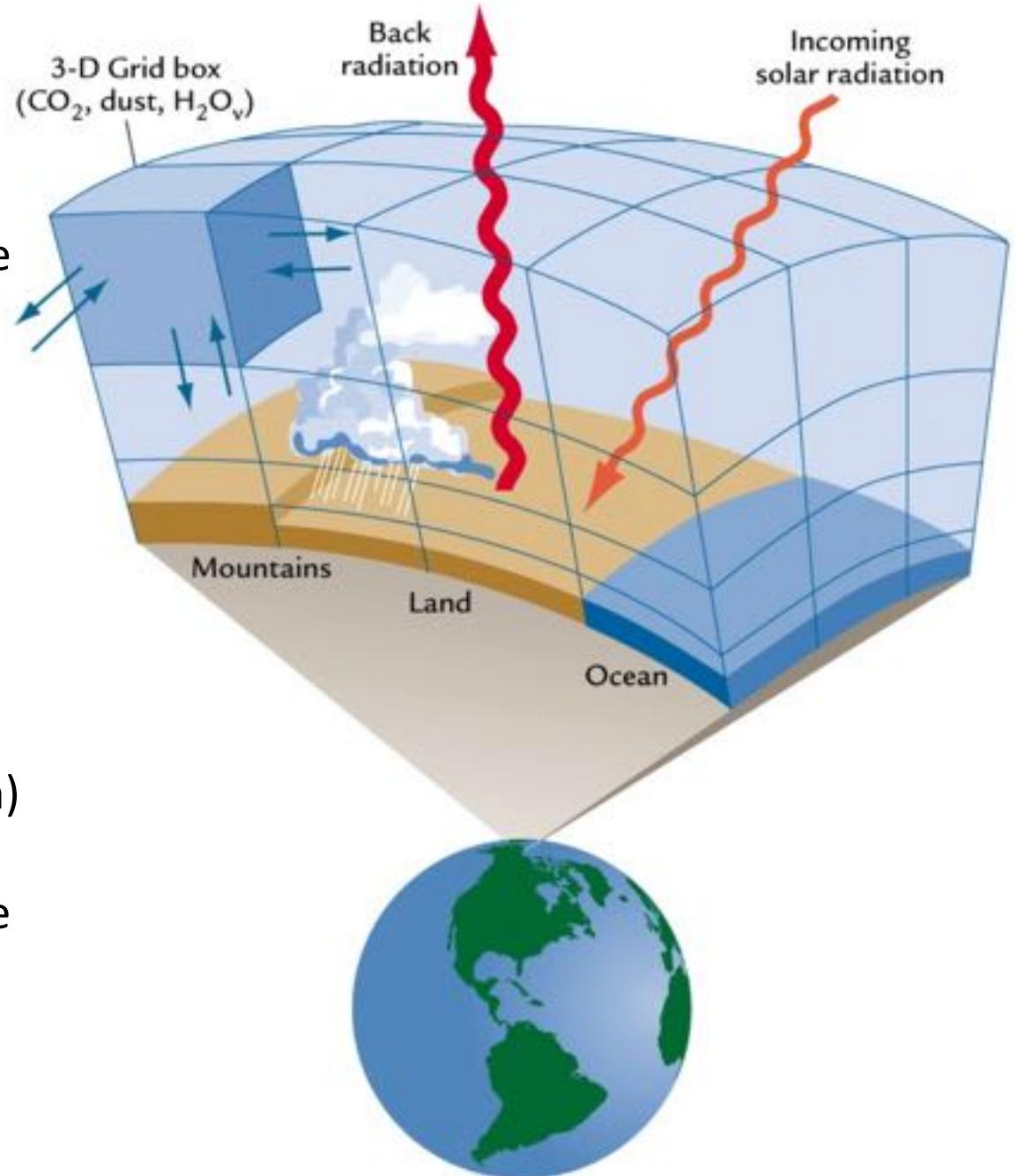


# Modelling the Climate System

Alex Sen Gupta; a.sengupta@unsw.edu.au

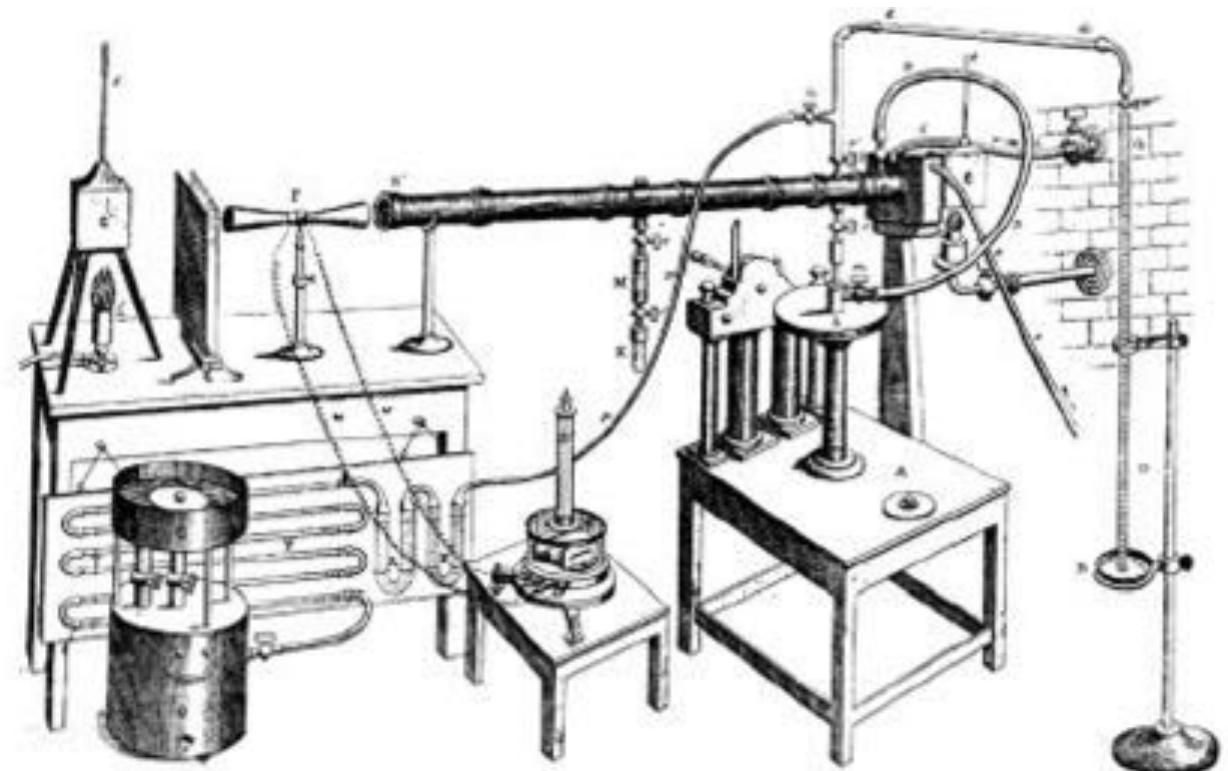
- Anthropogenic Climate Change
- Conceptual model of the Climate System
  - Governing equations
  - Solving the differential equations
  - Example 1- Climate Projections
  - Example 2- Geoengineering
- State-of-the-art climate models
  - Governing equations (ocean)
  - Solving the equations
  - Example: acceleration of the EAC – sending Nemo to Tasmania
- Climate Change research



*Fourier concluded that at the distance the earth is from the Sun the earth should be much colder than observed. Jean-Baptiste Joseph Fourier (1768-1830)*



*"The atmosphere admits of the entrance of the solar heat, but checks its exit; and the result is a tendency to accumulate heat at the surface of the planet". John Tyndall (1859)*



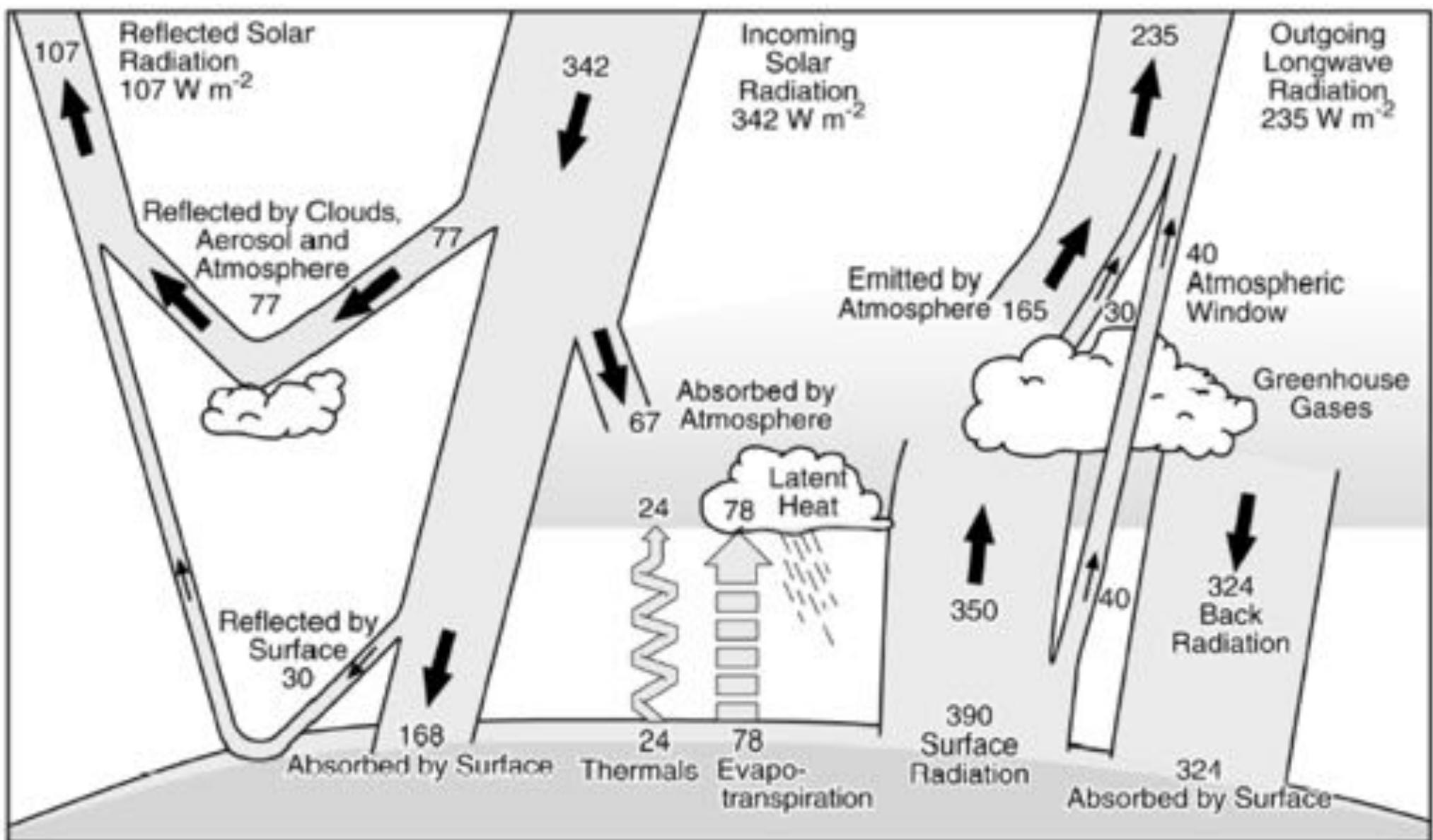
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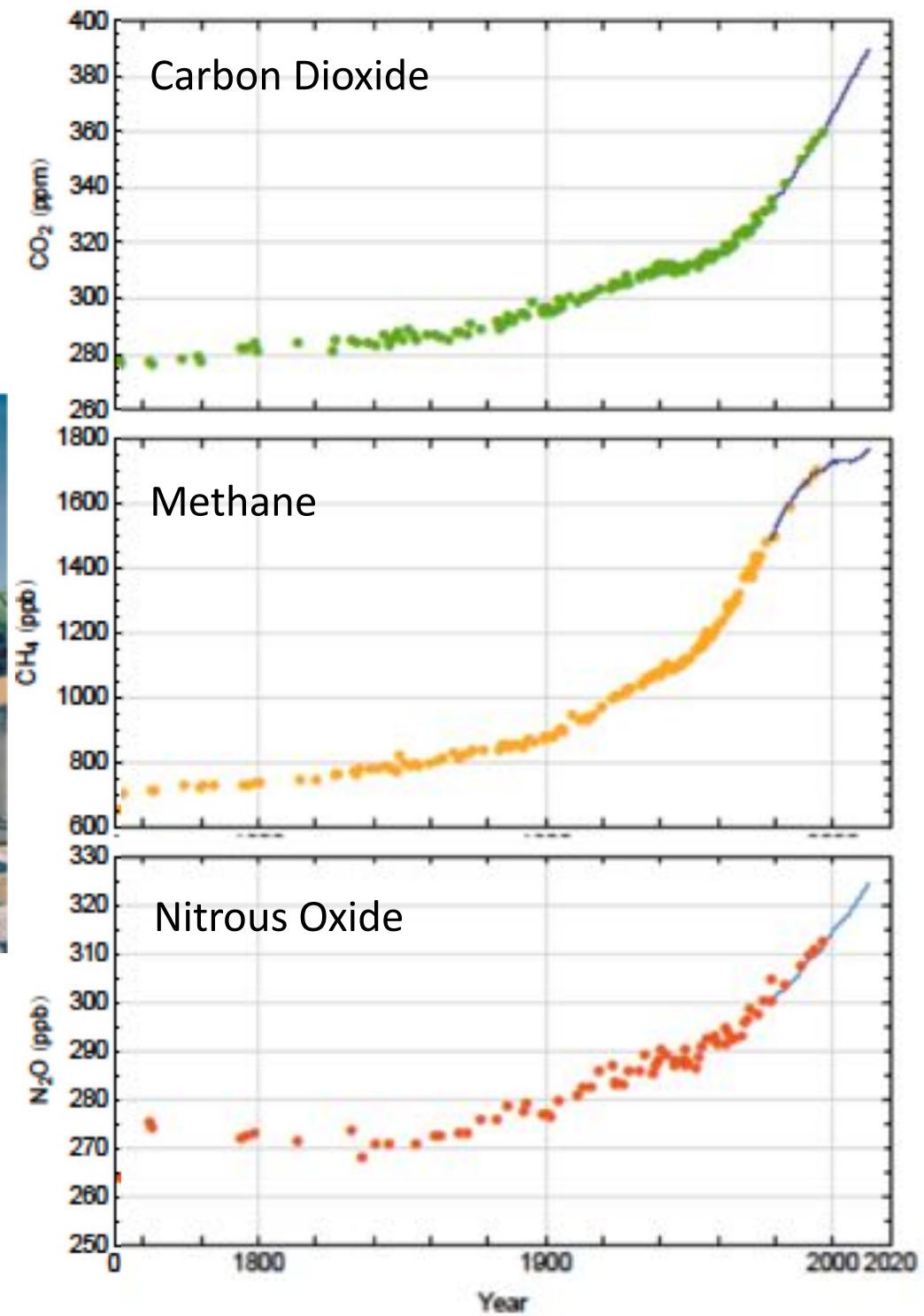
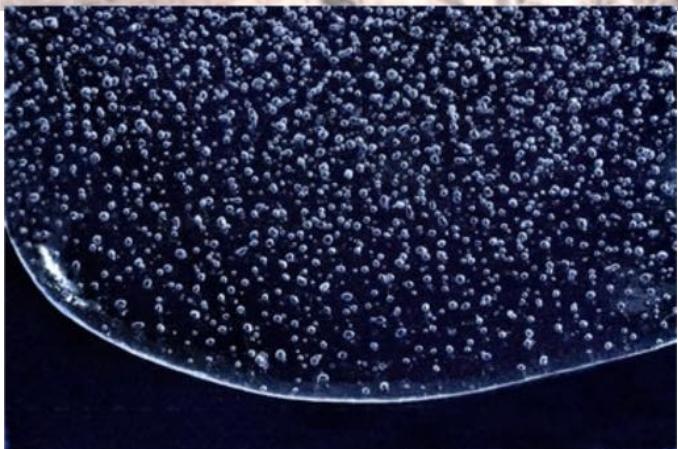
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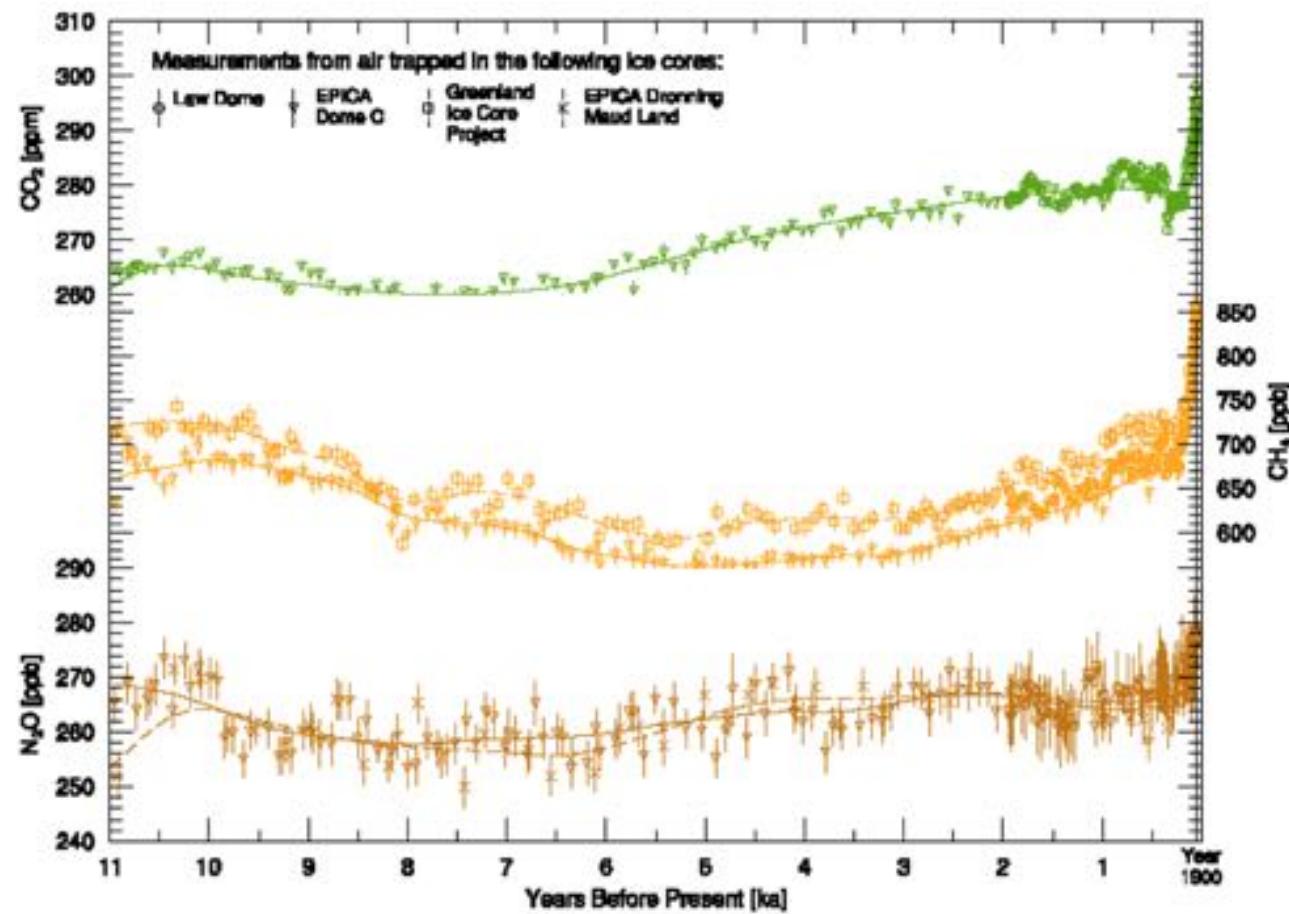
Arrhenius was the first person to predict that emissions of carbon dioxide from the burning of fossil fuels and other combustion processes would cause global warming. Svante August Arrhenius (1896)



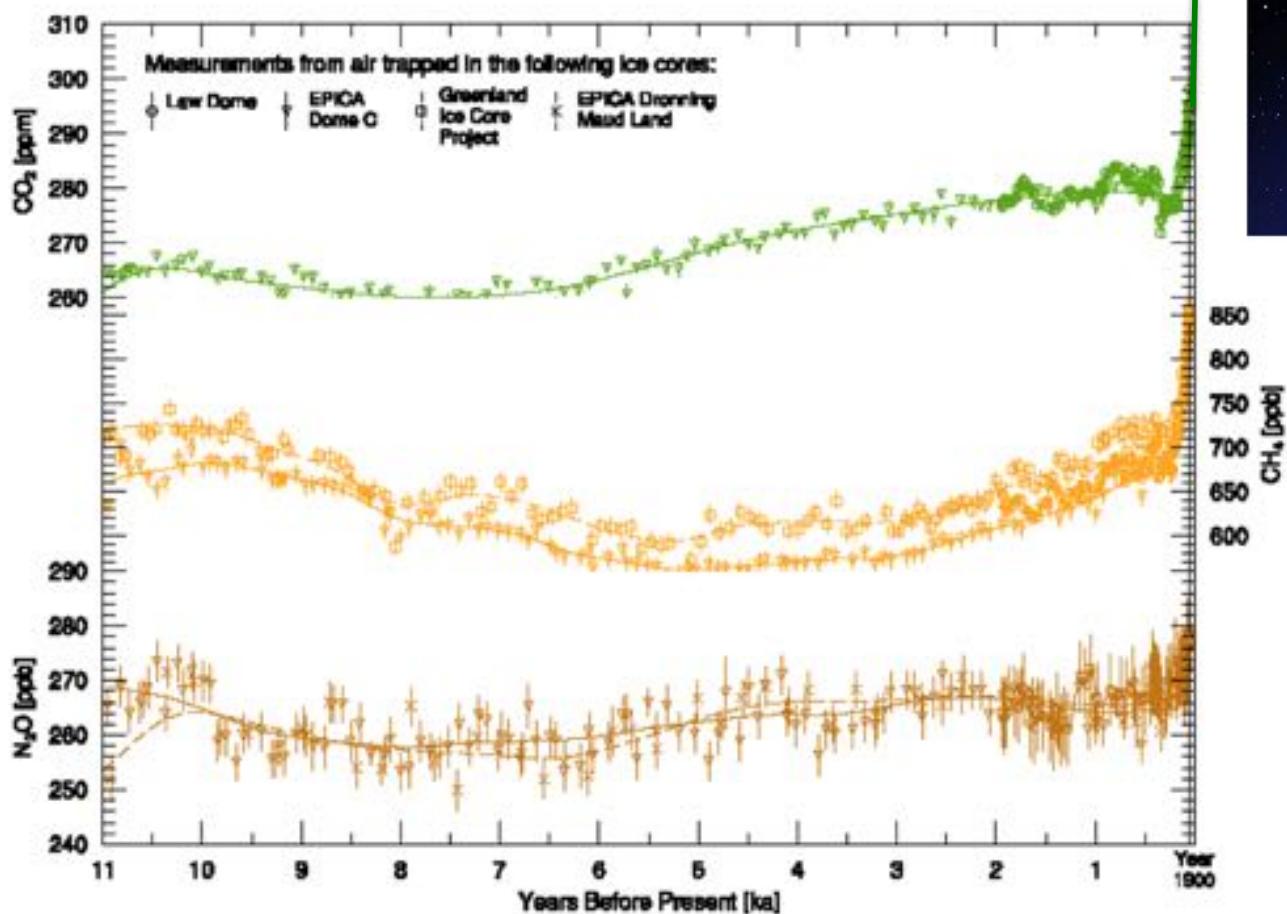


- Changes in greenhouse gas concentrations over the last 200yr can be directly linked to human activity (primarily burning of fossil fuels)

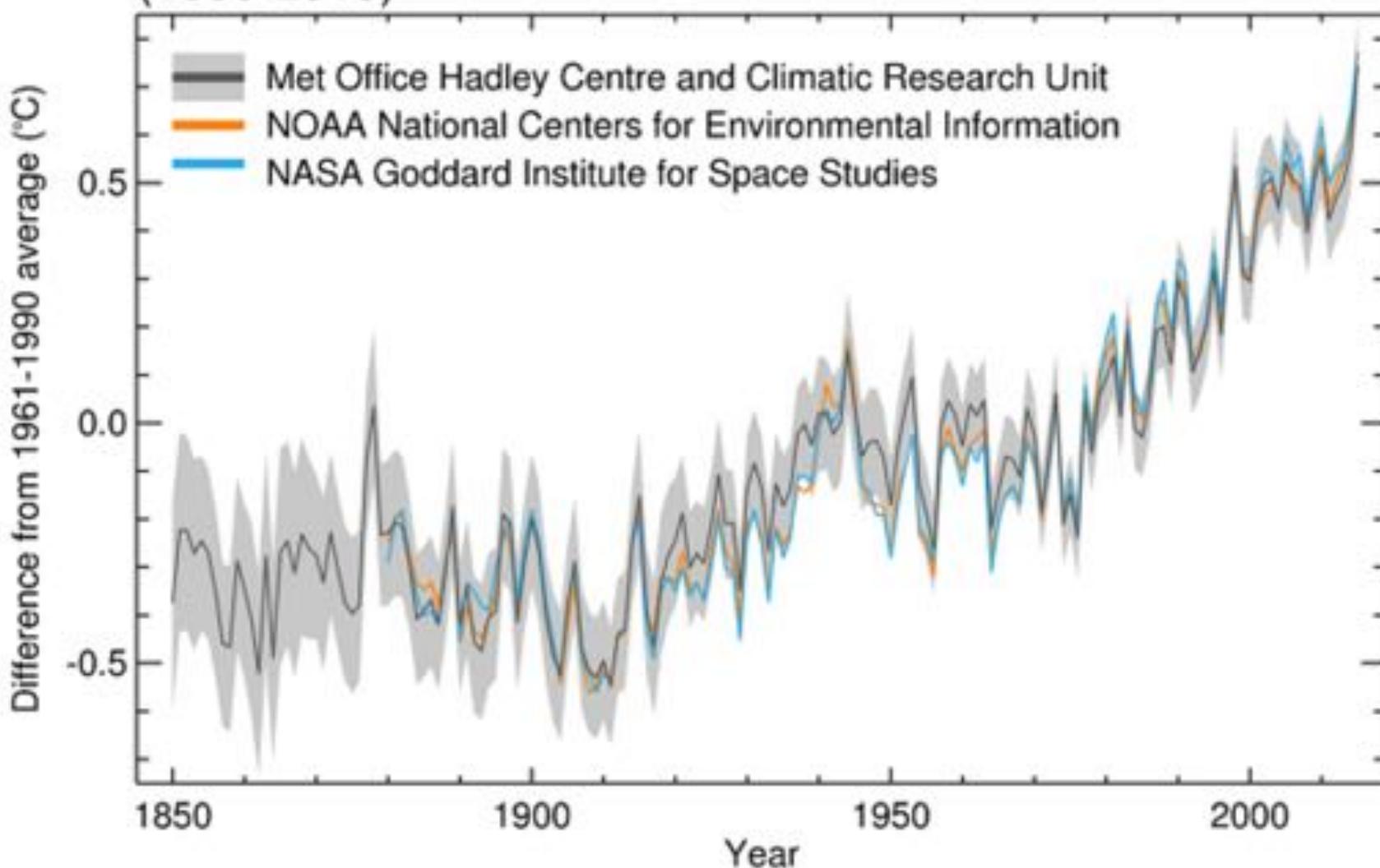


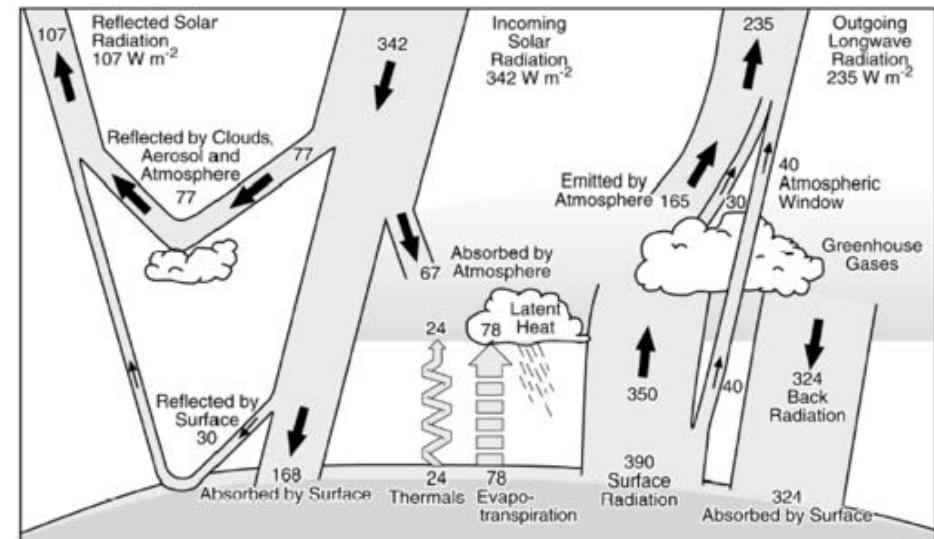


- Levels are unprecedented since the end of the last ice age and are well beyond natural fluctuations

