

LECTURE NOTES ON

ELECTRICAL

MACHINE-II

Subject Code - BEE 1401

For B-Tech 4th SEM EE & EEE

[Part-II]

[Module-III & IV]



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Syllabus of Bachelor of Technology in Electrical Engineering

(4th SEMESTER)

ELECTRICAL MACHINES-II

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MODULE-III

THREE PHASE INDCTION MOTOR

SYLLABUS/ TOPICS COVERED

Three Phase Induction Motors: Types, Construction and principle of operation, 3 phase Induction Motor, general phasor diagram, equivalent circuit, power and torque relations, condition for maximum torque, circle diagram, Performance characteristics, effect of rotor resistance on speed torque characteristics, stable & unstable region of operation, Operation with unbalanced supply voltage. Starting: Starting of 3 phase induction motors, high starting torque motors, speed control, rheostatic method, pole changing method cascade control of speed, Double cage induction motor, Cogging and Crawling of Induction motor, induction generator

[Topics are arranged as per above sequence]

Module-III

3.1 Three Phase Induction Motor

The most common type of AC motor being used throughout the work today is the "Induction Motor". Applications of three-phase induction motors of size varying from half a kilowatt to thousands of kilowatts are numerous. They are found everywhere from a small workshop to a large manufacturing industry.

The advantages of three-phase AC induction motor are listed below:

- Simple design
- Rugged construction
- Reliable operation
- Low initial cost
- Easy operation and simple maintenance
- Simple control gear for starting and speed control
- High efficiency.

Induction motor is originated in the year 1891 with crude construction (The induction machine principle was invented by *NIKOLA TESLA* in 1888.). Then an improved construction with distributed stator windings and a cage rotor was built.

The slip ring rotor was developed after a decade or so. Since then a lot of improvement has taken place on the design of these two types of induction motors. Lot of research work has been carried out to improve its power factor and to achieve suitable methods of speed control.

3.2 Types and Construction of Three Phase Induction Motor

Three phase induction motors are constructed into two major types:

1. Squirrel cage Induction Motors
2. Slip ring Induction Motors

3.2.1 Squirrel cage Induction Motors

(a) Stator Construction

The induction motor stator resembles the stator of a revolving field, three phase alternator. The stator or the stationary part consists of three phase winding held in place in the slots of a laminated steel core which is enclosed and supported by a cast iron or a steel frame as shown in Fig: 3.1(a).

The phase windings are placed 120 electrical degrees apart and may be connected in either star or delta externally, for which six leads are brought out to a terminal box mounted on the frame of the motor. When the stator is energized from a three phase voltage it will produce a rotating magnetic field in the stator core.

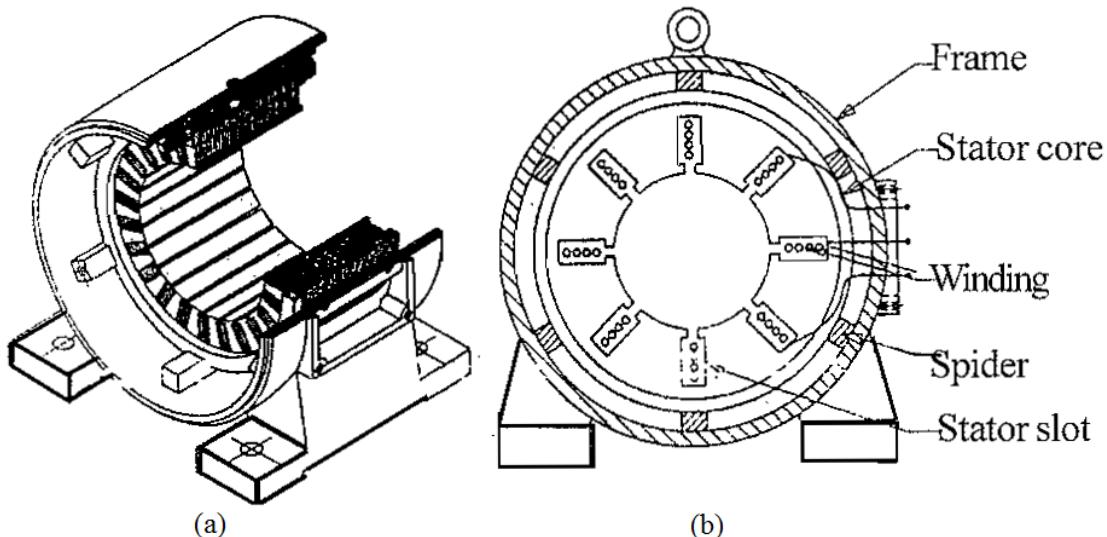


Fig: 3.1

(b) Rotor Construction

The rotor of the squirrel cage motor shown in Fig: 3.1(b) contains no windings. Instead it is a cylindrical core constructed of steel laminations with conductor bars mounted parallel to the shaft and embedded near the surface of the rotor core.

These conductor bars are short circuited by an end rings at both end of the rotor core. In large machines, these conductor bars and the end rings are made up of copper with the bars brazed or welded to the end rings shown in Fig: 3.1(b). In small machines the conductor bars and end rings are sometimes made of aluminium with the bars and rings cast in as part of the rotor core. Actually the entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived.

The rotor or rotating part is not connected electrically to the power supply but has voltage induced in it by transformer action from the stator. For this reason, the stator is sometimes called the primary and the rotor is referred to as the secondary of the motor since the motor operates on the principle of induction and as the construction of the rotor with the bars and end rings resembles a squirrel cage, the squirrel cage induction motor is used.

The rotor bars are not insulated from the rotor core because they are made of metals having less resistance than the core. The induced current will flow mainly in them. Also the rotor bars are usually not quite parallel to the rotor shaft but are mounted in a slightly skewed position. This feature tends to produce a more uniform rotor field and torque. Also it helps to reduce some of the internal magnetic noise when the motor is running.

(c) End Shields

The function of the two end shields is to support the rotor shaft. They are fitted with bearings and attached to the stator frame with the help of studs or bolts attention.

3.2.2 Slip ring Induction Motors

(a) Stator Construction

The construction of the slip ring induction motor is exactly similar to the construction of squirrel cage induction motor. There is no difference between squirrel cage and slip ring motors.

(b) Rotor Construction

The rotor of the slip ring induction motor is also cylindrical or constructed of lamination.

Squirrel cage motors have a rotor with short circuited bars whereas slip ring motors have wound rotors having "three windings" each connected in star.

The winding is made of copper wire. The terminals of the rotor windings of the slip ring motors are brought out through slip rings which are in contact with stationary brushes as shown in Fig: 3.2.

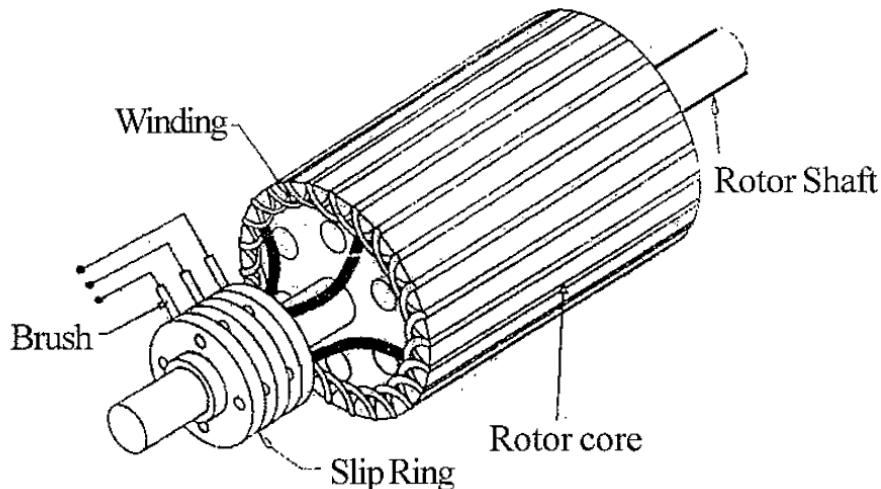


Fig: 3.2

THE ADVANTAGES OF THE SLIPRING MOTOR ARE

- It has susceptibility to speed control by regulating rotor resistance.
- High starting torque of 200 to 250% of full load value.
- Low starting current of the order of 250 to 350% of the full load current.

Hence slip ring motors are used where one or more of the above requirements are to be met.

3.2.3 Comparison of Squirrel Cage and Slip Ring Motor

Sl.No.	Property	Squirrel cage motor	Slip ring motor
1.	Rotor Construction	<i>Bars are used in rotor. Squirrel cage motor is very simple, rugged and long lasting. No slip rings and brushes</i>	<i>Winding wire is to be used.</i> <i>Wound rotor required attention.</i> <i>Slip ring and brushes are needed also need frequent maintenance.</i>
2.	Starting	<i>Can be started by D.O.L., star-delta, auto transformer starters</i>	<i>Rotor resistance starter is required.</i>
3.	Starting torque	<i>Low</i>	<i>Very high</i>
4.	Starting Current	<i>High</i>	<i>Low</i>
5.	Speed variation	<i>Not easy, but could be varied in large steps by pole changing or through smaller incremental steps through thyristors or by frequency variation.</i>	<i>Easy to vary speed.</i> <i>Speed change is possible by inserting rotor resistance using thyristors or by using frequency variation injecting emf in the rotor circuit cascading.</i>
6.	Maintenance	<i>Almost ZERO maintenance</i>	<i>Requires frequent maintenance</i>
7.	Cost	<i>Low</i>	<i>High</i>

3.3 Principle of Operation

The operation of a 3-phase induction motor is based upon the application of Faraday Law and the Lorentz force on a conductor. The behaviour can readily be understood by means of the following example.

