**MATHEMATICAL SCIENCES**

**(INCLUDING STATISTICS)**

# *SITUATIONS OF SUPPLY IN THE CLASSICAL NEWSBOY PROBLEM*


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# KEYWORDS:

**Optimal ordered quantity($q_{opt}$), Random Supply, Shortage Cost per unit short($c_1$), Excess Cost per unit excess($c_2$),Total Optimal Cost(T(q)), Mean Minimising(Classical) solution.**


# ABSTRACT:


**"DISTRIBUTING NEWSPAPERS IN THE MORNING…..NOBLE BUT MONOTONIC RIGHT?" , this is what newsboys start their day with.**

**Classical Newsboy Problem accepts the monotonicity of the job of newsboys but offers great profits. In this practical situation ; newsboys has a certain amount of newspapers with them ; *the demand(X) of newspapers being random.***

**In the following problem proposed considering two cases: *supply of newspapers- when it is same and random with respect to order quantity*, we derive the *optimal order quantity* using the cost function for different probability distributions which the demand follows; by *mean minimising technique* and thus finding out *total optimal cost.***

# 1. INTRODUCTION:

In this paper we consider the *Classical Newsboy (or Newsvendor) Problem*. In this problem, a newsboy starts his day with a *certain amount, say 'q'*, newspapers with him. The demand being random, at the end of the day he might face *shortage* or may be left with some *excess* newspapers in his hand. Accordingly, he has to incur *shortage* or *excess cost*. The demand being random, naturally, *the cost incurred will also be a random variable.* The problem is to determine the *optimal order quantity (q)* so that the *expected total cost is minimized.*

In this paper we have considered a more practical situation where whatever is ordered by the newspaper vendor, it is not same as what is received. In other words, *Supply is not same as the order quantity.* This problem often arises in production inventory situation. Here we have considered the situation where supply varies around the order quantity. Demand distribution of both finite range (viz., *Beta with varying parameters, Rectangular Distribution with it's parameters*) and infinite range (viz., *exponential*) have been considered, while supply has been considered to vary *uniformly* around the order quantity. Some numerical examples have also been worked out and a comparison of optimal order quantity and the optimal cost between the classical situation and the situation of random supply has been conducted.

## A. ##SUPPLY IS THE SAME AS THE ORDERED QUANTITY

***THE MODEL***

**X**→DEMAND

**q**→ORDERED QUANTITY

**f(x)**→PDF OF THE DEMAND RANDOM VARIABLE X

$c_1$→SHORTAGE COST PER UNIT SHORT

$c_2$→EXCESS COST PER UNIT EXCESS

***COST FUNCTION***

$$C(q)=\begin{cases} c_1(X-q), if\ X>q \\ c_2(q-X), if\ X<q \end{cases}$$

***EXPECTED TOTAL COST***

$$\Rightarrow T(q)=E[C(q)]=\int_q^{\infty} c_1(x-q)f(x)dx + \int_0^q c_2(q-x)f(x)dx$$

The objective is to obtain 'q' by minimising T(q) (the expected total cost) i.e. by equating ***T'(q) to 0.***

***MEAN MINIMISING( CLASSICAL) SOLUTIONS :***

1. **THE BETA (1,2) DISTRIBUTION: X~B(1,2)**

   The PDF is given by,

$$f(x)=\begin{cases} 2(1-x), 0<x<1 \\ 0, \quad otherwise \end{cases}$$

   The Expected Total Cost is given by,

$$\Rightarrow T(q)=E[C(q)]=\int_q^1 c_1(x-q)2(1-x)dx + \int_0^q c_2(q-x)2(1-x)dx$$

$\Rightarrow$T(q)= 2c$_1\int_q^1 (x - x^2 - q + qx)dx$ + 2c$_2\int_0^q (q - qx - x + x^2)dx$

$\Rightarrow$T(q)=2c$_1[\frac{x^2}{2} - \frac{x^3}{3} - qx + q\frac{x^2}{2}]_q^1$ + 2c$_2$[qx-q$\frac{x^2}{2}$ - $\frac{x^2}{2} + \frac{x^3}{3}]_0^q$

$\Rightarrow$T(q)=2c$_1[-\frac{q^3}{6} + \frac{q^2}{2} - \frac{q}{2} + \frac{1}{6}]$ + 2c$_2[\frac{q^2}{2} - \frac{q^3}{6}]$

$\boldsymbol{\Rightarrow T(q) = \frac{c_1}{3}\{(1-q)^3\} + \frac{c_2}{3}\{q^2(3-q)\}}$

Now to minimise the expected total cost, we take the equation T'(q)=0

$\Rightarrow$T'(q)=$-c_1(1-q)^2 + \frac{c_2}{3}2q(3-q) - \frac{c_2}{3}q^2$=0

$\Rightarrow$T'(q)=$c_1(1+q^2-2q) - \frac{c_2}{3}(6q-2q^2) + \frac{c_2}{3}q^2 = 0$

$\Rightarrow$T'(q)=$\{c_1 + \frac{2c_2}{3} + \frac{c_2}{3}\}q^2 + \{-2c_1 - 2c_2\} + c_1 = 0$

$\Rightarrow$T'(q)=(c$_1$+c$_2$)$q^2$- 2(c$_1$+c$_2$)q + c$_1$ = 0

Hence,

$$\Rightarrow q= \frac{2(c_1+c_2) \pm \sqrt{4(c_1+c_2)^2 - 4(c_1+c_2)c_1}}{2(c_1+c_2)}$$

$$\Rightarrow q=1\pm\sqrt{\frac{c_2}{c_1+c_2}}$$

Since, 0<x<1 $\Rightarrow$0<q<1

$$\boldsymbol{\Rightarrow q=1-\sqrt{\frac{c_2}{c_1+c_2}}}$$

Now let us consider,

$$\boldsymbol{\Rightarrow q_{opt} = 1-\sqrt{\frac{c_2}{c_1+c_2}}}$$

The Expected Total Optimal Cost is then given by,

$$\boldsymbol{\Rightarrow T(q_{opt}) = \frac{c_1}{3}\{(1-q_{opt})^3\} + \frac{c_2}{3}\{q_{opt}^2(3-q_{opt})\}}$$