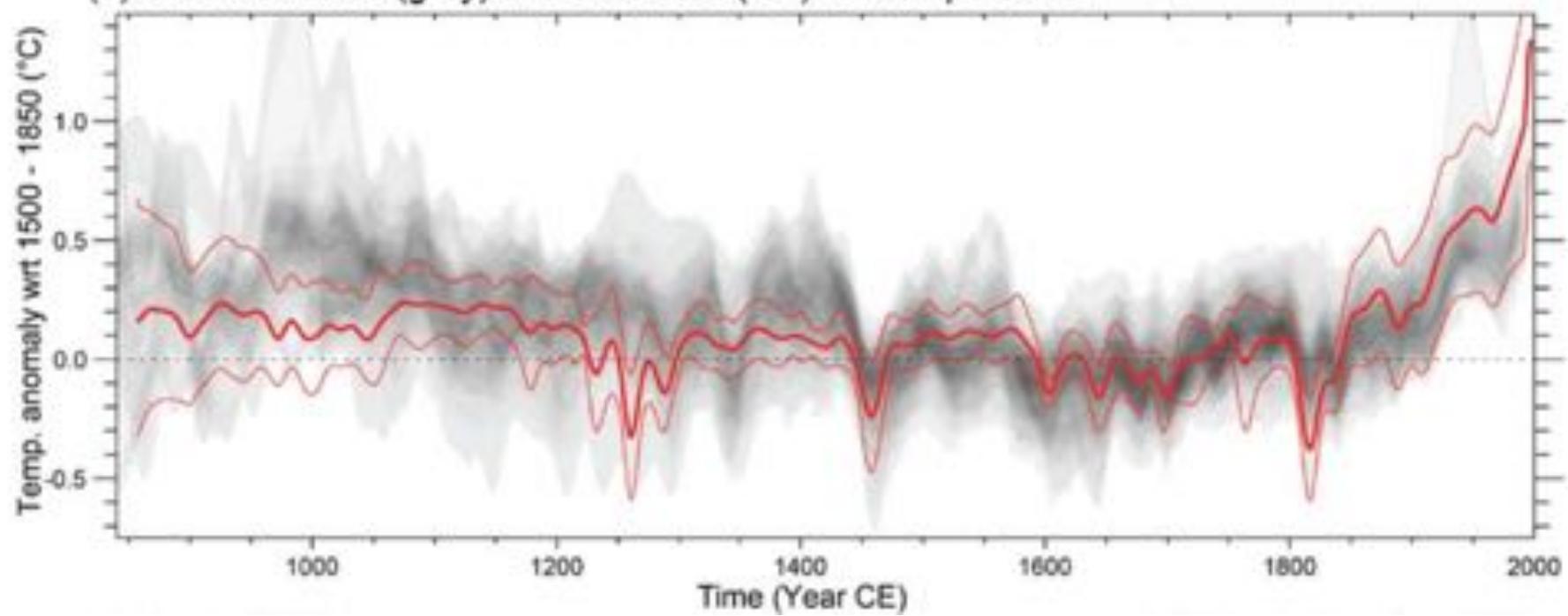


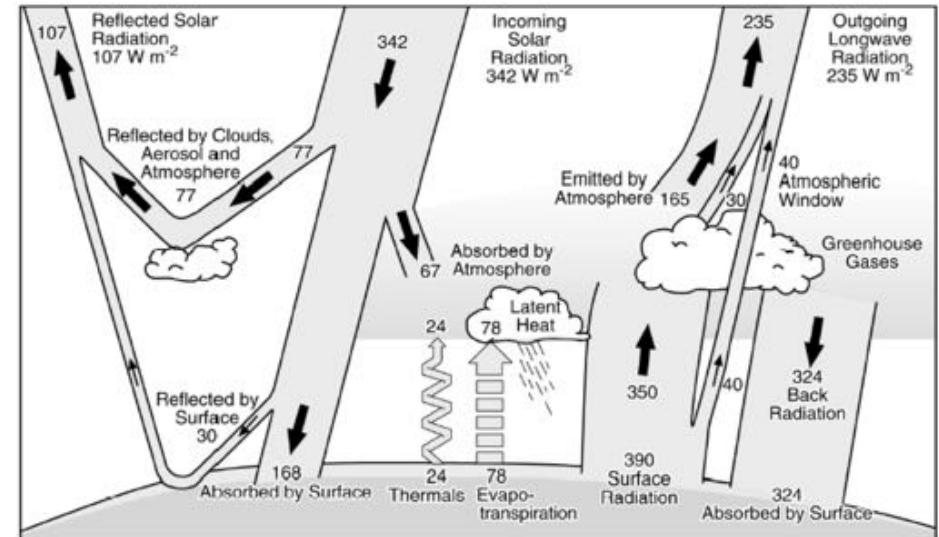
## Global average temperature anomaly (1850-2015)



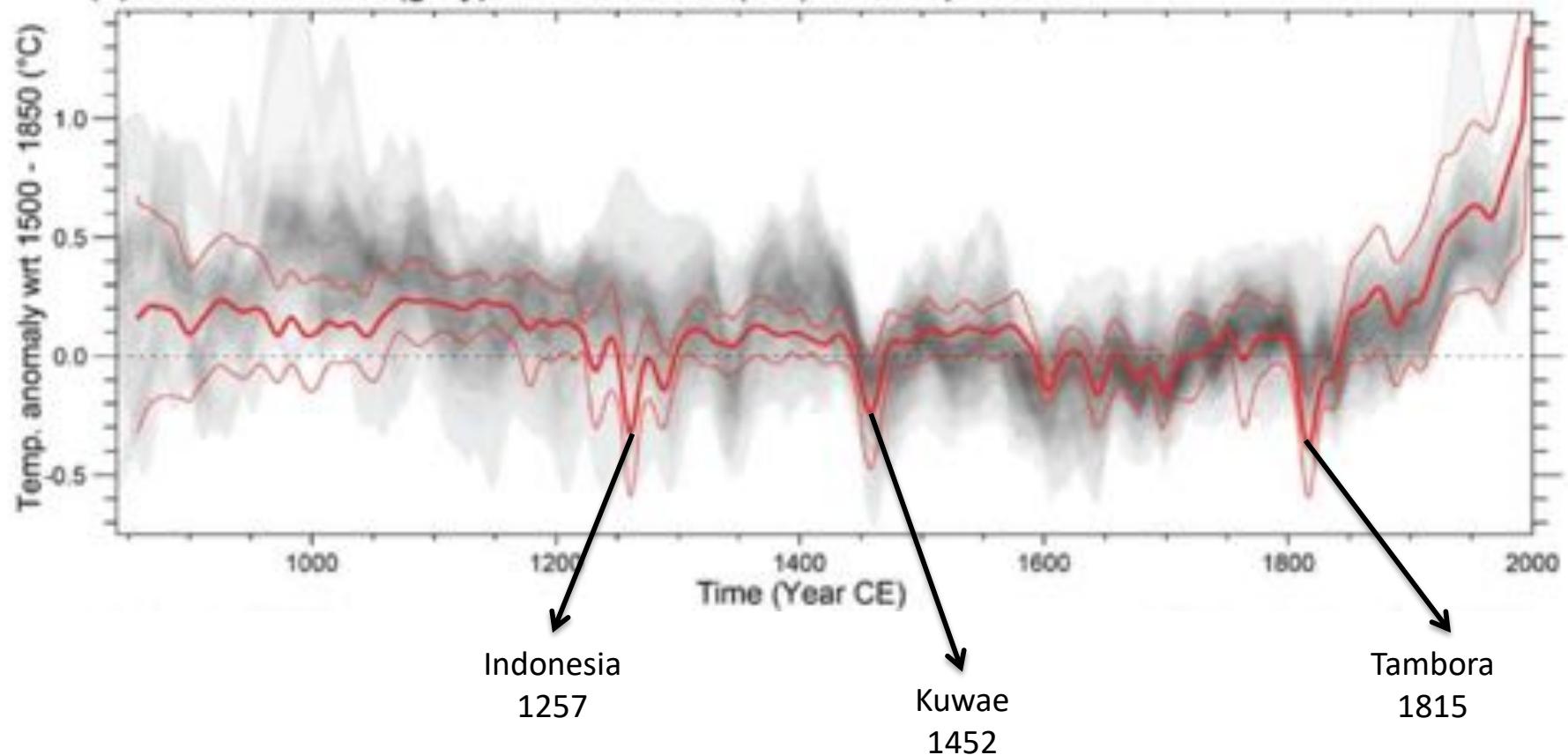


(b) Reconstructed (grey) and simulated (red) NH temperature





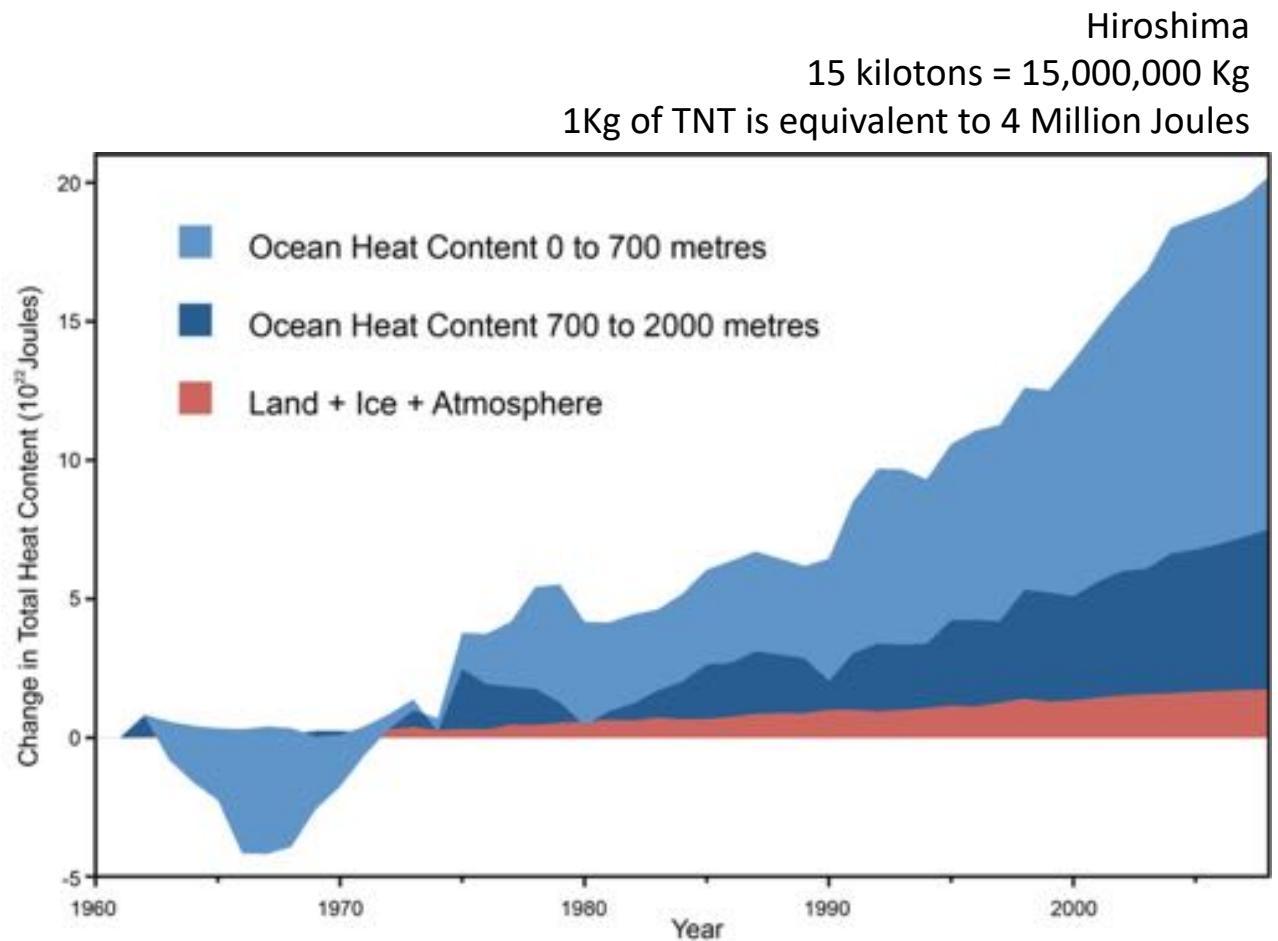
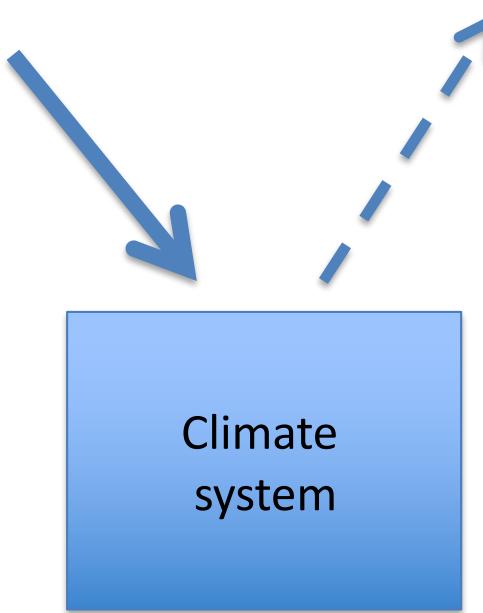
(b) Reconstructed (grey) and simulated (red) NH temperature



## Simplified [Box] model of the climate system

Assumptions:

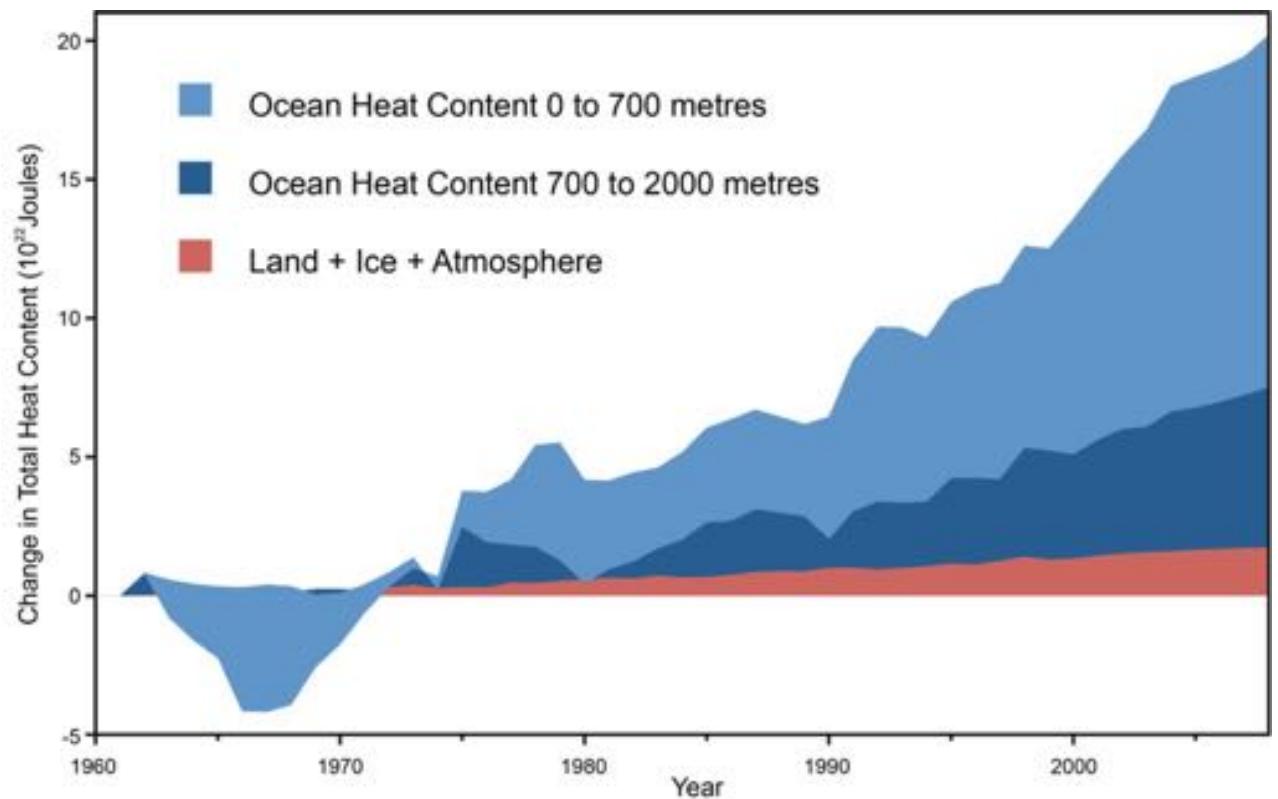
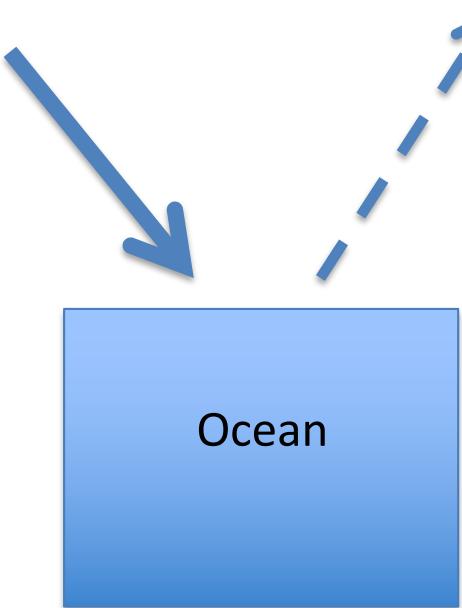
- Most of the thermal mass of the climate system is in the ocean



## Simplified [Box] model of the climate system

Assumptions:

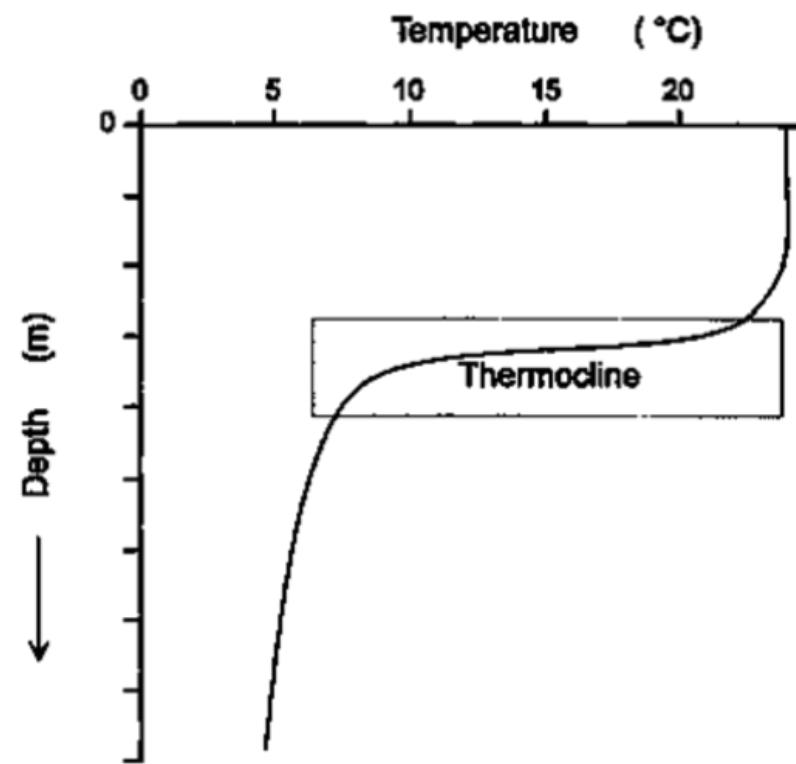
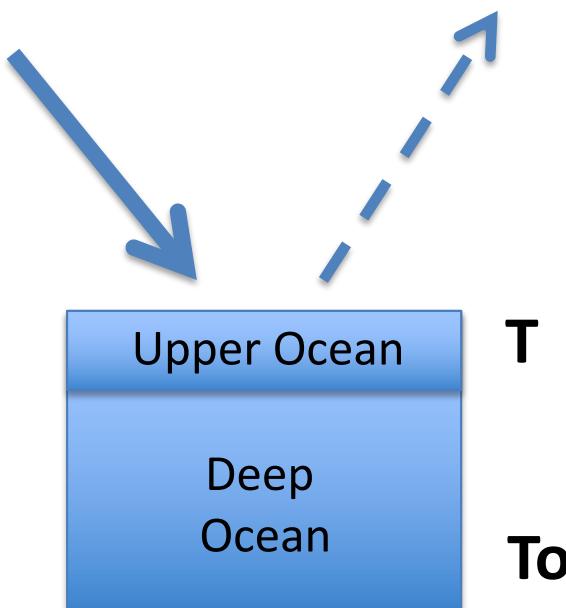
- Most of the thermal mass of the climate system is in the ocean
- Upper ocean is well mixed and responds quickly to surface energy changes
- Deep ocean interacts more slowly with the upper ocean
- Upper ocean and deep ocean are well mixed



## Simplified [Box] model of the climate system

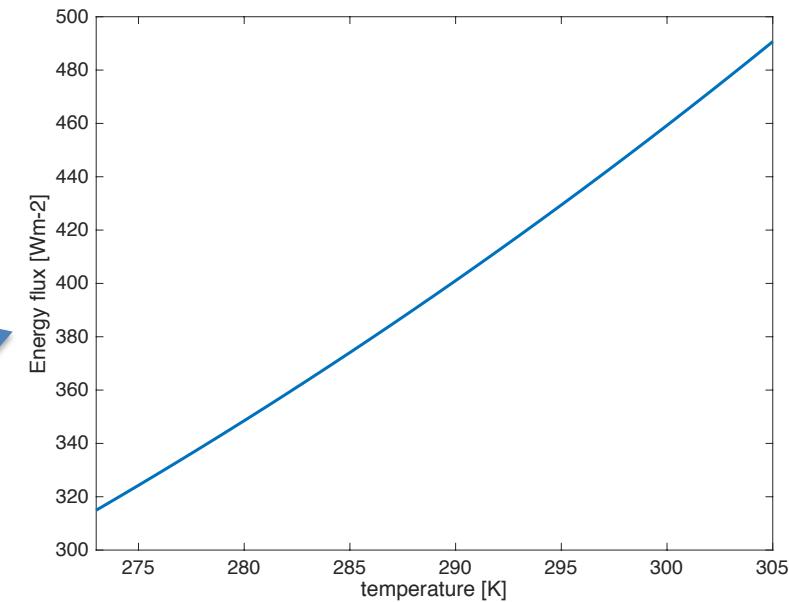
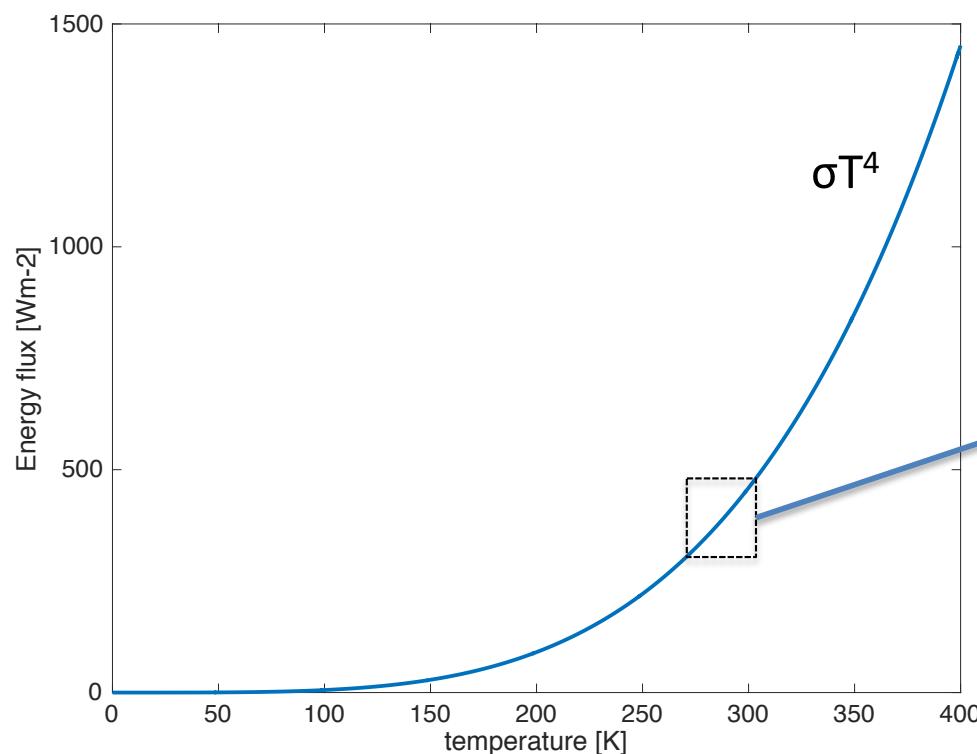
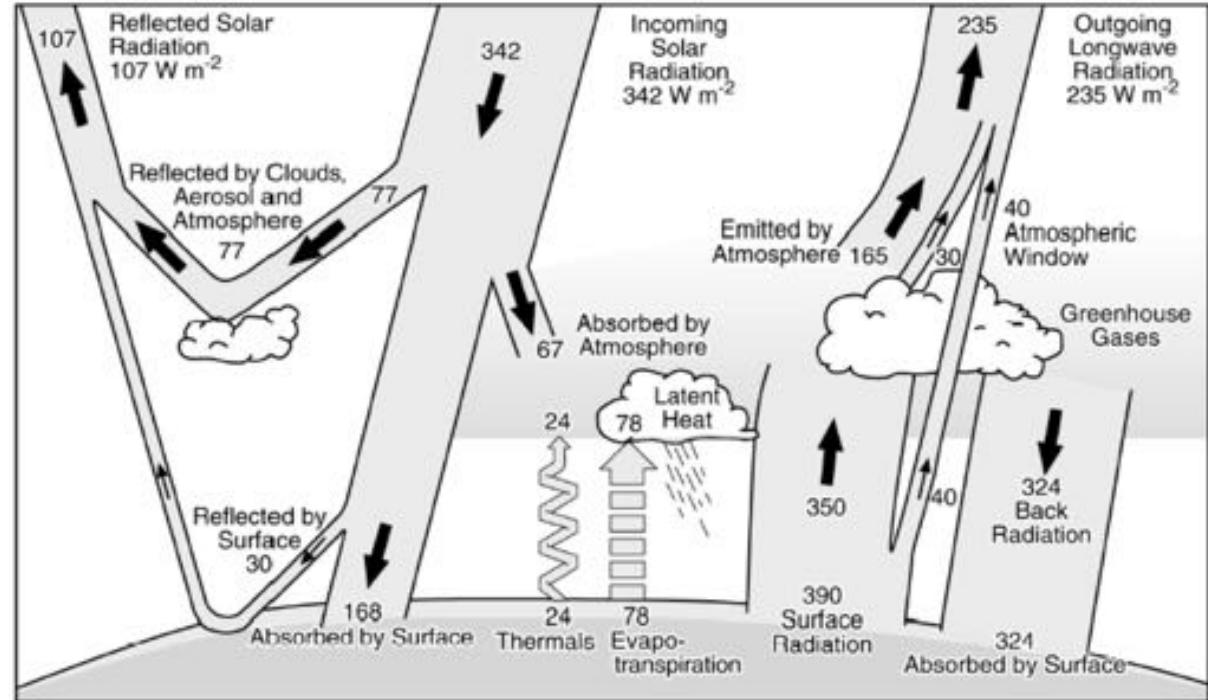
Assumptions:

- Most of the thermal mass of the climate system is in the ocean
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## Assumptions:

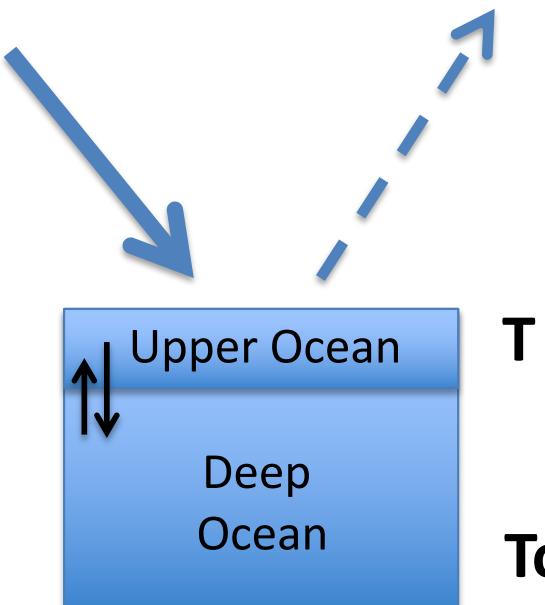
- Heat loss from the planet surface increases linearly with temperature (includes black body radiation, sensible and latent heat losses)



## Simplified [Box] model of the climate system

Assumptions:

- Most of the thermal mass of the climate system is in the ocean
- Upper ocean is well mixed and responds quickly to surface energy changes
- Deep ocean interacts more slowly with the upper ocean
- Upper ocean and deep ocean are well mixed
- Heat loss from the planet surface increases linearly with temperature (includes black body radiation, sensible and latent heat losses)
- Transfer of heat from the upper to the deep ocean just depends on the temperature difference



