1. Case 1. All blue or all red

$2\cdot\left(\frac{\cancel{10}}{\cancel{20}_2}\cdot\frac{9}{19}\cdot\frac{\cancel{8}^{2}}{\cancel{18}_{2}}\cdot\frac{7}{17}\right)$

Case 2. 1 b, 1 r

$4\left(\frac{\cancel{10}}{\cancel{20}_2}\cdot\frac{\cancel{10}^{5}}{19}\cdot\frac{9}{\cancel{18}_2}\cdot\frac{9}{17}\right)$

$2\cdot\left(\frac{14}{19\cdot17}\right)+4\left(\frac{45}{19\cdot17\cdot2}\right)$

$=\frac{28}{19\cdot17}+\frac{90}{19\cdot17}=\frac{118}{323}\Rightarrow 118+323=441$

RRRR

RBRB

RBBR

BRBR

BRRB

BBBB

2. $\binom{38}{2}$

$\binom{2}{2}$ if # of removed pair = 1

$9\cdot\binom{4}{2}=9\cdot\frac{4\cdot3}{2}=96=54$

$\frac{55}{\frac{38\cdot37}{2}}=\frac{55}{19\cdot37}=\frac{55}{703}\Rightarrow 55+703=758$

3. odd + odd + odd = odd

even + even + odd = odd

$\frac{3\cdot\left(\binom{5}{3}\cdot\binom{4}{2}\cdot\binom{2}{1}\cdot\binom{2}{2}\cdot\binom{1}{1}\right)}{\binom{9}{3}\cdot\binom{6}{3}\cdot\binom{3}{3}}=\frac{\cancel{3}\cdot\frac{\cancel{5}\cdot\cancel{4}}{\cancel{2}}\cdot\frac{\cancel{4}\cdot3}{\cancel{2}}\cdot\cancel{2}\cdot1\cdot1}{\cancel{3}\,\frac{\cancel{9}\cdot\cancel{8}\cdot7}{\cancel{3!}}\cdot\frac{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}}{\cancel{3!}}\cdot1}=\frac{3}{14}\Rightarrow 3+14=17$

4. 4 – 8 options

$4 \to 1, 12 \to \frac{2}{12} \cdot \frac{4}{11}$

$5 \to 2, 11 \to \frac{2}{12} \cdot \frac{5}{11}$

$6 \to 3, 10 \to \frac{2}{12} \cdot \frac{6}{11}$

$7 \to 4, 9 \to \frac{2}{12} \cdot \frac{7}{11}$

$8 \to 5, 6, 7, 8 \to \frac{4}{12} \cdot \frac{8}{11} \to \frac{1}{3} \cdot \frac{8}{11} = \frac{8}{33}$

$\frac{2}{12} \cdot \left(\frac{22}{11}\right) = \frac{4}{12}$

$\frac{44}{132} + \frac{32}{132} = \frac{76}{132}$

$= \frac{19}{33} \Rightarrow 52$

5.

$$5 \frac{x}{y} \cdot \left(\frac{y-x}{y}\right)^4 = 10 \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \left(\frac{y-x}{y}\right)^3$$

$$5 \frac{y-x}{y} = 10 \frac{x}{y}$$

$$y^2 - xy = 2xy$$

$$y^2 - 3xy = 0$$

$$y(y-3x) = 0$$

$y = 3x \Rightarrow$ heads is $\frac{1}{3}$

$$\binom{5}{2}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{40}{243} \Rightarrow 283$$

6. Case 1 1 distinct number

$$\binom{6}{1} \cdot \binom{1}{1} \cdot \binom{1}{1} \cdot \binom{1}{1} \cdot \binom{1}{1} = 6$$

Case 2 2 distinct numbers $\binom{6}{2} \cdot 3$

xxyy

xyyy

xxxy

$1 \to 5$

$2 \to 4$

$3 \to 3$

$4 \to 2$

$5 \to 1$

$\Rightarrow 15 \cdot 3 = 45$

Case 3. 3 distinct numbes $\binom{6}{3} \cdot 3$

$xxyz$ $\quad 1 \to (2 \to 4) + (3 \to 3) + (4 \to 2) + (5 \to 1) = 10$

$xyyz$ $\quad 2 \to 6 \quad 4 \to 1$ $\qquad 20 \cdot 3 = 60$

$xyzz$ $\quad 3 \to 3$

Case 4. 4 distinct

$wxyz$

$123 \to 3 \quad 134 \to 2 \quad 145 \to 1$

$124 \to 2 \quad 135 \to 1$

$345 \to 1$ $\quad 15$

$125 \to 1$

$234 \to 2 \quad 235 \to 1 \quad 245 \to 1$

$6 + 45 + 60 + 15 = 126$

$$\frac{126}{6^4} = \frac{126}{1296} = \frac{21}{216} = \frac{7}{72} \Rightarrow 79$$

