

Computational Physics
Roy Rinberg
Project 4

5.5.1 –
 $h = 0.01$

Cosine Error

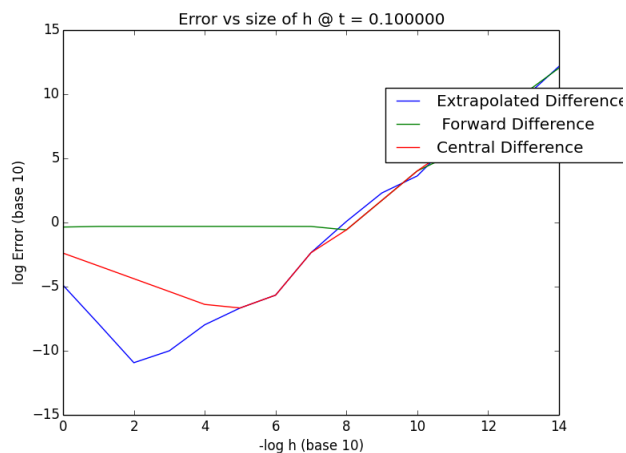
	Forward	Central	Extrapolated	
T = 0.1	0.497332	0.000042	0.000000	
T= 1.0	0.268746	0.000351	0.000000	
T=100.	0.432000	0.000211	0.000000	

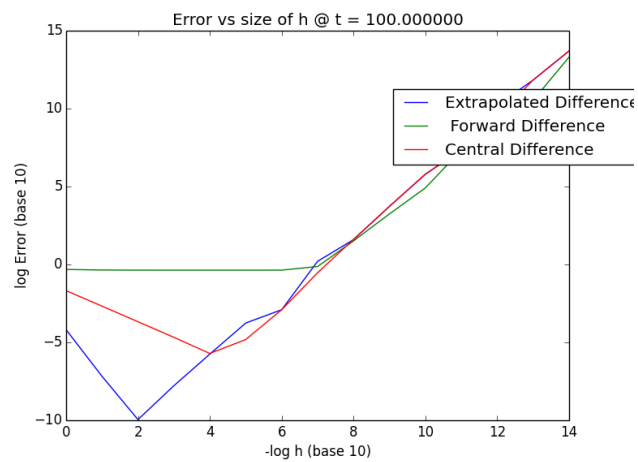
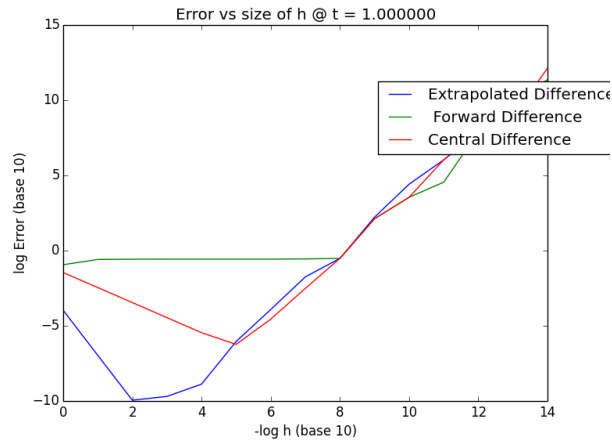
Exponent Error

	Forward	Central	Extrapolated
T = 0.1	0.554432	0.000460	0.000000
T= 1.0	1.363683	0.001133	0.000000
T=100.	1348549989209 2547921320749 4851927918772 22400.000000	11200499654636747843164176182 109693542400.000000	594953982880 553760116519 1020544000.0 00000

Plot $\log_{10} |\text{error}|$ vs. $\log_{10} h$ and check whether the number of decimal places obtained agrees with the estimates in the text

For cosine:





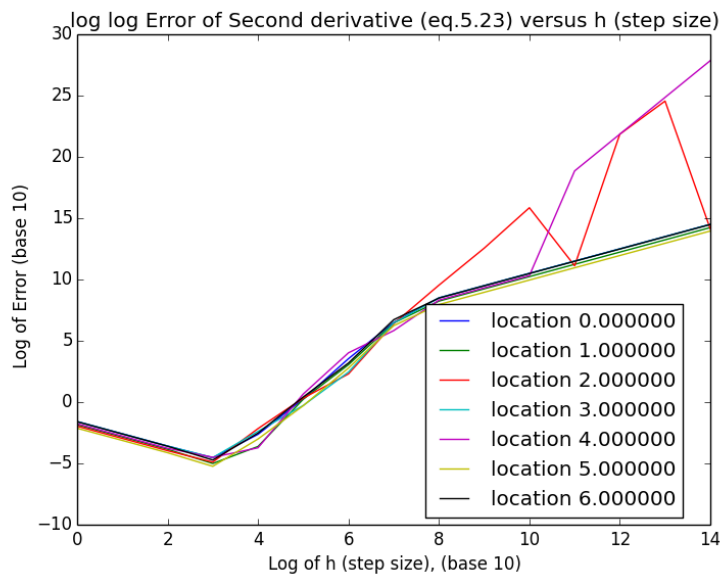
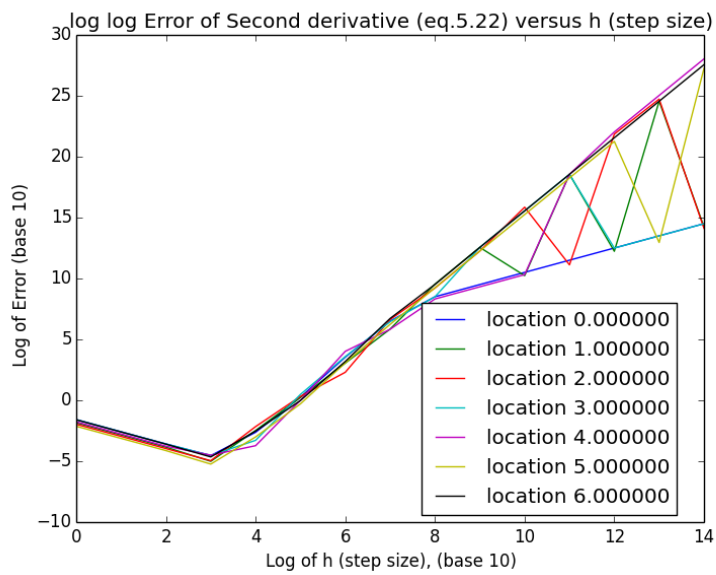
Are there regions that seem to work better for the different algorithms?