Consider a series of conductors of length l , whose extremities are short-circuited by two bars A and B (Fig.3.3 a). A permanent magnet placed above this conducting ladder, moves rapidly to the right at a speed v , so that its magnetic field B sweeps across the conductors. The following sequence of events then takes place:

1. A voltage $E = Blv$ is induced in each conductor while it is being cut by the flux (Faraday law).
2. The induced voltage immediately produces a current I , which flows down the conductor underneath the pole face, through the end-bars, and back through the other conductors.
3. Because the current carrying conductor lies in the magnetic field of the permanent magnet, it experiences a mechanical force (Lorentz force).
4. The force always acts in a direction to drag the conductor along with the magnetic field. If the conducting ladder is free to move, it will accelerate toward the right. However, as it picks up speed, the conductors will be cut less rapidly by the moving magnet, with the result that the induced voltage E and the current I will diminish. Consequently, the force acting on the conductors will also decrease. If the ladder were to move at the same speed as the magnetic field, the induced voltage E , the current I , and the force dragging the ladder along would all become zero.

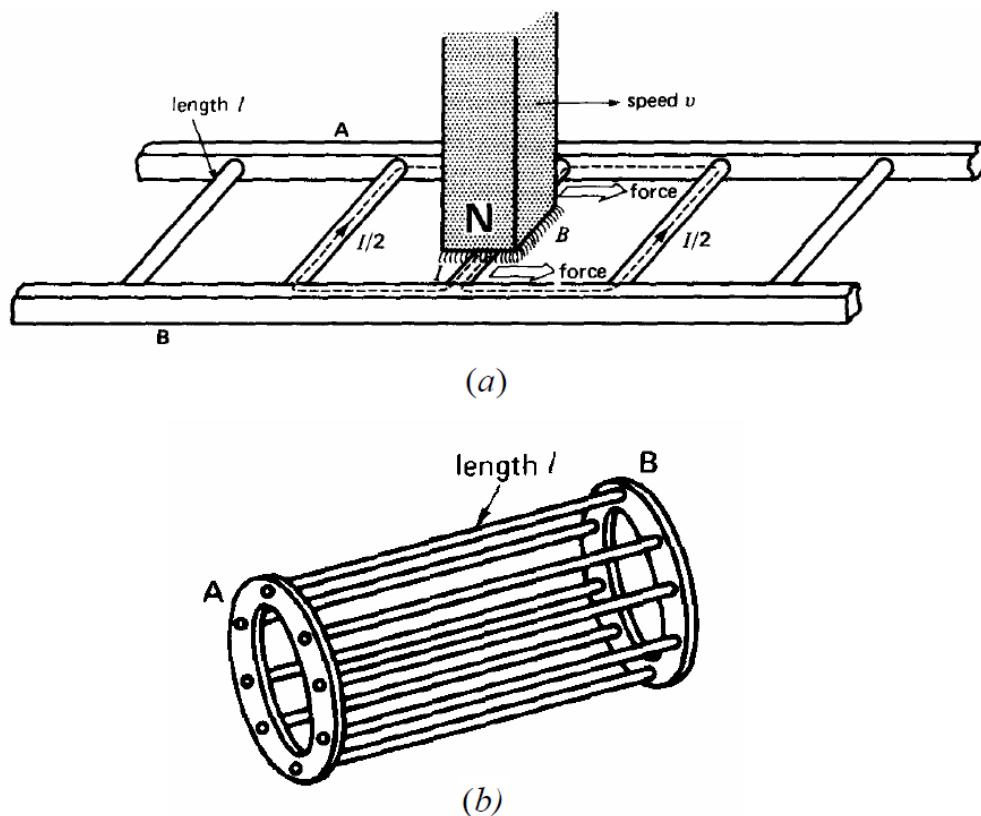


Fig: 3.3

In an induction motor the ladder is closed upon itself to form a squirrel-cage (Fig.3.3b) and the moving magnet is replaced by a rotating field. The field is produced by the 3-phase currents that flow in the stator windings.

3.4 Rotating Magnetic Field and Induced Voltages

Consider a simple stator having 6 salient poles, each of which carries a coil having 5 turns (Fig.3.4). Coils that are diametrically opposite are connected in series by means of three jumpers

that respectively connect terminals a-a, b-b, and c-c. This creates three identical sets of windings AN, BN, CN, which are mechanically spaced at 120 degrees to each other. The two coils in each winding produce magneto motive forces that act in the same direction.

The three sets of windings are connected in wye, thus forming a common neutral N. Owing to the perfectly symmetrical arrangement, the line to neutral impedances are identical. In other words, as regards terminals A, B, C, the windings constitute a balanced 3-phase system.

For a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the centre of the magnetic poles. The result is illustrated in Fig.3.5. The rotating field will induce voltages in the phase coils aa', bb', and cc'. Expressions for the induced voltages can be obtained by using Faraday laws of induction.

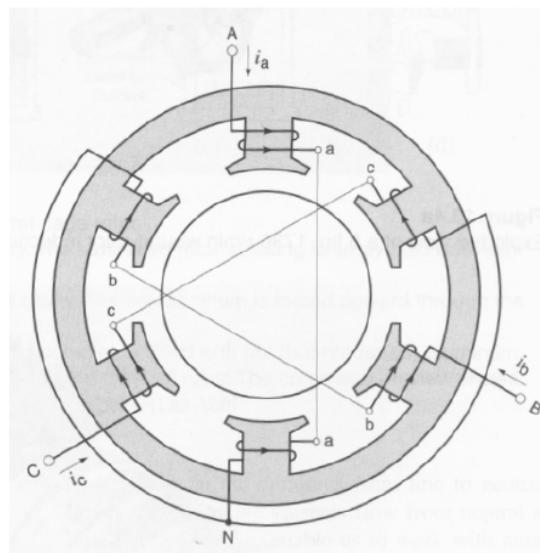


Fig: 3.4 Elementary stator having terminals A, B, C connected to a 3-phase source (not shown). Currents flowing from line to neutral are considered to be positive.

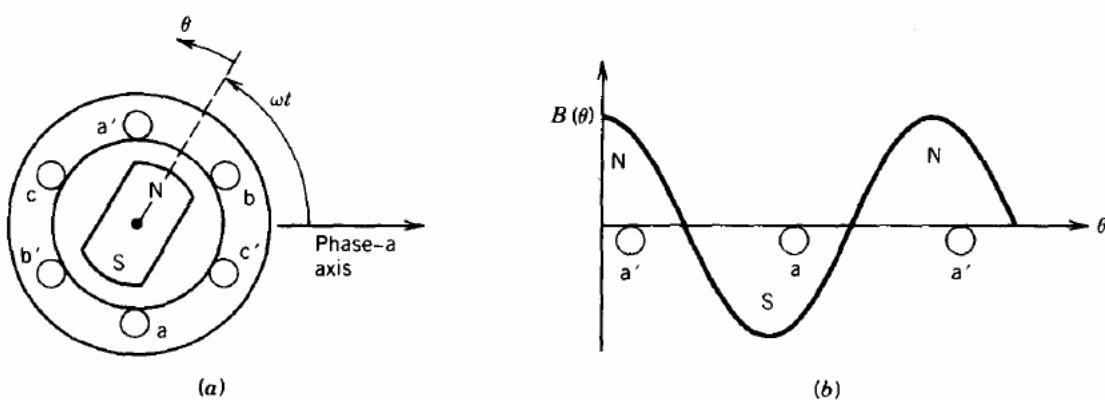


Fig: 3.5 Air gap flux density distribution.

The flux density distribution in the air gap can be expressed as:

$$B(\theta) = B_{\max} \cos \theta$$

The air gap flux per pole, ϕ_p , is:

$$\phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) l r d\theta = 2B_{\max} l r$$

Where,

l is the axial length of the stator.

r is the radius of the stator at the air gap.

Let us consider that the phase coils are full-pitch coils of N turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig.3.5). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil aa' will be maximum.

(= $N\phi_p$ at $\omega t = 0^\circ$) (Fig.3.5a) and zero at $\omega t = 90^\circ$. The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle ωt .

Hence,

$$\lambda_a(\omega t) = N\phi_p \cos \omega t$$

Therefore, the voltage induced in phase coil aa' is obtained from Faraday law as:

$$e_a = -\frac{d\lambda_a(\omega t)}{dt} = \omega N\phi_p \sin \omega t = E_{\max} \sin \omega t$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{\max} \sin(\omega t - 120)$$

$$e_c = E_{\max} \sin(\omega t + 120).$$

the rms value of the induced voltage is:

$$E_{rms} = \frac{\omega N\phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\phi_p = 4.44 f N\phi_p$$

Where f is the frequency in hertz. Above equation has the same form as that for the induced voltage in transformers. However, ϕ_p represents the flux per pole of the machine.

