Stopping Rule for Secretary Problem

Team Members

Zhaoqi Zhang
Gaochong Xu
Jeffrey Litfin
Zixian Lin

Problem Description

- Interview n candidates for a position
- Cannot recall
- After each can decide their rank
- Don't know the quality of rest

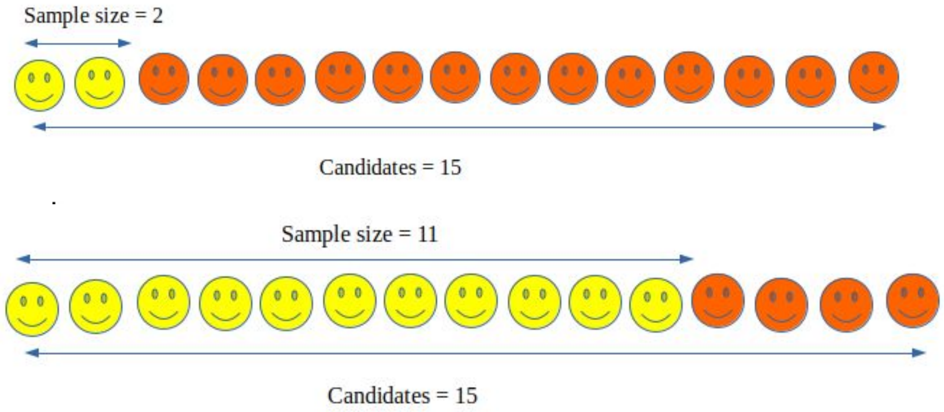
Goal: maximize the probability of choosing the best candidate.

Importance

- A frequent problem companies face is how to choose the best possible job candidate
- The secretary problem ensures the best possible candidate can be chosen by forcing the hiring manager to wait
  - Can be used on any problem when making a choice among a group of options when you can’t change your mind
  - Used by Mathematician Johannes Kepler in 1613 to successfully find his wife

Difficulty

- Need an effective way of choosing a control group
  - Taking the first candidate will rarely result in the best
  - Need an effective way of finding the optimal control group



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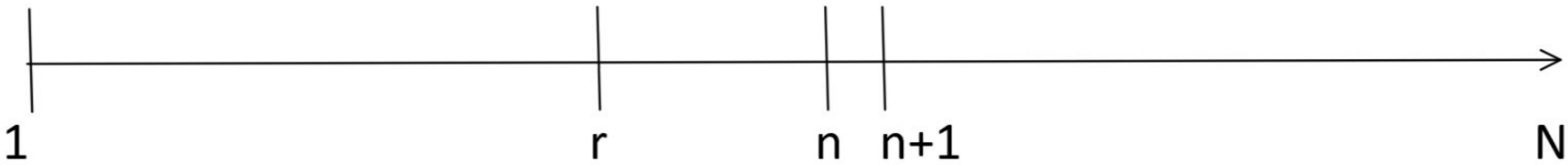
Solution

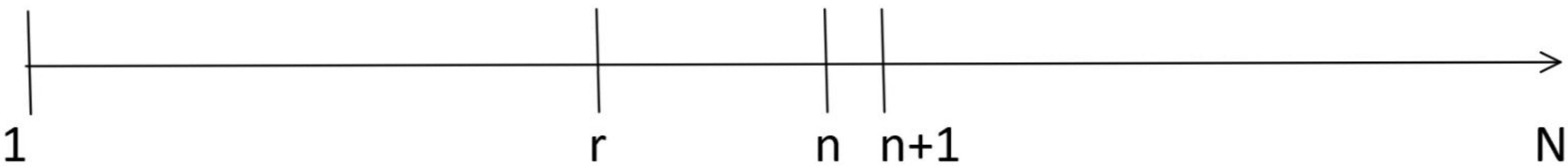


Interview and reject the first r applicants, for r < N.

Accept the very next applicant that is better than all the first r you interviewed.

P(Success) = P(r)

Solution



i $*$ (the best) occur at n+1. Do not pick it unless:

(1) $n \geq r$

(2) The highest applicant in [1, n] is the same highest applicant in [1, r]

The possibility of this is: $\frac{r}{n}\frac{1}{N}$

The probability of i $*$ occuring at n + 1 is 1/N

The probability of condition (2) is r/n

Solution

$$P(r) = \frac{1}{N}\left[\frac{r}{r} + \frac{r}{r+1} + \frac{r}{r+2} + \ldots + \frac{r}{N-1}\right] = \frac{r}{N}\sum_{n=r}^{N-1}\frac{1}{n}$$

P(r) is a Riemann approximation to an integral.

$$P(r) = \lim_{N\to\infty}\frac{r}{N}\sum_{n=r}^{N-1}\frac{N}{n}\frac{1}{N} = x\int_x^1 \frac{1}{t}dt = -x\ln x$$

Solving P'(r) = 0 for r gives us the optimal ratio and the probability of success P(r_optimal).

Solution

$$P'(r) = -\ln x - 1 = 0 \implies x = \frac{1}{e}$$

$$P\left(\frac{1}{e}\right) = \frac{1}{e}$$

$$\frac{1}{e} \approx .37$$

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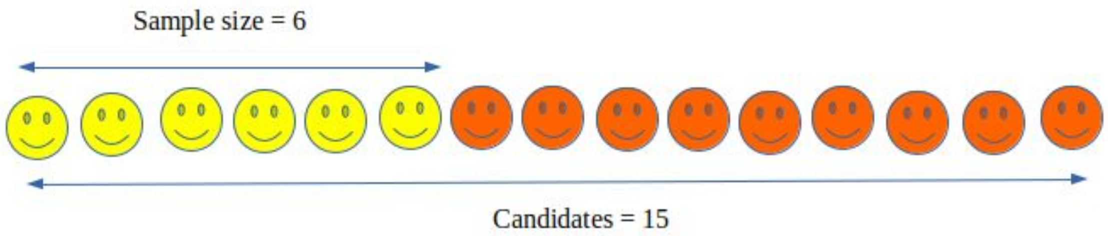
Solution

- The solution shows that an optimal control group makes up a size of 1/e or approximately 37%
- The first candidate better than the control group should be chosen
  - Results in the best probability of choosing the top candidate



Time Complexity

- O(n)
  - In the worst case will look at n candidates choosing the last candidate
    - This will occur when the best candidate was found in the first 1/e interviewed
- In most cases the best candidate will be found much earlier resulting in the code finishing in less than n iterations

Experimental Result

Number of candidates	Probability of success	Running time (sec/10000 times)
20	0.376	0.550550891
50	0.379	1.140982333
100	0.3786	2.301025147
200	0.3629	3.678622066
500	0.3754	11.11833581
1000	0.3763	20.32150593
2000	0.364	41.23882053
5000	0.3774	104.4833268
10000	0.3647	214.7531664

Conclusion

- The secretary problem shows that reviewing n/e candidates as part of a control group will maximize the chances are selecting the best
  - Does not guarantee the best is chosen
  - Reminds us to avoid rushing into a selection
- Can be used on any problem when a choice must be made without knowing all details.

Citations

https://www.geeksforgeeks.org/secretary-problem-optimal-stopping-problem/

https://thebryanhernandezgame.files.wordpress.com/2010/05/secretary-problem.pdf

Github

https://github.com/RoyZhang7/secretary-problem