<u>**MATHEMATICAL SCIENCES**</u>

<u>**(INCLUDING STATISTICS)**</u>

Newsvendor Problem:

Alternative Optimality Criteria

Aditya Pal Chaudhuri, Ishani Karmakar, Somjit Roy, Subharanjan Mandal

UG Students,

Department of Statistics,

St. Xavier's College,

30 Mother Teresa Sarani, Kolkata – 700 016.

West Bengal, India.

Email Id: somjit.roy2001@gmail.com

KEY WORDS

Classical Newsboy Problem, Cost Distribution, Penalized Mean, Modal Cost, Exceedance Probability, Random Supply, EOQ (Expected Optimal Quantity).

ABSTRACT

The Classical Newsboy Problem deals mainly with mean *minimizing solution to obtain the optimal order quantity*. In this paper we would resort to some *Alternative Optimality Criteria* to obtain the optimal order quantity; namely:

a) **Minimization of Standard Deviation of the Cost Distribution.**
b) **Minimization of Coefficient of Variation of the Cost Distribution.**
c) **Minimization of Standard Deviation Penalized Mean of the Cost Distribution.**
d) **Minimization of the Modal Cost.**
e) **Minimization of the probability of the Cost Distribution exceeding a given high value.**

Considering two realistic situations:

- *Supply is same*
- *Supply varies*

***with the order quantity*; we discuss the above procedures briefly with derivations and Numerical Illustrations.**

1. ***INTRODUCTION***

Let us consider the Newsvendor model where a newspaper vendor starts with *'q' newspapers*. Let the *demand be random*. As a result, he eventually incurs either *shortage cost or excess cost* corresponding to *excess demand or lack of demand.*

The classical solution: The optimal order quantity is determined by *mean minimizing the total cost*.

In this paper, some alternative methods of determining the optimal order quantity are considered, which are as follows:

1. **If the standard deviation of the distribution of cost is large enough, then mean minimization of the cost distribution may not give a satisfactory result as the actual cost incurred in any particular case may be higher than the expected cost.**
   Under this circumstance, the optimal order quantity may be obtained by,
   a. ***MINIMISING THE STANDARD DEVIATION OF THE COST DISTRIBUTION.***
   b. ***MINIMISING THE COEFFICIENT OF VARIATION OF THE COST DISTRIBUTION.***
2. **The optimal order quantity may also be obtained by minimizing the sum of expectation and 'k' times the standard deviation of the cost deviation, where k is a chosen constant, i.e. by *MINIMISING THE STANDARD DEVIATION PENALISED MEAN OF THE COST DISTRIBUION*.**
3. **Ensuring that the maximum possible cost is minimized (similar in principal to the Minimax criterion) the optimal orders may be obtained by *MINIMISING THE MODAL COST*.**
4. **Another approach to find out the optimal order quantity is to safeguard against the cost *by MINIMISING THE PROBABILITY OF THE COST DISTRIBUTION EXCEEDING A GIVEN HIGH VALUE.***

2. SUPPLY IS SAME AS THE ORDER QUANTITY

THE MODEL

Let X: Demand.

Y_q: Total Cost.

q: Order Quantity.

c_1: Shortage Cost.

c_2: Excess Cost.

Therefore,

$$\boldsymbol{Y_q} = \begin{cases} \boldsymbol{c_1(X-q)\ if\ X} > q \\ \boldsymbol{c_2(q-X)\ if\ X} < q \end{cases}$$

The probability density functions of X and Y will be denoted by f(x) and g(y) respectively.

Under this setup, the Expected Total Cost is given by

$$E[Y_q] = c_1 \int_q^{\infty} (x-q)f(x)dx + c_2 \int_0^{q} (q-x)f(x)dx$$

2.1. MEAN MINIMIZING (CLASSICAL) SOLUTIONS

Exponential Distribution:

Here we take $\boldsymbol{f(x) = \lambda e^{-\lambda x};\ x} > 0, \lambda > 0.$

Then,

$$E[Y_q] = c_1 \int_q^{\infty} (x-q)\lambda e^{-\lambda x} dx + c_2 \int_0^{q} (q-x)\lambda e^{-\lambda x} dx$$

$$= c_1 \frac{e^{-\lambda q}}{\lambda} + c_2\{q - \frac{1}{\lambda} + \frac{e^{-\lambda q}}{\lambda}\}$$

The classical solution consists in obtaining the optimal value of q, say q_{opt}, by minimizing the expected total cost. In this situation,

$q_{opt} = -\frac{1}{\lambda} ln \frac{c_2}{c_1+c_2}$.

The corresponding expected total cost is then given by

$$E\left[Y_{q_{opt}}\right] = c_1 \frac{e^{-\lambda q_{opt}}}{\lambda} c_2\{q_{opt} - \frac{1}{\lambda} + \frac{e^{-\lambda q_{opt}}}{\lambda}\}$$

Beta (2,1) Distribution:

Here $\boldsymbol{f(x) = 2x, 0} < x < 1.$

Then,

$$E[Y_q] = c_1 \int_q^{1} (x-q)2x dx + c_2 \int_0^{q} (q-x)2x dx$$

$$= 2c_1 \left\{\frac{1}{3} - \frac{q}{2} + \frac{q^3}{6}\right\} + 2c_2 \frac{q^3}{6}.$$

Therefore, $q_{opt} = \sqrt{\frac{c_1}{c_1+c_2}}$.

