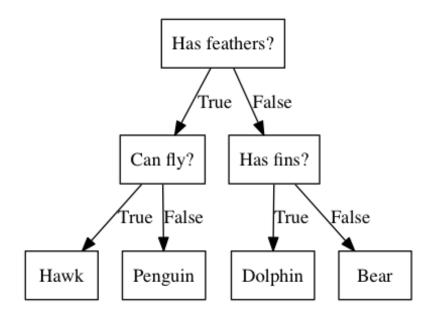
Trees and Ensembles

Trees

[144]: mglearn.plots.plot_animal_tree()



Building Decision Trees

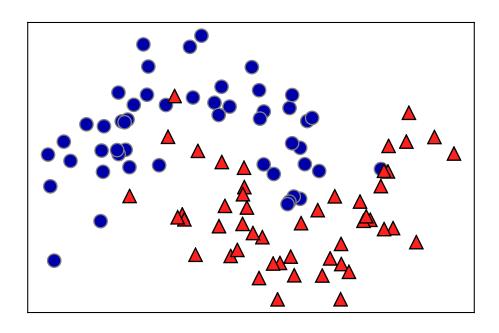
- Split the data in two (or more) parts
- Search over all possible splits and choose the one that is most *informative*
 - Many heuristics
 - E.g. information gain: how much does the entropy of the class labels decrease after the split (purer 'leafs')
- Repeat recursive partitioning

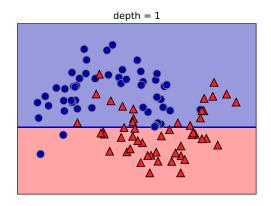
Making predictions:

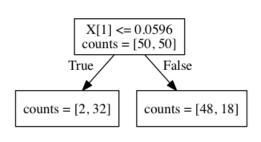
- Classification: find leaf for new data point, predict majority class (or class distribution)
- Regression: idem, but predict the *mean* of all values

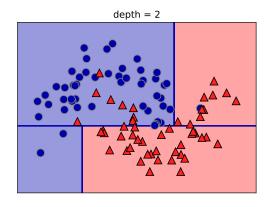
Decision Tree classification

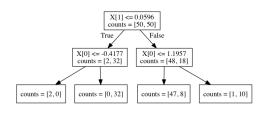
[145]: mglearn.plots.plot_tree_progressive()

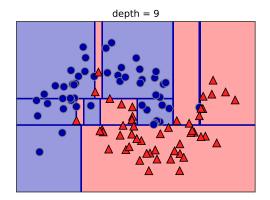


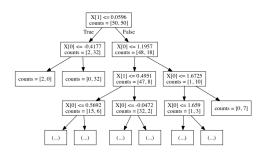












Heuristics

For classification $(X_i \rightarrow class_k)$: Impurity measures:

• Misclassification Error (leads to larger trees):

$$1 - \operatorname*{argmax} \hat{p}_k$$

• Gini-Index (probabilistic predictions):

$$\sum_{k \neq k'} \hat{p}_k \hat{p}_{k'} = \sum_{k=1}^K \hat{p}_k (1 - \hat{p}_k)$$

with \hat{p}_k = the relative frequency of class k in the leaf node

- Entropy (of the class attribute) measures *unpredictability* of the data:
 - How likely will random example have class k?

$$E(X) = -\sum_{k=1}^{K} \hat{p}_k \log_2 \hat{p}_k$$

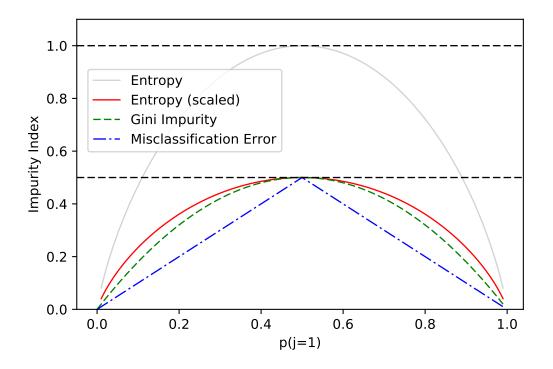
• Information Gain (a.k.a. Kullback–Leibler divergence) for choosing attribute X_i to split the data:

$$G(X, X_i) = E(X) - \sum_{v=1}^{V} \frac{|X_{i=v}|}{|X_i|} E(X_{i=v})$$

with \hat{p}_k = the relative frequency of class k in the leaf node, X = the training set, containing i features (variables) X_i , v a specific value for X_i , $X_{i=v}$ is the set of examples having value v for feature X_i : $\{x \in X | X_i = v\}$