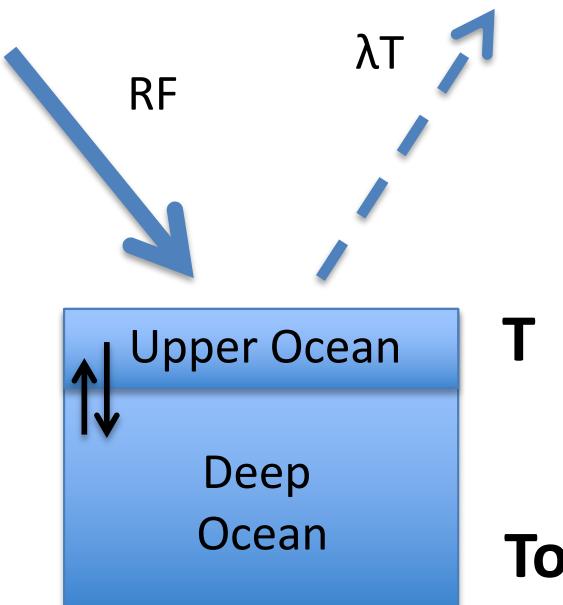
## Simplified [Box] model of the climate system

$$\text{Change in energy content/s} = \text{Heat capacity} \times \text{Change in temperature/s} = \text{Energy in/s} - \text{Energy out/s}$$

$$C \frac{dT}{dt} = RF - \lambda T - \gamma(T - T_o)$$

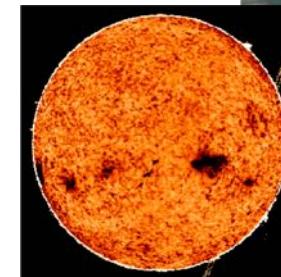
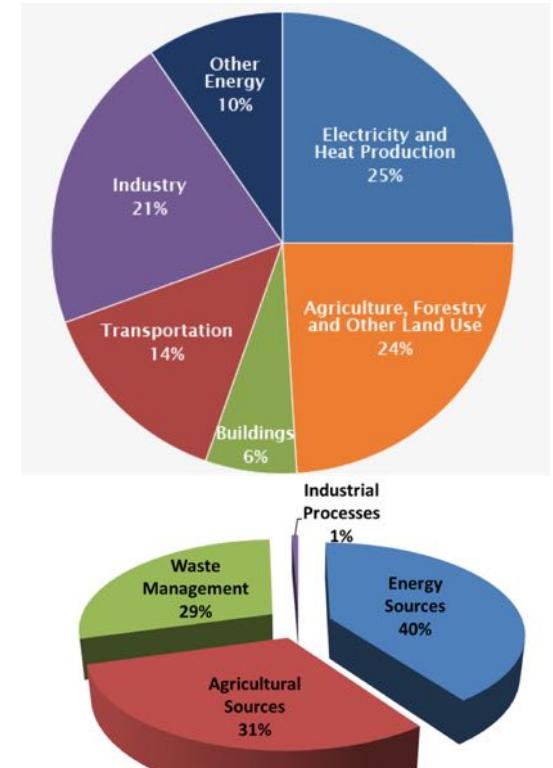
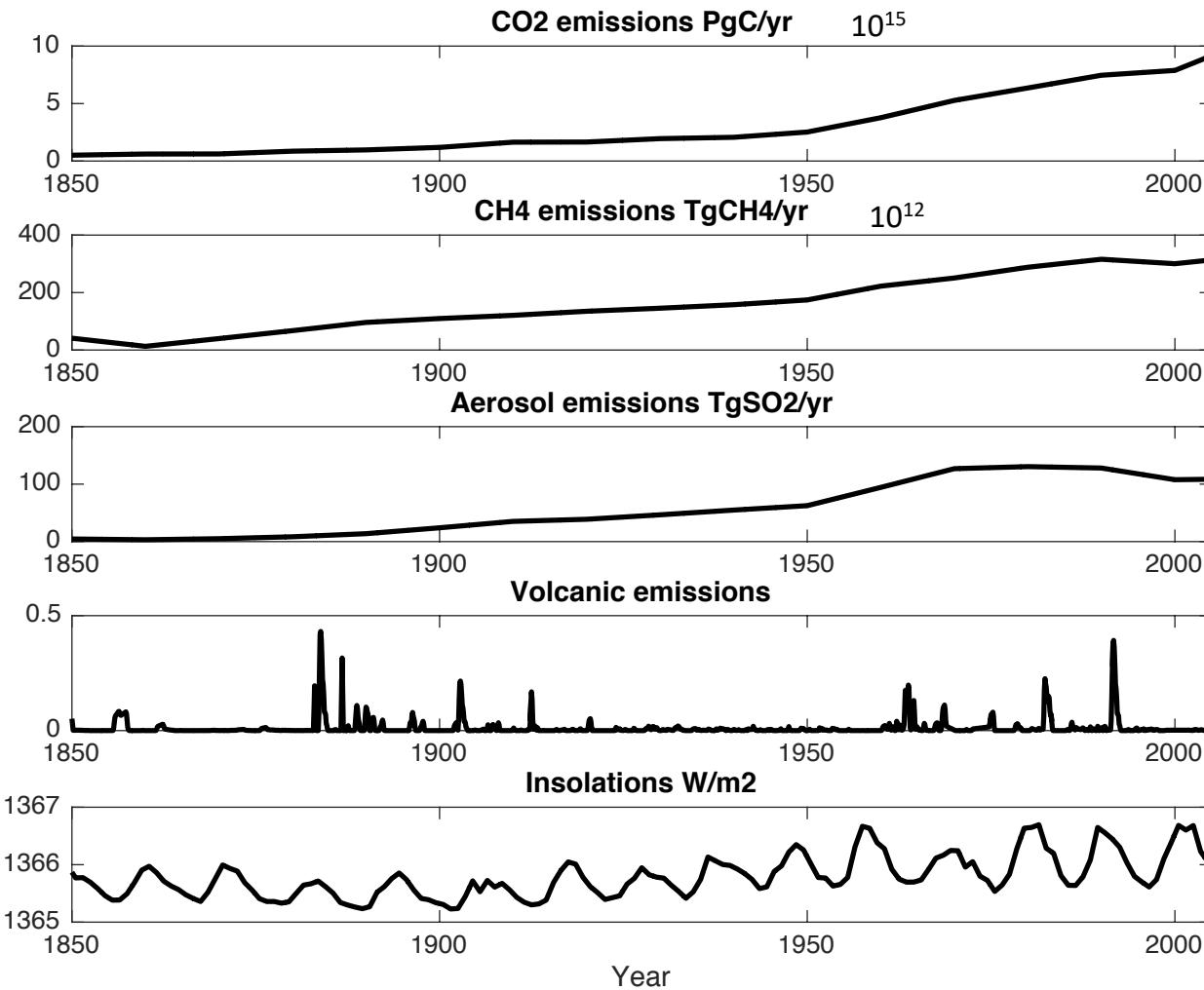
$$C_o \frac{dT_o}{dt} = \gamma(T - T_o)$$

$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + R^{sol}$$



## Model inputs

To calculate the radiative forcing we need certain information: model inputs



## Model inputs

$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + \boxed{R^{sol}}$$

### Solar radiation

- $S$  is the radiation flux per square meter at top of atmosphere
- Energy passing through a circle is distributed over a sphere
- A proportion  $\alpha$  of the surface energy is reflected back to space



$$\text{Insolation hitting earth} = S \times \pi R^2 [\text{W}]$$

This is distributed over total earth surface  $4\pi R^2 [\text{m}^2]$

$$\text{On average, insolation per square meter at surface} = S \times \pi R^2 / 4\pi R^2 = S/4$$

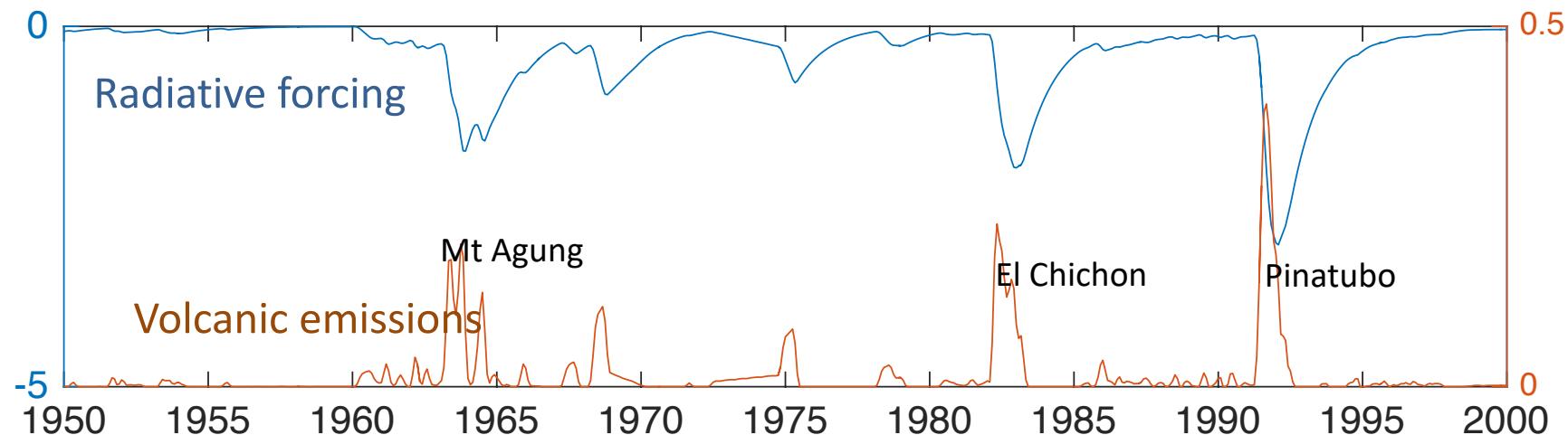
$$R^{sol} = \frac{S(t)}{4} (1 - \alpha)$$

## Model inputs

$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + R^{sol}$$

### Volcanic aerosols

- Particles from volcanic eruptions are ejected into the stratosphere
- These reflect away solar radiation (i.e. produce a negative radiative forcing)
- Volcanic aerosols fall out over a timescale of ~ 1-3 years



OT (Optical thickness) increases with emissions of volcanic aerosols and decay with timescale  $\tau^{volc}$  (2.5 years)  
 $R^{volc}$  is proportional to OT (constant is negative)

$$\frac{\partial(OT)}{\partial t} = E^{volc} - \frac{1}{\tau^{volc}}(OT)$$

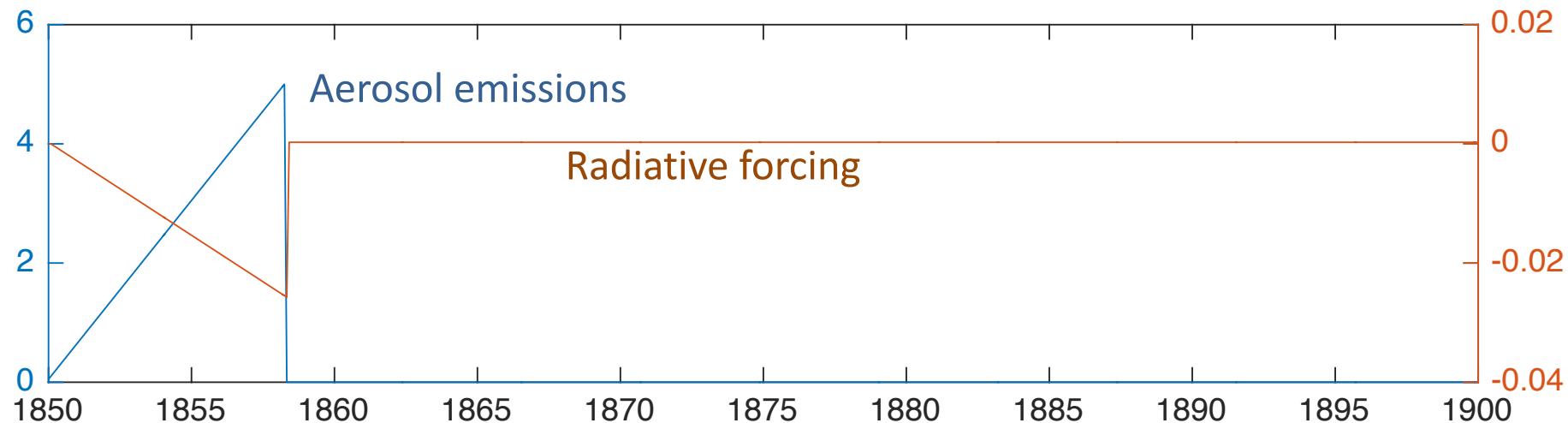
$$R^{volc} = c^{volc} OT$$

## Model inputs

$$RF = R^{CO_2} + R^{CH_4} + \boxed{R^{aer}} + R^{volc} + R^{sol}$$

### Human aerosols

- Aerosols (mainly  $SO_4$ ) released into lower atmosphere
- Particles are rained out almost immediately (few days)



$R^{volc}$  is proportional to the rate of emissions (constant is negative)

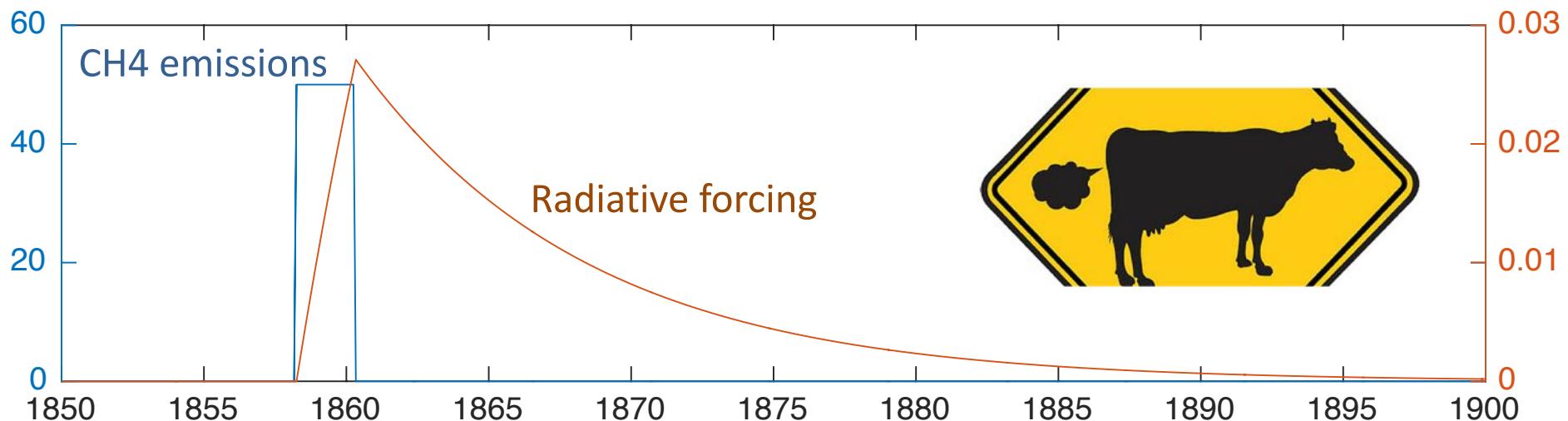
$$R^{aer} = c^{aer} E^{aer}$$

## Model inputs

$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + R^{sol}$$

### Methane emissions

- Methane in the atmosphere is emitted by energy production, agricultural processes and the decay of waste products
- It decays via oxidation in the atmosphere with a half life of  $\sim 10$  years (but the decay rate increases with increased methane concentrations)



$$\frac{\partial C^{CH_4}}{\partial t} = c^{CH_4} E^{CH_4} - \frac{1}{\tau^{CH_4}} C^{CH_4}$$

$$\tau^{CH_4} = \tau_{PI}^{CH_4} \left( \frac{C^{CH_4}}{C^{CH_4} + C_{PI}^{CH_4}} \right)^\alpha$$

$$R^{CH_4} = c^{CH_4} \log_e \left( \frac{C^{CH_4} + C_{PI}^{CH_4}}{C_{PI}^{CH_4}} \right)$$

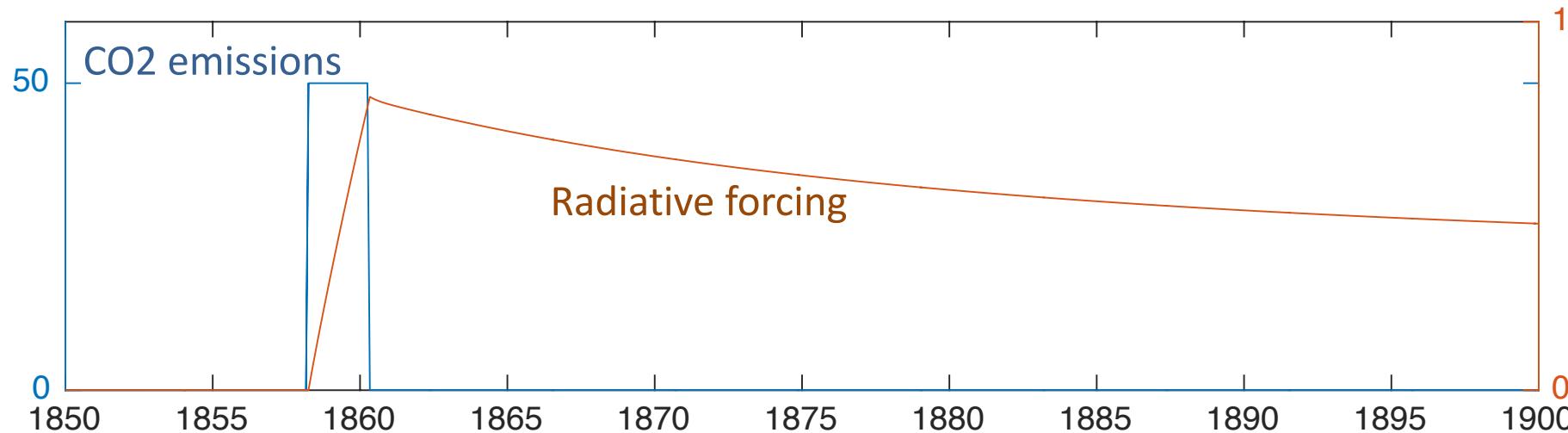
- Radiative forcing increases with the log of CH4 concentration

## Model inputs

$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + R^{sol}$$

### CO2 emissions

- CO2 in the atmosphere is primarily produced through the burning of fossil fuel, with a contribution from deforestation and agriculture
- Carbon cycle is complex and includes uptake and release of CO2 by the terrestrial biosphere and ocean. Half life 100's of years



$$R^{CO_2} = c^{CO_2} \log_e \left( \frac{C^{CO_2} + C_{PI}^{CO_2}}{C_{PI}^{CO_2}} \right)$$

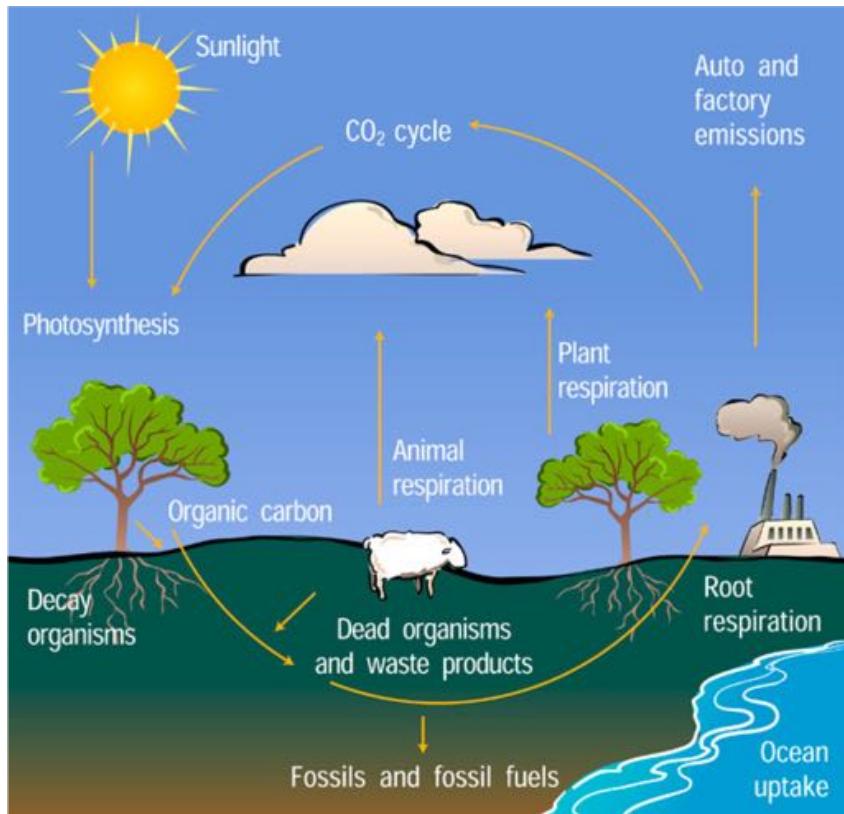
- Radiative forcing increases with the log of CO2 concentration

## Model inputs

$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + R^{sol}$$

### CO<sub>2</sub> emissions

- CO<sub>2</sub> is primarily produced through the burning of fossil fuel, with a contribution from deforestation and agriculture
- Carbon cycle is complex and includes uptake and release of CO<sub>2</sub> by the terrestrial biosphere and ocean. Half life 100's of years



$$\frac{dC^{at}(t)}{dt} = E^{co2} - k_a(C^{at} - A \cdot B \cdot C^{up}) + [(1 - \varepsilon)mN(t) + \delta S(t) - P(t, C^{at})] \quad (4)$$

$$\frac{dC^{up}(t)}{dt} = k_a(C^{at}(t) - A \cdot B \cdot C^{up}(t)) - k_d \left( C^{up}(t) - \frac{C^{lo}(t)}{d} \right) \quad (5)$$

$$\frac{dC^{lo}(t)}{dt} = k_d \left( C^{up}(t) - \frac{C^{lo}(t)}{d} \right) \quad (6)$$

$$\frac{dN(t)}{dt} = P(t, C^{at}) - mN(t) \quad (7)$$

$$\frac{dS(t)}{dt} = \varepsilon mN(t) - \delta S(t) \quad (8)$$

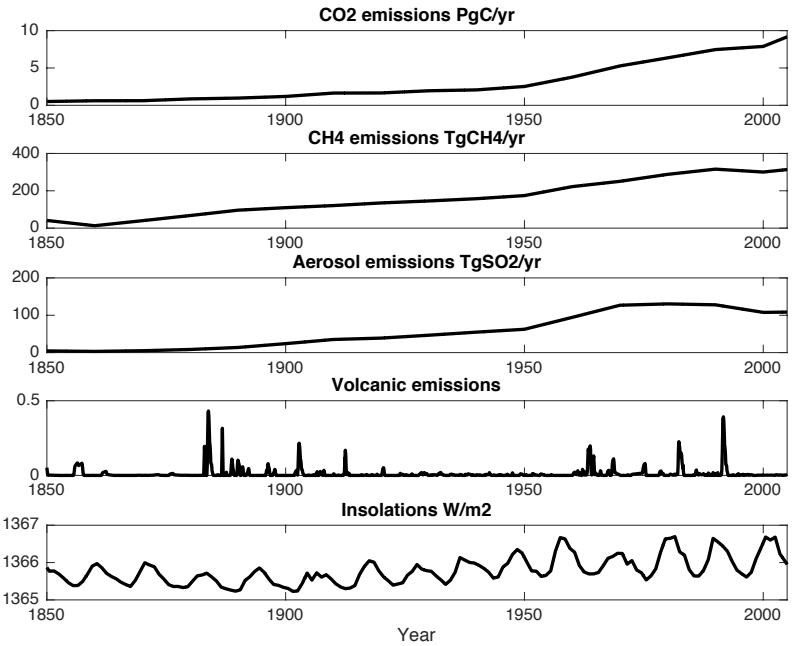
$$P(t) = P_0(1 - a_2(C^{at}(t) - C_0^{at})) \quad (9)$$

where  $C^{at}$ ,  $C^{up}$ ,  $C^{lo}$ ,  $N$  and  $S$  are the inventories of carbon dioxide (GtC) in the atmosphere, upper ocean and deep ocean, terrestrial vegetation and soil, respectively,  $P$  is net primary production by terrestrial plants (GtC/yr) and  $E$  is the human emissions of CO<sub>2</sub> (GtC/yr) See table for definition of constants.

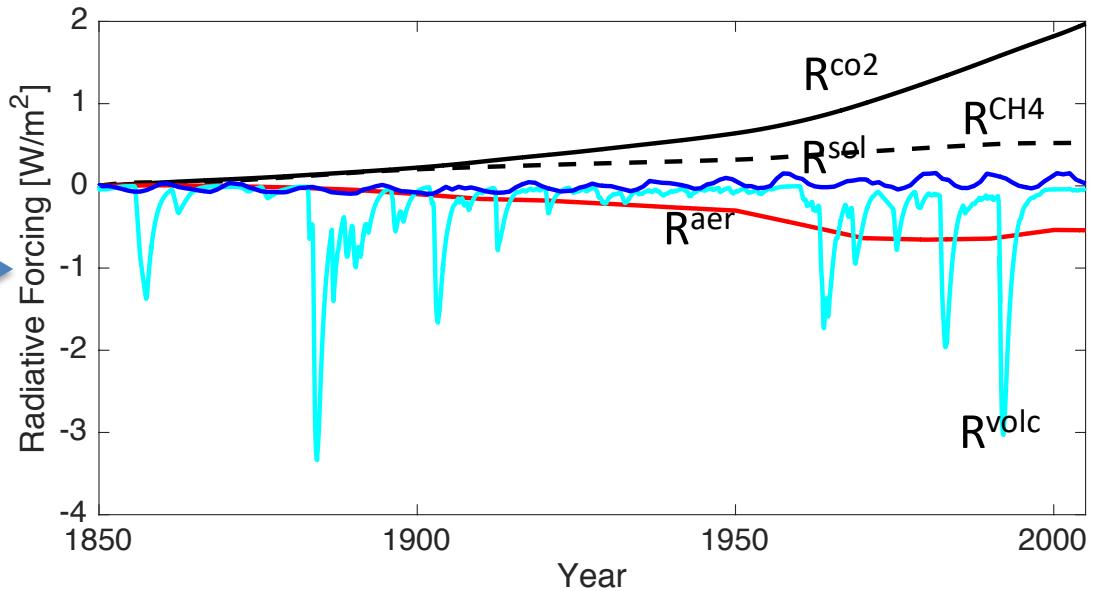
$$R^{CO_2} = c^{CO_2} \log_e \left( \frac{C^{CO_2} + C_{PI}^{CO_2}}{C_{PI}^{CO_2}} \right)$$

# Solving the Model

Model inputs...



Radiative Forcing...



$$RF = R^{CO_2} + R^{CH_4} + R^{aer} + R^{volc} + R^{sol}$$

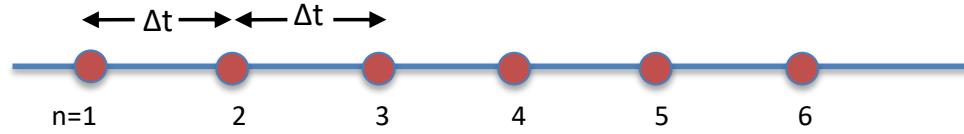


Energy  
balance  
model

$$C \frac{dT}{dt} = RF - \lambda T - \gamma(T - T_o)$$

$$C_o \frac{dT_o}{dt} = \gamma(T - T_o)$$

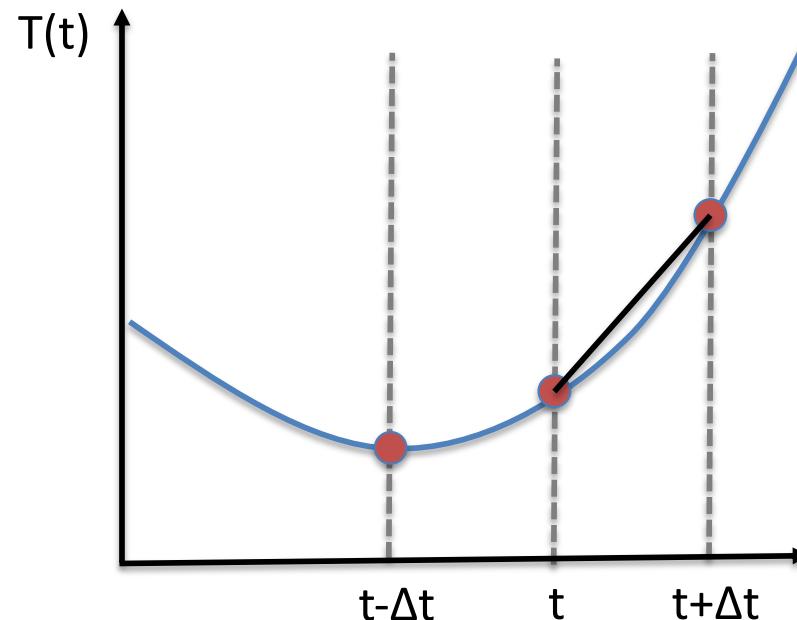
## Solving the Model



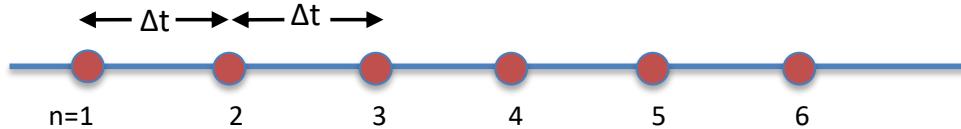
$$\left( \frac{dT}{dt} \right)_t \approx \frac{T_{(t+\Delta t)} - T_{(t)}}{\Delta t}$$

$$C \frac{dT(t)}{dt} = RF(t) - \lambda T(t) - \gamma(T(t) - T_o(t))$$

$$T(t + \Delta t) \approx T(t) + \frac{\Delta t}{C} [RF(t) - \lambda T(t) - \gamma(T(t) - T_o(t))]$$



## Solving the Model

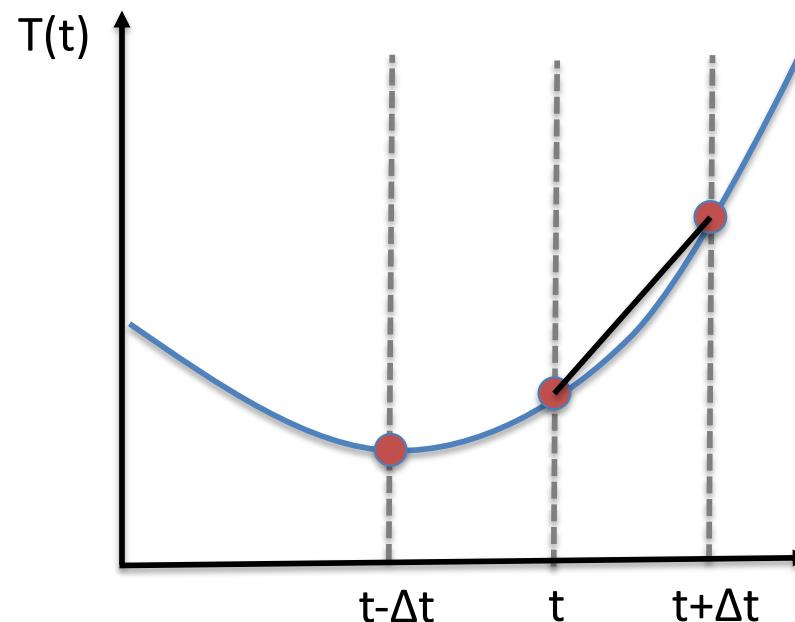


$$\left( \frac{dT}{dt} \right)_t \approx \frac{T_{(t+\Delta t)} - T_{(t)}}{\Delta t}$$

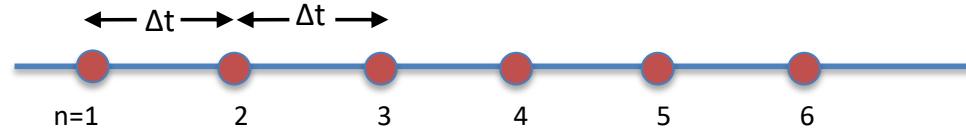
$$C \frac{dT(t)}{dt} = RF(t) - \lambda T(t) - \gamma(T(t) - T_o(t))$$

$$T(t + \Delta t) \approx T(t) + \frac{\Delta t}{C} [RF(t) - \lambda T(t) - \gamma(T(t) - T_o(t))]$$

$$T_o(t + \Delta t) \approx T_o(t) + \frac{\Delta t}{C_o} [\gamma(T(t) - T_o(t))]$$



## Solving the Model

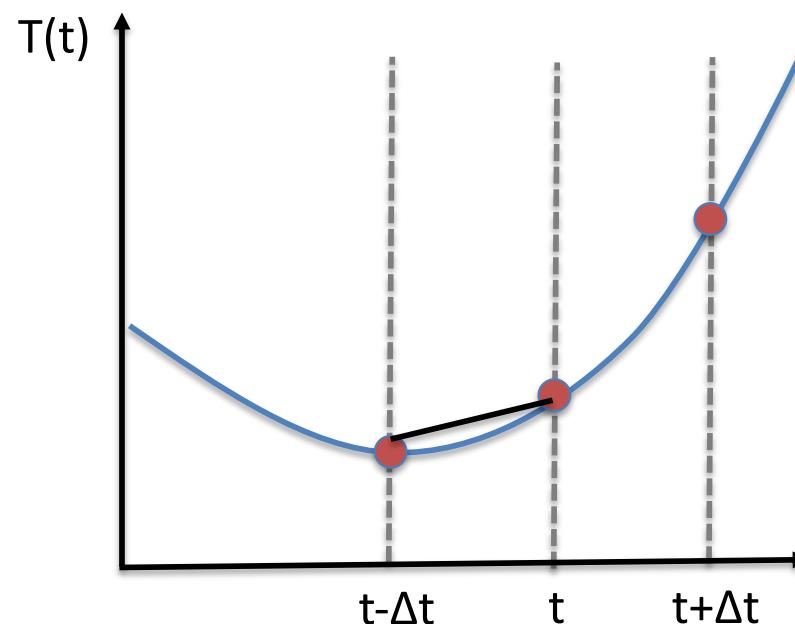


$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

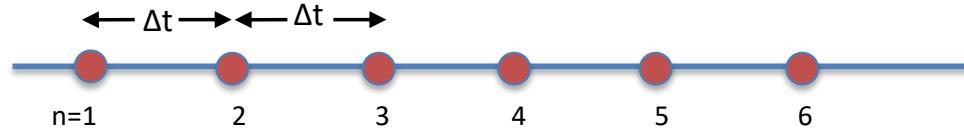
Forward difference

$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_t - T_{t-\Delta t}}{\Delta t}$$

