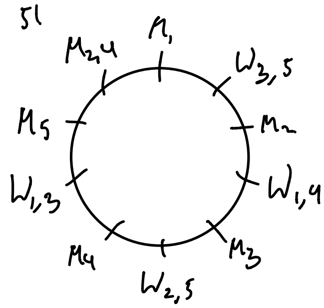
$\underbrace{5 \cdot 5 \cdot 5 \cdot \cdots \cdot 5}_{24 \text{ times}}$ $\quad 5^{24}$ ways

$2^6 - 2 = 64 - 2 = 62$

$\frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \cdot 8 \cdot 7$

5.



$2(\cdot 5! \cdot 2) = 4 \cdot 5! = 480$

$\binom{52}{8} = \frac{52!}{44! \cdot 8!}$

$\binom{n}{6} = 6 \cdot \binom{n}{3}$

$\frac{n!}{(n-6)! \cdot 6!} = \frac{\cancel{6} \cdot n!}{(n-3)! \cdot \cancel{3!}}$

$\frac{n!}{(n-6)! \cdot 6!} = \frac{n!}{(n-3)!}$

$(n-6)! \cdot 6! = (n-3)!$

$6! = (n-3)(n-4)(n-5)$

$720 = (10) \cdot (9) \cdot (8)$

$n = 13$

k types of identical objects

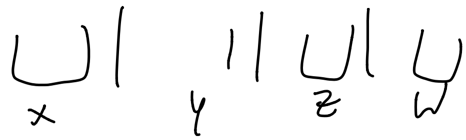
\# ways to make selection of n objects is $\binom{n+k-1}{k-1}$

$\binom{15}{6}$

How many x,y,z,w to satisfy

$x+y+z+w=2015$

Then



2015 stars, 3 bars $\Rightarrow \binom{2018}{3}$

$123-27=96$

$96 = |B \cup G \cup S| = |B| + |G| + |S| - |B \cup G| - |B \cup S| - |G \cup S|$

$+ |B \cup G \cup S|$

$96 + 32 + 21 + 15 - 11 = 3|G|$

$153 = 3|G|$

$\underline{|G| = 51}$

1. Case 1: No O's chosen

$$6\cdot5\cdot4\cdot3\cdot2 = 120\cdot6 = 720$$

Case 2: One O chosen

$$5\cdot(1\cdot 6\cdot 5\cdot 4\cdot 3) = 1800$$

Case 3: $\binom{5}{2}\cdot(1\cdot1\cdot6\cdot5\cdot4) = 10\cdot120 = 1200$

$3720 \Rightarrow \frac{n}{10} = 372$

2. 6 men 6 women

4 per dept

Choose 2 + 1 women, choose 1,1,1

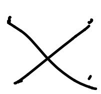
$$3\cdot\binom{2}{2}\cdot 2\binom{\cancel{1}\,2}{1}\cdot\binom{2}{1}\cdot\binom{2}{2} + \binom{2}{1}\cdot\binom{2}{1}\cdot\binom{2}{1}\cdot\binom{2}{1}\cdot\binom{3}{1}\cdot\binom{2}{1}$$

2 women   1 women 1 guy   2 guys

$$= 3\cdot1\cdot2\cdot\cancel{1}\,2\cdot2\cdot1 + 2^6$$

$$= \cancel{12}\,24 + 64 = \cancel{78\text{ ways}} = 88\text{ ways}$$

3. $\binom{26}{2}$ lines $-$ 60 edges $- 24 = 241$



24 quad diagonals

edges = lines

4.

2 2 2 $\frac{B}{B}$ $\frac{C}{F}$ $\frac{C}{F}$ $\frac{B}{C}$ $\frac{B}{C}$ $\frac{C}{C}$ $\frac{F}{B}$ $\frac{F}{B}$ $\frac{F}{F}$

B F C

B C F B C C B F F

$$9\cdot(2\cdot 3) + 9(2\cdot 3\cdot 3) = 9(6+18) = 9\cdot 24 = 216$$

5. Case 1: two 1's: $\binom{1}{1}\cdot\binom{3}{1}\cdot\binom{1}{1}\cdot\binom{9}{1}\cdot\binom{8}{1} = 3\cdot 8\cdot 9 = 216$

Case 2: two other: $\binom{1}{1}\cdot\binom{3}{2}\cdot\binom{9}{1}\cdot\binom{1}{1}\cdot\binom{8}{1} = 3\cdot 8\cdot 9 = 216$

