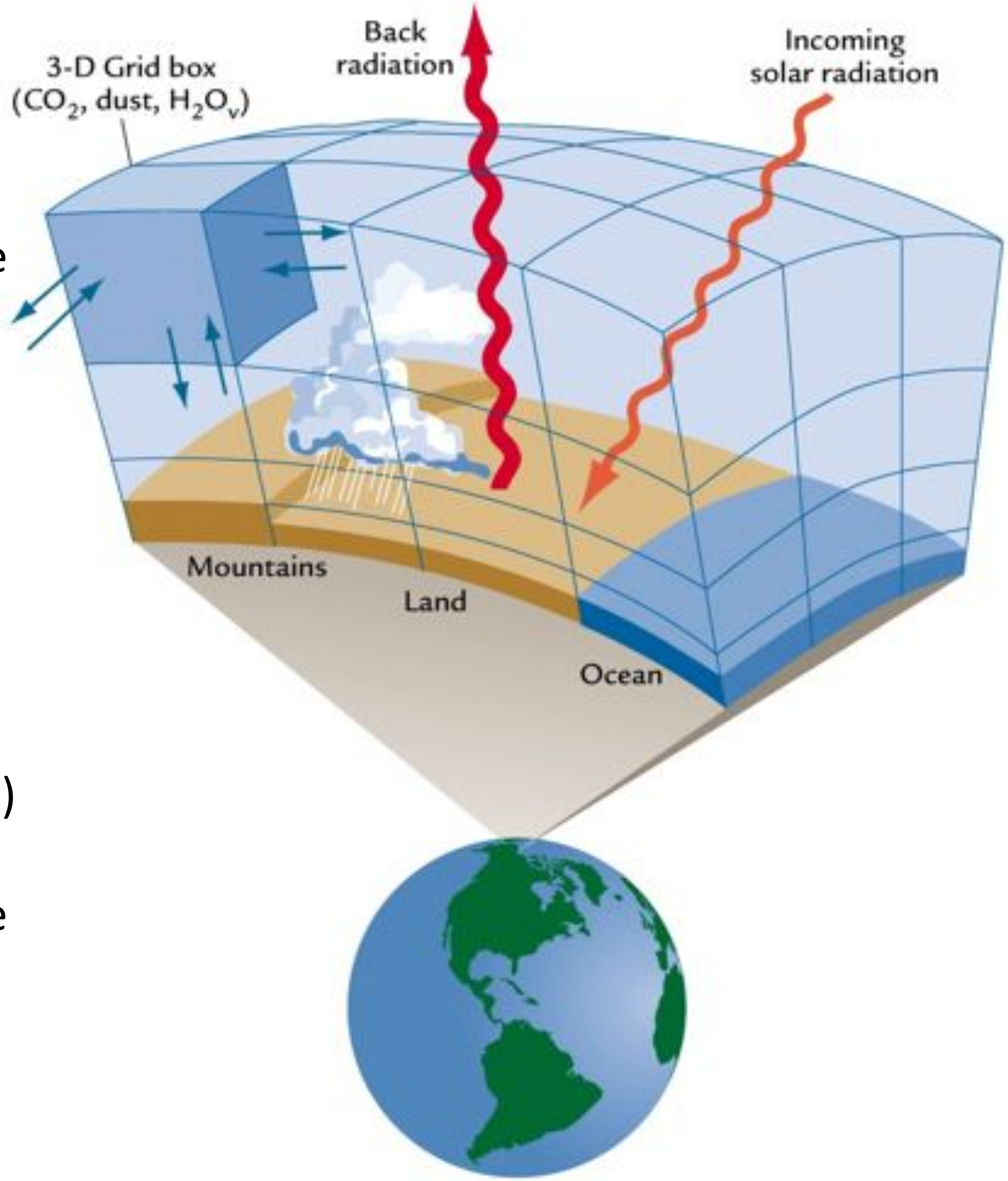
## Modelling the Climate System

**Alex Sen Gupta; a.sengupta@unsw.edu.au**

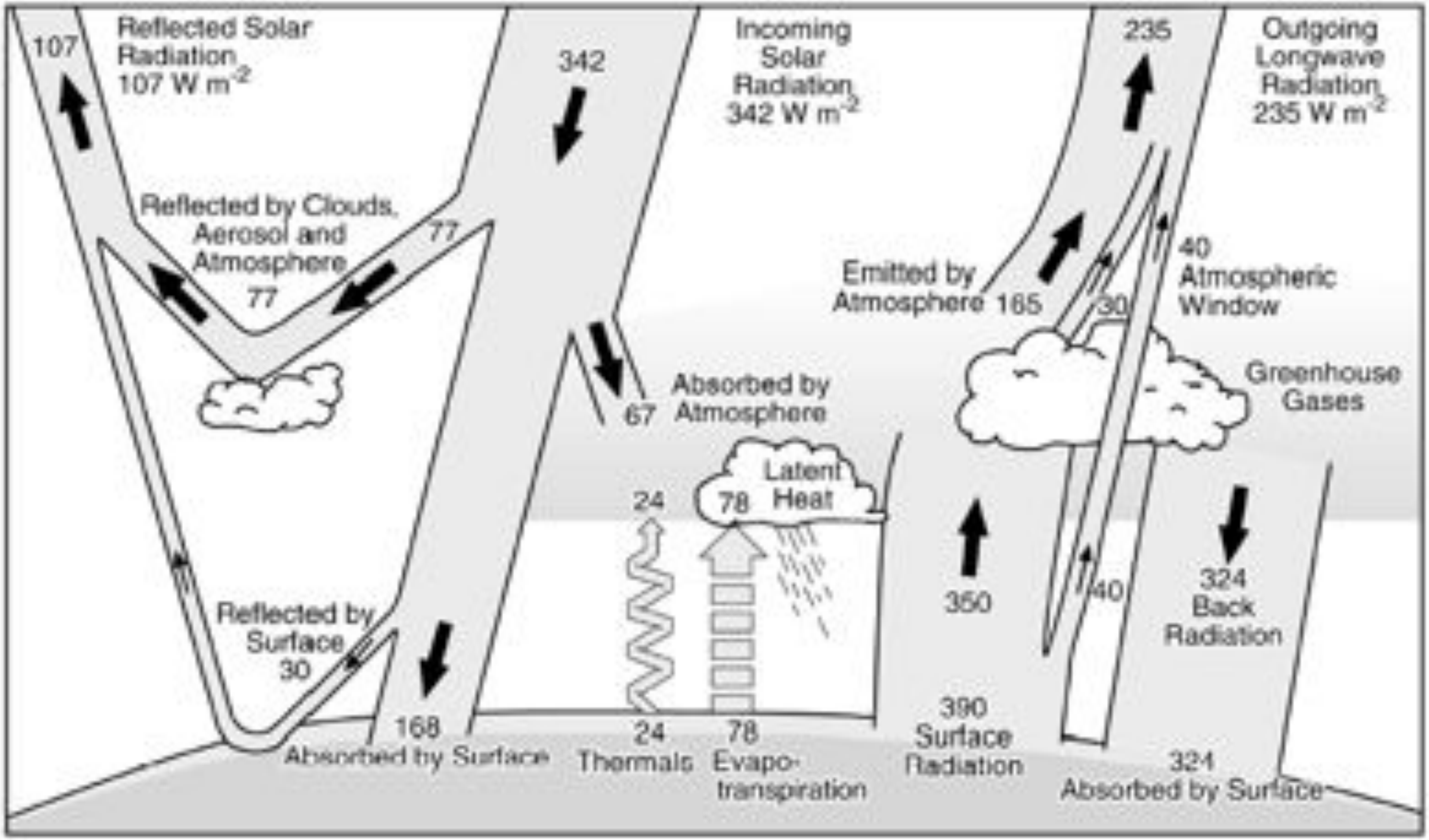
- Anthropogenic Climate Change
- Conceptual model of the Climate System
  - Governing equations
  - Solving the differential equations
  - Example 1- Climate Projections
  - Example 2- Geoengineering
- State-of-the-art climate models
  - Governing equations (ocean)
  - Solving the equations
  - Example: acceleration of the EAC – sending Nemo to Tasmania
- Climate Change research



*Fourier concluded that at the distance the earth is from the Sun the earth should be much colder than observed.* ***Jean-Baptiste Joseph Fourier (1768-1830)***

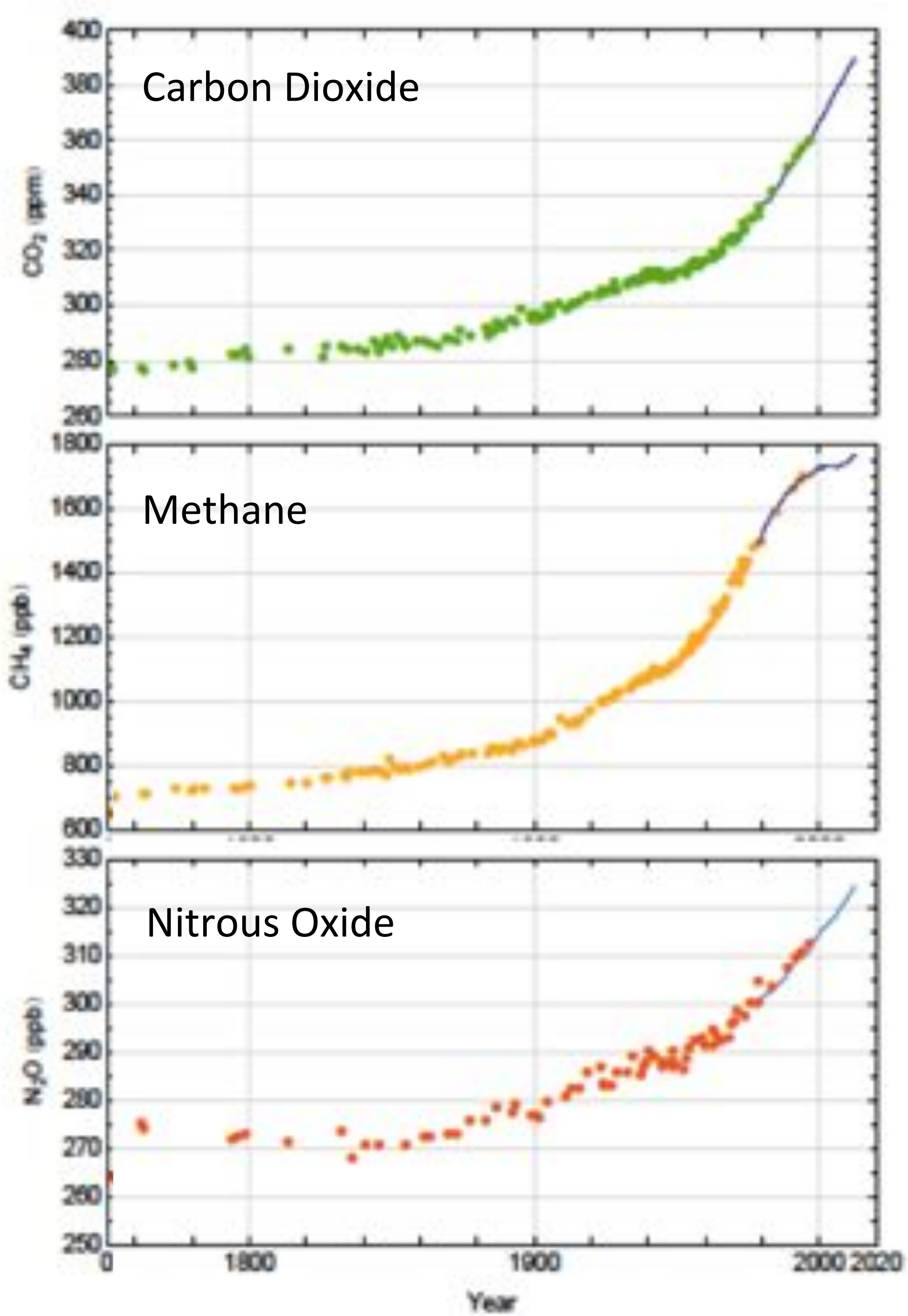
*"The atmosphere admits of the entrance of the solar heat, but checks its exit; and the result is a tendency to accumulate heat at the surface of the planet".* ***John Tyndall (1859)***

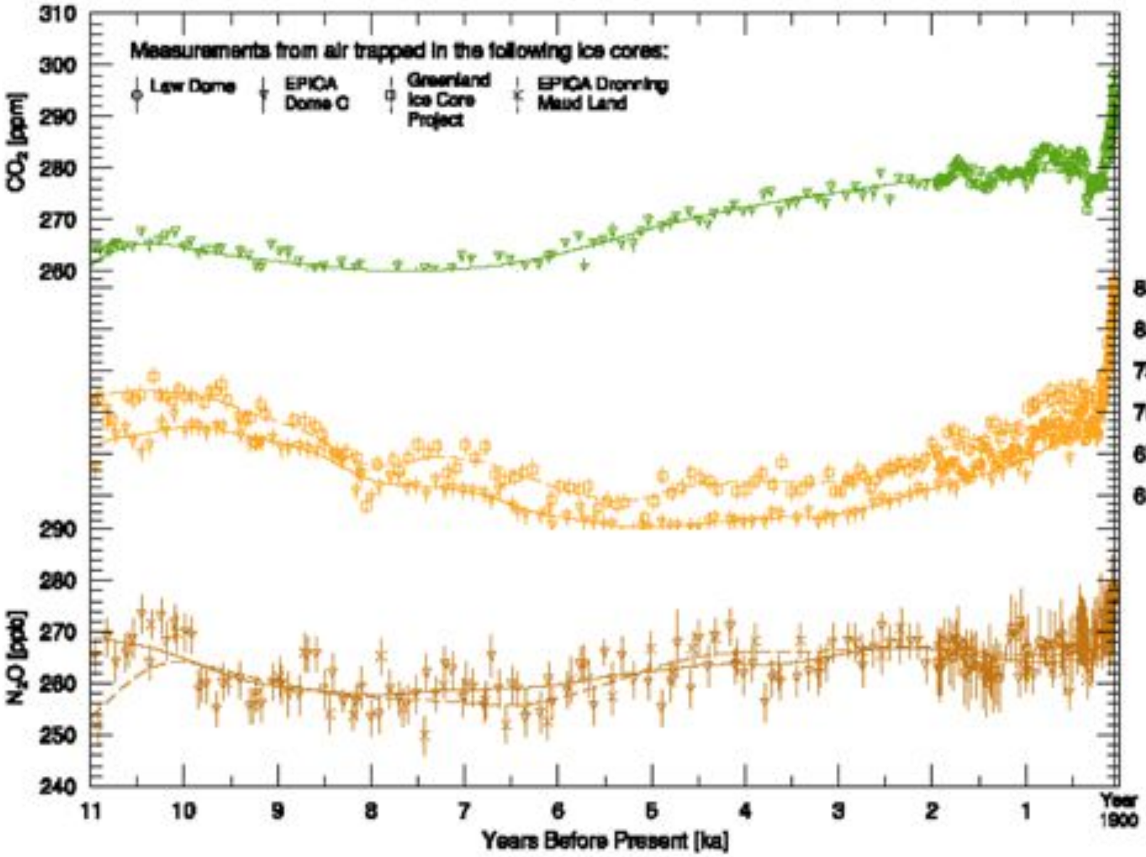
Arrhenius  was the first person to predict that emissions of carbon dioxide from the burning of fossil fuels and other combustion processes would cause global warming. **Svante August Arrhenius  (1896)**



107
Reflected Solar Radiation 107 W m-2
342
Incoming Solar Radiation 342 W m-2
235
Outgoing Longwave Radiation 235 W m-2
Reflected by Clouds, Aerosol and Atmosphere
77
77
Emitted by Atmosphere 165
30
40 Atmospheric Window
67 Absorbed by Atmosphere
Greenhouse Gases
24
78 Latent Heat
350
40
324 Back Radiation
Reflected by Surface 30
168 Absorbed by Surface
24 Thermals
78 Evapo-transpiration
390 Surface Radiation
324 Absorbed by Surface

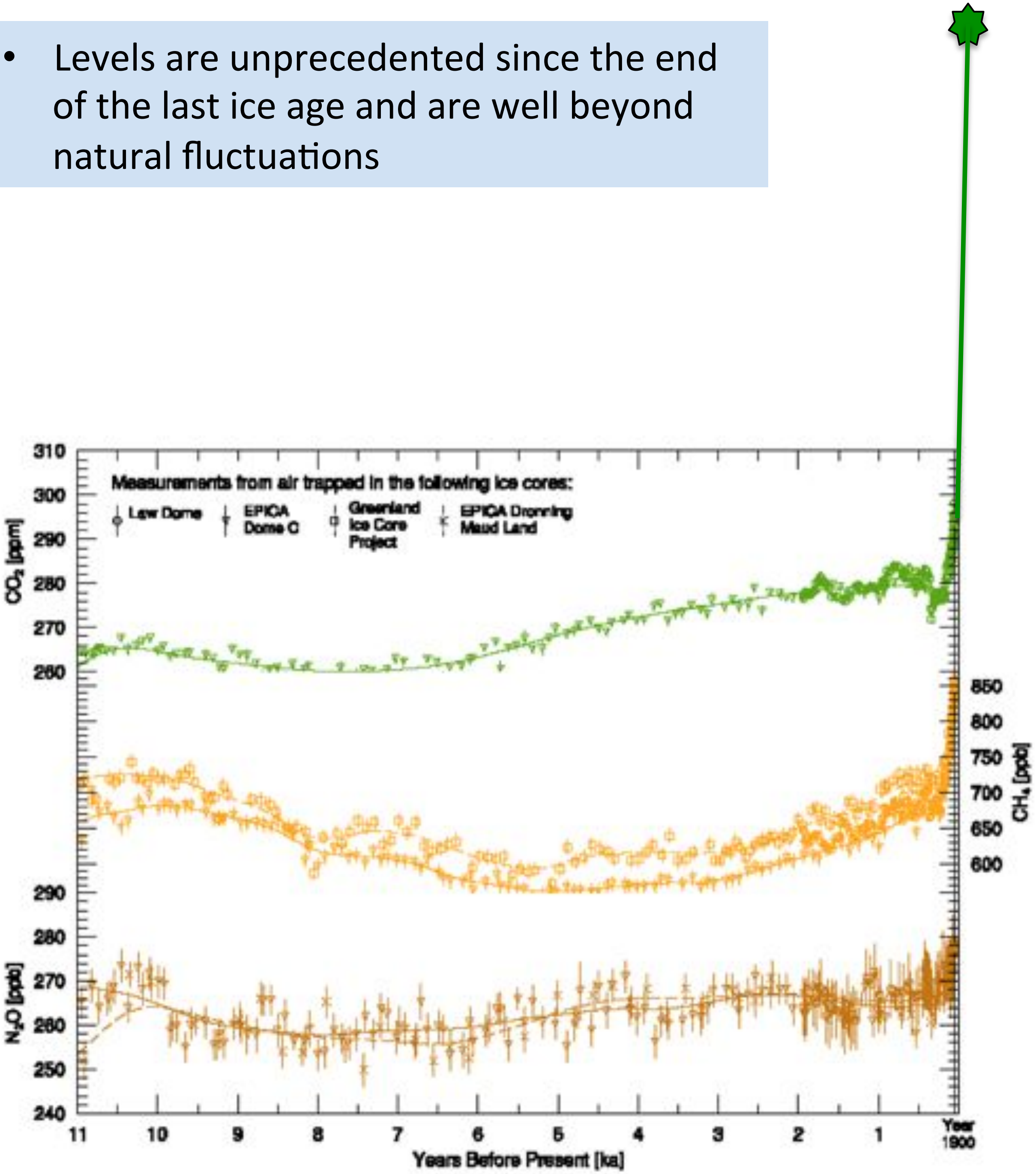
- Changes in greenhouse gas concentrations over the last 200yr can be directly linked to human activity (primarily burning of fossil fuels)

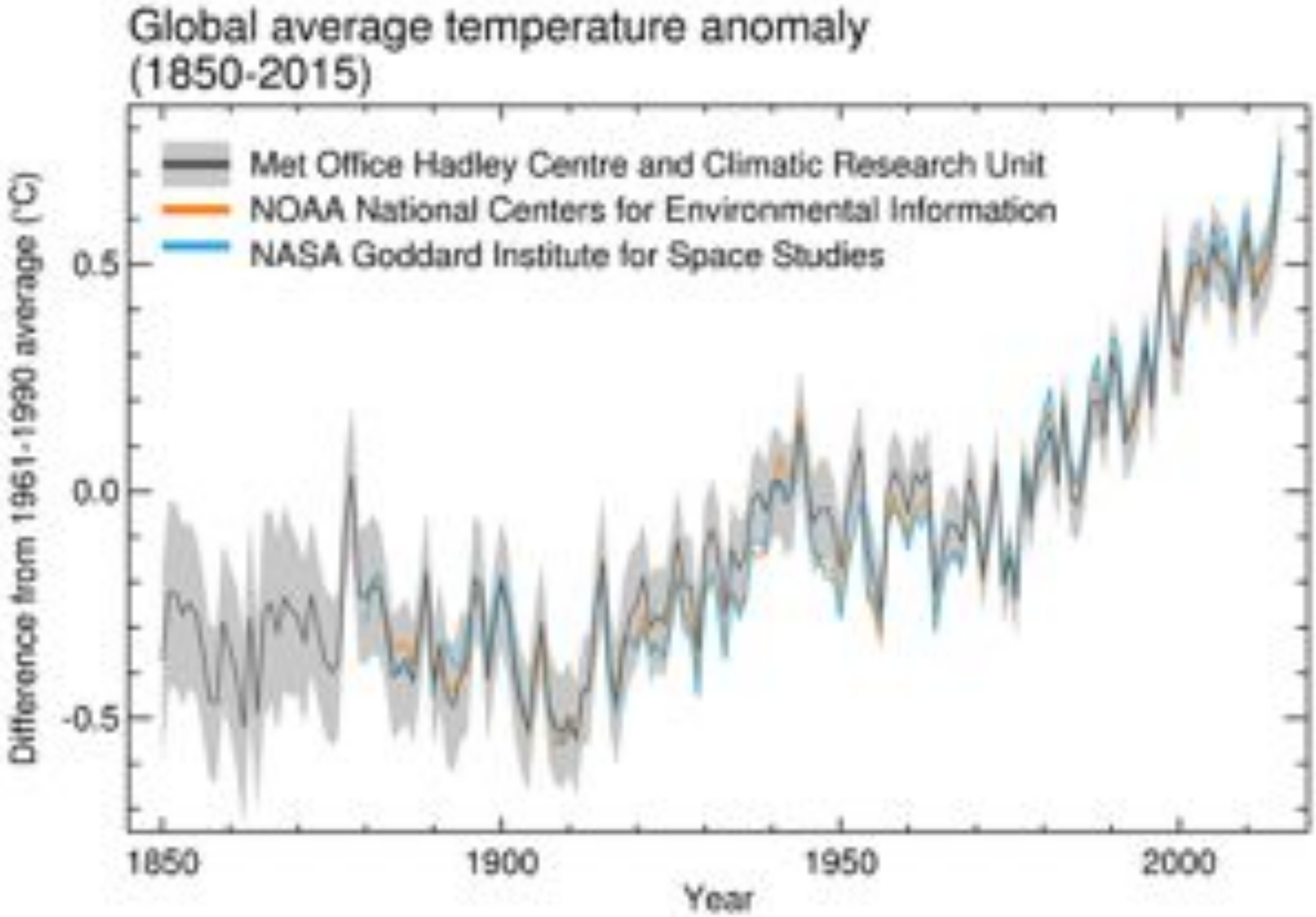




Measurements from air trapped in the following ice cores:
Law Dome
EPICA Dome C
Greenland Ice Core Project
EPICA Dronning Maud Land
$CO_2$ [ppm]
$CH_4$ [ppb]
$N_2O$ [ppb]
Years Before Present [ka]
Year 1900

- Levels are unprecedented since the end of the last ice age and are well beyond natural fluctuations



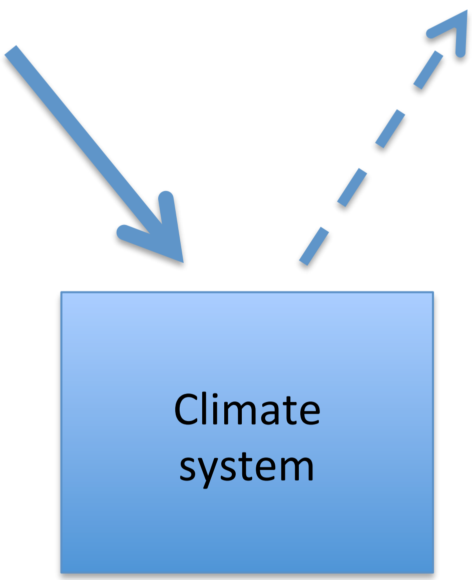


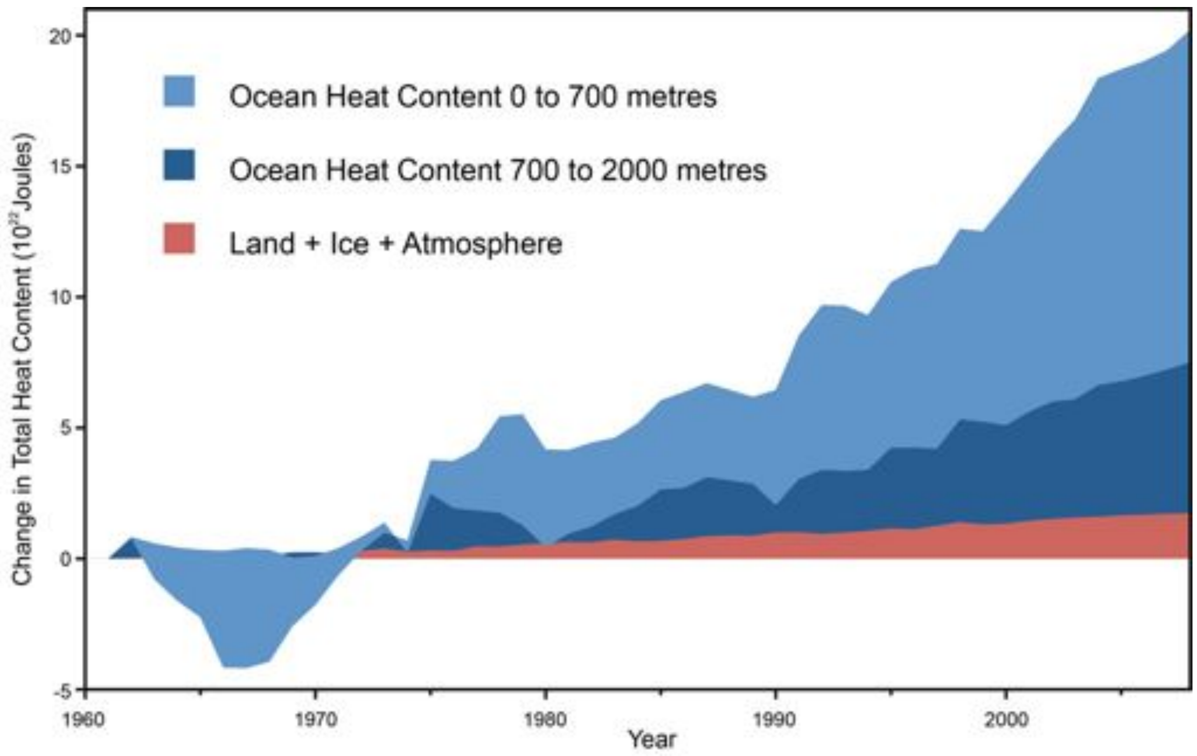
Global average temperature anomaly
(1850-2015)
Met Office Hadley Centre and Climatic Research Unit
NOAA National Centers for Environmental Information
NASA Goddard Institute for Space Studies
Difference from 1961-1990 average (°C)
0.5
0.0
-0.5
1850
1900
1950
2000
Year

**Simplified [Box] model of the climate system**

Assumptions:

- Most of the thermal mass of the climate system is in the ocean

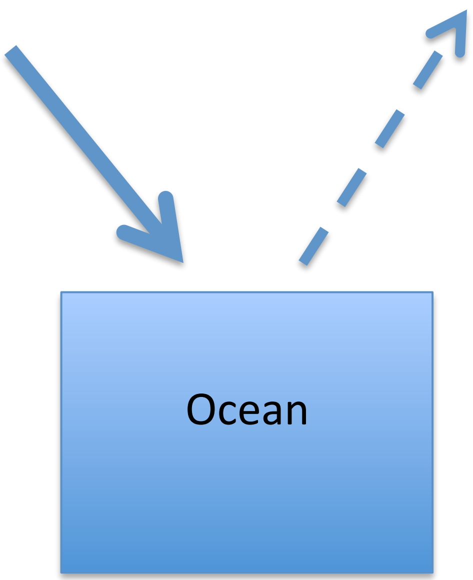


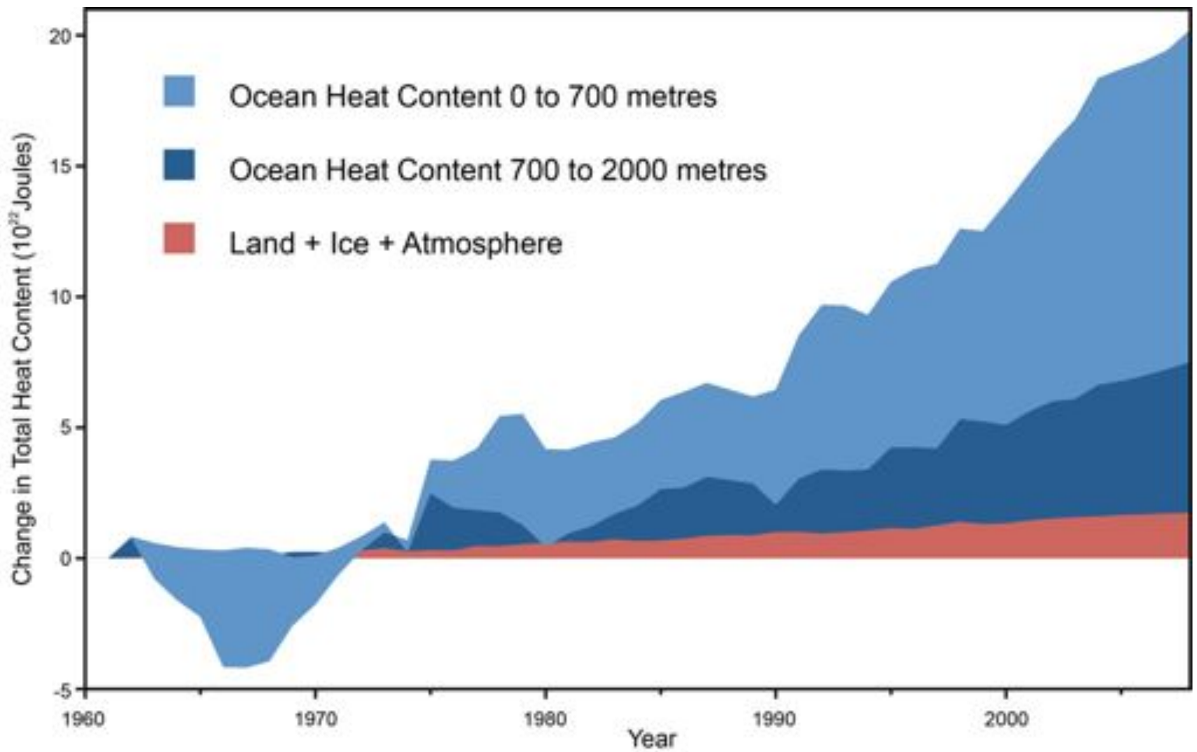



**Simplified [Box] model of the climate system**

Assumptions:

- Most of the thermal mass of the climate system is in the ocean
- Upper ocean is well mixed and responds quickly to surface energy changes
- Deep ocean interacts more slowly with the upper ocean
- Upper ocean and deep ocean are well mixed