The corresponding Expected total cost is given by

$$E[Y_{q_{opt}}] = 2c_1 \left\{\frac{1}{3} - \frac{q_{opt}}{2} + \frac{{q_{opt}}^3}{6}\right\} + 2c_2 \frac{{q_{opt}}^3}{6}.$$

Beta (1,2) Distribution:

Here $\boldsymbol{f(x) = 2(1-x), 0} < x < 1.$

Then,

$$E[Y_q] = c_1 \int_q^1 (x-q)2(1-x)dx + c_2 \int_0^q (q-x)2(1-x)dx$$

$$= (c_1+c_2)q^2 - (c_1+c_2)\frac{q^3}{3} + \frac{c_1}{3} - c_1 q$$

Therefore, $q_{opt} = 1 \pm \sqrt{1 - \frac{c_1}{(c_1+c_2)}}$

The corresponding Expected total cost is given by

$$E\left[Y_{q_{opt}}\right] = 2c_1\left\{\frac{1}{6} - \frac{q_{opt}}{2} + \frac{{q_{opt}}^2}{2} - \frac{{q_{opt}}^3}{6}\right\} + 2c_2\{\frac{{q_{opt}}^2}{2} - \frac{{q_{opt}}^3}{6}\}.$$

2.2. THE ALTERNATIVE METHODS

2.2.1. Minimization of the Standard Deviation of the Cost Distribution

Exponential Distribution:

In this case,

$$E[{Y_q}^2] = {c_1}^2 \int_q^\infty (x-q)^2 \lambda e^{-\lambda x} dx + {c_2}^2 \int_0^q (q-x)^2 \lambda e^{-\lambda x} dx$$

$$= {c_1}^2 \frac{2e^{-\lambda q}}{\lambda^2} + {c_2}^2 \{q^2 - \frac{2q}{\lambda} + \frac{2}{\lambda^2} - \frac{2e^{-\lambda q}}{\lambda^2}\}$$

Therefore,

$$Var[Y_q] = E[Y_q{}^2] - \{E[Y_q]\}^2$$
$$= e^{-\lambda q}\left\{\frac{2c_1{}^2}{\lambda^2} + \frac{2c_1c_2}{\lambda^2}\right\} - e^{-2\lambda q}\left\{\frac{c_1{}^2}{\lambda^2} + \frac{c_2{}^2}{\lambda^2} + \frac{2c_1c_2}{\lambda^2}\right\} - qe^{-\lambda q}\left\{\frac{2c_2{}^2}{\lambda^2} + \frac{2c_1c_2}{\lambda^2}\right\} + \frac{c_2{}^2}{\lambda^2}$$

<u>Beta (2,1) Distribution:</u>

Here, $E[Y_q{}^2] = 2c_1{}^2\left(\frac{1}{4} - \frac{2q}{3} + \frac{q^2}{2} - \frac{q^4}{12}\right) + c_2{}^2\frac{q^4}{6}.$

Therefore,

$$Var[Y_q] = \frac{1}{18}(-3(-1+q)^3(3+q)c_1{}^2 + 3q^4c_2{}^2$$
$$-2((2-3q+q^3)c_1 + q^3c_2)^2)$$

<u>Beta (1,2) Distribution:</u>

Here, $E[Y_q{}^2] = 2c_1{}^2\left(\frac{1}{12} - \frac{q}{3} + \frac{q^2}{2} - \frac{q^3}{3} + \frac{q^4}{12}\right) + 2c_2{}^2(\frac{q^3}{3} - \frac{q^4}{12}).$

Therefore,

$$Var[Y_q] = \frac{1}{18}(3(-1+q)^4c_1{}^2 - 3(-4+q)q^3c_2{}^2 - 2((-1+q)^3c_1$$
$$+(-3+q)q^2c_2)^2)$$

The optimal value of q, say q_1, may now be obtained for all the above demand distributions by minimizing $SD[Y_q]$ for specified values of c_1 and c_2. The corresponding optimal expected total cost will be denoted by Y_{q1}.

2.2.2. <u>Minimization of the Co-efficient of Variation of the Cost distribution</u>

Here the optimal value of q, say q_2 will be obtained by minimizing

$$c.v.\left[Y_q\right] = \frac{SD\left[Y_q\right]}{E\left[Y_q\right]}.$$

The corresponding optimal expected total cost will be denoted by Y_{q2}.

2.2.3. Here the optimal value of q, say q_3 will be obtained by minimizing

$$S(q) = E\left[Y_q\right] + k\,SD\left[Y_q\right],$$

k being a suitably chosen constant.

Exponential Distribution:

Here $S(q) = c_1 \frac{e^{-\lambda q}}{\lambda} + c_2\{q - \frac{1}{\lambda} + \frac{e^{-\lambda q}}{\lambda}\} + k\left\{e^{-\lambda q}\left\{\frac{2{c_1}^2}{\lambda^2} + \frac{2c_1c_2}{\lambda^2}\right\} - e^{-2\lambda q}\left\{\frac{{c_1}^2}{\lambda^2} + \frac{{c_2}^2}{\lambda^2} + \frac{2c_1c_2}{\lambda^2}\right\} - qe^{-\lambda q}\left\{\frac{2{c_2}^2}{\lambda^2} + \frac{2c_1c_2}{\lambda^2}\right\} + \frac{{c_2}^2}{\lambda^2}\right\}^{\frac{1}{2}}.$

The corresponding optimal expected total cost will be denoted by Y_{q3}.

Beta (2,1) Distribution:

Here $S(q) = 2c_1\left\{\frac{1}{3} - \frac{q}{2} + \frac{q^3}{6}\right\} + 2c_2\frac{q^3}{6} + k\left\{\frac{1}{18}\left(-3(-1+q)^3(3+q){c_1}^2 + 3q^4{c_2}^2 - 2\left((2-3q+q^3)c_1 + q^3c_2\right)^2\right)\right\}^{1/2}.$

The corresponding optimal expected total cost will be denoted by Y_{q3}.

Beta (1,2) Distribution:

Here $S(q) = 2c_1\left\{\frac{1}{6} - \frac{q}{2} + \frac{q^2}{2} - \frac{q^3}{6}\right\} + 2c_2\left\{\frac{q^2}{2} - \frac{q^3}{6}\right\} + k\{\frac{1}{18}(3(-1+q)^4{c_1}^2 - 3(-4+q)q^3{c_2}^2 - 2((-1+q)^3c_1 + (-3+q)q^2c_2)^2)\}^{1/2}$

The corresponding optimal expected total cost will be denoted by Y_{q3}.