7. $p(\overline{E_1 \cup E_2}) = 1 - (p(E_1) + p(E_2) - p(E_1 \cap E_2))$

$|E_1| \Rightarrow$ undefeated team

$1v2$ ✓ $\quad 2v3 \quad 3v4 \quad 4v5$

$1v3$ ✓ $\quad 2v4 \quad 3v5$

$1v4$ ✓ $\quad 2v5$

$1v5$ ✓

$$\binom{5}{1} \cdot \left(1^4 \cdot \binom{2}{1}^6\right) = 5 \cdot 2^6 = 5 \cdot 64 = 320$$

$|E_2| \Rightarrow$ winless $= |E_1|$

$$|E_1 \cap E_2| = \binom{5}{1} \cdot 1^4 \cdot \binom{4}{1} \cdot 1^3 \cdot \binom{2}{1}^3 = 5 \cdot 4 \cdot 8 = 160$$

$640-160=480$

$$\frac{\binom{2}{1}^{10}-480}{\binom{2}{1}^{10}} = \frac{544}{1024} = \frac{136}{256} = \frac{34}{64}$$

$$= \frac{17}{32} \Rightarrow 49$$

8. Case 0: 0 head

1 way $\frac{3}{28}$

$$\left(\frac{3}{28}\right)^2 + \left(\frac{5}{14}\right)^2 + \left(\frac{11}{28}\right)^2 + \left(\frac{1}{7}\right)^2$$

$$\frac{9}{28\cdot 28} + \frac{25}{14\cdot 14} + \frac{121}{28\cdot 28} + \frac{1}{7\cdot 7}$$

Case 1: 1 head

3 $\frac{5}{14}$

$$\Rightarrow \frac{9+100+121+16}{28\cdot 28} = \frac{146+100}{28\cdot 28}$$

$$= \frac{246}{28\cdot 28} = \frac{123}{14\cdot 28} = \frac{123}{392} \Rightarrow 515$$

Case 2: 2 head

3 way $\frac{11}{28}$

Case 3: 3 heads

1 way $\frac{1}{7}$

9. $\{1, 2, 3, 4, 5, 6\}$

1 2 3	1 2 4	1 2 5	1 3 4	1 3 5
4 5 6	3 5 6	3 4 6	2 5 6	2 4 6

5 of $\binom{6}{3}$ ways to group them are valid

thus $\frac{5}{\binom{6}{3}}$ of all groups of 6 numbers valid $\Rightarrow \frac{5}{\binom{6}{3}} = \frac{5}{20} = \frac{1}{4} \Rightarrow 5$

10. $\_\ \underline{\ \ }_H\ \_\ \underline{\ \ }_H\ \_\ \underline{\ \ }_H\ \_\ \underline{\ \ }_H\ \_\ \underline{\ \ }_H$

0 – 5 heads

Case 1 0 heads

1 way

Case 2. 1 head

$\binom{10}{1} = 10$ ways

Case 3 2 heads

$x_1+x_2+x_3 = 8$

$y_1+y_2+y_3 = 7$

$\binom{9}{2} = \frac{9 \cdot 8}{2} = 36$

Case 4. 3 heads

$x_1+x_2+x_3+x_4 = 10-3 = 7$

$y_1+y_2+y_3+y_4 = 5$

$\binom{5+3}{3} = \binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3!} = 8 \cdot 7 = 56$

Case 5. 4 heads

$y_1+y_2+y_3+y_4+y_5 = 10-4-3 = 3$

$\binom{7}{4} = \frac{7 \cdot 6 \cdot 5}{3!} = 35$

Case 6. 5 heads.

$y_1+y_2+y_3+y_4+y_5+y_6 = 10-5-4 = 1$

$\binom{6}{5} = \frac{6}{1!} = 6$

$$1+10+36+56+35+6=144$$

$$\frac{144}{2^{10}} = \frac{144}{1024} = \frac{9}{64} \Rightarrow 73$$

11. Each have 5 more matches (for k matches B wins, A can win $k, \dots, 5$ matches)

$$\binom{5}{0}\cdot\left(\binom{5}{0}+\cdots+\binom{5}{5}\right)+\binom{5}{1}\cdot\left(\binom{5}{1}+\cdots+\binom{5}{5}\right)+\binom{5}{2}\cdot\left(\binom{5}{2}+\cdots+\binom{5}{5}\right)$$

$$+\binom{5}{3}\cdot\left(\binom{5}{3}+\binom{5}{4}+\binom{5}{5}\right)+\binom{5}{4}\cdot\left(\binom{5}{4}+\binom{5}{5}\right)+\binom{5}{5}\cdot\binom{5}{5}$$

