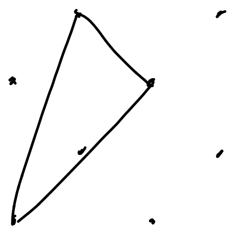
1. $PQ = \sqrt{1+4^2+9^2} = \sqrt{1+81+16} = \sqrt{98} = 7\sqrt{2}$

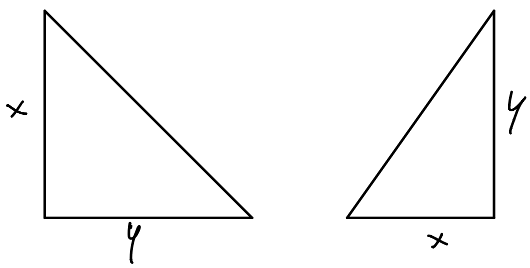
$PR = \sqrt{4^2+9^2+1^2} = 7\sqrt{2} = PQ$

$QR = \sqrt{3^2+5^2+8^2} = \sqrt{9+25+64} = \sqrt{98} = 7\sqrt{2}$



$49 \cdot 6 = 98 \cdot 3 = 196 + 98 = 294$

2.



$\frac{1}{3} y^2 \cancel{\pi} x = 1920\cancel{\pi}$ $\qquad$ $\frac{1}{3} x^2 \cancel{\pi} y = 800\cancel{\pi}$

$y^2 x = 5760$ $\qquad$ $x^2 y = 2400$

$\frac{y^{\cancel{2}} \cancel{x}}{x^{\cancel{2}} \cancel{y}} = \frac{576\cancel{0}}{240\cancel{0}}$

$\frac{y}{x} = \frac{24}{10} = \frac{12}{5}$

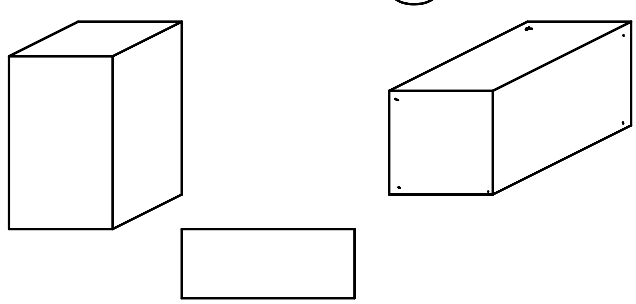
$5y = 12x \Rightarrow$ $\frac{1}{3} x^2 \cdot \frac{12}{5} x = 800$

$x^3 = 2400 \cdot \frac{5}{12} = 1000$

5-12-13 $\qquad$ $x = 10$

10-24-26

3.



236 blocks

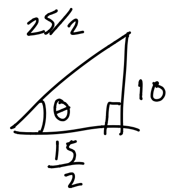
$N = \text{volume} = abc$

$(a-1)(b-1)(c-1) = 231 = 3 \cdot 7 \cdot 11$

$a-1$	$b-1$	$c-1$
231	1	1
77	3	1
33	7	1
21	11	1
11	7	$3 \Rightarrow 384$

4. $\frac{1}{3} A \cdot H$

$A = 225$

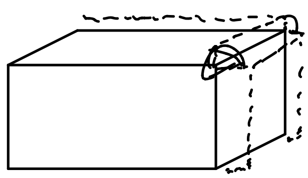


$\frac{625}{4} - \frac{225}{4} = \frac{400}{4} = 100$

$\frac{25}{2} \sin\theta$

$V = \frac{1}{3} 225 \cdot 10 = 75 \cdot 10 = 750$

5. Parallepiped $= 20 \times 3 = 60$



rectangular boxes $2 \cdot (3 \cdot 4) + 2 \cdot (4 \cdot 5) + 2 \cdot (4 \cdot 3) = 2(47) = 94$

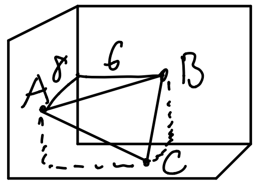
quarter cylinders $4 \cdot \frac{\pi}{4} 1^2 4 + 4 \cdot \frac{\pi}{4} 1^2 \cdot 3 + 4 \cdot \frac{\pi}{4} 1^2 \cdot 5 = 12\pi$

1/8 sphere $8 \cdot \left(\frac{1}{8} \cdot \frac{4}{3} \pi r^3\right) = \frac{4}{3}\pi$

$$60 + 94 + 12\pi + \frac{4}{3}\pi = \frac{154 \cdot 3 + 40\pi}{3} = \frac{462 + 40\pi}{3}$$

$\Rightarrow 505$

6.



