Computational Physics Project 3
Random Walks and Ising Models
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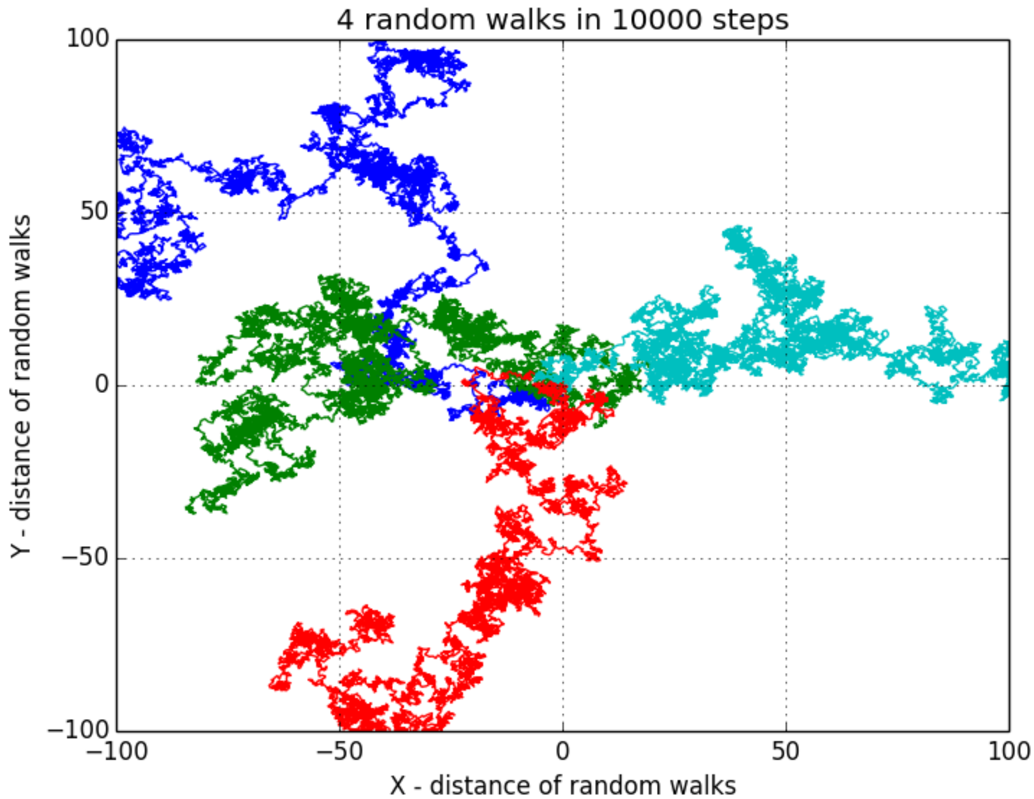
4.3 random walks

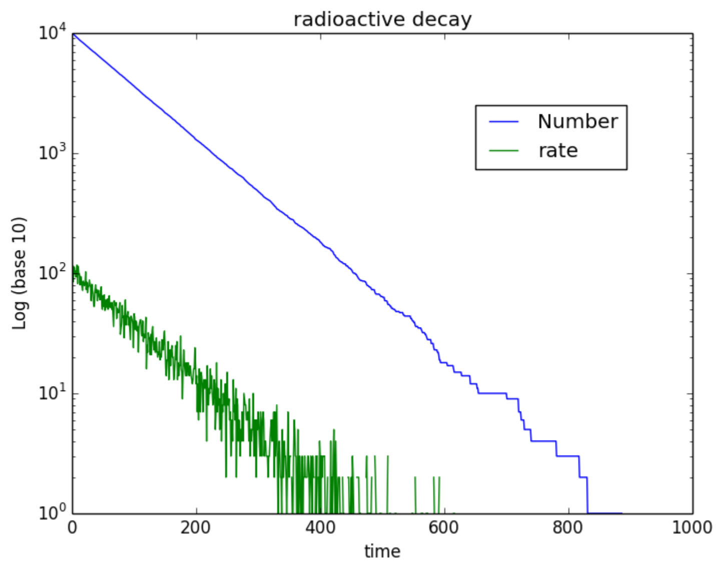
Random walks are a pretty simple model to make:



Essentially this takes a random walk for 1000 steps – the standard deviation of a random walk grows as $\sqrt{N}$ and nearly the entire plot is contained within a circle with radius of 100 (3 standard deviations away).