2.2.4. Minimization of the modal cost

Let f(x) be the PDF of the demand distribution and F(x) be the corresponding CDF. The CDF of the cost (Y) distribution is then given by

$$\boldsymbol{H_q(y) = F\left(q + \frac{y}{c_1}\right) - F\left(q - \frac{y}{c_2}\right)}.$$

Having obtained the CDF $H_q(y)$ and hence the PDF $h_q(y)$ of Y, we first obtain the mode of Y by solving the equation $h_q'(y) = 0$. Suppose a solution to this equation is $y_0(q)$. The optimal order quantity may then be obtained by minimizing $y_0(q)$ with respect to q. It may be pointed out that this method can be used in a wide range of situations apart from the cases when the demand has an exponential family of distributions or has a PDF which is not differentiable twice.

We take the PDF of demand distribution as

$$\boldsymbol{f(x) = 3x^2, 0} < x < 1.$$

The CDF of Y is given as,

$$F(y) = P(Y \leq y)$$

$$= (1 - q^3)\left\{\left(q + \frac{y}{c_1}\right)^3 - q^3\right\} + q^3\{q^3 - (q - \frac{y}{c_2})^3\}$$

Consequently, the PDF of Y is obtained as

$$f(y) = (1 - q^3)\{\frac{3}{c_1}\left(q + \frac{y}{c_1}\right)^2\} + q^3\{\frac{3}{c_2}\left(q - \frac{y}{c_2}\right)^2\}$$

The mode of Y is then obtained as below:

$$\frac{df(y)}{dy} = 0$$

$$\Rightarrow \frac{d}{dy}\{(\frac{3}{c_1})(1 - q^3)\left(q + \frac{y}{c_1}\right)^2 + (\frac{3q^3}{c_2})\left(q - \frac{y}{c_2}\right)^2\} = 0$$

$$\Rightarrow \frac{6(1-q^3)(q+\frac{y}{c_1})}{{c_1}^2} - \frac{6q^3(q-\frac{y}{c_2})}{{c_2}^2} = 0$$

$$\Rightarrow \frac{6q(1-q^3)}{{c_1}^2} + \frac{6y(1-q^3)}{{c_1}^3} = \frac{6q^4}{{c_2}^2} - \frac{6yq^3}{{c_2}^3}$$

$$\Rightarrow y\left\{\frac{6(1-q^3)}{{c_1}^3}+\frac{6q^3}{{c_2}^3}\right\}=\left\{\frac{6q^4}{{c_2}^2}-\frac{6q(1-q^3)}{{c_1}^2}\right\}$$

$$\Rightarrow y=\frac{\left\{\frac{q^4}{{c_2}^2}-\frac{q(1-q^3)}{{c_1}^2}\right\}}{\left\{\frac{(1-q^3)}{{c_1}^3}+\frac{q^3}{{c_2}^3}\right\}}.$$

Thus, the mode of Y is given by

$$\boldsymbol{M_0}=\frac{\left\{\frac{q^4}{{c_2}^2}-\frac{q(1-q^3)}{{c_1}^2}\right\}}{\left\{\frac{(1-q^3)}{{c_1}^3}+\frac{q^3}{{c_2}^3}\right\}}=\frac{q^4\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)-\frac{q}{{c_1}^2}}{q^3\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)+\frac{1}{{c_1}^3}}.$$

It may clearly be seen that the mode does not exist for $c_1=c_2$.

Now, to minimize mode, we differentiate M_0 with respect to q and equate it to zero.

Differentiating M_0 with respect to q, we get:

$$\frac{d[M_0]}{dq}=\frac{\left[q^3\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)+\frac{1}{{c_1}^3}\right]\left[4q^3\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)-\frac{1}{{c_1}^2}\right]-[q^4\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)-\frac{q}{{c_1}^2}][3q^2\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)]}{[q^3\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)+\frac{1}{{c_1}^3}]^2}$$

Now, $\frac{d[M_0]}{dq}=0$ gives

$$4q^6\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)+q^3\left[\left\{\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)\frac{4}{{c_1}^3}\right\}-\left\{\frac{1}{{c_1}^2}\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)\right\}\right]-\frac{1}{{c_1}^5}$$
$$-\quad 3q^6\left[\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)\right]+q^3\left[\frac{3}{{c_1}^2}\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)\right]=0$$

or, $q^6\left(\frac{1}{{c_2}^3}-\frac{1}{{c_1}^3}\right)\left(\frac{1}{{c_1}^2}+\frac{1}{{c_2}^2}\right)+q^3\left(\frac{2}{{c_1}^5}+\frac{2}{{c_1}^2{c_2}^3}+\frac{4}{{c_1}^3{c_2}^2}\right)-\frac{1}{{c_1}^5}=0.$

For varying values of c_1 *and* c_2, we obtain the value ofq (say, q_4) for which M_0 is minimized.

Let us now take the PDF of demand distribution as

$$f(x)=3(1-x)^2, 0<x<1.$$

The CDF of Y is given by,

$$F(y) = (1-q)^3\left[(1-q)^3 - \left\{(1-q) - \frac{y}{c_1}\right\}^3\right] +$$

$$\{1-(1-q)^3\}\left[\left\{(1-q) + \frac{y}{c_2}\right\}^3 - (1-q)^3\right]$$

Consequently, the PDF of Y is obtained as

$$f(y) z^3\{\tfrac{3}{c_2}\left(z - \tfrac{y}{c_2}\right)^2\} + (1-z^3)\left\{\tfrac{3}{c_1}\left(z + \tfrac{y}{c_1}\right)^2\right\}, z = 1-q.$$

The mode of Y is then obtained as below:

$$\boldsymbol{M_0} = \frac{\left\{\frac{z^4}{c_2{}^2} - \frac{z(1-z^3)}{c_1{}^2}\right\}}{\left\{\frac{(1-z^3)}{c_1{}^3} + \frac{z^3}{c_2{}^3}\right\}} = \frac{(1-q)^4\left(\frac{1}{c_1{}^2} + \frac{1}{c_2{}^2}\right) - \frac{(1-q)}{c_1{}^2}}{(1-q)^3\left(\frac{1}{c_2{}^3} - \frac{1}{c_1{}^3}\right) + \frac{1}{c_1{}^3}}$$

Here also, it may be seen that the mode does not exist for $c_1 = c_2$.

