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Exercise
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b) Complement of a recursive language is recursive

c) Recursive language may loop forever on Turing machine

d) String belongs to a recursive language either accepts or rejects on Turing machine.

6. Find the decidable problem regarding the DFA.

a) The problem that a set of null strings is accepted by a DFA M

b) The problem that a string w is accepted by a DFA M

c) The problem that two DFA, M1 and M2, satisfy the same language

d) All of these

7. Which is true for reducibility?

a) Converting one problem to another problem.

b) Converting one solved problem to another unsolved problem.

c) Converting one unsolved problem into another solved problem to solve the first problem.

d) Converting one unsolved problem to another unsolved problem.

Answers :

1. d 2. d 3. a 4. b 5. c 6. d 7. c

GATE Questions

1. Which of the following statements is false?

a) The halting problem of a Turing machine is undecidable.

b) Determining whether a context-free grammar is ambiguous is undecidable.

c) Given two arbitrary context-free grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$.

d) Given two regular grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$.

2. Which of the following is not decidable?

a) Given a Turing machine M, a string s, and an integer k, M accepts s within k steps

b) The equivalence of two given Turing machines

c) The language accepted by a given finite state machine is not empty.

d) The language generated by a context-free grammar is non-empty.

3. Consider the following decision problems:

   P_1: Does a given finite state machine accept a given string

   P_2: Does a given context-free grammar generate an infinite number of strings.

   Which of the following statements is true?

   a) Both P_1 and P_2 are decidable
   b) Neither P_1 nor P_2 are decidable
   c) Only P_1 is decidable
   d) Only P_2 is decidable

4. Consider the following problem X.

   Given a Turing machine M over the input alphabet Σ, any state q of M and a word $w \in \Sigma^*$, does the computation of M on w visit the state q?

a) X is decidable

b) X is undecidable but partially decidable

c) X is undecidable and not even partially decidable

d) X is not a decision problem

5. Which of the following is true?

a) The complement of a recursive language is recursive

b) The complement of a recursively enumerable language is recursively enumerable

c) The complement of a recursive language is either recursive or recursively enumerable

d) The complement of a context-free language is context free

6. L_1 is a recursively enumerable language over Σ. An algorithm A effectively enumerates its words as $w_1, w_2, w_3, \ldots$ Define another language L_2 over $\Sigma \cup \{\#\}$ as $\{w_i\#w_j : w_j \in L_1, i < j\}$. Here, $\#$ is a new symbol.

Consider the following assertions.

S_1 : L_1 is recursive implies that L_2 is recursive

S_2 : L_2 is recursive implies that L_1 is recursive Which of the following statements is true?

a) Both S_1 and S_2 are true
b) S_1 is true but S_2 is not necessarily true
c) S_2 is true but S_1 is not necessarily true
d) Neither is necessarily true

7. Consider three decision problems P_1, P_2, and P_3. It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is true?

a) P_3 is decidable if P_1 is reducible to P_3
b) P_3 is undecidable if P_3 is reducible to P_2
c) P_3 is undecidable if P_2 is reducible to P_3
d) P_3 is decidable if P_3 is reducible to P_2's complement

8. Let L1 be a recursive language, and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is true?

   a) $-L_1$ is recursive and $-L_2$ is recursively enumerable.

   b) $-L_1$ is recursive and $-L_2$ is not recursively enumerable.

   c) $-L_1$ and $-L_2$ are recursively enumerable.

   d) $-L_1$ is recursively enumerable and $-L_2$ is recursive

9. For $S \in (0+1)^*$, let $d(s)$ denote the decimal value of s [e.g., $d(101) = 5$]. Let $L = \{s \in (0+1)^* | d(s) \text{ mod } 5 = 2 \text{ and } d(s) \text{ mod } 7 \neq 4\}$

   Which one of the following statements is true?

   a) L is recursively enumerable, but not recursive

   b) L is recursive, but not context free

   c) L is context free, but not regular

   d) L is regular

10. Let L1 be a regular language, L2 be a deterministic context-free language, and L3 a recursively enumerable, but not recursive language. Which one of the following statements is false?
    a) $L_1 \cap L_2$ is a deterministic CFL
    b) $L_3 \cap L_1$ is recursive
    c) $L_1 \cup L_2$ is context free
    d) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

11. Which of the following problems is undecidable?
    a) Membership problem for CFGs.
    b) Ambiguity problem for CFGs.
    c) Finiteness problem for FSAs
    d) Equivalence problem for FSAs.

