Yixuan's research was motivated by modeling, analyzing, and simulating challenging problems with multiscale and singularity governed by physical equations. In his first year, they developed for the first time a state-of-the-art multiscale framework ExpMsFEM with theoretical guarantee of exponential scaling laws of accuracy for heterogenous elliptic and Helmholtz equations. Yixuan's later research was motivated by the Clay prize problem, singularity of Navier-Stokes equations in fluids. They studied a 1D modified model to understand the singularity phenomena. To mathematically establish singularity, they generalized the dynamic rescaling technique and established singularity of nonlinear heat and complex Ginzburg-Landau equations for the first time without spectral information, opening up the possibility of studying singularities beyond the self-similar assumption with computer-assisted proofs. One can first numerically search for a plausible candidate and upgrade the approximate profile to a rigorous proof via computer-assisted estimates. In the future they will study both challenging problems using this framework with computer-assisted proofs, and tackle open questions with more sophisticated type of singularities.

To model and simulate singularity, one could also resort to machine learning tools besides traditional numerical methods. With collaborators, Yixuan built upon the neural operator framework and constructed Fourier neural operators that can tackle non-periodic problems using Fourier continuation, and with symmetries encoded to boost the performance. Very recently, they proposed the KAN architecture for machine learning. Leveraging the Kolmogorov-Arnold representation theorem using a composition of sum of 1D functions to represent higher dimensional functions and generalizing into the perspective of modern machine learning, they proposed KANs with learnable activation functions between each neural pairs parameterized by splines. Compared with the prevalent multilayer perceptrons (MLPs), KANs are demonstrated to have much better scaling laws and interpretability via a much smaller network in many scientific problems, including function regression and PDE solving tasks. KANs also have nice theoretical properties like being universal approximators and suffer less from the spectral bias phenomenon. KANs have generated great interest and reached more than 200 citations within 5 months of arxiv release. They will continue to explore the applicability of KANs to larger scale problems, many of which have already been established by peers and domain experts, and further understanding of the mathematical properties of KANs.