$216 + 216 = 432$ numbers

6. 1 2 3 4 5 6 7 8 9 10 11 12 13 14

13 pairs of consecutive numbers

$$\binom{13}{1}\cdot\binom{12}{3} = 13\cdot\frac{12\cdot 11\cdot 10}{3!} = 13\cdot 11\cdot 10\cdot 2 = 26\cdot 11\cdot 10$$
$$= 2860$$

$$\binom{14}{5} - 252 = 2002 - 252 = 1750 \Rightarrow 1750 \,\%\, 1000 = 750$$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 9$ where $x_2, \dots, x_5 \geq 1$

$\Rightarrow y_1 + y_2 + y_3 + y_4 + \dots + y_6 = 5$

5 bars, 5 stars $= \binom{10}{5} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2} = 9\cdot 7\cdot 4 = 252$ (no consecutive #'s)

7. $a_6 = 1$

$1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9 < 10 < 11 < 12$

$\binom{11}{5}$ for a_1, a_2, a_3, a_4, a_5 (order predetermined) $\Rightarrow$ then $a_{7,8,9,10,11,12}$ also determined

Thus $\binom{11}{5} = 462$

$a_1 \; a_2 \; a_3 \; a_4$

8. 1 3 6 8 $\Leftarrow 5$ $\quad \binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{10}{1} = 64 \cdot 10 = 640$

1 3 6 8 $\Leftarrow 4$

1 4 6 8 $\Leftarrow 3$ 4 choices each time

1 4 6 8 $\Leftarrow 2$

2 4 6 8 $\Leftarrow 1$

9. $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$ (corrected after looking at answer, not sol'n, my approach = correct)

0 $\leftarrow$ 1 determined way

1 $\leftarrow$ $\leftarrow \cdot \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$ $\quad \binom{8}{4} = \frac{\cancel{8}^{2} \cdot 7 \cdot \cancel{6} \cdot 5}{\cancel{4!}} = 70$

2 $\leftarrow$ $\leftarrow \cdot \leftarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$ $\quad 9 \cdot 70 = 630$ ways

$\vdots$ $\leftarrow \cdot \leftarrow$

8 $\leftarrow$

10. Share 0 attributes: $\left(\binom{1}{1}\binom{1}{1}\binom{1}{1} \cdot 3! \cdot 3!\right) = 6 \cdot 6 = 36$

Share 1 attribute: 9 ways to permute the shared attribute. 3! ways to permute diff

$3 \cdot 3 \left(c \cdot 3 \cdot 1 \quad s \cdot 2 \cdot 1 \quad t \cdot 1 \cdot 1 \right) = 9 \cdot 6 = 54$

Share 2 attributes:

$\binom{3}{1}\binom{3}{1} \cdot \binom{3}{1} \left(c \cdot 1 \cdot 1 \quad s \cdot 1 \cdot 1 \quad t \cdot 1 \cdot 1 \right) = 27$

shared attributes

the diff attributes

$36 + 54 + 27 = 117$

11. M→V→E Once: 1 way (5M, 5V, 5E)

twice: (MMMVVVEEEMMVVEE) · (4·4·4)

3: (MVE MMVVEE MMVVEE) · 6·6·6

4: (MVE MVE MMVVEEMVE) · (4·4·4)

5: 1 (MVE·5)

1 + 64 + 216 + 64 + 1 = 2 + 128 + 216 = 130 + 216 = 346

346

4 1 3 2 15 ~~−8=7~~ −2 = 5

12.

TH TH TH TH

| | |

2 HH: $4 + \binom{4}{2} = 4 + 6 = 10$

5 TT: $\binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3!} = 8 \cdot 7 = 56$

5 T's, 3 bars

560 ways

0 0 10

13. Either in A, B or neither $|A \text{ empty}| = 2^{10}$ (Choose btw B and not in B) 1 9

2 7

⋮

10 0

A, B non-empty

⇓

3^{10} total

|A empty| + |B empty| − |A∪B both empty|

= 2|A empty| − |A∪B empty|

$= 2^{11} - 1$

Thus $3^{10} - 2^{11} + 1 = \frac{57002}{2}$ This double counts the sets (A={1,2}, B={3,4}

⇒ 28501 ⇒ 501 A={3,4}, B={1,2})

1 9

2 8

3 7

4 6

5 5

6 4

9 $a_1 + 9a_2 + 36a_3 + 84a_4 + 126a_5 + 126a_6 + 84a_7 + 36a_8 + 9a_9 + a_{10}$

14. . .