Now, $\frac{d[\mathrm{M_0}]}{dq} = 0$

$$\Rightarrow 4z^6\left(\frac{1}{c_2{}^3} - \frac{1}{c_1{}^3}\right)\left(\frac{1}{c_1{}^2} + \frac{1}{c_2{}^2}\right) + z^3\left[\left\{\left(\frac{1}{c_1{}^2} + \frac{1}{c_2{}^2}\right)\frac{4}{c_1{}^3}\right\} - \left\{\frac{1}{c_1{}^2}\left(\frac{1}{c_2{}^3} - \frac{1}{c_1{}^3}\right)\right\}\right] - \frac{1}{c_1{}^5}$$

$$-3z^6\left[\left(\frac{1}{c_2{}^3} - \frac{1}{c_1{}^3}\right)\left(\frac{1}{c_1{}^2} + \frac{1}{c_2{}^2}\right)\right] + z^3\left[\frac{3}{c_1{}^2}\left(\frac{1}{c_2{}^3} - \frac{1}{c_1{}^3}\right)\right] = 0$$

$$\Rightarrow z^6\left(\frac{1}{c_2{}^3} - \frac{1}{c_1{}^3}\right)\left(\frac{1}{c_1{}^2} + \frac{1}{c_2{}^2}\right) + z^3\left(\frac{2}{c_1{}^5} + \frac{2}{c_1{}^2 c_2{}^3} + \frac{4}{c_1{}^3 c_2{}^2}\right) - \frac{1}{c_1{}^5} = 0$$

For varying values of c_1 *and* c_2, we obtain the value of q (say, q_4) for which M_0 is minimized.

2.2.5. Minimization of the probability of the cost distribution exceeding a given high value

For f(x), the PDF of the demand distribution, we first obtain, $H_q(y)$, the CDF of the cost distribution. For a specified y, we obtain 'q' by minimizing $\overline{H}_q(y) = P\{Y > y\}$.

Here we take $\boldsymbol{f(x) = 12x(1-x)^2}$, $0 < x < 1$.

The CDF of the cost distribution is then given by

$$H_q(y) = F\left(q + \frac{y}{c1}\right) - F(q - \frac{y}{c2})$$

$$= \frac{12(c_1+c_2)(-1+q)^2 qy}{c_1 c_2} - \frac{6(c1^2 - c2^2)(1-4q+3q^2)y^2}{c_1^2 c_2^2} + \frac{4(c1^3+c2^3)(-2+3q)y^3}{c_1^3 c_2^3} + (\frac{3}{c_1^4} - \frac{3}{c_2^4})y^4.$$

Then, $\frac{d}{dq}\overline{H}_q(y) = -\frac{36(c_1+c_2)q^2 y}{c_1 c_2} + \frac{12(c_1+c_2)qy(4c_1c_2+3c_1y-3c_2y)}{c_1^2c_2^2} - \frac{12y(c_1^3c_2^3 - 2c_1c_2^3y + c_2^3y^2 + c_1^3(c_2+y)^2)}{c_1^3c_2^3}$.

For a given y, by solving $\frac{d}{dq}\overline{H}_q(y) = 0$, one can find ‘q’. We shall denote this by q_5.

2.3. SOME NUMERICAL EXAMPLES

2.3.1. Exponential Demand:

Model Parameters				Optimal Criteria							
λ	c_1	c_2	k	3.2.1 Classical		3.2.2.1 SD		3.2.2.2 CV		3.2.2.3 Penalized Mean	
				EOQ	Cost	EOQ	Cost	EOQ	Cost	EOQ	Cost
0.1	***1***	***2***	***1***	***4.05***	***8.10***	***8.75***	***10.01***	***2.4***	***8.40***	***5.86***	***8.42***
			2							***6.78***	***8.79***
	1	***0.5***	***1***	***10.98***	***5.49***	***28.3***	***10.03***	***6.7***	***6.02***	***16.57***	***6.15***
			2							***19.5***	***6.88***
0.05	***1***	***2***	***1***	***8.11***	***16.22***	***17.5***	***20.01***	***4.8***	***16.80***	***11.72***	***16.83***

			2							13.57	17.58
	1	0.5	1	21.97	10.99	56.5	76.56	13.3	12.08	33.13	12.29
			2							39.00	13.77

2.3.2. Beta (2,1) Demand:

Model Parameters			Optimal Criteria							
c_1	c_2	k	3.2.1 Classical		3.2.2.1 SD		3.2.2.2 CV		3.2.2.3 Penalized Mean	
			EOQ	Cost	EOQ	Cost	EOQ	Cost	EOQ	Cost
1	2	1	0.577	0.282	0.455	0.306	0.785	0.365	1.139	1.0053
		2							1.093	0.8800
1	0.5	1	0.816	0.122	0.757	0.343	0.917	0.135	1.087	0.2222
		2							1.053	0.1980

2.3.3. Beta (1,2) Demand:

Model Parameters			Optimal Criteria							
c_1	c_2	k	3.2.1 Classical		3.2.2.1 SD		3.2.2.2 CV		3.2.2.3 Penalized Mean	
			EOQ	Cost	EOQ	Cost	EOQ	Cost	EOQ	Cost
1	2	1	0.184	0.245	0.241	0.252	0.629	0.330	0.412	0.361
		2							0.577	0.563
1	0.5	1	0.423	0.140	0.545	0.153	1.158	0.410	0.491	0.145
		2							0.510	0.147

2.3.4. <u>*Minimization of the modal cost:*</u>

B(3,1) Distribution:

c_1	c_2	q_4
1	*2*	*0.58*
1	*0.5*	*0.93*

B(1,3) Distribution:

c_1	c_2	q_4
1	*2*	*0.69*
1	*0.5*	*1.03*

2.3.5. <u>*Minimization of the probability of the cost distribution exceeding a given high value*</u>

It may be observed that, for $\boldsymbol{c_1 = 1, c_2 = 2}$, the mean minimizing ***EOQ*** and the ***corresponding cost*** come as ***0.29*** and ***0.22*** respectively. Hence, we have chosen the value of 'y' around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.22}, \textit{we have } \boldsymbol{q = 0.29}$.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.32}, \textit{we have } \boldsymbol{q = 0.28}$.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.12}, \textit{we have } \boldsymbol{q = 0.31}$.

