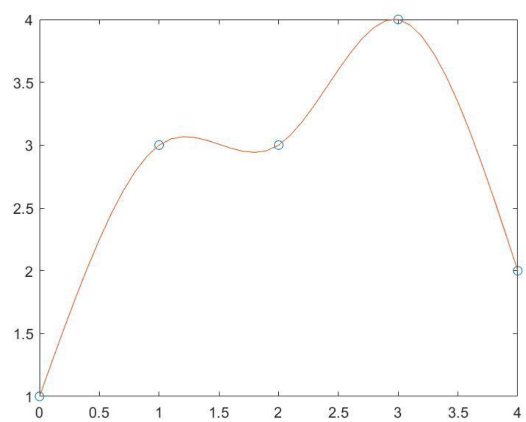
$S_1(x) = 1 + 2.6607x - 0.6607x^3$ is the function on interval [0,1]

$S_2(x) = 3 + 0.6786(x-1) - 1.9821(x-1)^2 + 1.3036(x-1)^3$ is the function on interval [1,2]

$S_3(x) = 3 + 0.6250(x-2) + 1.9286(x-2)^2 - 1.5536(x-2)^3$ is the function on interval [2,3]

$S_3(x) = 4 - 0.1786(x-3) - 2.7321(x-3)^2 + 0.9107(x-3)^3$ is the function on interval [3,4]

The plot is as follows:



Problem 6 section 5.2 CP 1 (b) & (h)

Function:

```
function y = com_trap_rule(f,a,b,m)
h = (b - a)/m;
x = linspace(a, b, m+1);
sumy = 0;
for i = 2:m
    sumy = sumy +f(x(i));
end
y =h/2 * (f(a) + f(b) + 2 * sumy);
end
```

Main:

```
f1 = @(x)x^3/(x^2 + 1);
yb16 = com_trap_rule(f1,0,1,16)
yb32 = com_trap_rule(f1,0,1,32)
syms x
vpa(abs(int(x^3/(x^2 + 1),0,1) - yb16),4)
vpa(abs(int(x^3/(x^2 + 1),0,1) - yb32),4)
f2 = @(x)x/sqrt(x^4 + 1);
yh16 = com_trap_rule(f2,0,1,16)
yh32 = com_trap_rule(f2,0,1,32)
```

```
syms x
vpa(abs(int(x/sqrt(x^4 + 1),0,1) - yh16),4)
vpa(abs(int(x/sqrt(x^4 + 1),0,1) - yh32),4)
```

Result:

(b) For m = 16, the approximate of definite integral is 0.153752089736523, the error with correct integral is 0.0003257
For m = 32, the approximate of definite integral is 0.153507799866167, the error with correct integral is 8.139e-5
The correct integral is 1/2 - log(2)/2
(h) For m = 16, the approximate of definite integral is 0.440361182629694, the error with correct integral is 0.0003256
For m = 32, the approximate of definite integral is 0.440605407679783, the error with correct integral is 8.139e-5
The correct integral is log(2^(1/2) + 1)/2

Problem 7 section 5.3 CP 1 (a) & (c)

Function:

```
function r=romberg(f,a,b,n)
h=(b-a)./(2.^(0:n-1));
r(1,1)=(b-a)*(f(a)+f(b))/2;
for j=2:n
subtotal = 0;
for i=1:2^(j-2)
subtotal = subtotal + f(a+(2*i-1)*h(j));
end
r(j,1) = r(j-1,1)/2+h(j)*subtotal;
for k=2:j
r(j,k)=(4^(k-1)*r(j,k-1)-r(j-1,k-1))/(4^(k-1)-1);
end
end
```

Main:

```
f1 = @(x)x/sqrt(x^2 + 9);
ya = romberg(f1,0,4,5);
ya(5,5)
syms x
vpa(abs(int(x/sqrt(x^2 + 9),0,4) - ya(5,5)),4)
int(x/sqrt(x^2 + 9),0,4)
f2 = @(x)x * exp(x)
yc = romberg(f2,0,1,5);
yc(5,5)
syms x
vpa(abs(int(x * exp(x),0,1) - yc(5,5)),4)
int(x * exp(x),0,1)
```

(a) The Romberg Integration approximation of R_{55} is 2.0000, the correct integral is 2, and the error is 1.041e-7

(c) The Romberg Integration approximation of R_{55} is 1.0000 the correct integral is 1, and the error is 3.477e-13