Heuristics visualized (binary class) - Note that gini != entropy/2

```
[146]: def gini(p):
          return (p) * (1 - (p)) + (1 - p) * (1 - (1-p))
       def entropy(p):
          return - p*np.log2(p) - (1 - p)*np.log2((1 - p))
       def classification_error(p):
          return 1 - np.max([p, 1 - p])
       x = np.arange(0.0, 1.0, 0.01)
       ent = [entropy(p) if p != 0 else None for p in x]
       scaled_ent = [e*0.5 if e else None for e in ent]
       c_err = [classification_error(i) for i in x]
      fig = plt.figure()
       ax = plt.subplot(111)
       for j, lab, ls, c, in zip(
             [ent, scaled_ent, gini(x), c_err],
             ['Entropy', 'Entropy (scaled)', 'Gini Impurity', 'Misclassification
             ['-', '-', '--', '-.'],
             ['lightgray', 'red', 'green', 'blue']):
          line = ax.plot(x, j, label=lab, linestyle=ls, lw=1, color=c)
       ax.legend(loc='upper left', bbox_to_anchor=(0.01, 0.85),
                ncol=1, fancybox=True, shadow=False)
       ax.axhline(y=0.5, linewidth=1, color='k', linestyle='--')
       ax.axhline(y=1.0, linewidth=1, color='k', linestyle='--')
      plt.ylim([0, 1.1])
      plt.xlabel('p(j=1)')
      plt.ylabel('Impurity Index')
      plt.show()
```



Example

Ex.	1	2	3	4	5	6
a1	T	T	T	F	F	F
a2	T	T	F	F	T	T
class	+	+	-	+	-	-

E(X) ? $G(X,X_{a2})$? $G(X,X_{a1})$? $E(X) = -(\frac{1}{2}*log_2(\frac{1}{2}) + \frac{1}{2}*log_2(\frac{1}{2})) = 1 \text{ (classes have equal probabilities)}$ $G(X,X_{a2}) = 0 \text{ (after split, classes still have equal probabilities, entropy stays 1)}$

$$E(X) = -\sum_{k=1}^{K} \hat{p}_k \log \hat{p}_k \quad , \quad G(X, X_i) = E(X) - \sum_{v=1}^{V} \frac{|X_{i=v}|}{|X_i|} E(X_{i=v})$$

$$E(X_{a1=T}) = -\frac{2}{3} \log_2(\frac{2}{3}) - \frac{1}{3} \log_2(\frac{1}{3}) = 0.9183 \quad (= E(X_{a1=F}))$$

$$G(X, X_{a1}) = 1 - \frac{1}{2} 0.9183 - \frac{1}{2} 0.9183 = 0.0817$$

hence we split on a1

Heuristics in scikit-learn The splitting criterion can be set with the criterion option in DecisionTreeClassifier

- gini (default): gini impurity index
- entropy: information gain

Best value depends on dataset, as well as other hyperparameters

```
[147]: import plot_classifiers as pc
    names = ["Decision tree - gini", "Decision tree - entropy"]
    classifiers = [
        DecisionTreeClassifier(),
        DecisionTreeClassifier(criterion="entropy")
        ]
    pc.plot_classifiers(names, classifiers, figuresize=(20,8))
```

Handling many-valued features

What happens when a feature has (almost) as many values as examples? - Information Gain will select it

One approach: use Gain Ratio instead (not available scikit-learn):

$$GainRatio(X,X_i) = \frac{Gain(X,X_i)}{SplitInfo(X,X_i)}$$

$$SplitInfo(X, X_i) = -\sum_{v=1}^{V} \frac{|X_{i=v}|}{|X|} log_2 \frac{|X_{i=v}|}{|X|}$$

where $X_{i=v}$ is the subset of examples for which feature X_i has value v.

SplitInfo will be big if X_i fragments the data into many small subsets, resulting in a smaller Gain Ratio.