$$=1\cdot(1+5+10+10+5+1)+5(5+10+10+5+1)+10(10+10+5+1)$$

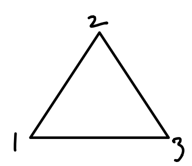
$$+10(10+5+1)+5(5+1)+1\cdot 1 = 1\cdot 32+5\cdot 31+10\cdot 26+10\cdot 16+5\cdot 6+1$$

$$=32+155+260+160+30+1$$

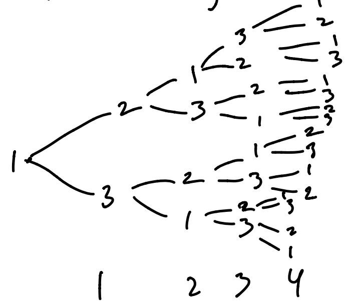
$$=420+31+187=420+218=638$$

$$\frac{638}{32\cdot 32} = \frac{319}{16\cdot 32} = \frac{319}{512} \Rightarrow 831$$

12.



Probability it is @ 2 vertices after 9 moves



2^{10}

$p_0=1 \quad q_0=0 \quad r_0=0$

$p_1=0 \quad q_1=\frac{1}{2} \quad r_1=\frac{1}{2}$

$p_2=\frac{1}{2} \quad q_2=\frac{1}{4} \quad r_2=\frac{1}{4}$

$p_3=\frac{1}{4} \quad q_3=\frac{3}{8} \quad r_3=\frac{3}{8}$ $\qquad \frac{6}{16} \quad \frac{5}{16}$

$p_4=\frac{3}{8} \quad q_4=\frac{5}{16} \quad r_4=\frac{5}{16}$

$p_5=\frac{5}{16} \quad q_5=\frac{11}{32} \quad r_5=\frac{11}{32}$

$p_6 = \frac{11}{32}$ $q_6 = \frac{21}{64}$ $r_6 = \frac{21}{64}$ $\frac{6}{16}$ $\frac{5}{16}$

$p_7 = \frac{21}{64}$ 10 $\frac{21}{32}$

$p_8 = \frac{43}{128}$ $22+21$ $\frac{43}{64}$

$p_9 = \frac{85}{216}$

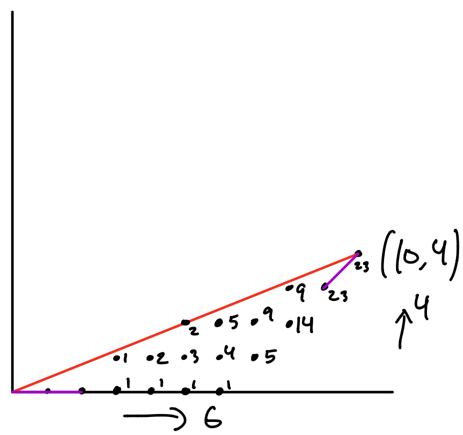
$43+42 = 85$

$p_{10} = \frac{171}{512}$

$\frac{85+86}{216} = 171$

$171 + 512 = 683$

13.



23 total ways

$$\left(\frac{3}{5}\right)^6 \cdot \left(\frac{2}{5}\right)^4 \cdot 23 = \frac{3^6 \cdot 2^4 \cdot 23}{5^{10}}$$

$$(3+2+23+5)\cdot(6+4+10) = 33 \cdot 20 = 660$$

14. $0, 1, \ldots, 39$

$40!$ to assign teams # of wins

$1+2+\cdots+39 = \frac{39 \cdot 40}{2} = 780$ games played

$$\frac{1}{2^{780}} \cdot 40! = \frac{40!}{2^{780}} \Rightarrow \frac{m}{2^{780-38}} = \frac{m}{2^{742}} \Rightarrow \log(2^{742}) = 742$$

15. Case 1. Starts w/ head

$p_T = \frac{1}{2} \cdot p_H$

HHHHH HHHHT HHHT HHT HT

$$p_H = \frac{1}{32} + \frac{1}{16} \cdot p_T + \frac{1}{8} p_T + \frac{1}{4} p_T + \frac{1}{2} p_T = \frac{1}{32} + \frac{15}{16} p_T$$

$$p_H = \frac{1}{32} + \frac{15}{16} \cdot \frac{1}{2} p_H$$

$$= \frac{1}{32} + \frac{15}{32} p_H$$

$$\frac{17}{32} p_H = \frac{1}{32}$$

$$p_H = \frac{32}{32 \cdot 17} = \frac{1}{17}$$

$$p_H + p_T = \frac{1}{17} + \frac{1}{34} = \frac{3}{34} \Rightarrow 37$$