The above equation also shows the rms voltage per phase. The N is the total number of series turns per phase with the turns forming a concentrated full-pitch winding. In an actual AC machine each phase winding is distributed in a number of slots for better use of the iron and copper and to improve the waveform. For such a distributed winding, the EMF induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the EMF is less than their numerical sum when they are connected in series for the phase winding. A reduction factor K_w , called the winding factor, must therefore be applied. For most three-phase machine windings K_w is about 0.85 to 0.95.

Therefore, for a distributed phase winding, the rms voltage per phase is

$$E_{rms} = 4.44fN_{ph}\phi_p K_w$$

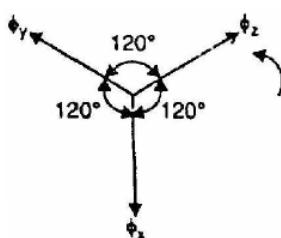
Where N_{ph} is the number of turns in series per phase.

3.5 Alternate Analysis for Rotating Magnetic Field

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. It can be shown that magnitude of this rotating field is constant and is equal to $1.5 \phi_m$ where ϕ_m is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3-phase winding as shown in Fig. 3.6 (i). The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as I_x , I_y and I_z [See Fig. 3.6 (ii)]. Referring to Fig. 3.6 (ii), the fluxes produced by these currents are given by:

$$\begin{aligned}\phi_x &= \phi_m \sin \omega t \\ \phi_y &= \phi_m \sin (\omega t - 120^\circ) \\ \phi_z &= \phi_m \sin (\omega t - 240^\circ)\end{aligned}$$



Here ϕ_m is the maximum flux due to any phase. Above figure shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to $1.5 \phi_m$.

At instant 1 [See Fig. 3.6 (ii) and Fig. 3.6 (iii)], the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward

in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to $1.5 \phi_m$ as proved under:

At instant 1, $\omega t = 0^\circ$. Therefore, the three fluxes are given by;

$$\begin{aligned}\phi_x &= 0; & \phi_y &= \phi_m \sin(-120^\circ) = -\frac{\sqrt{3}}{2} \phi_m; \\ \phi_z &= \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2} \phi_m\end{aligned}$$

The phasor sum of $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

So,

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2} = 2 \times \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2} = 1.5 \phi_m$$

At instant 2 [Fig: 3.7 (ii)], the current is maximum (negative) in ϕ_y phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is $1.5 \phi_m$ as proved under:

At instant 2, $\omega t = 30^\circ$. Therefore, the three fluxes are given by;

$$\begin{aligned}\phi_x &= \phi_m \sin 30^\circ = \frac{\phi_m}{2} \\ \phi_y &= \phi_m \sin (-90^\circ) = -\phi_m \\ \phi_z &= \phi_m \sin (-210^\circ) = \frac{\phi_m}{2}\end{aligned}$$

The phasor sum of ϕ_x , $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

$$\text{Phasor sum of } \phi_x \text{ and } \phi_z, \phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$$

$$\text{Phasor sum of } \phi'_r \text{ and } -\phi_y, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that resultant flux is displaced 30° clockwise from position 1.

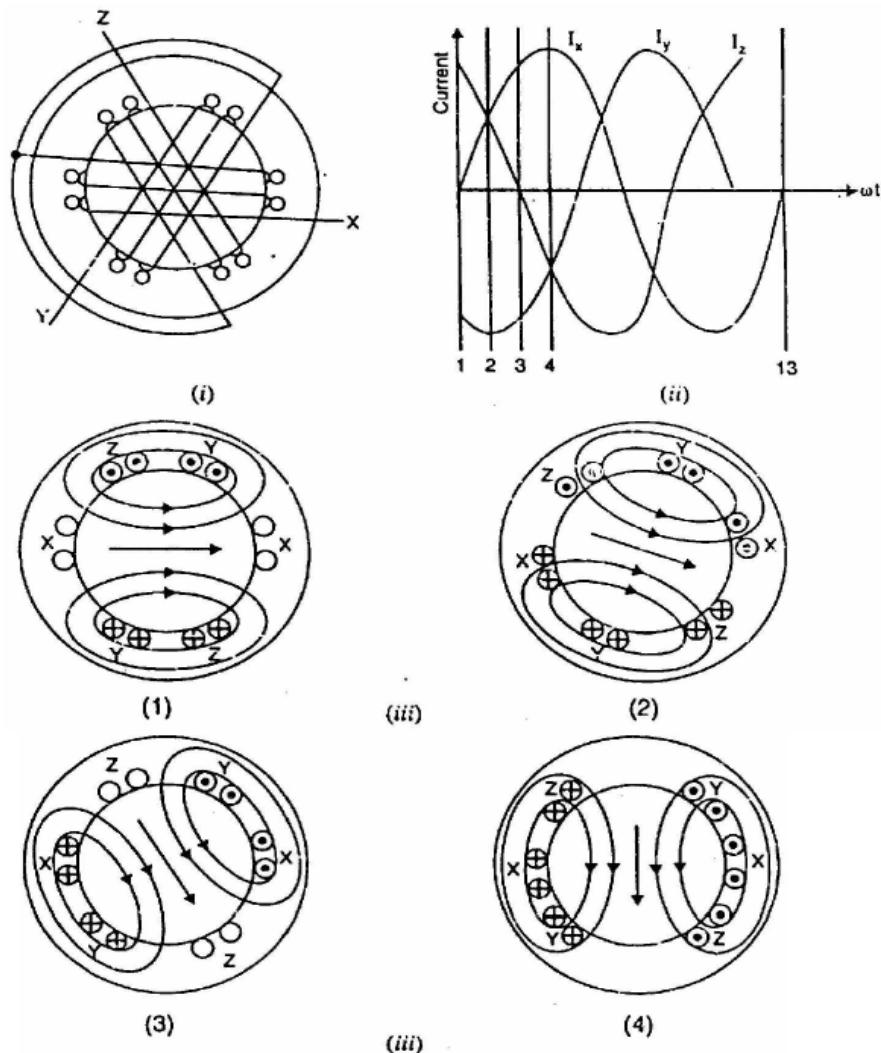


Fig: 3.6

At instant 3[Fig: 3.7 (iii)], current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y are $0.866 \times$ max. value). The magnitude of resultant flux is $1.5 \phi_m$ as proved under:

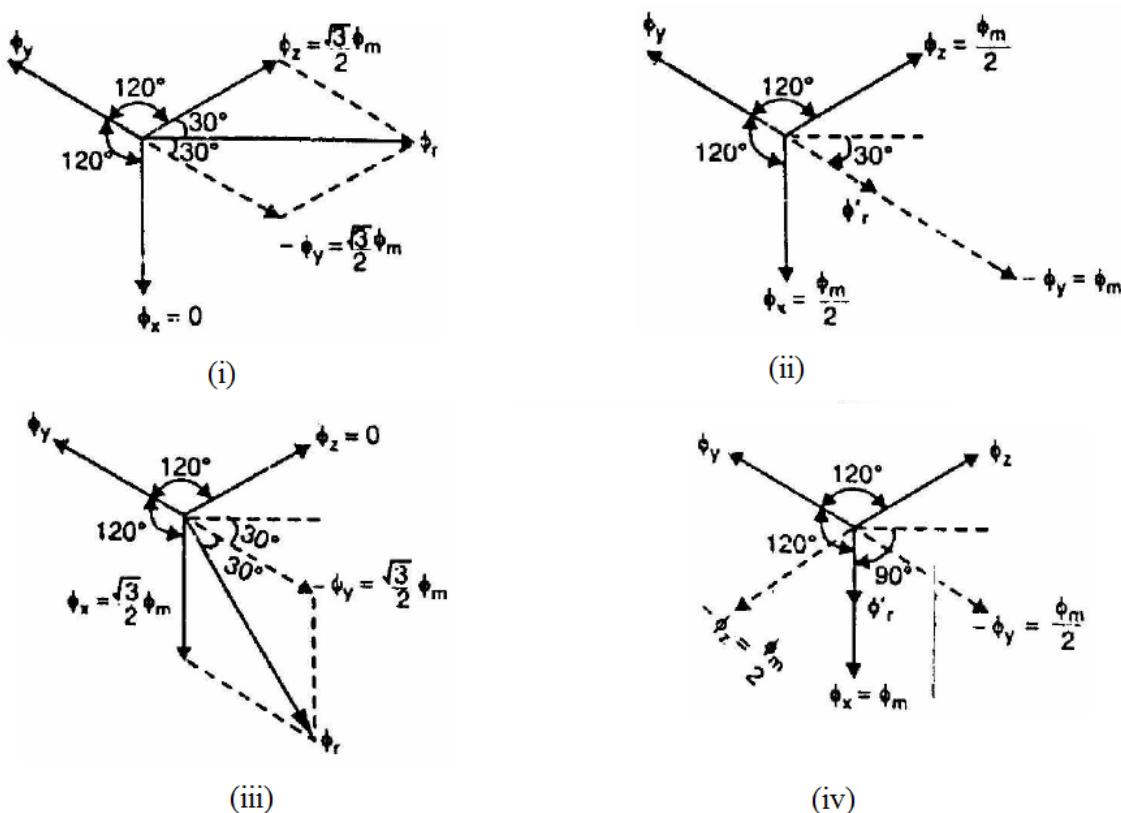


Fig: 3.7

At instant 3, $\omega t = 60^\circ$. Therefore, the three fluxes are given by;

$$\begin{aligned}\phi_x &= \phi_m \sin 60^\circ = \frac{\sqrt{3}}{2} \phi_m; \\ \phi_y &= \phi_m \sin(-60^\circ) = -\frac{\sqrt{3}}{2} \phi_m; \\ \phi_z &= \phi_m \sin(-180^\circ) = 0\end{aligned}$$

The resultant flux ϕ_r is the phasor sum of ϕ_x and $-\phi_y$ ($\because \phi_z = 0$).

$$\phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2} = 1.5 \phi_m$$

Note that resultant flux is displaced 60° clockwise from position 1.