Overfitting: Controlling complexity of Decision Trees

Decision trees can very easily overfit the data. Regularization strategies:

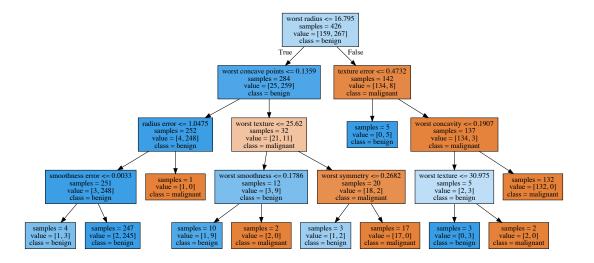
- Pre-pruning: stop creation of new leafs at some point
 - Limiting the depth of the tree, or the number of leafs
 - Requiring a minimal leaf size (number of instances)
- Post-pruning: build full tree, then prune (join) leafs
 - Reduced error pruning: evaluate against held-out data
 - Many other strategies exist.
 - scikit-learn supports none of them (yet)

Effect of pre-pruning: default tree overfits, setting max_depth=4 is better

```
[148]: from sklearn.datasets import load_breast_cancer
       from sklearn.tree import DecisionTreeClassifier
       from sklearn.model_selection import train_test_split
       cancer = load_breast_cancer()
       X_train, X_test, y_train, y_test = train_test_split(
           cancer.data, cancer.target, stratify=cancer.target, random_state=42)
       tree = DecisionTreeClassifier(random_state=0)
       tree.fit(X_train, y_train)
       print ("Accuracy on training set: {:.3f}".format(tree.score(X_train, y_tra
       print("Accuracy on test set: {:.3f}".format(tree.score(X_test, y_test)))
Accuracy on training set: 1.000
Accuracy on test set: 0.937
[149]: tree = DecisionTreeClassifier(max_depth=4, random_state=0)
      tree.fit(X_train, y_train)
       print("Accuracy on training set: {:.3f}".format(tree.score(X_train, y_tra
      print("Accuracy on test set: {:.3f}".format(tree.score(X_test, y_test)))
Accuracy on training set: 0.988
Accuracy on test set: 0.951
```

Analyzing Decision Trees manually

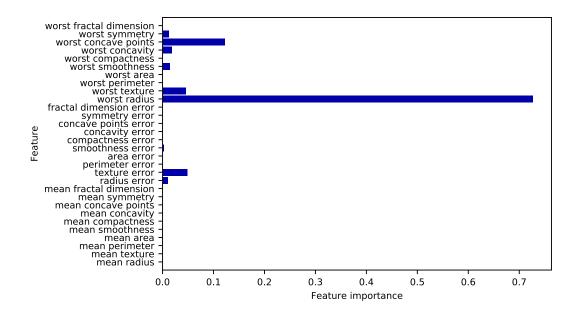
Visualize and find the path that most data takes



DecisionTreeClassifier also returns feature importances

- In [0,1], sum up to 1
- High values for features selected by the algorithm
- Other features may also be relevant, but don't contribute new information given the selected features

```
[151]: # Feature importances sum up to 1
       print("Feature importances:\n{}".format(tree.feature_importances_))
Feature importances:
[ 0.
         0.
                0.
                                      0.
                                             0.
                                                     0.
                       0.
                               0.
                                                            0.
                                                                   0.
                                                                          0.01
                                                            0.
                                                                   0.727
  0.048
                0.
                       0.002
                                      0.
                                             0.
         0.
                               0.
                                                    0.
  0.046 0.
                0.
                       0.014
                               0.
                                      0.018
                                             0.122 0.012
                                                           0.
                                                                 ]
[152]: def plot_feature_importances_cancer(model):
           n_features = cancer.data.shape[1]
           plt.barh(range(n_features), model.feature_importances_, align='center
           plt.yticks(np.arange(n_features), cancer.feature_names)
           plt.xlabel("Feature importance")
           plt.ylabel("Feature")
           plt.ylim(-1, n_features)
       plt.rcParams.update({'font.size': 8})
       plot_feature_importances_cancer(tree)
```



Decision tree regression

Heuristic for regression ($x_i \rightarrow y_i \in \mathbb{R}$): Minimal quadratic distance

- Consider splits at every data point for every variable (or halfway between data points)
- Dividing the data on split variable X_i at splitpoint s leads to the following half-spaces:

$$R_1(j,s) = X : X_j \le s \quad and \quad R_2(j,s) = X : X_j > s$$

• The best split variable and the corresponding splitpoint, with predicted value c_i and actual value Y_i :

$$\min_{j,s} \left(\min_{c_1} \sum_{x_i \in R_1(j,s)} (y_i - c_1)^2 + \min_{c_2} \sum_{x_i \in R_2(j,s)} (y_i - c_2)^2 \right)$$