2. **<u>THE BETA (2,1) DISTRIBUTION</u> : X~B(2,1)**

The PDF is given by,

$$\mathbf{f(x)}=\begin{cases} \mathbf{2x}, \mathbf{if}\ \mathbf{0} < x < 1 \\ \mathbf{0},\ \mathbf{otherwise} \end{cases}$$

The Expected Total Cost is given by,

$\Rightarrow$T(q)=E[C(q)]=$\int_q^1 c_1(x-q)2xdx$ + $\int_0^q c_2(q-x)2xdx$

$\Rightarrow$T(q)= 2c$_1\int_q^1(x^2-qx)dx$ + 2c$_2\int_0^q(qx-x^2)dx$

$\Rightarrow$T(q)=2c$_1[\frac{x^3}{3}-q\frac{x^2}{2}]_q^1$ + 2c$_2$[q$\frac{x^2}{2}$ - $\frac{x^3}{3}]_0^q$

$\Rightarrow$T(q)=2$c_1\left\{\frac{q^3}{6}-\frac{q}{2}+\frac{1}{3}\right\}$ +2$c_2\left\{\frac{q^3}{6}\right\}$

$\Rightarrow$T(q)=$\frac{c_1}{3}\{q^3-3q+2\}+\frac{c_2}{3}q^3$

**$\Rightarrow$T(q)=($\boldsymbol{\frac{c_1+c_2}{3}}$)$\boldsymbol{q^3-c_1q+\frac{2}{3}c_1}$**

Now to minimise the expected total cost, we take the equation T'(q)=0

$\Rightarrow$T'(q)=(c$_1$+c$_2$)$q^2$- c$_1$ = 0

$\Rightarrow q^2=\frac{c_1}{c_1+c_2}$

Hence,

$\Rightarrow$q=$\pm\sqrt{\frac{c_1}{c_1+c_2}}$

Since, 0<x<1 $\Rightarrow$0<q<1

$$\boldsymbol{\Rightarrow q=\sqrt{\frac{c_1}{c_1+c_2}}}$$

Now let us consider,

$$\boldsymbol{\Rightarrow q_{opt}=\sqrt{\frac{c_1}{c_1+c_2}}}$$

The Expected Total Optimal Cost is then given by,

**$\Rightarrow$T($\boldsymbol{q_{opt}}$) = ($\boldsymbol{\frac{c_1+c_2}{3}}$)$\boldsymbol{{q_{opt}}^3-c_1q_{opt}+\frac{2}{3}c_1}$**

3. **THE BETA (1,1) DISTRIBUTION : X~B(1,1)**

The PDF is given by,

$$\textbf{f(x)=}\begin{cases}\mathbf{1}, \boldsymbol{if}\ \mathbf{0}<x<1\\ \mathbf{0}, \boldsymbol{otherwise}\end{cases}$$

The Expected Total Cost is given by,

$\Rightarrow$T(q)=E[C(q)]=$\int_q^1 c_1(x-q)1dx$ + $\int_0^q c_2(q-x)1dx$

$\Rightarrow$T(q)= $c_1\int_q^1 (x-q)dx$ + $c_2\int_0^q (q-x)dx$

$\Rightarrow$T(q)= $c_1[\frac{x^2}{2} - qx]_q^1$ + $c_2[qx - \frac{x^2}{2}]_0^q$

$\Rightarrow$T(q)=$c_1\{\frac{1}{2} - \frac{q^2}{2} - q + q^2\}$ +$c_2\{q^2 - \frac{q^2}{2}\}$

$\Rightarrow$T(q)=$c_1\{\frac{q^2}{2} - q + \frac{1}{2}\}$ +$c_2\{\frac{q^2}{2}\}$

$\Rightarrow$***T(q)=*** $\boldsymbol{(\frac{c_1+c_2}{2})q^2 - c_1 q + \frac{1}{2}c_1}$

Now to minimise the expected total cost, we take the equation T’(q)=0

$\Rightarrow$T’(q)=$(c_1+c_2)q - c_1 = 0$

Hence,

$$\boldsymbol{q=\frac{c_1}{c_1+c_2}}$$

Now let us consider,

$$\boldsymbol{\Rightarrow q_{opt} = \frac{c_1}{c_1+c_2}}$$

The Expected Total Optimal Cost is then given by,

$\Rightarrow$**T**$\boldsymbol{(q_{opt}) = (\frac{c_1+c_2}{2})q_{opt}{}^2 - c_1 q_{opt} + \frac{1}{2}c_1}$

4. **<u>THE EXPONENTIAL DISTRIBUTION</u>: X~exp(λ)**

The PDF is given by,

$$\textbf{f(x)}=\begin{cases} \boldsymbol{\lambda e^{-\lambda x}, if\ x} > 0, & \boldsymbol{\lambda} > 0 \\ \boldsymbol{0}, & \boldsymbol{otherwise} \end{cases}$$

The Expected Total Cost is given by,

$\Rightarrow$T(q)=E[C(q)]=$\int_q^\infty c_1(x-q)\lambda e^{-\lambda x}dx$ + $\int_0^q c_2(q-x)\lambda e^{-\lambda x}dx$

$\Rightarrow$T(q)=$c_1\lambda\int_q^\infty (xe^{-\lambda x} - qe^{-\lambda x})dx + c_2\lambda\int_0^q (qe^{-\lambda x} - xe^{-\lambda x})dx$