At instant 4 [Fig: 3.7 (iv)], the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are $0.5 \times$ max. value). This establishes a resultant flux downward as shown under:

At instant 4, $\omega t = 90^\circ$. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90^\circ = \phi_m$$

$$\phi_y = \phi_m \sin (-30^\circ) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin (-150^\circ) = -\frac{\phi_m}{2}$$

The phasor sum of ϕ_x , $-\phi_y$ and $-\phi_z$ is the resultant flux ϕ_r

$$\text{Phasor sum of } -\phi_z \text{ and } -\phi_y, \phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$$

$$\text{Phasor sum of } \phi'_r \text{ and } \phi_x, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value ($= 1.5 \phi_m$, where ϕ_m is the maximum flux due to any phase).

3.5.1 Speed of rotating magnetic field

The speed at which the rotating magnetic field revolves is called the synchronous speed (N_s). Referring to Fig. 3.6 (ii), the time instant 4 represents the completion of one-quarter cycle of alternating current I_x from the time instant 1. During this one quarter cycle, the field has rotated through 90° . At a time instant represented by 13 [Fig. 3.6 (ii)] or one complete cycle of current I_x from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, for P poles, the rotating field makes one revolution in $P/2$ cycles of current.

$$\therefore \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

$$\text{or } \text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}$$

Since revolutions per second is equal to the revolutions per minute (N_s) divided by 60 and the number of cycles per second is the frequency f ,

$$\therefore f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

$$\text{or } N_s = \frac{120 f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

3.5.2 Direction of rotating magnetic field

The phase sequence of the three-phase voltage applied to the stator winding in Fig. 3.6 (ii) is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counter clockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of

the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

3.5.3 Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the stator field speed (N_s). This difference in speed depends upon load on the motor. The difference between the synchronous speed N_s of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.

$$\% \text{ age slip}, s = \frac{N_s - N}{N_s} \times 100$$

- (i) The quantity $N_s - N$ is sometimes called slip speed.
- (ii) When the rotor is stationary (i.e., $N = 0$), slip, $s = 1$ or 100 %.
- (iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

3.5.4 Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a rotating field and a magnetic field is given by the general formula;

$$\text{Frequency} = \frac{NP}{120}$$

where N = Relative speed between magnetic field and the winding
 P = Number of poles

For a rotor speed N , the relative speed between the rotating flux and the rotor is $N_s - N$. Consequently, the rotor current frequency f' is given by;

$$\begin{aligned} f' &= \frac{(N_s - N)P}{120} \\ &= \frac{s N_s P}{120} \quad \left(\because s = \frac{N_s - N}{N_s} \right) \\ &= sf \quad \left(\because f = \frac{N_s P}{120} \right) \end{aligned}$$

i.e., Rotor current frequency = Fractional slip x Supply frequency

- (i) When the rotor is at standstill or stationary (i.e., $s = 1$), the frequency of rotor current is the same as that of supply frequency ($f' = sf = 1 \times f = f$).
- (ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

3.6 Phasor Diagram of Three Phase Induction Motor

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor winding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. 3.8 shows the equivalent circuit per phase for an induction motor. Let discuss the stator and rotor circuits separately.

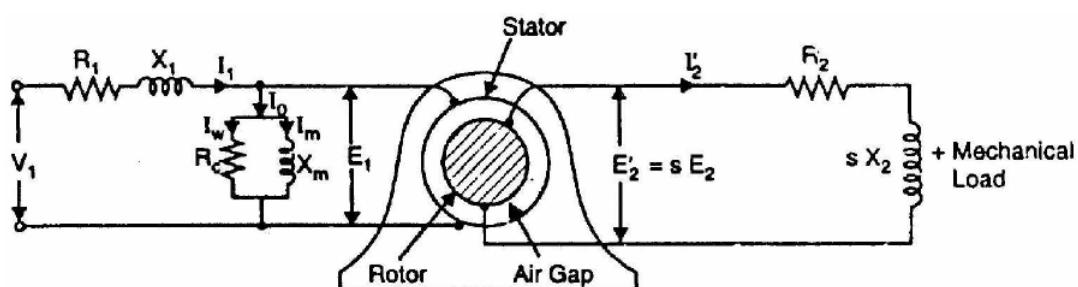


Fig: 3.8

Stator circuit. In the stator, the events are very similar to those in the transformer primary. The applied voltage per phase to the stator is V_1 and R_1 and X_1 are the stator resistance and leakage reactance per phase respectively. The applied voltage V_1 produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-induced e.m.f. E_1 is induced in the stator winding and mutually induced e.m.f.

$E'2 (= s E_2 = s K E_1$ where K is transformation ratio) is induced in the rotor winding. The flow of stator current I_1 causes voltage drops in R_1 and X_1 .

$$\therefore V_1 = -E_1 + I_1 (R_1 + j X_1) \dots \text{phasor sum}$$

When the motor is at no-load, the stator winding draws a current I_0 . It has two components viz., (i) which supplies the no-load motor losses and (ii) magnetizing component I_m which sets up magnetic flux in the core and the air gap. The parallel combination of R_c and X_m , therefore, represents the no-load motor losses and the production of magnetic flux respectively.

$$\therefore I_0 = I_w + I_m$$

Rotor circuit. Here R_2 and X_2 represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip s , the rotor reactance will be X_2 . The induced voltage/phase in the rotor is $E'_2 = s E_2 = s K E_1$. Since the rotor winding is short-circuited, the whole of e.m.f. E'_2 is used up in circulating the rotor current I'_2 .

$$\therefore E'_2 = I'_2 (R_2 + j s X_2)$$

The rotor current I'_2 is reflected as $I''_2 (= K I'_2)$ in the stator. The phasor sum of I''_2 and I_0 gives the stator current I_1 .

It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.

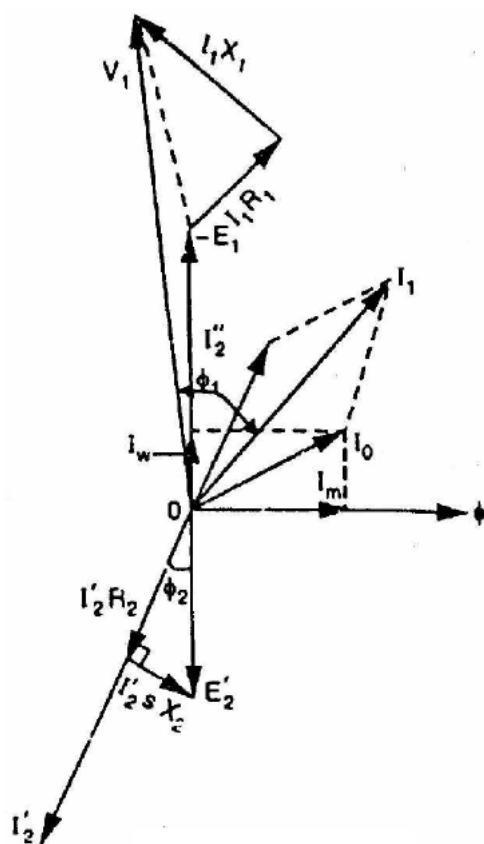


Fig: 3.9

It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed N_s . The stator currents produce a magnetic flux which rotates at a speed N_s . At slip s , the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$= \frac{120 f'}{P} = \frac{120 s f}{P} = s N_s$$

But the rotor is revolving at a speed of N relative to the stator core. Therefore, the speed of rotor field relative to stator core

$$= s N_s + N = (N_s - N) + N = N_s$$

Thus no matter what the value of slip s , the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings. Fig. 3.9 shows the phasor diagram of induction motor.

3.7 Equivalent Circuit of Three Phase Induction Motor

Fig. 3.10 (i) shows the equivalent circuit per phase of the rotor at slip s . The rotor phase current is given by;

$$I'_2 = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}}$$

Mathematically, this value is unaltered by writing it as:

$$I'_2 = \frac{E_2}{\sqrt{(R_2/s)^2 + (X_2)^2}}$$

As shown in Fig. 3.10 (ii), we now have a rotor circuit that has a fixed reactance X_2 connected in series with a variable resistance R_2/s and supplied with constant voltage E_2 . Note that Fig. 3.10 (ii) transfers the variable to the resistance without altering power or power factor conditions.