• Assuming that the tree predicts y_i as the average of all x_i in the leaf:

$$\hat{c}_1 = \text{avg}(y_i | x_i \in R_1(j, s))$$
 and $\hat{c}_2 = \text{avg}(y_i | x_i \in R_2(j, s))$

with x_i being the i-th example in the data, with target value y_i

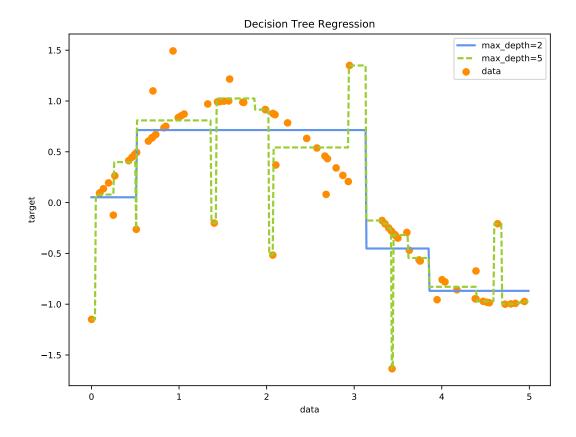
In scikit-learn Regression is done with DecisionTreeRegressor

```
[153]: def plot_decision_tree_regression(regr_1, regr_2):
    # Create a random dataset
    rng = np.random.RandomState(1)
    X = np.sort(5 * rng.rand(80, 1), axis=0)
    y = np.sin(X).ravel()
    y[::5] += 3 * (0.5 - rng.rand(16))
```

```
# Fit regression model
           regr_1.fit(X, y)
           regr_2.fit(X, y)
           # Predict
          X_{test} = np.arange(0.0, 5.0, 0.01)[:, np.newaxis]
          y_1 = regr_1.predict(X_test)
          y_2 = regr_2.predict(X_test)
           # Plot the results
          plt.figure(figsize=(8,6))
          plt.scatter(X, y, c="darkorange", label="data")
          plt.plot(X_test, y_1, color="cornflowerblue", label="max_depth=2", li
          plt.plot(X_test, y_2, color="yellowgreen", label="max_depth=5", linew
          plt.xlabel("data")
          plt.ylabel("target")
          plt.title("Decision Tree Regression")
          plt.legend()
          plt.show()
[171]: from sklearn.tree import DecisionTreeRegressor
      regr_1 = DecisionTreeRegressor(max_depth=2)
```

regr_2 = DecisionTreeRegressor(max_depth=5)

plot_decision_tree_regression(regr_1, regr_2)

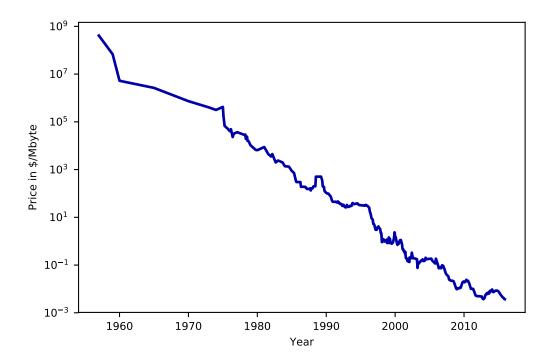


Note that decision trees do not extrapolate well.

- The leafs return the same *mean* value no matter how far the new data point lies from the training examples.
- Example on the ram_price forecasting dataset

```
[155]: ram_prices = pd.read_csv('data/ram_price.csv')

plt.semilogy(ram_prices.date, ram_prices.price)
plt.xlabel("Year")
plt.ylabel("Price in $/Mbyte");
```



```
[156]: from sklearn.tree import DecisionTreeRegressor
    from sklearn.linear_model import LinearRegression

# Use historical data to forecast prices after the year 2000
    data_train = ram_prices[ram_prices.date < 2000]
    data_test = ram_prices[ram_prices.date >= 2000]

# predict prices based on date:
    X_train = data_train.date[:, np.newaxis]
    # we use a log-transform to get a simpler relationship of data to target
    y_train = np.log(data_train.price)

tree = DecisionTreeRegressor().fit(X_train, y_train)
    linear_reg = LinearRegression().fit(X_train, y_train)

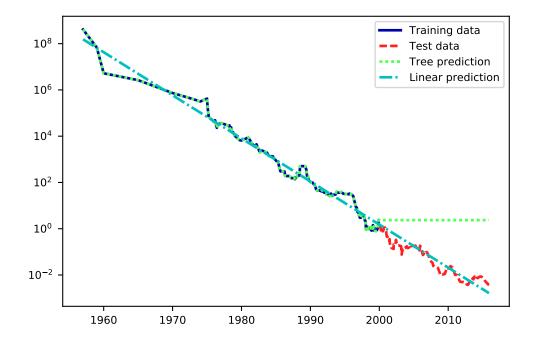
# predict on all data
```