$AB = \sqrt{100} = 10$

$AC = \sqrt{6^2 + \left(\frac{h}{2}\right)^2} = \sqrt{36 + \frac{h^2}{4}} = \frac{1}{2}\sqrt{h^2+144}$

$BC = \sqrt{8^2 + \left(\frac{h}{2}\right)^2} \quad \sqrt{\frac{h^2}{4} + 64} = \frac{1}{2}\sqrt{h^2+256}$

Heron's $\Rightarrow$ $s = \dfrac{AB + AC + BC}{2}$

$$= \frac{10 + \frac{1}{2}\sqrt{h^2+144} + \frac{1}{2}\sqrt{h^2+256}}{2}$$

$$= 5 + \frac{1}{4}\sqrt{h^2+144} + \frac{1}{4}\sqrt{h^2+256}$$

$$\Rightarrow [ABC]^2 = s\cdot(s-AB)(s-AC)(s-BC)$$

$$= \left(5 + \frac{1}{4}\sqrt{h^2+144} + \frac{1}{4}\sqrt{h^2+256}\right)$$
$$\cdot\left(-5 + \frac{1}{4}\sqrt{h^2+144} + \frac{1}{4}\sqrt{h^2+256}\right)$$
$$\cdot\left(5 - \frac{1}{4}\sqrt{h^2+144} + \frac{1}{4}\sqrt{h^2+256}\right)$$
$$\cdot\left(5 + \frac{1}{4}\sqrt{h^2+144} - \frac{1}{4}\sqrt{h^2+256}\right)$$

$$\Rightarrow \left(-25 + \frac{1}{16}(h^2+144) + \frac{1}{8}\sqrt{h^2+144}\sqrt{h^2+256} + \frac{1}{16}(h^2+256)\right)$$
$$\cdot\left(25 - \frac{1}{16}(h^2+144) + \frac{1}{8}\sqrt{h^2+144}\sqrt{h^2+256} - \frac{1}{16}(h^2+256)\right)$$

$$\Rightarrow \left(\frac{1}{8}h^2 + \frac{1}{8}\sqrt{h^2+144}\sqrt{h^2+256}\right)\left(-\frac{1}{8}h^2 + \frac{1}{8}\sqrt{h^2+144}\sqrt{h^2+256}\right) = -\frac{1}{64}h^4 + \frac{1}{64}(h^2+144)(h^2+256)$$

$$= \frac{1}{64}\left(-h^4 + h^4 + 400h^2 + 144\cdot 256\right)$$

$$= \frac{\overset{25}{\cancel{400}}h^2 + 144\cdot\overset{16}{\cancel{256}}}{\cancel{64}\,4} = 900$$

$$25h^2 = 3600 - 144\cdot 16$$

$$= 1296$$

$$h^2 = \frac{1296}{25}$$

$$\Rightarrow h = \frac{36}{5} \Rightarrow 36 + 5 = 41$$

7. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\cdot 12\cdot 25\pi = 100\pi$

$V_e = \frac{1}{3}\pi\cdot\left(\frac{3}{4}\cdot 5\right)^2\cdot 9 = 3\pi\cdot\frac{225}{16} = \frac{675\pi}{16}$

$100\pi - \frac{675\pi}{16} = \frac{925\pi}{16} = \text{Volume}_{air}$

$\frac{1}{3}\pi r^2 h = \frac{925\pi}{16}$

$\Rightarrow \frac{1}{3}\cancel{\pi}\cdot\left(\frac{5}{12}h\right)^2\cdot h = \frac{925\cancel{\pi}}{16}$

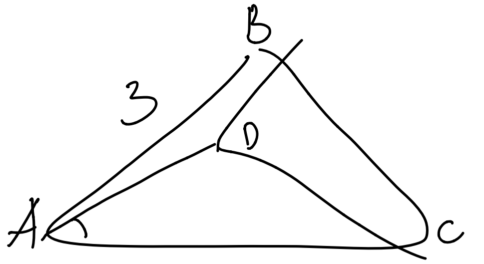
$\frac{\cancel{25}}{\cancel{144}\,9}\cdot\frac{1}{3}h^3 = \frac{\cancel{925}^{37}}{\cancel{16}}$

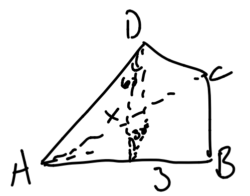
$h^3 = 37\cdot 27$

$h = 3\sqrt[3]{37}$

$\Rightarrow 12 - 3\sqrt[3]{37} \Rightarrow 12+40 = 52$

8.