## Simplified [Box] model of the climate system

Assumptions:
- Most of the thermal mass of the climate system is in the ocean
- Upper ocean is well mixed and responds quickly to surface energy changes
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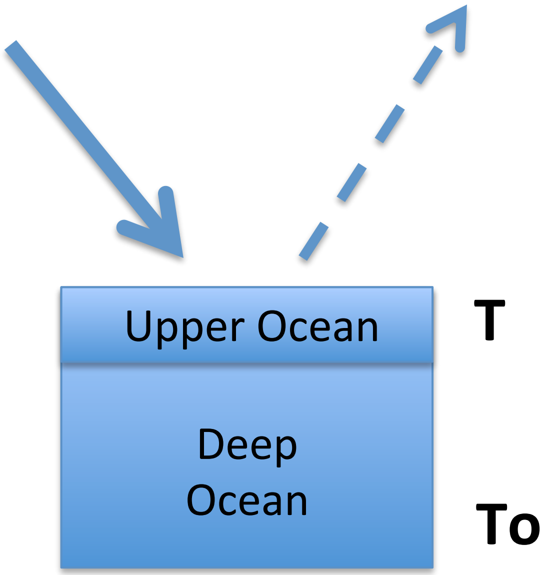
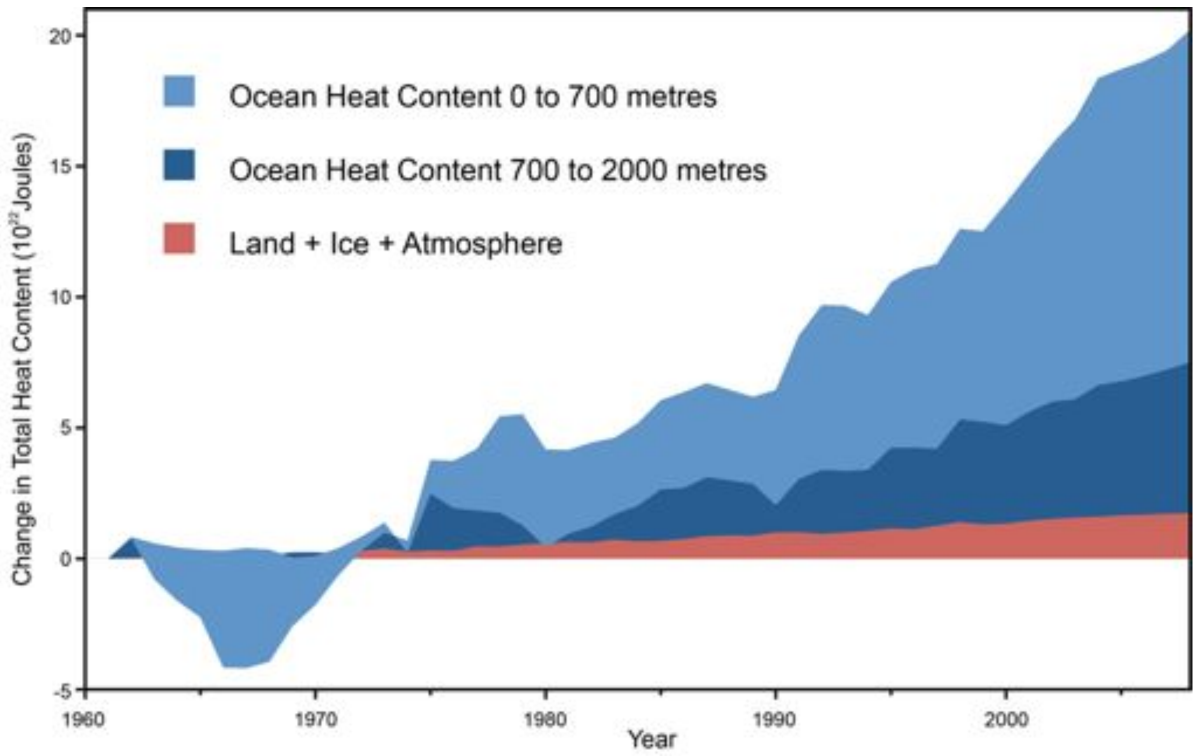
- Upper ocean and deep ocean are well mixed
- Heat loss from the planet surface increases linearly with temperature (includes black body radiation, sensible and latent heat losses)



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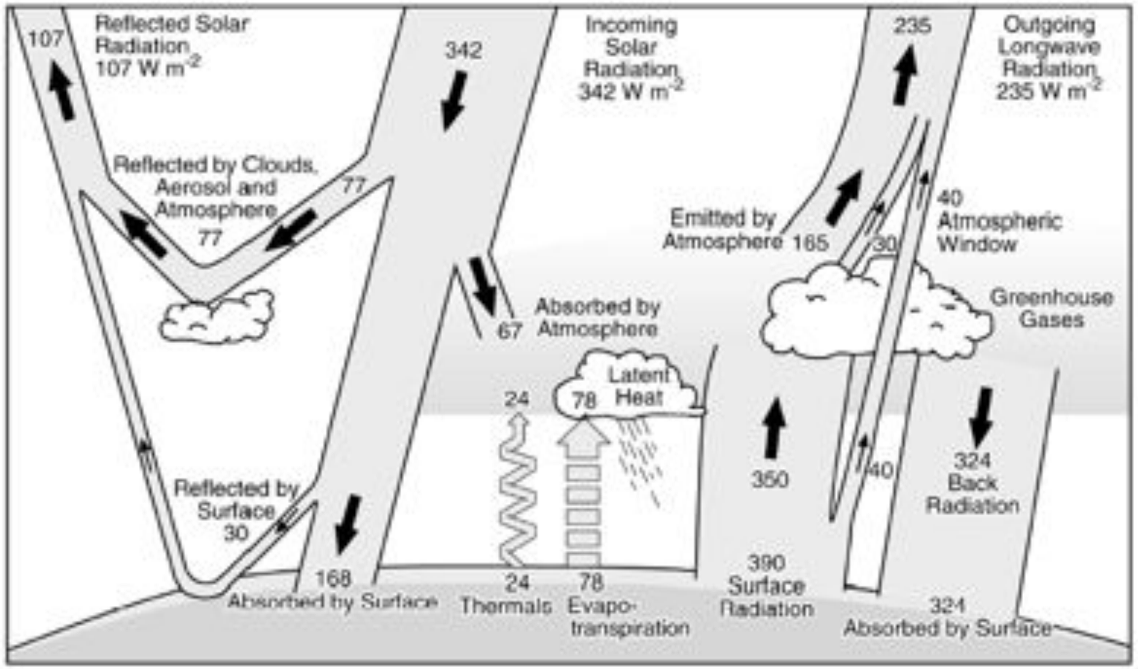
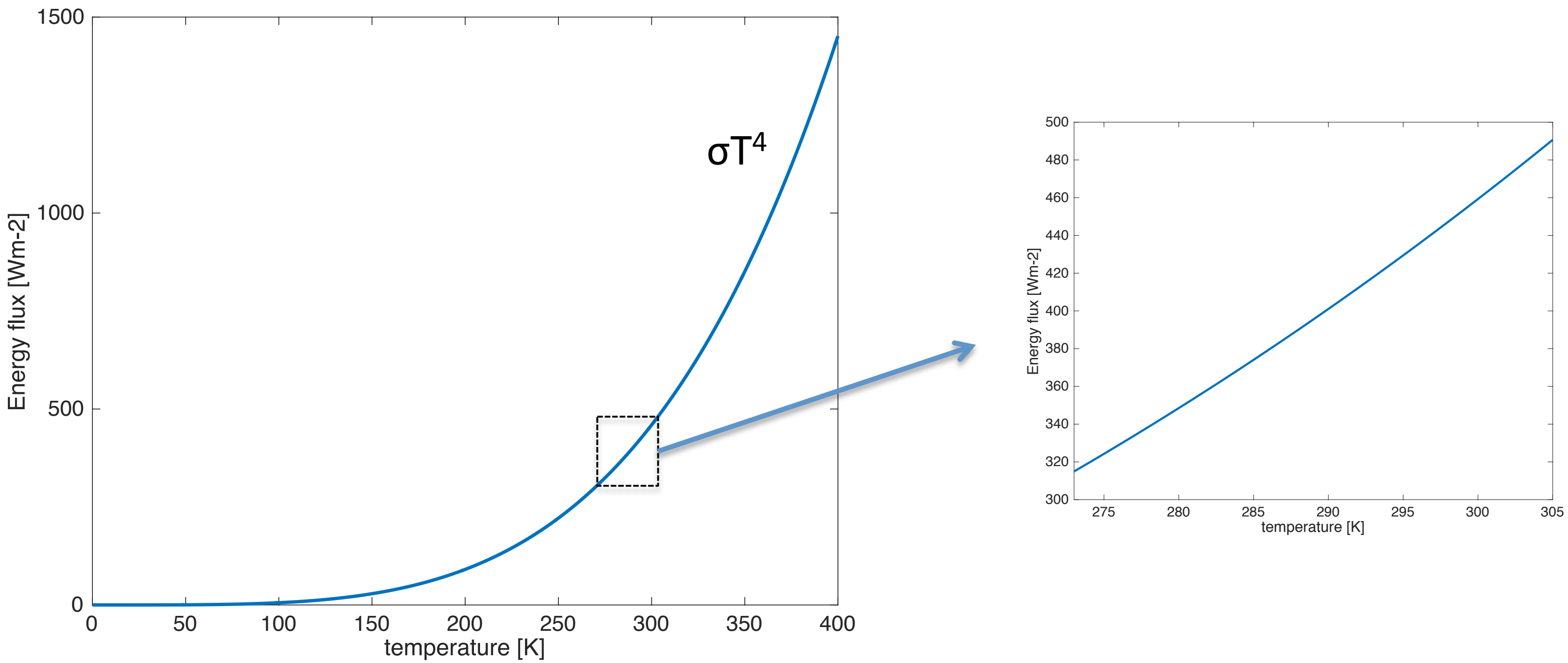
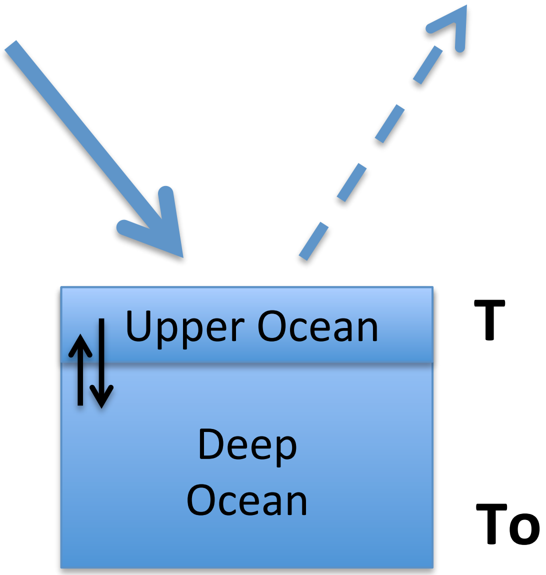


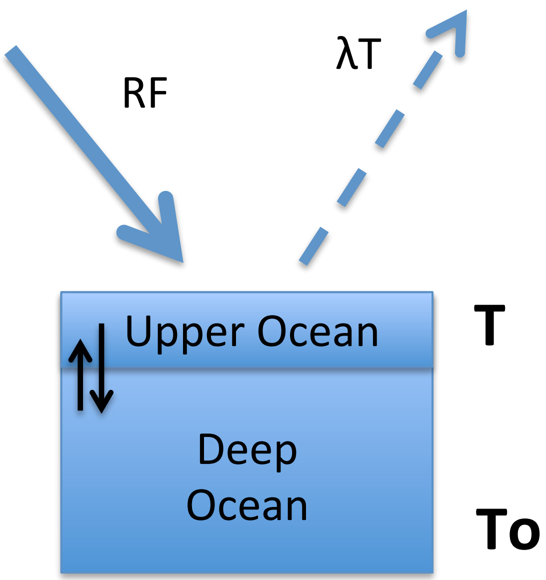
**Simplified [Box] model of the climate system**

Assumptions:
- Most of the thermal mass of the climate system is in the ocean
- Upper ocean is well mixed and responds quickly to surface energy changes
- Deep ocean interacts more slowly with the upper ocean
- Upper ocean and deep ocean are well mixed
- Heat loss from the planet surface increases linearly with temperature (includes black body radiation, sensible and latent heat losses)
- Transfer of heat from the upper to the deep ocean just depends on the temperature difference



## Simplified [Box] model of the climate system

| Change in energy content/s | = | Heat capacity | x | Change in temperature/s | = | Energy in/s | - | Energy out/s |
|---|---|---|---|---|---|---|---|---|



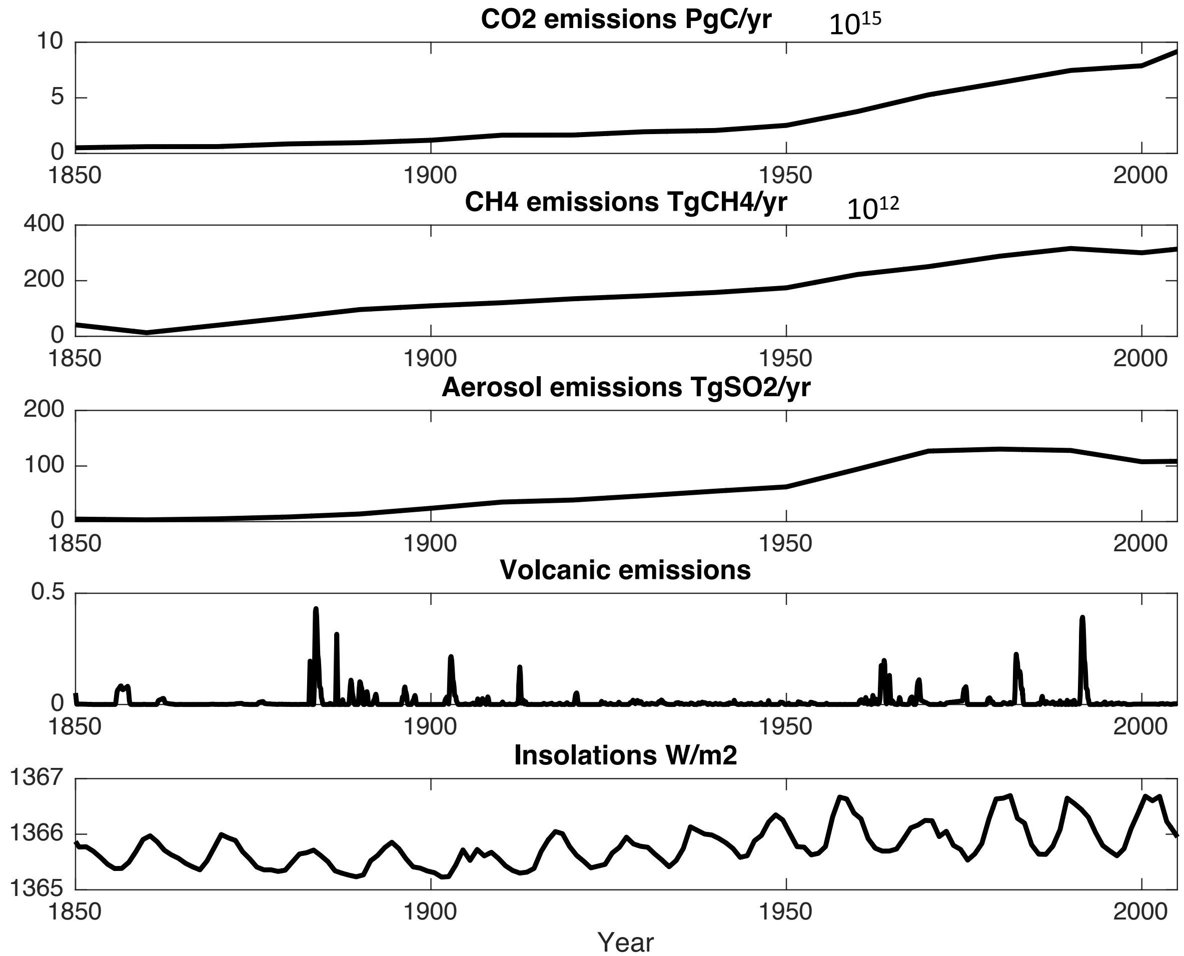

$$C\frac{dT}{dt} = RF - \lambda T - \gamma(T - T_o)$$

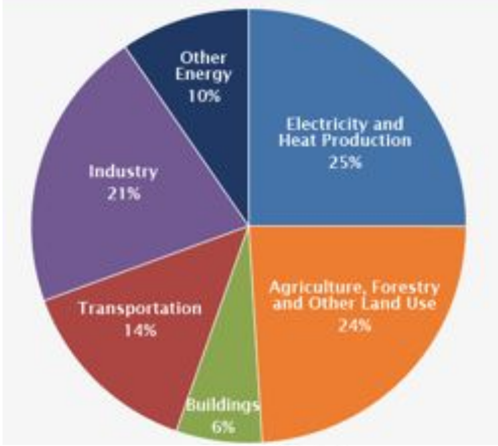
$$C_o\frac{dT_o}{dt} = \gamma(T - T_o)$$

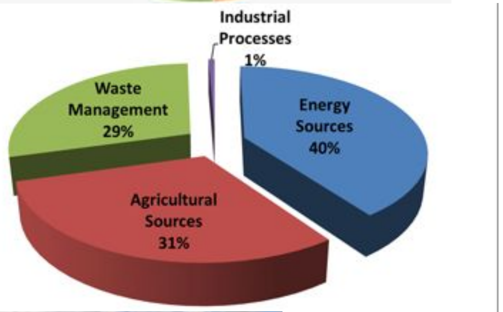
$$RF = R^{CO2} + R^{CH4} + R^{aer} + R^{volc} + R^{sol}$$

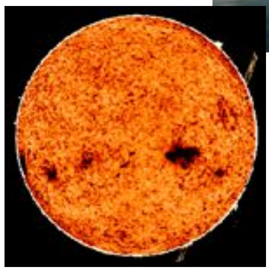
# Model inputs

To calculate the radiative forcing we need certain information: model inputs



**Model inputs**

$$RF = R^{CO2} + R^{CH4} + R^{aer} + R^{volc} + \boxed{R^{sol}}$$

Solar radiation

- S is the radiation flux per square meter at top of atmosphere
- Energy passing through a circle is distributed over a sphere
- A proportion α of the surface energy is reflected back to space


Insolation hitting earth = S x $\pi R^2$ [W]
This is distributed over total earth surface $4\pi R^2$ [$m^2$]
On average, insolation per square meter at surface = S x $\pi R^2$ / $4\pi R^2$ = S/4

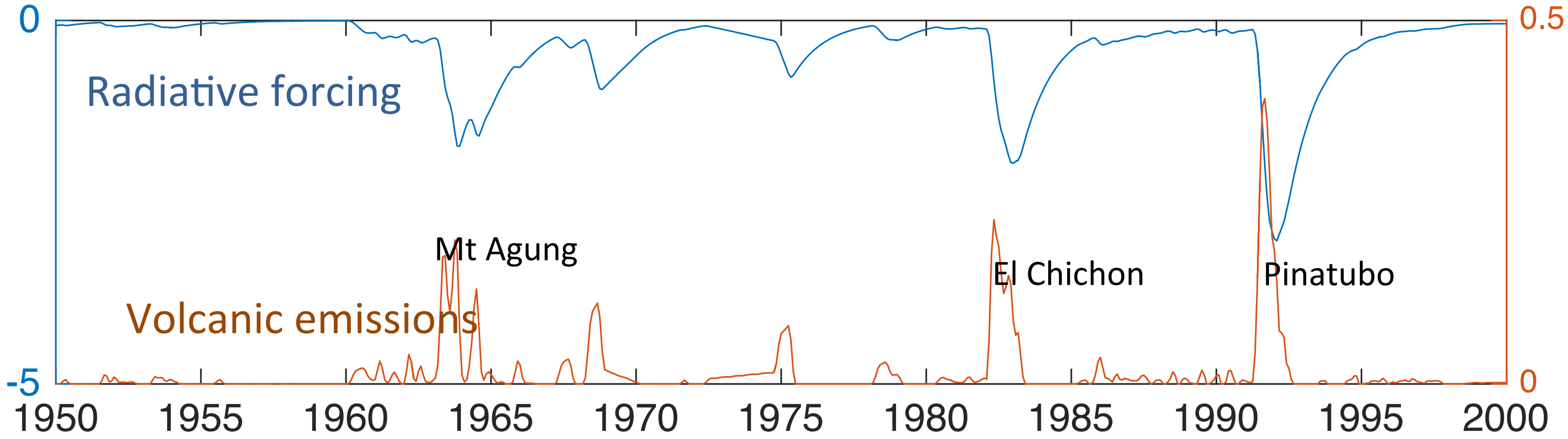
$$R^{sol} = \frac{S(t)}{4}(1-\alpha)$$

**Model inputs**

$$RF = R^{CO2} + R^{CH4} + R^{aer} + \boxed{R^{volc}} + R^{sol}$$

Volcanic aerosols

- Particles from volcanic eruptions are ejected into the stratosphere
- These reflect away solar radiation (i.e. produce a negative radiative forcing)
- Volcanic aerosols fall out over a timescale of ~ 1-3 years




OT (Optical thickness) increases with emissions of volcanic aerosols and decay with timescale $\tau^{volc}$ (2.5 years)
$R^{volc}$ is proportional to OT (constant is negative)

$$\frac{\partial(OT)}{\partial t} = E^{volc} - \frac{1}{\tau^{volc}}(OT)$$

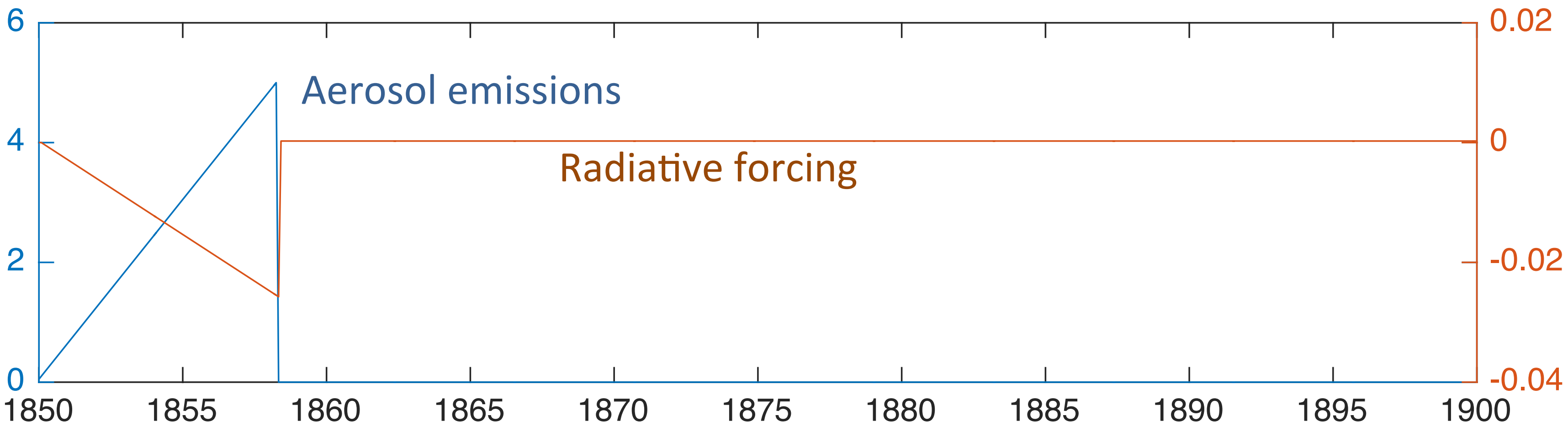
$$R^{volc} = c^{volc}\ OT$$

**Model inputs**

$$RF = R^{CO2} + R^{CH4} + \boxed{R^{aer}} + R^{volc} + R^{sol}$$

Human aerosols

- Aerosols (mainly $SO_4$) released into lower atmosphere
- Particles are rained out almost immediately (few days)




$R^{volc}$ is proportional to the rate of emissions (constant is negative)

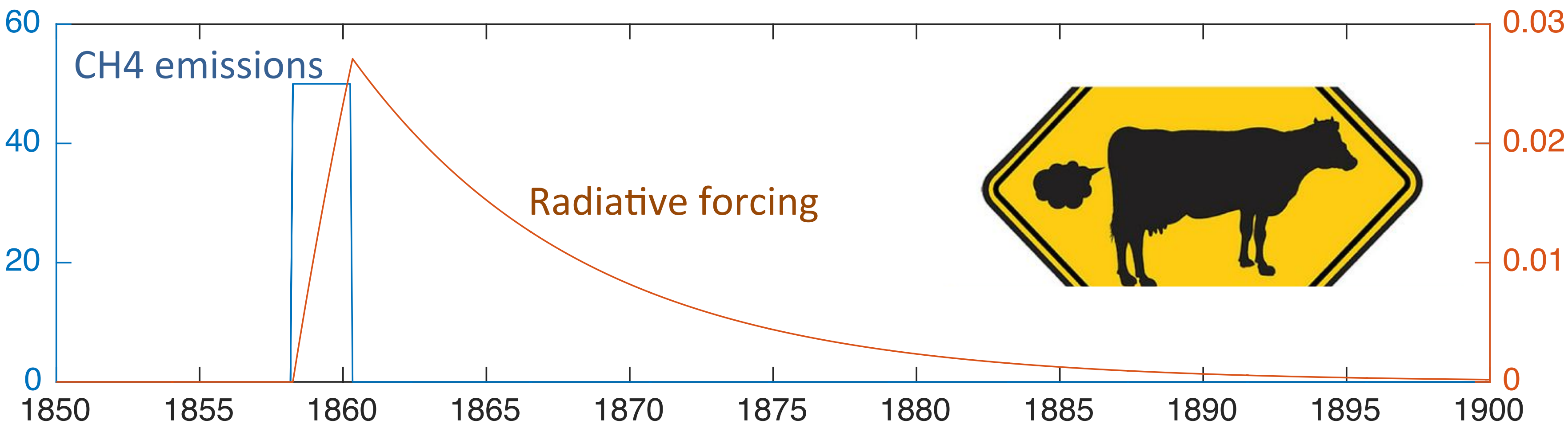
$$R^{aer} = c^{aer} E^{aer}$$

**Model inputs**

$$RF = R^{CO2} + \boxed{R^{CH4}} + R^{aer} + R^{volc} + R^{sol}$$

Methane emissions

- Methane in the atmosphere is emitted by energy production, agricultural processes and the decay of waste products
- It decays via oxidation in the atmosphere with a half life of ~ 10 years (but the decay rate increases with increased methane concentrations)




$$\frac{\partial C^{CH4}}{\partial t} = c^{CH4} E^{CH4} - \frac{1}{\tau^{CH4}} C^{CH4}$$

$$\tau^{CH4} = \tau_{PI}^{CH4} \left( \frac{C^{CH4}}{C^{CH4} + C_{PI}^{CH4}} \right)^{\alpha}$$

$$R^{CH4} = c^{CH4} \, log_e \left( \frac{C^{CH4} + C_{PI}^{CH4}}{C_{PI}^{CH4}} \right)$$

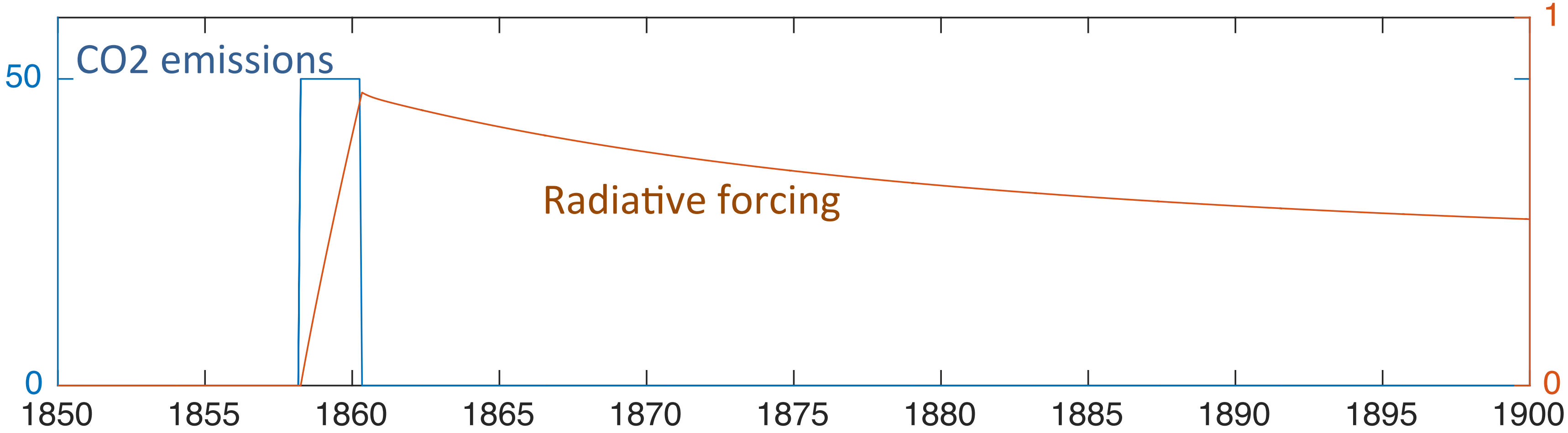
- Radiative forcing increases with the log of CH4 concentration

**Model inputs**

$$RF = \boxed{R^{CO2}} + R^{CH4} + R^{aer} + R^{volc} + R^{sol}$$

CO2 emissions

- CO2 in the atmosphere is primarily produced through the burning of fossil fuel, with a contribution from deforestation and agriculture
- Carbon cycle is complex and includes uptake and release of CO2 by the terrestrial biosphere and ocean. Half life 100's of years




$$R^{CO2} = c^{CO2} \log_e\left(\frac{C^{CO2} + C_{PI}^{CO2}}{C_{PI}^{CO2}}\right)$$

- Radiative forcing increases with the log of CO2 concentration

**Model inputs**

$$RF = \boxed{R^{CO2}} + R^{CH4} + R^{aer} + R^{volc} + R^{sol}$$

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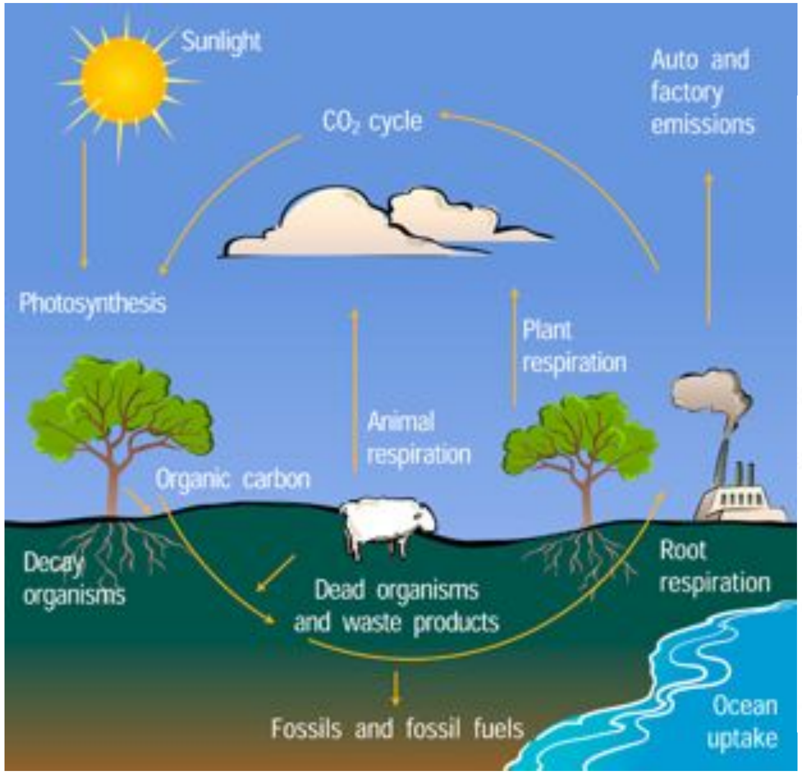



$$\frac{dC^{at}(t)}{dt} = E^{co2} - k_a(C^{at} - A \cdot B \cdot C^{up}) + [(1-\varepsilon)mN(t) + \delta S(t) - P(t, C^{at})] \quad (4)$$

$$\frac{dC^{up}(t)}{dt} = k_a(C^{at}(t) - A \cdot B \cdot C^{up}(t)) - k_d\left(C^{up}(t) - \frac{C^{lo}(t)}{d}\right) \quad (5)$$

$$\frac{dC^{lo}(t)}{dt} = k_d\left(C^{up}(t) - \frac{C^{lo}(t)}{d}\right) \quad (6)$$

$$\frac{dN(t)}{dt} = P(t, C^{at}) - mN(t) \quad (7)$$

$$\frac{dS(t)}{dt} = \varepsilon mN(t) - \delta S(t) \quad (8)$$

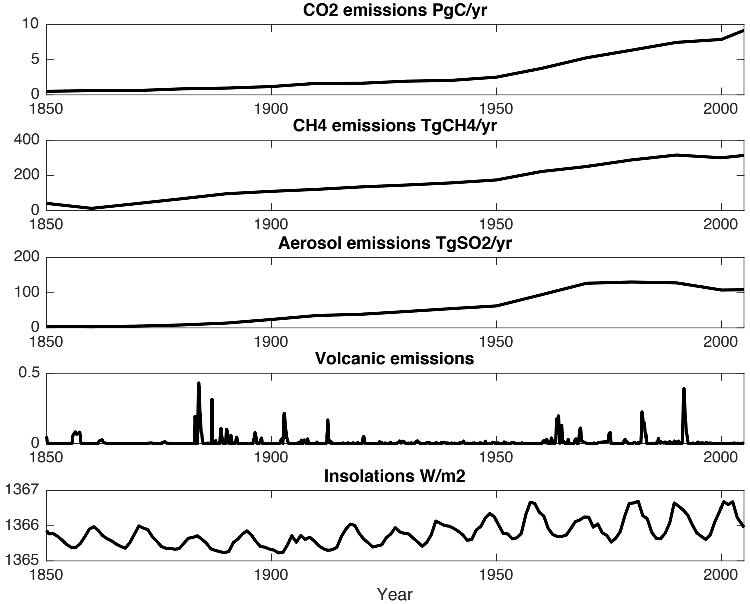
$$P(t) = P_0(1 - a_2(C^{at}(t) - C_0^{at}) \quad (9)$$

where $C^{at}$, $C^{up}$, $C^{lo}$, N and S are the inventories of carbon dioxide (GtC) in the atmosphere, upper ocean and deep ocean, terrestrial vegetation and soil, respectively, P is net primary production by terrestrial plants (GtC/yr) and E is the human emissions of CO2 (GtC/yr) See table for definition of constants.