It may be observed that, for $\boldsymbol{c_1 = 1, c_2 = 0.5}$, the mean minimizing ***EOQ*** and the ***corresponding cost*** come as ***0.49*** and ***0.11*** respectively. Hence, we have chosen the value of 'y' around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.11}, \textit{we have } \boldsymbol{q = 0.40}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.21}, \textit{we have } \boldsymbol{q = 0.49}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.01}, \textit{we have } \boldsymbol{q = 0.34}$.

It may be observed that, for $\boldsymbol{c_1 = 1, c_2 = 1}$, the mean minimizing ***EOQ*** and the ***corresponding cost*** come as ***0.39*** and ***0.17*** respectively. Hence, we have chosen the value of 'y' around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.17}, \textit{we have } \boldsymbol{q = 0.35}$.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.27}, \textit{we have } \boldsymbol{q = 0.37}$.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.07}, \textit{we have } \boldsymbol{q = 0.34}$.

3. SUPPLY IS A RANDOM, DEPENDING ON THE ORDER QUANTITY

THE MODEL

Let X: Demand.

Y_q: Total Cost.

q: Order Quantity.

c_1: Shortage Cost.

c_2: Excess Cost.

S: Supply.

Therefore,

$$\boldsymbol{Y_q} = \begin{cases} \boldsymbol{c_1(X - S)\ if\ X} > S \\ \boldsymbol{c_2(S - X)\ if\ X} < S \end{cases}$$

The probability density functions of X and Y will be denoted by f(x) and g(y) respectively.

We assume f ~ B(m,n). We take m = 2, n = 1.

The p.d.f. of 'f' is then given by $h(s) = \frac{2s}{q^2}, 0 < s < q$.

3.1. THE METHODS

3.1.1. Minimization of the Standard Deviation of the Cost distribution

Exponential Distribution:

In this case,

$$E\left[{Y_q}^2\right] = {c_1}^2 \int_0^q \int_s^\infty (x - s)^2 \lambda e^{-\lambda x} \frac{2s}{q^2} dx\, ds + {c_2}^2 \int_0^q \int_0^s (s - x)^2 \lambda e^{-\lambda x} \frac{2s}{q^2} dxds$$

$$= \frac{4c1^2\left(1-e^{-q\lambda}(1+q\lambda)\right)}{q^2\lambda^4} + \frac{c2^2 e^{-q\lambda}(24(1+q\lambda)+e^{q\lambda}(-24+q^2\lambda^2(12+q\lambda(-8+3q\lambda))))}{6q^2\lambda^4}$$

Therefore,

$$Var[Y_q] = E[Y_q{}^2] - \{E[Y_q]\}^2$$

$$= \frac{4c1^2(1-e^{-q\lambda}(1+q\lambda))}{q^2\lambda^4} + \frac{c2^2 e^{-q\lambda}(24(1+q\lambda)+e^{q\lambda}(-24+q^2\lambda^2(12+q\lambda(-8+3q\lambda))))}{6q^2\lambda^4} - \left(\frac{2}{q^2\lambda^3}(1-e^{-\lambda q})(c_1+c_2) - \frac{2e^{-\lambda q}}{q\lambda^2}(c_1+c_2) + \frac{2c_2 q}{3} - c_2\right)^2.$$

Beta (2,1) Distribution:

Here, $E[Y_q{}^2] = \frac{c2^2q^4}{18} - \frac{1}{18}c1^2(-9+16q-9q^2+q^4)$.

Therefore,

$$Var[Y_q] = \frac{c2^2q^4}{18} - \frac{1}{18}c1^2(-9+16q-9q^2+q^4) - \left(\frac{2c2q^3}{15} + \frac{2}{15}c1(5-5q+q^3)\right)^2 .$$

Beta (1,2) Distribution:

Here, $E[Y_q{}^2] = \frac{1}{630}c_2^2(35-12q)q^4 + \frac{1}{630}c_1^2(63+q(-140+105q-35q^3+12q^4)).$

Therefore,

$$Var[Y_q] = \frac{1}{630}c_2^2(35-12q)q^4 + \frac{1}{630}c_1^2\left(63+q(-140+105q-35q^3+12q^4)\right) - \left(\frac{1}{90}c_2(12-5q)q^3 + \frac{1}{90}c_1(15-20q+12q^3-5q^4)\right)^2$$

Here also, the optimal value of q, say q_1, may now be obtained by minimizing $SD[Y_q]$ for specified values of c_1 and c_2. The corresponding optimal expected total cost will be denoted by Y_{q1}.

3.1.2. Minimization of the Co-efficient of Variation of the Cost distribution

Here the optimal value of q, say q_2 will be obtained by minimizing

$$c.v.\left[Y_q\right] = \frac{SD\left[Y_q\right]}{E\left[Y_q\right]}.$$

The corresponding optimal expected total cost will be denoted by Y_{q2}.

3.1.3. Here the optimal value of q, say q_3 will be obtained by minimizing

$$S(q) = E\left[Y_q\right] + k\,SD\left[Y_q\right],$$

k being a suitably chosen constant.

3.1.4. Minimization of the modal cost

In this case the CDF of the cost distribution is given by

$$H_q(y) = \int_0^q \left\{F\left(s + \frac{y}{c_1}\right) - F\left(s - \frac{y}{c_2}\right)\right\} g(s)ds$$

We shall here assume that the supply varies around the order quantity following a Rectangular distribution i.e. $S \sim R(q - a, q + a)$. Thus,

$$f_S(s) = \begin{cases} \frac{1}{2a} & if\ q - a < s < q + a \\ 0 & otherwise \end{cases}$$

Here we shall consider the demand to follow a Beta distribution with parameters 3 and 1 i.e. X~ B (3, 1).