$\Rightarrow$T(q)=$c_1\lambda\,[\frac{xe^{-\lambda x}}{\lambda} + \frac{1}{\lambda^2}e^{-\lambda x} - \frac{qe^{-\lambda x}}{\lambda}]\,{}^q_\infty + c_2\lambda\,[-\frac{qe^{-\lambda x}}{\lambda} + \frac{xe^{-\lambda x}}{\lambda} + \frac{1}{\lambda^2}e^{-\lambda x}]\,{}^q_0$

$\Rightarrow$T(q)=$c_1\lambda\left\{\frac{1}{\lambda}qe^{-\lambda q} + \frac{1}{\lambda^2}e^{-\lambda q} - \frac{1}{\lambda}qe^{-\lambda q}\right\} + c_2\lambda\{\frac{q}{\lambda} - \frac{1}{\lambda}qe^{-\lambda q} + \frac{1}{\lambda}qe^{-\lambda q} + \frac{1}{\lambda^2}e^{-\lambda q} + \frac{1}{\lambda^2}\}$

$\Rightarrow$***T(q)=*** $\boldsymbol{c_1\frac{e^{-\lambda q}}{\lambda} + c_2[q + \frac{e^{-\lambda q}}{\lambda} + \frac{1}{\lambda}]}$

Now to minimise the expected total cost, we equate T'(q) to 0.

⇒T'(q)= $-c_1 e^{-\lambda q} + c_2 - c_2 e^{-\lambda q}$=0

⇒$e^{-\lambda q}(c_1 + c_2) = c_2$

⇒$e^{-\lambda q} = \frac{c_2}{c_1+c_2}$

⇒$-\lambda q$=ln $\frac{c_2}{c_1+c_2}$

⇒q=$\frac{1}{\lambda}$ ln $\frac{c_1+c_2}{c_2}$

Hence,

$$\boldsymbol{q = \frac{1}{\lambda} \ln \frac{c_1+c_2}{c_2}}$$

Now let us consider,

$$\Rightarrow \boldsymbol{q_{opt} = \frac{1}{\lambda} \ln \frac{c_1+c_2}{c_2}}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow \mathbf{T}(\boldsymbol{q_{opt}}) = \boldsymbol{c_1 \frac{e^{-\lambda q_{opt}}}{\lambda} + c_2[q_{opt} + \frac{e^{-\lambda q_{opt}}}{\lambda} + \frac{1}{\lambda}]}$$

## 5. <u>THE RECTANGULAR OR UNIFORM DISTRIBUTION</u> : X~R(α,β)

The PDF is given by,

$$\mathbf{f(x)} = \begin{cases} \boldsymbol{\frac{1}{\beta-\alpha}}, \textbf{ if } \alpha < x < \beta \\ \mathbf{0}, \textbf{ otherwise} \end{cases}$$

The Expected Total Cost is given by,

⇒T(q)=E[C(q)]=$\int_q^\beta c_1(x-q)\frac{1}{\beta-\alpha}dx + \int_\alpha^q c_2(q-x)\frac{1}{\beta-\alpha}dx$

⇒T(q)=$c_1\frac{1}{\beta-\alpha}\int_q^\beta (x-q)dx + c_2\frac{1}{\beta-\alpha}\int_\alpha^q (q-x)dx$

⇒T(q)=$c_1\frac{1}{\beta-\alpha}[x^2 - qx]_q^\beta + c_2\frac{1}{\beta-\alpha}[qx - \frac{x^2}{2}]_\alpha^q$

⇒T(q)=$c_1\frac{1}{\beta-\alpha}\left\{\frac{\beta^2}{2} - \frac{q^2}{2} - q\beta + q^2\right\} + c_2\frac{1}{\beta-\alpha}\{q^2 - q\alpha - \frac{q^2}{2} + \frac{\alpha^2}{2}\}$

$$\Rightarrow T(q)=c_1\frac{1}{\beta-\alpha}\{\frac{q^2}{2}-q\beta+\frac{\beta^2}{2}\}+\ c_2\frac{1}{\beta-\alpha}\{\frac{q^2}{2}-q\alpha+\frac{\alpha^2}{2}\}$$

Now to minimise the expected total cost, we equate T'(q) to 0.

$$\Rightarrow T'(q)=c_1\frac{1}{\beta-\alpha}\{q-\beta\}+c_2\frac{1}{\beta-\alpha}\{q-\alpha\}=0$$

$$\Rightarrow q\{c_1\frac{1}{\beta-\alpha}+c_2\frac{1}{\beta-\alpha}\}-\frac{c_1\beta+c_2\alpha}{\beta-\alpha}=0$$

$$\Rightarrow q=\frac{c_1\beta+c_2\alpha}{c_1+c_2}$$

Hence,

$$q=\frac{c_1\beta+c_2\alpha}{c_1+c_2}$$

Now let us consider,

$$\Rightarrow q_{opt}=\frac{c_1\beta+c_2\alpha}{c_1+c_2}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow T(q_{opt})=\ c_1\frac{1}{\beta-\alpha}\{\frac{{q_{opt}}^2}{2}-q_{opt}\beta+\frac{\beta^2}{2}\}+\ c_2\frac{1}{\beta-\alpha}\{\frac{{q_{opt}}^2}{2}-q_{opt}\alpha+\frac{\alpha^2}{2}\}$$

## B. ## SUPPLY IS RANDOM, DEPENDING UPON THE ORDER QUANTITY

***THE MODEL***

**X**→DEMAND

**q**→ORDERED QUANTITY

**f(x)**→PDF OF THE DEMAND RANDOM VARIABLE X

**$c_1$**→SHORTAGE COST PER UNIT SHORT

**$c_2$**→EXCESS COST PER UNIT EXCESS

**S**→SUPPLY

$$S \sim R(q - a\,,\, q + a)$$

The PDF of S is given by,

$$g(s)=\begin{cases} \frac{1}{2a}, & \text{if } q-a < s < q+a \\ 0 & ,\text{otherwise} \end{cases}$$