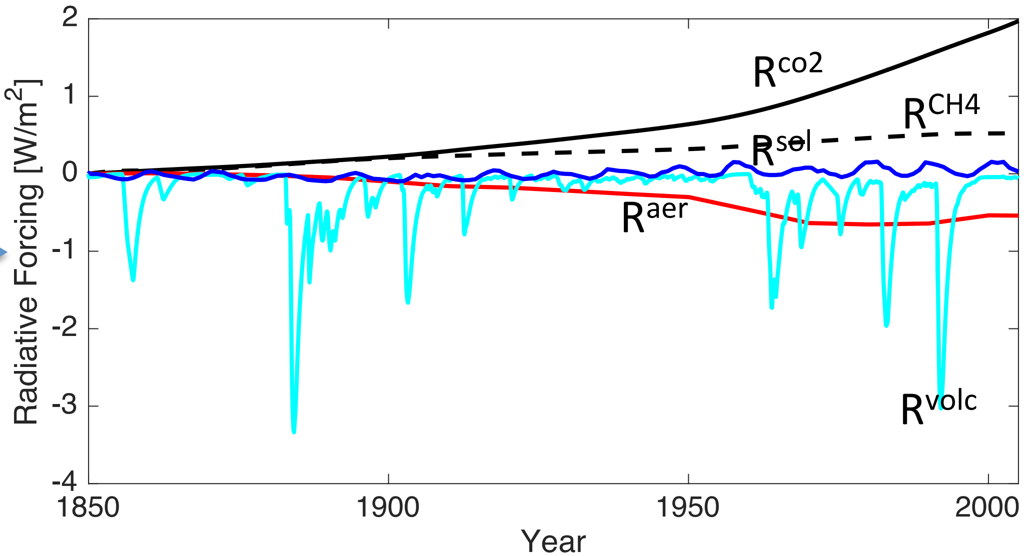
$$R^{CO2} = c^{CO2} \log_e\left(\frac{C^{CO2} + C^{CO2}_{PI}}{C^{CO2}_{PI}}\right)$$

## Solving the Model

Model inputs…




Radiative Forcing…



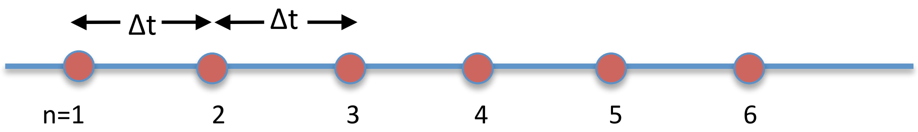

$$RF = R^{CO2} + R^{CH4} + R^{aer} + R^{volc} + R^{sol}$$

Energy balance model

$$C\frac{dT}{dt} = RF - \lambda T - \gamma(T - T_o)$$

$$C_o\frac{dT_o}{dt} = \gamma(T - T_o)$$

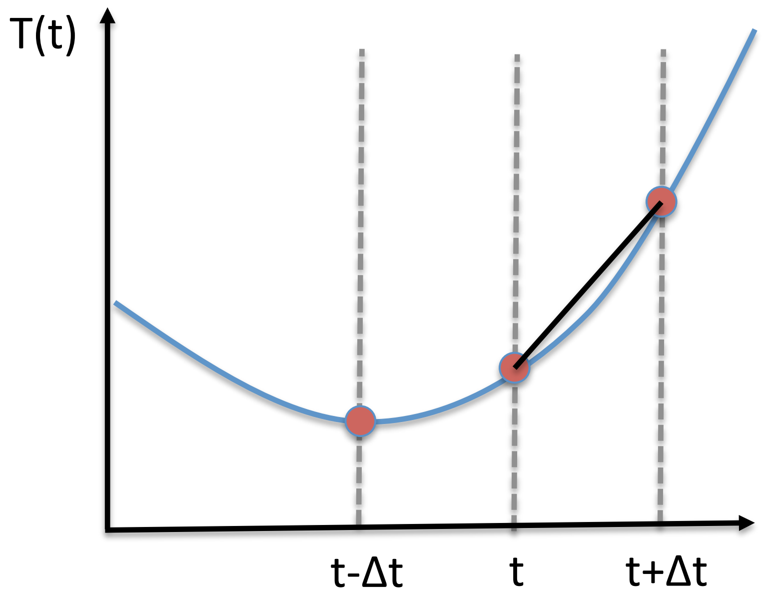
## Solving the Model




$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$$C\frac{dT(t)}{dt} = RF(t) - \lambda T(t) - \gamma\big(T(t) - T_o(t)\big)$$

$$T(t+\Delta t) \approx T(t) + \frac{\Delta t}{C}\Big[RF(t) - \lambda T(t) - \gamma\big(T(t) - T_o(t)\big)\Big]$$



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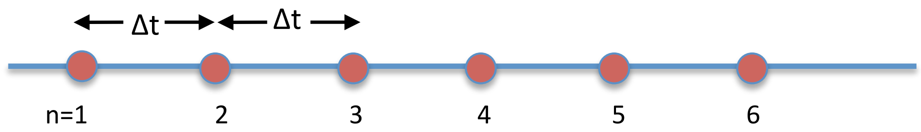


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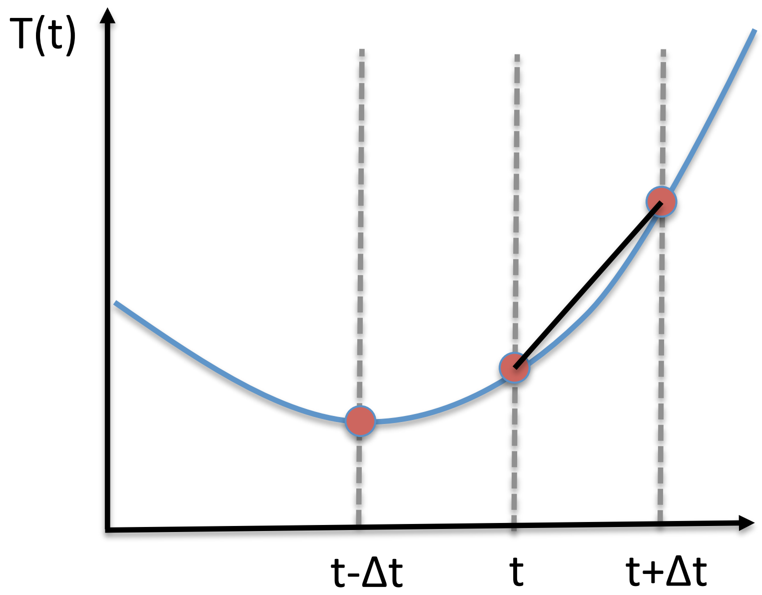
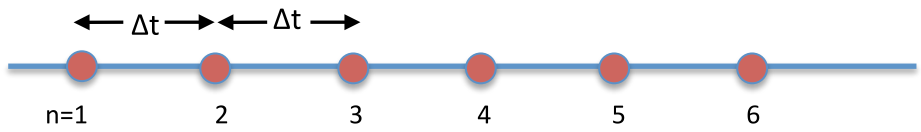


## Solving the Model

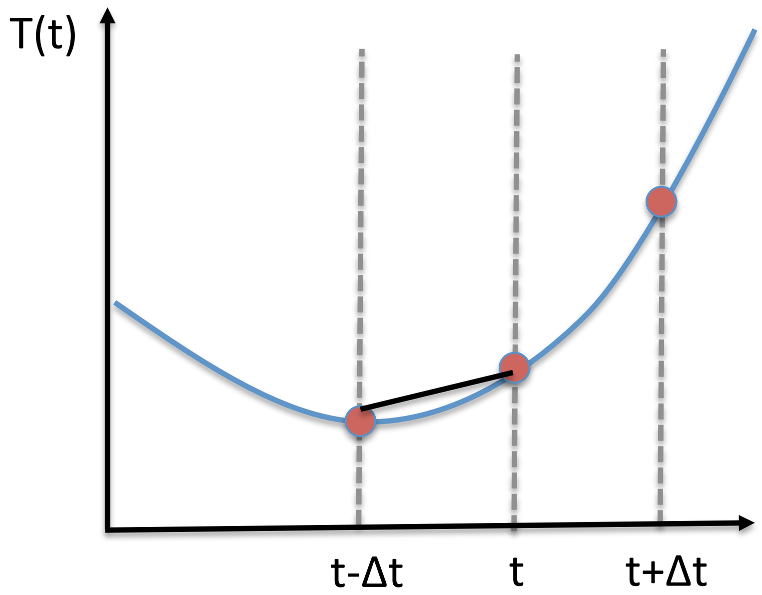



$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

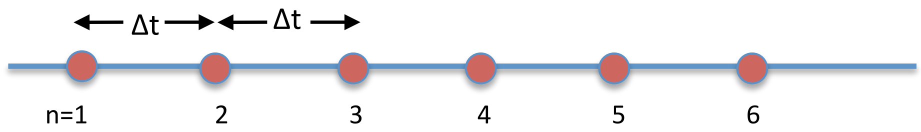
Forward difference

$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_t - T_{t-\Delta t}}{\Delta t}$$

Backward difference



## Solving the Model

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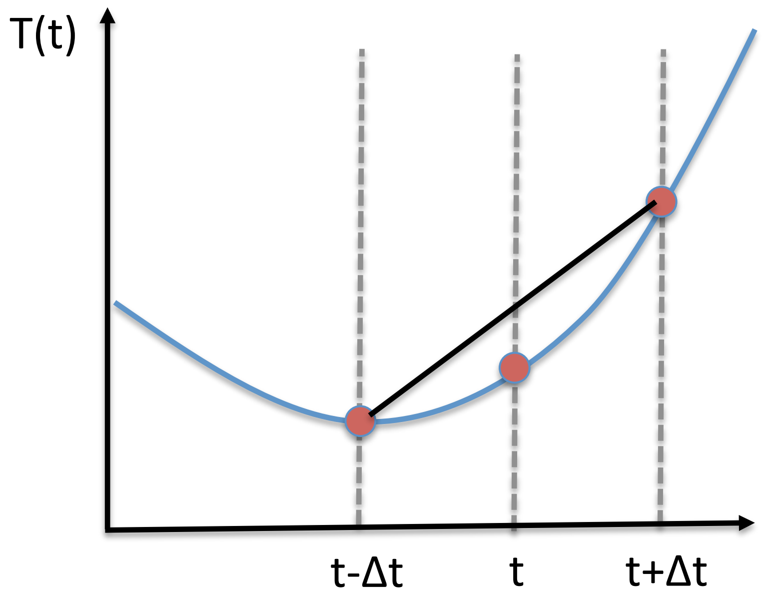
Forward difference

$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_t - T_{t-\Delta t}}{\Delta t}$$

Backward difference

$$\left(\frac{dT}{dt}\right)_t \approx \frac{T_{t+\Delta t} - T_{t-\Delta t}}{2\Delta t}$$

Centred difference



**Solving the Model**

Taylor expansion of T(t+$\Delta$ t)

$$T(t+\Delta t) = T(t) + \Delta t\left(\frac{dT}{dt}\right)_t + \frac{(\Delta t)^2}{2}\left(\frac{d^2T}{dt^2}\right)_t + \frac{(\Delta t)^3}{6}\left(\frac{d^3T}{dt^3}\right)_t + \cdots$$

$$T(t-\Delta t) = T(t) - \Delta t\left(\frac{dT}{dt}\right)_t + \frac{(\Delta t)^2}{2}\left(\frac{d^2T}{dt^2}\right)_t - \frac{(\Delta t)^3}{6}\left(\frac{d^3T}{dt^3}\right)_t + \cdots$$

$$\left(\frac{dT}{dt}\right)_t = \frac{T(t+\Delta t) - T(t)}{\Delta t} - \frac{(\Delta t)}{2}\left(\frac{d^2T}{dt^2}\right)_t - \frac{(\Delta t)^2}{6}\left(\frac{d^3T}{dt^3}\right)_t + \cdots$$

Forward difference

$$\left(\frac{dT}{dt}\right)_t = \frac{T(t) - T(t-\Delta t)}{\Delta t} + \frac{(\Delta t)}{2}\left(\frac{d^2T}{dt^2}\right)_t - \frac{(\Delta t)^2}{6}\left(\frac{d^3T}{dt^3}\right)_t + \cdots$$

Backward difference

$$\left(\frac{dT}{dt}\right)_t = \frac{T_{t+1} - T_{t-1}}{2\Delta t} - \frac{(\Delta t)^2}{6}\left(\frac{d^3T}{dt^3}\right)_t + \cdots$$

Centred difference

- The error is proportional to the time step Δt
- Error is proportional to Δt for forward and backward approximations
- Error is proportional to $\Delta t^2$ for central approximation
- For small Δt central difference error will be smaller

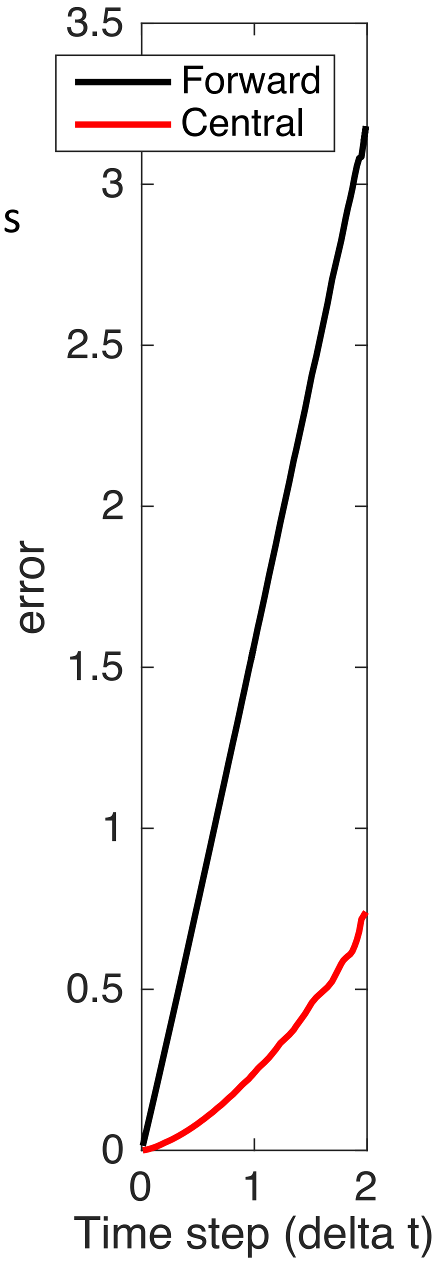
**Solving the Model**

- The error is proportional to the time step Δt
- Error is proportional to Δt for forward and backward approximations
- Error is proportional to $\Delta t^2$ for central approximation
- For small Δt central difference error will be smaller

$$\frac{dT}{dt} = \sqrt{t}$$

$$T(t + \Delta t) = T(t) + \Delta t\sqrt{t}$$ Forward difference

$$T(t + \Delta t) = T(t - \Delta t) + 2\Delta t\sqrt{t}$$ Centred difference



**Solving the Model**

Other more sophisticated schemes provide higher order correction terms
e.g a commonly used scheme is the fourth order Runge Kutta

$$\frac{dT}{dt} = F(T, t)$$

$$k_1 = \Delta t\, F(T(t), t)$$
$$k_2 = \Delta t\, F(T + \frac{k_1}{2}, t + \frac{\Delta t}{2})$$
$$k_3 = \Delta t\, F(T + \frac{k_2}{2}, t + \frac{\Delta t}{2})$$
$$k_4 = \Delta t\, F(T + k_3, t + \Delta t)$$

$$\mathrm{T}(\mathrm{t} + \Delta t) = T(t) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O[\Delta t^5]$$

## Solving the Model

$$T(t + \Delta t) = T(t) + \frac{\Delta t(RF - \lambda T - \gamma(T - T_o))}{C}$$

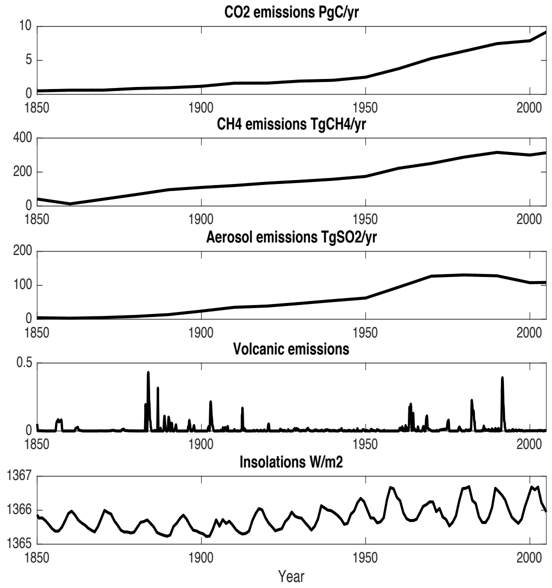
$$T_o(t + \Delta t) = T_o(t) + \frac{\Delta t(\gamma(T - T_o))}{C_o}$$

Forward difference

Code for simplest forward differencing:

```
T(1)=0; % initialise the upper ocean temperature
To(1)=0; % initialise the lower ocean temperature

for t=2:N  % loop over time steps
   RF = R_CO2(t-1) + R_CH4(t-1) + R_SO2(t-1) + R_volc(t-1) R_sol(t-1);
   % Solve for T and To at time t based on T and To at time t-1
   T(t)   = T(t-1) + DT*( RF - L*T(t-1) - g*(T(t-1)-To(t-1)) )/C ;
   To(t) = To(t-1) + DT*( g*(T(t-1)-To(t-1)) )/Co;
end
```




# **Using the model:** Evaluation

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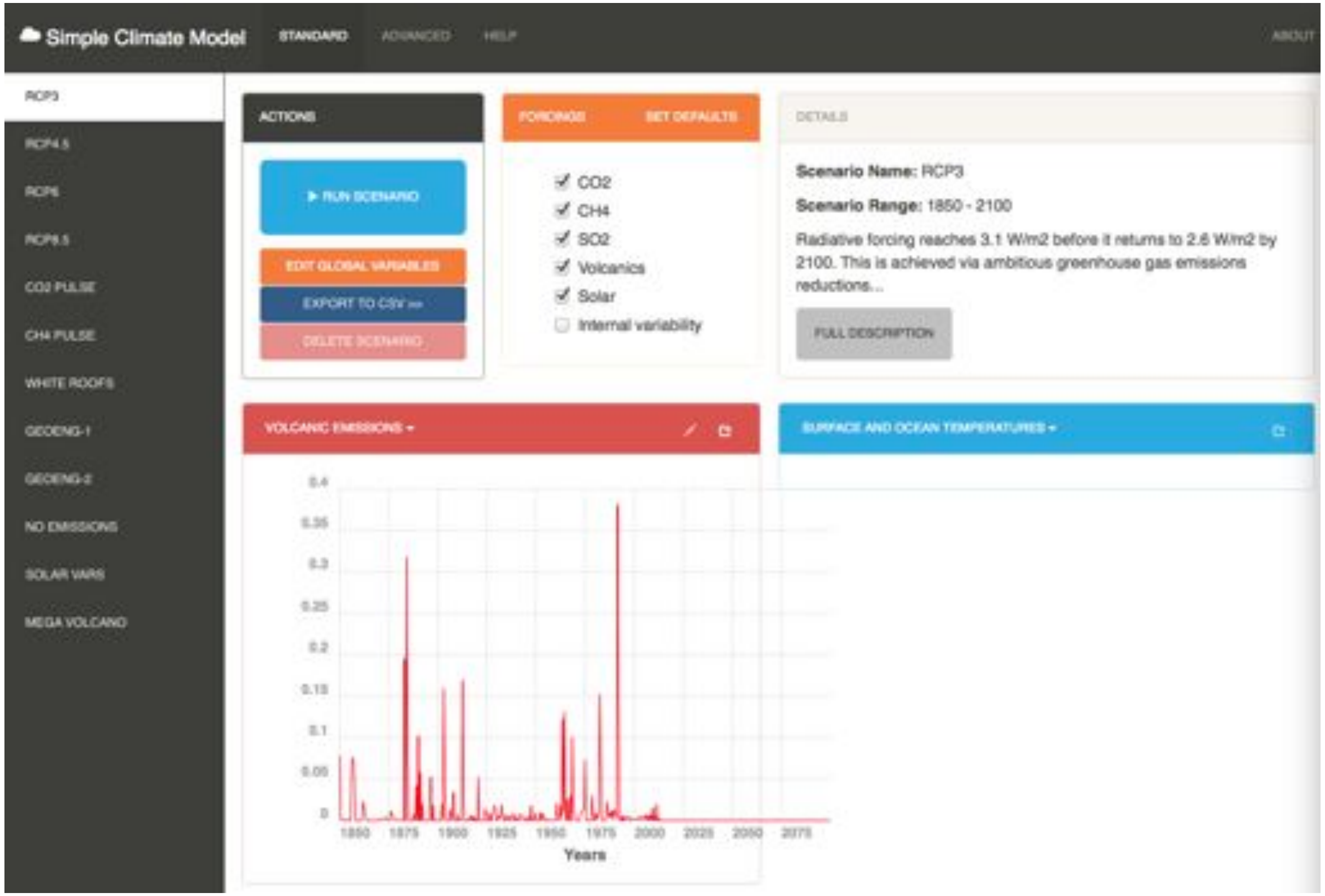

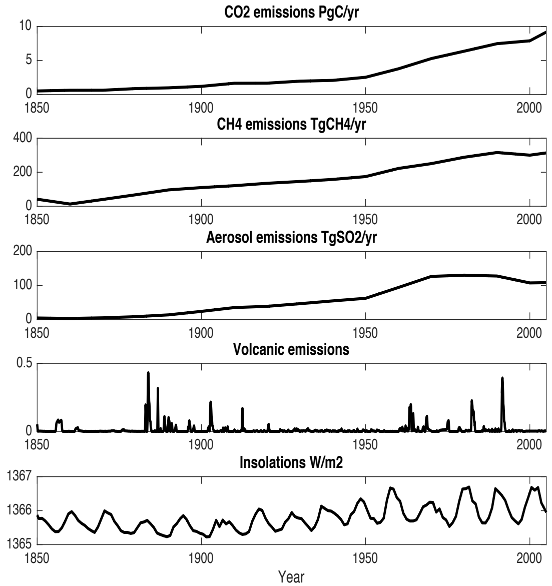


**Using the model:** Evaluation

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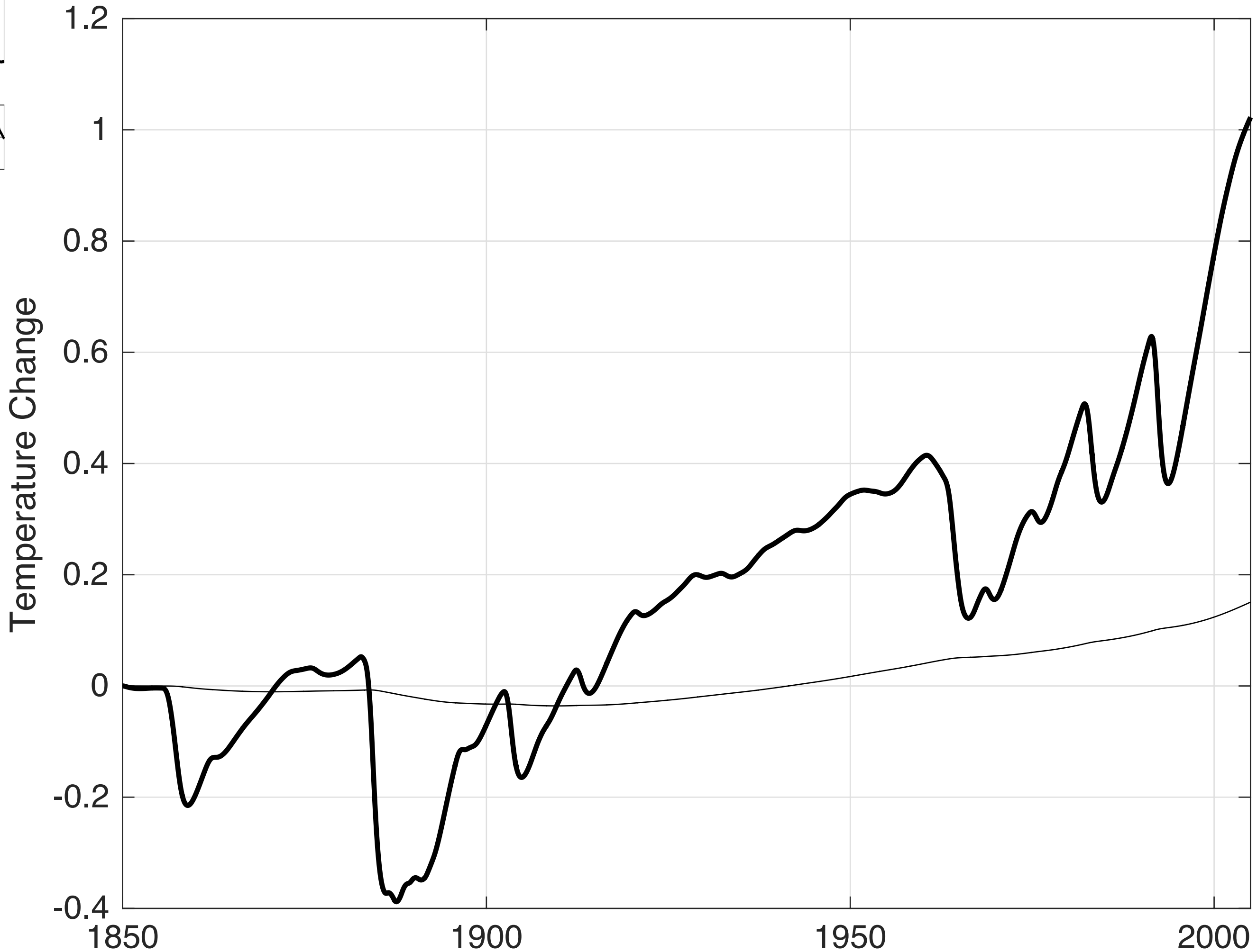
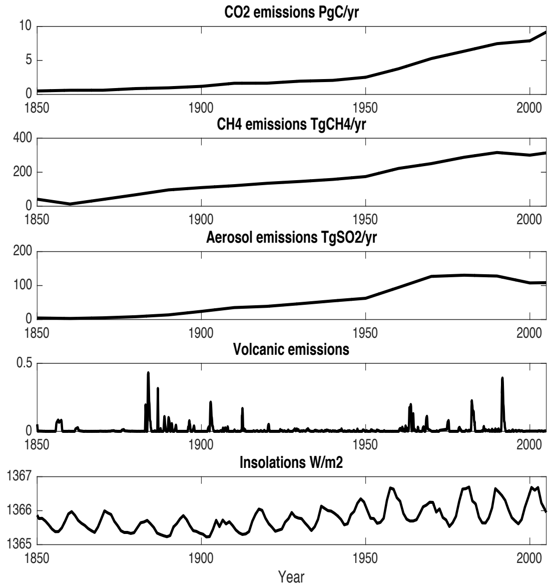
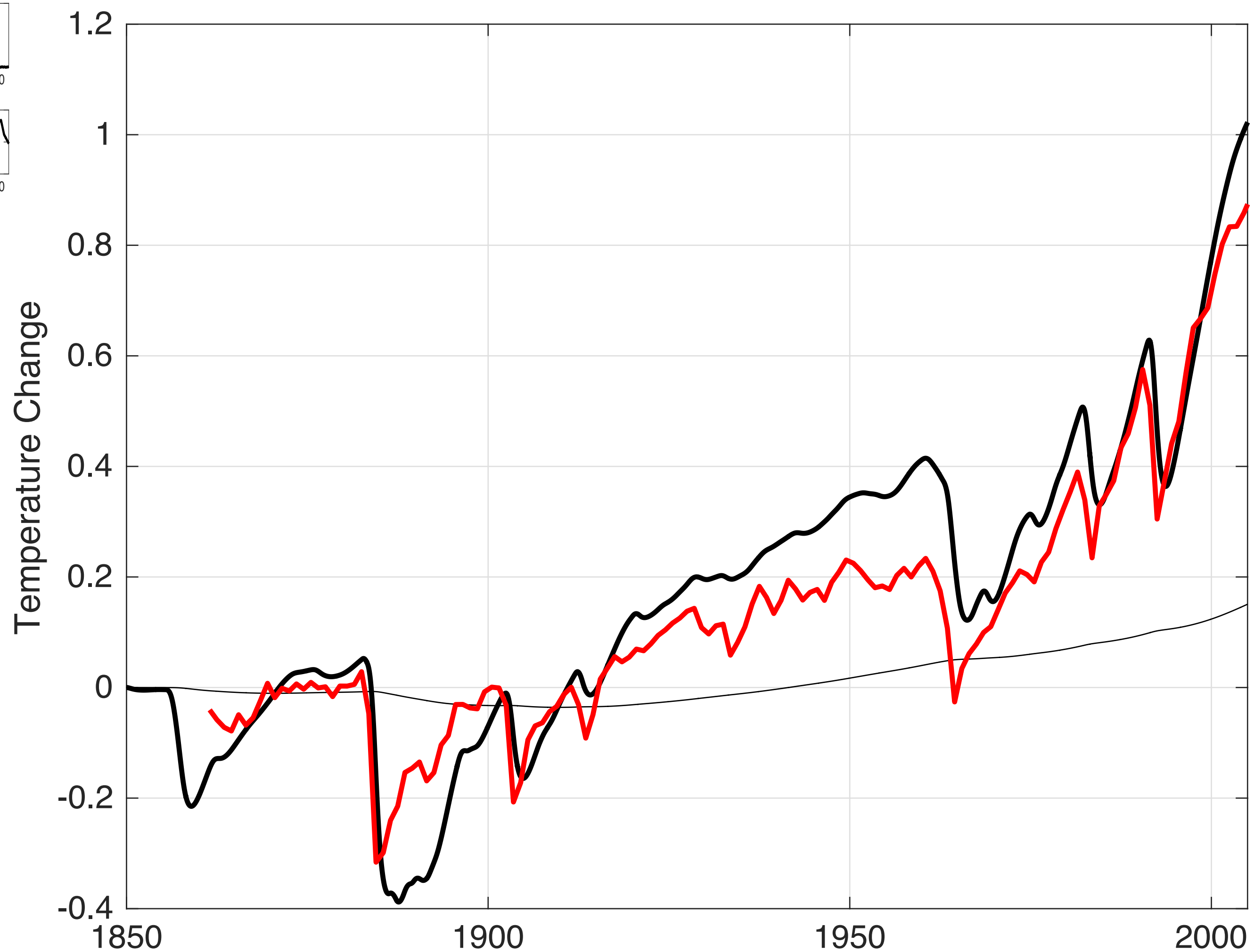
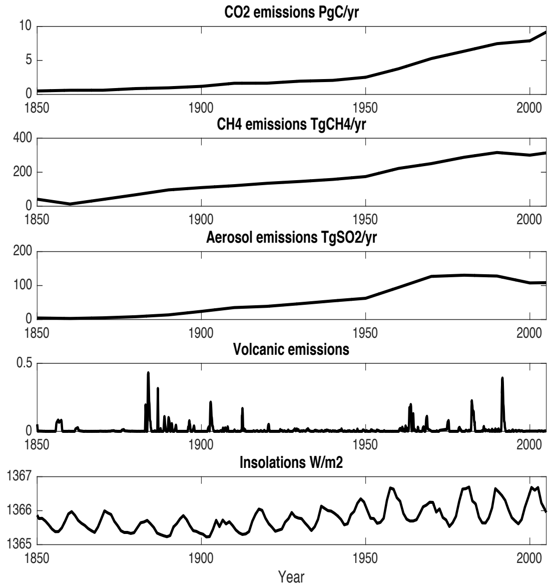