Backward difference



## Solving the Model



$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

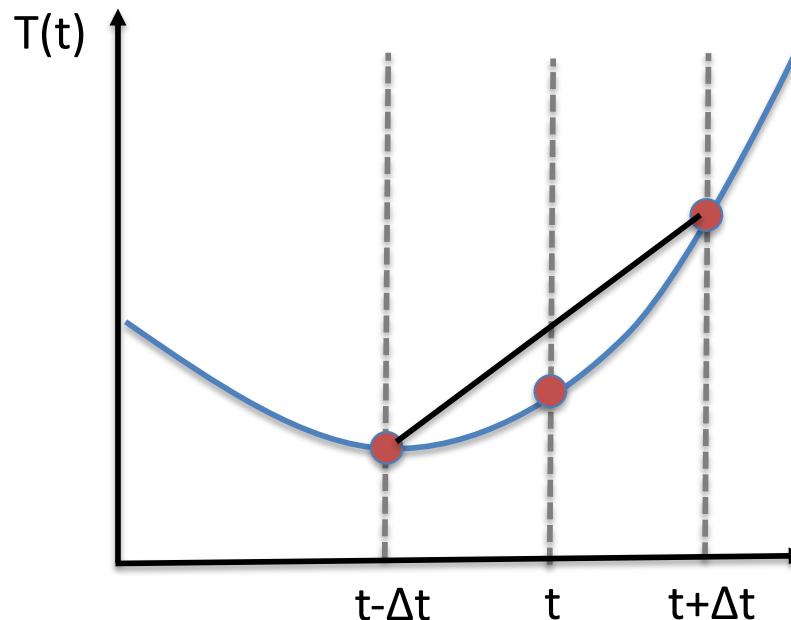
Forward difference

$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_t - T_{t-\Delta t}}{\Delta t}$$

Backward difference

$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_{t+\Delta t} - T_{t-\Delta t}}{2\Delta t}$$

Centred difference



## Solving the Model

Taylor expansion of  $T(t + \Delta t)$

$$T(t + \Delta t) = T(t) + \Delta t \left( \frac{dT}{dt} \right)_t + \frac{(\Delta t)^2}{2} \left( \frac{d^2T}{dt^2} \right)_t + \frac{(\Delta t)^3}{6} \left( \frac{d^3T}{dt^3} \right)_t + \dots$$

$$T(t - \Delta t) = T(t) - \Delta t \left( \frac{dT}{dt} \right)_t + \frac{(\Delta t)^2}{2} \left( \frac{d^2T}{dt^2} \right)_t - \frac{(\Delta t)^3}{6} \left( \frac{d^3T}{dt^3} \right)_t + \dots$$

$$\left( \frac{dT}{dt} \right)_t = \frac{T(t + \Delta t) - T(t)}{\Delta t} - \frac{(\Delta t)}{2} \left( \frac{d^2T}{dt^2} \right)_t - \frac{(\Delta t)^2}{6} \left( \frac{d^3T}{dt^3} \right)_t + \dots \quad \text{Forward difference}$$

$$\left( \frac{dT}{dt} \right)_t = \frac{T(t) - T(t - \Delta t)}{\Delta t} + \frac{(\Delta t)}{2} \left( \frac{d^2T}{dt^2} \right)_t - \frac{(\Delta t)^2}{6} \left( \frac{d^3T}{dt^3} \right)_t + \dots \quad \text{Backward difference}$$

$$\left( \frac{dT}{dt} \right)_t = \frac{T(t + \Delta t) - T(t - \Delta t)}{2\Delta t} - \frac{(\Delta t)^2}{6} \left( \frac{d^3T}{dt^3} \right)_t + \dots \quad \text{Centred difference}$$

- The error increases with the time step  $\Delta t$  size
- Error is proportional to  $\Delta t$  for forward and backward approximations
- Error is proportional to  $\Delta t^2$  for central approximation
- For small  $\Delta t$  central difference error will be smaller

## Solving the Model

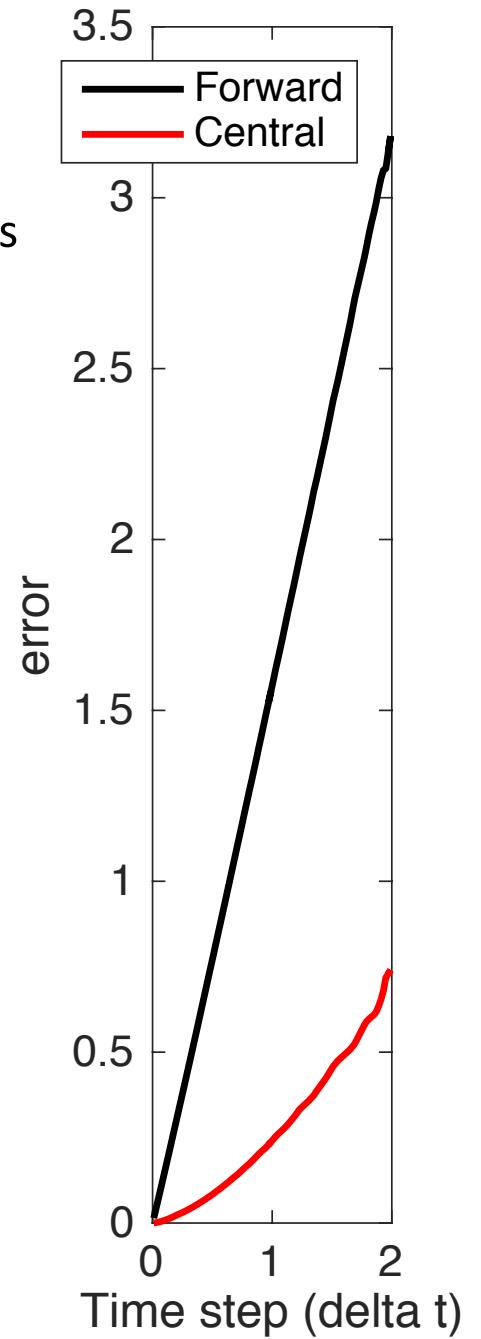
- The error is proportional to the time step  $\Delta t$
- Error is proportional to  $\Delta t$  for forward and backward approximations
- Error is proportional to  $\Delta t^2$  for central approximation
- For small  $\Delta t$  central difference error will be smaller

$$\frac{dT}{dt} = \sqrt{t}$$

$$T(t + \Delta t) = T(t) + \Delta t \sqrt{t}$$

$$T(t + \Delta t) = T(t - \Delta t) + 2\Delta t \sqrt{t}$$

Forward difference  
Centred difference



## Solving the Model

Other more sophisticated schemes provide higher order correction terms  
e.g a commonly used scheme is the fourth order Runge Kutta

$$\frac{dT}{dt} = F(T, t)$$

$$k_1 = \Delta t F(T(t), t)$$

$$k_2 = \Delta t F\left(T + \frac{k_1}{2}, t + \frac{\Delta t}{2}\right)$$

$$k_3 = \Delta t F\left(T + \frac{k_2}{2}, t + \frac{\Delta t}{2}\right)$$

$$k_4 = \Delta t F(T + k_3, t + \Delta t)$$

$$T(t + \Delta t) = T(t) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O[\Delta t^5]$$

## Solving the Model

$$T(t + \Delta t) = T(t) + \frac{\Delta t(RF - \lambda T - \gamma(T - T_o))}{C}$$

Forward difference

$$T_o(t + \Delta t) = T_o(t) + \frac{\Delta t(\gamma(T - T_o))}{C_o}$$

Code for simplest forward differencing:

*T(1)=0; % initialise the upper ocean temperature*

*To(1)=0; % initialise the lower ocean temperature*

*for t=1:N-1 % loop over time steps*

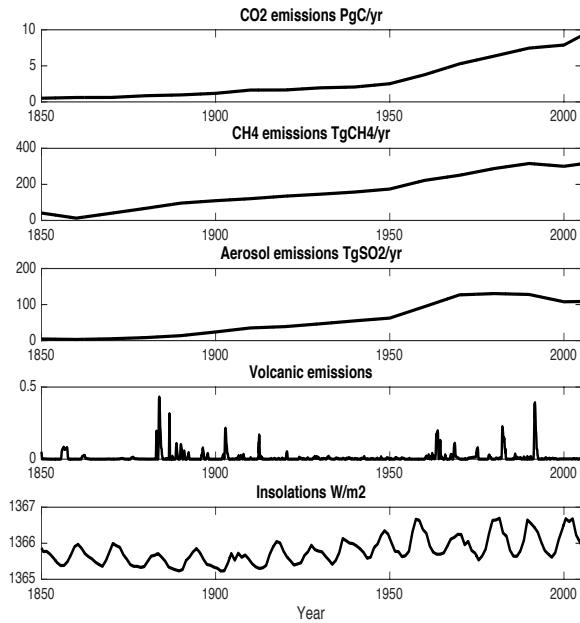
*RF = R\_CO2(t) + R\_CH4(t) + R\_SO2(t) + R\_vvolc(t) + R\_sol(t);*

*% Solve for T and To at time t+1 based on T and To at time t*

*T(t+1) = T(t) + DT\*( RF - L\*T(t) - g\*(T(t)-To(t)) )/C ;*

*To(t+1) = To(t) + DT\*( g\*(T(t)-To(t)) )/Co;*

*end*



## Simple Climate Model

STANDARD ADVANCED HELP ABOUT

**RCPs**

- RCP4.5
- RCP8.5
- RCP6.0

**GLOBAL FORCINGS**

- CO<sub>2</sub>
- CH<sub>4</sub>
- SO<sub>2</sub>
- Volcanics
- Solar
- Internal variability

**ACTIONS**

- 
- 
- 
-

RCP3

1850 - 2100

Radiative forcing reaches 3.1 W/m<sup>2</sup> before it returns to 2.6 W/m<sup>2</sup> by 2100. This is achieved via ambitious greenhouse gas emissions reductions...

Scenario Name: RCP3

Scenario Range: 1850 - 2100

Radiative forcing reaches 3.1 W/m<sup>2</sup> before it returns to 2.6 W/m<sup>2</sup> by 2100. This is achieved via ambitious greenhouse gas emissions reductions...

FURTHER DETAILS

VOLCANIC EMISSIONS

1850 - 2100

Radiative forcing reaches 3.1 W/m<sup>2</sup> before it returns to 2.6 W/m<sup>2</sup> by 2100. This is achieved via ambitious greenhouse gas emissions reductions...

Scenario Name: RCP3

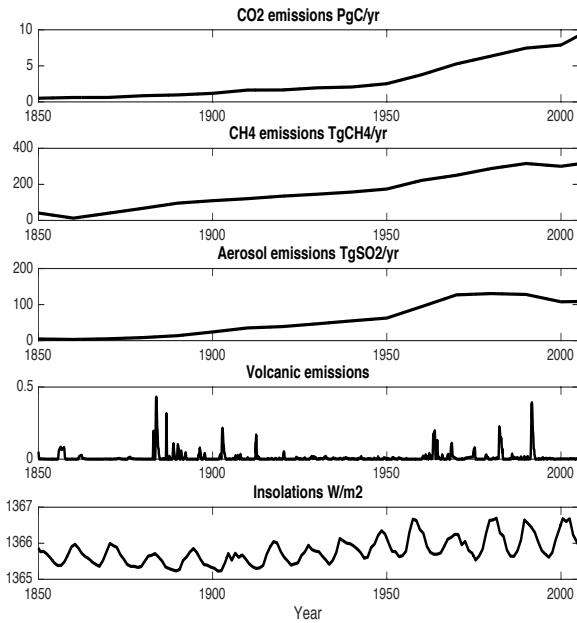
Scenario Range: 1850 - 2100

Radiative forcing reaches 3.1 W/m<sup>2</sup> before it returns to 2.6 W/m<sup>2</sup> by 2100. This is achieved via ambitious greenhouse gas emissions reductions...

FURTHER DETAILS

## Using the model: Evaluation

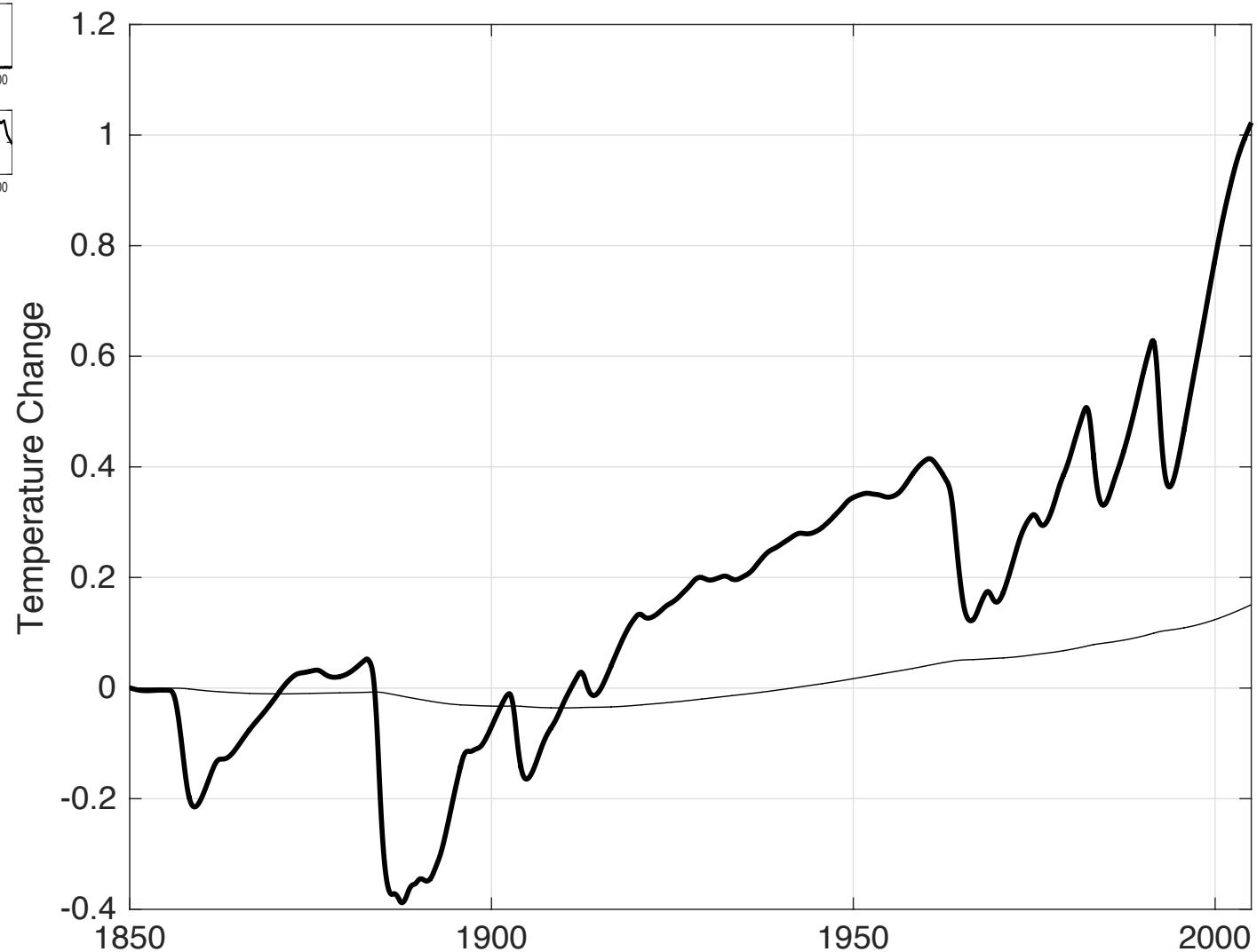
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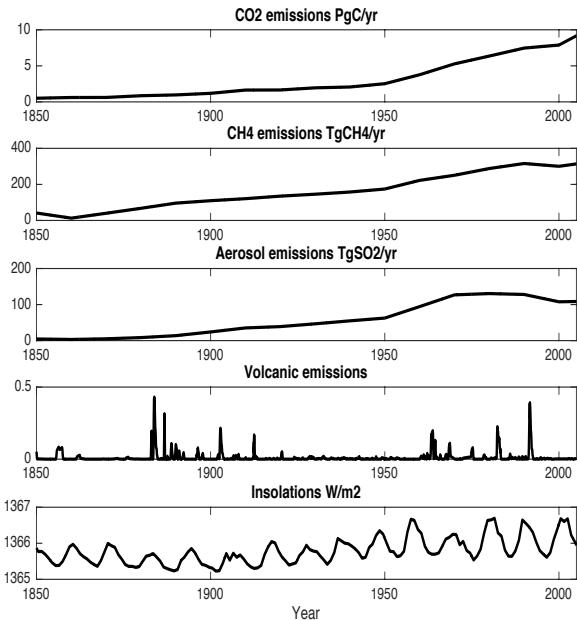
## Using the model: Evaluation

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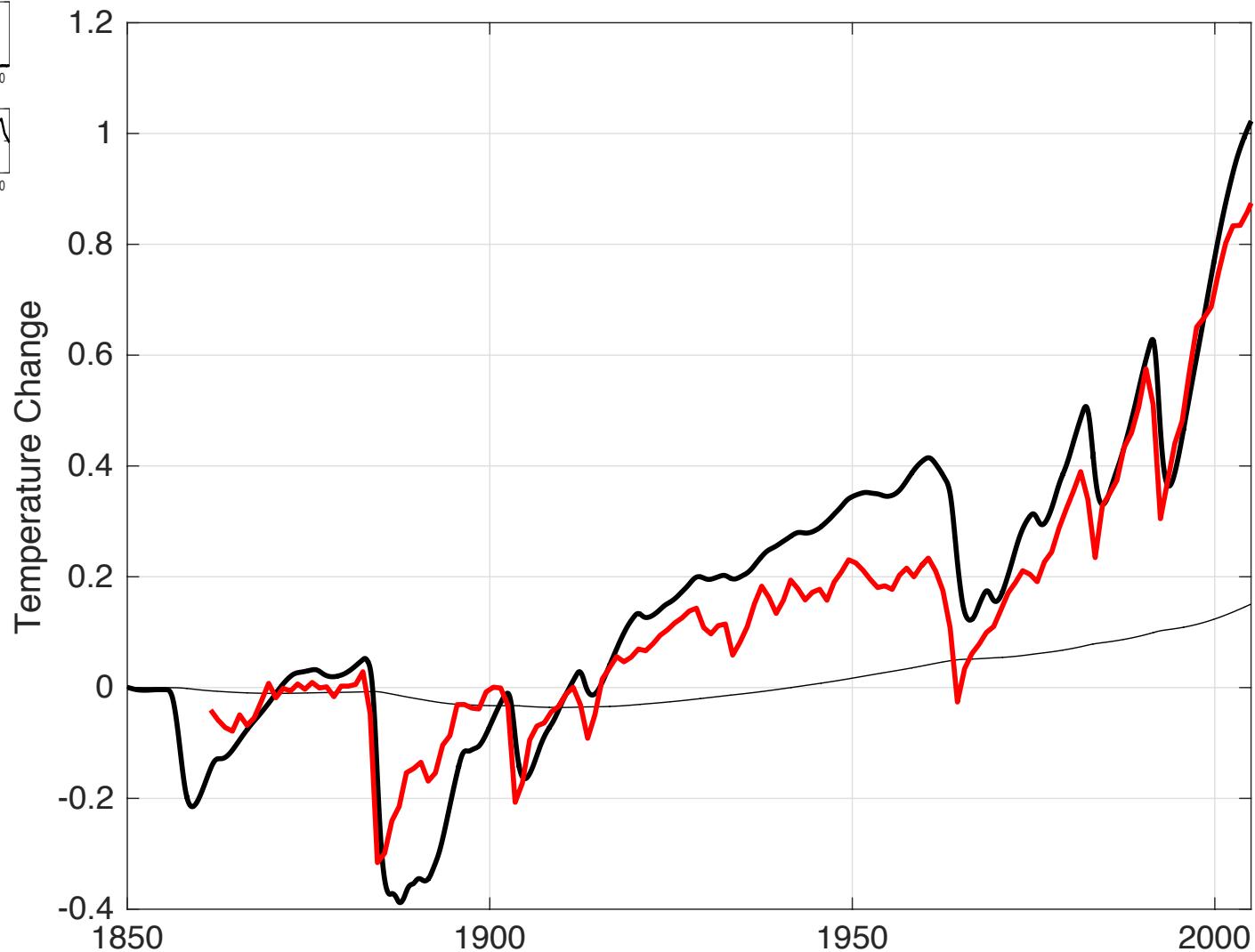
Black: simple model output



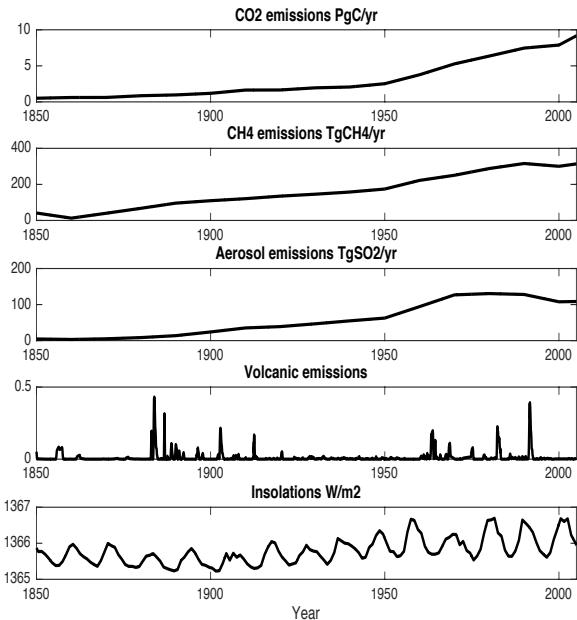
## Using the model: Evaluation



Black: simple model output  
Red: Average of 20 sophisticated climate models



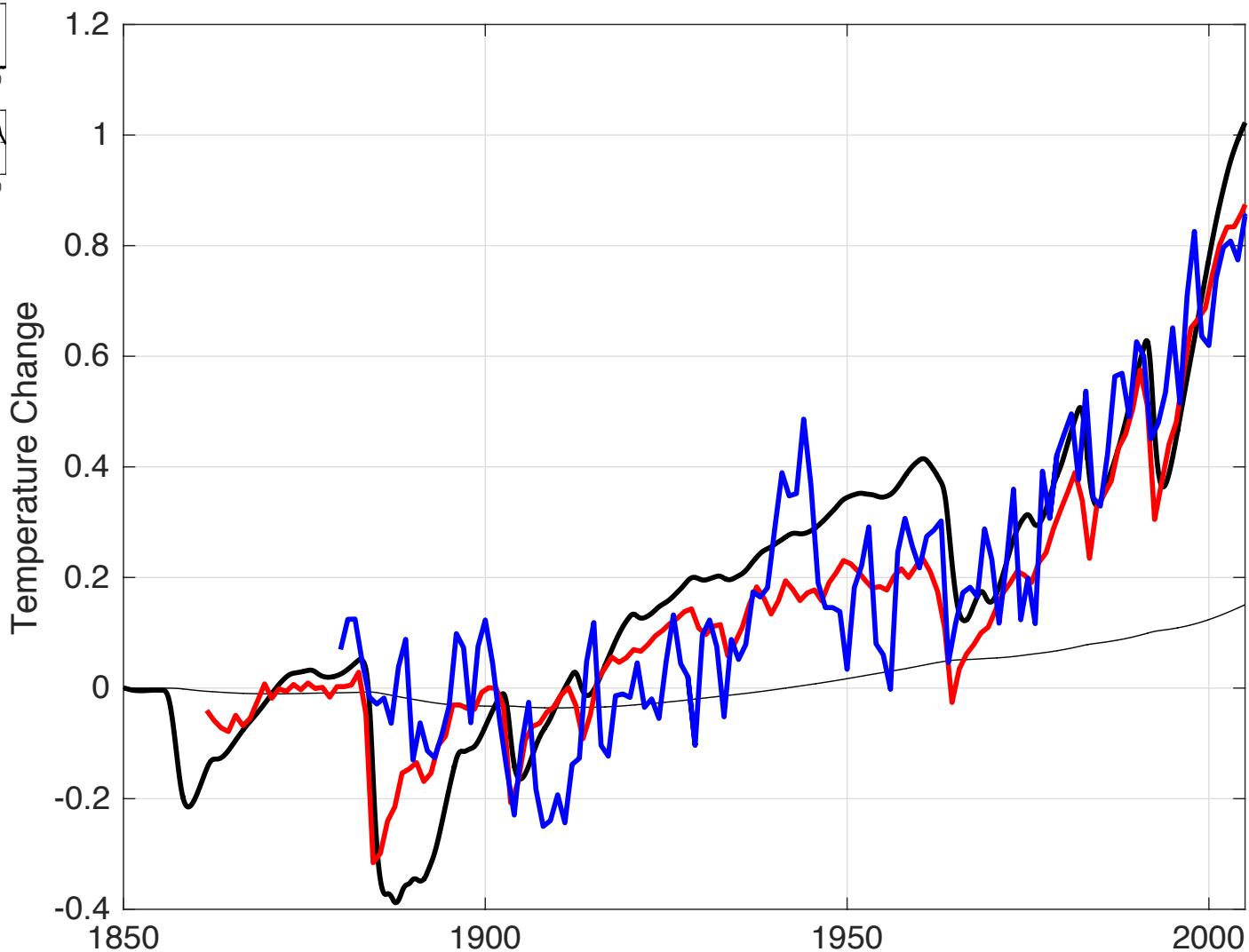
## Using the model: Evaluation



Black: simple model output

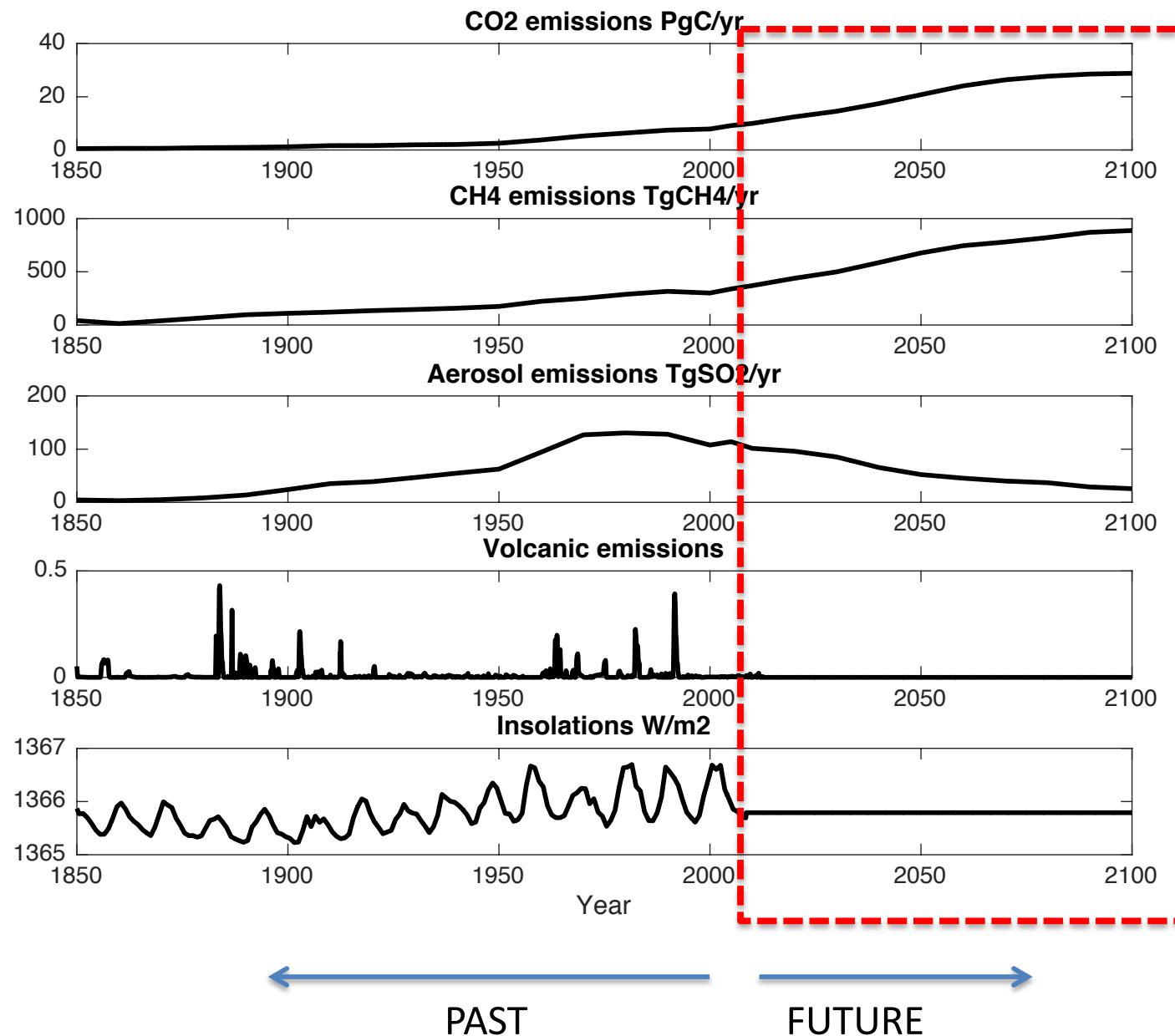
Red: Average of 20 sophisticated climate models

