

(a) **false**. הפונקציה  $g$  מקבלת את  $a$  שהוא מסוג **NUMBER** אבל היא מוגדרת לקבל משתנה מטיפוס אחר-  $1T$ . לא הוגדר לנו ש  $T1=number$ .  
לכן שני הצדדים בביטוי זה שונים אז יחזיר **false**.

(b) **false**. הפונקציה  $f$  מקבלת את  $x$  שמוגדר על ידי הטיפוס  $1T$  לעומת זאת, כמוגדרת לקבל פרמטר מטיפוס  $2T$ .

(c) **true**. הפונקציה  $f$  מקבלת את המשתנה  $x$  מטיפוס  $1T$  בדיוק כמו שהיא מצפה. כמו כן  $f$  מחזירה משתנה מטיפוס  $2T$  כפי שהיא אמורה להחזיר.

(d) **True**. הפונקציה  $f$  אמורה לקבל שני טיפוסים  $1T$  ו- $2T$ . לא הוגדר הטיפוס של  $x$  אז ניתן להסיק שטיפוסו הוא  $1T$  והטיפוס של  $y$  מוגדר להיות  $2T$  כנדרש כלומר **TRUE**

שאלה 2

(a)

שלב 1 :  $((\lambda (f \ x) (f \ 1 \ x)) + \#t) \Rightarrow ((\lambda (f \ x) (f \ 1 \ x)) + \#t)$

שלב 2 :

Expression	Variable
$((\lambda (f \ x) (f \ 1 \ x)) + \#t)$	<b>T0</b>
$(\lambda (f \ x) (f \ 1 \ x))$	<b>T1</b>
<b>+</b>	<b>T+</b>
<b>#t</b>	<b>Ttrue</b>
$(f \ x)$	<b>T2</b>
$(f \ 1 \ x)$	<b>T3</b>
<b>f</b>	<b>Tf</b>
<b>f1</b>	<b>Tf1</b>
<b>x</b>	<b>Tx</b>
<b>1</b>	<b>Tnum1</b>

שלב 3:

Expression	Equation
$((\lambda (f \ x) (f \ 1 \ x)) + \#t)$	<b>T1 = [T+*Ttrue-&gt;T0]</b>
$(\lambda (f \ x) (f \ 1 \ x))$	<b>T1 = [Tf*T<sub>x</sub>-&gt;T3]</b>
$(f \ x)$	<b>Tf = [Tx -&gt; T2]</b>
$(f \ 1 \ x)$	<b>Tf1 = [Tnum1*T<sub>x</sub>-&gt;T3]</b>

  

Expression	Equation
<b>1</b>	<b>Tnum1= Number</b>
<b>+</b>	<b>T+= [Number *Number -&gt;Number]</b>
<b>#t</b>	<b>Ttrue = Boolean</b>

שלב 4: נפתור את מערכת המשוואות

Equation	Substitution
$T1 = [T+*Ttrue \rightarrow T0]$	$\{ \}$
$T1 = [Tf*Tx \rightarrow T3]$	
$Tf = [Tx \rightarrow T2]$	
$Tf1 = [Tnum1*Tx \rightarrow T3]$	
$Tnum1 = \text{Number}$	
$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
$Ttrue = \text{Boolean}$	

Equation	Substitution
$T1 = [Tf*Tx \rightarrow T3]$	$\{ T1 = [T+*Ttrue \rightarrow T0] \}$
$Tf = [Tx \rightarrow T2]$	
$Tf1 = [Tnum1*Tx \rightarrow T3]$	
$Tnum1 = \text{Number}$	
$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
$Ttrue = \text{Boolean}$	

Equation	Substitution
$Tf = [Tx \rightarrow T2]$	$\{ T1 = [T+*Ttrue \rightarrow T0] \}$
$Tf1 = [Tnum1*Tx \rightarrow T3]$	
$Tnum1 = \text{Number}$	
$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
$Ttrue = \text{Boolean}$	
$T0 = T3$	
$Ttrue = Tx$	
$T+ = Tf$	

Equation	Substitution
$Tf1 = [Tnum1*Tx \rightarrow T3]$	$\{ T1 = [T+*Ttrue \rightarrow T0],$ $Tf = [Tx \rightarrow T2] \}$
$Tnum1 = \text{Number}$	
$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
$Ttrue = \text{Boolean}$	
$T0 = T3$	
$Ttrue = Tx$	
$T+ = Tf$	

Equation	Substitution
$Tnum1 = \text{Number}$	$\{ T1 = [T+*Ttrue \rightarrow T0],$ $Tf = [Tx \rightarrow T2],$ $Tf1 = [Tnum1*Tx \rightarrow T3] \}$
$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
$Ttrue = \text{Boolean}$	
$T0 = T3$	

$T += [\text{Number} * \text{Number} \rightarrow \text{Number}]$
$T_{\text{true}} = T_x$
$T_+ = T_f$