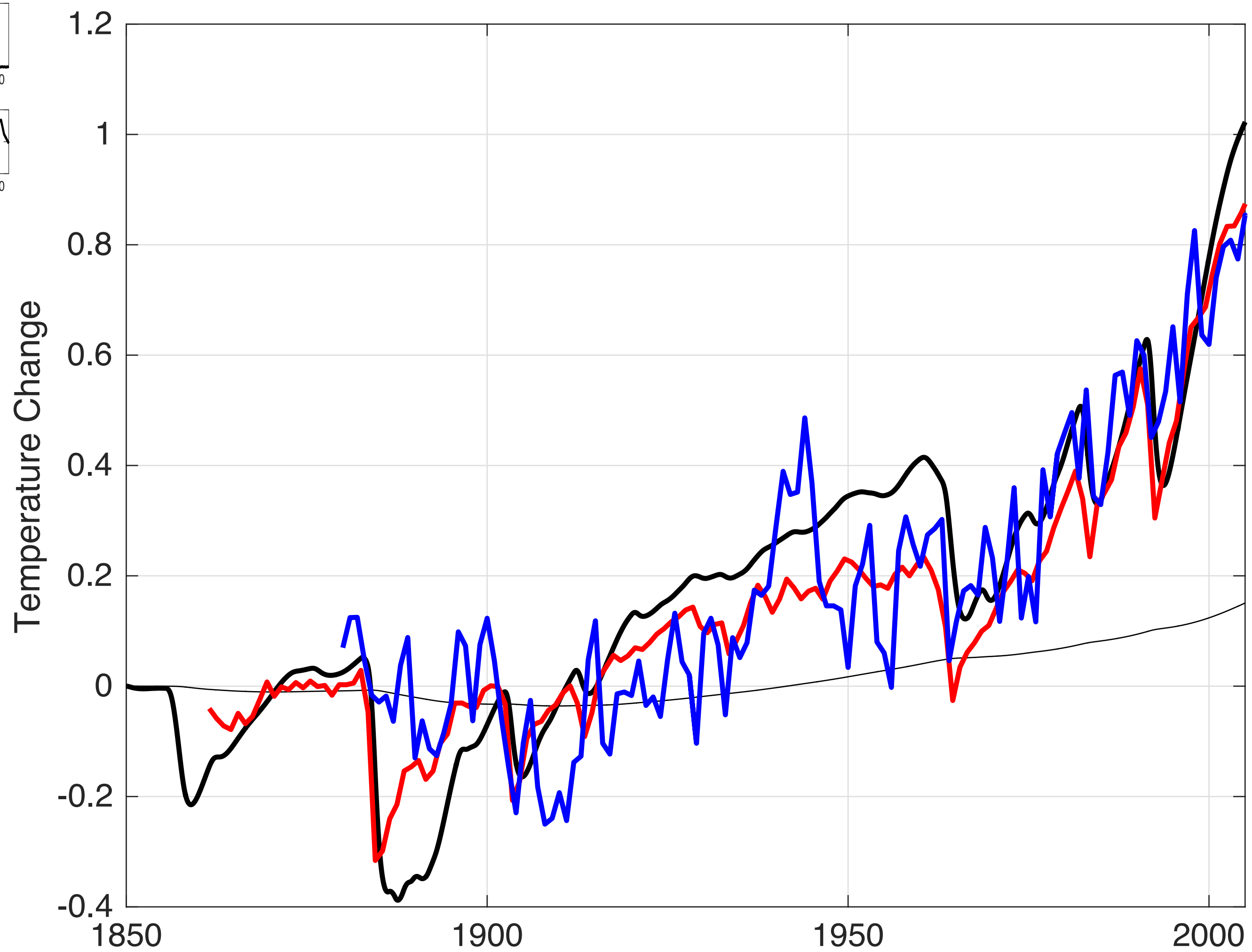
# **Using the model:** Evaluation



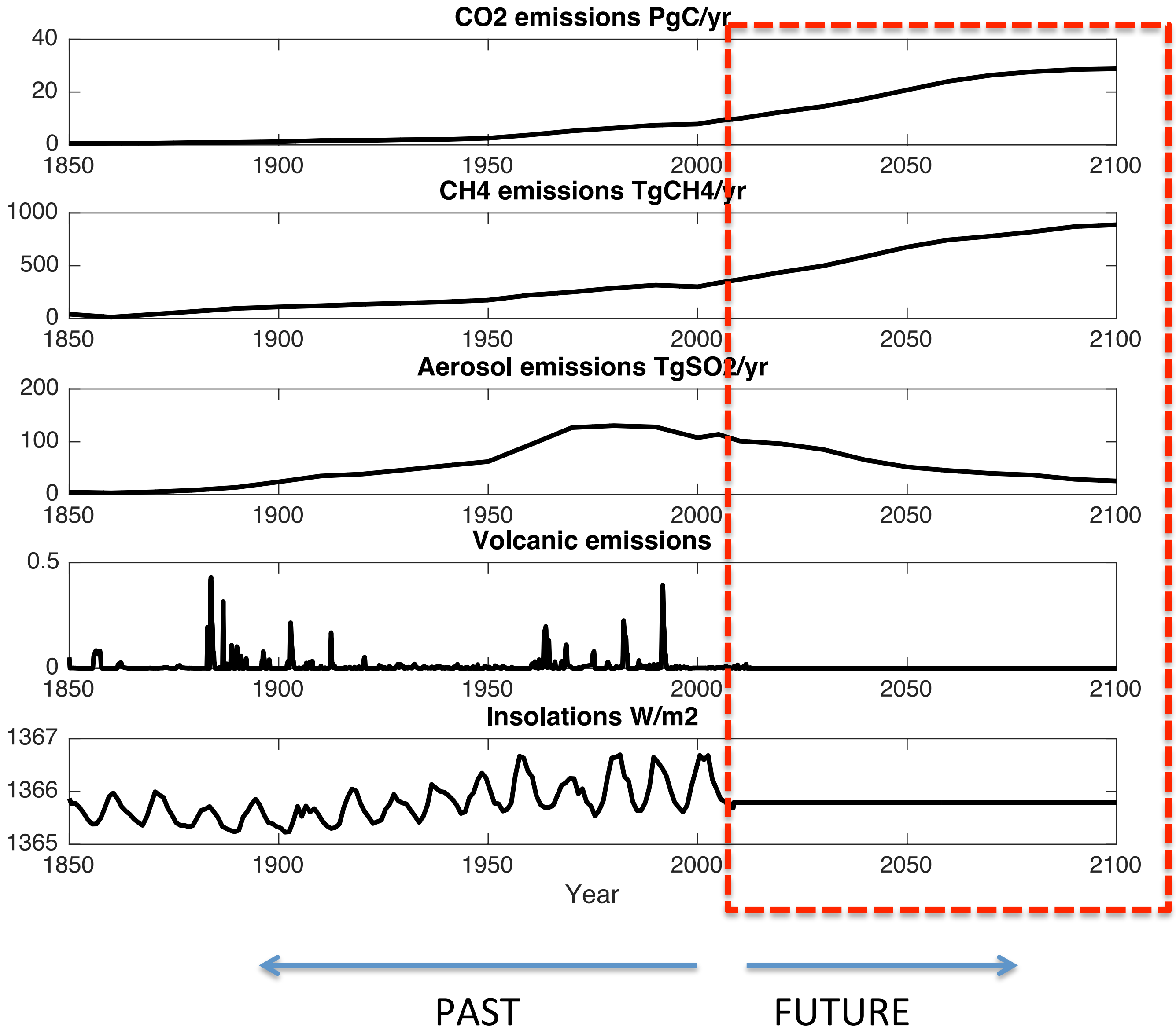
# **Using the model:** Evaluation



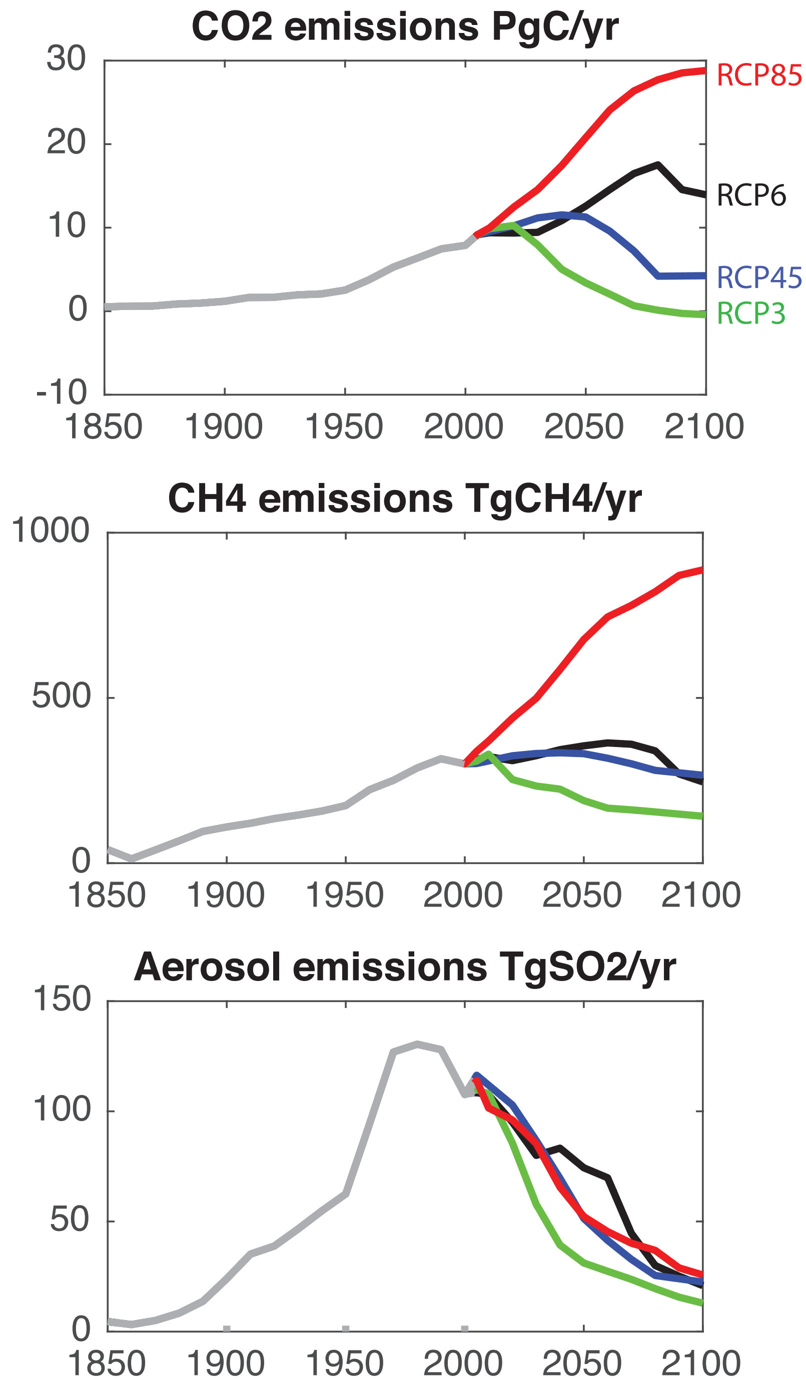



**Using the model:** Projections

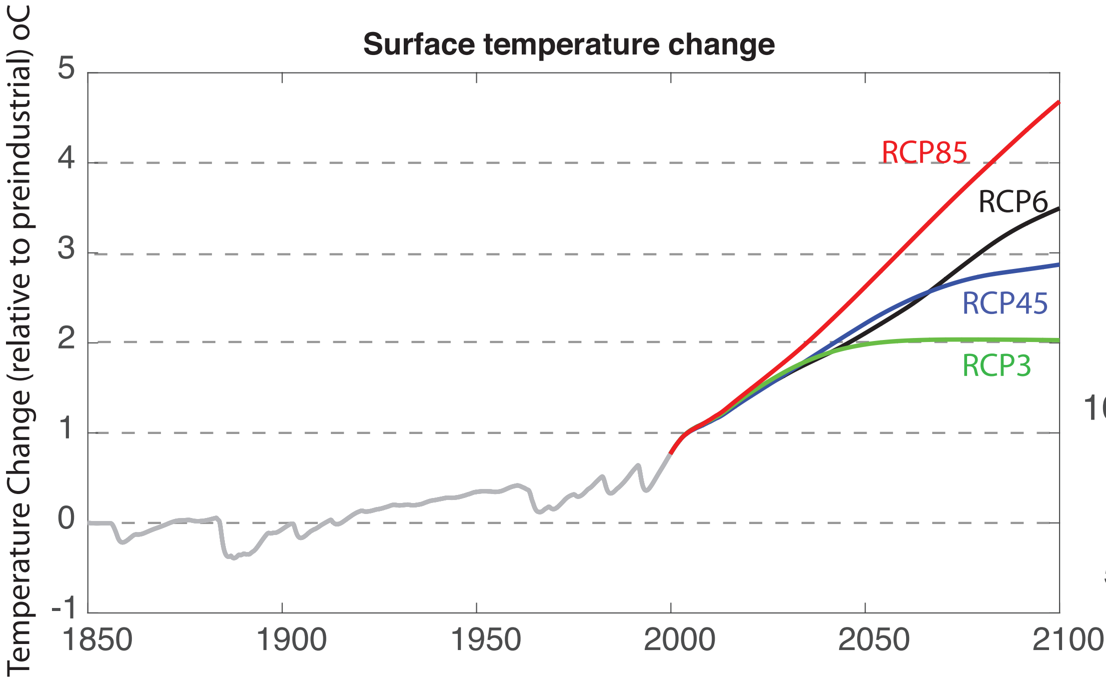
Scenario of possible future emissions

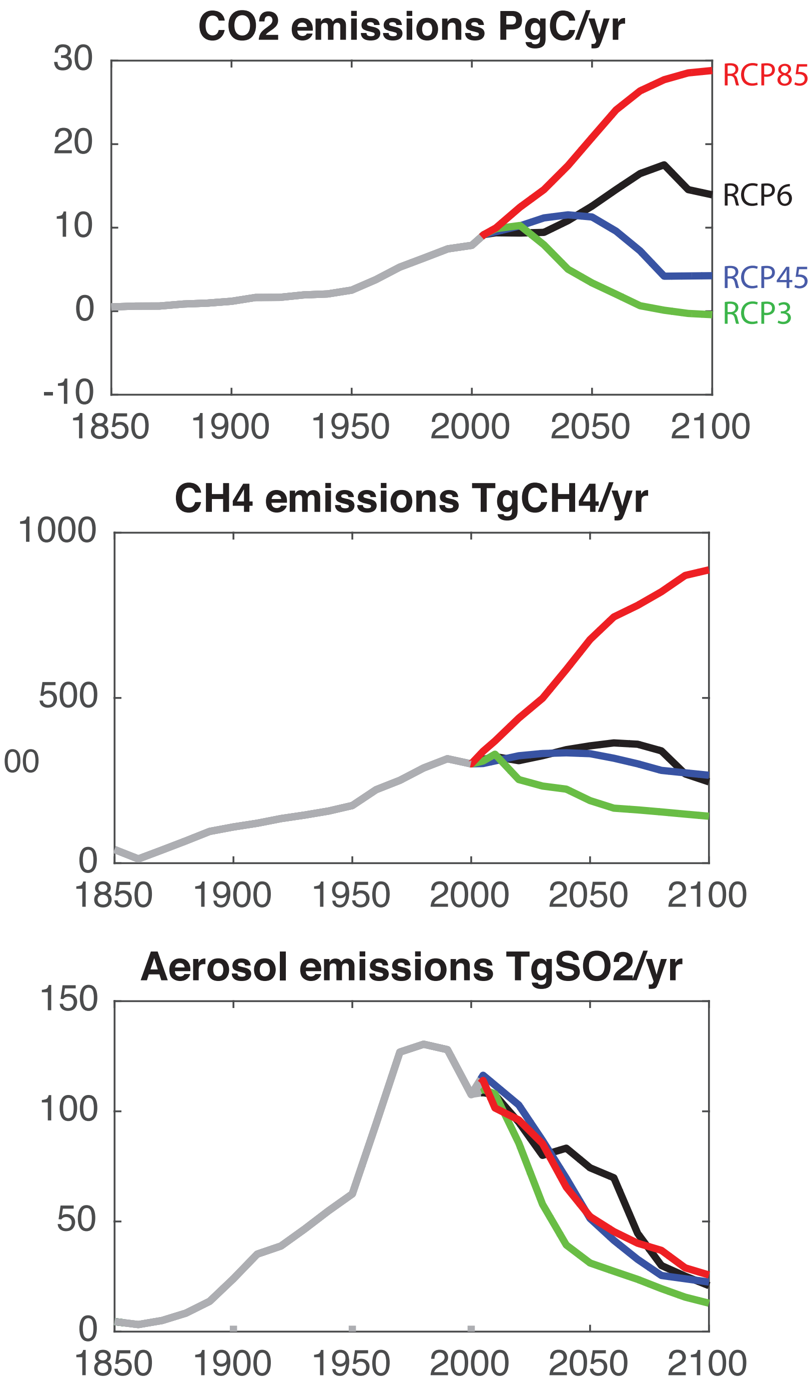


**Using the model:** Projections

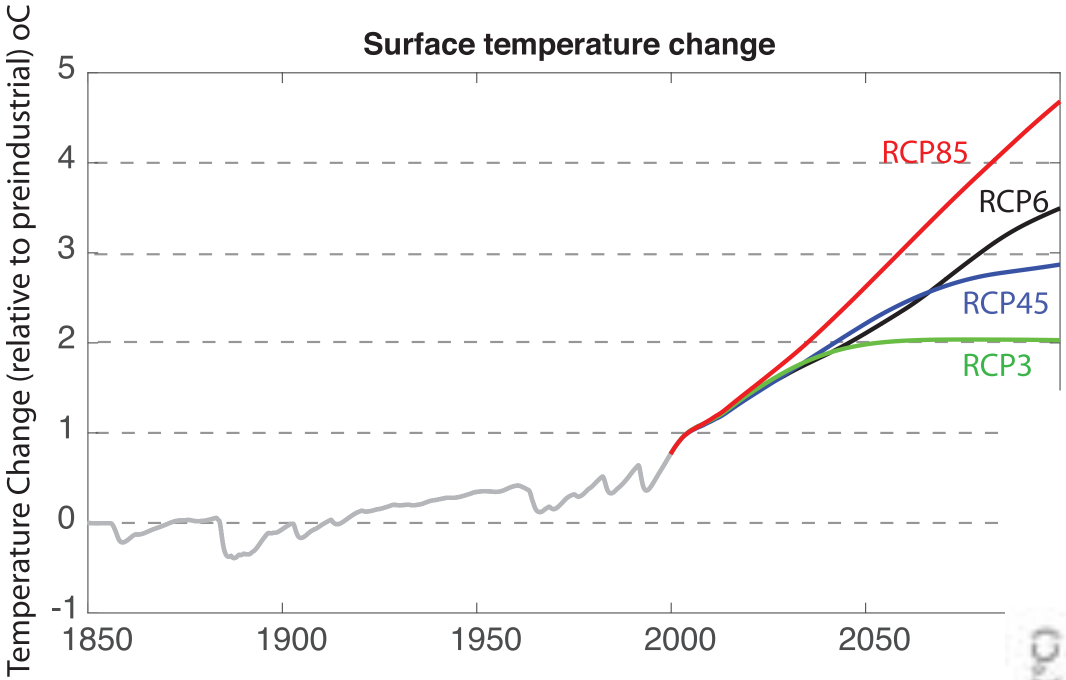


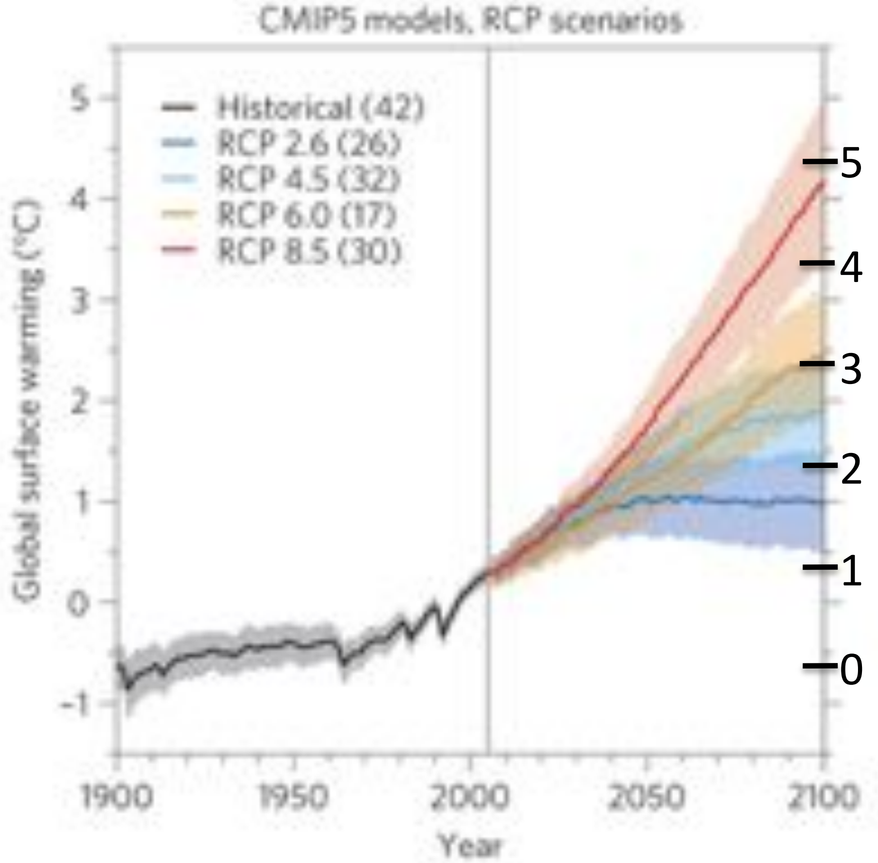
## **Using the model:** Projections



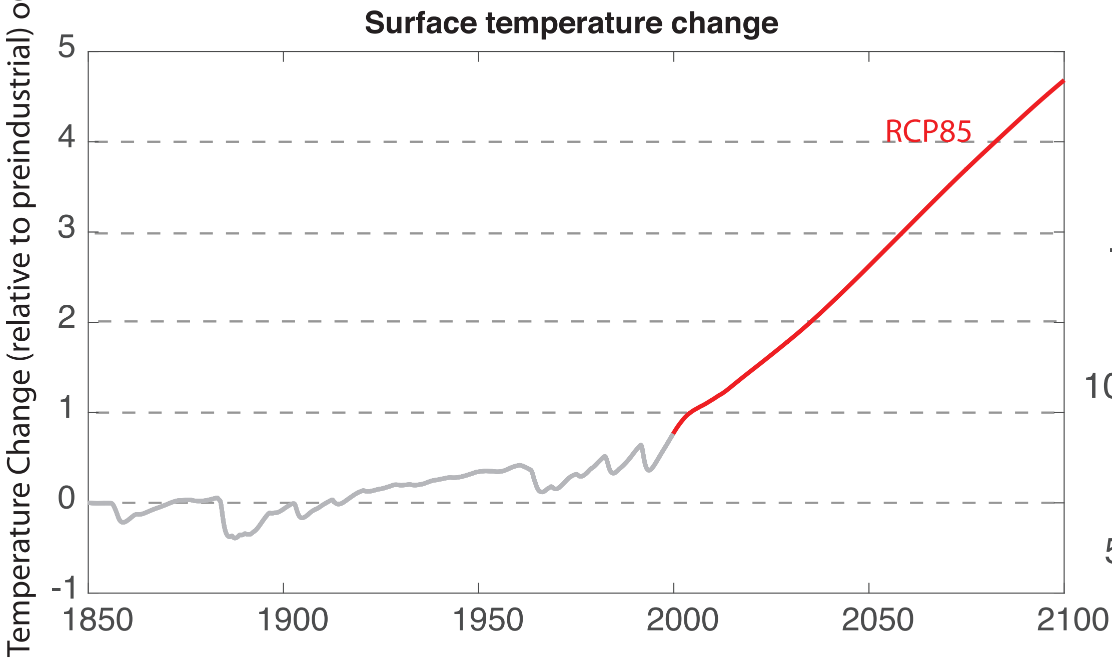



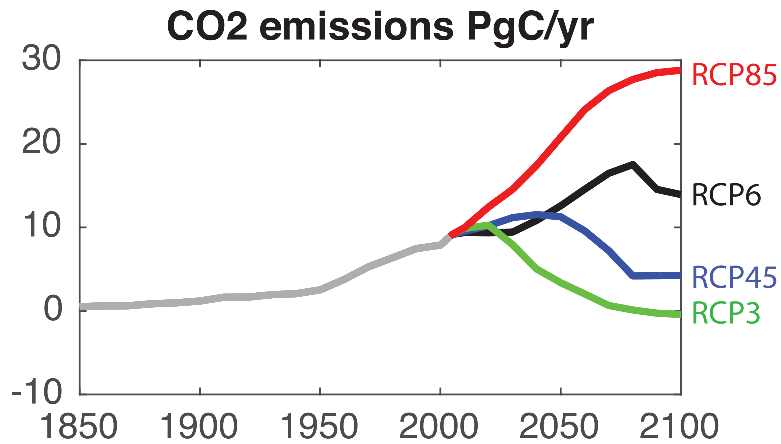
**Using the model:** Projections

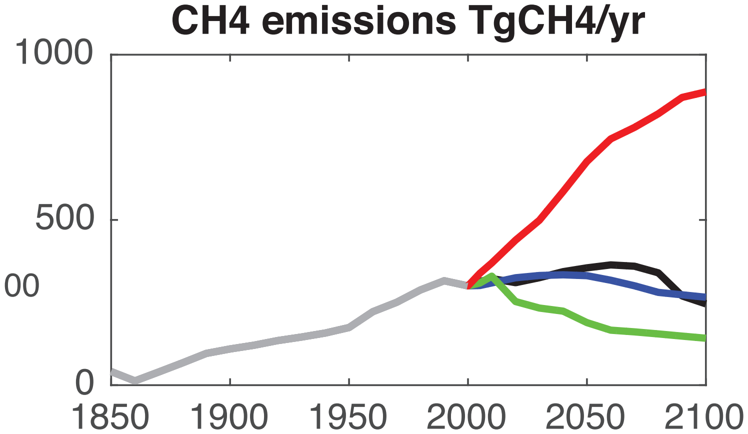


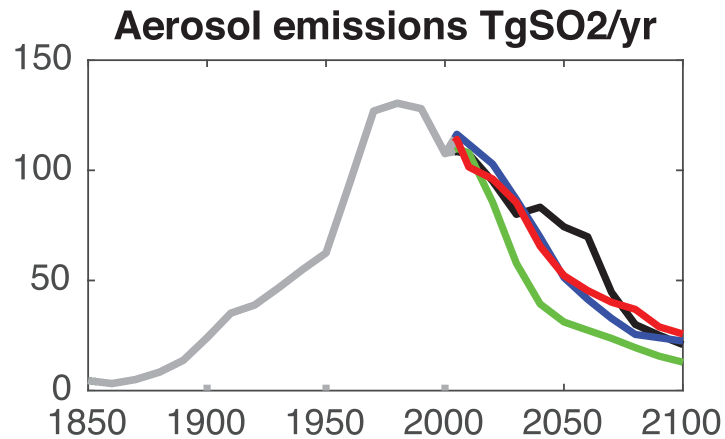



## **Using the model:** Geoengineering

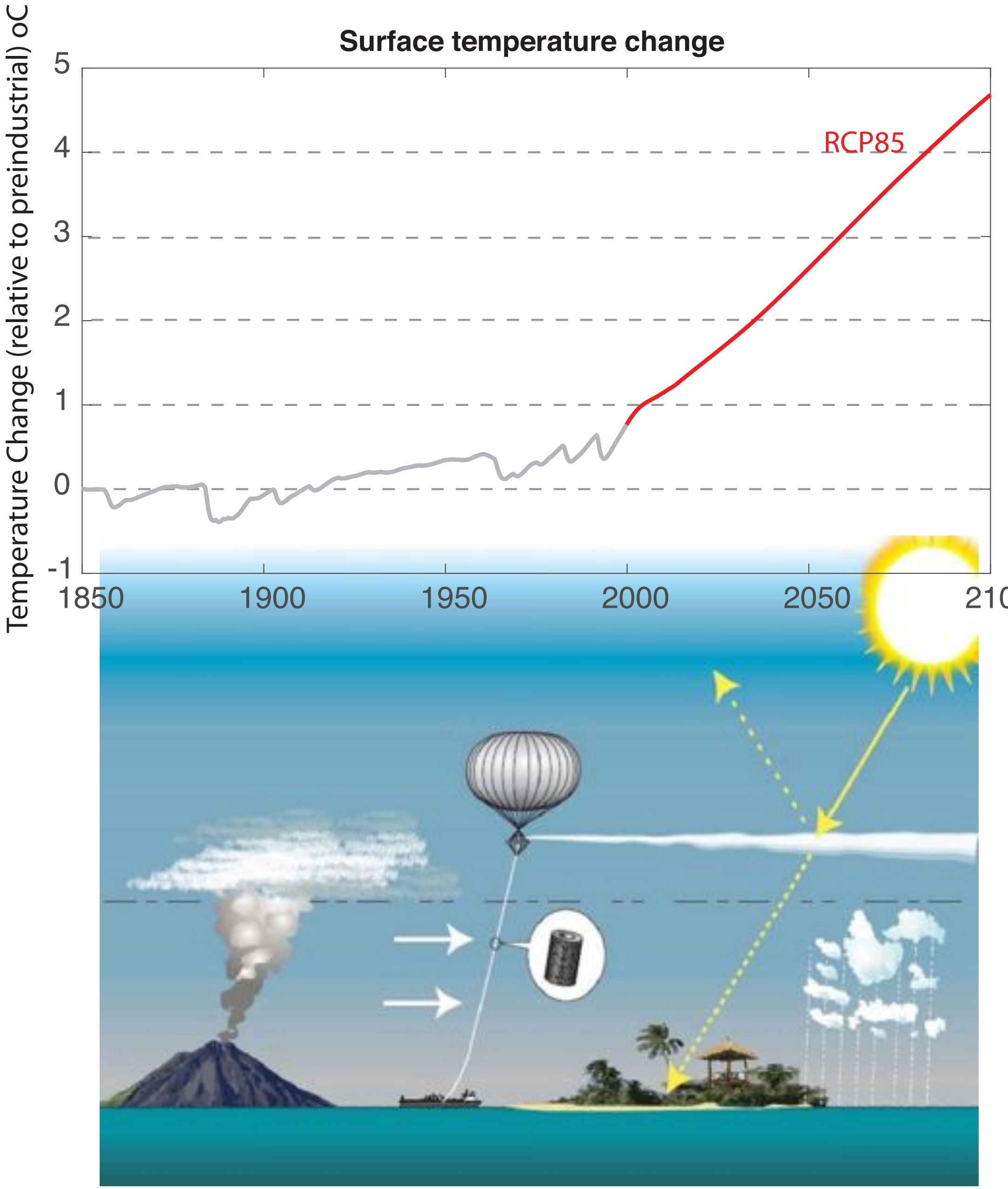


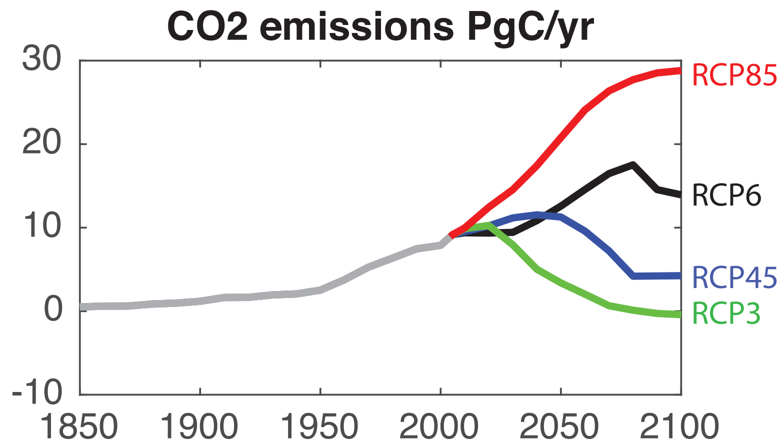



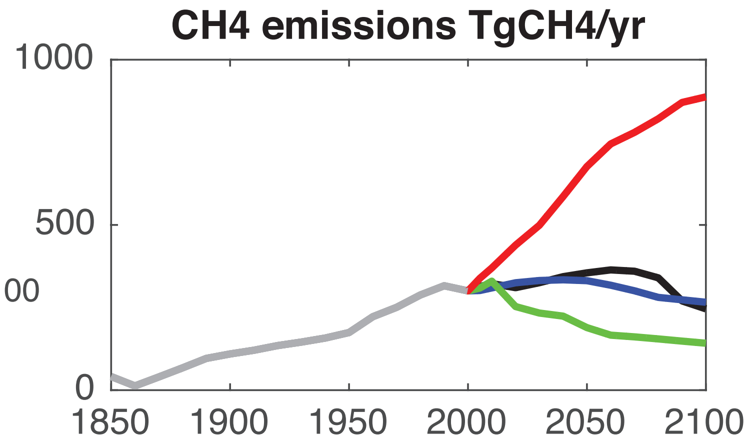



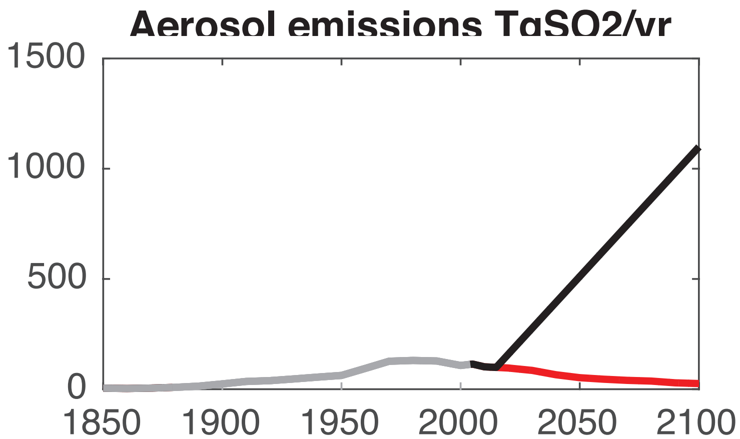



## **Using the model:** Geoengineering

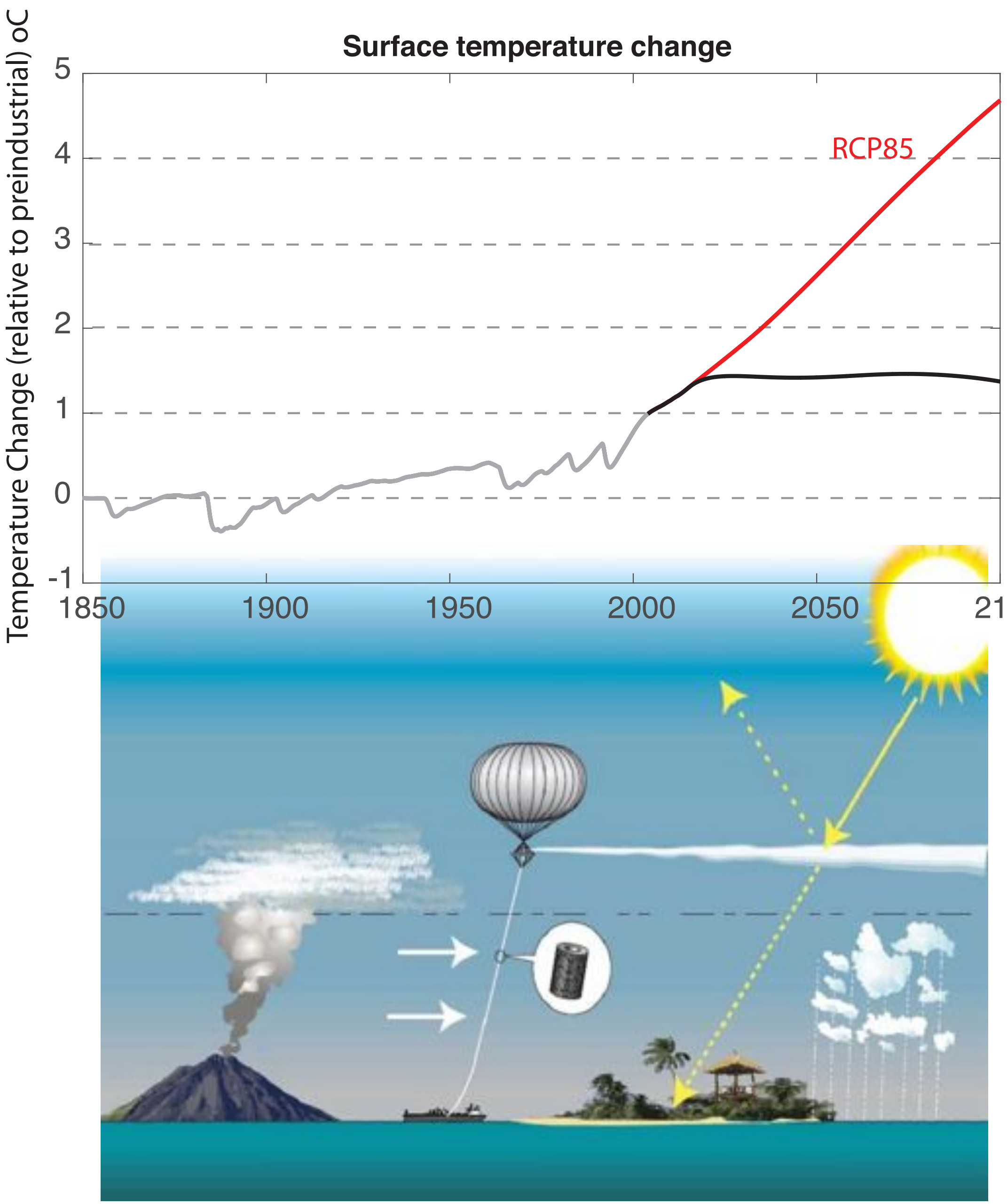


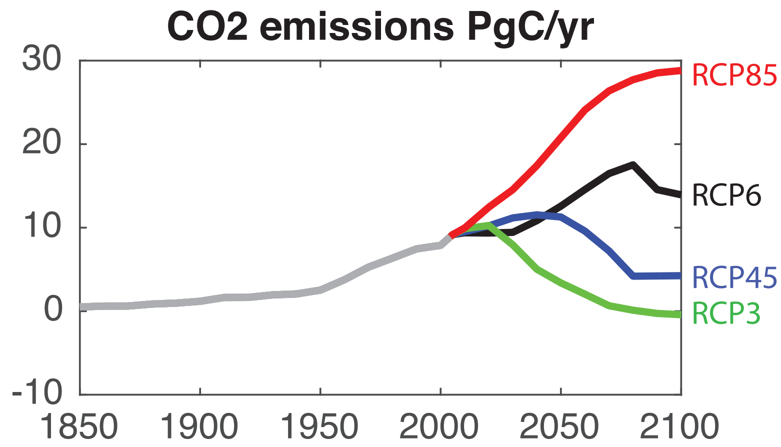



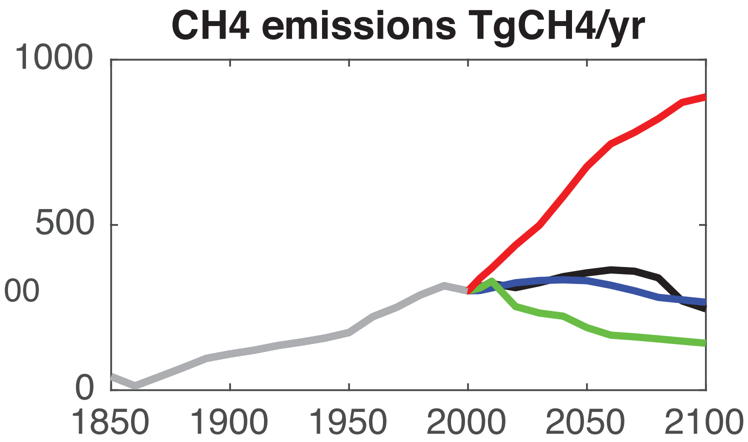



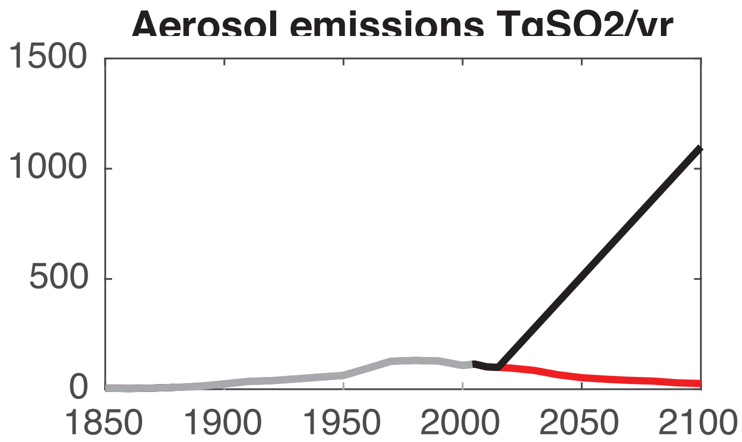



## **Using the model:** Geoengineering

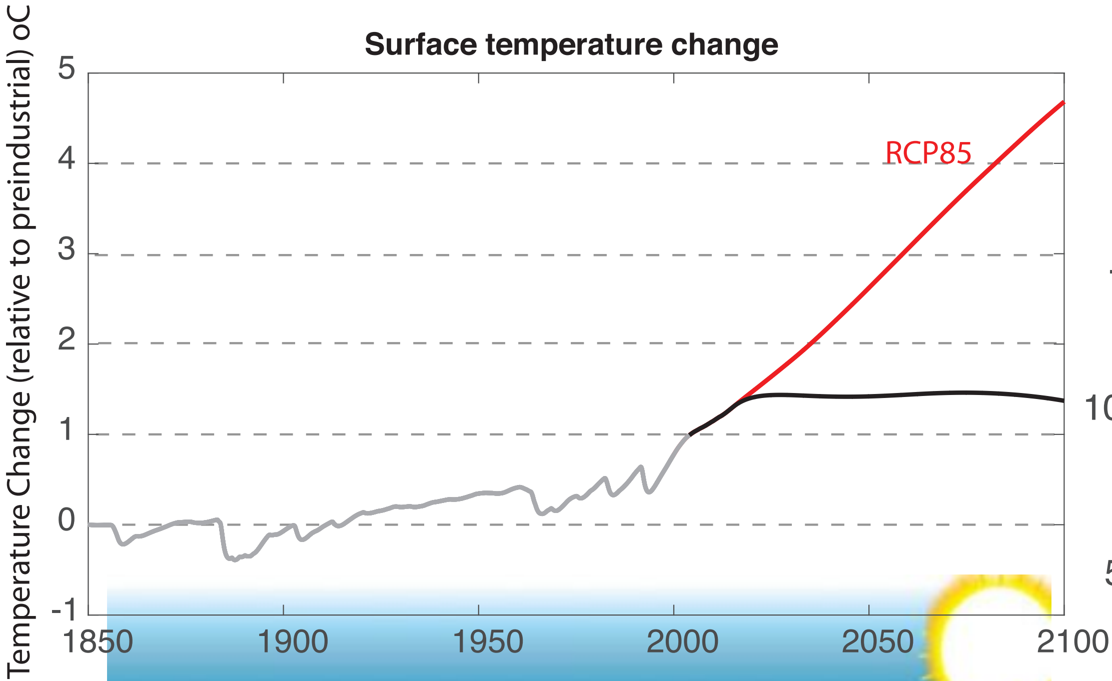


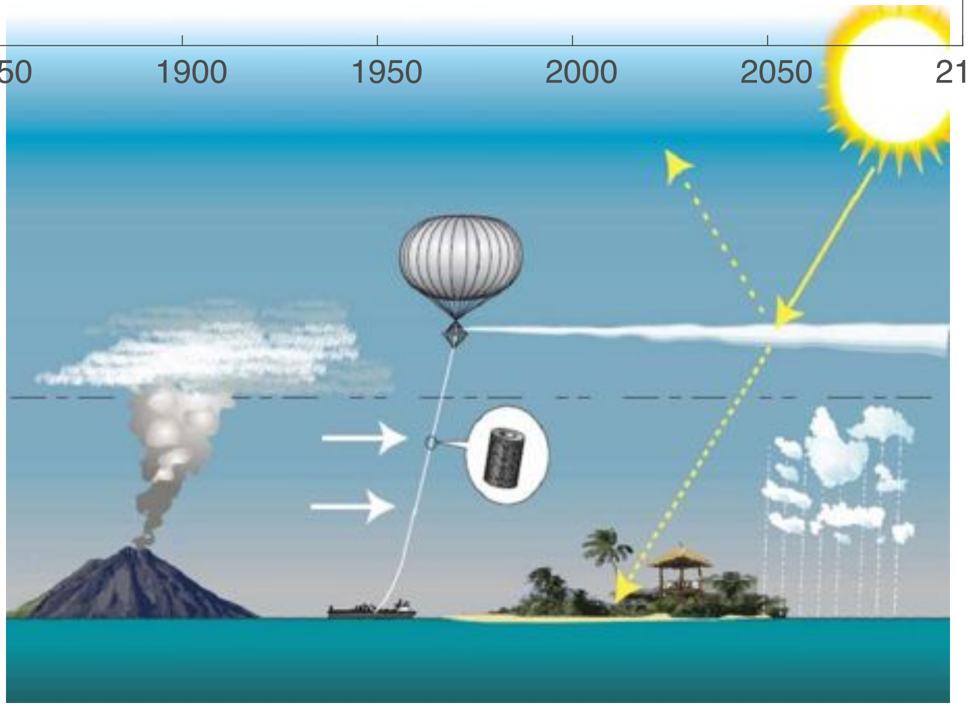



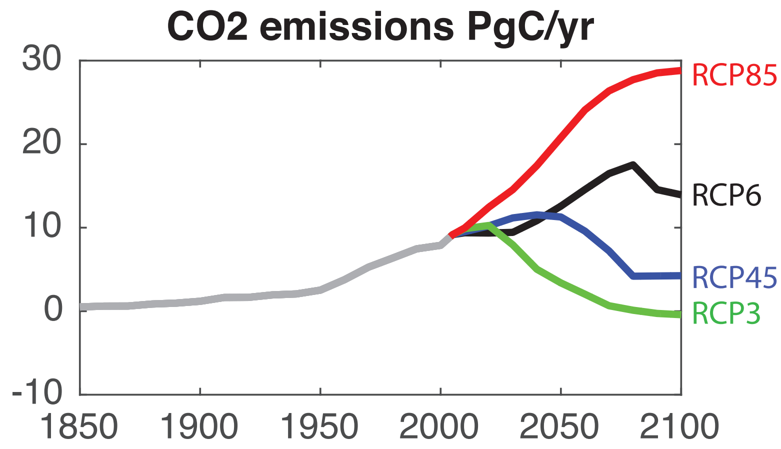



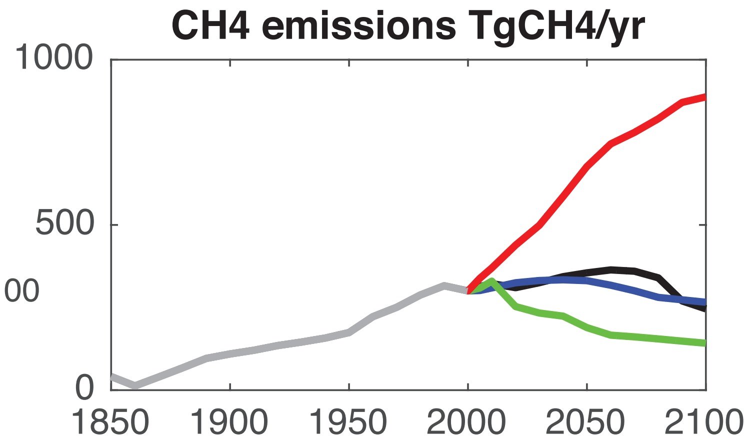



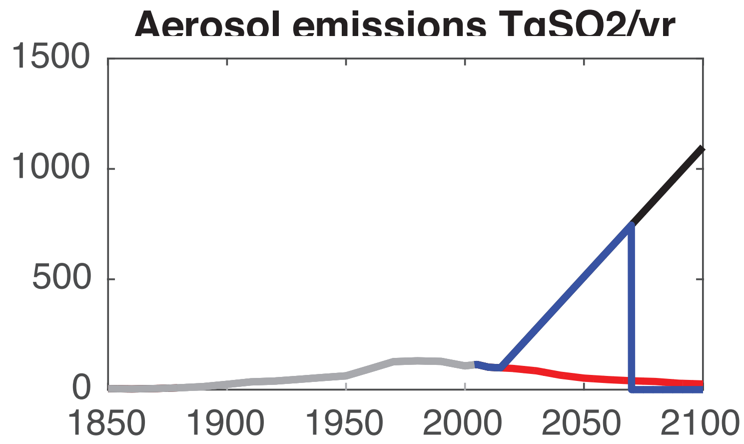
## **Using the model:** Geoengineering



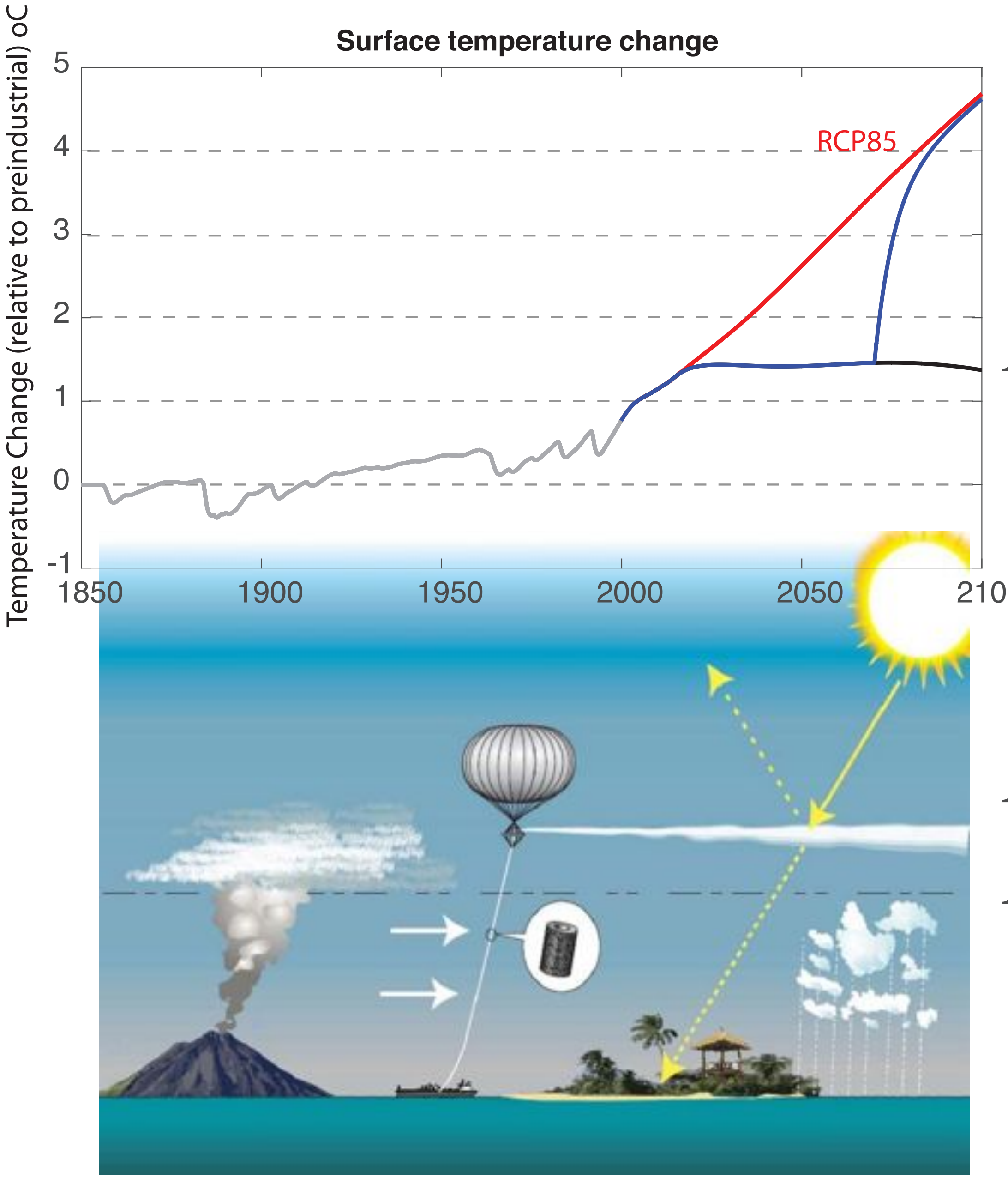



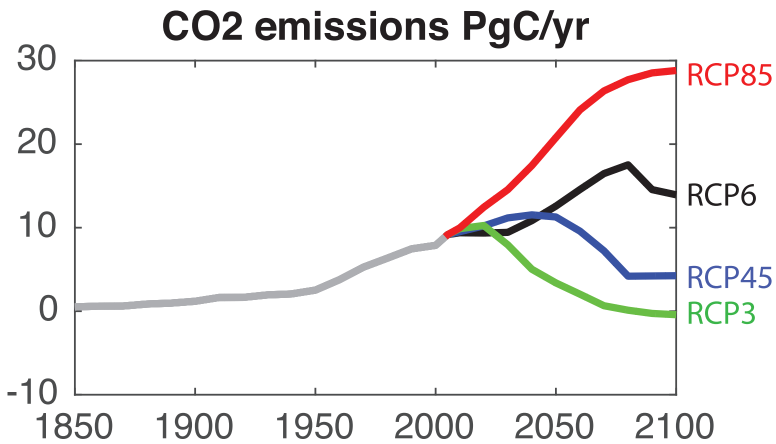


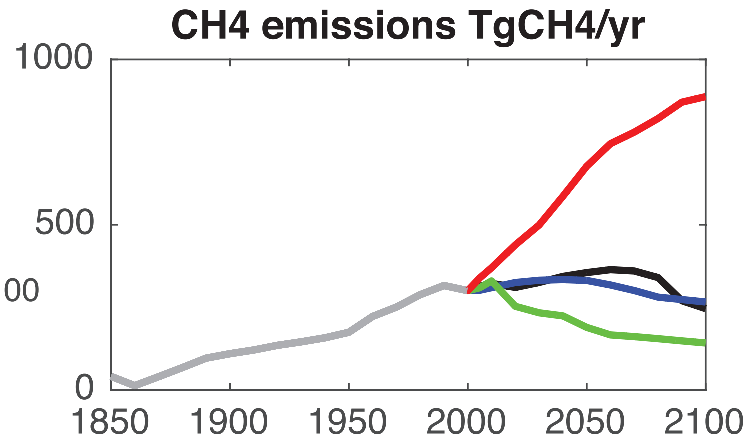



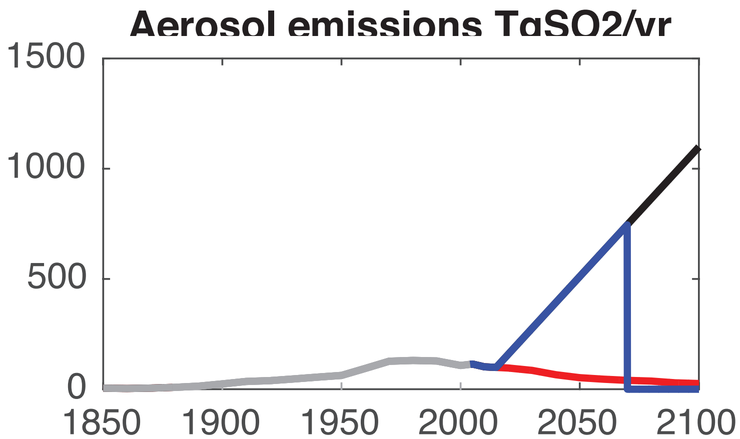



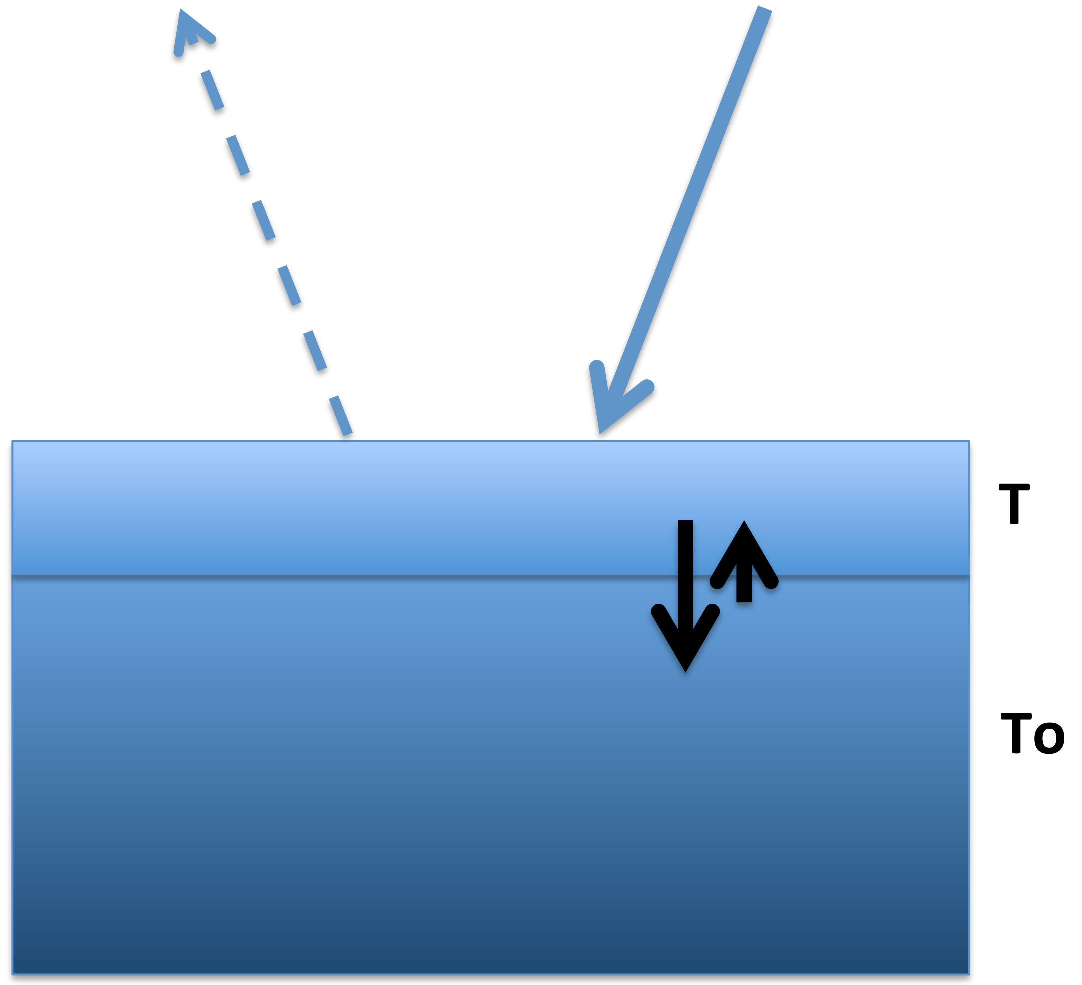
**Using the model:** Geoengineering

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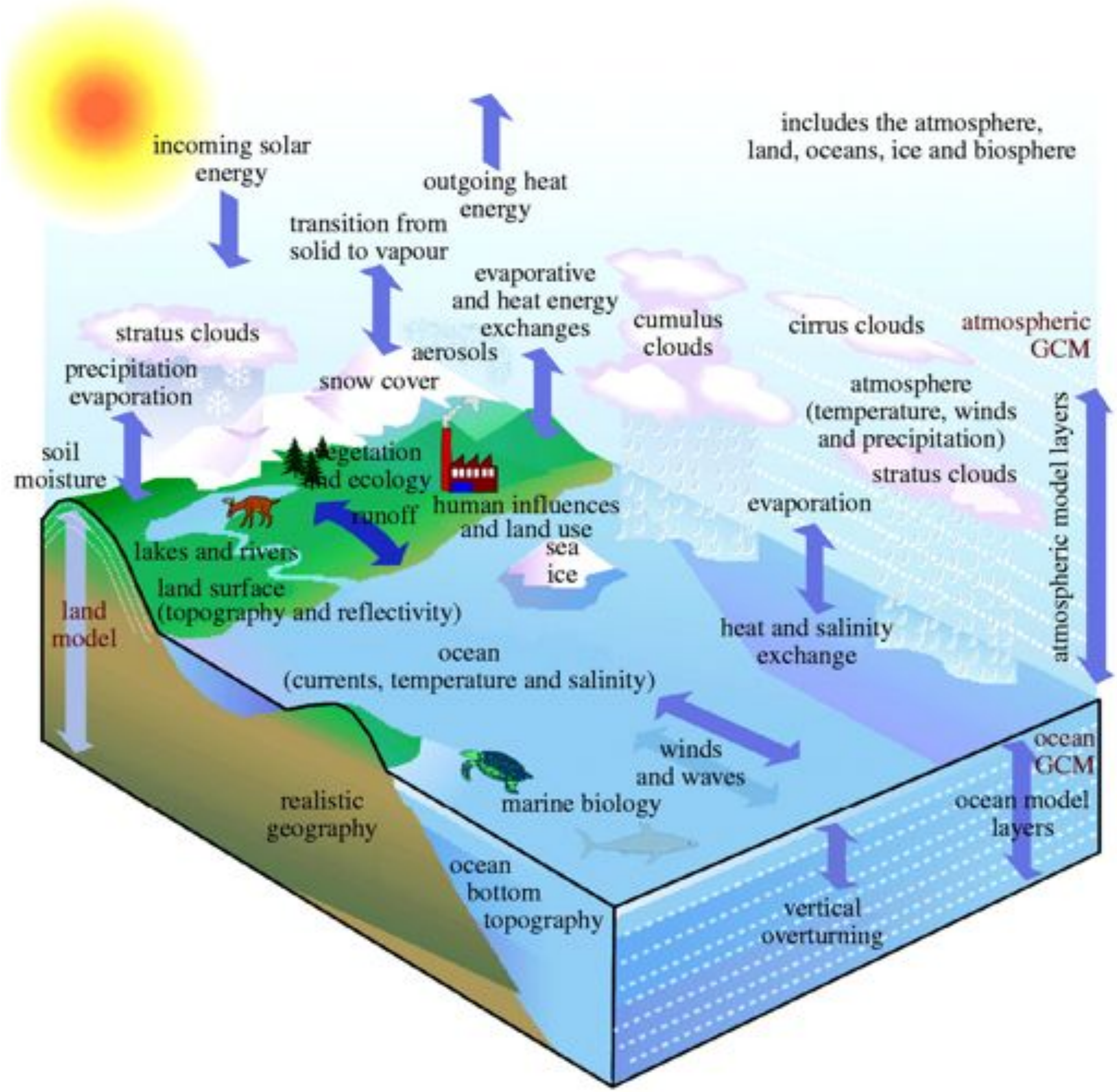

incoming solar energy
outgoing heat energy
includes the atmosphere, land, oceans, ice and biosphere
transition from solid to vapour
evaporative and heat energy exchanges
stratus clouds
cumulus clouds
cirrus clouds
atmospheric GCM
aerosols
precipitation evaporation
snow cover
atmosphere (temperature, winds and precipitation)
soil moisture
vegetation and ecology
stratus clouds
runoff
human influences and land use
evaporation
atmospheric model layers
lakes and rivers
sea ice
land surface (topography and reflectivity)
land model