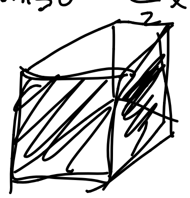




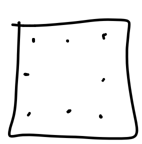
$x = 12\cdot\frac{2}{3} = 8$

$[ABD] = \frac{1}{2}\cdot 3\cdot x = 12$

$V = \frac{1}{2}Bh = \frac{15}{3}h = 5\cdot x\cdot \sin 30 = \frac{5}{2}x = 20$



9. $\frac{2}{3}$ probability to be facing orange



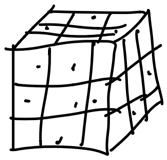
Center cube: $\left(\frac{2}{3}\right)^6$

Edge center: $\left(\frac{5}{12}\right)^{12}$

Corner edge: $\frac{4}{6}\cdot\frac{3}{5}\cdot\frac{2}{\cancel{4}} = \left(\frac{3}{12}\right)^8$

$\frac{2^6\cdot 5^{12}\cdot 1^8}{3^6\cdot 12^{12}\cdot 4^8} = \frac{\cancel{2^6}\cdot 5^{12}}{3^{18}\cdot 2^{18}\cdot 2^{16}}$

$= \frac{5^{12}}{3^{18}\cdot 2^{34}}$



Inside: 1

$17+21+36=57+17$

$=74$

10. $|8-x|+y\le 10$

$3y-x\ge 15$

$x\le 8:\ 8-x+y\le 10$

$\Rightarrow\ y\le x+2$

$x\ge 8:\ x-8+y\le 10$

$y\le 18-x$

$y=x+2$

$y=18-x$

$18-x=x+2$

$2x=16$

$x=8$

$y=10$

intersect $(8,10)$

$3y-x=15$

$y=\frac{1}{3}x+5$

$x+2=\frac{1}{3}x+5$

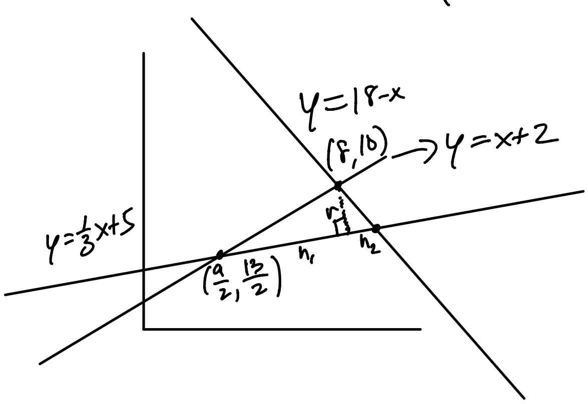
$\frac{2}{3}x=3$

$x=\frac{9}{2},\ y=\frac{13}{2}$ $\quad\left(\frac{9}{2},\frac{13}{2}\right)$

$18-x=\frac{1}{3}x+5$

$\frac{4}{3}x=13$ $\quad x=\frac{39}{4},\ y=\frac{13}{4}+5=\frac{33}{4}$

$\left(\frac{39}{4},\frac{33}{4}\right)$



$$h = h_1 + h_2 = \sqrt{\left(\frac{39}{4} - \frac{18}{4}\right)^2 + \left(\frac{33}{4} - \frac{26}{4}\right)^2}$$

$$= \sqrt{\left(\frac{21}{4}\right)^2 + \left(\frac{7}{4}\right)^2} = \sqrt{\frac{441 + 49}{16}} = \sqrt{\frac{490}{16}} = \frac{7\sqrt{10}}{4}$$

perp. line to $y = \frac{1}{3}x + 5$ @ $(8, 10)$

$$y - 10 = -3(x - 8)$$

$$y = -3x + 34$$

$$\Rightarrow -3x + 34 = \frac{1}{3}x + 5$$

$$29 = \frac{10}{3}x$$

$$x = \frac{87}{10} = 8.7$$

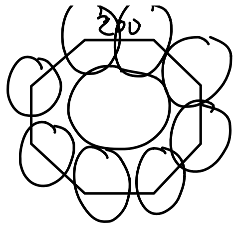
$$y = \frac{29}{10} + 5 = 7.9$$

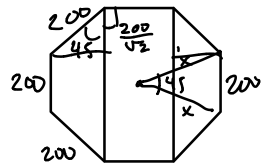
$r =$ distance from $(8, 10)$ to $y = -3x + 34$

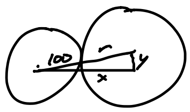
$$= \sqrt{.7^2 + (-2.1)^2} = \sqrt{.49 + 4.41} = \sqrt{4.90} = \frac{7}{10}\sqrt{10}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot \frac{49}{\cancel{100}\ \cancel{10}\ \sqrt{10}} \cdot \cancel{10} \cdot \frac{7\cancel{\sqrt{10}}}{4} = \frac{343\pi}{12\sqrt{10}} \Rightarrow 343 + 22 = 365$$

11.