***COST FUNCTION***

$$C(q)=\begin{cases} c_1(X-S), \text{if } X > S \\ c_2(S-X), \text{if } X < S \end{cases}$$

***EXPECTED TOTAL COST***

$$\Rightarrow T(q)=E[C(q)]=\int_{q-a}^{q+a}\int_{s}^{\infty} c_1(x-s)f(x)g(s)dxds + \int_{q-a}^{q+a}\int_{0}^{s} c_2(s-x)f(x)g(s)dxds$$

To obtain ‘q’ by minimising T(q) (the expected total cost), we take the equation ***T’(q)=0***

***MEAN MINIMISING( CLASSICAL) SOLUTIONS*** ***:***

1. **THE BETA (1,2) DISTRIBUTION: X~B(1,2)**

   The PDF is given by ,

$$f(x)=\begin{cases} 2(1-x), 0 < x < 1 \\ 0, \text{otherwise} \end{cases}$$

The Expected Total Cost is given by,

$$\Rightarrow T(q)=E[C(q)]=\int_{q-a}^{q+a}\int_{s}^{1} c_1(x-s)2(1-x)\frac{1}{2a}dxds + \int_{q-a}^{q+a}\int_{0}^{s} c_2(s-x)2(1-x)\frac{1}{2a}dxds$$

$$\Rightarrow T(q)=\frac{c_1}{a}\int_{q-a}^{q+a}\{\int_{s}^{1}(x-s+sx-x^2)dx\}ds + \frac{c_2}{a}\int_{q-a}^{q+a}\{\int_{0}^{s}(s-x+x^2-sx)dx\}ds$$

$$\Rightarrow T(q)=\frac{c_1}{a}\int_{q-a}^{q+a}[\frac{x^2}{2}-sx+s\frac{x^2}{2}-\frac{x^3}{3}]_{s}^{1}ds + \frac{c_2}{a}\int_{q-a}^{q+a}[sx-\frac{x^2}{2}+\frac{x^3}{3}-s\frac{x^2}{2}]_{0}^{s}ds$$

$$\Rightarrow T(q)=\frac{c_1}{a}\int_{q-a}^{q+a}[\frac{1}{6}-\frac{s}{2}+\frac{s^2}{2}-\frac{s^3}{6}]ds + \frac{c_2}{a}\int_{q-a}^{q+a}\left[\frac{s^2}{2}-\frac{s^3}{6}\right]ds$$

$$\Rightarrow T(q)=\frac{c_1}{a}\int_{q-a}^{q+a}[\frac{1}{6}-\frac{s}{2}]ds + \frac{c_1+c_2}{a}\int_{q-a}^{q+a}\left[\frac{s^2}{2}-\frac{s^3}{6}\right]ds$$

$$\Rightarrow T(q)=\frac{c_1}{a}\left[\frac{s}{6}-\frac{s^2}{4}\right]_{q-a}^{q+a} + \frac{c_1+c_2}{a}[\frac{s^3}{6}-\frac{s^4}{24}]_{q-a}^{q+a}$$

$$\Rightarrow T(q)=\frac{c_1}{a}\left[\frac{q+a}{6}-\frac{(q+a)^2}{4}-\frac{q-a}{6}+\frac{(q-a)^2}{4}\right] + \frac{c_1+c_2}{2a}[\frac{(q+a)^3}{6}-\frac{(q+a)^4}{24}-\frac{(q-a)^3}{6}+\frac{(q-a)^4}{24}]$$

$\Rightarrow$T(q)=$\frac{c_1}{a}\left[\frac{a}{3}-qa\right]+\frac{c_1+c_2}{2a}[\frac{2}{3}(3q^2a+a^3)-\frac{2}{3}(q^3a+qa^3)]$

$\Rightarrow$***T(q)***=$\boldsymbol{\frac{c_1}{3}-qc_1+\frac{c_1+c_2}{3}(3q^2+a^2-q^3-qa^2)}$

Now to minimise the expected total cost, we equate T'(q) to 0.

$\Rightarrow$T'(q)=-$c_1+\frac{c_1+c_2}{3}(6q-3q^2-a^2)=0$

$\Rightarrow\frac{c_1+c_2}{3}(6q-3q^2)=c_1+a^2(\frac{c_1+c_2}{3})$

$\Rightarrow$2q-$q^2=\frac{c_1}{c_1+c_2}+\frac{a^2}{3}$

$\Rightarrow q^2-2q+1=1-\frac{c_1}{c_1+c_2}-\frac{a^2}{3}$

$\Rightarrow(q-1)^2=\frac{c_2}{c_1+c_2}-\frac{a^2}{3}$

$\Rightarrow$q=1±$\sqrt{\frac{c_2}{c_1+c_2}-\frac{a^2}{3}}$

Since, 0<x<1 $\Rightarrow$0<q<1

$$\boldsymbol{q=1-\sqrt{\frac{c_2}{c_1+c_2}-\frac{a^2}{3}}}$$

Now let us consider,

$$\boldsymbol{\Rightarrow q_{opt}=1-\sqrt{\frac{c_2}{c_1+c_2}-\frac{a^2}{3}}}$$

The Expected Total Optimal Cost is then given by,

$$\boldsymbol{\Rightarrow T(q_{opt})=\frac{c_1}{3}-q_{opt}c_1+\frac{c_1+c_2}{3}\left(3{q_{opt}}^2+a^2-{q_{opt}}^3-q_{opt}a^2\right)}$$

2. **<u>THE BETA (2,1) DISTRIBUTION</u> : X~B(2,1)**

The PDF is given by,

$$\textbf{f(x)}=\begin{cases}\boldsymbol{2x, if\ 0}<x<1\\ \boldsymbol{0, otherwise}\end{cases}$$

