Matrix Multiplication and Dot Product

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November 2, 2021

1 Dot Product

Given two vectors $\vec{a}$ and $\vec{b}$ we can form a dot product:

$$c = \vec{a} \cdot \vec{b}$$

c here is a scalar value, not another vector.

In other notation:

$$c = (a_1, a_2, a_3, ...) \cdot (b_1, b_2, b_3...)$$

This is calculated by multiplying corresponding elements and summing.

$$c = a_1 b_1 + a_2 b_2 + a_3 b_3 + ...$$

Here's an example, where I've arbitrarily written one vector horizontally and the other vertically.

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = 1.4 + 2.5 + 3.6 = 32$$

2 Matrix Multiplication

Two matrices can be multiplied to produce a third matrix, provided they are suitably shaped.

$$AB = C \tag{1}$$

Note that matrix multiplication is not commutative. In other words, in general:

$$AB \neq BA$$

Before starting:

- Check that the number of columns of the first matrix in the multiplication is equal to the number of rows of the second matrix in the multiplication.
- The result has the same number of rows as the first matrix and the same number of columns as the second matrix.

Rule for calculating result:

$$c_{ij} = \sum_n a_{in} b_{nj} \tag{2}$$

> To calculate the ijth entry of the result, we take the dot product of the ith row in the first matrix (the multiplicand) with the jth column in the second matrix (the multiplier).

i here is the row number of the entry, and j is the column number.

n is the number of columns in the multiplicand, or equivalently, the number of rows in the multiplier.

3 Matrix Multiplication Examples

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 8 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 11 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 21 \\ 37 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 21 \\ 3 & 37 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 24 \\ 5 & 43 \end{pmatrix}$$