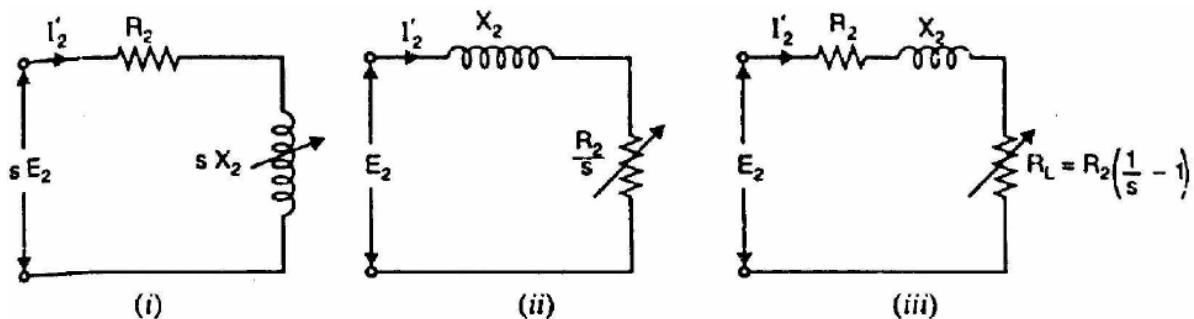


Fig: 3.10

The quantity R_2/s is greater than R_2 since s is a fraction. Therefore, R_2/s can be divided into a fixed part R_2 and a variable part $(R_2/s - R_2)$ i.e.,

$$\frac{R_2}{s} = R_2 + R_2 \left(\frac{1}{s} - 1 \right)$$

- (i) The first part R_2 is the rotor resistance/phase, and represents the rotor Cu loss.
- (ii) The second part $R_2\left(\frac{1}{s}-1\right)$ is a variable-resistance load. The power delivered to this load represents the total mechanical power developed in the rotor. Thus mechanical load on the induction motor can be replaced by a variable-resistance load of value $R_2\left(\frac{1}{s}-1\right)$. This is

$$\therefore R_L = R_2\left(\frac{1}{s}-1\right)$$

Fig. 3.10 (iii) shows the equivalent rotor circuit along with load resistance R_L .

Now Fig: 3.11 shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance R_L given by;

$$R_L = R_2\left(\frac{1}{s}-1\right) \quad \text{----- (i)}$$

The circuit shown in Fig. 3.11 is similar to the equivalent circuit of a transformer with secondary load equal to R_2 given by eq. (i). The rotor e.m.f. in the equivalent circuit now depends only on the transformation ratio K ($= E_2/E_1$).

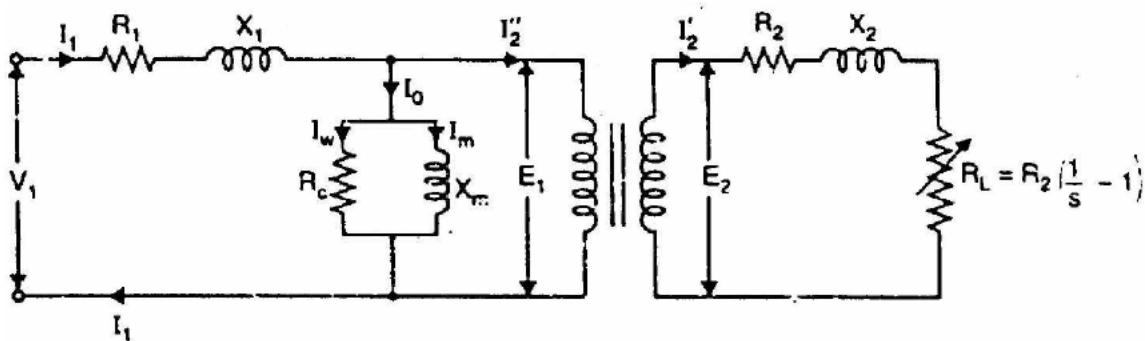


Fig: 3.11

Therefore; induction motor can be represented as an equivalent transformer connected to a variable-resistance load R_L given by eq. (i). The power delivered to R_L represents the total mechanical power developed in the rotor. Since the equivalent circuit of Fig. 3.11 is that of a transformer, the secondary (i.e., rotor) values can be transferred to primary (i.e., stator) through the appropriate use of transformation ratio K. Recall that when shifting resistance/reactance from secondary to primary, it should be divided by K^2 whereas current should be multiplied by K. The equivalent circuit of an induction motor referred to primary is shown in Fig. 3.12.

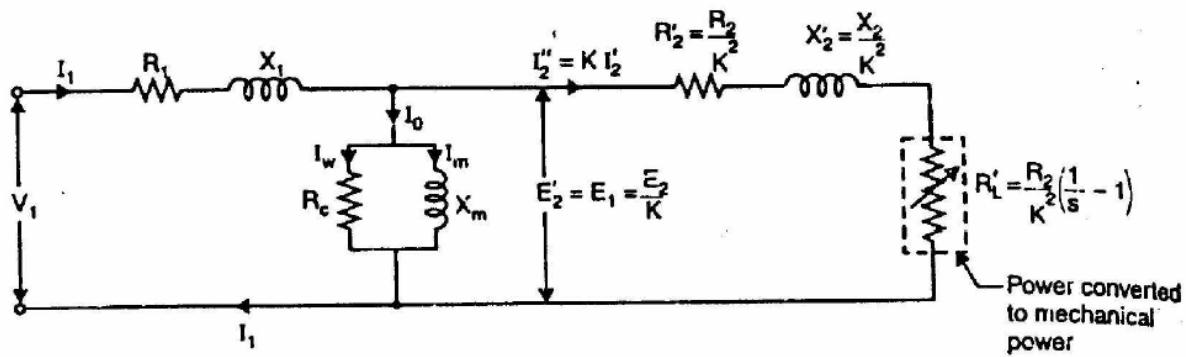


Fig: 3.12

Note that the element (i.e., R'_L) enclosed in the dotted box is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

- (i) At no-load, the slip is practically zero and the load R'_L is infinite. This condition resembles that in a transformer whose secondary winding is open-circuited.
- (ii) At standstill, the slip is unity and the load R'_L is zero. This condition resembles that in a transformer whose secondary winding is short-circuited.
- (iii) When the motor is running under load, the value of R'_L will depend upon the value of the slip s . This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.
- (iv) The equivalent electrical resistance R'_L related to mechanical load is slip or speed dependent. If the slip s increases, the load R'_L decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load on the motor shaft.

3.8 Power and Torque Relations of Three Phase Induction Motor

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. 3.13 shows the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).

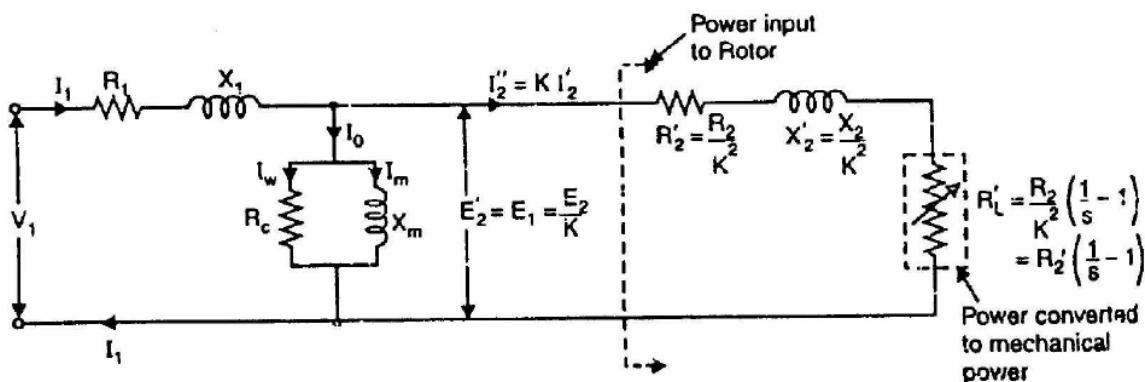


Fig: 3.13

(i) Total electrical load $= R'_2 \left(\frac{1}{s} - 1 \right) + R'_2 = \frac{R'_2}{s}$

Power input to stator $= 3V_1 I_1 \cos\phi_1$

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

(ii) Rotor input $= \frac{3(I''_2)^2 R'_2}{s}$

Rotor Cu loss $= 3(I''_2)^2 R'_2$

Total mechanical power developed by the rotor is

$$P_m = \text{Rotor input} - \text{Rotor Cu loss}$$

$$= \frac{3(I''_2)^2 R'_2}{s} - 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2 \left(\frac{1}{s} - 1 \right)$$

This is quite apparent from the equivalent circuit shown in Fig: 3.13.

(iii) If T_g is the gross torque developed by the rotor, then,

$$P_m = \frac{2\pi N T_g}{60}$$

$$\text{or } 3(I''_2)^2 R'_2 \left(\frac{1}{s} - 1 \right) = \frac{2\pi N T_g}{60}$$

$$\text{or } 3(I''_2)^2 R'_2 \left(\frac{1-s}{s} \right) = \frac{2\pi N T_g}{60}$$

$$\text{or } 3(I''_2)^2 R'_2 \left(\frac{1-s}{s} \right) = \frac{2\pi N_s (1-s) T_g}{60} \quad [\because N = N_s (1-s)]$$

$$\therefore T_g = \frac{3(I''_2)^2 R'_2 / s}{2\pi N_s / 60} \text{ N-m}$$

$$\text{or } T_g = 9.55 \frac{3(I''_2)^2 R'_2 / s}{N_s} \text{ N-m}$$

Note that shaft torque T_{sh} will be less than T_g by the torque required to meet windage and frictional losses.