The Expected Total Cost is given by,

$\Rightarrow$T(q)=E[C(q)]=$\int_{q-a}^{q+a}\int_s^1 c_1(x-s)2x\frac{1}{2a}dxds$ + $\int_{q-a}^{q+a}\int_0^s c_2(s-x)2x\frac{1}{2a}dxds$

$\Rightarrow$T(q)=$\frac{c_1}{a}\int_{q-a}^{q+a} ds \int_s^1 (x^2 - sx)dx\ + \frac{c_2}{a}\int_{q-a}^{q+a} ds \int_0^s (sx - x^2)dx$

$\Rightarrow$T(q)=$\frac{c_1}{a}\int_{q-a}^{q+a}[\frac{x^3}{3} - s\frac{x^2}{2}]_s^1 ds+\frac{c_2}{a}\int_{q-a}^{q+a}[s\frac{x^2}{2} - \frac{x^3}{3}]_0^s ds$

$\Rightarrow$T(q)=$\frac{c_1}{a}\int_{q-a}^{q+a}\left[\frac{1}{3} - \frac{s^3}{3} - \frac{s}{2} + \frac{s^3}{2}\right] ds+\frac{c_2}{a}\int_{q-a}^{q+a}[\frac{s^3}{2} - \frac{s^3}{3}]ds$

$\Rightarrow$T(q)=$\frac{c_1}{a}\int_{q-a}^{q+a}\left[\frac{s^3}{6} - \frac{s}{2} + \frac{1}{3}\right] ds + \frac{c_2}{a}\int_{q-a}^{q+a}[\frac{s^3}{6}]ds$

$\Rightarrow$T(q)=$\frac{c_1}{a}[\frac{s^4}{24} - \frac{s^2}{4} + \frac{s}{3}]_{q-a}^{q+a} + \frac{c_2}{a}[\frac{s^4}{24}]_{q-a}^{q+a}$

$\Rightarrow$T(q)=$\frac{c_1}{3}(q^2 + a^2)q - c_1 q + \frac{2c_1}{3} + \frac{c_2}{3}(q^2 + a^2)q$

$\Rightarrow \boldsymbol{T(q)=\frac{c_1+c_2}{3}(q^3 + qa^2) - c_1 q + \frac{2c_1}{3}}$

Now to minimise the expected total cost, we equate T'(q) to 0.

$\Rightarrow$T'(q)=$\frac{c_1+c_2}{3}(3q^2 + a^2) - c_1 = 0$

$\Rightarrow (c_1 + c_2)q^2 + \frac{c_1+c_2}{3}a^2 - c_1 = 0$

$\Rightarrow q^2 = \frac{c_1}{c_1+c_2} - \frac{a^2}{3}$

$\Rightarrow q = \pm\sqrt{\frac{c_1}{c_1+c_2} - \frac{a^2}{3}}$

Since, 0<x<1 $\Rightarrow$0<q<1

$$\boldsymbol{q = \sqrt{\frac{c_1}{c_1 + c_2} - \frac{a^2}{3}}}$$

Now let us consider,

$$\Rightarrow \boldsymbol{q_{opt} = \sqrt{\frac{c_1}{c_1+c_2} - \frac{a^2}{3}}}$$

The Expected Total Optimal Cost is then given by,

$\Rightarrow \mathbf{T}(\boldsymbol{q_{opt}}) = \boldsymbol{\frac{c_1+c_2}{3}(q_{opt}{}^3 + q_{opt}a^2) - c_1 q_{opt} + \frac{2c_1}{3}}$

3. **THE BETA (1,1) DISTRIBUTION : X~B(1,1)**

The PDF is given by,

$$\mathbf{f(x)}=\begin{cases}\mathbf{1}, \boldsymbol{if}\ \mathbf{0} < x < 1 \\ \mathbf{0}, \boldsymbol{otherwise}\end{cases}$$

The Expected Total Cost is given by ,

$$\Rightarrow T(q)=E[C(q)]=\int_{q-a}^{q+a} \int_{s}^{1} c_1(x-s)1\frac{1}{2a}dxds + \int_{q-a}^{q+a} \int_{0}^{s} c_2(s-x)1\frac{1}{2a}dxds$$

$$\Rightarrow T(q)=\frac{c_1}{2a}\int_{q-a}^{q+a} ds\int_{s}^{1}(x-s)dx + \frac{c_2}{2a}\int_{q-a}^{q+a} ds\int_{0}^{s}(s-x)dx$$

$$\Rightarrow T(q)=\frac{c_1}{2a}\int_{q-a}^{q+a}\left[\frac{x^2}{2}-sx\right]_{s}^{1}ds + \frac{c_2}{2a}\int_{q-a}^{q+a}[sx-\frac{x^2}{2}]_{0}^{s}ds$$

$$\Rightarrow T(q)=\frac{c_1}{2a}\int_{q-a}^{q+a}\left[\frac{1}{2}-\frac{s^2}{2}-s+s^2\right]ds + \frac{c_2}{2a}\int_{q-a}^{q+a}\left[\frac{s^2}{2}\right]ds$$

$$\Rightarrow T(q)=\left(\frac{c_1+c_2}{2a}\right)[\frac{s^3}{6}]_{q-a}^{q+a} + \frac{c_1}{2a}\left(\frac{s}{2}-\frac{s^2}{2}\right)_{q-a}^{q+a}$$

$$\boldsymbol{\Rightarrow T(q)}=\left(\frac{\boldsymbol{c_1+c_2}}{\mathbf{6}}\right)(\mathbf{3}\boldsymbol{q}^{\mathbf{2}}+\boldsymbol{a}^{\mathbf{2}})+\frac{\boldsymbol{c_1}}{\mathbf{2}}-\boldsymbol{c_1 q}$$

Now to minimise the expected total cost we equate T’(q) to 0.