$200^2 = 2x^2 - 2x^2\cos 45° = 2x^2 - \cancel{2}x^2\cdot\frac{\sqrt2}{\cancel{2}}$

$= 2x^2 - \sqrt2 x^2$

$x^2 = \frac{200^2}{2-\sqrt2}$ $\quad x = \frac{200}{\sqrt{2-\sqrt2}}$

$(r+100)^2 = x^2+y^2 = x^2+(r-100)^2$

$r^2+200r+10000 = x^2+r^2-200r+10000$

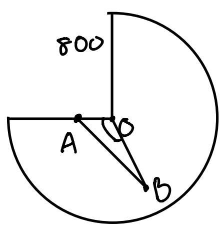
$400r = x^2$

$r = \frac{x^2}{400} = \frac{\left(\frac{200}{\sqrt{2-\sqrt2}}\right)^2}{400} = \frac{\cancel{400}00}{2-\sqrt2}\cdot\frac{1}{\cancel{400}} = \frac{100}{2-\sqrt2} = \frac{100(2+\sqrt2)}{2}$

$= 100+50\sqrt2$

$= 152$

12.



$\sqrt{600^2+(200\sqrt7)^2} = \sqrt{360000+280000}$

$= \sqrt{640000} = 800$

270° sector $= 2\pi\cdot\cancel{8}^{2}00\times\frac{3}{\cancel{4}} = 1200\pi$

m<AOB = 135° (half of total sector angle since on opposite sides of cone)

Law of cosines

$AB^2 = AO^2+OB^2-2AO\cdot OB\cos(135°)$

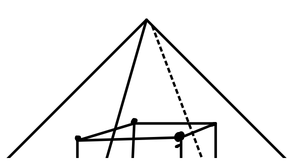
$\Rightarrow AB = \sqrt{125^2+375^2\cdot2-2\cdot125\cdot375\sqrt2\cdot-\frac{\sqrt2}{2}}$

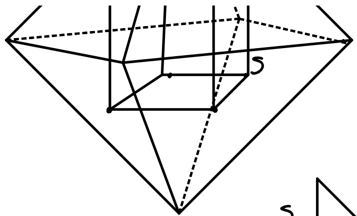
$= \sqrt{125^2+375^2\cdot2+2\cdot125\cdot375}$

$= \sqrt{125^2(1+3^2\cdot2+6)}$

$= 125\sqrt{25} = 125\cdot5 = 625$

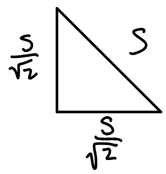
13.





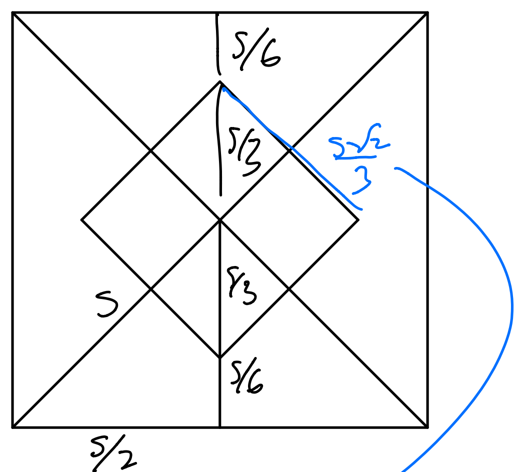
2 tetrahedron: $V=2\cdot\frac{1}{3}Bh$

$V=2\cdot\frac{1}{3}\cdot\left(\frac{s}{\sqrt{2}}\right)^2\cdot\frac{s}{\sqrt{2}}$



$\rightarrow h=\frac{s}{\sqrt{2}}$

$=2\cdot\frac{1}{3}\cdot\frac{s^2}{2\sqrt{2}}=\frac{s^3}{3\sqrt{2}}$



$\frac{\cancel{s^3}}{\cancel{3}\sqrt{2}} : \frac{\cancel{s^3}\cdot 2\sqrt{2}}{\cancel{27}\,9}$

9:2

$\frac{2}{9}\Rightarrow 2+9=11$

$\left(\frac{s\sqrt{2}}{3}\right)^3=\frac{s^3\cdot 2\sqrt{2}}{27}$