Yes, definitely. It's not surprising that the extrapolated differentiation technique is generally the best, and the forward differentiation is generally the worst. Additionally, it falls generally in line with the ratios that the ideal extrapolated is $\sim 1000\times$ larger than it is for central, and 10^6 times larger than it is for forward. However, it is surprising the exact numbers – I expected smaller h 's.

5.6.1

Write a program to calculate the second derivative of $\cos t$ using the central difference algorithms (5.22) and (5.23). Test it over four cycles. Start with $h \approx \pi/10$ and keep reducing h until you reach machine precision. Is there any noticeable differences between (5.22) and (5.23)?

Here are the plots of Cosine being evaluated at 6 locations, and their corresponding Errors



It seems there are not really hugely different results for the errors as a function of h – perhaps there is at larger x evaluations.

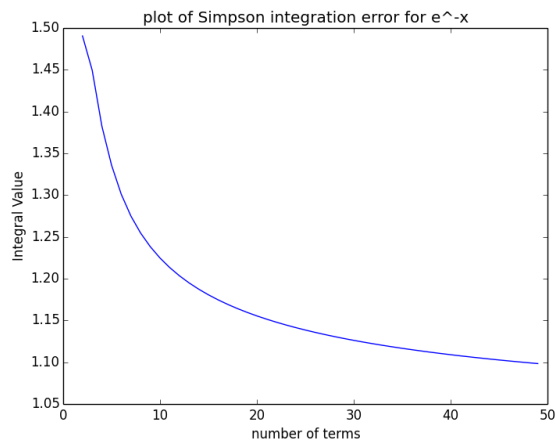
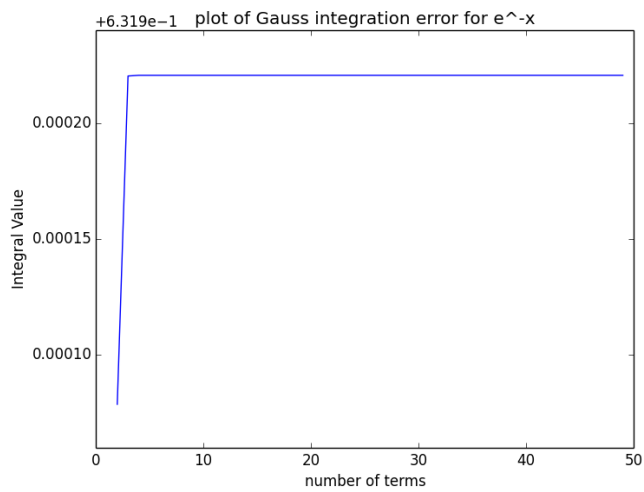
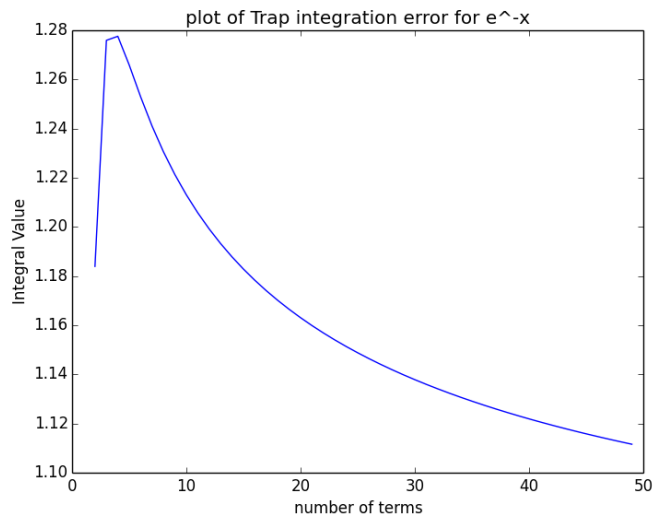
5.12.3

Numerical Integrations

Compute the relative error $\epsilon = |(\text{numerical} - \text{exact}) / \text{exact}|$ in each case. Present your data in the tabular form

	Trapezoid	Simpson	Gaussian
N =2	0.872965	1.357946	0.000224
N =10	0.919018	0.690694	0.000000

How strange, that the error in the trapezoid actually increases (but later it decreases again)



What is odd, is that the plot of the error tells me that the rate of increase for all the errors identical – this does not make sense to me.

I was unable to plot the error of the gaussian – I guess because it was taking the log of 0, and that was causing an error.

Additionally, this plot of the error of the trapezoidal and Simpson's looks correct in shape, but not in scale, so I don't know what to say about that.