Thus,

$$\boldsymbol{f_X(x)} = \begin{cases} \boldsymbol{3x^2 if\ 0} < x < 1 \\ \boldsymbol{0\ otherwise} \end{cases}$$

Now,

$$P(X < S + K) = \int_{q-a}^{q+a} \int_{0}^{s+k} f(x) f_S(s) dx ds$$

$$= \frac{3}{2a} \int_{q-a}^{q+a} \int_{0}^{s+k} x^2 dx ds$$

$$= \frac{1}{2a} \int_{q-a}^{q+a} [x^3]_0^{s+k} ds$$

$$= \frac{1}{2a} \int_{q-a}^{q+a} (s+k)^3 ds$$

$$= \frac{1}{8a} [(s+k)^4]_{q-a}^{q+a}$$

$$= (q+k)^3 + a^2(q+k)$$

The CDF of Y is given by,

$$F_y(y) = \left\{q^3 + a^2 q - \left(q - \frac{y}{c_2}\right)^3 - a^2\left(q - \frac{y}{c_2}\right)\right\}(q^3 + a^2 q)$$

$$+ \left\{\left(q + \frac{y}{c_1}\right)^3 + a^2\left(q + \frac{y}{c_2}\right) - q^3 - a^2 q\right\}(1 - q^3 - a^2 q)$$

$$= y^3\left[\left(\frac{1}{C_2^3} - \frac{1}{C_1^3}\right)(q^3 + a^2 q) + \frac{1}{C_1^3}\right] + 3qy^2\left[\frac{1}{C_1^2} - \left(\frac{1}{C_1^2} + \frac{1}{C_2^2}\right)(q^3 + a^2 q)\right]$$

$$+ y(3q^2 + a^2)\left[\frac{1}{c_1} + \left(\frac{1}{c_2} - \frac{1}{c_1}\right)(q^3 + a^2 q)\right]$$

Hence, the PDF of Y is given by

$f_Y(y) = 3y^2\left[\left(\frac{1}{C_2^3}-\frac{1}{C_1^3}\right)(q^3+a^2q)+\frac{1}{C_1^3}\right]+6qy\left[\frac{1}{C_1^2}-\left(\frac{1}{C_1^2}+\frac{1}{C_2^2}\right)(q^3+a^2q)\right]$

$$+(3q^2+a^2)\left[\frac{1}{c_1}+\left(\frac{1}{c_2}-\frac{1}{c_1}\right)(q^3+a^2q)\right]$$

Here the mode is obtained as

$$\boldsymbol{M_0}=\frac{(q^4+a^2q^2)\left(\frac{1}{c_1^2}+\frac{1}{c_2^2}\right)-\frac{q}{c_1^2}}{\left(\frac{1}{c_2^3}-\frac{1}{c_1^3}\right)(q^3+a^2q)+\frac{1}{c_1^3}}.$$

Here also, it can be seen that the mode does not exist for $c_1 = c_2$.

Now by solving the equation $\frac{dM_0}{dq} = 0$ we get

$$M_0\left[\left(\frac{1}{C_2^3}-\frac{1}{C_1^3}\right)(3q^2+a^2)\right]=(4q^3+2a^2q)\left(\frac{1}{C_1^2}+\frac{1}{C_2^2}\right)-\frac{1}{C_1^2}$$

$$or,\frac{(q^4+a^2q^2)\left(\frac{1}{C_1^2}+\frac{1}{C_2^2}\right)-\frac{q}{C_1^2}}{\left(\frac{1}{C_2^3}-\frac{1}{C_1^3}\right)(q^3+a^2q)+\frac{1}{C_1^3}}=\frac{(4q^3+2a^2q)\left(\frac{1}{C_1^2}+\frac{1}{C_2^2}\right)-\frac{1}{C_1^2}}{\left(\frac{1}{C_2^3}-\frac{1}{C_1^3}\right)(3q^2+a^2)}$$

For varying values of c_1, c_2 *and* a we obtain the value of q (q_4) for which the mode of Y is minimized.

Let us now consider demand to follow a Beta distribution with parameters 1 and 3 i.e. X~ B (1, 3).

Thus,

$$\boldsymbol{f_X(x)} = \begin{cases} \boldsymbol{3(1-x)^2}\ , \boldsymbol{if\ 0} < x < 1 \\ \boldsymbol{0}\ , \quad \boldsymbol{otherwise} \end{cases}$$

Now,

$$P(X < S + k) = \int_{q-a}^{q+a}\int_0^{s+k} f(x)f_S(s)dsdx$$

$$= \frac{3}{2a}\int_{q-a}^{q+a}\int_0^{s+k}(1-x)^2dsdx$$

$$= \frac{3}{2a}\int_{q-a}^{q+a}\left[\frac{(1-x)^3}{3}\right]_{s+k}^{0} ds$$

$$= \frac{1}{2a}\int_{q-a}^{q+a}\{1-(1-s-k)^3\}ds$$

$$= 1 + \frac{1}{8a}\,[(s+k-1)^4]_{q-a}^{q+a}$$

$$= 1 + \frac{1}{8a}\,[(q-1+a+k)^4 - (q-1-a+k)^4]$$

Letz=q-1.