$\Rightarrow T'(q)=(c_1+c_2)q-c_1=0$

Hence,

$$\boldsymbol{q}=\frac{\boldsymbol{c_1}}{\boldsymbol{c_1+c_2}}$$

Now let us consider,

$$\Rightarrow\boldsymbol{q_{opt}}=\frac{\boldsymbol{c_1}}{\boldsymbol{c_1+c_2}}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow\mathbf{T}(\boldsymbol{q_{opt}})=\left(\frac{\boldsymbol{c_1+c_2}}{\mathbf{6}}\right)\left(\mathbf{3}\boldsymbol{q_{opt}}^{\mathbf{2}}+\boldsymbol{a}^{\mathbf{2}}\right)+\frac{\boldsymbol{c_1}}{\mathbf{2}}-\boldsymbol{c_1 q_{opt}}$$

4. **THE EXPONENTIAL DISTRIBUTION: X~exp(λ)**

The PDF is given by,

$$\mathbf{f(x)}=\begin{cases}\lambda \boldsymbol{e}^{-\lambda x}, \boldsymbol{if}\ \boldsymbol{x} > 0, \quad \lambda > 0 \\ \mathbf{0}, \quad \boldsymbol{otherwise}\end{cases}$$

The Expected Total Cost is given by ,

$\Rightarrow$T(q)=E[C(q)]=$\int_{q-a}^{q+a} \int_{s}^{\infty} c_1(x-s)\lambda e^{-\lambda x}\frac{1}{2a}dxds + \int_{q-a}^{q+a} \int_{0}^{s} c_2(s-x)\lambda e^{-\lambda x}\frac{1}{2a}dxds$

$\Rightarrow$T(q)=$\frac{c_1}{2a}\int_{q-a}^{q+a} ds \int_{s}^{\infty}(x\lambda e^{-\lambda x} - s\lambda e^{-\lambda x})dx \; + \frac{c_2}{2a}\int_{q-a}^{q+a} ds \int_{0}^{s}(s\lambda e^{-\lambda x} - x\lambda e^{-\lambda x})dx$

<u>Now let us first compute</u>:

$\Rightarrow I_1 = \int_{s}^{\infty}(x\lambda e^{-\lambda x} - s\lambda e^{-\lambda x})dx \; = \lambda[\{\frac{xe^{-\lambda x}}{\lambda}\}_{\infty}^{s} + \frac{1}{\lambda}\int_{s}^{\infty} e^{-\lambda x}dx] + [se^{-\lambda x}]_{s}^{\infty}$

$\Rightarrow I_1 = se^{-\lambda s} + [\frac{1}{\lambda}e^{-\lambda x}]_{\infty}^{s} - se^{-\lambda s}$

$\Rightarrow I_1 = \frac{1}{\lambda}e^{-\lambda s}$

<u>Now let us compute</u>:

$\Rightarrow I_2 = \int_{0}^{s}(s\lambda e^{-\lambda x} - x\lambda e^{-\lambda x})dx$

$\Rightarrow I_2 = \left[se^{-\lambda x}\right]_{s}^{0} - \lambda[\{\frac{xe^{-\lambda x}}{\lambda}\}_{s}^{0} + \frac{1}{\lambda}\int_{0}^{s} e^{-\lambda x}dx]$

$\Rightarrow I_2 = s - se^{-\lambda s} + se^{-\lambda s} + [\frac{1}{\lambda}e^{-\lambda x}]_{0}^{s}$

$\Rightarrow I_2 = s + \frac{1}{\lambda}e^{-\lambda s} - \frac{1}{\lambda}$

Hence,

$\Rightarrow$T(q)=$\frac{c_1}{2a}\int_{q-a}^{q+a}(\frac{1}{\lambda}e^{-\lambda s})ds + \frac{c_2}{2a}\int_{q-a}^{q+a}(s + \frac{1}{\lambda}e^{-\lambda s} - \frac{1}{\lambda})ds$

$\Rightarrow$T(q)=$\frac{c_1+c_2}{2a}\frac{1}{\lambda^2}\left[e^{-\lambda s}\right]_{q+a}^{q-a} + \frac{c_2}{2a}\left[\frac{s^2}{2} - \frac{s}{\lambda}\right]_{q-a}^{q+a}$

$\Rightarrow$T(q)=$\frac{c_1+c_2}{2a}\frac{1}{\lambda^2}\left[e^{-\lambda q}e^{\lambda a} - e^{-\lambda q}e^{-\lambda a}\right] + c_2 q - \frac{c_2}{\lambda}$

$\boldsymbol{\Rightarrow T(q)=\frac{c_1+c_2}{2a}\frac{1}{\lambda^2}e^{-\lambda q}\left[e^{\lambda a} - e^{-\lambda a}\right] + c_2 q - \frac{c_2}{\lambda}}$

Now to minimise the expected total cost, we equate T'(q) to 0

$\Rightarrow$T'(q)=$\frac{c_1+c_2}{2a}\frac{1}{\lambda}e^{-\lambda q}\left[e^{-\lambda a} - e^{\lambda a}\right] + c_2 = 0$

$\Rightarrow e^{-\lambda q} = \frac{c_2 2a\lambda}{(c_1+c_2)\left[e^{\lambda a} - e^{-\lambda a}\right]}$

$\Rightarrow e^{\lambda q} = \frac{(c_1+c_2)\left[e^{\lambda a} - e^{-\lambda a}\right]}{c_2 2a\lambda}$

$\Rightarrow \lambda q = \ln\frac{c_1+c_2}{c_2} + \ln\frac{\left[e^{\lambda a} - e^{-\lambda a}\right]}{2a\lambda}$

Hence,

$$q = \frac{1}{\lambda} ln \frac{c_1 + c_2}{c_2} + \frac{1}{\lambda} ln \frac{[e^{\lambda a} - e^{-\lambda a}]}{2a\lambda}$$