X_all = ram_prices.date[:, np.newaxis]

```
pred_tree = tree.predict(X_all)
pred_lr = linear_reg.predict(X_all)

# undo log-transform
price_tree = np.exp(pred_tree)
price_lr = np.exp(pred_lr)

[157]: plt.rcParams['lines.linewidth'] = 2
plt.semilogy(data_train.date, data_train.price, label="Training data")
plt.semilogy(data_test.date, data_test.price, label="Test data")
plt.semilogy(ram_prices.date, price_tree, label="Tree prediction")
plt.semilogy(ram_prices.date, price_lr, label="Linear prediction")
plt.legend();
```



Strengths, weaknesses and parameters Pre-pruning: regularize by:

- Setting a low max_depth, max_leaf_nodes
- Setting a higher min_samples_leaf (default=1)

Decision trees:

- Work well with features on completely different scales, or a mix of binary and continuous features
 - Does not require normalization
- Interpretable, easily visualized
- Still tend to overfit easily. Use ensembles of trees.

Ensemble learning

Ensembles are methods that combine multiple machine learning models to create more powerful models. Most popular are:

- RandomForests: Build randomized trees on random samples of the data
- **Gradient boosting machines**: Build trees iteratively, giving higher weights to the points misclassified by previous trees

In both cases, predictions are made by doing a vote over the members of the example. **Stacking** is another technique that builds a (meta)model over the predictions of each member.

RandomForests

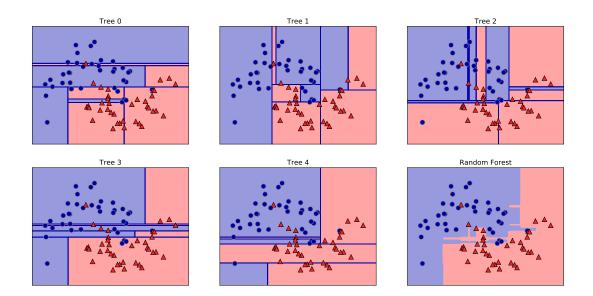
Reduce overfitting by averaging out individual predictions (variance reduction)

- Take a bootstrap sample of your data
 - Randomly sample with replacement
 - Build a tree on each bootstrap
- Repeat n_estimators times
 - Higher values: more trees, more smoothing
 - Make prediction by aggrating the individual tree predictions
 - * a.k.a. Bootstrap aggregating (Bagging)
- RandomForest: Randomize trees by considering only a random subset of features of size max_features in each node
 - Small max_features yields more different trees, more smoothing
 - Default: $sqrt(n_features)$ for classification, $log2(n_features)$ for regression

Making predictions: * Classification: soft voting * Every member returns probability for each class * After averaging, the class with highest probability wins * Regression:

```
* Return the mean of all predictions
```

```
[158]: from sklearn.ensemble import RandomForestClassifier
      from sklearn.datasets import make_moons
      X, y = make_moons(n_samples=100, noise=0.25, random_state=3)
      X_train, X_test, y_train, y_test = train_test_split(X, y, stratify=y,
                                                           random state=42)
      forest = RandomForestClassifier(n_estimators=5, random_state=2)
      forest.fit(X_train, y_train)
      plt.rcParams.update({'font.size': 12})
      fig, axes = plt.subplots(2, 3, figsize=(20, 10))
      for i, (ax, tree) in enumerate(zip(axes.ravel(), forest.estimators_)):
          ax.set_title("Tree {}".format(i))
          mglearn.plots.plot_tree_partition(X_train, y_train, tree, ax=ax)
      mglearn.plots.plot_2d_separator(forest, X_train, fill=True, ax=axes[-1, -
                                       alpha=.4)
      axes[-1, -1].set_title("Random Forest")
      mglearn.discrete_scatter(X_train[:, 0], X_train[:, 1], y_train);
```



Scikit-learn algorithms:

- RandomForestClassifier (or Regressor)
- ExtraTreesClassifier: Grows deeper trees, faster

Most important parameters:

- n_estimators (higher is better, but diminishing returns)
- max_features (default is typically ok)
 - Set smaller to reduce space/time requirements
- parameters of trees, e.g. max_depth (less effect)

n_jobs sets the number of parallel cores to run random_state should be fixed for reproducibility

```
Accuracy on training set: 1.000 Accuracy on test set: 0.951
```