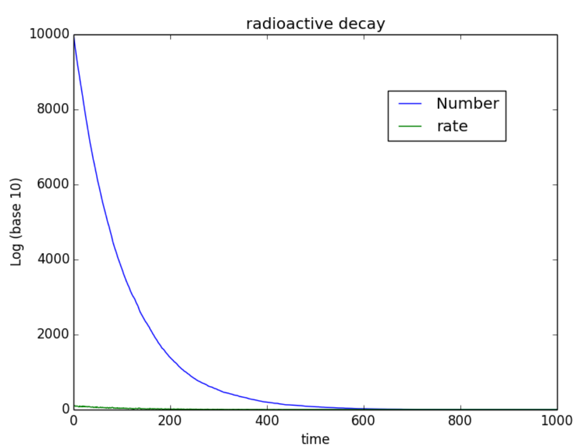
4.5 modelling radioactive decay

4.6 modelling radioactive decay



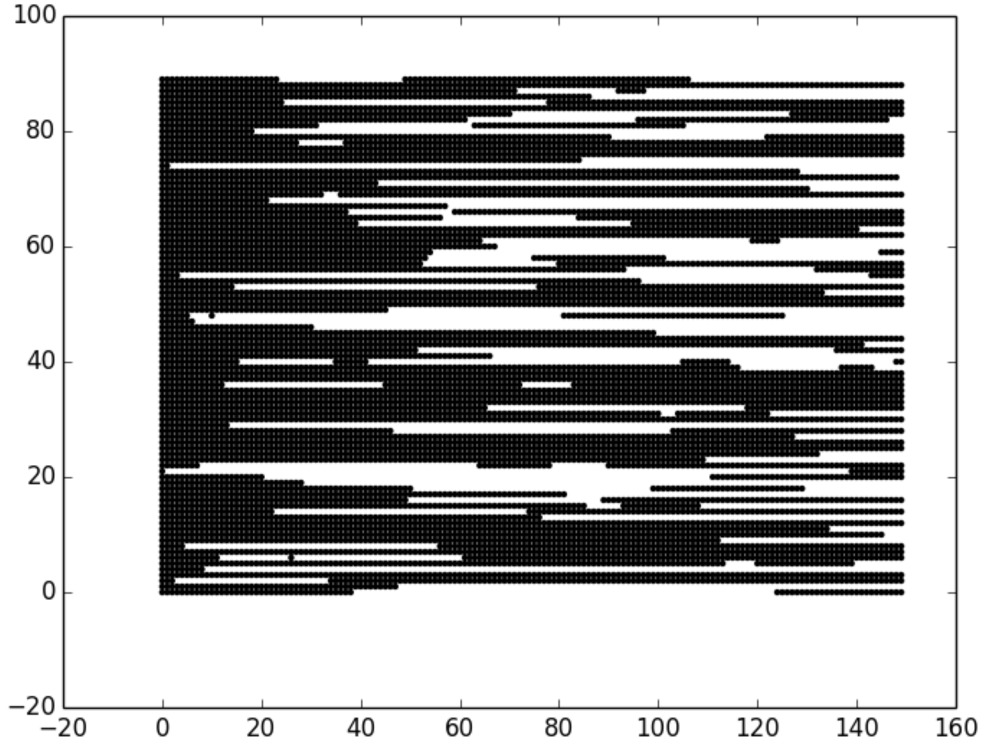
Here is a semi-log plot of radioactive decay, where we have superimposed the plots of the rate of decay (R = -a N(t)) versus the number (N(t) = No – a N(t-1))
As you can see, they generally mirror each other and have the same slope (as they should).

Note the difference between a real radioactive decay, and a continuous function, namely in the more noticeable discretizations when the count of N grows low.



17.4 Metropolis

I made my own metropolis implementation of the 1-D ising model



Here is a plot of the standard numbers plugged in (from the text), for 100 time steps. Metropolis algorithm seems odd, in that it looks for local minima by changing one term (in a discrete set) at a time, and so it seems intrinsically slow.

Metropolis works by changing 1 term, evaluating energy, and then if it is lower, then going down that path, otherwise keeping the old arrangement with some probability (as a function of temperature).

The explicit steps of the Metropolis algorithm are:

1. Start with an arbitrary spin configuration $\alpha_k = \{s_1, s_2, \ldots, s_N\}$.
2. Generate a trial configuration α_{k+1} by
   a) picking a particle i randomly and
   b) flipping its spin.[1)]
3. Calculate the energy $E_{\alpha_{\mathrm{tr}}}$ of the trial configuration.
4. If $E_{\alpha_{\mathrm{tr}}} \leq E_{\alpha_k}$, accept the trial by setting $\alpha_{k+1} = \alpha_{\mathrm{tr}}$.
5. If $E_{\alpha_{\mathrm{tr}}} > E_{\alpha_k}$, accept with relative probability $\mathcal{R} = \exp(-\Delta E/k_{\mathrm{B}} T)$:
   a) Choose a uniform random number $0 \leq r_i \leq 1$.
   b) Set $\alpha_{k+1} = \begin{cases} \alpha_{\mathrm{tr}}\,, & \text{if} \quad \mathcal{R} \geq r_j \quad \text{(accept)}\,, \\ \alpha_k\,, & \text{if} \quad \mathcal{R} < r_j \quad \text{(reject)}\,. \end{cases}$

I am confident that there have to be better algorithms to calculate local minima, but I don’t know what they are.