heat and salinity exchange
ocean (currents, temperature and salinity)
winds and waves
ocean GCM
marine biology
ocean model layers
realistic geography
ocean bottom topography
vertical overturning

## Governing equations of the Ocean

$$\frac{D}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + A_Z(\frac{\partial^2 u}{\partial z^2})$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + A_H(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + A_Z(\frac{\partial^2 v}{\partial z^2})$$

$$\frac{Dw}{Dt} + hu = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + A_H(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) + A_Z(\frac{\partial^2 w}{\partial z^2})$$

F=ma

$f = 2\Omega \sin\theta,$ $h = 2\Omega \cos\theta$

$$\frac{DS}{dt} = K_s\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}\right)$$

Conservation of salt

$$\rho C_p\frac{D\theta}{dt} = K_\theta\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) + Q$$

Conservation of energy

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of mass

**Governing equations of the Ocean**

$$\frac{D}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Acceleration Coriolis Pressure gradients Horizontal friction Vertical friction

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + A_Z\left(\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + A_H\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + A_Z\left(\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{Dw}{Dt} + hu = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + A_H\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + A_Z\left(\frac{\partial^2 w}{\partial z^2}\right)$$

F=ma

$$f = 2\Omega\sin\theta, \qquad h = 2\Omega\cos\theta$$

$$\frac{DS}{dt} = K_s\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}\right)$$

Conservation of salt

$$\rho C_p\frac{D\theta}{dt} = K_\theta\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) + Q$$

Conservation of energy

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

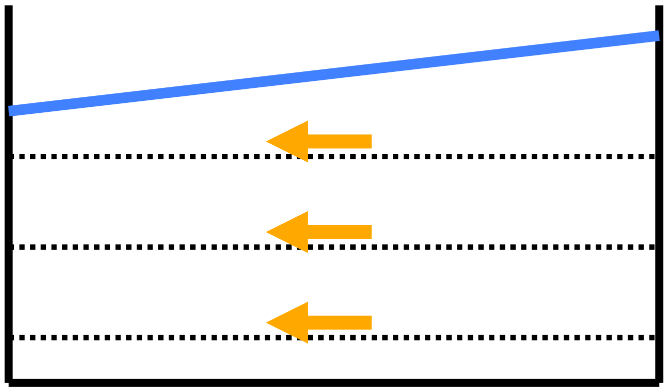
Conservation of mass

**Governing equations**

$$\frac{Du}{Dt} - \cancel{fv} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right) + A_Z\left(\cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

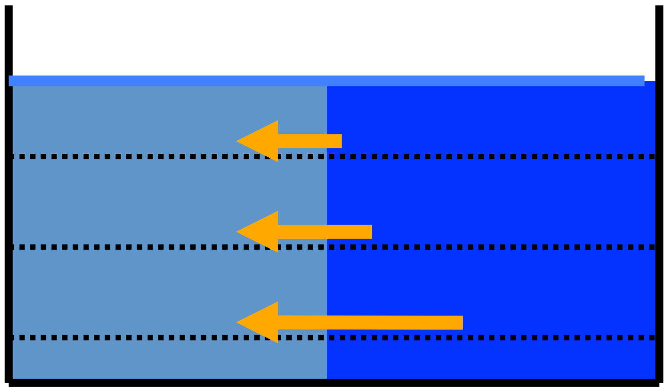
Pressure force:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$