Blue: Observations

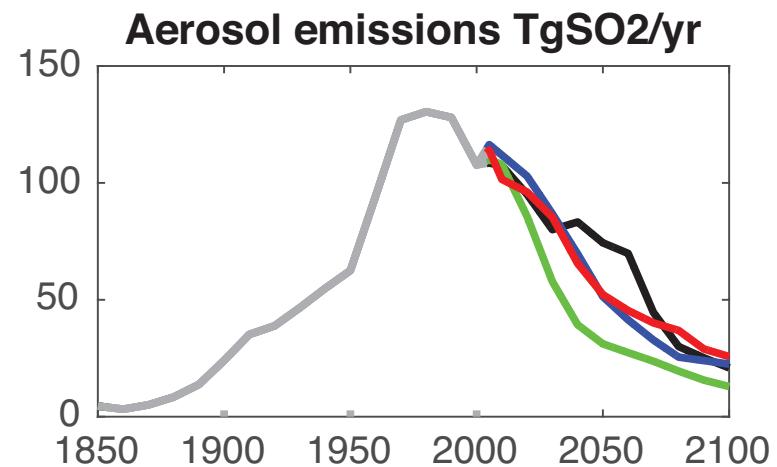
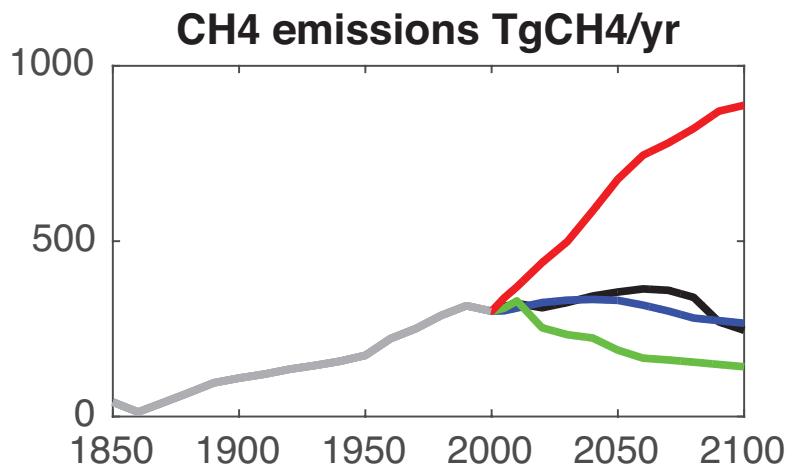
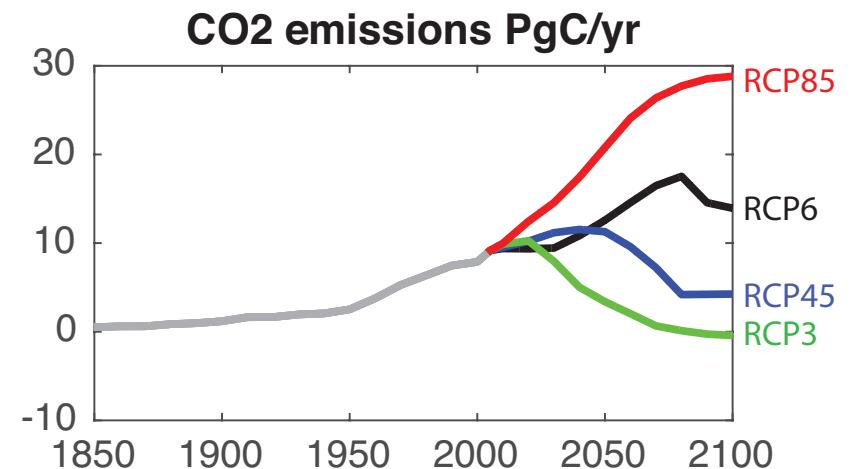


## Using the model: Projections

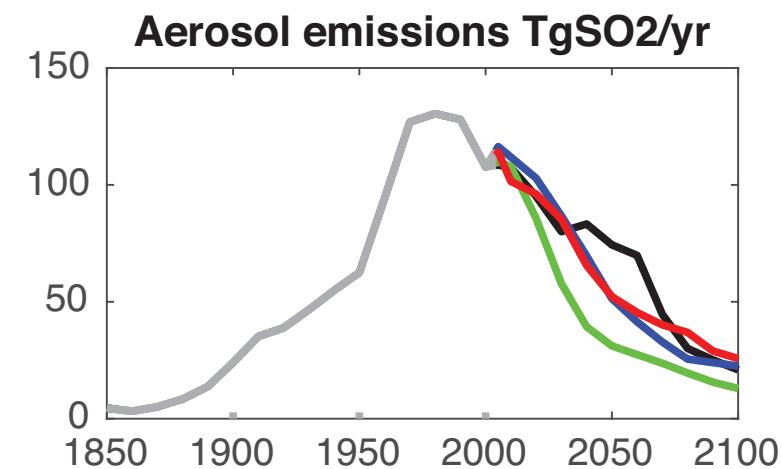
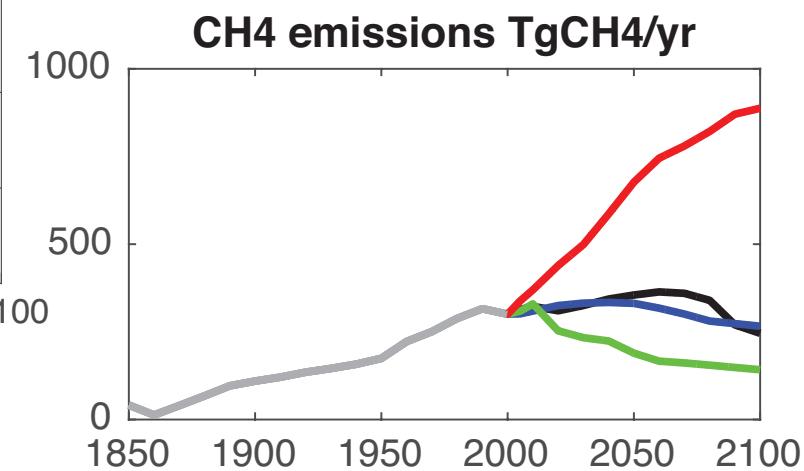
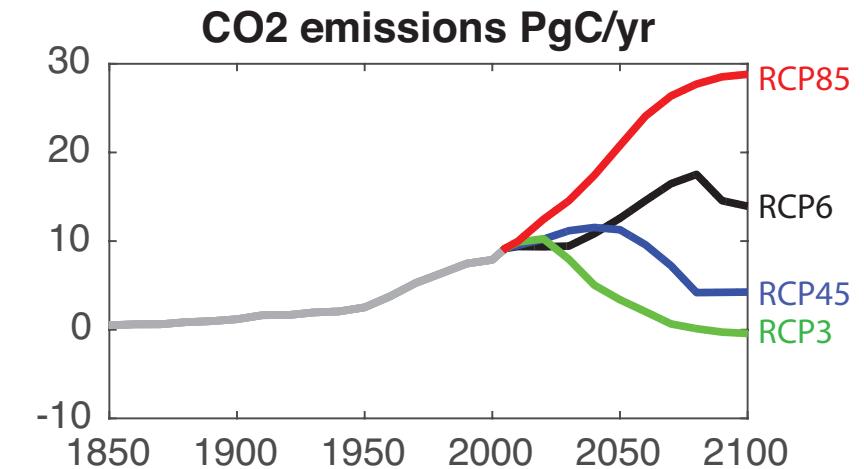
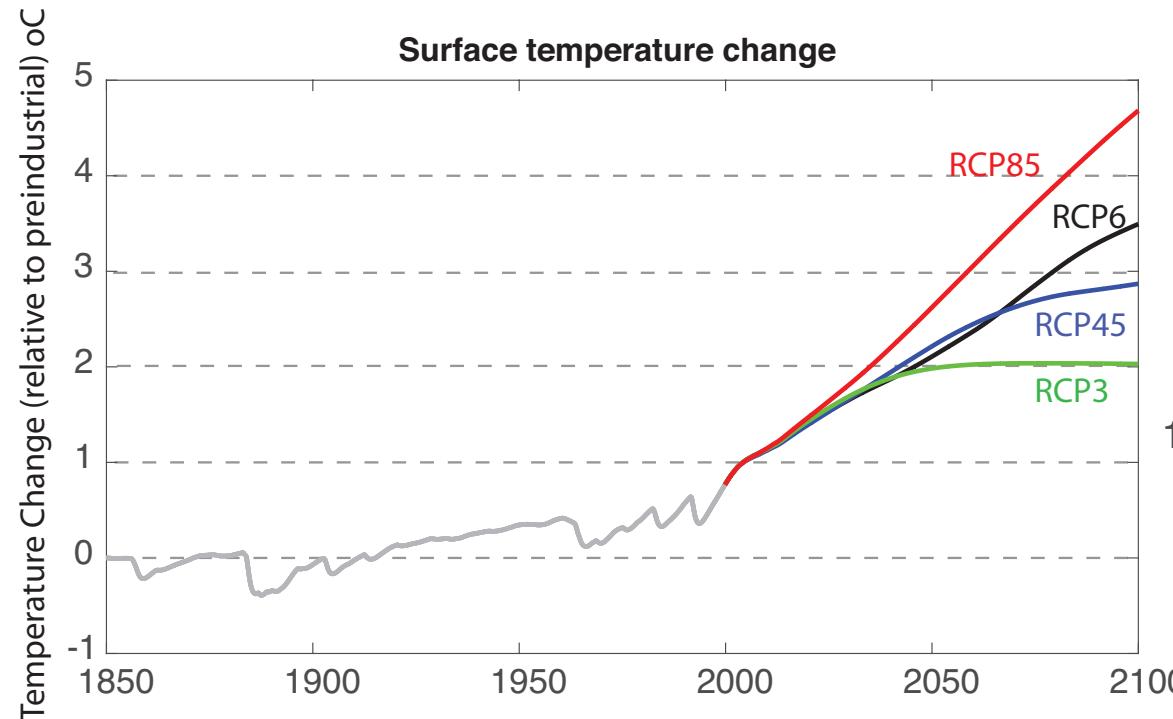
Scenario of possible future emissions



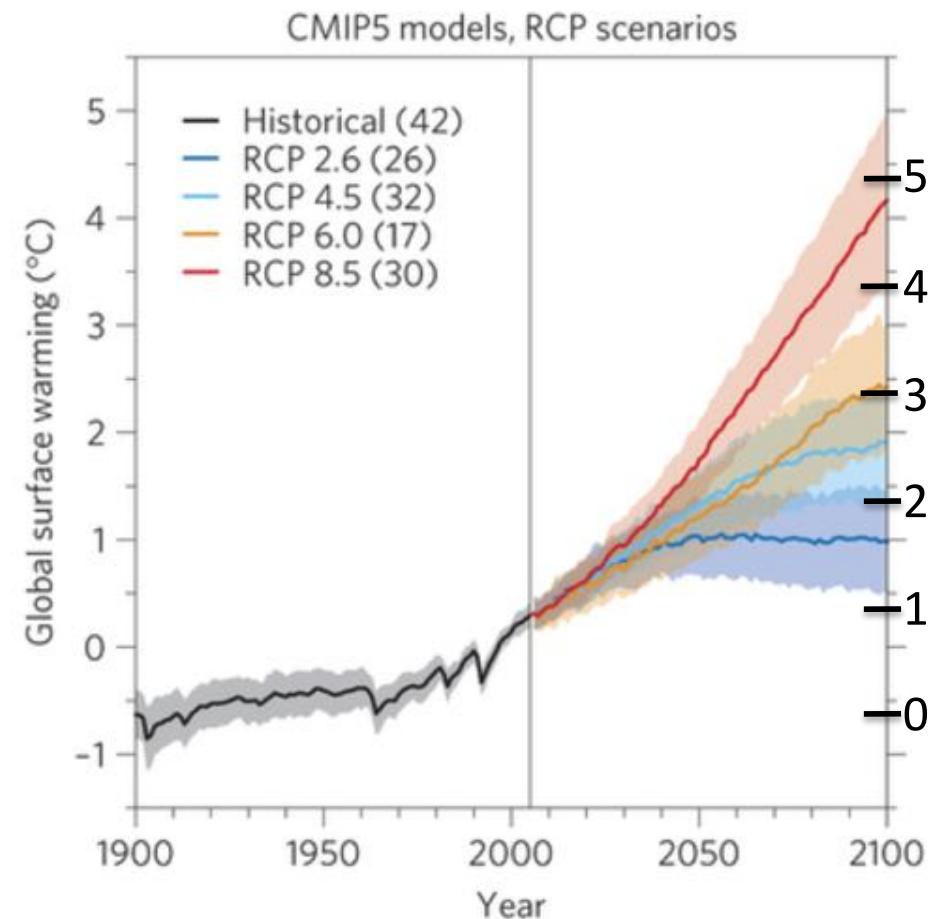
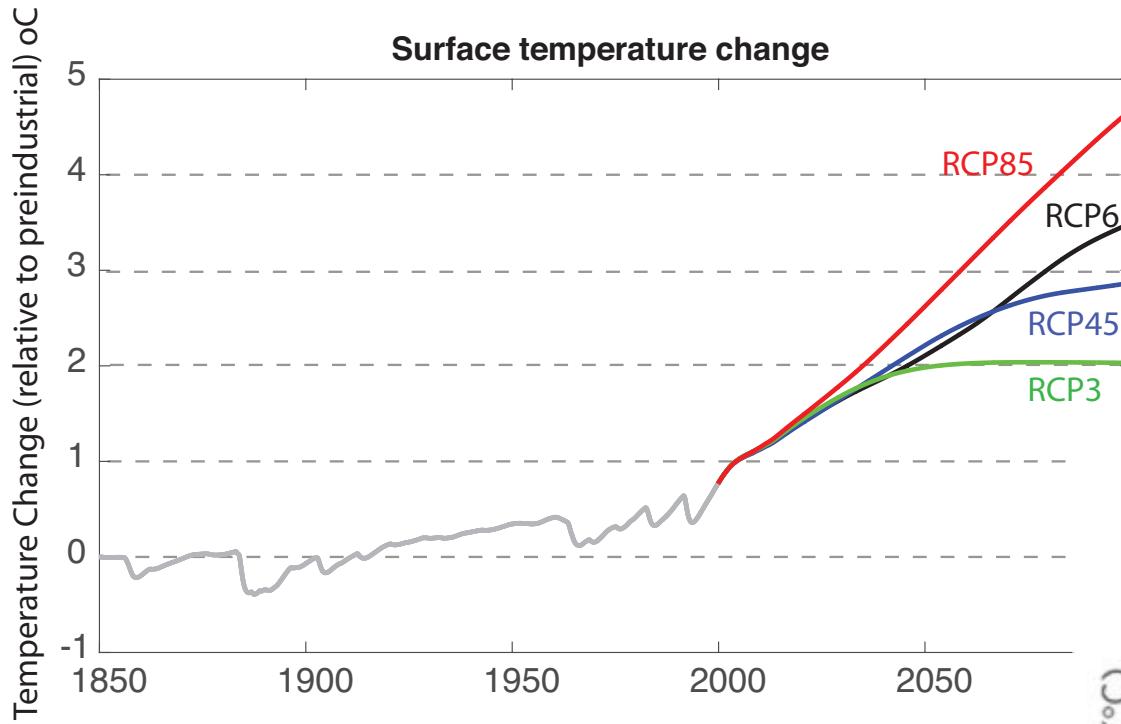
## Using the model: Projections



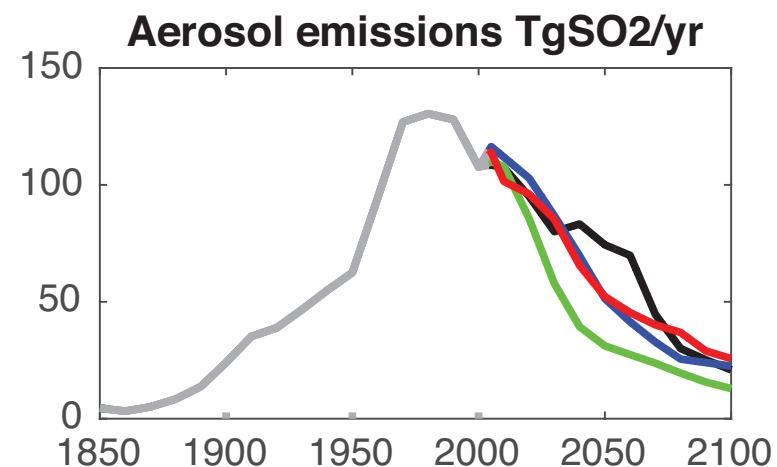
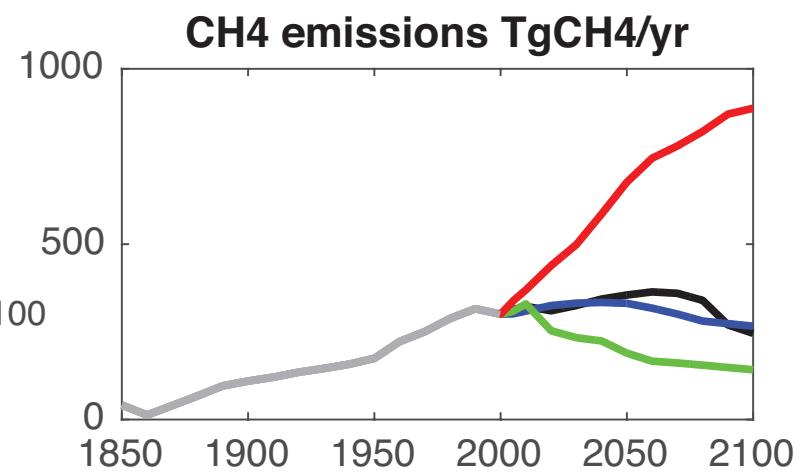
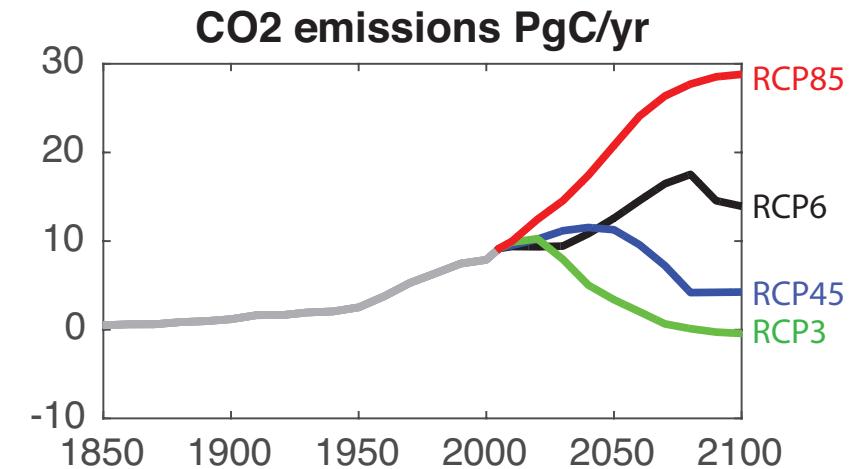
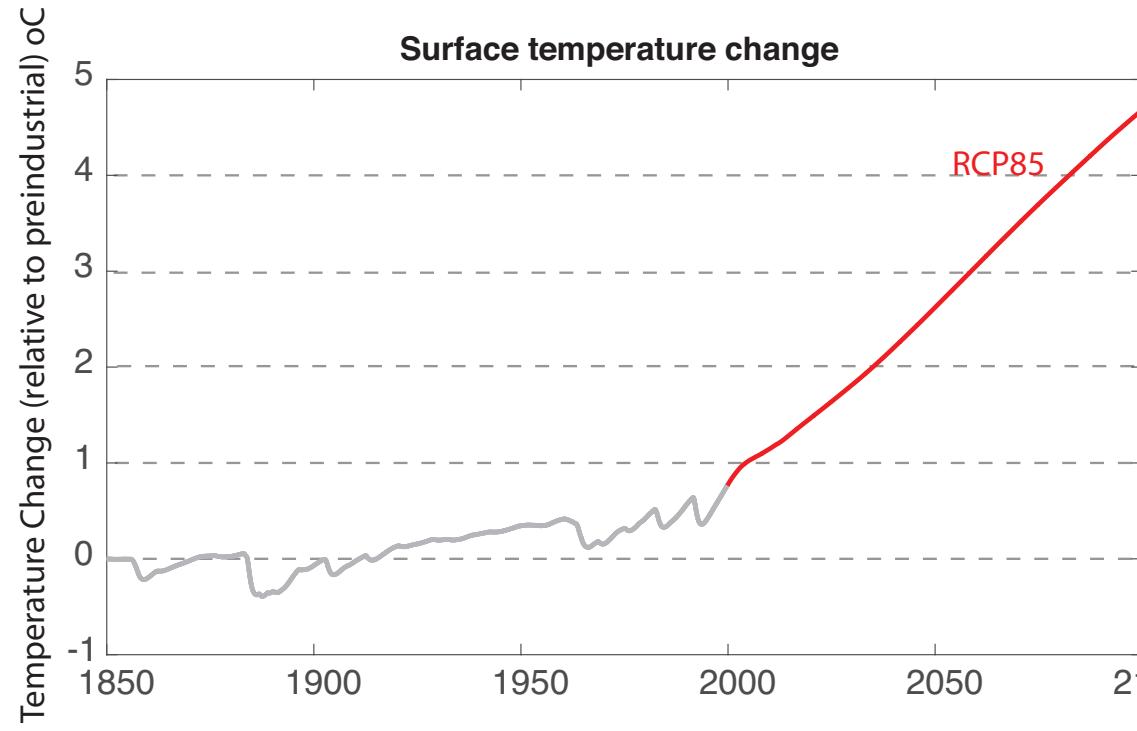
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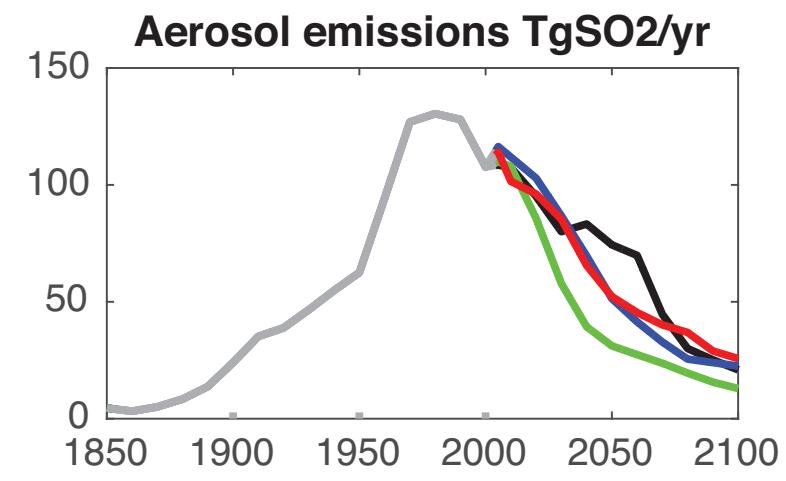
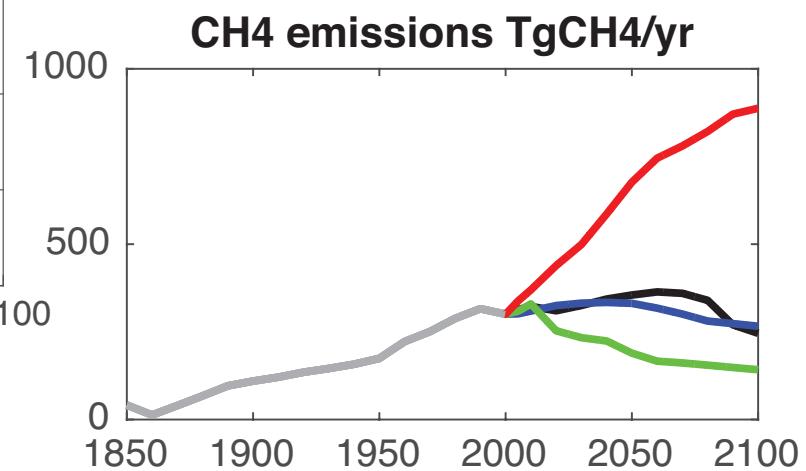
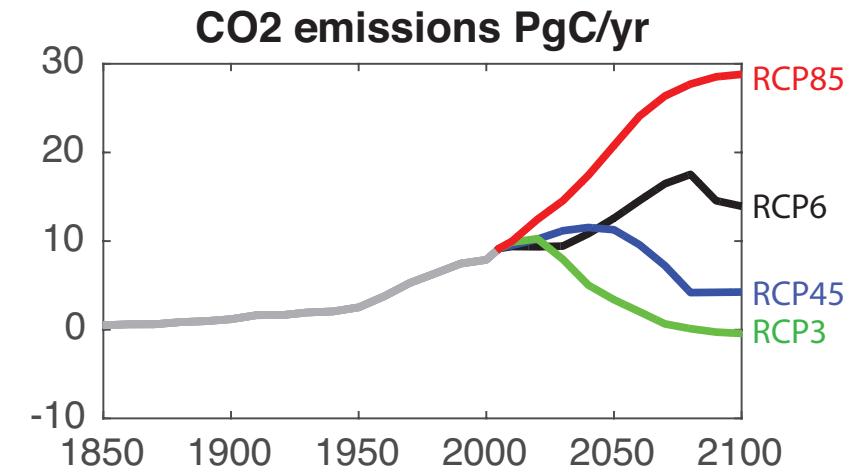
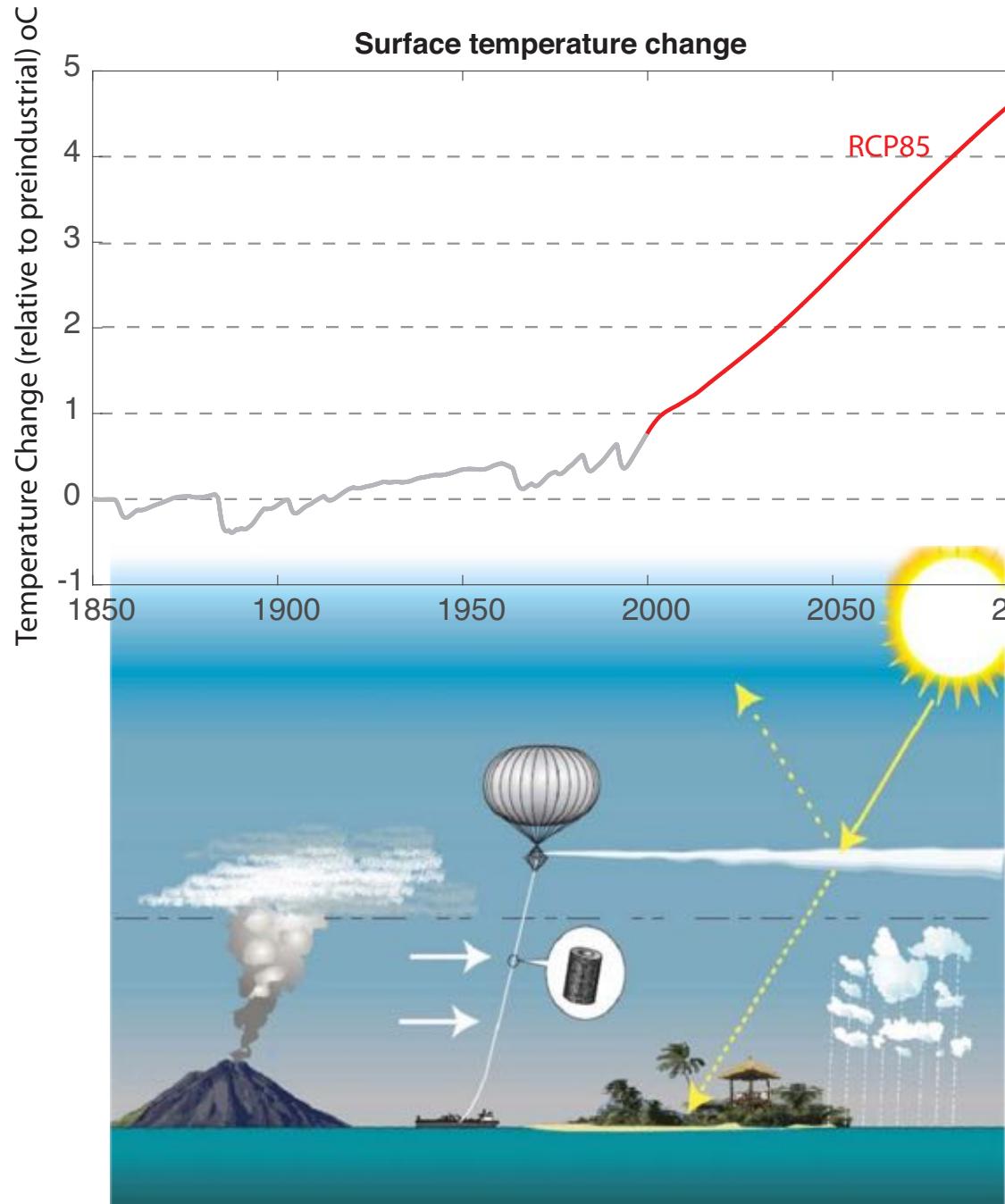
## Using the model: Projections