Now let us consider,

$$\Rightarrow q_{opt} = \frac{1}{\lambda} ln \frac{c_1+c_2}{c_2} + \frac{1}{\lambda} ln \frac{[e^{\lambda a} - e^{-\lambda a}]}{2a\lambda}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow T(q_{opt}) = \frac{c_1+c_2}{2a} \frac{1}{\lambda^2} e^{-\lambda q_{opt}} [e^{\lambda a} - e^{-\lambda a}] + c_2 q_{opt} - \frac{c_2}{\lambda}$$

5. **THE RECTANGULAR OR UNIFORM DISTRIBUTION : X~R(α,β)**

The PDF is given by,

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & if\ \alpha < x < \beta \\ 0, & otherwise \end{cases}$$

The Expected Total Cost is given by,

$$\Rightarrow T(q)=E[C(q)]=\int_{q-a}^{q+a} \int_{s}^{\beta} c_1(x-s) \frac{1}{\beta-\alpha} \frac{1}{2a} dxds + \int_{q-a}^{q+a} \int_{\alpha}^{s} c_2(s-x) \frac{1}{\beta-\alpha} \frac{1}{2a} dxds$$

$$\Rightarrow T(q)=\frac{c_1}{(\beta-\alpha)2a} \int_{q-a}^{q+a} [\frac{x^2}{2} - sx]_{s}^{\beta} ds + \frac{c_2}{(\beta-\alpha)2a} \int_{q-a}^{q+a} [sx - \frac{x^2}{2}]_{\alpha}^{s} ds$$

$$\Rightarrow T(q)=\frac{c_1}{(\beta-\alpha)2a} \int_{q-a}^{q+a} [\frac{\beta^2}{2} - \frac{s^2}{2} - \beta s + s^2] ds + \frac{c_2}{(\beta-\alpha)2a} \int_{q-a}^{q+a} [s^2 - s\alpha - \frac{s^2}{2} + \frac{\alpha^2}{2}] ds$$

$$\Rightarrow T(q)=\frac{c_1}{(\beta-\alpha)2a} \left[\frac{s^3}{6} - \beta \frac{s^2}{2} + \frac{\beta^2}{2} s\right]_{q-a}^{q+a} + \frac{c_2}{(\beta-\alpha)2a} \left[\frac{s^3}{6} - \alpha \frac{s^2}{2} + \frac{\alpha^2}{2} s\right]_{q-a}^{q+a}$$

$$\Rightarrow T(q)=\frac{1}{\beta-\alpha} \left\{(c_1 + c_2)\left(\frac{q^2}{2} + a^2\right) + \frac{c_1\beta^2 + c_2\alpha^2}{2} - (c_1\beta + c_2\alpha)q\right\}$$

Now to minimise the expected total cost we equate T'(q) to 0.

$$\Rightarrow T'(q)=\frac{1}{\beta-\alpha} \{(c_1 + c_2)q - c_1\beta - c_2\alpha\} = 0$$

$$\Rightarrow (c_1 + c_2)q = c_1\beta + c_2\alpha$$

Hence,

$$q = \frac{c_1\beta + c_2\alpha}{(c_1 + c_2)}$$

Now let us consider,

$$\Rightarrow q_{opt} = \frac{c_1\beta + c_2\alpha}{(c_1 + c_2)}$$

The Expected Total Optimal Cost is then given by,

$$\Rightarrow \mathrm{T}(\boldsymbol{q_{opt}}) = \frac{\mathbf{1}}{\boldsymbol{\beta-\alpha}}\left\{(\boldsymbol{c_1}+\boldsymbol{c_2})\left(\frac{\boldsymbol{q_{opt}}^{\mathbf{2}}}{\mathbf{2}}+\boldsymbol{a}^{2}\right)+\frac{\boldsymbol{c_1\beta^2+c_2\alpha^2}}{\mathbf{2}}-(\boldsymbol{c_1\beta}+\boldsymbol{c_2\alpha})\boldsymbol{q_{opt}}\right\}$$

## C. NUMERICAL ILLUSTRATIONS

### C.1 ##SUPPLY IS THE SAME AS THE ORDERED QUANTITY

***NOTE: *) For Exponential Distribution we take the parameter $\lambda$=1/15.***

****) For Rectangular Distribution we take the parameters $\alpha$=10, $\beta$=20***

**1. THE BETA (1,2) DISTRIBUTION: X~B(1,2)**

| $CHOICE\ OF\ C_1 AND\ C_2$ | $\boldsymbol{q_{opt}}$ | $\mathrm{T}(\boldsymbol{q_{opt}})$ |
|---|---|---|
| $\boldsymbol{c_1=1, c_2=2}$ | ***0.18350*** | ***0.24467*** |
| $\boldsymbol{c_1=1, c_2=0.5}$ | ***0.42265*** | ***0.14088*** |
| $\boldsymbol{c_1=1, c_2=1}$ | ***0.29289*** | ***0.19526*** |

2. **THE BETA (2,1) DISTRIBUTION : X~B(2,1)**

| $CHOICE\ OF\ C_1 AND\ C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | ***0.57735*** | ***0.28177*** |
| $c_1 = 1, c_2 = 0.5$ | ***0.81650*** | ***0.12234*** |
| $c_1 = 1, c_2 = 1$ | ***0.70711*** | ***0.19526*** |

3. **THE BETA (1,1) DISTRIBUTION : X~B(1,1)**

| $CHOICE\ OF\ C_1 AND\ C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | ***0.33333*** | ***0.33333*** |
| $c_1 = 1, c_2 = 0.5$ | ***0.66666*** | ***0.16666*** |
| $c_1 = 1, c_2 = 1$ | ***0.50000*** | ***0.25000*** |

4. **THE EXPONENTIAL DISTRIBUTION: X~exp(λ)**

| $CHOICE\ OF\ C_1 AND\ C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | ***6.0820*** | ***72.164*** |
| $c_1 = 1, c_2 = 0.5$ | ***16.4792*** | ***23.240*** |
| $c_1 = 1, c_2 = 1$ | ***10.3972*** | ***40.397*** |