$\rho_0$

Motion due to surface slopes



$\rho_1 < \rho_2$

Motion due to density differences

$$\frac{Du}{Dt} - fv = -\cancel{\frac{1}{\rho}\frac{\partial p}{\partial x}} + A_H(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}) + A_Z(\cancel{\frac{\partial^2 u}{\partial z^2}})$$

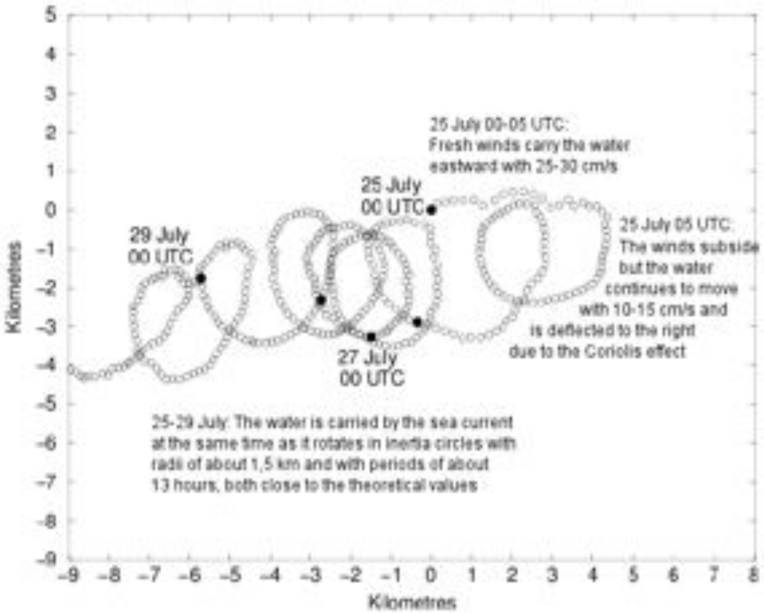
**Governing equations**

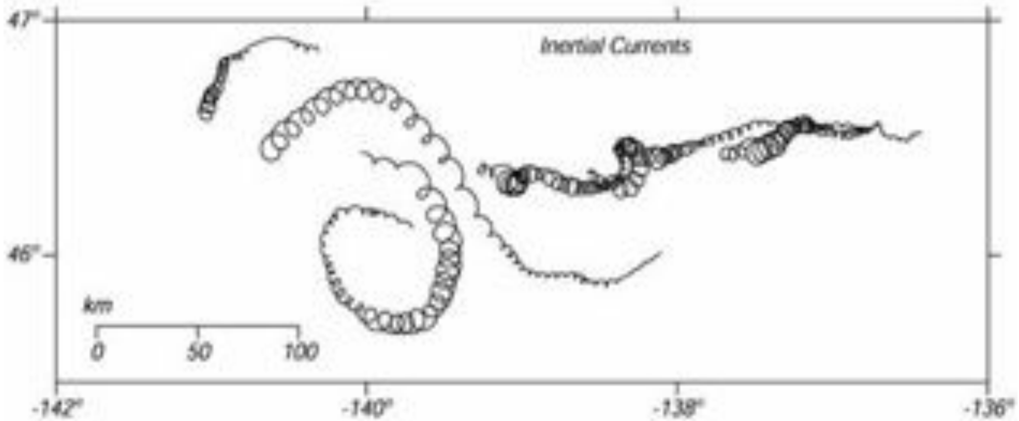
$$\frac{Du}{Dt} - fv = -\cancel{\frac{1}{\rho}\frac{\partial p}{\partial x}} + A_H(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}) + A_Z(\cancel{\frac{\partial^2 u}{\partial z^2}})$$

Inertial Oscillations:

$$\frac{du}{dt} = fv, \qquad \frac{dv}{dt} = -fu, \qquad where\ f = 2\Omega \sin\,(latitude)$$

$$u = V\sin(ft), \qquad v = V\sin(ft), V^2 = u^2 + v^2$$



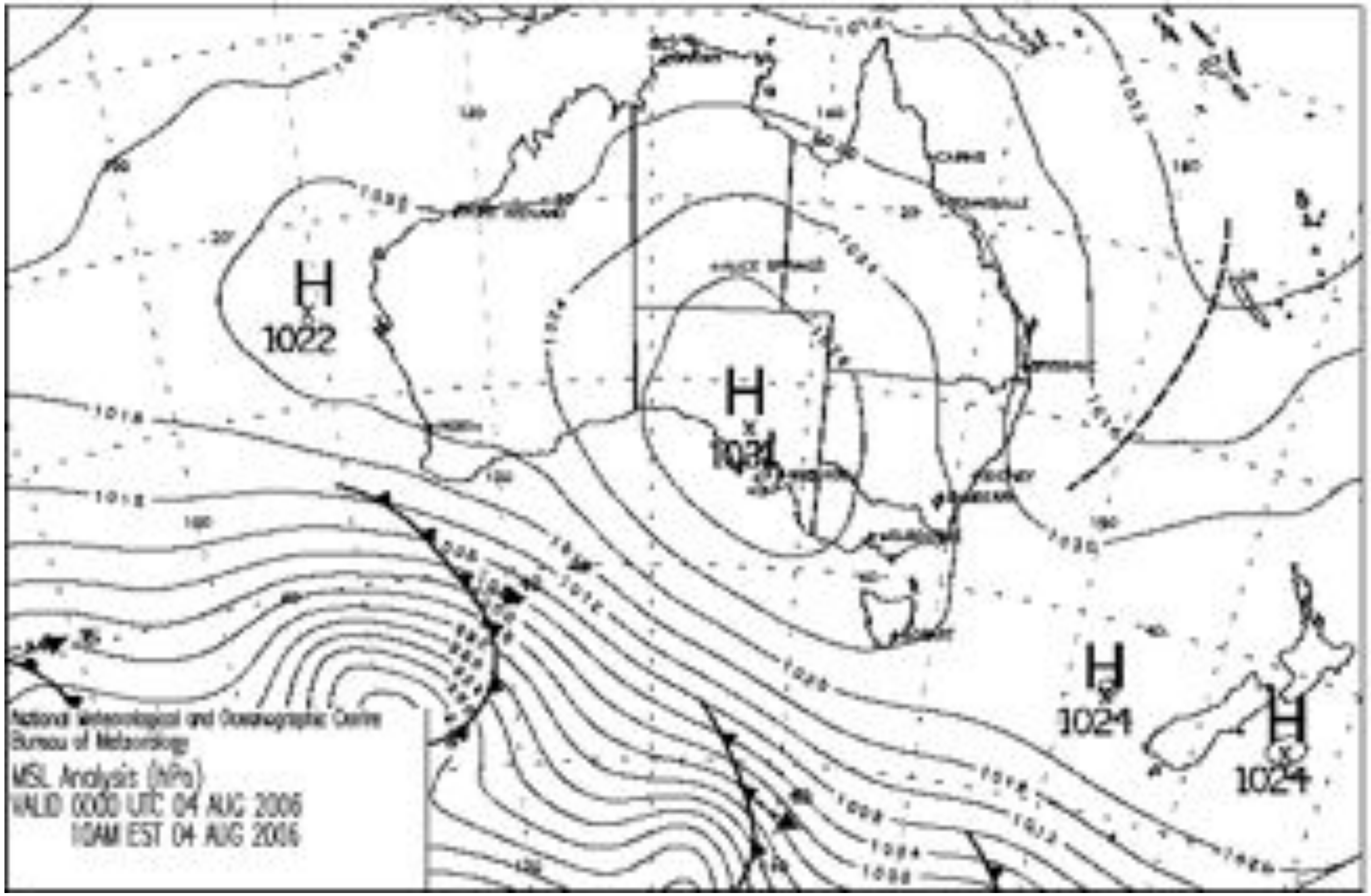
**Governing equations**

$$\xcancel{\frac{Du}{Dt}} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H\left(\xcancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right) + A_Z\left(\xcancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

Geostrophic flow:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv, \quad \frac{1}{\rho}\frac{\partial p}{\partial y} = -fu$$

$$where\ f = 2\Omega \sin\,(latitude)$$



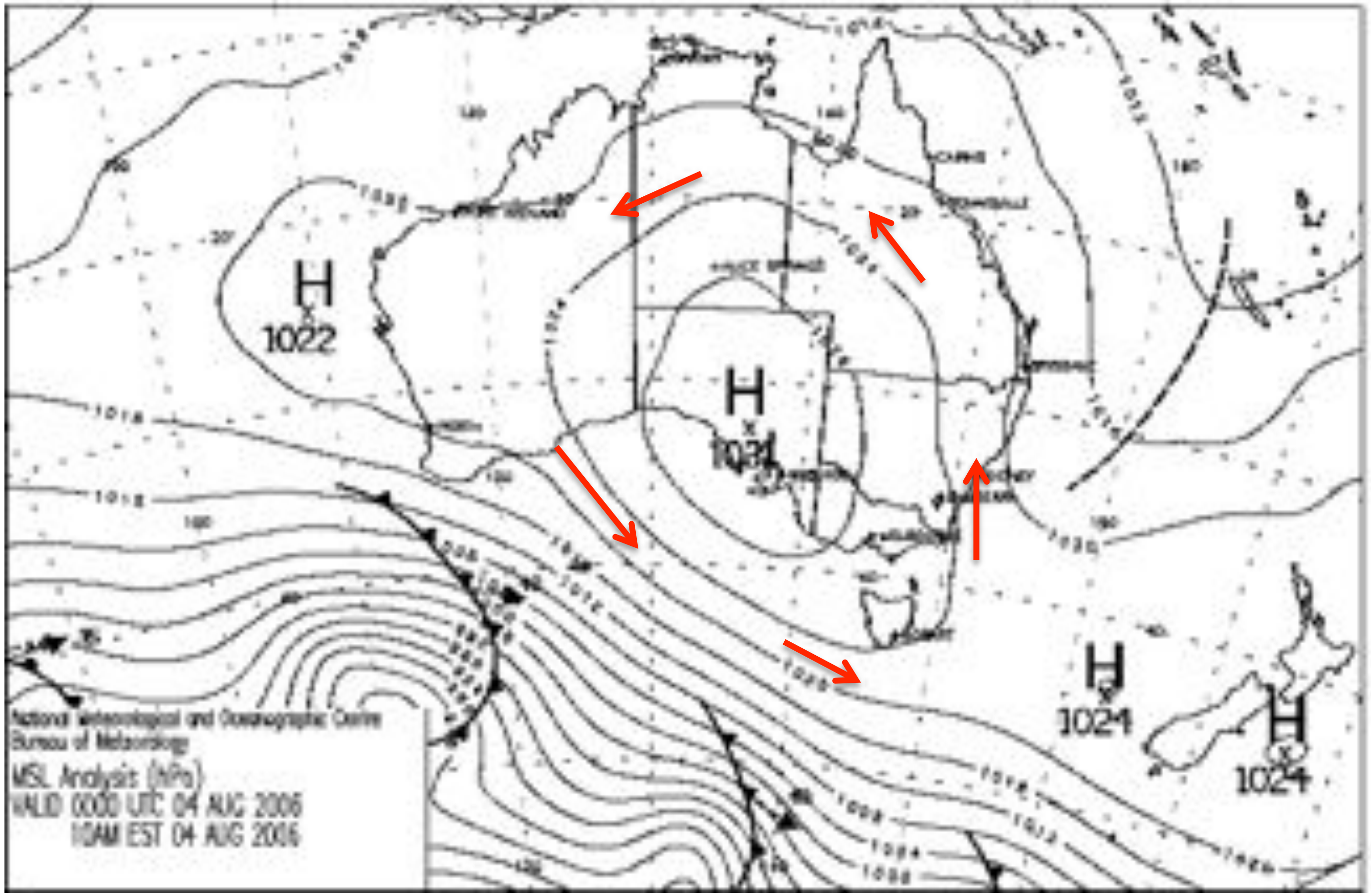
**Governing equations**

$$\cancel{\frac{Du}{Dt}} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right) + A_Z\left(\cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

Geostrophic flow:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv, \quad \frac{1}{\rho}\frac{\partial p}{\partial y} = -fu$$

$$where\ f = 2\Omega \sin\,(latitude)$$



**Governing equations of the Ocean**

$$\frac{D}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + A_Z(\frac{\partial^2 u}{\partial z^2})$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + A_H(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + A_Z(\frac{\partial^2 v}{\partial z^2})$$

$$\frac{Dw}{Dt} + hu = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + A_H(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}) + A_Z(\frac{\partial^2 w}{\partial z^2})$$

F=ma

$$f = 2\Omega \sin\theta, \qquad h = 2\Omega\cos\theta$$

$$\frac{DS}{dt} = K_s\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}\right)$$

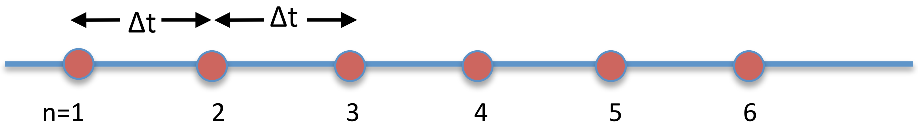
Conservation of salt

$$\rho C_p\frac{D\theta}{dt} = K_\theta\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) + Q$$

Conservation of energy

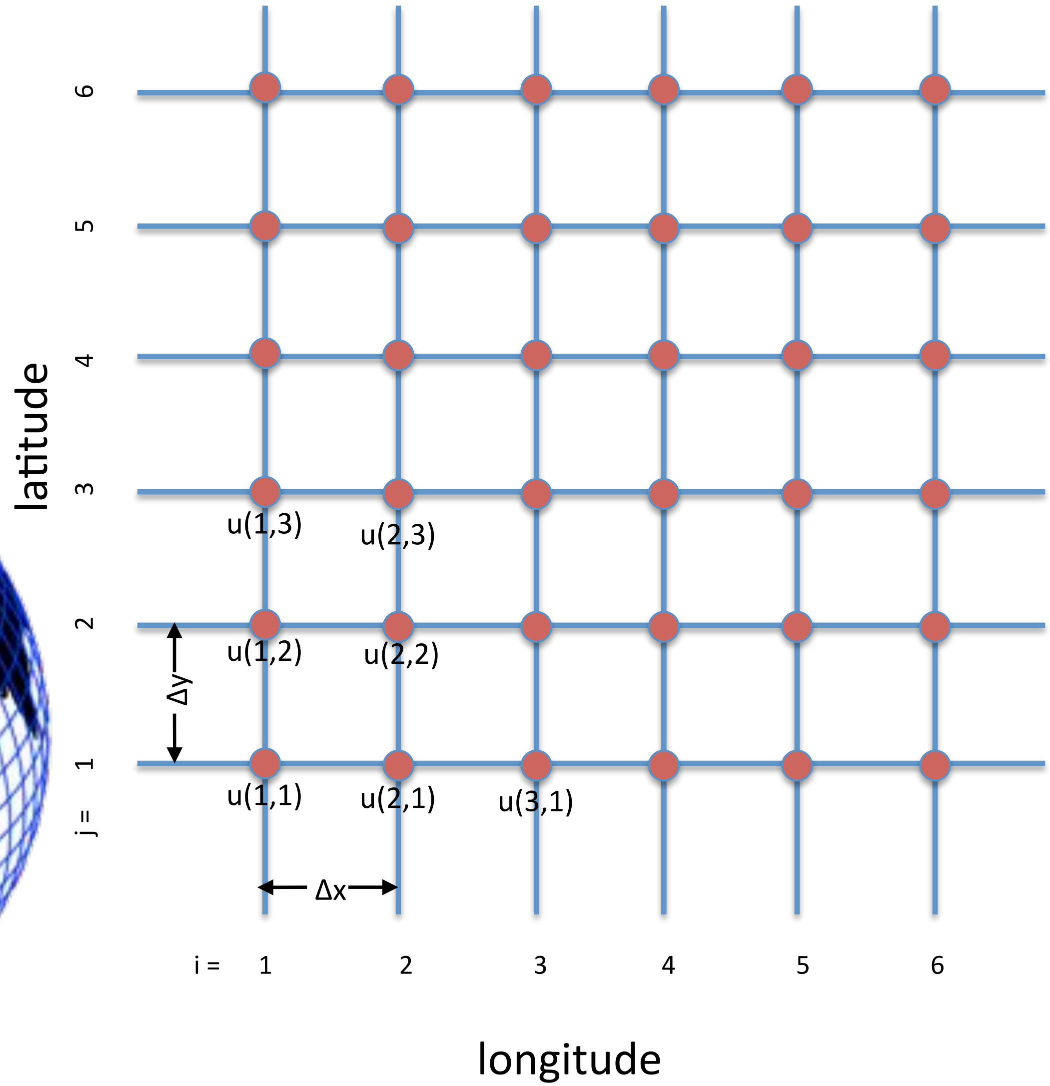
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of mass

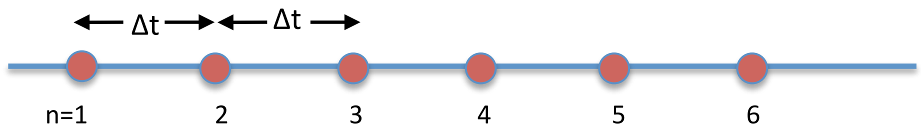



$$\frac{du}{dt}$$

$$\frac{d^2u}{dx^2}, \qquad T\frac{du}{dx}$$

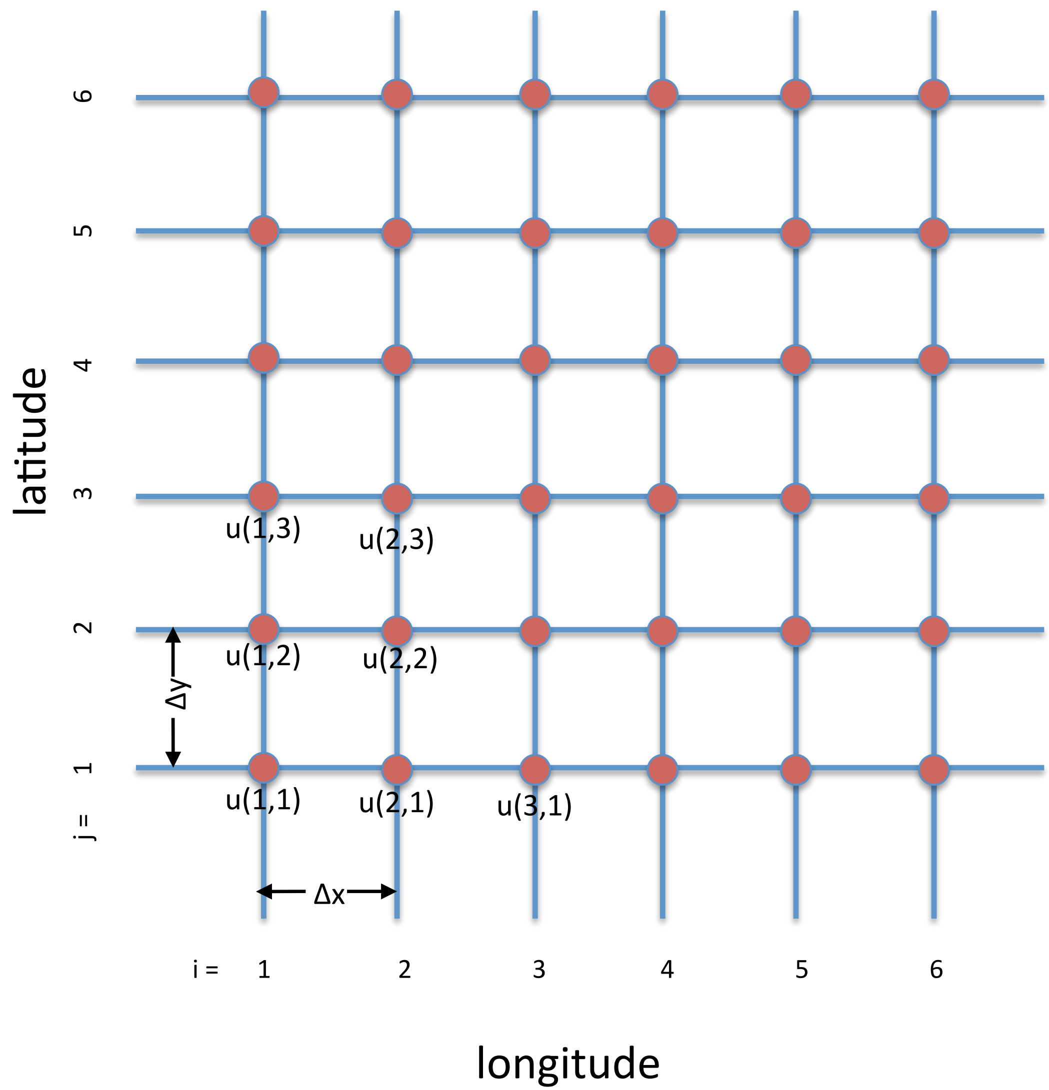


$$\frac{du}{dt}$$



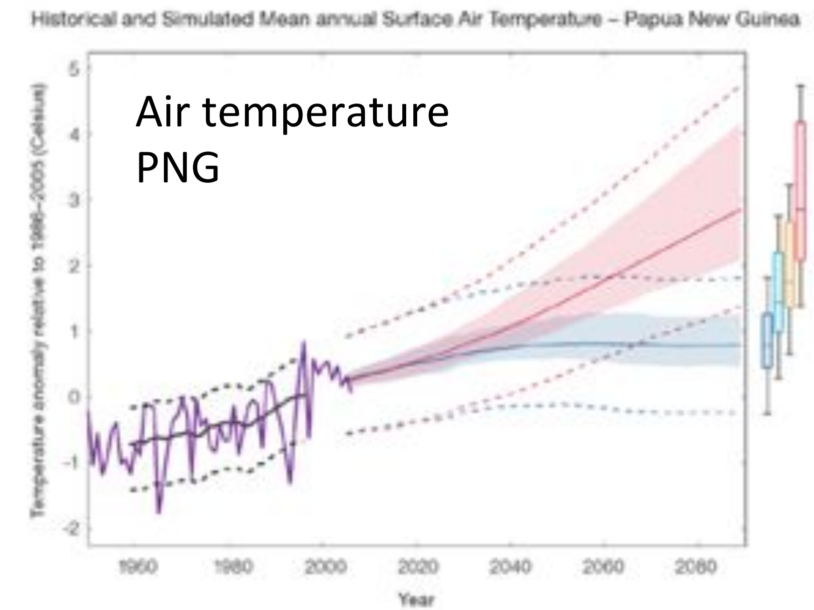

$$\frac{d^2u}{dx^2}, \qquad T\frac{du}{dx}$$

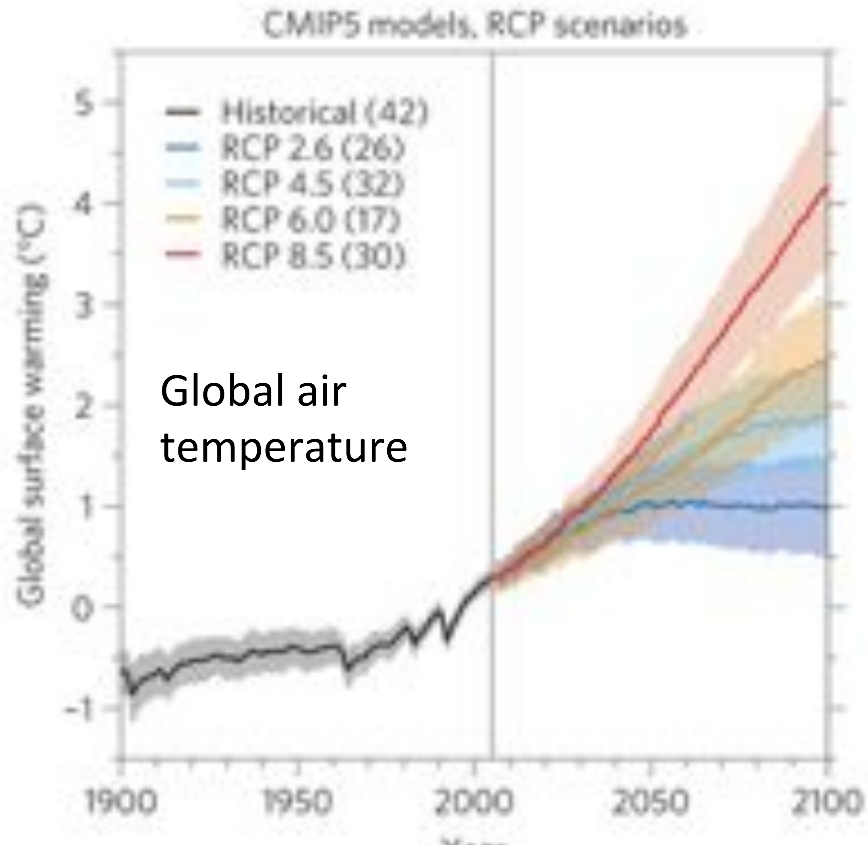
$$\frac{du}{dx} \approx \frac{u(2,1) - u(1,1)}{\Delta x}$$

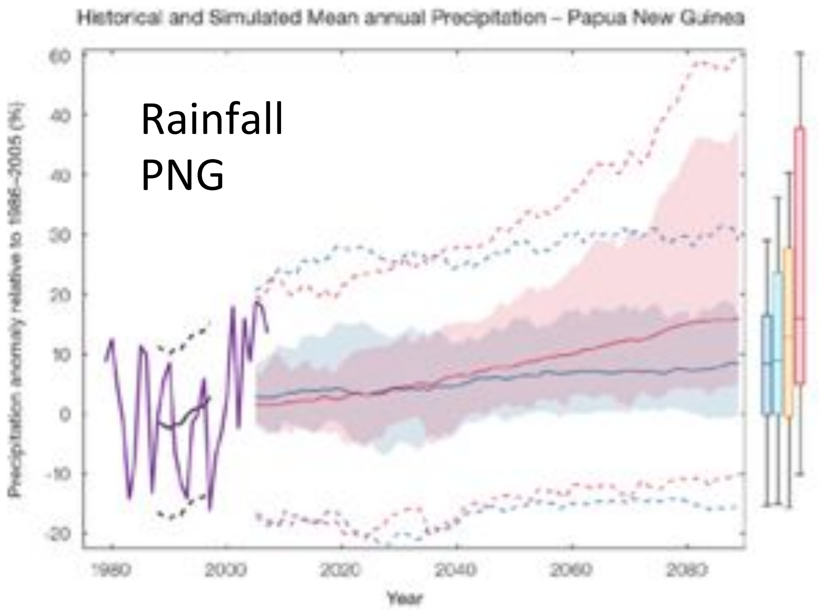


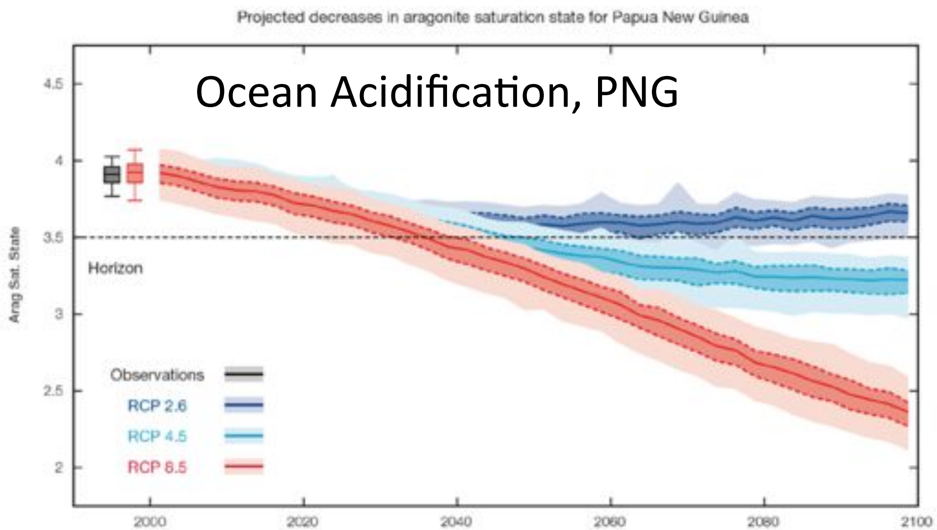
[redacted]

# Example: Using an ocean/climate model

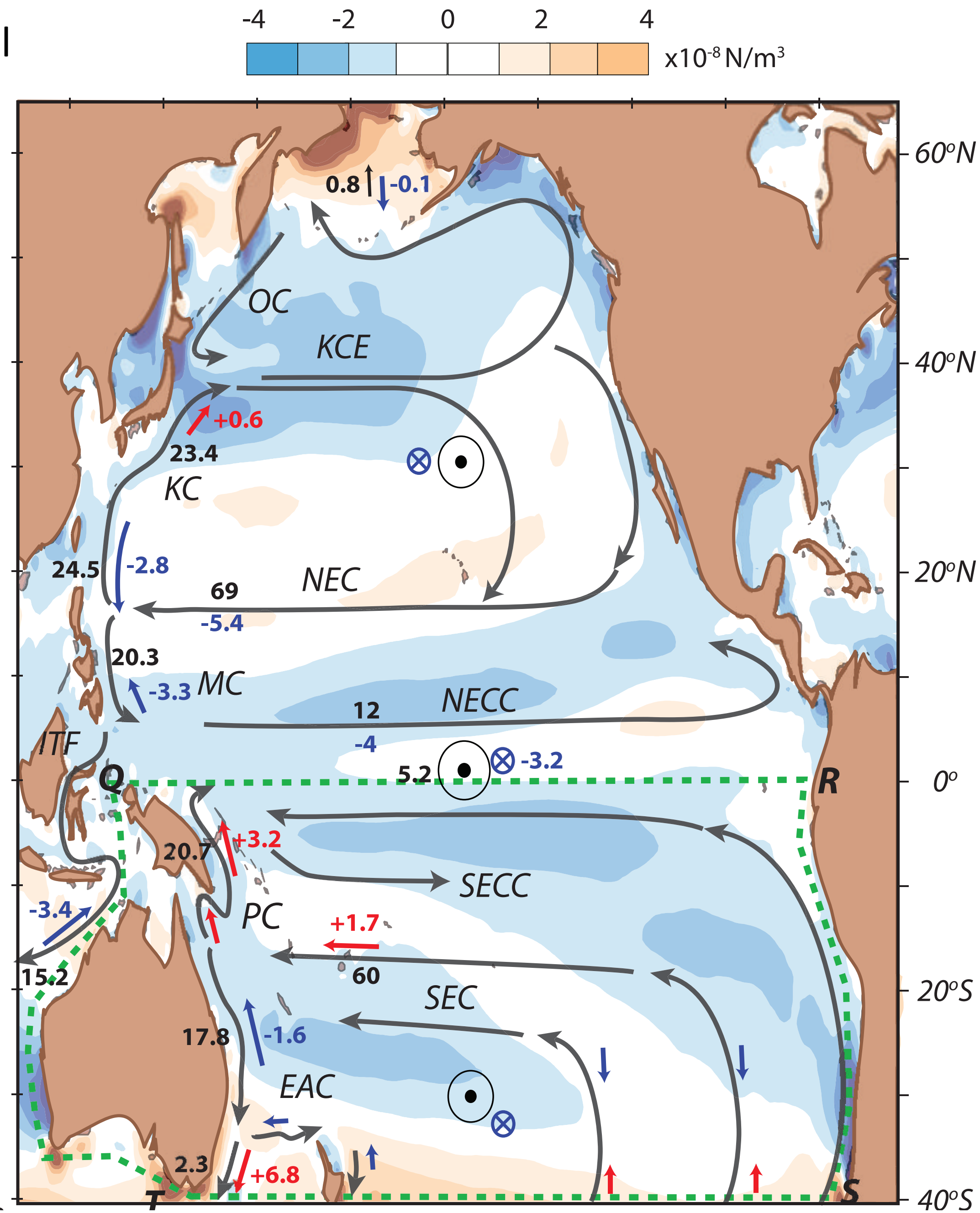


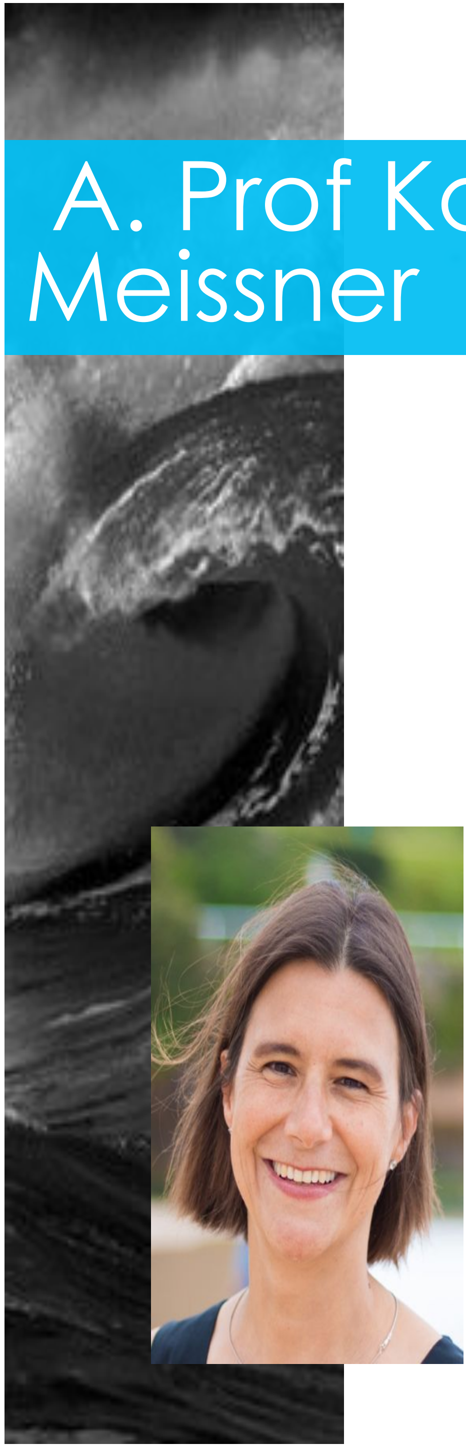



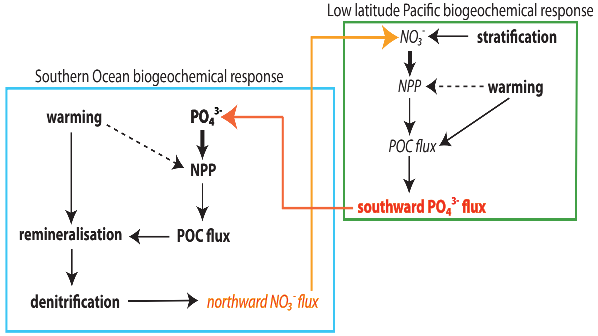
Example: Using an ocean/climate model



- Simple models can capture important features of the climate system
- Can recreate past climate change (global average) and be used to look at future changes.
- More complex models (e.g. based on Navier Stokes) are needed to resolve local features and the dynamics of the climate system
- Equations usually require numerical solution. Finite difference methods discretise space and time
- Interested in modelling the climate system or researching climate change, come talk to us at the CCRC