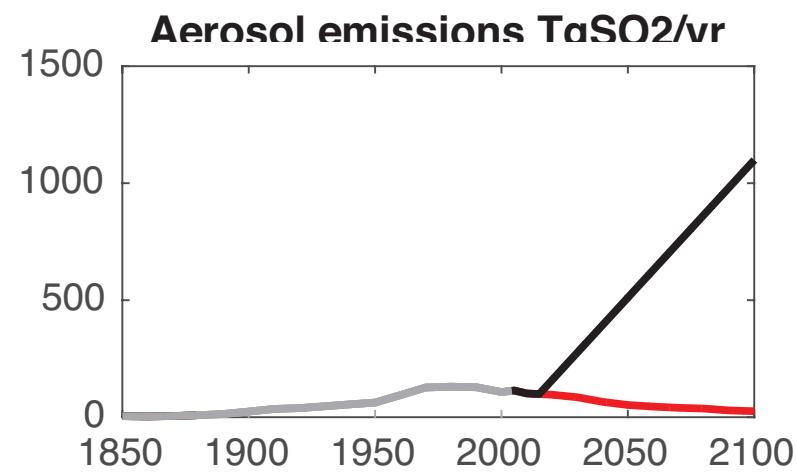
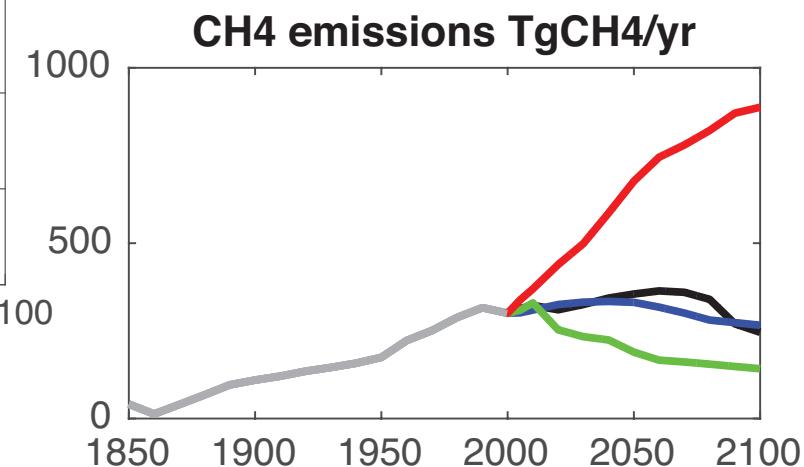
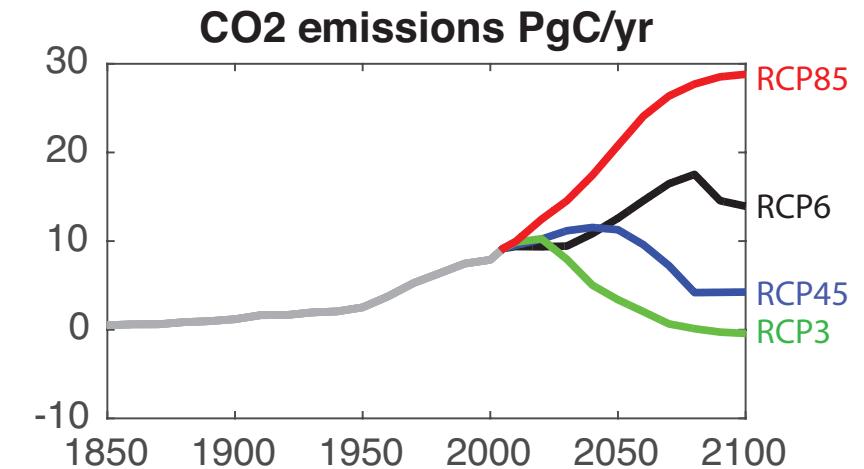
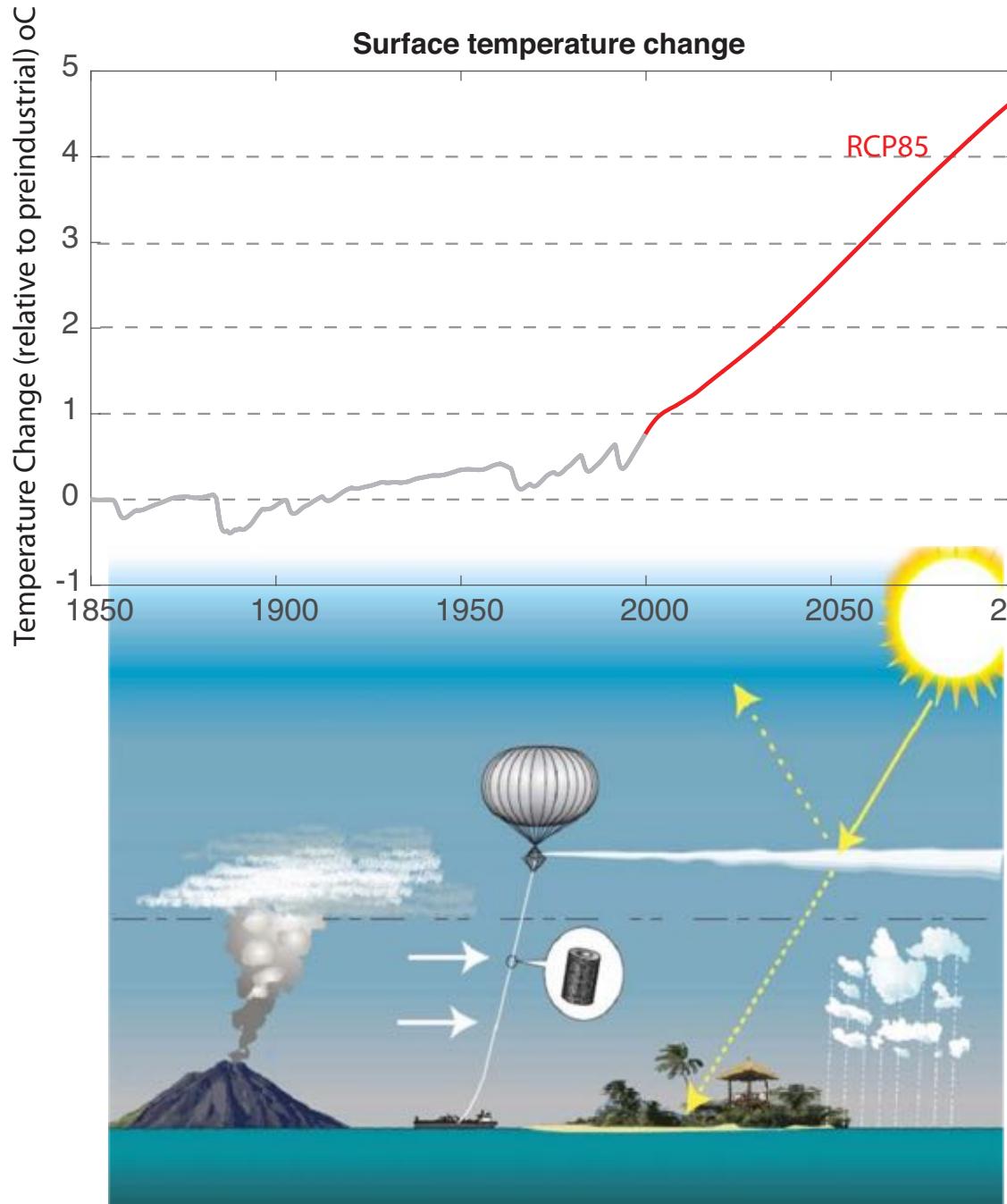
## Using the model: Geoengineering



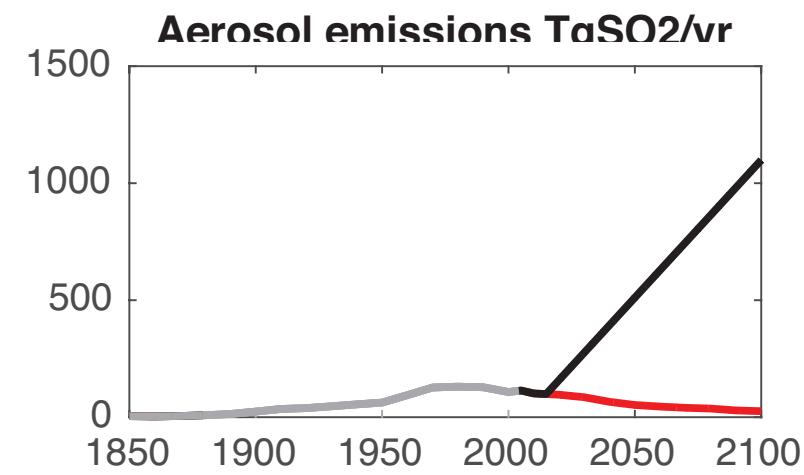
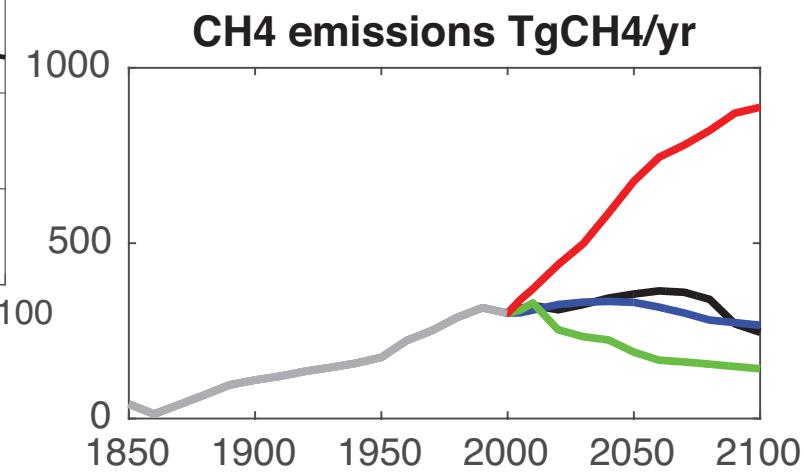
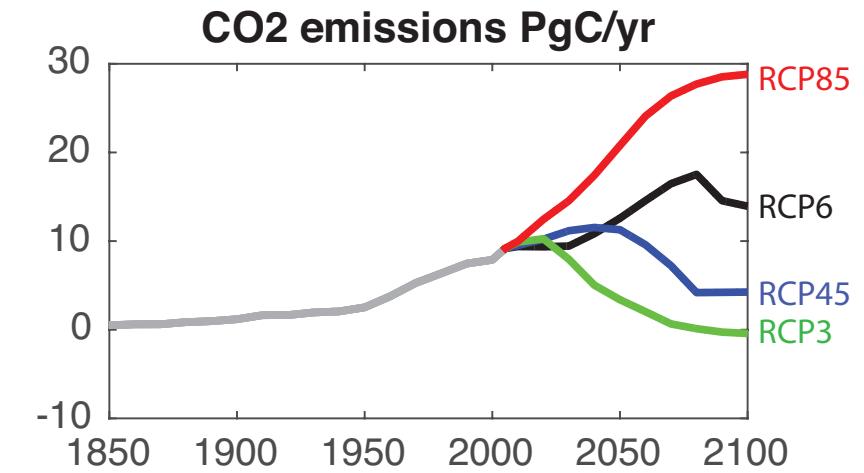
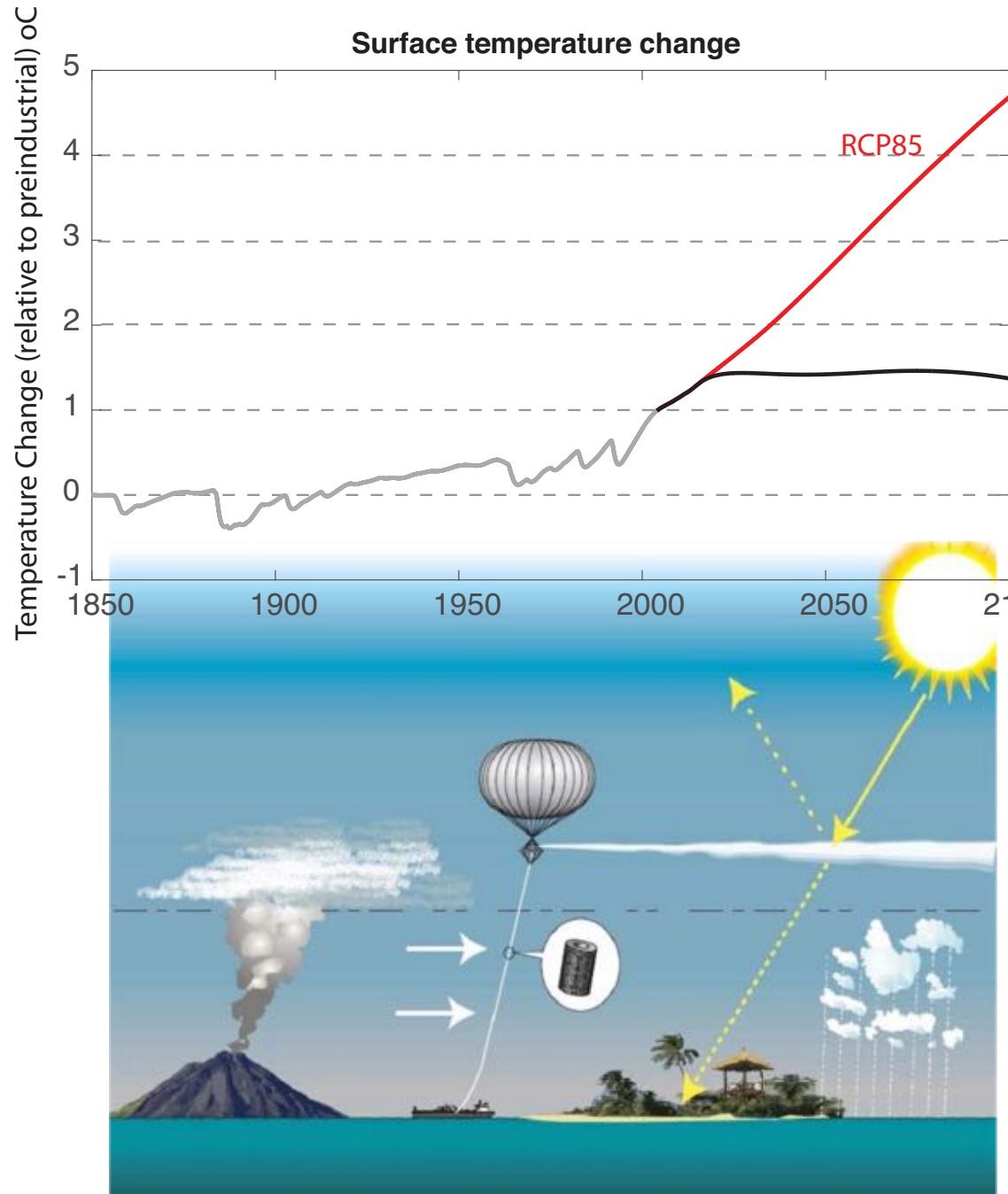
## Using the model: Geoengineering



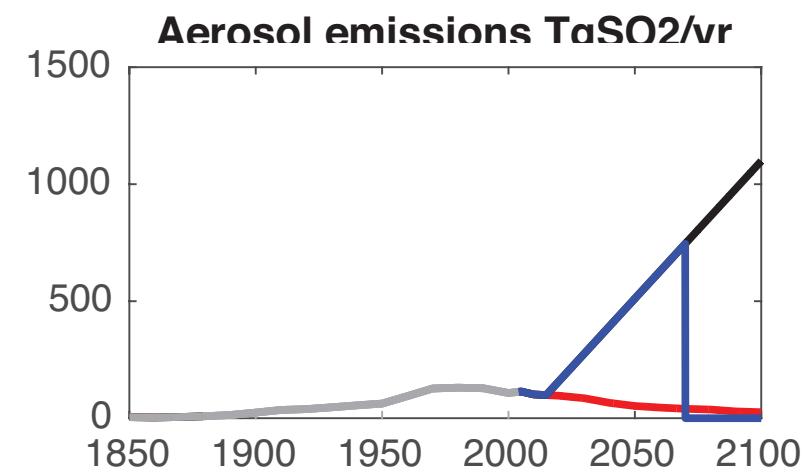
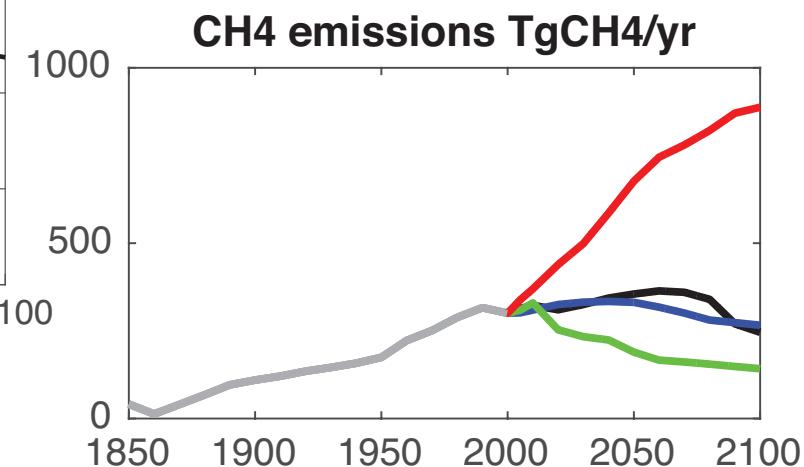
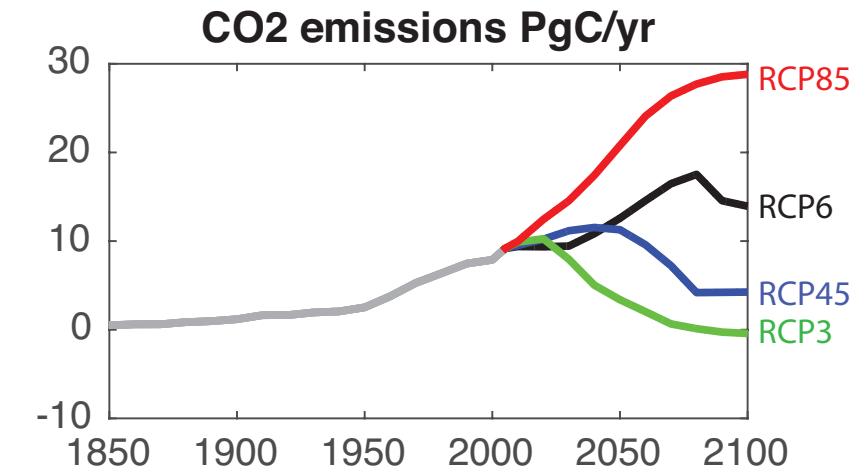
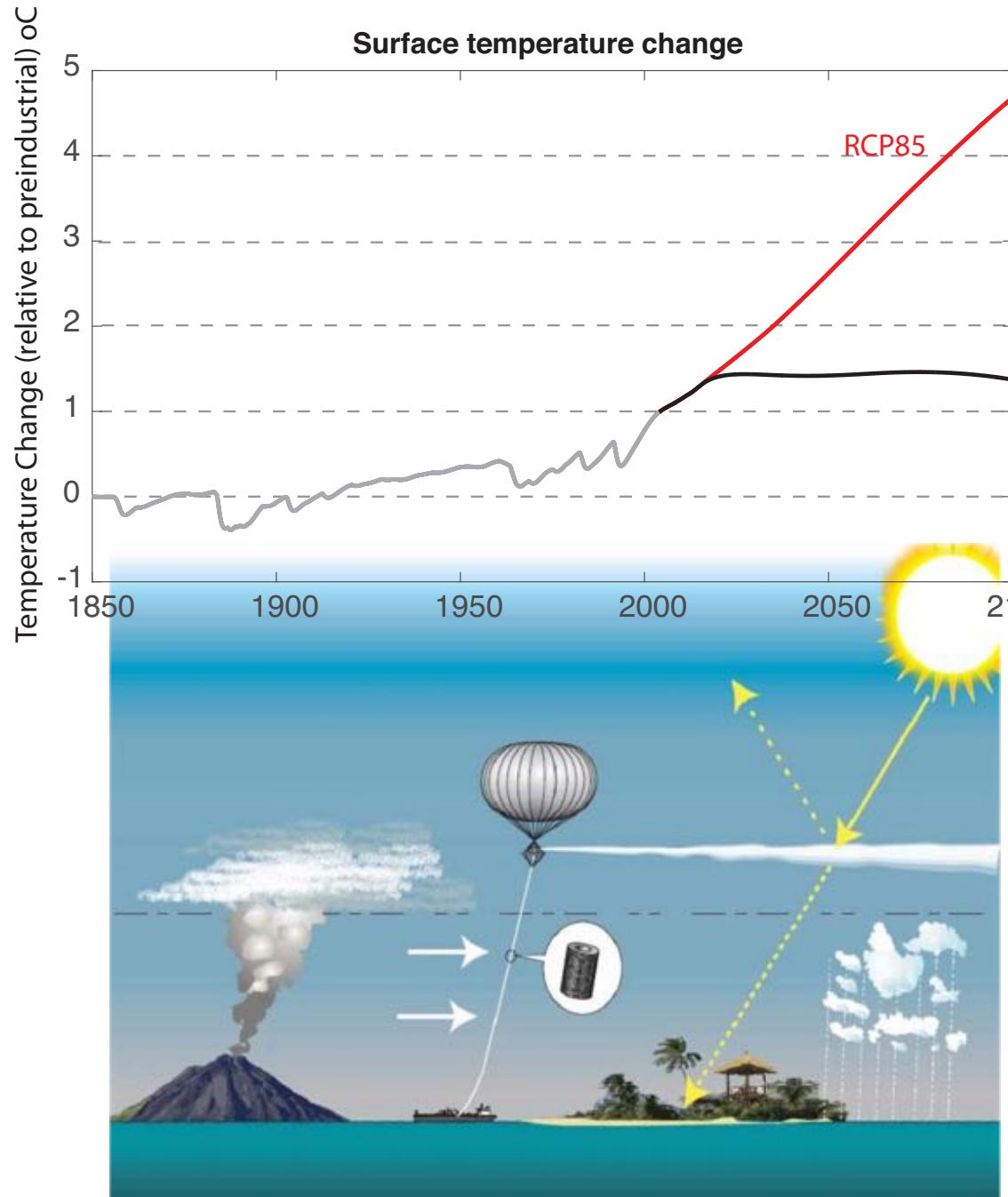
# Using the model: Geoengineering



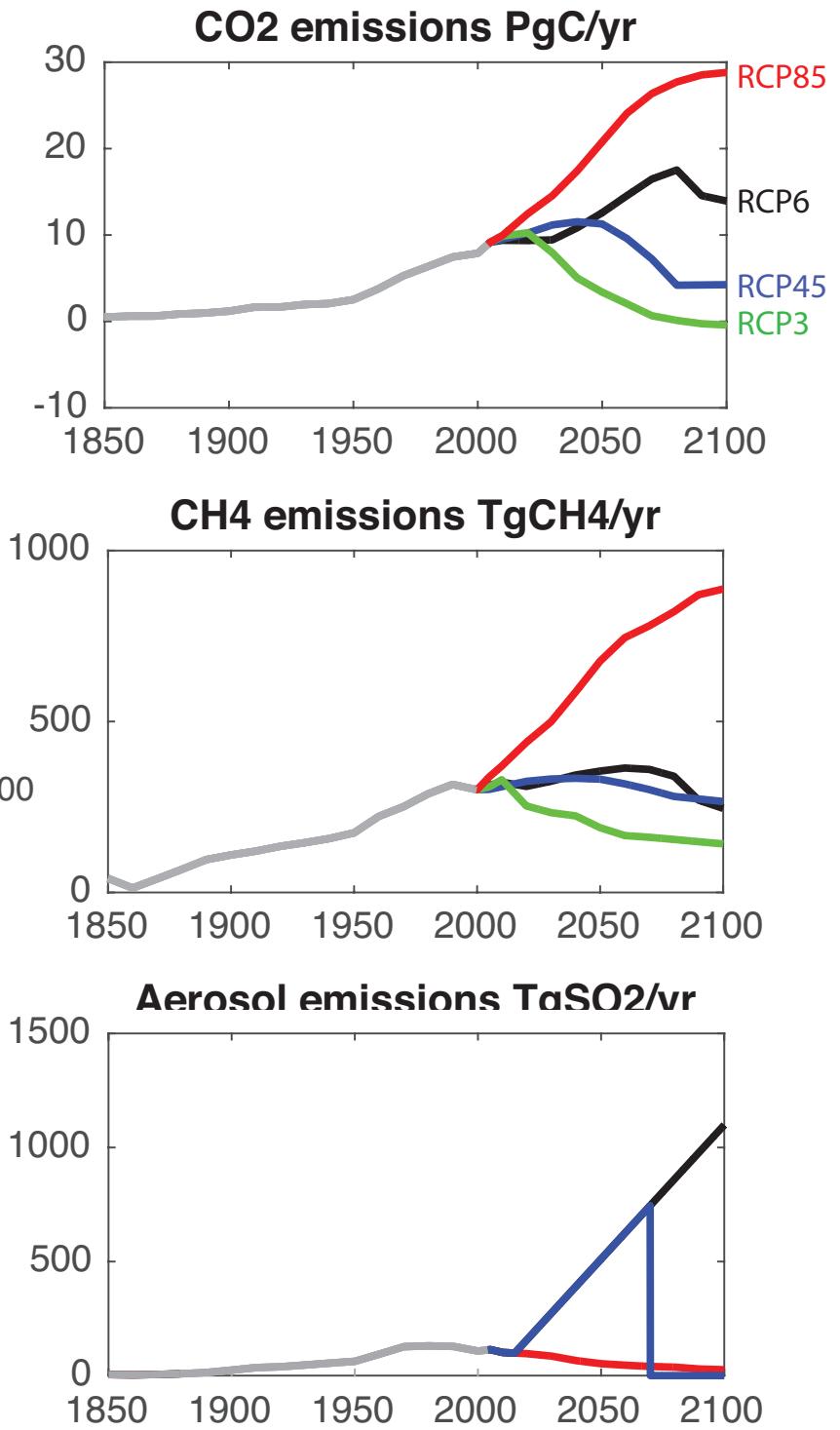
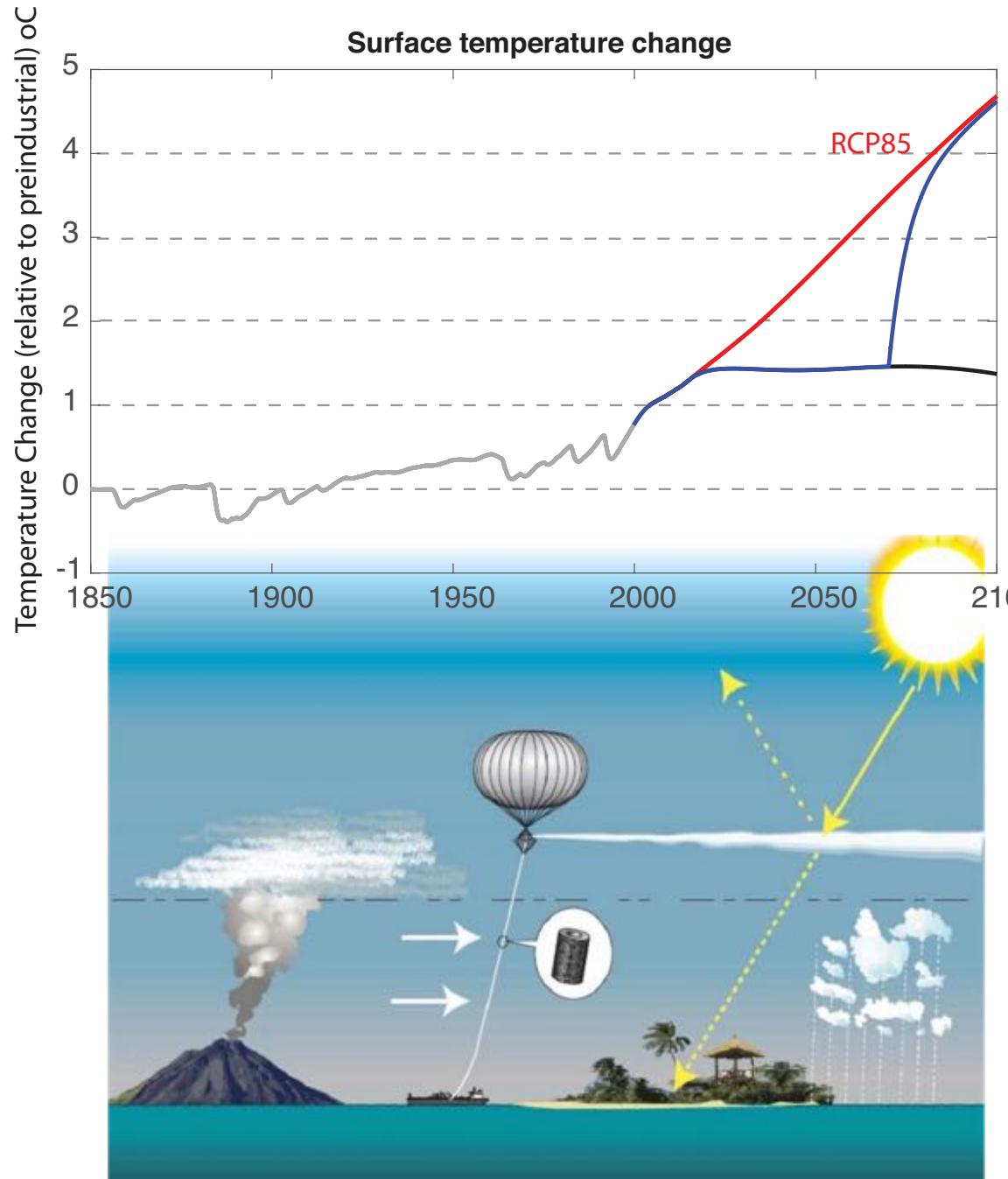
## Using the model: Geoengineering