RandomForest allow another way to evaluate performance: out-of-bag (OOB) error

- While growing forest, estimate test error from training samples
- For each tree grown, 33-36% of samples are not selected in bootstrap
 - Called the 'out of bootstrap' (OOB) samples
 - Predictions are made as if they were novel test samples
 - Through book-keeping, majority vote is computed for all OOB samples from all trees
- OOB estimated test error is rather accurate in practice
 - As good as CV estimates, but can be computed on the fly (without repeated model fitting)
 - Tend to be slightly pessimistic

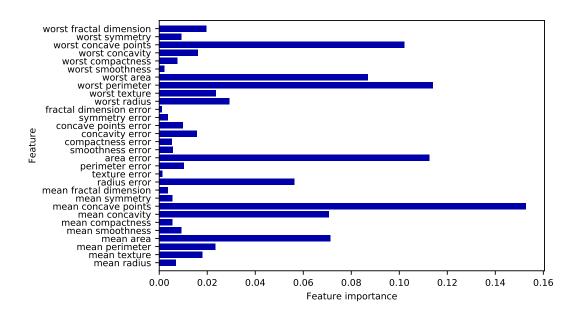
In scikit-learn OOB error are returned as follows:

```
oob_error = 1 - clf.oob_score_
[161]: from collections import OrderedDict
       from sklearn.datasets import make_classification
       from sklearn.ensemble import RandomForestClassifier, ExtraTreesClassifier
       RANDOM\_STATE = 123
       # Generate a binary classification dataset.
       X, y = make_classification(n_samples=500, n_features=25,
                                  n_clusters_per_class=1, n_informative=15,
                                  random_state=RANDOM_STATE)
       # NOTE: Setting the `warm_start` construction parameter to `True` disable
       # support for parallelized ensembles but is necessary for tracking the OC
       # error trajectory during training.
       ensemble\_clfs = [
           ("RandomForestClassifier, max_features='sqrt'",
               RandomForestClassifier(warm_start=True, oob_score=True,
                                      max_features="sqrt",
                                      random_state=RANDOM_STATE)),
           ("RandomForestClassifier, max_features='log2'",
               RandomForestClassifier(warm_start=True, max_features='log2',
                                      oob_score=True,
                                      random_state=RANDOM_STATE)),
           ("RandomForestClassifier, max_features=None",
               RandomForestClassifier(warm_start=True, max_features=None,
                                      oob_score=True,
                                      random_state=RANDOM_STATE))
       1
       # Map a classifier name to a list of (<n_estimators>, <error rate>) pairs
```

```
error_rate = OrderedDict((label, []) for label, _ in ensemble_clfs)
  # Range of `n_estimators` values to explore.
  min estimators = 15
  max_estimators = 175
  for label, clf in ensemble_clfs:
      for i in range(min_estimators, max_estimators + 1):
          clf.set_params(n_estimators=i)
          clf.fit(X, y)
          # Record the OOB error for each `n_estimators=i` setting.
          oob_error = 1 - clf.oob_score_
          error_rate[label].append((i, oob_error))
  # Generate the "OOB error rate" vs. "n_estimators" plot.
  for label, clf_err in error_rate.items():
      xs, ys = zip(*clf_err)
      plt.plot(xs, ys, label=label)
  plt.xlim(min_estimators, max_estimators)
  plt.xlabel("n_estimators")
  plt.ylabel("OOB error rate")
  plt.legend(loc="upper right")
  plt.show()
                RandomForestClassifier, max features='sqrt'
  0.14
                RandomForestClassifier, max features='log2'
                RandomForestClassifier, max_features=None
  0.12
JOB error rate
  0.10
  0.08
  0.06
        20
               40
                     60
                            80
                                  100
                                         120
                                                140
                                                      160
                            n_estimators
```

Feature importance RandomForests provide more reliable feature importances, based on many alternative hypotheses (trees)

```
[162]: plt.rcParams.update({'font.size':8})
    plot_feature_importances_cancer(forest)
```



Strengths, weaknesses and parameters RandomForest are among most widely used algorithms:

- Don't require a lot of tuning
- Typically very accurate models
- Handles heterogeneous features well
- Implictly selects most relevant features

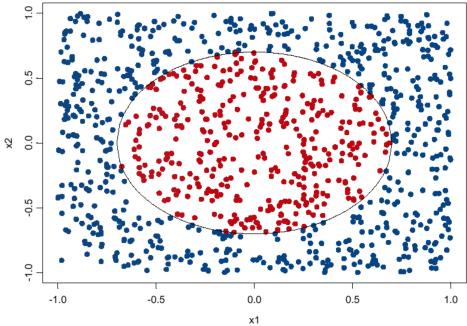
Downsides:

- less interpretable, slower to train (but parallellizable)
- don't work well on high dimensional sparse data (e.g. text)

Gradient Boosted Regression Trees (Gradient Boosting Machines)

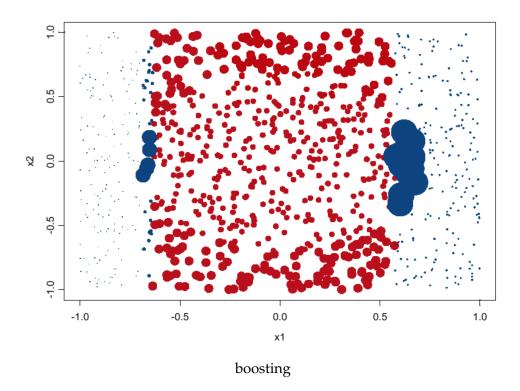
Instead of reducing the variance of overfitted models, reduce the bias of underfitted models

- Use strong pre-pruning to build very shallow trees
 - Default max_depth=3
- Iteratively build new trees by increasing weights of points that were badly predicted
 - learning rate controls how strongly the weights are altered in each iteration (default 0.1)
 - Gradient descent (covered later) finds optimal set of weights
- Repeat n_estimators times (default 100)



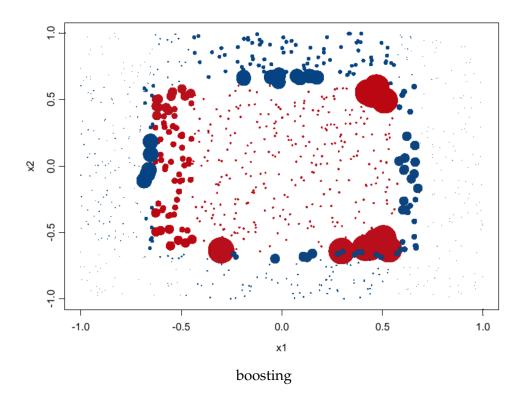
Example: After 1 iteration

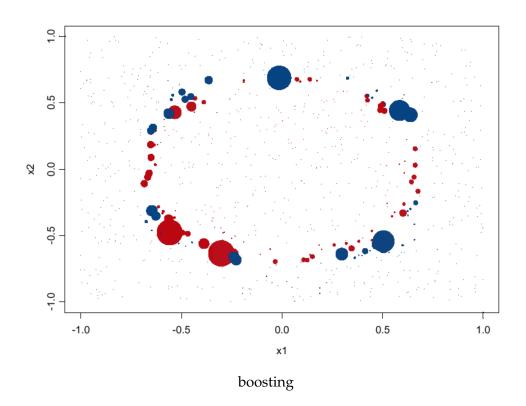
- The simple decision tree divides space
- Misclassified points get higher weight (larger dots)



After 3 iterations
After 20 iterations
Each tree provides good predictions on part of the data, use voting for final prediction

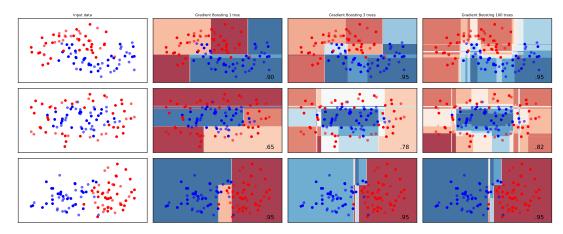
• Soft voting for classification, mean values for regression





```
[163]: import plot_classifiers as pc
    names = ["Gradient Boosting 1 tree", "Gradient Boosting 3 trees", "Gradient
    classifiers = [
        GradientBoostingClassifier(n_estimators=1, random_state=0, learning_r
        GradientBoostingClassifier(n_estimators=3, random_state=0, learning_r
        GradientBoostingClassifier(n_estimators=100, random_state=0, learning_r
        ]
```

pc.plot_classifiers(names, classifiers, figuresize=(20,8))