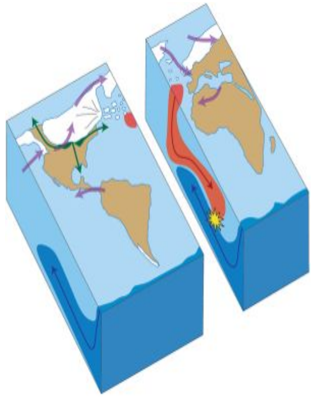


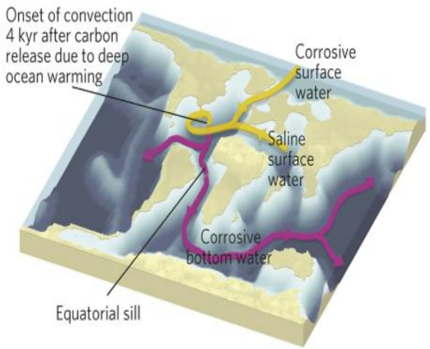
# A. Prof Katrin Meissner

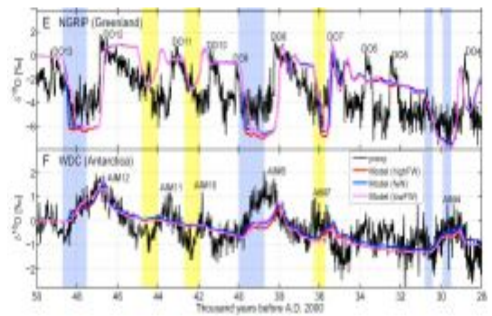



Director Climate Change Research Centre

- Climate change and variability
- Feedbacks between climate components
- Ocean circulation
- Carbon cycle
- Biogeochemical cycles
- Past climate change
- Isotopes

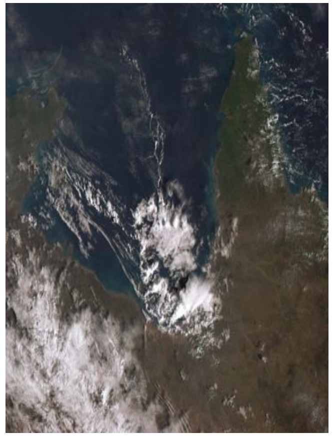




# Prof Steven Sherwood

ARC Laureate Fellow
Deputy Director Climate Change Research Centre



Himawari 8 image showing triggering of cloud lines over Gulf of Carpentaria

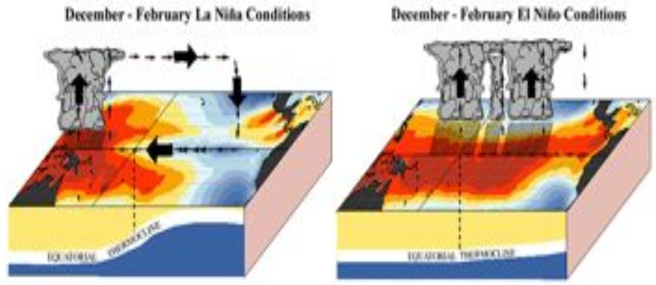
Research profile:

- Atmospheric water, cloud and convective processes
- Atmospheric radiation and thermodynamics
- Climate change and feedbacks
- Climate data analysis and homogenisation
- Heat stress and climate adaptability

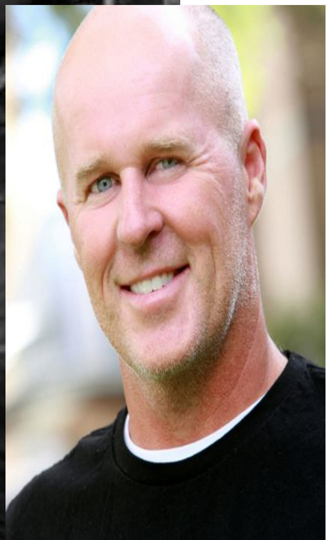
Current group topics:

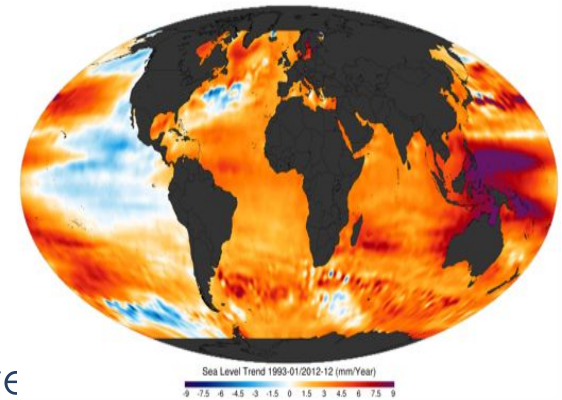
- Understanding the mid-Holocene “Green Sahara”
- Low-cloud feedbacks on global temperature
- Convective triggering and persistence
- Fluctuation-dissipation approaches to cloud-climate relationships
- Aerosol impacts on precipitating clouds

# Scientia Prof Matthew England



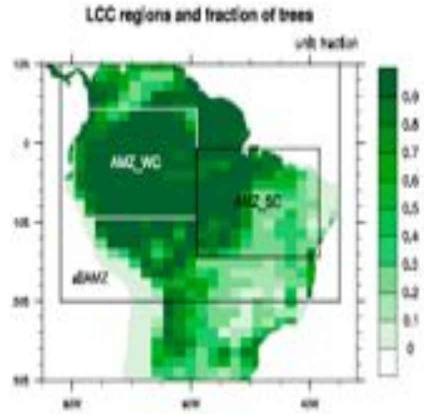

Scientia Professor of Climate Dynamics
CCRC and ARC Centre of Excellence in Climate System Science (ARCCSS)





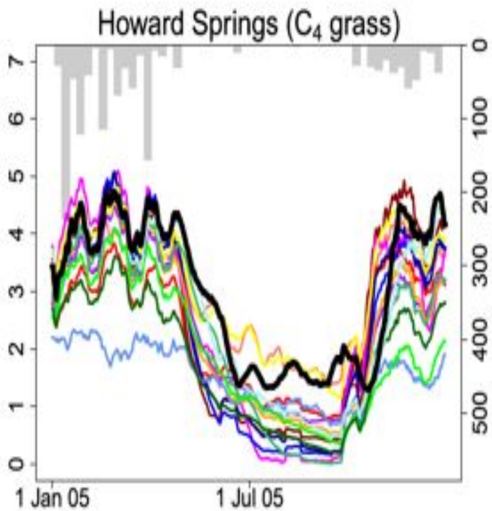

- •Antarctic climate
- •The Southern Ocean
- •El Niño and tropical modes
- •Ocean heat uptake and sea-leve
- •Ocean drivers of climate extremes (floods, drought)

# Prof Andy Pitman



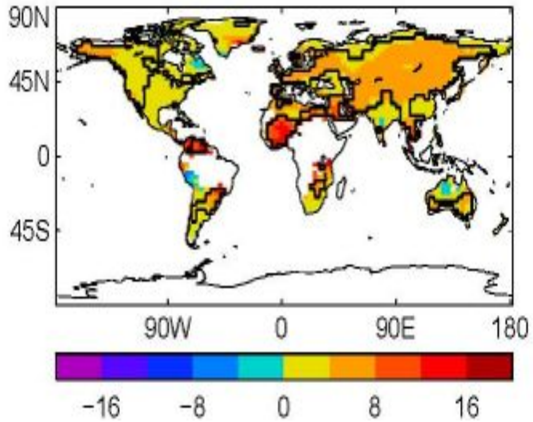

Director ARC Centre of Excellence for Climate System Science (ARCCSS)
Director ARC Centre of Excellence for Climate Extremes (ARC CLEX)

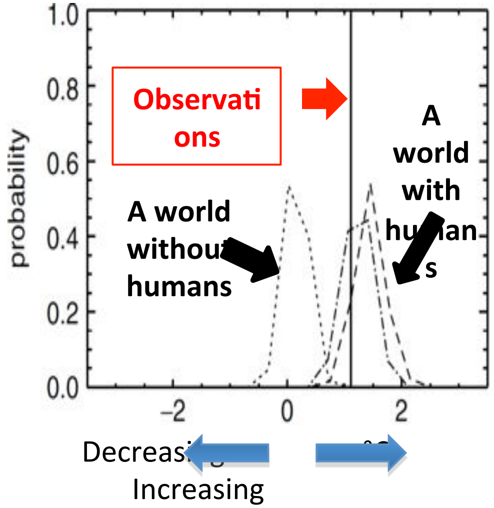
- Land Surface Processes
- How the land amplifies climate extremes
- Climate and regional modelling
- Water, energy and carbon processes
- How land use change affects climate
- Use of climate models for risk assessment
- Engagement of business with climate risk



# A. Prof Lisa Alexander

- Changes in the frequency and/or severity of extreme climate events have the potential to have profound societal and ecological impacts.
- Lisa's work primarily focuses on improving our understanding of observed changes in these events using multiple research tools ranging from station observations to climate model output.
- Much of her work has been focused on the creation of high quality global datasets and comparison with state of the art climate models.




Increasing trends in the hottest night of the year
– humans are to blame

# A. Prof Jason Evans

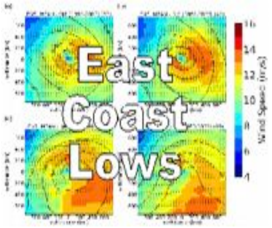
Journal of Climate Editor
Co Chair of GEWEX Hydroclimate Panel
Coordinator of CORDEX Australasia


Expertise:

- Regional Climate Models, Processes & Projections
- Water cycle over land
- Land-atmosphere interactions
- Remote sensing of land degradation

Major collaborations:

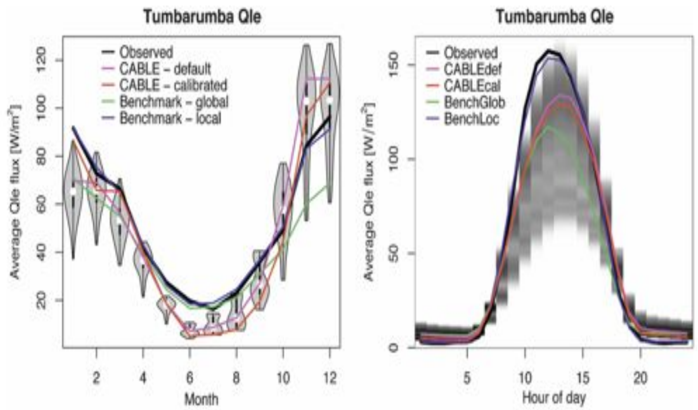
- NSW State Government
- National Environmental Science
- Program Earth System & Climate Change Hub
- ARC Centre of Excellence for Climate Extremes




# A. Prof Donna Green

- Climate change and energy policy
- Climate change and human health
- Air pollution and environmental justice
- Climate adaptation for Indigenous Australians

# Dr. Gabriel Abramowitz




CCRC Post Graduate Coordinator

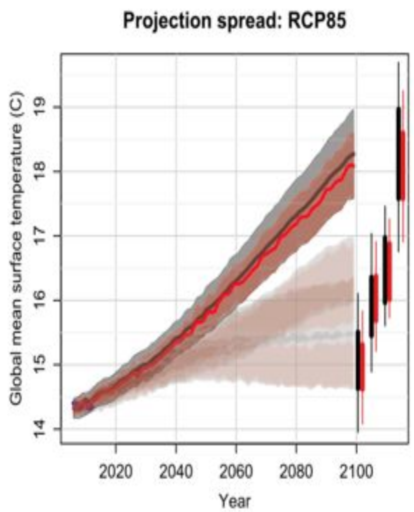
Co-chair of the GEWEX Global Land-Atmosphere System Study (GLASS) panel

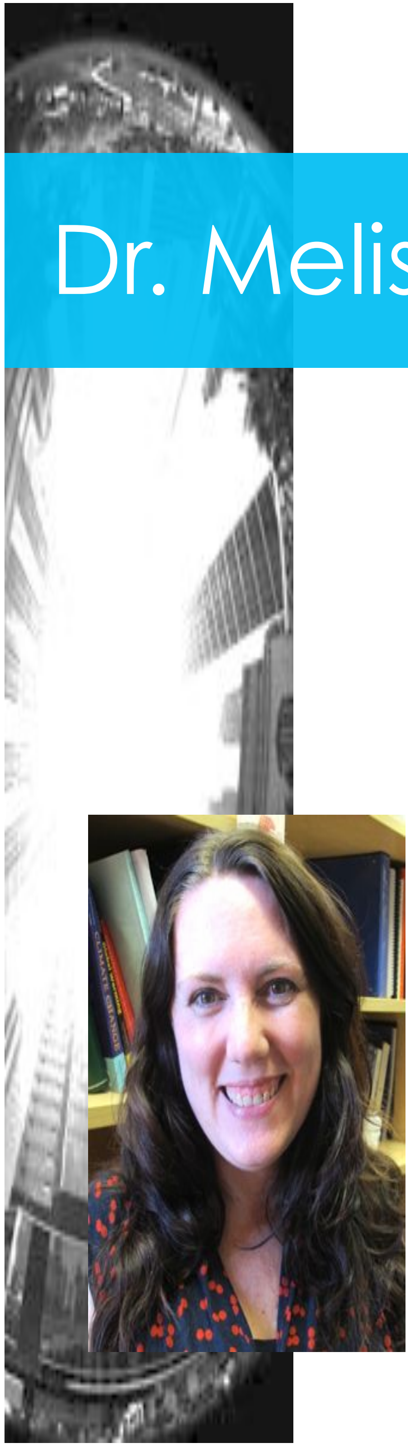
Committee member for the Australian community land surface model, CABLE

Chair of the Australian Energy & Water Cycle Experiment Benchmarking Working Group

Research areas:

- Model evaluation and benchmarking (including climate, hydrological, ecological models)
- Model dependence in ensemble prediction
- Machine learning applications in climate
- Land surface model process representation





# Dr. Melissa Hart

Graduate Director - ARC Centre of Excellence for Climate System Science

Developed and co-ordinated a national graduate program in climate science across 5 universities -120 PhD students

**Urban Climate**

- Quantification of the urban heat island magnitude
- Impact of land-use and anthropogenic activities on the climate of cities

**Air Pollution Meteorology**

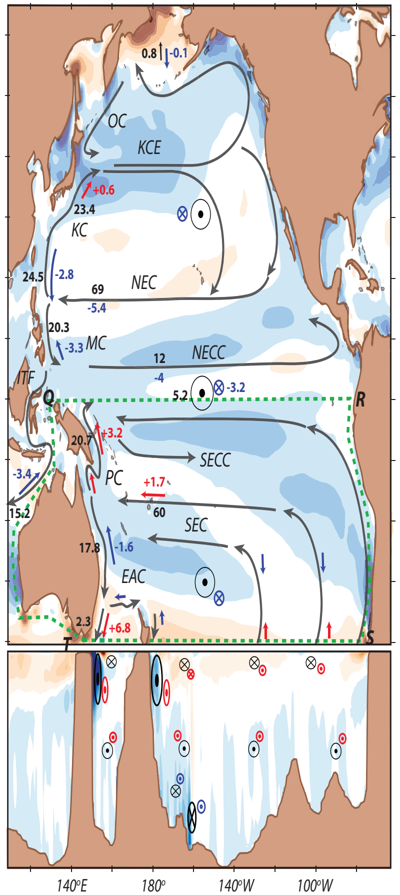
- Synoptic and mesoscale controls on air pollution
- Air pollution impacts from prescribed burns and wildfires

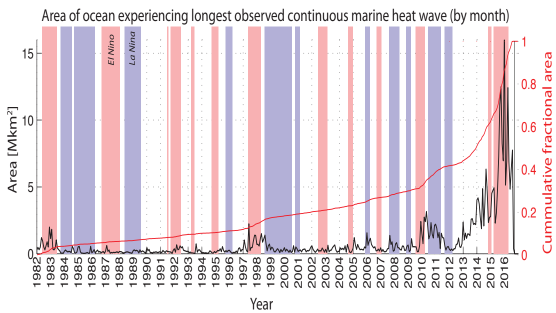
# Dr. Alex Sen Gupta

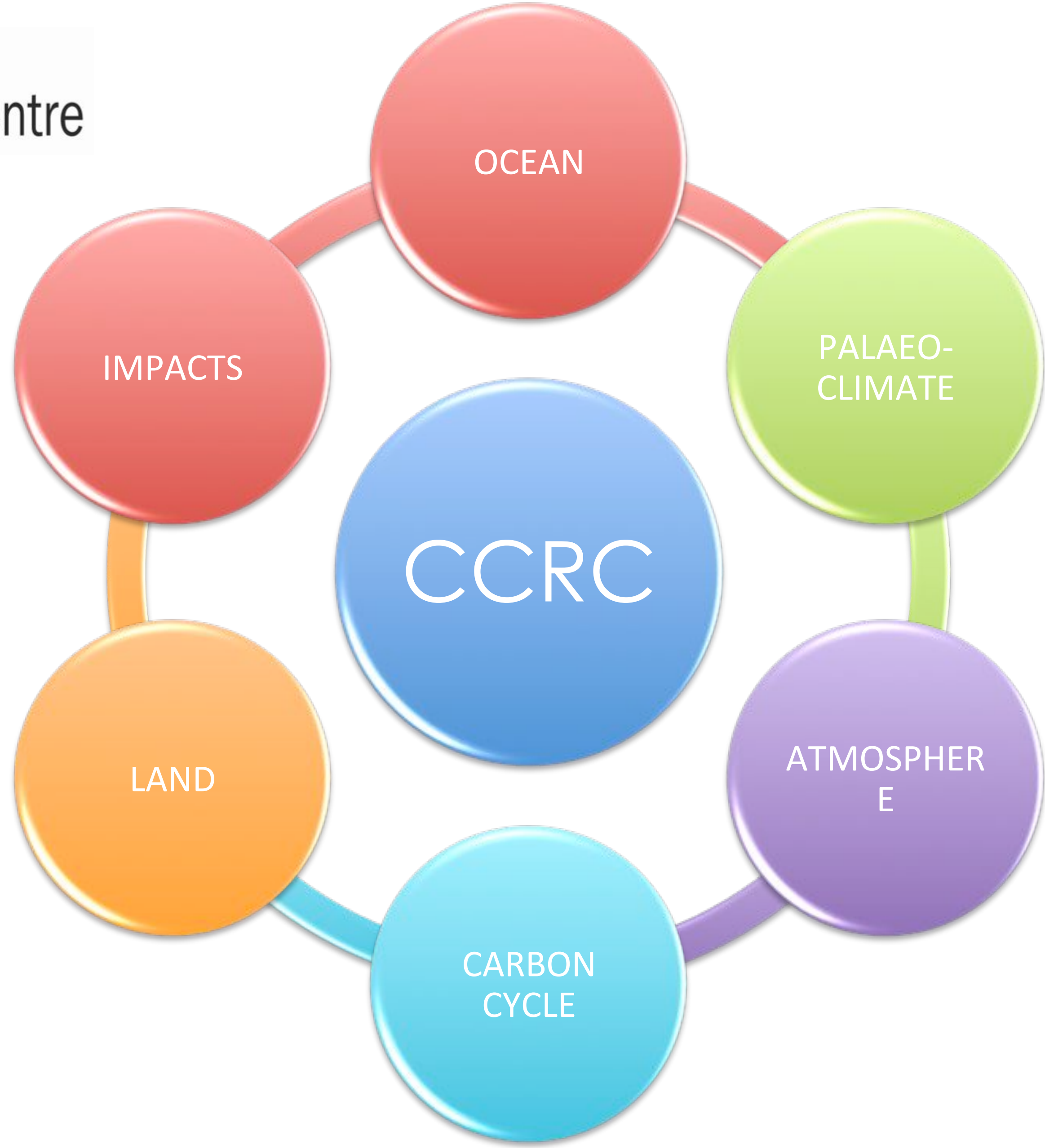
Senior Lecturer
BEES Postgraduate Coordinator (Thesis Examination)

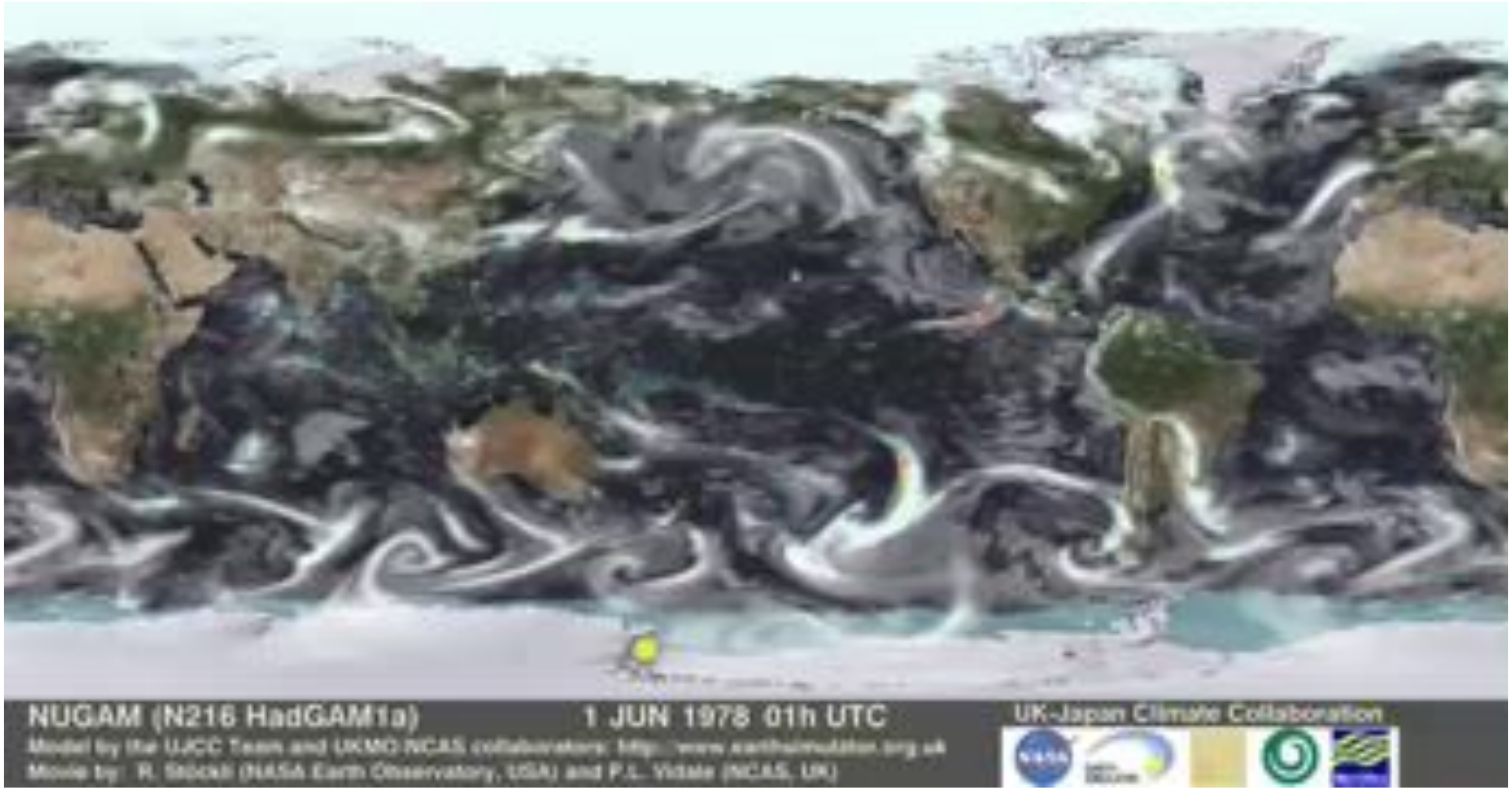
Climate variability and change can dramatically effect the ocean, its ecosystems and the industries that rely on these. Related projects include:

- Understanding the effect of anthropogenic warming on the ocean circulation, temperatures and marine critters!
- Processes and drivers of marine heat waves
- Numerical simulation of tropical tuna
- Climate model evaluation and climate projections




OCEAN
PALAEO-
CLIMATE
ATMOSPHER
E
CARBON
CYCLE
LAND
IMPACTS
CCRC


NUGAM (N216 HadGAM1a)
1 JUN 1978 01h UTC
UK-Japan Climate Collaboration

# Further reading …

## Ocean equations

http://www.rsmas.miami.edu/users/lbeal/MPO603/Lectures%207%268.html

https://www.google.com.au/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwi7qdmKqI_TAhWjxlQKHahxC24QFggbMAA&url=http%3A%2F%2Fwww.colorado.edu%2Foclab%2Fstewart-textbook&usg=AFQjCNHJWgoycQtkTfVW0B3WSIYvCUAM9w&sig2=L5q99UGfDsnDbQ1rdSltiQ