3.9 Induction Motor Torque

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi N T}{60} \text{ watts}$$

where N = speed of the motor in r.p.m.
 T = torque developed in N-m

$$\therefore T = \frac{60}{2\pi} \frac{P}{N} = 9.55 \frac{P}{N} \text{ N-m}$$

If the gross output of the rotor of an induction motor is P_m and its speed is N r.p.m., then gross torque T developed is given by:

$$T_g = 9.55 \frac{P_m}{N} \text{ N-m}$$

$$\text{Similarly, } T_{sh} = 9.55 \frac{P_{out}}{N} \text{ N-m}$$

Note. Since windage and friction loss is small, $T_g = T_{sh}$. This assumption hardly leads to any significant error.

3.10 Rotor Output

If T_g newton-metre is the gross torque developed and N r.p.m. is the speed of the rotor, then,

$$\text{Gross rotor output} = \frac{2\pi N T_g}{60} \text{ watts}$$

If there were no copper losses in the rotor, the output would equal rotor input and the rotor would run at synchronous speed N_s .

$$\therefore \text{Rotor input} = \frac{2\pi N_s T_g}{60} \text{ watts}$$

$$\begin{aligned} \therefore \text{Rotor Cu loss} &= \text{Rotor input} - \text{Rotor output} \\ &= \frac{2\pi T_g}{60} (N_s - N) \end{aligned}$$

$$(i) \quad \frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s - N}{N_s} = s$$

$$\therefore \text{Rotor Cu loss} = s \times \text{Rotor input}$$

$$\begin{aligned} (ii) \quad \text{Gross rotor output, } P_m &= \text{Rotor input} - \text{Rotor Cu loss} \\ &= \text{Rotor input} - s \times \text{Rotor input} \\ \therefore \quad P_m &= \text{Rotor input} (1 - s) \end{aligned}$$

$$(iii) \quad \frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s}$$

$$(iv) \quad \frac{\text{Rotor Cu loss}}{\text{Gross rotor output}} = \frac{s}{1 - s}$$

It is clear that if the input power to rotor is “Pr” then “s.Pr” is lost as rotor Cu loss and the remaining $(1 - s)$ Pr is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

Note.

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses are neglected, then,

$$\text{Gross rotor output} = \text{Useful output}$$

$$\text{Rotor input} = \text{Stator input}$$

$$\therefore \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is $1 - s$. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is $= 1 - 0.125 = 0.875$ or 87.5%.

3.11.1 Torque Equations

The gross torque T_g developed by an induction motor is given by;

$$T_g = \frac{\text{Rotor input}}{2\pi N_s} \quad \dots N_s \text{ is r.p.s.}$$

$$= \frac{60 \times \text{Rotor input}}{2\pi N_s} \quad \dots N_s \text{ is r.p.s.}$$

Now Rotor input = $\frac{\text{Rotor Cu loss}}{s} = \frac{3(I'_2)^2 R_2}{s}$ (i)

As shown in Sec. 8.16, under running conditions,

$$I'_2 = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} = \frac{s K E_1}{\sqrt{R_2^2 + (s X_2)^2}}$$

where $K = \frac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \text{Rotor input} = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of I'_2 in eq.(i))

$$\text{Also Rotor input} = 3 \times \frac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of I'_2 in eq.(i))

$$\therefore T_g = \frac{\text{Rotor input}}{2\pi N_s} = \frac{3}{2\pi N_s} \times \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \quad \dots \text{in terms of } E_2$$

$$= \frac{3}{2\pi N_s} \times \frac{s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \quad \dots \text{in terms of } E_1$$

Note that in the above expressions of T_g , the values E_1 , E_2 , R_2 and X_2 represent the phase values.

3.11.2 Rotor Torque

The torque T developed by the rotor is directly proportional to:

- (i) rotor current
- (ii) rotor e.m.f.
- (iii) power factor of the rotor circuit

$$\therefore T \propto E_2 I_2 \cos \phi_2$$

$$\text{or } T = K E_2 I_2 \cos \phi_2$$

where I_2 = rotor current at standstill
 E_2 = rotor e.m.f. at standstill
 $\cos \phi_2$ = rotor p.f. at standstill

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

3.11.3 Starting Torque (T_s)

Let,

$$E_2 = \text{rotor e.m.f. per phase at standstill}$$

$$X_2 = \text{rotor reactance per phase at standstill}$$

$$R_2 = \text{rotor resistance per phase}$$

$$\text{Rotor impedance/phase, } Z_2 = \sqrt{R_2^2 + X_2^2} \quad \dots \text{at standstill}$$

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \quad \dots \text{at standstill}$$

$$\text{Rotor p.f., } \cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \quad \dots \text{at standstill}$$

$$\therefore \text{Starting torque, } T_s = K E_2 I_2 \cos \phi_2$$

$$\begin{aligned} &= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \\ &= \frac{K E_2^2 R_2}{R_2^2 + X_2^2} \end{aligned}$$

Generally, the stator supply voltage V is constant so that flux per pole ϕ set up by the stator is also fixed. This in turn means that e.m.f. E_2 induced in the rotor will be constant.

$$\therefore T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of R_2 and X_2 i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that $K = 3/2 \pi N_s$.

$$\therefore T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here N_s is in r.p.s.

3.11.4 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

$$\text{Now } T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \quad (\text{i})$$

Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$$\text{or } R_2^2 + X_2^2 = 2R_2^2$$

$$\text{or } R_2 = X_2$$

Hence starting torque will be maximum when:

$$\text{Rotor resistance/phase} = \text{Standstill rotor reactance/phase}$$

Under the condition of maximum starting torque, $\phi_2 = 45^\circ$ and rotor power factor is 0.707 lagging.

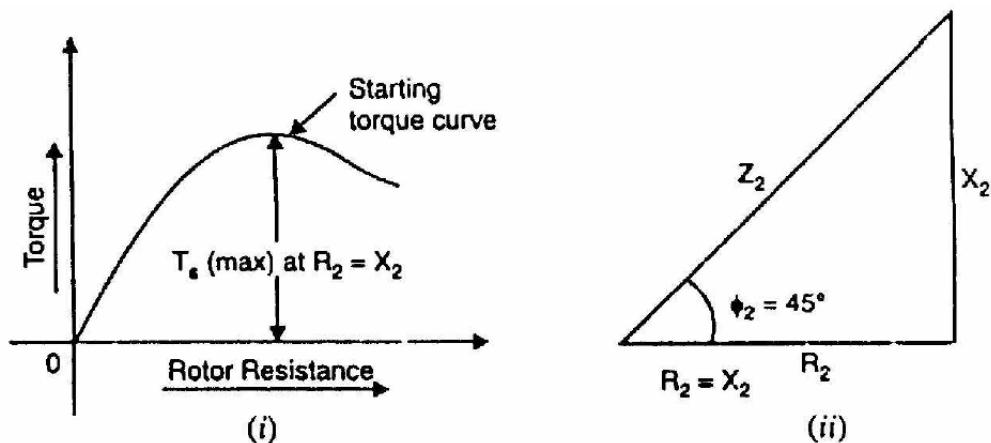


Fig: 3.14

Fig. 3.14 shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when $R_2 = X_2$. If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

3.11.5 Effect of Change of Supply Voltage

$$T_s = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Since $E_2 \propto$ Supply voltage V

$$\therefore T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

where K_2 is another constant.

$$\therefore T_s \propto V^2$$

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20%. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plus friction torque.

3.12 Circle Diagram

To analyse the three phase induction motor performance using circle diagram we need to determine the equivalent circuit parameters of the machine.

3.12.1 Approximate Equivalent Circuit of Induction Motor

As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch ($R_c - X_m$) to the input terminals as shown in Fig. 3.15. This step has been taken on the assumption that voltage drop in R_1 and X_1 is small and the terminal voltage V_1 does not appreciably differ from the induced voltage E_1 . Fig. 3.15 shows the approximate equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).

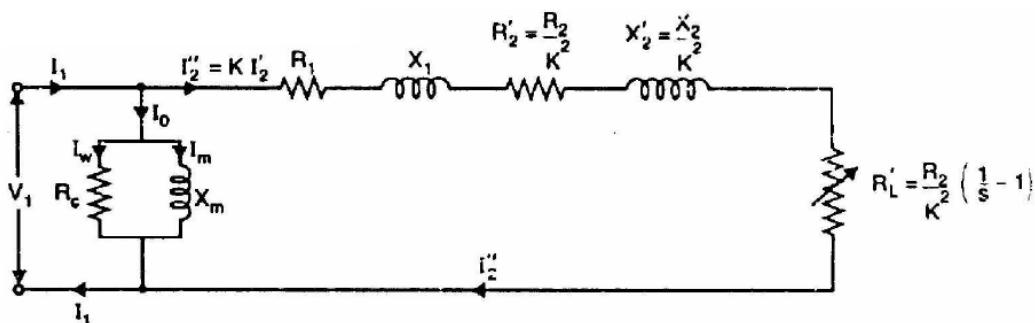


Fig: 3.15

The above approximate circuit of induction motor is not so readily justified as with the transformer. This is due to the following reasons:

- (i) Unlike that of a power transformer, the magnetic circuit of the induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the power transformer. Consequently, the exact equivalent circuit must be used for accurate results.
- (ii) The relative values of X_1 and X_2 in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit.
- (iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.