12. The language $L = \{0^i 2 1^i | i \geq 0\}$ over alphabet $\{0, 1, 2\}$ is
    a) not recursive
    b) recursive and is a deterministic CFL.
    c) a regular language.
    d) not a deterministic CFL but a CFL.

13. Which of the following are decidable?

    i) Whether the intersection of two regular languages is infinite

    ii) Whether a given context-free language is regular

    iii) Whether two pushdown automata accept the same language

    iv) Whether a given grammar is context free

    a) (i) and (ii) b) (i) and (iv) c) (ii) and (iii) d) (ii) and (iv)

14. If L and$-\overline{L}$ are recursively enumerable, then L is

    a) Regular b) Context free c) Context sensitive d) Recursive

15. Which of the following statements is false?

    a) Every NFA can be converted to an equivalent DFA

    b) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine.

    c) Every regular language is also a context-free language

    d) Every subset of a recursively enumerable set is recursive.

16. Let L_1 be a recursive language. Let L_2 and L_3 be the languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

   a) $L_2 - L_1$ is recursively enumerable
   b) $L_1 - L_3$ is recursively enumerable
   c) $L_2 \cap L_1$ is recursively enumerable
   d) $L_2 \cup L_1$ is recursively enumerable

17. Let $L = L_1 \cap L_2$, where L_1 and L_2 are languages as defined in the following:

$$L^1 = \{ambmcanbn, m, n \geq 0), L^2 = \}a^i b^i c^k, i, j, k \geq 0)$$

Then, L is

   a) not recursive
   b) regular
   c) context-free but not regular
   d) recursively enumerable but not context free

18. Which of the following problems are decidable?

    i) Does a given program ever produce an output?

    ii) If L is a context-free language, then, is $-L$ also context free?

    iii) If L is a regular language, then is $-L$ also regular?

    iv) If L is a recursive language, then is $-L$ also recursive?

    a) (i), (ii), (iii), (iv) b) (i), (ii) c) (ii), (iii), (iv) d) (iii), (iv)

Answers: 1. d 2. b 3. a 4. b 5. a 6. b 7. b 8. b 9. b 10. b 11. b 12. b 13. b 14. d 15. d 16. b 17. a 18. d

Hints:

1. From a regular grammar, FA can be designed. It can be tested whether two FA are equivalent or not.

3. A given CFG is infinite if there is at least one cycle in the directed graph generated from the production rules of the given CFG in CNF.

4. The problem is undecidable. But if the state is the beginning state, it must be traversed. Thus, it is partially decidable.

8. The complement of a recursively enumerable language is not recursively enumerable.

9. $L = 5n + 2$ but $L \neq 7n + 4$. Hence, we can design a machine which halts and accepts if $L = 5n + 2$ and halts and rejects if $L \neq 7n + 4$. So, it is decidable.

10. L_1 and L_2 are recursive. The intersection of two recursive languages is recursive. But the intersection of the recursive and the recursively enumerable languages are recursively enumerable and not recursive.

12. A DPDA can be designed which PUSH X for each appearance of '0'. No stack operation for traversing 2 and POP X for '1'. A Turing machine can be designed for it where for each '0' it traverses the rightmost '1' by replacing them by X and Y, respectively. If after X, 2 appears and after 2, Y appears, then it halts. Thus, it is recursive.

13. The intersection of two regular language is regular, i.e., CFL. Using $Q3$, we can find infiniteness.

16. The size of the L_1 set is less than the size of the L_3 set.

17. Two languages are CFL. The intersection of the two CFL is not a CFL, so not regular. (b and c are false). The answer will be 'a' or 'd'.

Exercise

1. Prove that any decision cannot be taken for $A \cup B^C$, if A is recursive and B is recursively enumerable.

2. Prove that $A \cup B^C$ is recursively enumerable if A is recursive and B is recursively enumerable.