Equation	Substitution
$T += [\text{Number} * \text{Number} \rightarrow \text{Number}]$	$\{T_1 = [T_+ * T_{\text{true}} \rightarrow T_0],$ $T_f = [T_x \rightarrow T_2],$ $T_{f1} = [\text{Number} * T_x \rightarrow T_3],$ $T_{\text{num}1} = \text{Number} \}$
$T_{\text{true}} = \text{Boolean}$	
$T_0 = T_3$	
$T_{\text{true}} = T_x$	
$T_+ = T_f$	

Equation	Substitution
$T_{\text{true}} = \text{Boolean}$	$\{T_1 = [[\text{Number} * \text{Number} \rightarrow \text{Number}] * T_{\text{true}} \rightarrow T_0],$ $T_f = [T_x \rightarrow T_2],$ $T_{f1} = [\text{Number} * T_x \rightarrow T_3],$ $T_{\text{num}1} = \text{Number},$ $T_+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]\}$
$T_0 = T_3$	
$T_{\text{true}} = T_x$	
$T_f = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	

Equation	Substitution
$T_0 = T_3$	$\{T_1 = [[\text{Number} * \text{Number} \rightarrow \text{Number}] * \text{Boolean} \rightarrow T_0],$ $T_f = [T_x \rightarrow T_2],$ $T_{f1} = [\text{Number} * T_x \rightarrow T_3],$ $T_{\text{num}1} = \text{Number},$ $T_+ = [\text{Number} * \text{Number} \rightarrow \text{Number}],$ $T_{\text{true}} = \text{Boolean} \}$
$T_x = \text{Boolean}$	
$T_f = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	

Equation	Substitution
$T_x = \text{Boolean}$	$\{T_1 = [[\text{Number} * \text{Number} \rightarrow \text{Number}] * \text{Boolean} \rightarrow T_0],$ $T_f = [T_x \rightarrow T_2],$ $T_{f1} = [\text{Number} * T_x \rightarrow T_3],$ $T_{\text{num}1} = \text{Number},$ $T_+ = [\text{Number} * \text{Number} \rightarrow \text{Number}],$ $T_{\text{true}} = \text{Boolean}$ $T_0 = T_3 \}$
$T_f = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	

Equation	Subtitution
Tf = [Number *Number ->Number],	{T1 = [[Number *Number ->Number]* Boolean ->T0], Tf = [Boolean-> T2], Tf1 = [Number*Tx->T3], Tnum1= Number, T+= [Number *Number ->Number], Ttrue = Boolean, T0 = T3, Tx = Boolean , }

אז סתירה מפני שTf מוגדר לקבל Boolean ובנוסף מוגדר לקבל number\* number כמובן שהדבר לא יתכן.

(b)

שלב 1:

$((\text{lambda } (f1 \ x1) (f1 \ x1 \ 1)) + *) \Rightarrow ((\text{lambda } (f \ x) (f1 \ x \ 1)) + *)$

שלב 2:

Expression	Variable
$((\text{lambda } (f \ x) (f1 \ x \ 1)) + *)$	T0
$(\text{lambda } (f \ x) (f1 \ x \ 1))$	T1
$(f \ x)$	T2
$(f1 \ x \ 1)$	T3
f1	Tf1
f	Tf
X	Tx
1	Tnum1
+	T+
*	T*

שלב 3:

Expression	Equation
$((\text{lambda } (f \ x) (f1 \ x \ 1)) + *)$	T1 = [T+*T*->T0]
$(\text{lambda } (f \ x) (f1 \ x \ 1))$	T1 = [Tf*Tx->T3]
$(f \ x)$	Tf= [Tx->.T2]
$(f1 \ x \ 1)$	Tf1 = [Tx*Tnum1->T3]
1	Tnum1 = [Number]
+	T+=[Number *Number ->Number]
*	T*=[Number *Number ->Number]

שלב 4:

Equation	Substitution
$T1 = [T+ * T^* \rightarrow T0]$	$\{ \}$
$T1 = [Tf * Tx \rightarrow T3]$	
$Tf = [Tx \rightarrow T2]$	
$Tf1 = [Tx * Tnum1 \rightarrow T3]$	
$Tnum1 = [Number]$	
$T+ = [Number * Number \rightarrow Number]$	
$T^* = [Number * Number \rightarrow Number]$	

Equation	Substitution
$T1 = [Tf * Tx \rightarrow T3]$	$\{ T1 = [T+ * T^* \rightarrow T0] \}$
$Tf = [Tx \rightarrow T2]$	
$Tf1 = [Tx * Tnum1 \rightarrow T3]$	
$Tnum1 = [Number]$	
$T+ = [Number * Number \rightarrow Number]$	
$T^* = [Number * Number \rightarrow Number]$	

Equation	Substitution
$Tf = [Tx \rightarrow T2]$	$\{ T1 = [T+ * T^* \rightarrow T0] \}$
$Tf1 = [Tx * Tnum1 \rightarrow T3]$	
$Tnum1 = [Number]$	
$T+ = [Number * Number \rightarrow Number]$	
$T^* = [Number * Number \rightarrow Number]$	
$T0 = T3$	
$Tf = T+$	
$Tx = T^*$	