In spite of the above drawbacks of approximate equivalent circuit, it yields results that are satisfactory for large motors. However, approximate equivalent circuit is not justified for small motors.

3.12.2 Tests to Determine the Equivalent Circuit Parameters

In order to find values for the various elements of the equivalent circuit, tests must be conducted on a particular machine, which is to be represented by the equivalent circuit. In order to do this, we note the following.

1. When the machine is run on no-load, there is very little torque developed by it. In an ideal case where there is no mechanical losses, there is no mechanical power developed at no-load. Recalling the explanations in the section on torque production, the flow of current in the rotor is indicative of the torque that is produced. If no torque is produced, one may conclude that no current would be flowing in the rotor either. The rotor branch acts like an open circuit. This conclusion may also be reached by reasoning that when there is no load, an ideal machine will run up to its synchronous speed where the slip is zero resulting in an infinite impedance in the rotor branch.
2. When the machine is prevented from rotation, and supply is given, the slip remains at unity. The elements representing the magnetizing branch R_m & X_m are high impedances much larger than R'_r & X'_{lr} in series. Thus, in the exact equivalent circuit of the induction machine, the magnetizing branch may be neglected.

From these considerations, we may reduce the induction machine equivalent circuit of Fig.3.13 & Fig: 3.15 to those shown in Fig: 3.16.

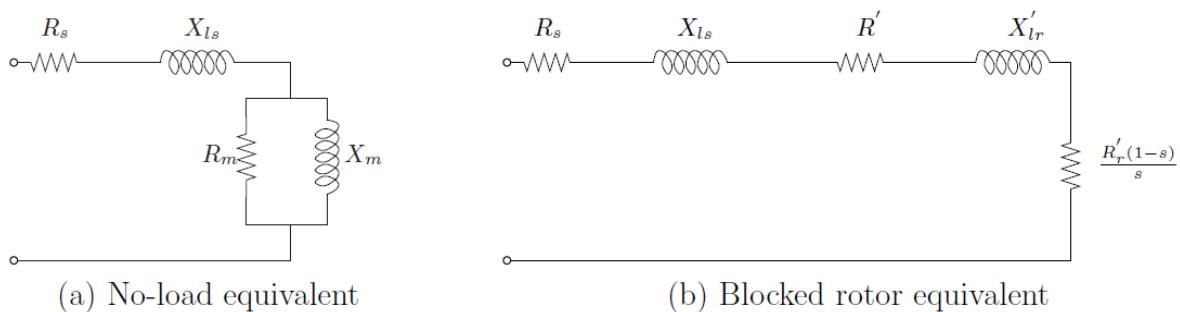


Fig: 3.16

These two observations and the reduced equivalent circuits are used as the basis for the two most commonly used tests to find out the equivalent circuit parameters — the blocked rotor test and no load test. They are also referred to as the short circuit test and open circuit test respectively in conceptual analogy to the transformer.

1. No-load test

The behaviour of the machine may be judged from the equivalent circuit of Fig: 3.16 (a). The current drawn by the machine causes a stator-impedance drop and the balance voltage is applied across the magnetizing branch. However, since the magnetizing branch impedance is large, the current drawn is small and hence the stator impedance drop is small compared to the applied voltage (rated value). This drop and the power dissipated in the stator resistance are therefore neglected and the total power drawn is assumed to be consumed entirely as core loss. This can also be seen from the approximate equivalent circuit, the use of which is justified by the foregoing arguments. This test therefore enables us to compute the resistance and inductance of the magnetizing branch in the following manner.

Let applied voltage = V_s . Then current drawn is given by

$$I_s = \frac{V_s}{R_m} + \frac{V_s}{jX_m}$$

The power drawn is given by

$$P_s = \frac{V_s^2}{R_m} \Rightarrow R_m = \frac{V_s^2}{P_s}$$

V_s , I_s and P_s are measured with appropriate meters. With R_m known by above equation, X_m also can be found. The current drawn is at low power factor and hence a suitable wattmeter should be used.

2. Blocked-rotor Test

In this test the rotor is prevented from rotation by mechanical means and hence the name. Since there is no rotation, slip of operation is unity, $s = 1$. The equivalent circuit valid under these conditions is shown in Fig: 3.16 (b). Since the current drawn is decided by the resistance and leakage impedances alone, the magnitude can be very high when rated voltage is applied. Therefore in this test, only small voltages are applied — just enough to cause rated current to flow. While the current magnitude depends on the resistance and the reactance, the power drawn depends on the resistances.

The parameters may then be determined as follows. The source current and power drawn may be written as -

$$\begin{aligned} I_s &= \frac{V_s}{(R_s + R'_r) + j(X_s + X'_r)} \\ P_s &= |I_s|^2 (R_s + R'_r) \end{aligned}$$

In the test V_s , I_s and P_s are measured with appropriate meters. Above equation enables us to compute $(R_s + R'r)$. Once this is known, $(X_s + X'r)$ may be computed from the above equation.

Note that this test only enables us to determine the series combination of the resistance and the reactance only and not the individual values. Generally, the individual values are assumed to be equal; the assumption $R_s = R'r$, and $X_s = X'r$ suffices for most purposes.

In practice, there are differences. If more accurate estimates are required IEEE guidelines may be followed which depend on the size of the machine.

These two tests determine the equivalent circuit parameters in a ‘Stator-referred’ sense, i.e., the rotor resistance and leakage inductance are not the actual values but what they ‘appear to be’ when looked at from the stator. This is sufficient for most purposes as interconnections to the external world are generally done at the stator terminals.

3.12.3 Construction of Circle Diagram

Conduct No load test and blocked rotor test on the induction motor and find out the per phase values of no load current I_0 , short circuit current I_{SC} and the corresponding phase angles Φ_0 and Φ_{SC} . Also find short circuit current I_{SN} corresponding to normal supply voltage. With this data, the circle diagram can be drawn as follows see Fig: 3.17.

1. With suitable scale, draw vector OA with length corresponding to I_0 at an angle Φ_0 from the vertical axis. Draw a horizontal line AB.
2. Draw OS equal to I_{SN} at an angle Φ_{SC} and join AS.
3. Draw the perpendicular bisector to AS to meet the horizontal line AB at C.
4. With C as centre, draw a portion of circle passing through A and S. This forms the circle diagram which is the locus of the input current.
5. From point S, draw a vertical line SL to meet the line AB.
6. Divide SL at point K so that $SK : KL = \text{rotor resistance} : \text{stator resistance}$.
7. For a given operating point P, draw a vertical line PEFGD as shown. then PE = output power, EF = rotor copper loss, FG = stator copper loss, GD = constant loss (iron loss + mechanical loss)
8. To find the operating points corresponding to maximum power and maximum torque, draw tangents to the circle diagram parallel to the output line and torque line respectively. The points at which these tangents touch the circle are respectively the maximum power point and maximum torque point.

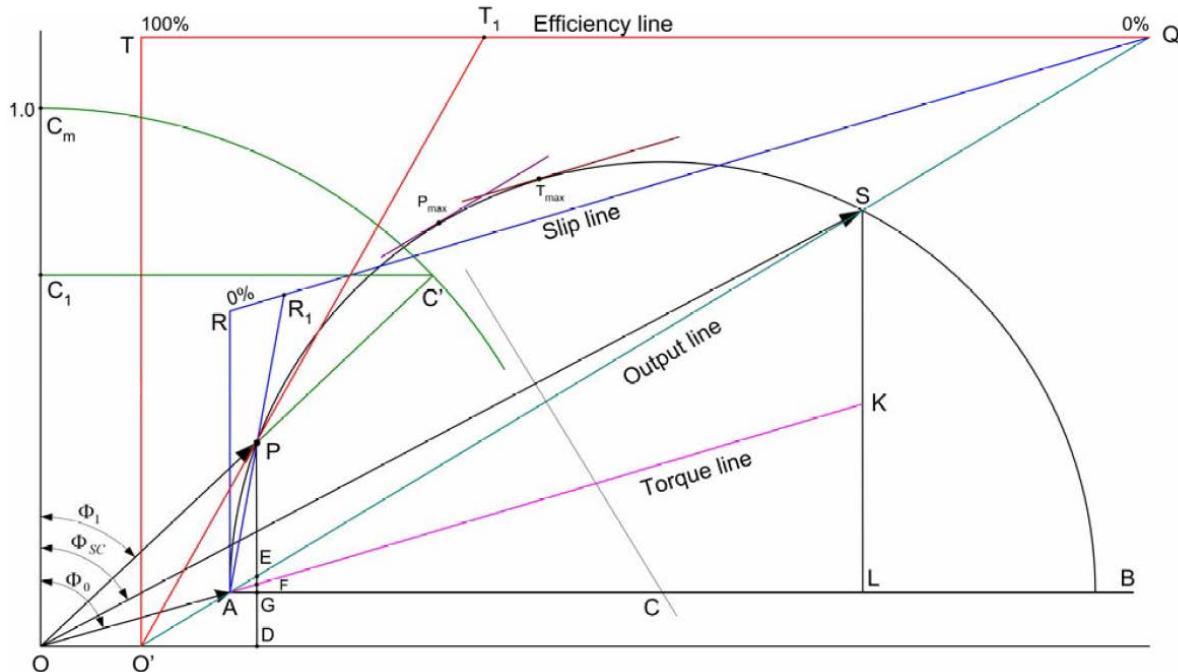


Fig: 3.17 Construction of Circle Diagram

Efficiency line

1. The output line AS is extended backwards to meet the X-axis at O'.
2. From any convenient point on the extended output line, draw a horizontal line QT so as to meet the vertical from O'. Divide the line QT into 100 equal parts.
3. To find the efficiency corresponding to any operating point P, draw a line from O' to the efficiency line through P to meet the efficiency line at T₁. Now QT₁ is the efficiency.