5. **<u>THE RECTANGULAR OR UNIFORM DISTRIBUTION</u> : X~R(α,β)**

| *CHOICE OF* $C_1$ *AND* $C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | ***13.33333*** | ***3.33333*** |
| $c_1 = 1, c_2 = 0.5$ | ***16.66666*** | ***1.66666*** |
| $c_1 = 1, c_2 = 1$ | ***15.00000*** | ***2.50000*** |

## ***C.2 <u>## SUPPLY IS RANDOM, DEPENDING UPON THE ORDER QUANTITY</u>***

***NOTE: *) For Exponential Distribution we take the parameter $\lambda$=1/15.***

****) For Rectangular Distribution we take the parameters $\alpha$=10, $\beta$=20.***

****) we take a=1***

1. **THE BETA (1,2) DISTRIBUTION: X~B(1,2)**

| *CHOICE OF* $C_1$ *AND* $C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | **0.42265** | **0.94843** |
| $c_1 = 1, c_2 = 0.5$ | **1.00000** | **0.33333** |
| $c_1 = 1, c_2 = 1$ | **0.59175** | **0.57594** |

2. **THE BETA (2,1) DISTRIBUTION : X~B(2,1)**

| *CHOICE OF* $C_1$ *AND* $C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | **0.00000** | **0.66666** |
| $c_1 = 1, c_2 = 0.5$ | **0.57735** | **0.47422** |
| $c_1 = 1, c_2 = 1$ | **0.40825** | **0.57594** |

3. **THE BETA (1,1) DISTRIBUTION : X~B(1,1)**

| *CHOICE OF* $C_1$ *AND* $C_2$ | $q_{opt}$ | $T(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | **0.33333** | **0.83333** |
| $c_1 = 1, c_2 = 0.5$ | **0.66666** | **0.41666** |

| | | |
|---|---|---|
| $c_1 = 1, c_2 = 1$ | ***0.50000*** | ***0.58333*** |

## 4. THE EXPONENTIAL DISTRIBUTION: X~exp(λ)

| $CHOICE\ OF\ C_1 AND\ C_2$ | $q_{opt}$ | $\mathrm{T}(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | ***6.0931*** | ***301.880*** |
| $c_1 = 1, c_2 = 0.5$ | ***16.4903*** | ***146.471*** |
| $c_1 = 1, c_2 = 1$ | ***10.4083*** | ***200.512*** |

## 5. THE RECTANGULAR OR UNIFORM DISTRIBUTION : X~R(α,β)

| $CHOICE\ OF\ C_1 AND\ C_2$ | $q_{opt}$ | $\mathrm{T}(q_{opt})$ |
|---|---|---|
| $c_1 = 1, c_2 = 2$ | ***13.33333*** | ***3.63333*** |
| $c_1 = 1, c_2 = 0.5$ | ***16.66666*** | ***1.81666*** |
| $c_1 = 1, c_2 = 1$ | ***15.00000*** | ***2.70000*** |

## 2. <u>FIELD OF APPLICATIONS:</u>

- **"Want to know how the International Air Transport (IATA) hosted a net profit of $35.5 billions in 2019 slightly ahead of 2018's net profit of $32.3 billions!!!"**

  **Well the above figures have been greatly achieved due to a concept known as the "*<u>Policy of Overbooking in Airlines</u>*" .**

***<u>Overbooking</u>*** involves controlling the level of reservations to balance the potential risks of denied service against the rewards of increased sales and hiked profits.

***<u>Overview of the practical situation</u>*** **:**

**Suppose a customer books a ticket on a particular flight but eventually does not show up during the scheduled departure, the Airlines end up in a vacant seat resulting a loss in revenue.**

**On the other hand suppose the demands are higher than the actual number of seats available and also there are no shows of passengers to be expected, then the Airlines may deprive itself from acquiring an added profit.**

**Hence the Policy of Overbooking based on the exact problem proposed above centralized on the Classical Newsboy Model helps the Airlines to earn greater revenues and strengthened profits.**

**The simplicity of this model stretches its applicability beyond imagination . Not only overbooking policies in Airlines witnesses the utility of Classical Newsboy**

**Problem but it is also used in "*Ticket booking procedure in IPL" , "Car Rentals"*, etc.**

- **In the case of selling tickets to high voltage cricket matches , like the IPL final or World Cup Matches , the *Classical Newsboy Setup provides immense rewards in terms of profits and revenues*.
*The total number of tickets* , subject to demand , which is huge in these cases , *is inflated to a number far above the actual availability* in order to prevent losses from excessive cancellations , and at the same time , a check is made on the inflated number so that every person who pays for a ticket gets one . This *maximizes* the IPL franchises , The BCCI and also leads to a *highlighted promotion* as; based on the *demand and supply function* under the Newsboy Setup the *optimal Quantity of tickets* to be sold is meticulously calculated.**

- ***Newly manufactured brand new cars* highly draw the attention of the public because of the publicity created by the various Car Companies.
Automatically this results in an *increased demand* in the purchase of car but it may so happen that at the time of purchase few of the buyers back out because of insufficient fund as the prices of the newly launched model of the car is sky high. So if the initial demand is taken into account by the car companies , it leads it to automatic losses due to *unwanted excess supply*. Hence the *demand is forcibly reduced by a factor and then supply is made in order to optimize sale and thus optimize profit* without wasted supply.
These calculations are nothing but the branches or roundabouts of *The Classical Newsboy Problem.***

## 3. CONCLUSION:

From the numerical computations, we see that, the ***optimal order quantity and the associated cost is more when the supply is random***. This excess Cost needs to be incurred for allowing the uncertainty in supply.

## 4. REFERENCE:


- *Some inventory model with random supply , Chandra . A(2002) – IAPQR Transactions.*