# First part for Multigrid.

This describes the modules that you need for multi grid. These are the building blocks that are needed. There are several modules needed, a sweep, residual, coarsen and refine. To reflect the physical setup, the operator should be the second derivative operator, i.e. it divides the difference by $h^2$. This makes the coarsen and refinement operators more intuitive but will affect both the residual and sweep operator where you have to multiply or divide by $h^2$. Since the step sizes are different at different resolution you can't factor them completely out so you either do this or have a factor of 4 put in the right places.

**Sweep:** Takes in a state `x`, right hand side `b` and relaxation parameter $\omega$. Performs a relaxed Jacobi iteration. Note that here the right hand side has to be scaled by $h^2$.

**Residual:** Takes in a state `x`, right hand side `b`. Similar to Jacobi, and you have a scale here as well but this time you have to divide by $h^2$. Because division is slower than multiplication, define a constant as $1/h^2$ and multiply by that instead.

**Coarsen:** Takes in a fine grid and returns a coarse grid. If the input grid is $(2M+1) \times (2N+1)$ the output is $(M+1) \times (N+1)$. This is typically picked so that it proportional to the adjoint of the refinement operator. That is instead of just picking up every other value in $x$ and $y$ you take a weighted average of the neighbors. To check this calculation, hand it in the test functions $x$ and $y$ since the coarsening operator is exact on those functions. To handle the step size, either use the `Mesh2D` class or hand in both the array and the step size. The `Mesh2D` class is really just an array with a grid class attached and exist in both a const and mutable version.

**Refine:** Takes in a coarse grid and returns a fine grid. Here the points in between the grid points are averages of neighboring coarse grid points. This operation should be the transpose of he coarsening operation.

Test each part on your own. The sweep can be tested by using a coarse grid and make sure that the iteration will converge to the exact solution. The residual should be tested by comparing it to a matlab implementation on a coarse grid. The coarsen and refinement operations can be tested by using linear functions like $x + 5y$ and make sure that both operations are exact up to the boundary.