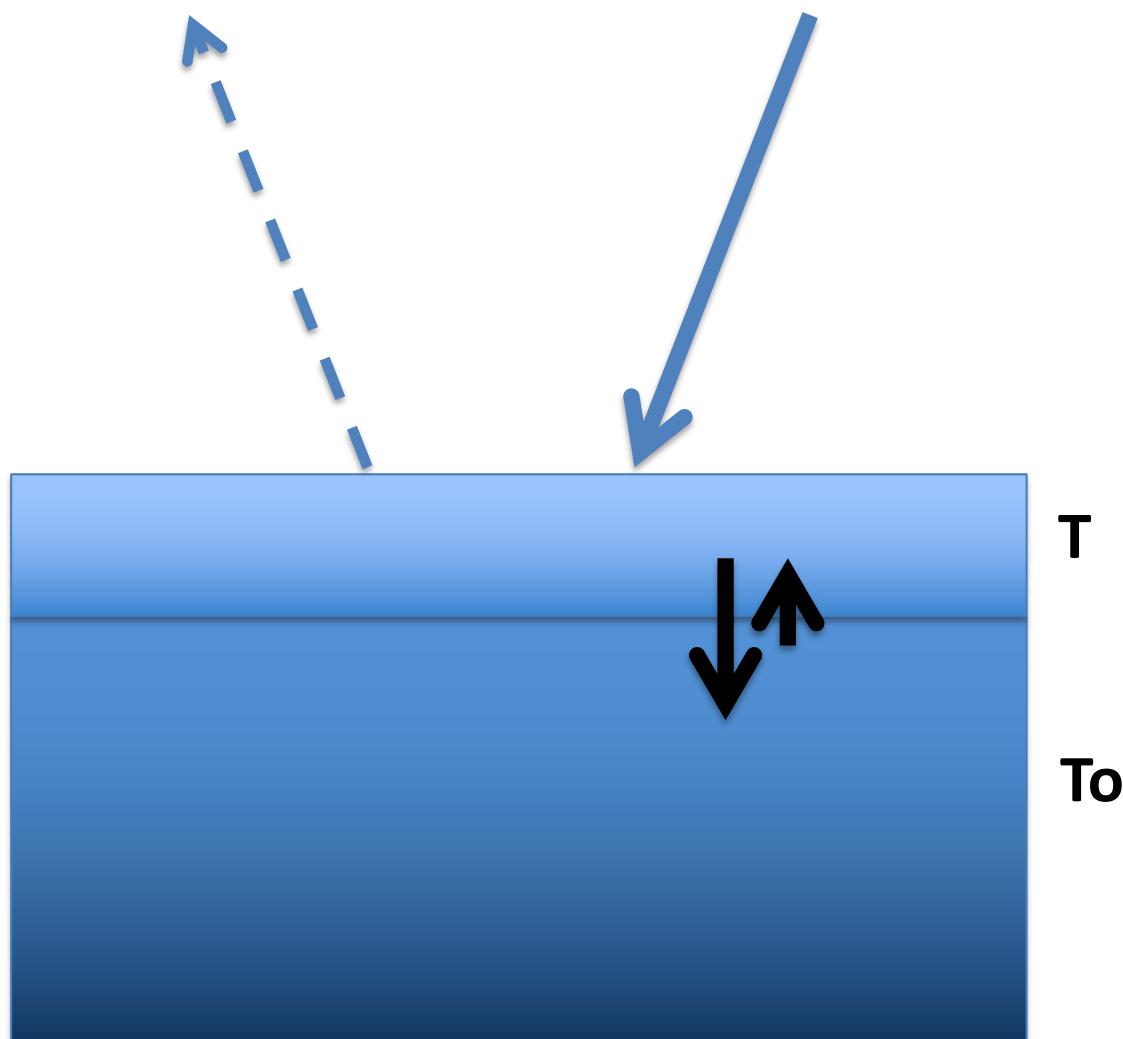


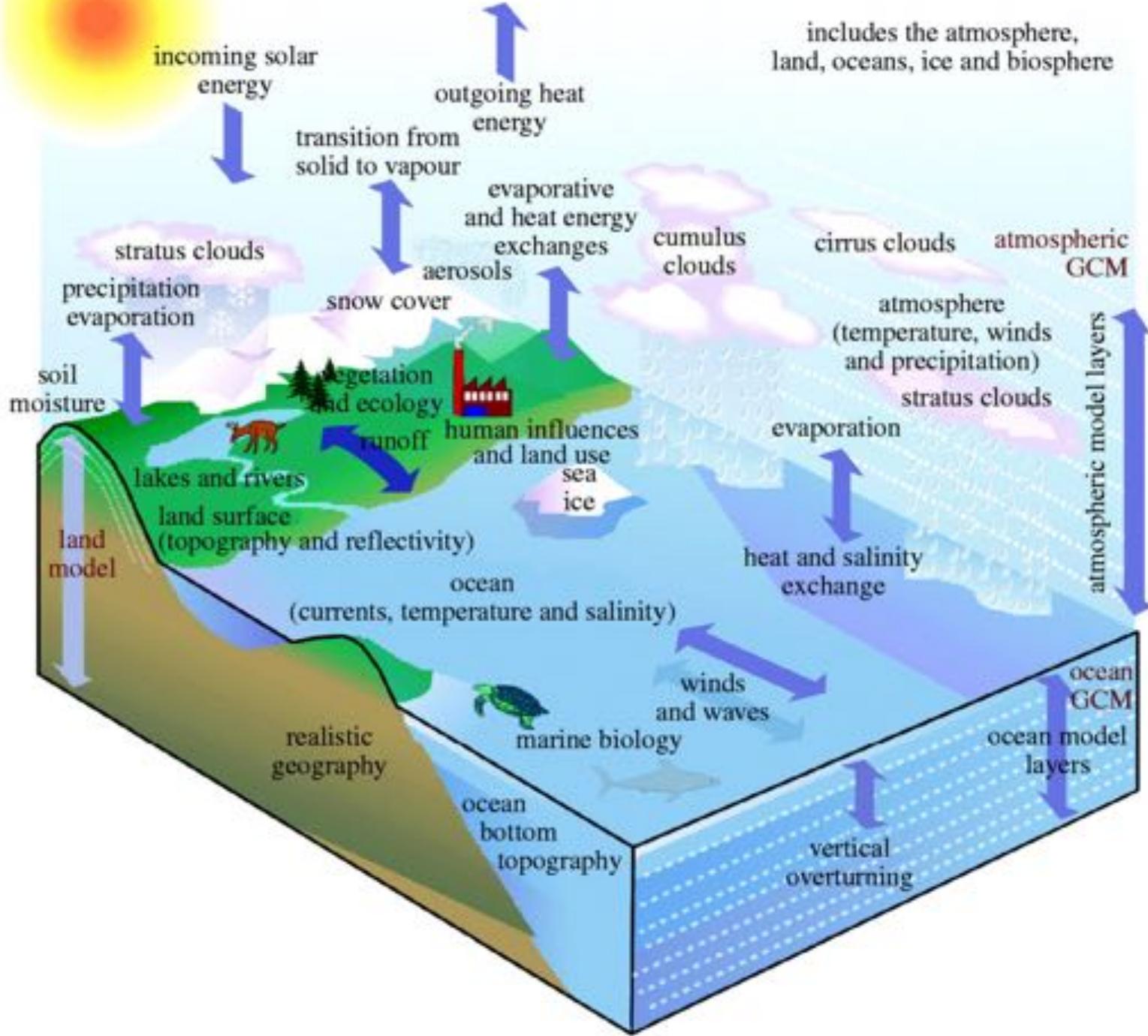
## Using the model: Geoengineering



## Using the model: Geoengineering







## Governing equations of the Ocean

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + A_H \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 u}{\partial z^2} \right)$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} + A_H \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 v}{\partial z^2} \right)$$

$$\frac{Dw}{Dt} + hu = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + A_H \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 w}{\partial z^2} \right)$$

$$f = 2\Omega \sin \theta, \quad h = 2\Omega \cos \theta$$

$$\frac{DS}{Dt} = K_s \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} \right)$$

Conservation of salt

$$\rho C_p \frac{D\theta}{Dt} = K_\theta \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + Q$$

Conservation of energy

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of mass

$$\rho = \rho(T, S, p)$$

Equation of state

## Governing equations of the Ocean

Acceleration      Coriolis      Pressure  
gradients      friction      friction

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + A_H \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 u}{\partial z^2} \right)$$

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Conservation of energy

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of mass

$$\frac{D}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

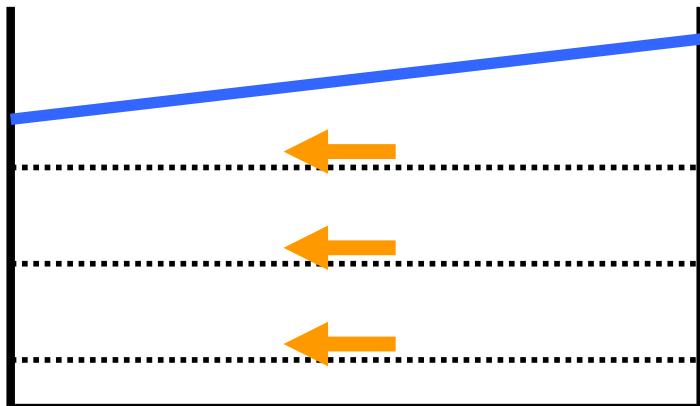
F=ma

## Governing equations

$$\frac{Du}{Dt} - \cancel{\dot{v}} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + A_H \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 u}{\partial z^2} \right)$$

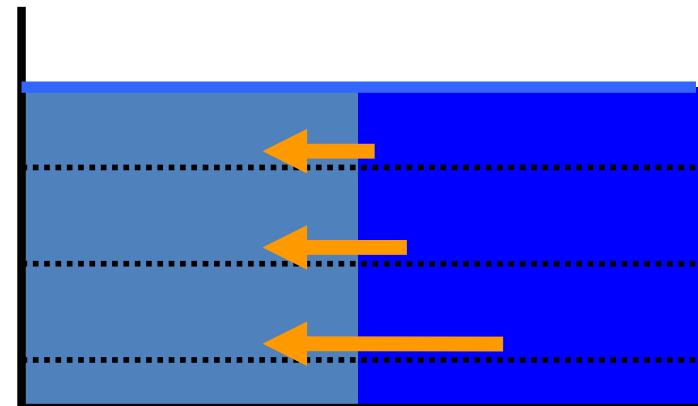
Pressure force:

$$\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$



$$\rho_0$$

Motion due to surface slopes



$$\rho_1 < \rho_2$$

Motion due to density differences

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho} \cancel{\frac{\partial p}{\partial x}} + A_H \left( \frac{\partial^2 v}{\partial x^2} \cancel{\frac{\partial u}{\partial x}} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \cancel{\frac{\partial^2 u}{\partial z^2}} \right)$$



## Example #1:

A cannon is placed on a rotating disc.  
The cannonballs fly in straight lines  
since no forces act on them.

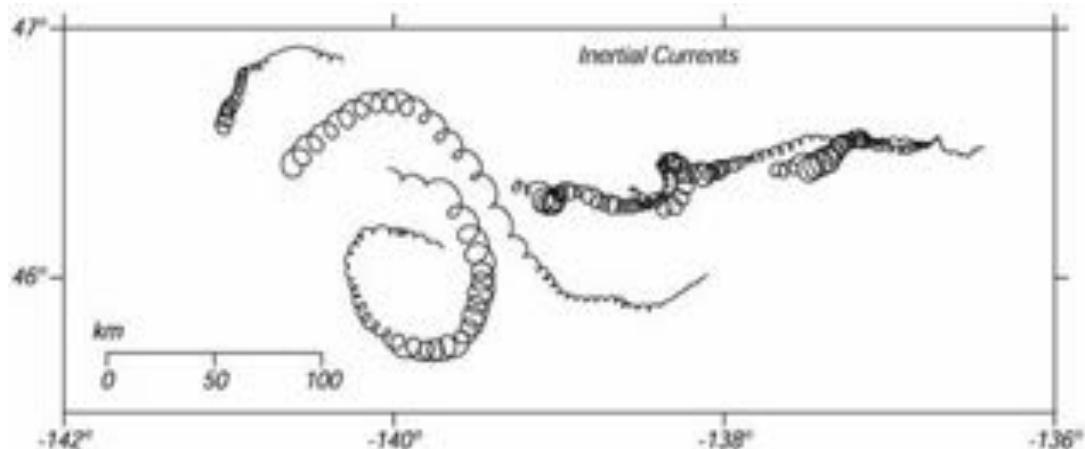
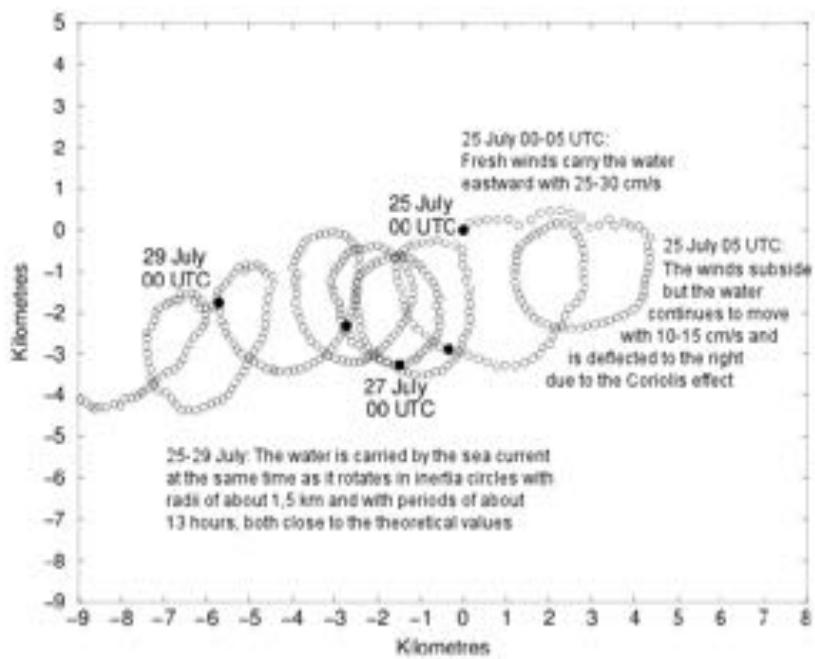
## Governing equations

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho} \cancel{\frac{\partial p}{\partial x}} + A_H \left( \cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 u}{\partial y^2}} \right) + A_Z \left( \cancel{\frac{\partial^2 u}{\partial z^2}} \right)$$

## Inertial Oscillations:

$$\frac{du}{dt} = fv, \quad \frac{dv}{dt} = -fu, \quad \text{where } f = 2\Omega \sin(\text{latitude})$$

$$u = V \sin(ft), \quad v = V \cos(ft), \quad V^2 = u^2 + v^2$$



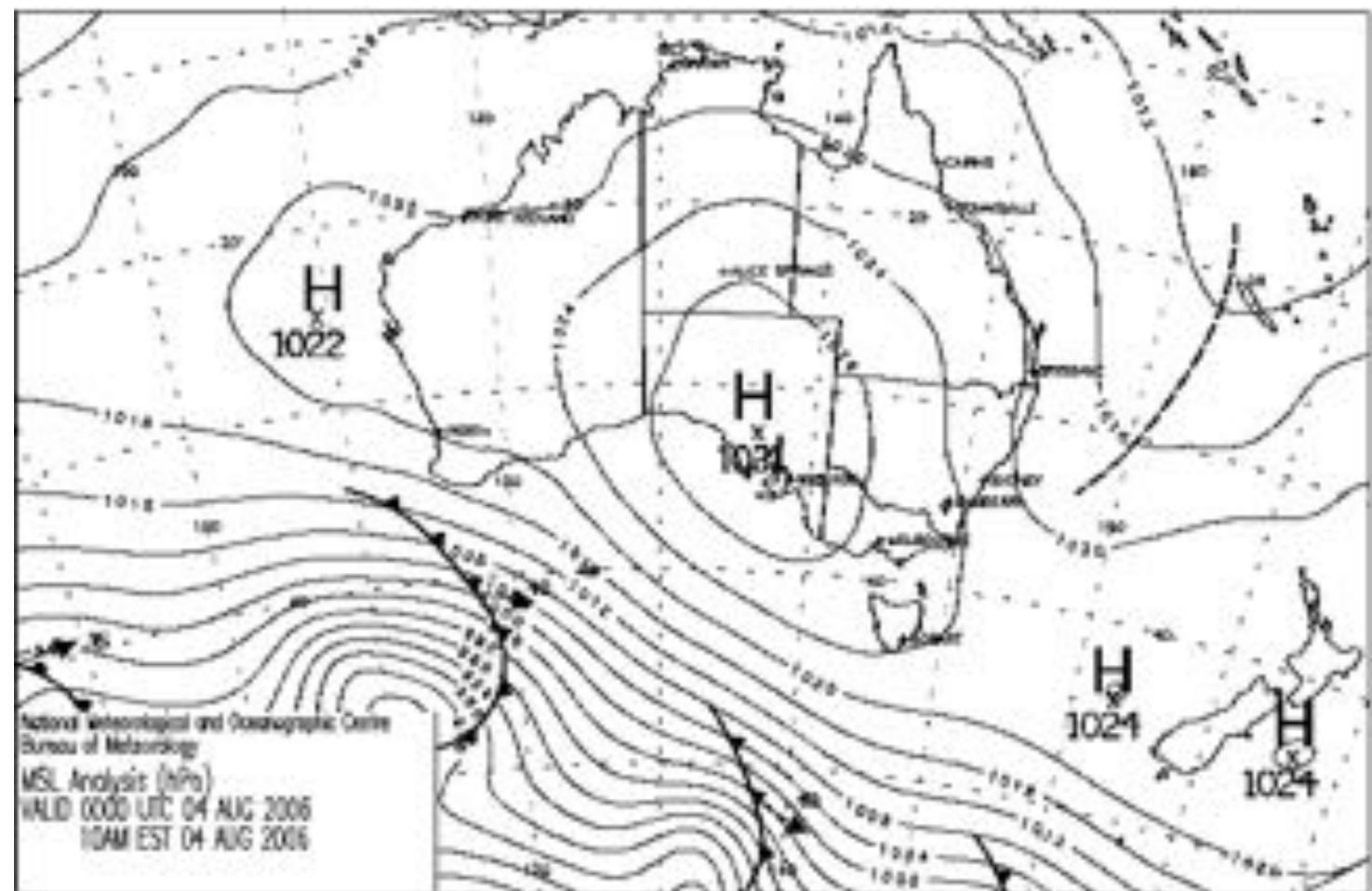
## Governing equations

Geostrophic flow:

$$\frac{D\mathbf{X}}{Dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + A_H \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 u}{\partial z^2} \right)$$

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = fv, \quad \frac{1}{\rho} \frac{\partial p}{\partial y} = -fu$$

where  $f = 2\Omega \sin(\text{latitude})$



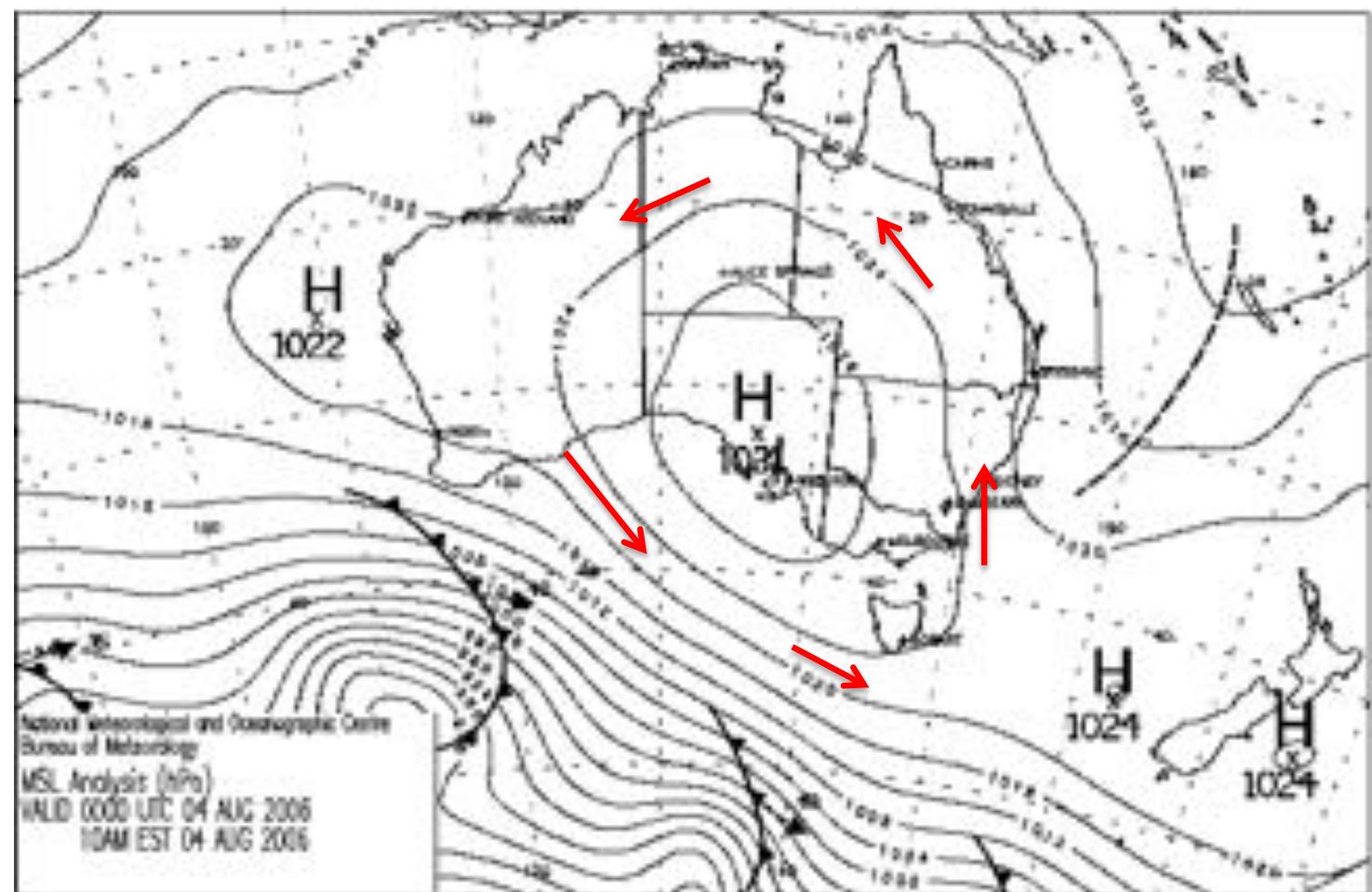
## Governing equations

Geostrophic flow:

$$\frac{D\mathbf{X}}{Dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + A_H \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 u}{\partial z^2} \right)$$

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where  $f = 2\Omega \sin(\text{latitude})$



## Governing equations of the Ocean

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + A_H \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 u}{\partial z^2} \right)$$

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$$\frac{Dw}{Dt} + hu = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + A_H \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + A_Z \left( \frac{\partial^2 w}{\partial z^2} \right)$$

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$$\frac{DS}{Dt} = K_s \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} \right)$$

Conservation of salt

$$\rho C_p \frac{D\theta}{Dt} = K_\theta \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + Q$$

Conservation of energy

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

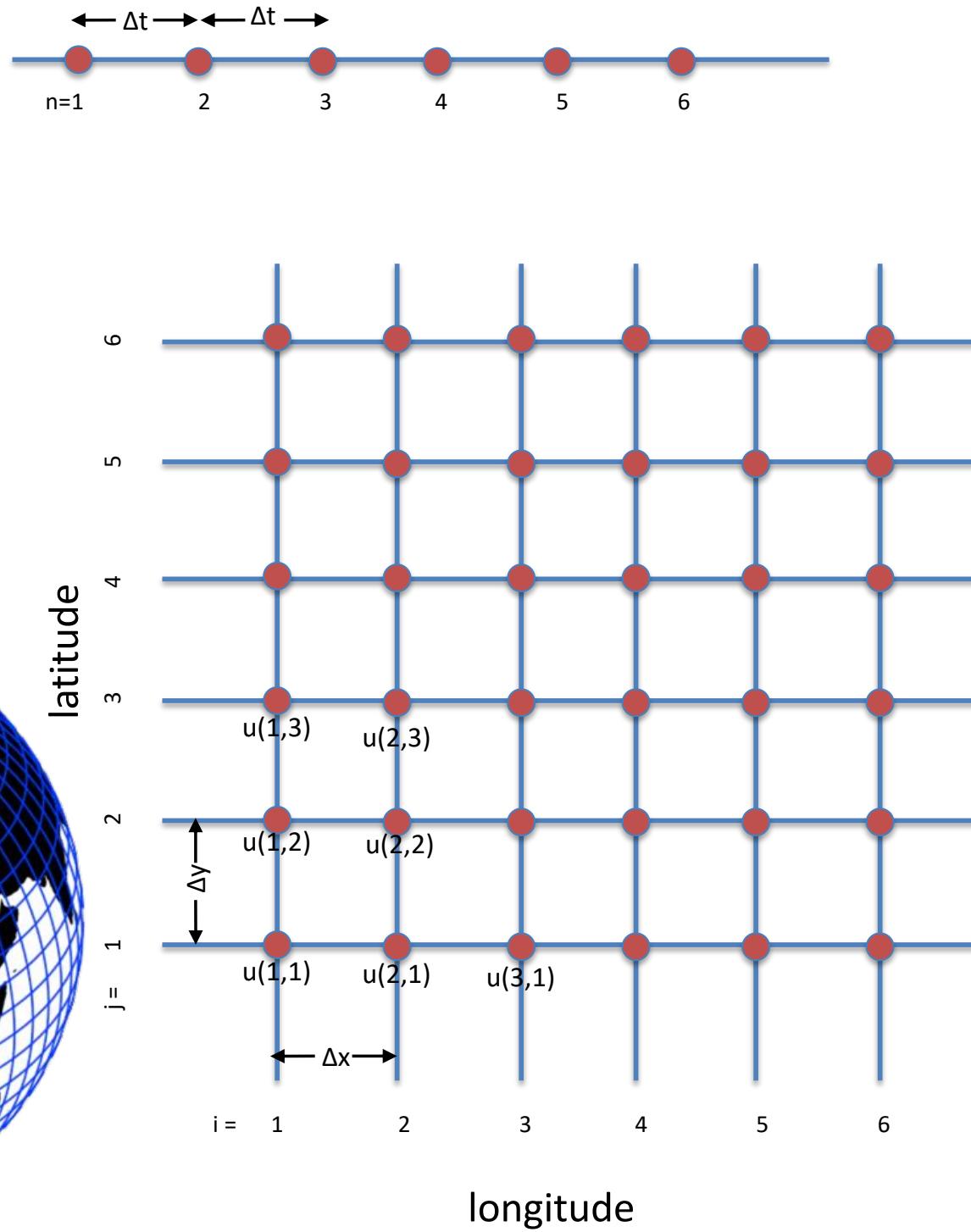
Conservation of mass

$$\rho = \rho(T, S, p)$$

Equation of state

$$\frac{du}{dt}$$

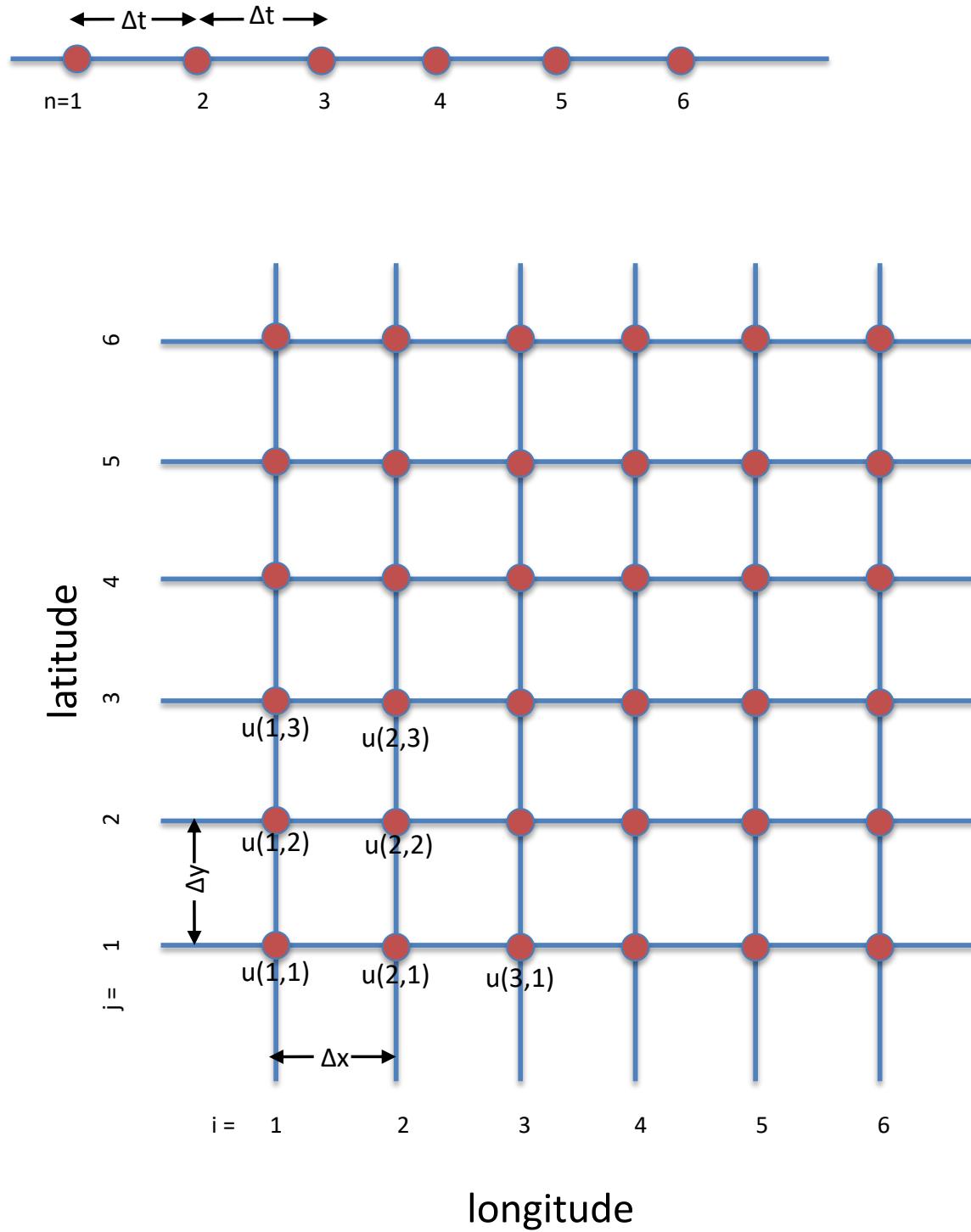
$$\frac{d^2u}{dx^2}, \quad T \frac{du}{dx}$$



