Introduction

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VI Tutorial @ IST

https://vitutorial.github.io/tour/ist2019

Why are we here today?

Because NN models work

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but they may struggle with

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- partial supervision
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As we progress we will

- develop a shared vocabulary to talk about generative models powered by NNs
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Goal

- you should be able to navigate through fresh literature
- and start combining probabilistic models and NNs

Supervised problems

We have data $x^{(1)}, \dots, x^{(N)}$ e.g. sentences, images
generated by some **unknown** procedure which we assume can be captured by a probabilistic model

- with **known** probability (mass/density) function e.g.

$$X \sim \mathrm{Cat}(\theta_1, \dots, \theta_K) \quad \text{or} \quad X \sim \mathcal{N}(\theta_\mu, \theta_\sigma^2)$$

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and estimate parameters θ that assign maximum likelihood $p(x^{(1)}, \ldots, x^{(N)}|\theta)$ to observations

Supervised NN models

Let y be all side information available
e.g. deterministic *inputs/features/predictors*

Have neural networks predict parameters of our probabilistic model

$$X|y \sim \mathrm{Cat}(\pi_\theta(y)) \quad \text{or} \quad X|y \sim \mathcal{N}(\mu_\theta(y), \sigma_\theta(y)^2)$$

and proceed to estimate parameters θ of the NNs

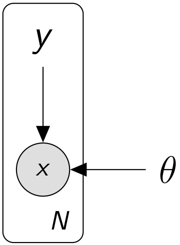
Graphical model

Random variables

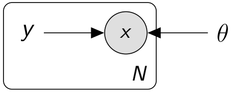
- observed data
  $x^{(1)}, \ldots, x^{(N)}$

Deterministic variables

- inputs or predictors
  $y^{(1)}, \ldots, y^{(N)}$
- model parameters θ



Multiple problems, same language



(Conditional) Density estimation

	Side information (y)	Observation (x)
Parsing	a sentence	its syntactic/semantic parse tree/graph
Translation	a sentence	its translation
Captioning	an image	caption in English
Entailment	a text and hypothesis	entailment relation

Task-driven feature extraction

Often our side information is itself some high dimensional data

- y is a sentence and x a tree
- y is the source sentence and x is the target
- y is an image and x is a caption

and part of the job of the NNs that parametrise our models is to also deterministically encode that input in a low-dimensional space

NN as efficient parametrisation

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Prediction is done by a decision rule outside the statistical model

- e.g. argmax, beam search

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MLE via gradient-based optimisation

If the log-likelihood is **differentiable** and **tractable** then backpropagation gives us the gradient

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and we can update θ in the direction

$$\gamma \nabla_\theta \mathcal{L}(\theta|x^{(1:N)})$$

to attain a local maximum of the likelihood function

Big Data

For large N, computing the gradient is inconvenient

$$\nabla_\theta \mathcal{L}(\theta | x^{(1:N)}) = \underbrace{\sum_{s=1}^{N} \nabla_\theta \log p(x^{(s)} | \theta)}_{\text{too many terms}}$$

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&= \sum_{s=1}^{N} \mathcal{U}(s|{}^1\!/\!{}_N) N \nabla_\theta \log p(x^{(s)}|\theta) \\
&= \mathbb{E}_{S\sim\mathcal{U}({}^1\!/\!{}_N)} \left[N \nabla_\theta \log p(x^{(S)}|\theta) \right]
\end{aligned}$$

S selects data points uniformly at random

Stochastic optimisation

For large N, we can use a gradient estimate

$$\nabla_\theta \mathcal{L}(\theta | x^{(1:N)}) = \underbrace{\mathbb{E}_{S \sim \mathcal{U}(1/N)} \left[N \nabla_\theta \log p(x^{(S)} | \theta) \right]}_{\text{expected gradient :)}}$$

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and take a step in the direction

$$\gamma \frac{N}{M} \underbrace{\nabla_\theta \mathcal{L}(\theta|x^{(s_1:s_M)})}_{\text{stochastic gradient}}$$

where $x^{(s_1:s_M)}$ is a random mini-batch of size M

DL in NLP recipe

Maximum likelihood estimation

- tells you which loss to optimise
  (i.e. negative log-likelihood)

Automatic differentiation (*backprop*)

- "give me a tractable forward pass and I will give you gradients"

Stochastic optimisation powered by backprop

- general purpose gradient-based optimisers

Constraints

Differentiability

- intermediate representations must be continuous
- activations must be differentiable

Tractability

- the likelihood function must be evaluated exactly, thus it's required to be tractable

When do we have intractable likelihood?

Unsupervised problems contain unobserved random variables

$$p(x, z|\theta)$$

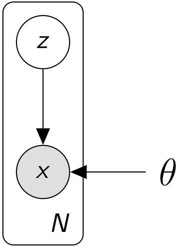
thus assessing the marginal likelihood requires marginalisation of latent variables

$$p(x|\theta) = \int p(x, z|\theta)\, \mathrm{d}z$$

Latent variable model

Latent random variables

- unobserved
- or unobservable



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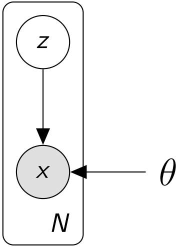

A joint distribution over data and unknowns

$$p(x, z|\theta) = p(z)p(x|z, \theta)$$

Examples of latent variable models

Discrete latent variable, continuous observation

$$\underbrace{p(x|\theta) = \sum_{c=1}^{K} \mathrm{Cat}(c|\pi_1, \ldots, \pi_K) \underbrace{\mathcal{N}(x|\mu_\theta(c), \sigma_\theta(c)^2)}_{\text{forward pass}}}_{\textbf{too many forward passes}}$$

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Continuous latent variable, discrete observation

$$p(x|\theta) = \underbrace{\int \mathcal{N}(z|0, I) \underbrace{\mathrm{Cat}(x|\pi_\theta(z))}_{\text{forward pass}} \, \mathrm{d}z}_{\textbf{infinitely many forward passes}}$$

Intractable gradient

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- uncertainty quantification
  e.g. Bayesian NNs

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a mixture model whose mixture components are labelled by words marginalisation $O(|x||y|)$

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A factor model whose factors are labelled by words

marginalisation $O(2^{|y|})$

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