7 3 $a_1 + 3a_2 + 3a_3$

2 $a_1+2a_2+a_3$ $a_2+2a_3+a_4$ $a_3+2a_4+a_5$

1 a_1+a_2 a_2+a_3 a_3+a_4 a_4+a_5 a_5+a_6

0 a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10}

1 8 28 56 70 56 28 8 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

a_1 a_2 $\underbrace{a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8 \quad a_9}_{2^7}$ a_{10} a_{11}

$\boxed{0 \quad 1 \qquad 1 \quad 1}$ = 4 ways

+

0 0 0 0 = 1 way

$2^7 \cdot 5 = 5 \cdot 128 = 640$ ways

15.

6 - 10 houses get mail everyday

Case 1: 6 houses get mail

$x_1 + x_2 + \cancel{x_3} + \dots + x_7 = 19-6=13$

For 6, neither x_1 nor x_7 can be 0

Then only $6\cdot 2+1\cdot 1=13$, so $\binom{7}{1}=7$ ways $=7$

Case 2: 7 houses get mail

$x_1+x_2+\dots+x_8=12$

x_1 AND x_8 0: $6\cdot 2+2\cdot 0=1$ case

Neither 0: $4\cdot 2+4\cdot 1=\binom{8}{4}=\frac{\cancel{8}^{2}\cdot 7\cdot \cancel{6}\cdot 5}{\cancel{4!}}=70$

83

Case 3: 8

$x_1+\dots+x_9=11$

x_1 AND x_9 0: $4\cdot 2+3\cdot 1=\binom{7}{3}=\frac{7\cdot 6\cdot 5}{3!}=7\cdot 5=35$

Neither 0: $2\cdot 2+7\cdot 1=\binom{9}{2}=\frac{9\cdot 8}{2}=36$

Either is 0: $3\cdot 2+5\cdot 1+1\cdot 0=2\cdot\binom{8}{3}=2\cdot\frac{8\cdot 7\cdot 6}{3!}=8\cdot 7\cdot 2=112$

183

Case 4: 9

$x_1+\dots+x_{10}=10$ 8

x_1 AND x_{10} 0: $2\cdot 2+6\cdot 1=\binom{8}{2}=\frac{8\cdot 7}{2}=28$

Neither 0: $10\cdot 1=1$

Either 0: $1\cdot 0+1\cdot 2+8\cdot 1=2\binom{9}{1}=2\cdot 9=18$

$46+1=47$

Case 5: 10

$x_1 + \cdots + x_{11}$: 9

x_1 AND x_{11} 0: $9 \cdot 1 = 1 \Longrightarrow 1$

Neither: Impossible

Either Or: Impossible

$$7 + 83 + 183 + 47 + 1 = 90 + 230 + 1 = 321$$

15. $x_1 + x_2 + \cdots + x_{k+1} = 19 - k$, convert using $y_i = x_i - 1$ (then $y_2, \ldots, y_k \le 1$)

$\Rightarrow y_1 + y_2 + \cdots + y_{k+1} = 19 - k - (k-1) = 20 - 2k$

Case 1. $y_1 = y_{k+1} = 2$

Then $y_2 + \cdots + y_k = 20 - 2k - 4 = 16 - 2k$

Thus for $6 \le k \le 8$, $\binom{k-1}{16-2k}$ for this case

(Choose $16-2k$ y_i's to be 1)

Case 2. Either $y_1 = 2$ or $y_2 = 2$

Then, wlog, $y_1 = 2$

So $y_2 + \cdots + y_k + y_{k+1} = 18 - 2k$

Thus $2 \cdot \binom{k}{18-2k}$ for $6 \le k \le 9$

Case 3. $y_1 \ne 2$, $y_{k+1} \ne 2$

Then $y_1 + \cdots + y_{k+1} = 20 - 2k$

$\binom{k+1}{20-2k} \Rightarrow$ for $7 \le k \le 10$

($\binom{6}{8}$ invalid)

Thus, $\left(\binom{5}{4} + \binom{6}{2} + \binom{7}{0}\right) + 2\left(\binom{6}{6} + \binom{7}{4} + \binom{8}{2} + \binom{9}{0}\right) + \left(\binom{8}{6} + \binom{9}{4} + \binom{10}{2} + \binom{11}{0}\right)$

$$= \left(5 + \frac{6 \cdot 5}{2} + 1\right) + 2\left(1 + \frac{7 \cdot \cancel{6} \cdot 5}{\cancel{3!}} + \frac{\overset{4}{\cancel{8}} \cdot 7}{\cancel{2}} + 1\right) + \left(\frac{8 \cdot 7}{2} + \frac{9 \cdot \overset{2}{\cancel{8}} \cdot 7 \cdot \cancel{6}}{4!} + \frac{10 \cdot 9}{2} + 1\right)$$

$$= 21 + 2(\underbrace{37 + 28}_{65}) + (28 + 126 + 45 + 1)$$

$$= 21 + 130 + 200 = 351$$