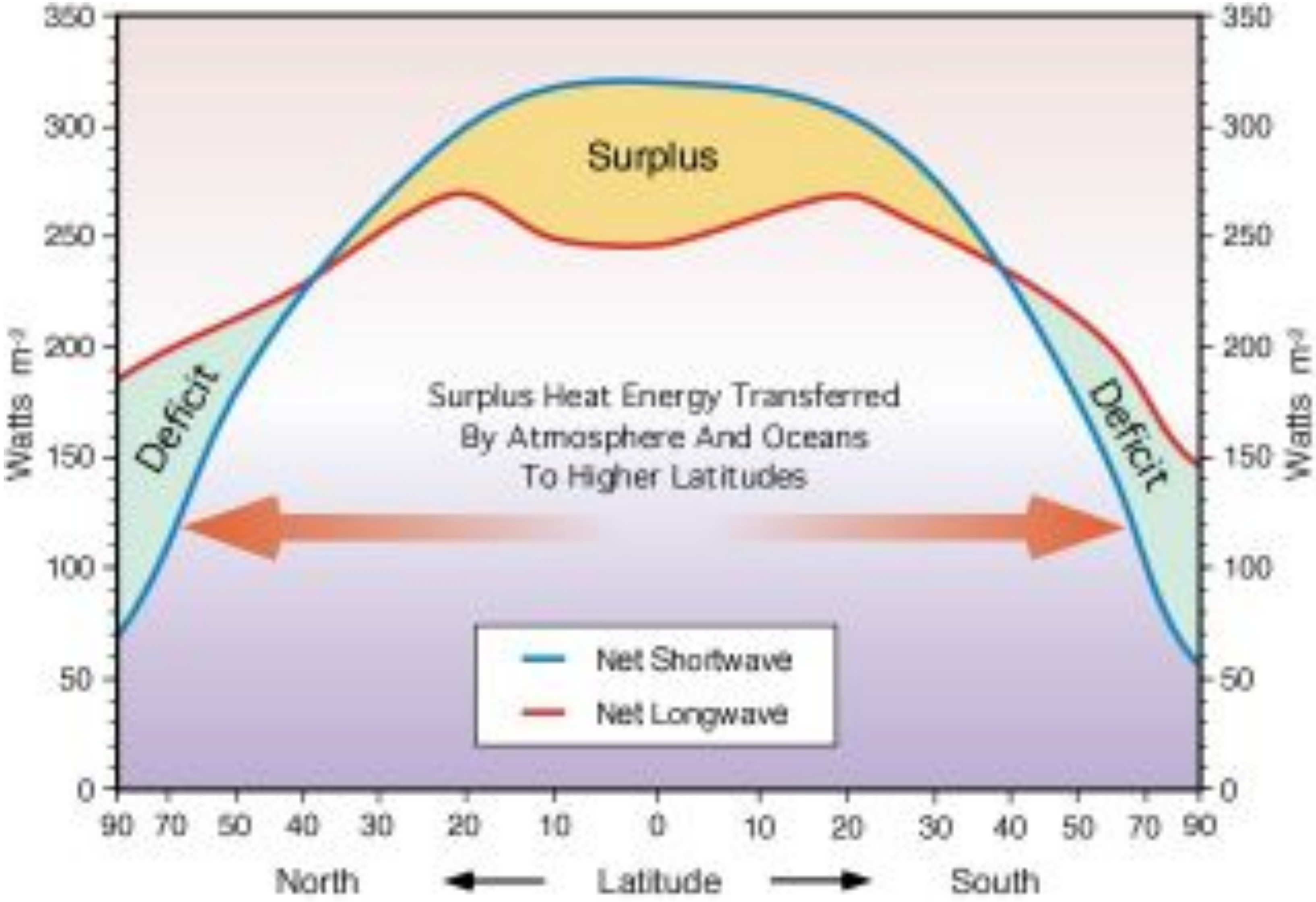
## Simple Climate Model

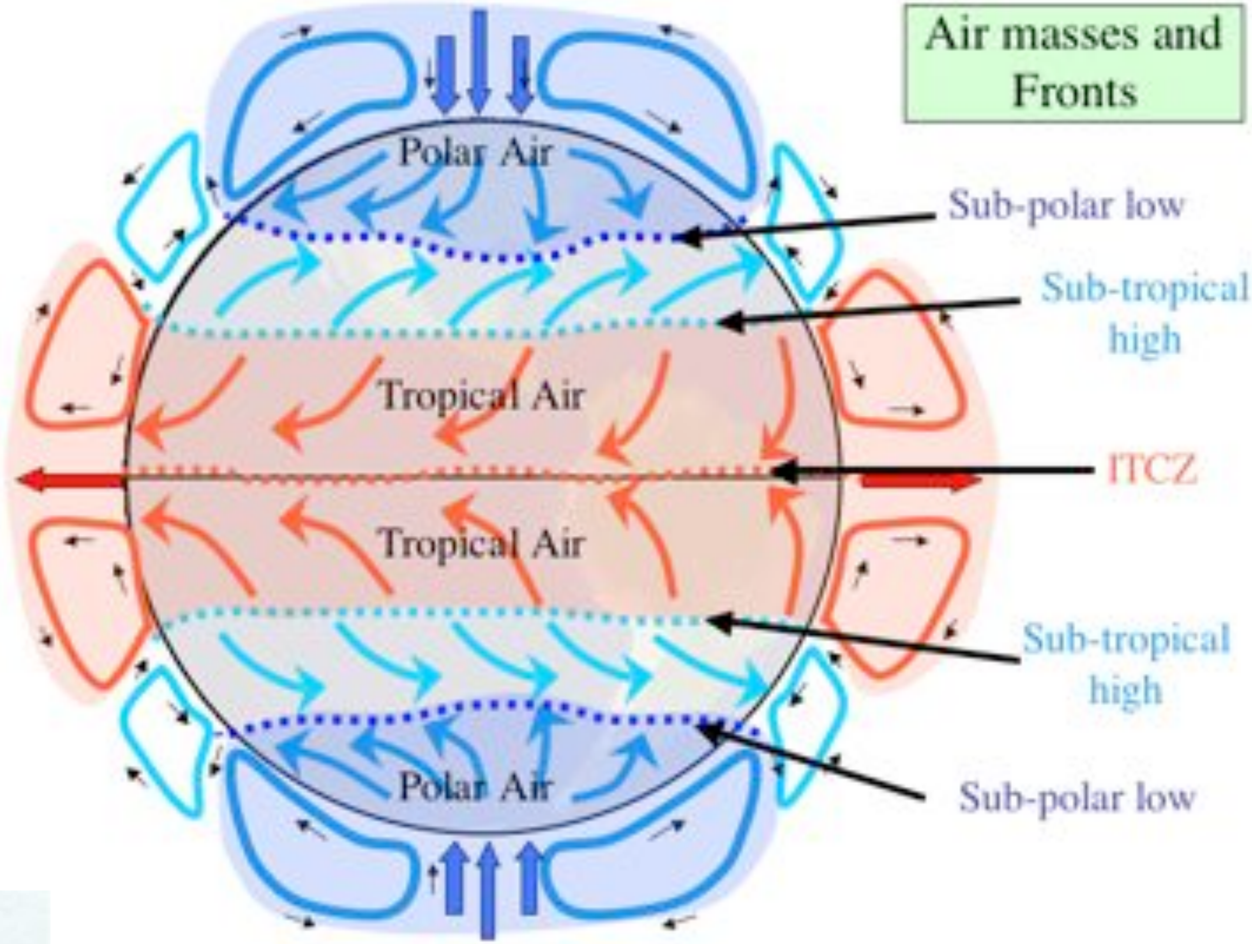
http://45.55.211.78:8080

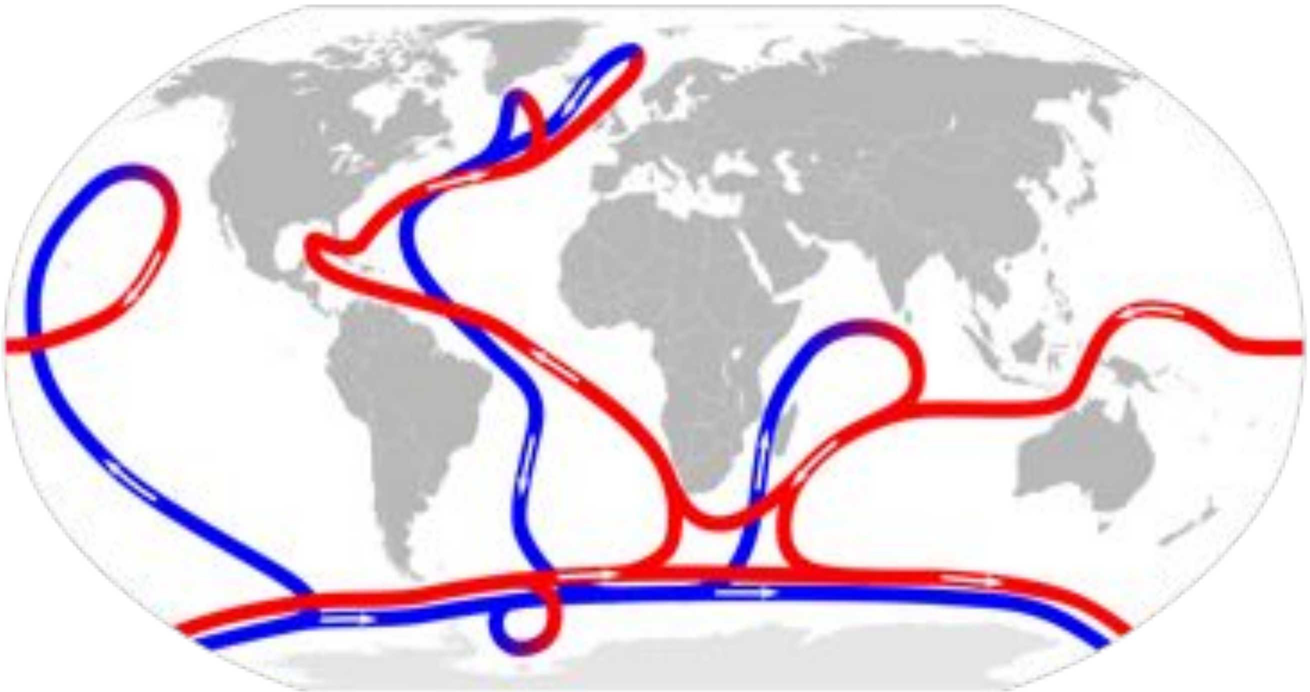
## Climate models

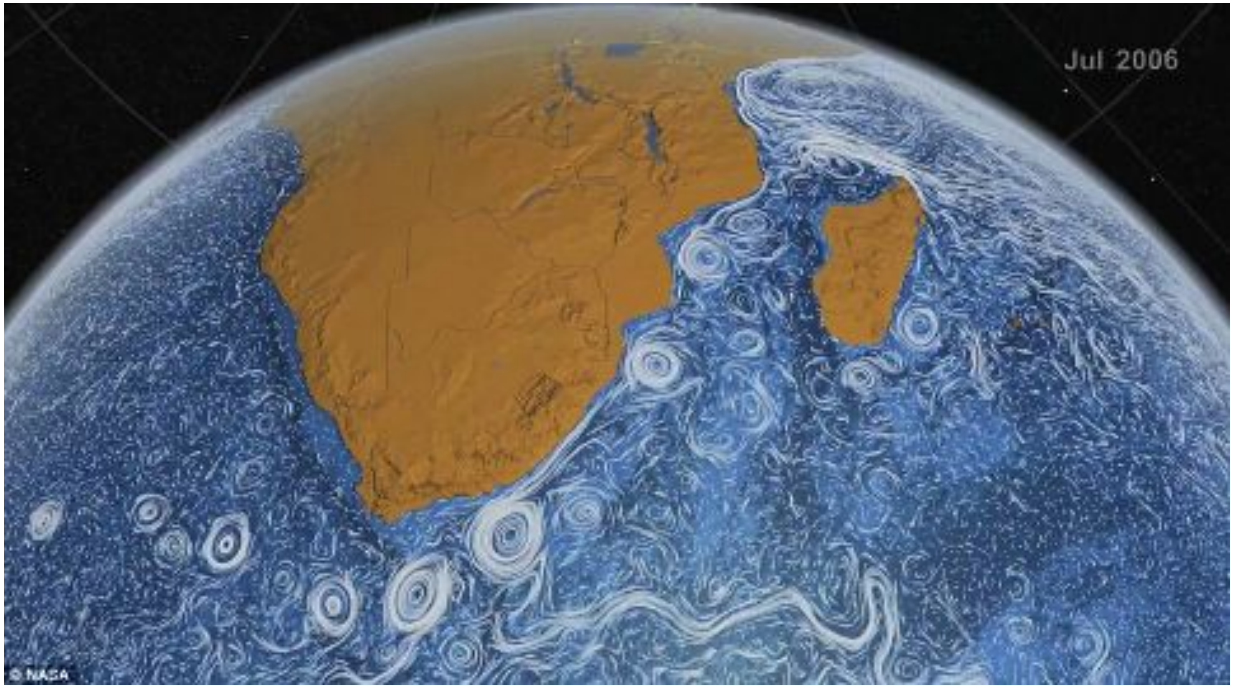
https://www.google.com.au/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwjnraC4jZHTAhUHlZQKHfq2DIAQFgggMAA&url=http%3A%2F%2Fwww.climatescience.org.au%2Fsites%2Fdefault%2Ffiles%2FWaterman_Ocean_Modelling_no_movies.pdf&usg=AFQjCNHc4yKjCkJta3VAhxtFf4aBqs0fUA&sig2=H-LWoeMuEeNJt-eipwDw9g

# Further information on CCRC …


Surplus
Deficit
Deficit
Surplus Heat Energy Transferred
By Atmosphere And Oceans
To Higher Latitudes
Net Shortwave
Net Longwave
Watts m⁻²
350
300
250
200
150
100
50
0
90 70 50 40 30 20 10 0 10 20 30 40 50 70 90
North
Latitude
South

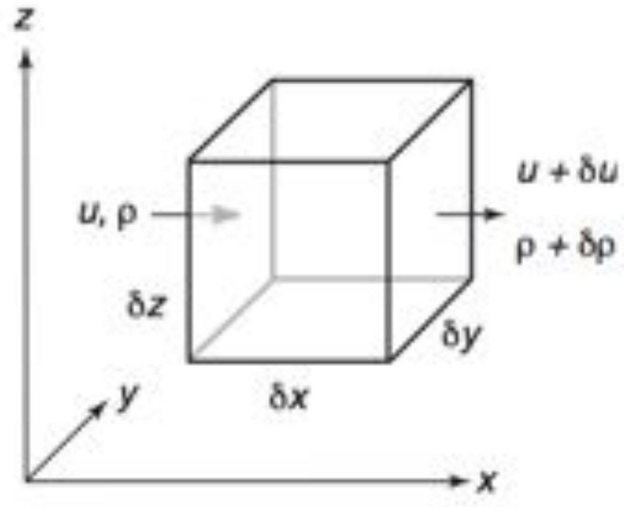

Air masses and Fronts
Polar Air
Sub-polar low
Sub-tropical high
Tropical Air
ITCZ
Tropical Air
Sub-tropical high
Polar Air
Sub-polar low




Jul 2006
© NASA

**Governing equations**

Conservation of mass




Change in mass = (mass in) – (mass out)

$$\delta x \delta y \delta z \frac{\partial \rho}{\partial t} = (u \delta y \delta z)\rho - \big((u+\delta u)\delta y \delta z\big)(\rho + \delta)$$

Density changes are small in the ocean (i.e. water is incompressible)

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

**Continuity equation:**
Volume of water going into a region equals the water going out

**Governing equations**

$$\frac{D}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Conservation of heat & salt

$$\frac{DS}{dt} = K_s\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}\right)$$

The amount is salt in the ocean is constant (unless we consider geological timescales) so salt is either moved around as a body (advected) or diffused (K is the diffusion coefficient)

$$\rho C_p \frac{D\theta}{dt} = K_\theta\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) + Q$$

Conservation of heat is expressed in terms of temperature (θ). Temperature can be advected or diffused in the ocean or can be gained from or lost to the atmosphere (Q)

**Governing equations of the Ocean**

Based on:

- Conservation of mass (continuity equation) and salt
- Conservation of energy
- Conservation of momentum (Navier-Stokes equations)
- Conservation of angular momentum (vorticity conservation)

Important forces:

- Gravity. Weight of water (earths gravity) generates pressure. Gravitational changes from sum and moon generate tides, tidal currents and tidal mixing
- Pressure forces. Due to horizontal differences in water surface height or water density
- Buoyancy forces. Vertical forces resulting from water with density higher or lower than its surroundings (Archimedes)
- Frictional forces. Associated with boundaries, shear in water velocity
- Wind stress. Transfer of momentum from the wind to the surface waters
- Coriolis force. Pseudo force associated with being on a rotating frame of reference

**Governing equations**

Conservation of momentum

F = ma

$$m\frac{d\boldsymbol{v}}{dt} = F$$

$$\frac{d\boldsymbol{v}}{dt} = f^m$$

Acceleration is proportional to the net force per unit mass

The important forces are:

Pressure force

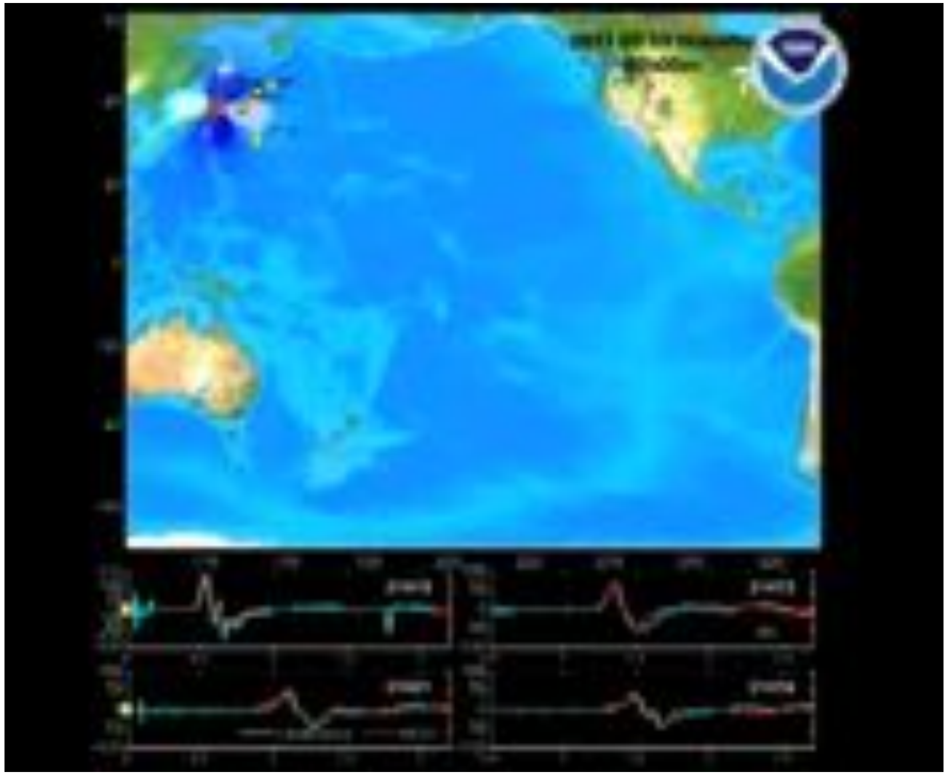
Coriolis force

Gravitational force

Frictional force

$$\frac{d\boldsymbol{v}}{dt} = -\frac{1}{\rho}\nabla p - 2\boldsymbol{\Omega}\times\boldsymbol{v} + \boldsymbol{g} + \boldsymbol{F_r}$$

$\boldsymbol{v}$ is the velocity vector (u,v,w)
$\rho$ is density
p is pressure
$\boldsymbol{\Omega}$ is the rotation rate of the earth
**g** is acceleration due to gravity
$\mathbf{F_r}$ is the frictional force



Disclaimer

" The results presented here are intended for research guidance only. They are not a forecast of where the oil will go, but a scenario of what might happen if the loop current is in a typical configuration. "

Model Resolution (what values for Δx and Δy)

Add picture of eddi

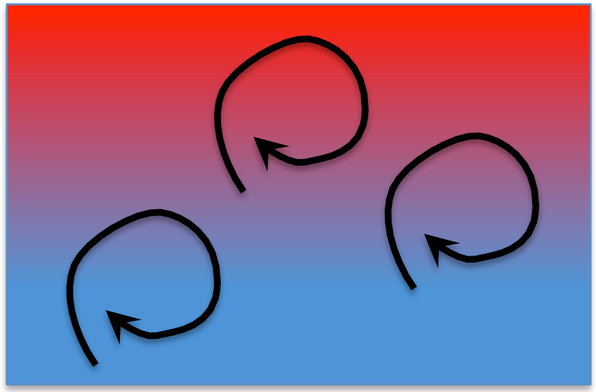
- We want to capture physical processes/features that are important to ocean circulation. But the higher the resolution the more calculations are required (2x increase in resolution > 8x increase in calculations + need smaller time step).
- So resource constrained – how big is your supercomputer
- Most of the ocean's kinetic energy is in ocean eddies, which have scales of a few 10s of km
- Most climate models have resolutions of O[100km]. They cannot 'see' eddies – 'coarse model' – need to PARAMETRISE
- Big push to increase resolution to O[10km] – 'eddy resolving model'

## Model Parametrisation

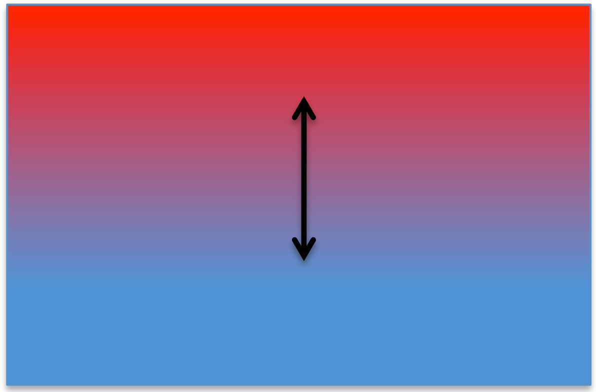
- Small scale processes that are not resolved by models can have a major impact on large scale conditions
- E.g. eddies are important for mixing of heat, biology, pollutants, but are unresolved in state-of-the art climate model
- Need to 'parametrise' the effect of unresolved processes on the large scale conditions

- Parametrised processes:
    - Mesoscale eddies
    - Sub-mesoscale circulation
    - Dense overflows
    - Coastal processes
    - Surface vertical mixing
    - Ocean-ice interaction

Eddies advect water around



$$\frac{\partial T}{\partial t} = K \frac{\partial T}{\partial y}$$

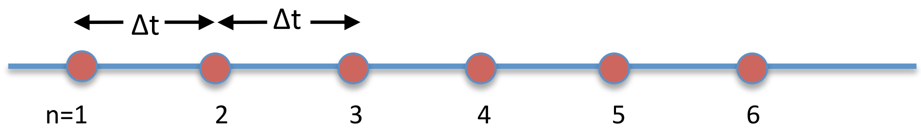


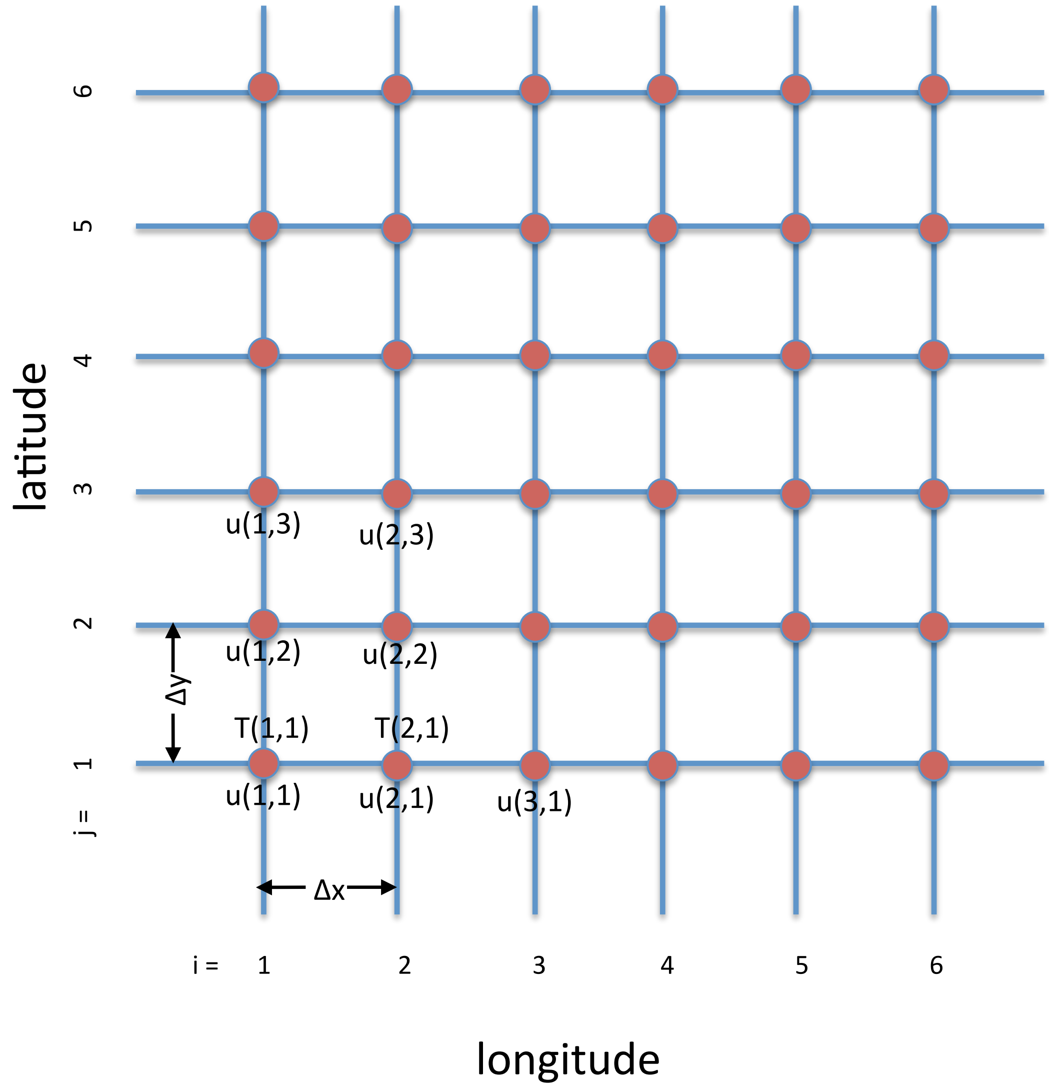
This effect can be parametrised by treating this as diffusion (but with a much larger diffusivity than associated with molecular diffusion)

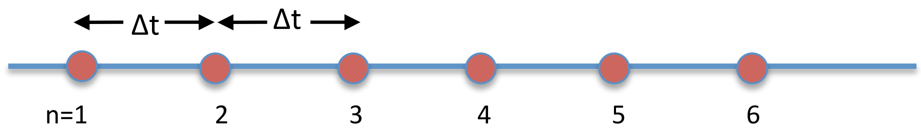
$$\frac{du}{dt}$$

$$\frac{d^2u}{dx^2}, \qquad T\frac{du}{dx}$$

$$\frac{du}{dx} \approx \frac{u(2,1) - u(1,1)}{\Delta x}$$





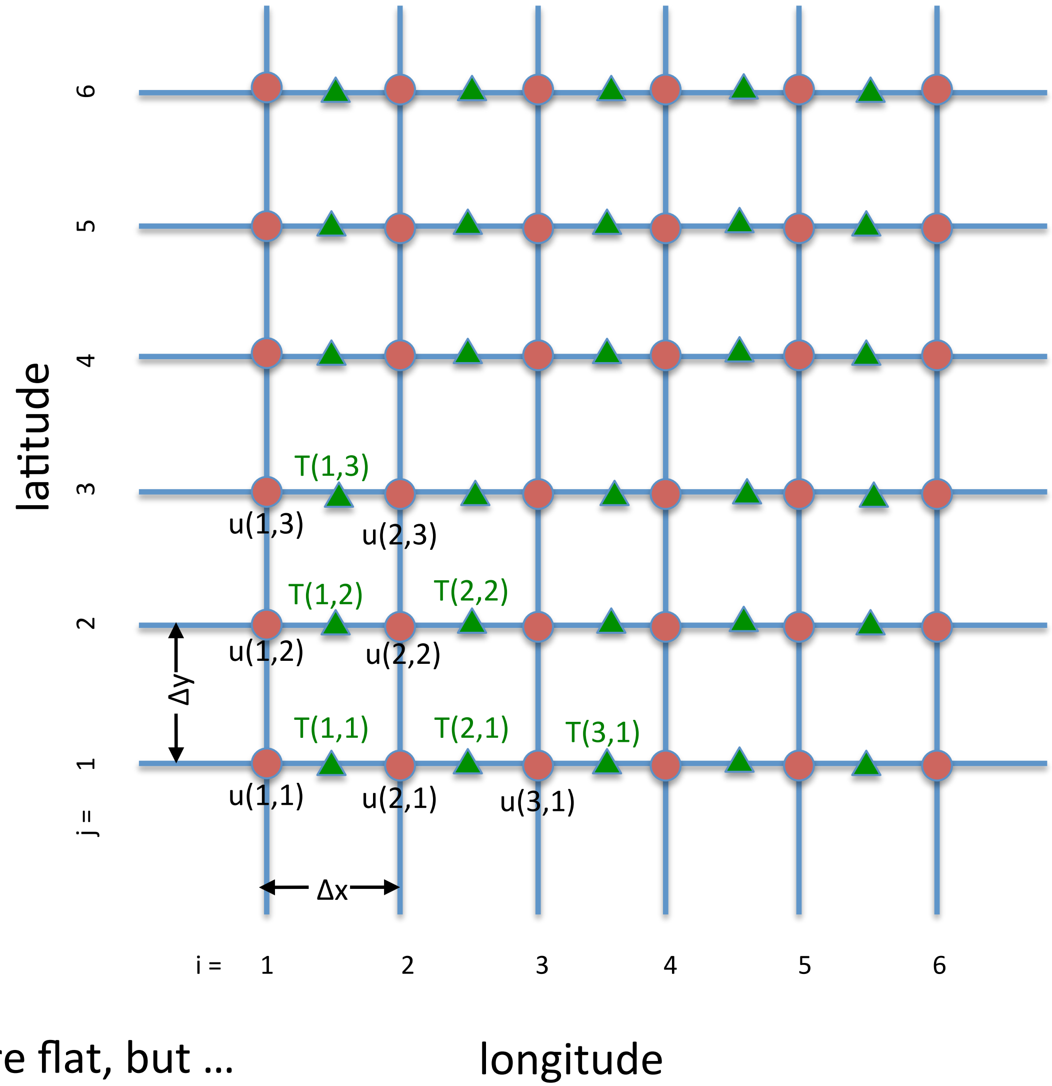

$$\frac{du}{dt}$$

$$\frac{d^2u}{dx^2}, \qquad T\frac{du}{dx}$$

$$\frac{du}{dx} \approx \frac{u(2{,}1) - u(1{,}1)}{\Delta x}$$

$$T\frac{du}{dx} \approx T(1{,}1)\left(\frac{u(2{,}1) - u(1{,}1)}{\Delta x}\right)$$

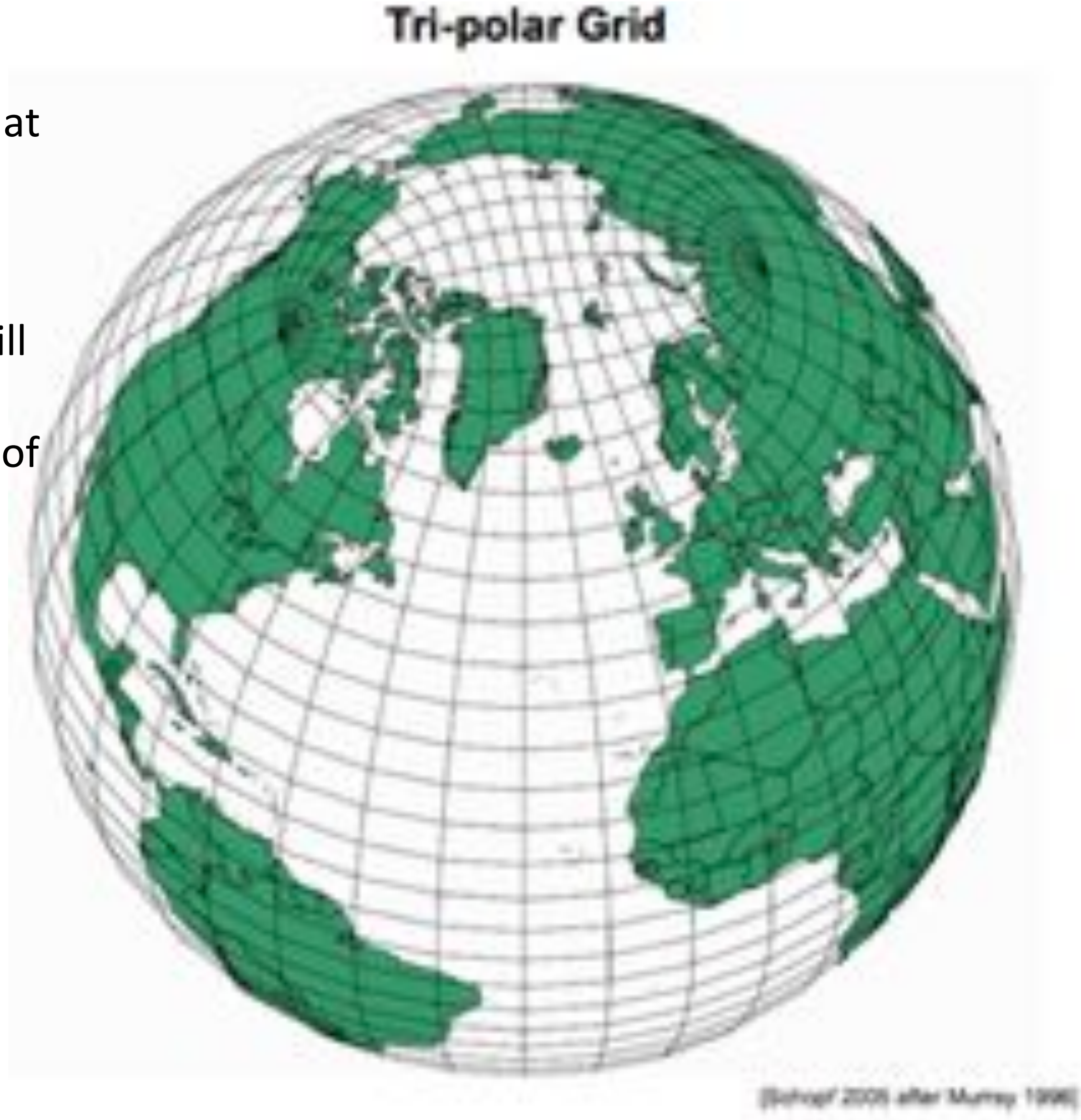
**Staggered grid** allows for much more accurate finite difference schemes



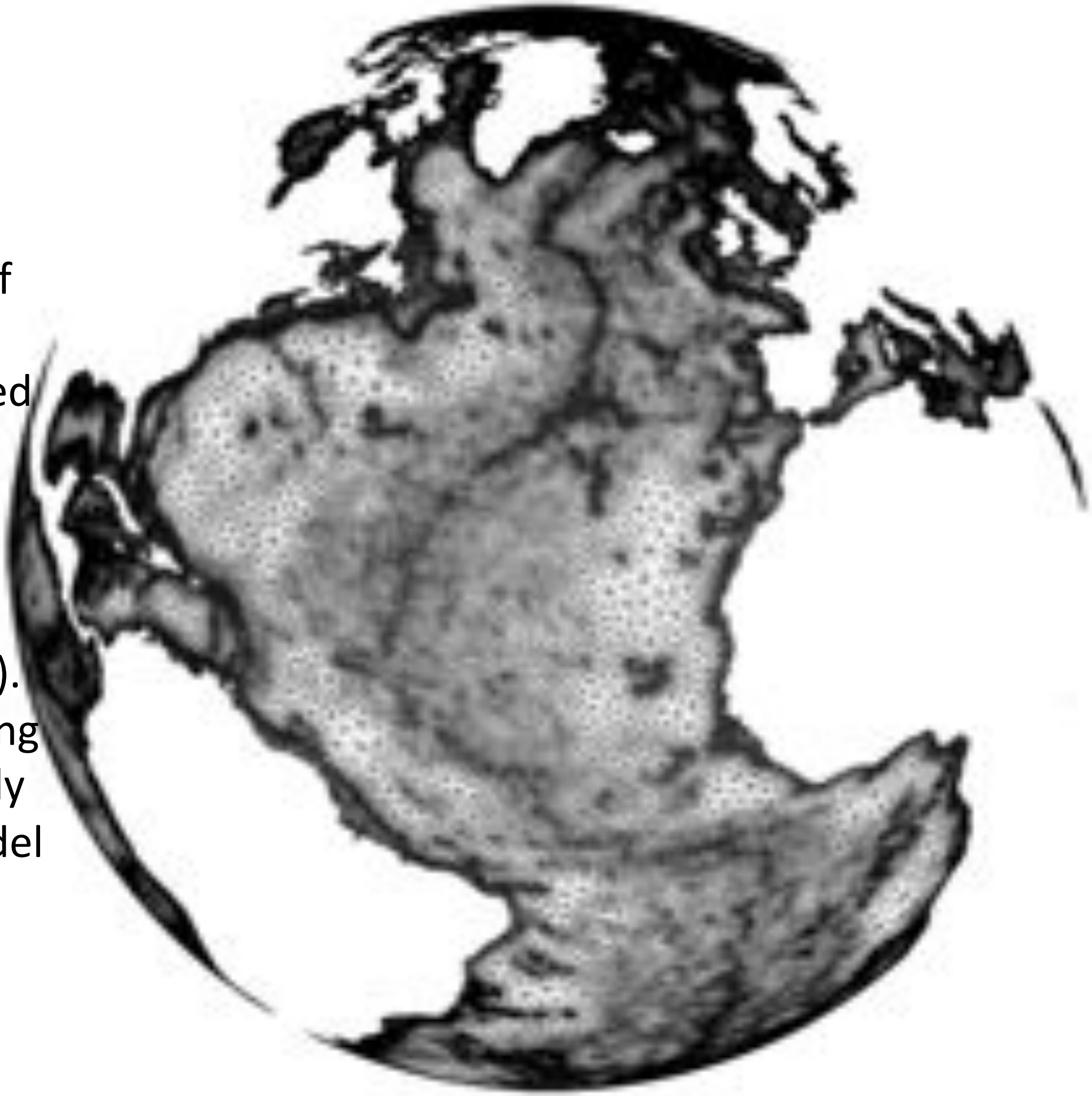

This would be OK if the world were flat, but ...

- Area of grid cells change with location and we have a problem at the pole where the grid lines converge (smaller grids need smaller time steps)

- Area of grid cells change with location and we have a problem at the pole where the grid lines converge (smaller grids need smaller time steps)
- One solution is a tripolar grid. Still have convergence, but it occurs over land. But, it makes analysis of model data more difficult



- Area of grid cells change with location and we have a problem at the pole where the grid lines converge (smaller grids need smaller time steps)
- One solution is a tripolar grid. Still have convergence, but it occurs over land. But, it makes analysis of model data more difficult
- Another solution is an unstructured grid (made up of interlocking triangles). This allows changing resolution in different regions (so you can resolve small-scale processes where they are needed). And can even include a time varying grid resolution. But mathematically complex, and hard to analyse model output



## The Full Model