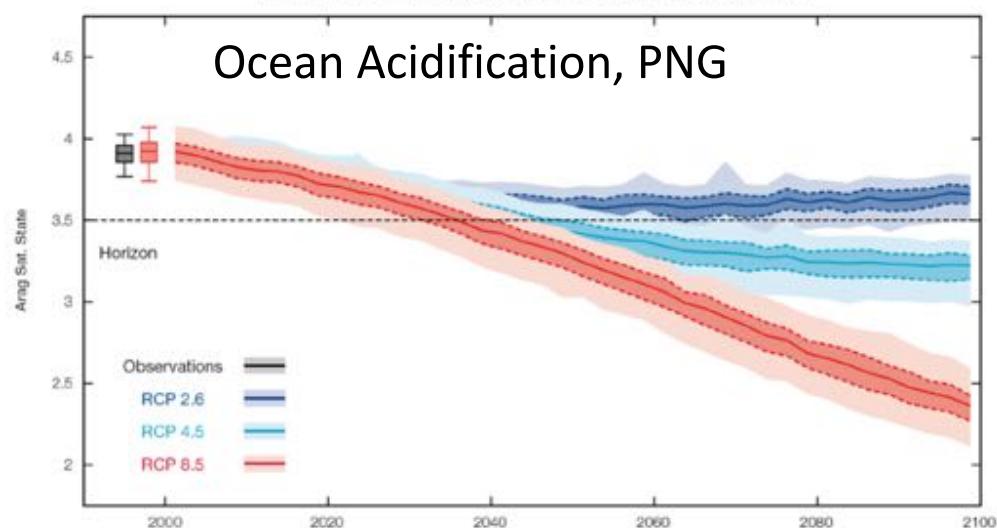
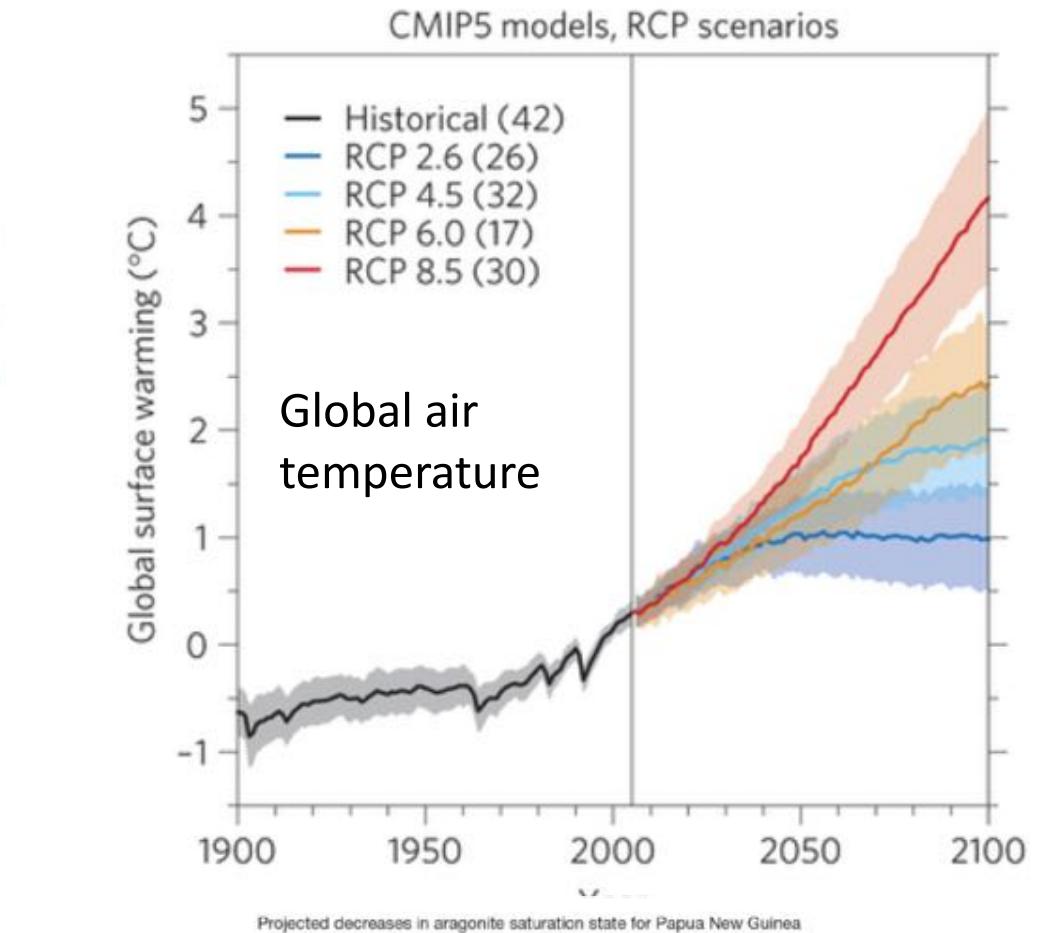
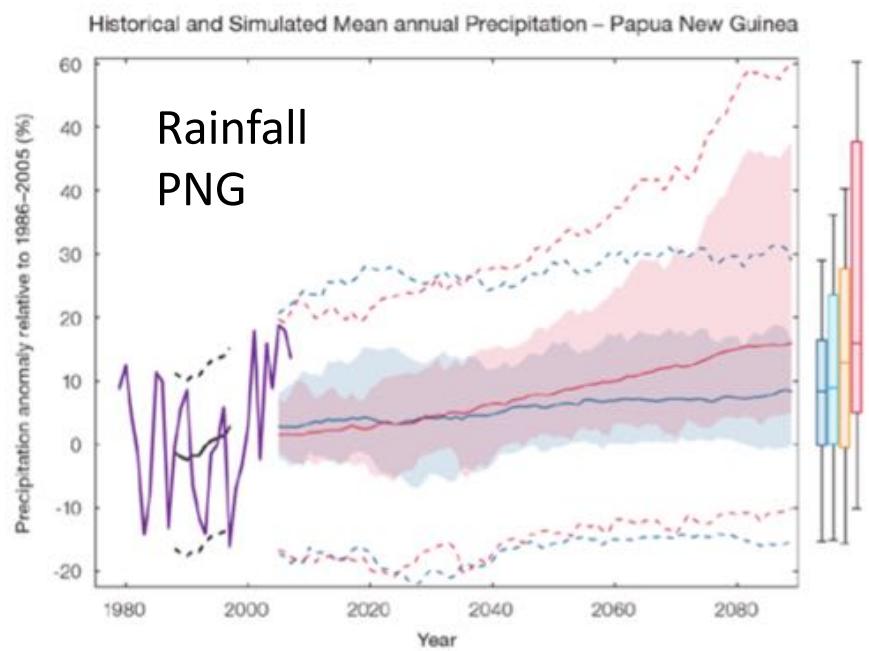
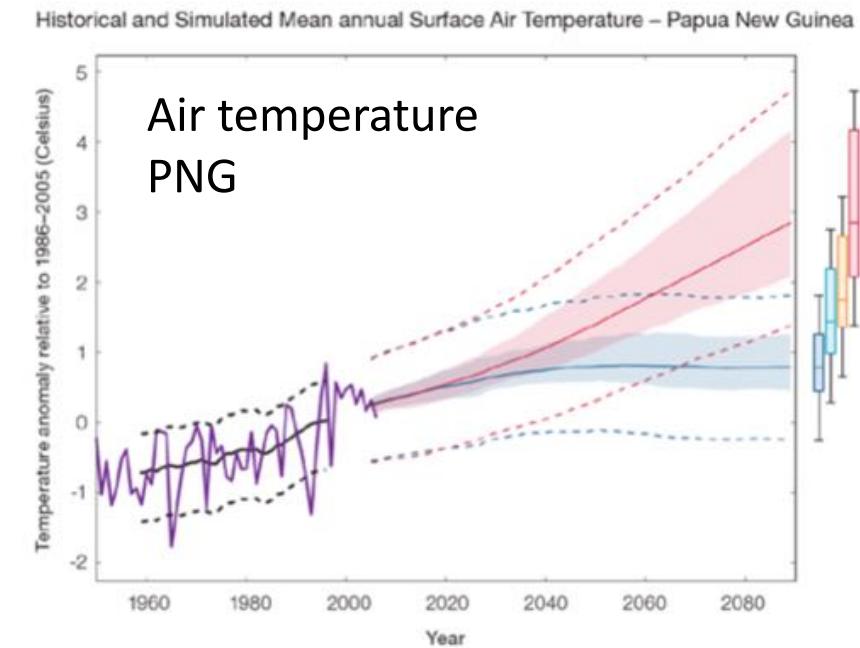
$$\frac{du}{dt}$$

$$\frac{d^2u}{dx^2}, \quad T \frac{du}{dx}$$

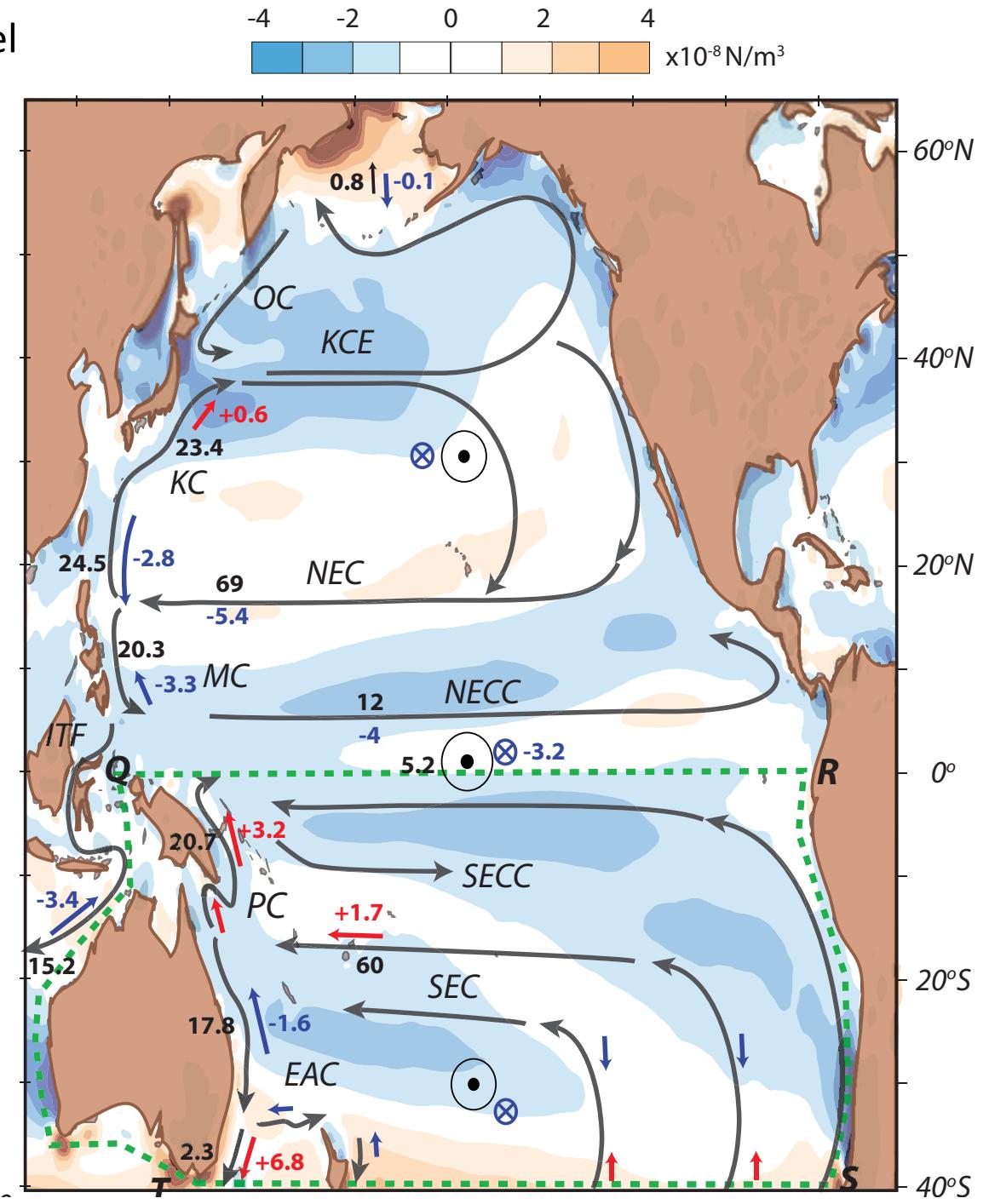
$$\frac{du}{dx} \approx \frac{u(2,1) - u(1,1)}{\Delta x}$$



## Example: Using an ocean/climate model



## Example: Using an ocean/climate model



- Simple models can capture important features of the climate system
- Can recreate past climate change (global average) and be used to look at future changes.
- More complex models (e.g. based on Navier Stokes) are needed to resolve local features and the dynamics of the climate system
- Equations usually require numerical solution. Finite difference methods discretise space and time
- Interested in modelling the climate system or researching climate change, come talk to us at the CCRC

# Analogue example to the energy balance model (bathtub with constant inflow and variable outflow of water)

```
clear
% MATLAB code with simple finite difference solution
% consider a bathtuv with a constant inflow and an outflow that
varies
% depending on the depth (or equivalently volume) of water
% $\frac{dV}{dt} = \text{input} - \text{output}$ 
%where output is proportional to the volume (or depth)

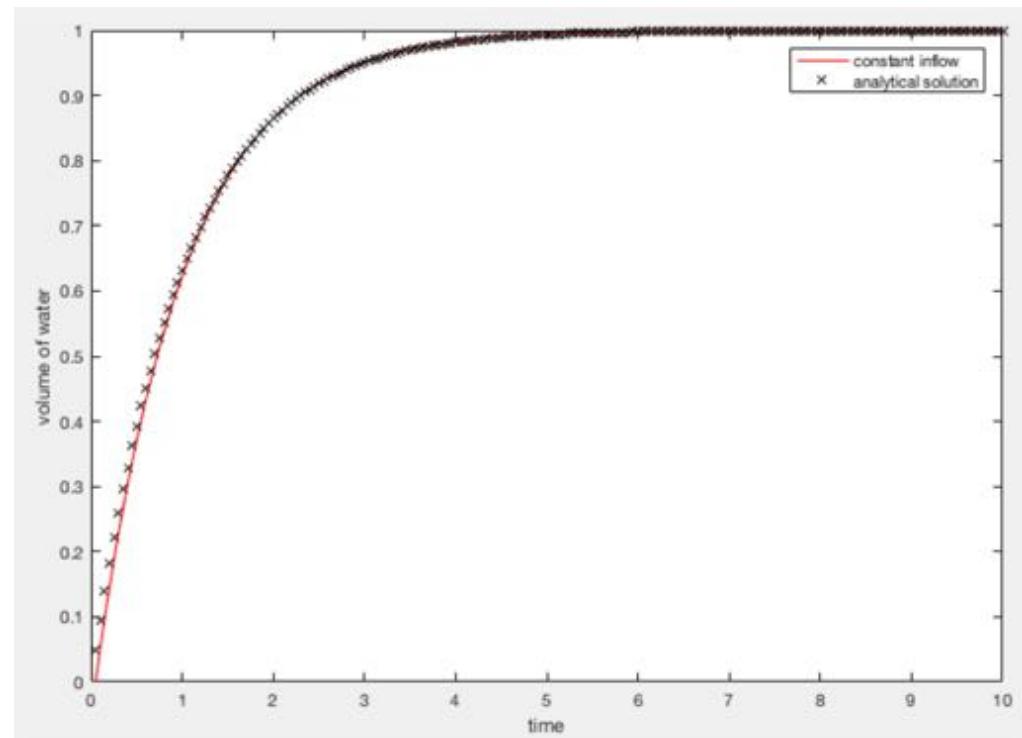
inp = 1; %litre/second
N=200; % no. of timesteps
DT=0.05; %time step [seconds]
time=(1:N)*DT; % vector of times [seconds]
const=1; %constant of proportionality

V(1)=0; %initial condition; depth at t=0

% numerical Solution
for t=2:N
V(t) = V(t-1) +DT*( inp - const*V(t-1) );
end

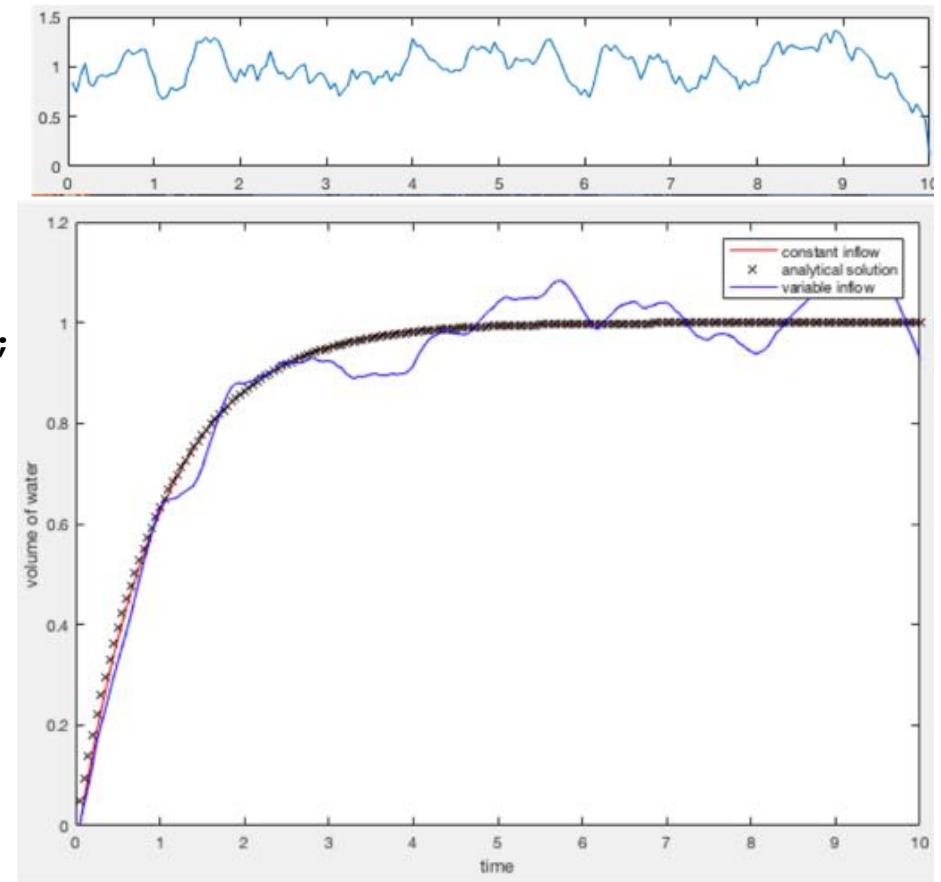
figure(1);clf
plot(time , V)
hold on

% Analytic solution
Vanalytic = inp - exp(-time);
plot(time,Vanalytic, 'kx')
```



# Analogue example to the energy balance model (bathtub with constant inflow and variable outflow of water)

```
%% what about a time varying input  
inp2=smooth(rand(1,N),10)*2;%generate random inflow  
timesereies  
  
figure(2);clf  
plot(time,inp2);  
  
v2(1)=0; %depth at t=0  
  
% numerical Solution  
for t=2:N  
v2(t) = v2(t-1) +DT*( inp2(t-1) - const*v2(t-1) );  
end
```



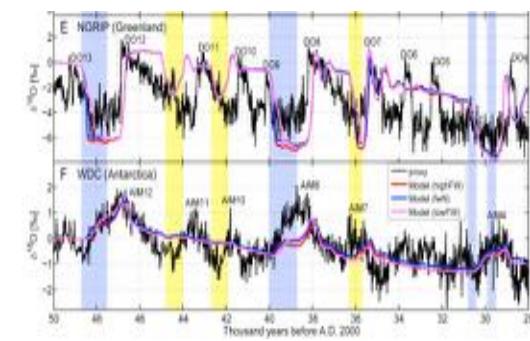
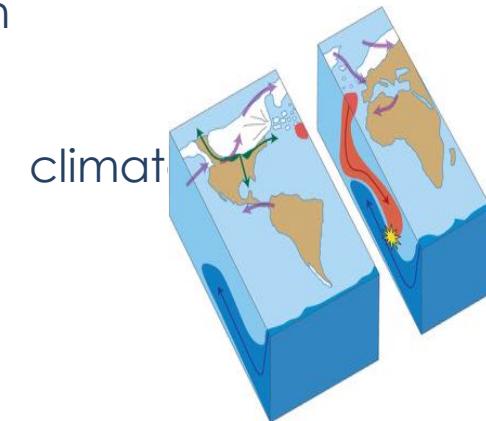
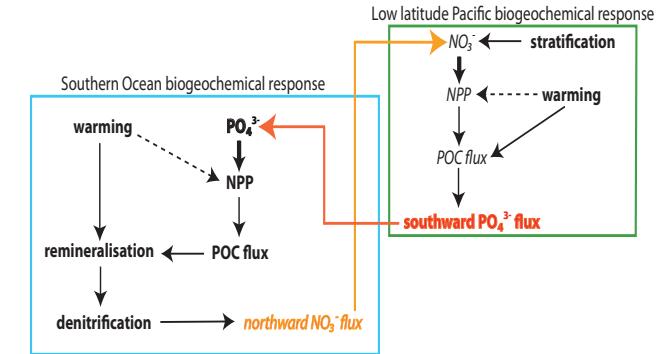
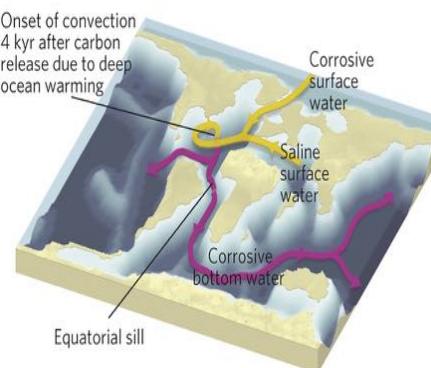
# A. Prof Katrin Meissner



Director Climate Change Research

Centre

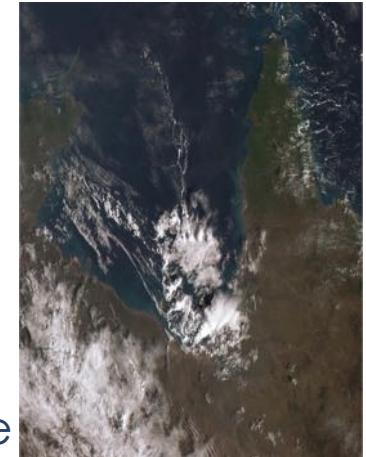
- Climate change and variability
- Feedbacks between components
- Ocean circulation
- Carbon cycle
- Biogeochemical cycles
- Past climate change
- Isotopes



# Prof Steven Sherwood



ARC Laureate Fellow  
Deputy Director Climate Change Research Centre



## Research profile:

- Atmospheric water, cloud and convective processes
- Atmospheric radiation and thermodynamics
- Climate change and feedbacks
- Climate data analysis and homogenisation
- Heat stress and climate adaptability

## Current group topics:

- Understanding the mid-Holocene “Green Sahara”
- Low-cloud feedbacks on global temperature
- Convective triggering and persistence
- Fluctuation-dissipation approaches to cloud-climate relationships
- Aerosol impacts on precipitating clouds

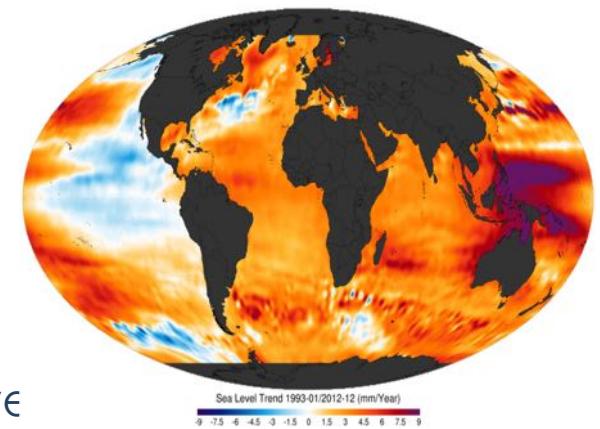
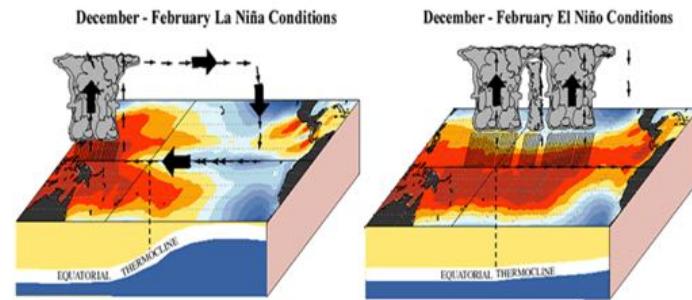
Himawari 8 image showing triggering of cloud lines over Gulf of Carpentaria

# Scientia Prof Matthew England



Scientia Professor of Climate Dynamics  
CCRC and ARC Centre of Excellence in Climate System  
Science (ARCCSS)

- Antarctic climate
- The Southern Ocean
- El Niño and tropical modes
- Ocean heat uptake and sea-level
- Ocean drivers of climate extremes (floods, drought)

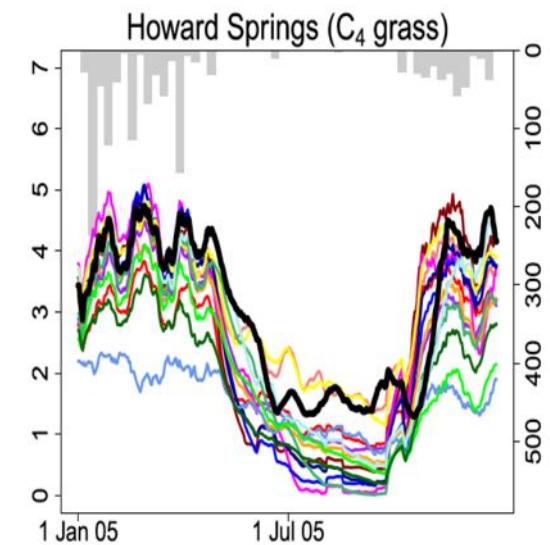
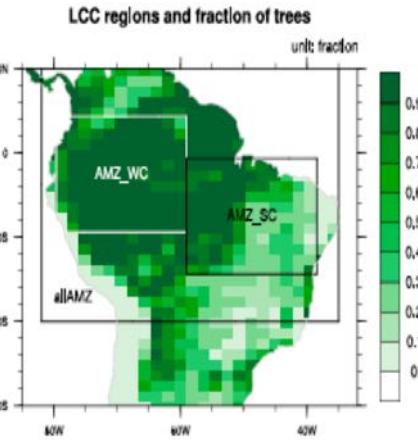


# Prof Andy Pitman



Director ARC Centre of Excellence for Climate System Science (ARCCSS)  
Director ARC Centre of Excellence for Climate Extremes (ARC CLEX)

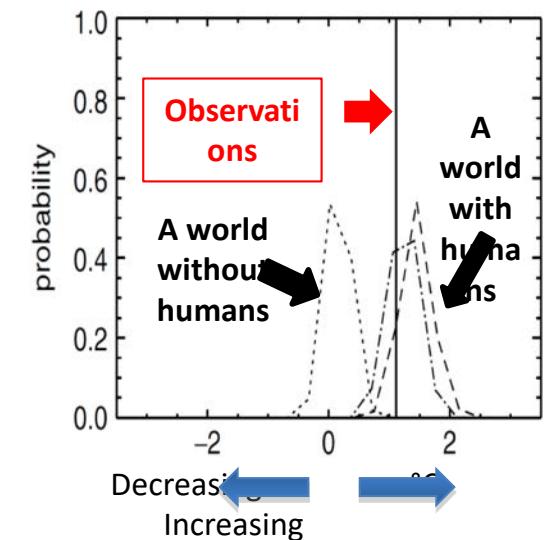
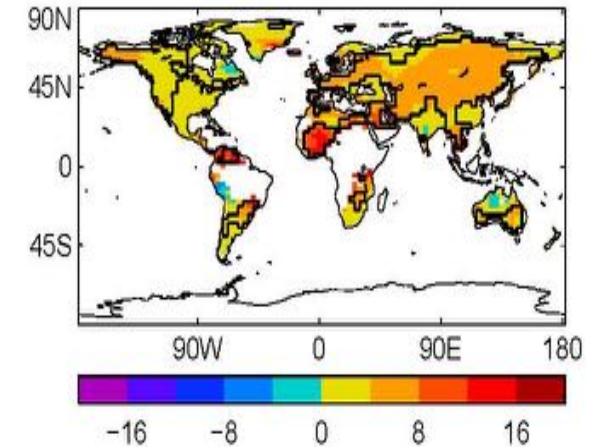
- Land Surface Processes
- How the land amplifies climate extremes
- Climate and regional modelling
- Water, energy and carbon processes
- How land use change affects climate
- Use of climate models for risk assessment
- Engagement of business with climate risk



# A. Prof Lisa Alexander



- Changes in the frequency and/or severity of extreme climate events have the potential to have profound societal and ecological impacts.
- Lisa's work primarily focuses on improving our understanding of observed changes in these events using multiple research tools ranging from station observations to climate model output.
- Much of her work has been focused on the creation of high quality global datasets and comparison with state of the art climate models.



Increasing trends in the hottest night  
of the year  
– humans are to blame

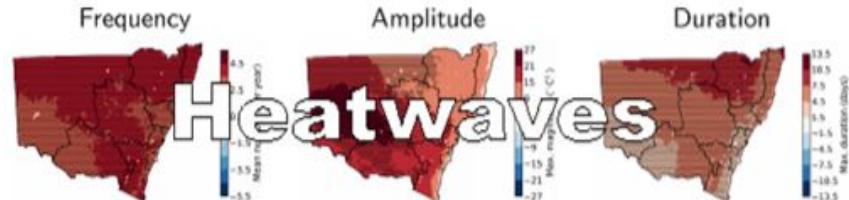
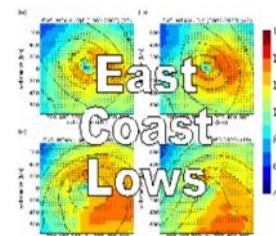
# A. Prof Jason Evans



Journal of Climate Editor  
Co Chair of GEWEX Hydroclimate Panel  
Coordinator of CORDEX Australasia

## Expertise:

- Regional Climate Models, Processes & Projections
- Water cycle over land
- Land-atmosphere interactions
- Remote sensing of land degradation



NSW Climate projections map for 2060-2079  
Rainfall: Change in rainfall (%)



## Major collaborations:

- NSW State Government
- National Environmental Science
- Program Earth System & Climate Change Hub
- ARC Centre of Excellence for Climate Extremes

# A. Prof Donna Green



- Climate change and energy policy
- Climate change and human health
- Air pollution and environmental justice
- Climate adaptation for Indigenous Australians

# Dr. Gabriel Abramowitz

CCRC Post Graduate Coordinator

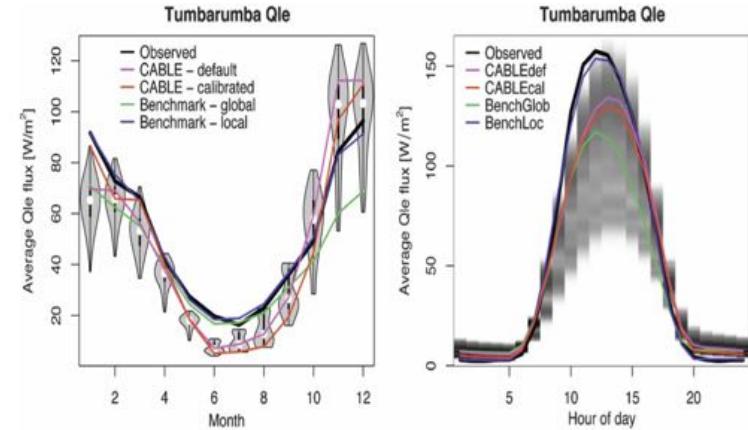
Co-chair of the GEWEX Global Land-Atmosphere System Study (GLASS) panel

Committee member for the Australian community land surface model, CABLE

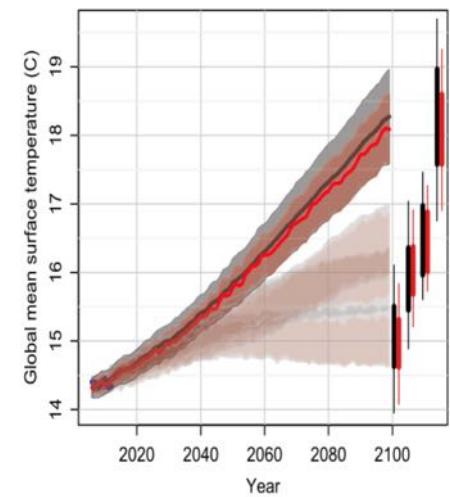
Chair of the Australian Energy & Water Cycle Experiment Benchmarking Working Group

Research areas:

- Model evaluation and benchmarking (including climate, hydrological, ecological models)
- Model dependence in ensemble prediction
- Machine learning applications in climate
- Land surface model process representation



Projection spread: RCP85





# Dr. Melissa Hart



Graduate Director - ARC Centre of Excellence for Climate System Science

Developed and co-ordinated a national graduate program in climate science across 5 universities -120 PhD students

## Urban Climate

- Quantification of the urban heat island magnitude
- Impact of land-use and anthropogenic activities on the climate of cities



## Air Pollution Meteorology

- Synoptic and mesoscale controls on air pollution
- Air pollution impacts from prescribed burns and wildfires

# Dr. Alex Sen Gupta



Senior Lecturer  
BEES Postgraduate Coordinator (Thesis Examination)

Climate variability and change can dramatically effect the ocean, its ecosystems and the industries that rely on these. Related projects include:

- Understanding the effect of anthropogenic warming on the ocean circulation, temperatures and marine critters!
- Processes and drivers of marine heat waves
- Numerical simulation of tropical tuna
- Climate model evaluation and climate projections