$$P(X < S+k) = 1 + \frac{1}{8a}\,[(z+a+k)^4 - (z-a+k)^4]$$

$$= 1 + (z+k)^3 + a^2(z+k)$$

The CDF of Y is given by,

$$F_y(y) = \left\{z^3 + a^2 z - \left(z - \frac{y}{c_2}\right)^3 - a^2\left(z - \frac{y}{c_2}\right)\right\}(1 + z^3 + a^2 z\,)$$

$$-\left\{\left(z + \frac{y}{c_1}\right)^3 + a^2\left(z + \frac{y}{c_2}\right) - z^3 - a^2 z\right\}(z^3 + a^2\,z)$$

$$= y^3\left[\left(\frac{1}{C_2^3} - \frac{1}{C_1^3}\right)(z^3 + a^2 z) + \frac{1}{C_1^3}\right] - 3zy^2\left[\frac{1}{C_2^2} + \left(\frac{1}{C_1^2} + \frac{1}{C_2^2}\right)(z^3 + a^2 z)\right]$$

$$+y(3z^2 + a^2)\left[\frac{1}{c_2} + \left(\frac{1}{c_2} - \frac{1}{c_1}\right)(z^3 + a^2 z)\right]$$

Hence, the PDF of Y is

$$f_Y(y) = 3y^2\left[\left(\frac{1}{C_2^3} - \frac{1}{C_1^3}\right)(z^3 + a^2 z) + \frac{1}{C_2^3}\right] - 6zy\left[\frac{1}{C_2^2} + \left(\frac{1}{C_1^2} + \frac{1}{C_2^2}\right)(z^3 + a^2 z)\right]$$

$$+(3z^2 + a^2)\left[\frac{1}{c_2} + \left(\frac{1}{c_2} - \frac{1}{c_1}\right)(z^3 + a^2 z)\right]$$

Here the mode is obtained as

$$M_0 = \frac{(z^4 + a^2 z^2)\left(\frac{1}{c_1^2} + \frac{1}{c_2^2}\right) + \frac{z}{c_2^2}}{\left(\frac{1}{c_2^3} - \frac{1}{c_1^3}\right)(z^3 + a^2 z) + \frac{1}{c_2^3}},\ z = q-1.$$

Solving $\frac{dM_0}{dq} = 0$ we get

$$M_0\left[\left(\frac{1}{C_2^3} - \frac{1}{C_1^3}\right)(3z^2 + a^2)\right] = (4z^3 + 2a^2z)\left(\frac{1}{C_1^2} + \frac{1}{C_2^2}\right) + \frac{1}{c_2^2}$$

$$or, \frac{(z^4+a^2z^2)\left(\frac{1}{c_1^2}+\frac{1}{c_2^2}\right)+\frac{z}{c_2^2}}{\left(\frac{1}{c_2^3}-\frac{1}{c_1^3}\right)(z^3+a^2z)+\frac{1}{c_2^3}} = \frac{(4z^3+2a^2z)\left(\frac{1}{c_1^2}+\frac{1}{c_2^2}\right)+\frac{1}{c_2^2}}{\left(\frac{1}{c_2^3}-\frac{1}{c_1^3}\right)(3z^2+a^2)}.$$

For varying values of c_1, c_2 *and* a we may now obtain the value of q (q_4).

3.1.5 Minimization of the probability of the cost distribution exceeding a given high value

Keeping the demand distribution unaltered, here we take the PDF of the Supply distribution as

$$\boldsymbol{g(s) = \frac{1}{q}, \frac{q}{2}} < s < \boldsymbol{\frac{3q}{2}}.$$

Consequently,

$$H_q(y) = \frac{(\mathrm{c}_1+\mathrm{c}_2)q(12-26q+15.q^2)y}{\mathrm{c}_1\mathrm{c}_2} + (\frac{-6+24q-19.5}{\mathrm{c}_2^2} + \frac{6-24q+19.5q^2}{\mathrm{c}_1^2})y^2 + \frac{(\mathrm{c}_1^3+\mathrm{c}_2^3)(-8+12q)y^3}{\mathrm{c}_1^3\mathrm{c}_2^3} + (\frac{3}{\mathrm{c}_1^4} - \frac{3.}{\mathrm{c}_2^4})y^4.$$

Then,

$$\frac{d}{dq}\overline{H}_q(y) = qy\left(-\frac{52}{\mathrm{c}_1} + \frac{-52\mathrm{c}2 - 39y}{\mathrm{c}_2^2} + \frac{39y}{\mathrm{c}_1^2}\right) + q^2\left(\frac{45y}{\mathrm{c}_1} + \frac{45y}{\mathrm{c}_2}\right)$$

$$+y(\frac{12}{\mathrm{c}_1} - \frac{24y}{\mathrm{c}_1^2} + \frac{12y^2}{\mathrm{c}_1^2} + \frac{12\mathrm{c}2^2 + 24\mathrm{c}2y + 12y^2}{\mathrm{c}_2^3})$$

Solving the equation $\frac{d}{dq}\overline{H}_q(y) = 0$ one can now obtain the value of ‘q’ (denoted by q_5).

Here we consider supply to be a fraction of the order quantity q and take

$$\boldsymbol{g(s) = \frac{2s}{q^2}, 0} < s < q.$$

Hence,

$$H_q(y) = \frac{4(c_1+c_2)q(10-15q+6q^2)y}{5c_1c_2} - \frac{(c_1^2-c_2^2)(6-16q+9q^2)y^2}{c_1^2c_2^2} + \frac{8(c_1^3+c_2^3)(-1+q)y^3}{c_1^3c_2^3} + (\frac{3}{c_1^4} - \frac{3}{c_2^4})y^4.$$

Then,

$$\frac{d}{dq}\overline{H}_q(y) = -\frac{72(c_1+c_2)q^2y}{5c1c2} + \frac{6(c_1+c_2)qy(4c_1c_2+3c_1y-3c_2y)}{c1^2c2^2} - \frac{8y(c_1^2c_2^3-2c_1c_2^3y+c_2^3y^2+c_1^3(c_2+y)^2)}{c_1^3c_2^3}$$

Here also, solving $\frac{d}{dq}\overline{H}_q(y) = 0$, the value of q ($q_5$) may be worked out.