Equation	Substitution
$Tf1 = [Tx * Tnum1 \rightarrow T3]$	$\{ T1 = [T+ * T^* \rightarrow T0], Tf = [Tx \rightarrow T2] \}$
$Tnum1 = [Number]$	
$T+ = [Number * Number \rightarrow Number]$	
$T^* = [Number * Number \rightarrow Number]$	
$T0 = T3$	
$Tf = T+ = Tx \rightarrow T2$	
$Tx = T^*$	

Equation	Substitution
$Tnum1 = [Number]$	$\{ T1 = [T+ * T^* \rightarrow T0], Tf = [Tx \rightarrow T2], Tf1 = [Tx * Tnum1 \rightarrow T3] \}$

T+ =[Number *Number ->Number]
T* =[Number *Number ->Number]
T0=T3
Tf=T+ = Tx->T2
Tx= T*

Equation	Substitution
T+ =[Number *Number ->Number]	{T1 =[T+*T*->T0], Tf= [Tx->T2], Tf1 = [Tx*Number->T3] , Tnum1 = [Number]}
T* =[Number *Number ->Number]	
T0=T3	
Tf=T+ = Tx->T2	
Tx= T*	

Equation	Substitution
T* =[Number *Number ->Number]	{T1 =[ [Number *Number ->Number]*T*->T0], Tf= [Tx->T2], Tf1 = [Tx*Number->T3] , Tnum1 = [Number], T+ =[Number *Number ->Number] }
T0=T3	
Tf=T+ = Tx->T2 = Number *Number ->Number	
Tx= T*	

Equation	Substitution
T0=T3	{T1 =[ [Number *Number ->Number]* [Number *Number ->Number] ->T0], Tf= [Tx->T2], Tf1 = [Tx*Number->T3] , Tnum1 = [Number], T+ =[Number *Number ->Number], T* =[Number *Number ->Number] }
Tf=T+ = Tx->T2 = Number *Number ->Number	
Tx= T* = [Number *Number ->Number]	

Equation	Substitution
Tf=T+ = Tx->T2 = Number *Number ->Number	{T1 =[ [Number *Number ->Number]* [Number *Number ->Number] ->T0], Tf= [Tx->T2], Tf1 = [Tx*Number->T3] , Tnum1 = [Number], T+ =[Number *Number ->Number], T* =[Number *Number ->Number] , T0 =T3 }

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$$Tx = T^* = [Number * Number \rightarrow Number]$$


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Equation	Substitution
$Tf = T+ = Tx \rightarrow T2 = Number * Number -$ $\rightarrow Number$	$\{T1 = [ [Number * Number \rightarrow Number]^*$ $[Number * Number \rightarrow Number] \rightarrow T0],$ $Tf = [Tx \rightarrow T2],$ $Tf1 = [Tx * Number \rightarrow T3] ,$ $Tnum1 = [Number],$ $T+ = [Number * Number \rightarrow Number],$ $T^* = [Number * Number \rightarrow Number] ,$ $T0 = T3 \}$
$Tx = T^* = [Number * Number \rightarrow Number]$	

יש שגיאה בהגדרת הטיפוס  $Tx$  מפני שהגדרתו בשתי השורות בטבלה שונות.

### 3.1.1 Typing rules :

#### **Set-Exp :**

For every:            type environment  $\_Tenv$ ,  
                         Variable reference  $\_r$ ,  
                         Expression  $\_e$   
                         Type expressions  $\_S$  :

If  $\_Tenv \mid - e : \_S$  and

$\_Tenv \mid - \_r : \_S$

Then  $\_Tenv \mid - (set! \_r \_e) : void$

#### **Lit-Exp :**

For every:            type environment  $\_Tenv$   
                         Expressions  $e$

If  $\_Tenv \mid - e : T$

Then  $(e) : T$

### 3.2.2 Typing rules :

#### **Define-type :**

For every:            type environment  $\_Tenv$   
                         Define\_type  $\_dt$

$\_Tenv \mid - (\_dt) : void$

#### **Type-case :**

Expressions :  $rec1, \dots, recn$  (record  $\rightarrow$  for each record in the UDT)

Expressions :  $f11, \dots, f1t, \dots, fnn$  (record.field  $\rightarrow$  for each record,  $f_{ij}$  is record  $i$  in the UDT at field  $j$ )

Expressions:  $\_t, u$  (TypeCase, UDT)

Expressions:  $c1, \dots, cn$  (cases.case  $\rightarrow$  for each case )

Expressions:  $c11, \dots, c1t, \dots, cnn$  (case.vardecl  $\rightarrow$  for each case,  $c_{ij}$  is case  $i$  in cases at vardecl  $j$ )

Type expressions  $\_S11, \dots, Snn, Sa1, \dots, San$

If  $\_Tenv \circ \{ f11 : S11, \dots, fnn : Snn, \dots, c1.body : Sa1, \dots, cn.body : San \}$

$\_Tenv \mid - c11 : S11, \dots, cnn : Snn$

Then  $\_Tenv \mid - (TypeCase \_t (c1(c11, \dots, c1t)) c1.body) \dots (cn(c11, \dots, cnn) cn.body)) :$

$checkCoverType(Sa1, \dots, San).value$