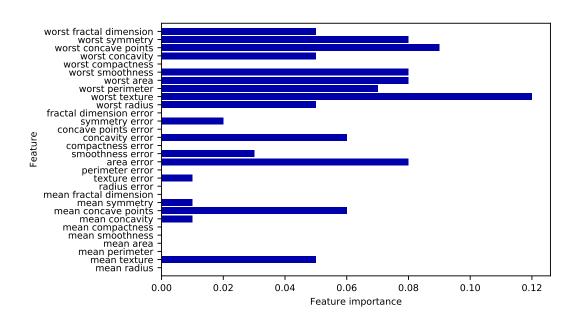


Tuning

- n_estimators: Higher is better, but will start to overfit
- learning_rate: Lower rates mean more trees are needed to get more complex models
 - Main regularizer, also known as 'shrinkage'
 - Set n_estimators as high as possible, then tune learning_rate
- max_depth: typically kept low (<5), reduce when overfitting
- loss: Loss function used for gradient descent (defaults OK)
 - Classification:
 - * deviance (default): log-likelihood loss (as in logistic regression)
 - * exponential: exponential loss (AdaBoost algorithm)
 - Regression:
 - * 1s: Least squares (typically the best option)

```
Accuracy on training set: 1.000
Accuracy on test set: 0.958
[165]: # We are overfitting. We can decrease max_depth
       gbrt = GradientBoostingClassifier(random_state=0, max_depth=1)
       gbrt.fit(X_train, y_train)
       print("Accuracy on training set: {:.3f}".format(gbrt.score(X_train, y_tra
       print("Accuracy on test set: {:.3f}".format(gbrt.score(X_test, y_test)))
Accuracy on training set: 0.991
Accuracy on test set: 0.972
[166]: # or decrease the learning rate (less effect)
       gbrt = GradientBoostingClassifier(random_state=0, learning_rate=0.01)
       gbrt.fit(X_train, y_train)
       print("Accuracy on training set: {:.3f}".format(gbrt.score(X_train, y_tra
       print("Accuracy on test set: {:.3f}".format(gbrt.score(X_test, y_test)))
Accuracy on training set: 0.988
Accuracy on test set: 0.965
```

Gradient boosting machines use much simpler trees - Hence, tends to completely ignore some of the features



Strengths, weaknesses and parameters

- Among the most powerful and widely used models
- · Work well on heterogeneous features and different scales
- Require careful tuning, take longer to train.
- Does not work well on high-dimensional sparse data

Main hyperparameters:

- n_estimators: Higher is better, but will start to overfit
- learning_rate: Lower rates mean more trees are needed to get more complex models
 - Set n_estimators as high as possible, then tune learning_rate
- max_depth: typically kept low (<5), reduce when overfitting

XGBoost

XGBoost is another python library for gradient boosting (install separately).

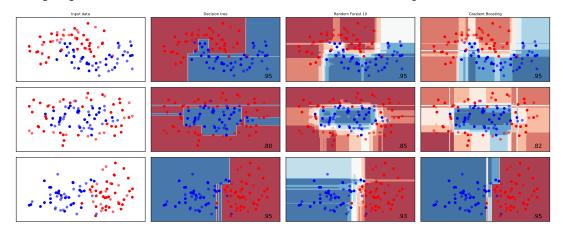
- The main difference lies the use of approximation techniques to make it faster.
 - Hence, you can do 10x (or 100x) more boosting iterations in same amount of time
- Sketching: Given 10000 possible splits, it will only consider 300 "good enough" splits by default
 - Controlled by the sketch_eps parameter (default 0.03)
- Loss function approximation with Taylor Expansion: more efficient way to evaluate splits

Further reading: XGBoost Documentation Paper Comparison

```
[168]: import plot_classifiers as pc
    names = ["Decision tree", "Random Forest 10", "Gradient Boosting"]

classifiers = [
    DecisionTreeClassifier(),
    RandomForestClassifier(max_depth=5, n_estimators=100, max_features=1)
    GradientBoostingClassifier(random_state=0, learning_rate=0.5)
    ]
```

pc.plot_classifiers(names, classifiers, figuresize=(20,8))



Summary

- Bagging / RandomForest is a variance-reduction technique
 - Build many high-variance (overfitting) models
 - * Typically deep (randomized) decision trees
 - * The more different the models, the better
 - Aggregation (soft voting or averaging) reduces variance
 - Parallellizes easily
- Boosting is a bias-reduction technique
 - Build many high-bias (underfitting) models
 - * Typically shallow decision trees
 - * Sample weights are updated to create different trees
 - Aggregation (soft voting or averaging) reduces bias
 - Doesn't parallelize easily
 - * Approximation techniques exist to speed up calculation
- You can build ensembles with other models as well
 - Especially if they show high variance or bias
- It is also possible to build *heterogeneous* ensembles
 - Models from different algorithms
 - Are combined by letting each algorithm predict
 - Often a meta-classifier/regressor is trained on the predictions: Stacking