3.2. SOME NUMERICAL EXAMPLES

3.2.1. Exponential Demand:

Model Parameters				Optimal Criteria					
λ	c_1	c_2	K	3.2.2.1 SD		3.2.2.2 CV		3.2.2.3 Penalized Mean	
				EOQ	Cost	EOQ	Cost	EOQ	Cost
0.1	***1***	***2***	***1***	***10.75***	***14.03***	***4.1***	***11.40***	***6.92***	***9.15***
			2					***7.82***	***9.19***
	1	***0.5***	***1***	***31.30***	***15.08***	***8.2***	***8.32***	***18.51***	***8.27***
			2					***21.22***	***8.34***
0.05	***1***	***2***	***1***	***22.5***	***26.01***	***6.5***	***19.01***	***12.97***	***17.44***
			2					***15.72***	***19.03***
	1	***0.5***	***1***	***59.2***	***79.26***	***15.3***	***14.04***	***35.93***	***14.67***
			2					***41.22***	***15.33***

3.2.2. ***Beta (2,1) Demand:***

Model Parameters			Optimal Criteria					
c_1	c_2	K	3.2.2.1 SD		3.2.2.2 CV		3.2.2.3 Penalized Mean	
			EOQ	Cost	EOQ	Cost	EOQ	Cost
1	***2***	***1***	***0.638***	***0.501***	***0.905***	***0.552***	***1.544***	***1.453***
		2					***1.172***	***1.002***
1	***0.5***	***1***	***0.812***	***0.607***	***1.241***	***0.389***	***1.847***	***0.319***
		2					***1.902***	***0.216***

3.2.3. ***Beta (1,2) Demand:***

Model Parameters			Optimal Criteria					
c_1	c_2	K	3.2.2.1 SD		3.2.2.2 CV		3.2.2.3 Penalized Mean	
			EOQ	Cost	EOQ	Cost	EOQ	Cost
1	***2***	***1***	***0.321***	***0.352***	***0.851***	***0.511***	***0.607***	***0.519***
		2					***0.703***	***0.668***
1	***0.5***	***1***	***0.651***	***0.311***	***1.981***	***0.496***	***0.802***	***0.382***
		2					***0.916***	***0.462***

3.2.4. Minimization of the modal cost:

Beta (3,1) Distribution:

c_1	c_2	q_4
1	*2*	*0.19*
1	*0.5*	*0.28*

Beta (1,3) Distribution:

c_1	c_2	q_4
1	*2*	*0.41*
1	*0.5*	*0.58*

3.2.5. Minimization of the probability of the cost distribution exceeding a given high value

3.2.5.1. Here for $\boldsymbol{c_1 = 1, c_2 = 2,}$ the mean minimizing ***EOQ*** and the ***corresponding cost*** come out as ***0.27*** and ***0.23*** respectively. We have chosen the value of 'y' around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.23,}$ *we have* $\boldsymbol{q = 0.28.}$

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.33}, we\ have\ \boldsymbol{q = 0.27}$.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.13}, we\ have\ \boldsymbol{q = 0.29}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5,}$ the mean minimizing ***EOQ*** and the ***corresponding cost*** come out as 0.46 and 0.14 respectively. We have chosen the value of ‘y’ around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.14}, we\ have\ \boldsymbol{q = 0.41}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.24}, we\ have\ \boldsymbol{q = 0.53}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.04}, we\ have\ \boldsymbol{q = 0.34}$.

For $\boldsymbol{c_1 = 1, c_2 = 1,}$ the mean minimizing ***EOQ*** and the ***corresponding cost*** come out as ***0.36*** and ***0.18*** respectively. We have chosen the value of ‘y’ around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.18}, we\ have\ \boldsymbol{q = 0.34}$.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.28}, we\ have\ \boldsymbol{q = 0.36}$.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.08}, we\ have\ \boldsymbol{q = 0.32}$.

3.2.5.2. In this case, for $\boldsymbol{c_1 = 1, c_2 = 2,}$ the mean minimizing ***EOQ*** and the ***corresponding cost*** come out as 0.38 and 0.24 respectively. We have chosen the value of ‘y’ around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.24}, we\ have\ \boldsymbol{q = 0.40}$.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.34}, we\ have\ \boldsymbol{q = 0.39}$.

For $\boldsymbol{c_1 = 1, c_2 = 2, y = 0.14}, we\ have\ \boldsymbol{q = 0.42}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5}$, the mean minimizing ***EOQ*** and the ***corresponding cost*** come out as ***0.66*** and ***0.15*** respectively. We have chosen the value of 'y' around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.15}, we\ have\ \boldsymbol{q = 0.61}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.25}, we\ have\ \boldsymbol{q = 0.78}$.

For $\boldsymbol{c_1 = 1, c_2 = 0.5, y = 0.05}, we\ have\ \boldsymbol{q = 0.50}$.

For $\boldsymbol{c_1 = 1, c_2 = 1}$, the mean minimizing ***EOQ*** and the corresponding cost come out as ***0.52*** and ***0.19*** respectively. We have chosen the value of 'y' around this cost.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.19}, we\ have\ \boldsymbol{q = 0.49}$.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.29}, we\ have\ \boldsymbol{q = 0.53}$.

For $\boldsymbol{c_1 = 1, c_2 = 1, y = 0.09}, we\ have\ \boldsymbol{q = 0.47}$.

4. CONCLUSION

Considering the aim and the field of practical application of the 'Newsvendor problem', *mean minimization of total cost* is not always sufficient to obtain the *optimal order quantity* in all situations. Further the *optimality criterion depends* on the *aim* under consideration. Thus, the optimality criteria obtained by *minimizing the standard deviation*, *coefficient of variation*, *penalized mean of the cost distribution*, *minimizing the modal cost and the probability of the cost distribution exceeding a given high value* as discussed in this paper should be appropriately applied.

5. REFERENCES

1. ***Chakraborty R. (1979). Robustness of the Total Cost in a Static Risk. IAPQR Transactions, 4(1), 9-20.***

2. ***Chandra A. (2002). Some Inventory Models with Random Supply. IAPQR Transactions, 27(2), 101-109.***

3. ***Chandra A. and Mukherjee S.P. (2005). Some Alternative Methods of Finding the Optimal Order Quantity in Inventory Models. Calcutta Statistical Association Bulletin, 57, 121-128.***