Slip Line

1. Draw line QR parallel to the torque line, meeting the vertical through A at R. Divide RQ into 100 equal parts.
2. To find the slip corresponding to any operating point P, draw a line from A to the slip line through P to meet the slip line at R₁. Now RR₁ is the slip.

Power Factor Curve

1. Draw a quadrant of a circle with O as centre and any convenient radius. Divide OC_m into 100 equal parts.
2. To find power factor corresponding to P, extend the line OP to meet the power factor curve at C'. Draw a horizontal line C'C₁ to meet the vertical axis at C₁. Now OC₁ represents power factor.

3.13 Performance Characteristics of Three phase Induction Motor

The equivalent circuits derived in the preceding section can be used to predict the performance characteristics of the induction machine. The important performance characteristics in the steady state are the efficiency, power factor, current, starting torque, maximum (or pull-out) torque.

3.13.1 The complete torque-speed characteristic

In order to estimate the speed torque characteristic let us suppose that a sinusoidal voltage is impressed on the machine. Recalling that the equivalent circuit is the per-phase representation of the machine, the current drawn by the circuit is given by

$$I_s = \frac{V_s}{(R_s + \frac{R'_r}{s}) + j(X_{ls} + X'_{lr})}$$

Where, V_s is the phase voltage phasor and I_s is the current phasor. The magnetizing current is neglected. Since this current is flowing through R'_r/s , the air-gap power is given by

$$\begin{aligned} P_g &= |I_s|^2 \frac{R'_r}{s} \\ &= \frac{V_s}{(R_s + \frac{R'_r}{s})^2 + (X_{ls} + X'_{lr})^2} \frac{R'_r}{s} \end{aligned}$$

The mechanical power output was shown to be $(1-s)P_g$ (power dissipated in R'_r/s). The torque is obtained by dividing this by the shaft speed ω_m . Thus we have,

$$\frac{P_g(1-s)}{\omega_m} = \frac{P_g(1-s)}{\omega_s(1-s)} = |I_s|^2 \frac{R'_r}{s\omega_s}$$

where ω_m is the synchronous speed in radians per second and s is the slip. Further, this is the torque produced per phase. Hence the overall torque is given by

$$T_e = \frac{3}{\omega_s} \cdot \frac{V_s^2}{(R_s + \frac{R'_r}{s})^2 + (X_{ls} + X'_{lr})^2} \cdot \frac{R'_r}{s}$$

The torque may be plotted as a function of ‘ s ’ and is called the torque-slip (or torque- speed, since slip indicates speed) characteristic a very important characteristic of the induction machine.

A typical torque-speed characteristic is shown in Fig: 3.18. This plot corresponds to a 3 kW, 4 pole, and 60 Hz machine. The rated operating speed is 1780 rpm.

Further, this curve is obtained by varying slip with the applied voltage being held constant. Coupled with the fact that this is an equivalent circuit valid under steady state, it implies that if this characteristic is to be measured experimentally, we need to look at the torque for a given speed after all transients have died down. One cannot, for example, try to obtain this curve by directly starting the motor with full voltage applied to the terminals and measuring the torque and speed dynamically as it runs up to steady speed.

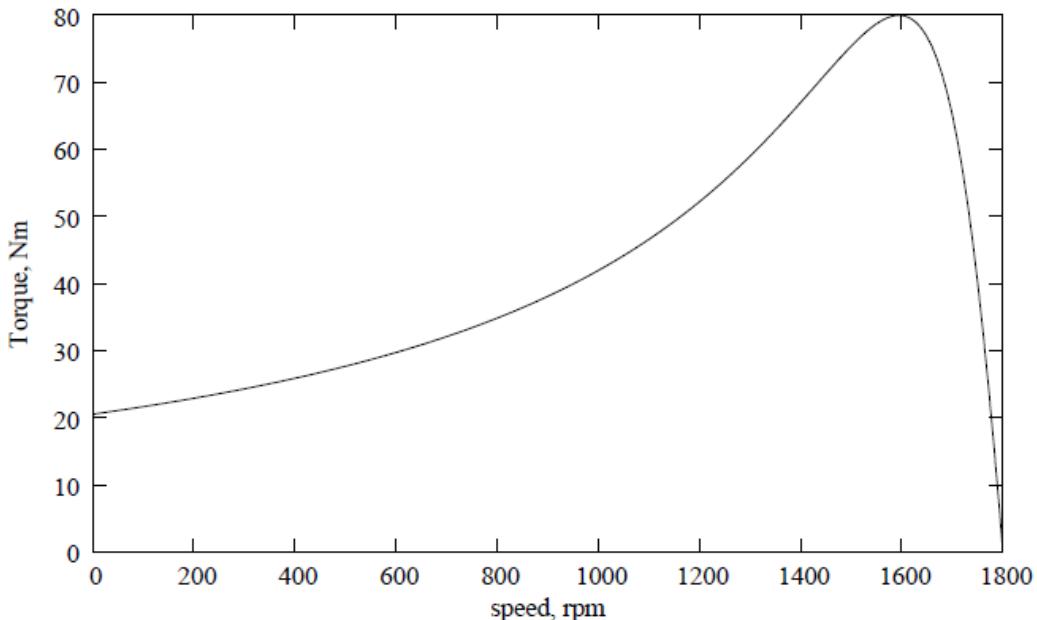


Fig: 3.18

With respect to the direction of rotation of the air-gap flux, the rotor maybe driven to higher speeds by a prime mover or may also be rotated in the reverse direction. The torque-speed relation for the machine under the entire speed range is called the complete speed-torque characteristic. A typical curve is shown in Fig: 3.19 for a four-pole machine, the synchronous speed being 1500 rpm. Note that negative speeds correspond to slip values greater than 1, and speeds greater than 1500 rpm correspond to negative slip. The plot also shows the operating modes of the induction machine in various regions. The slip axis is also shown for convenience.

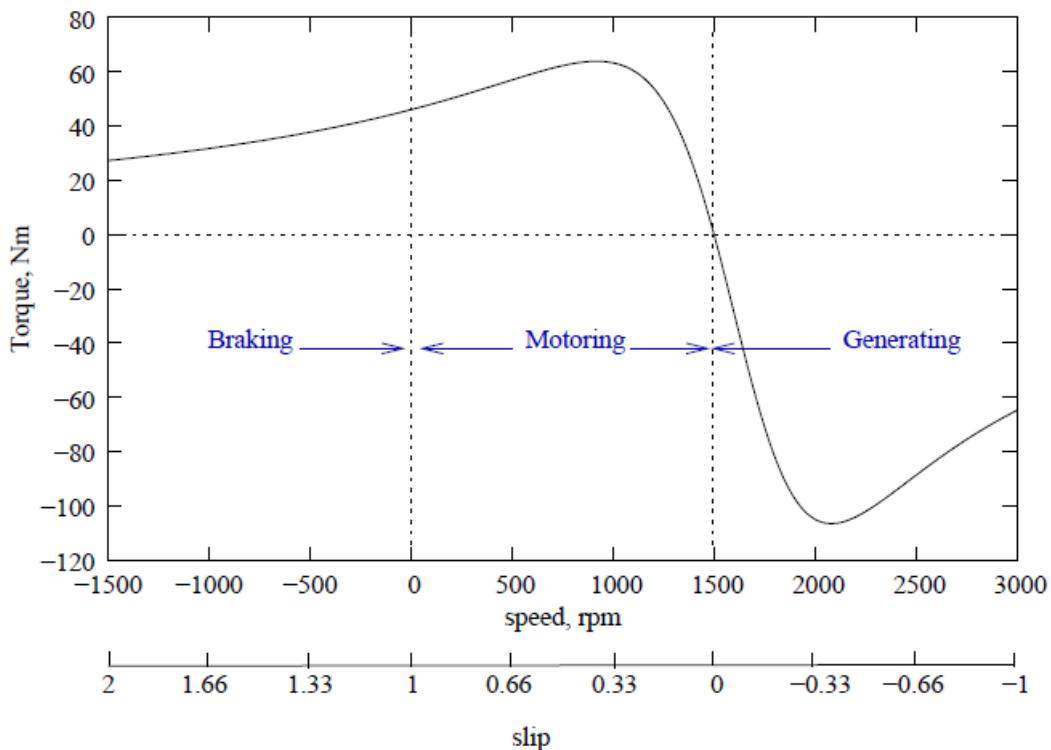


Fig: 3.19

3.13.2 Effect of Rotor Resistance on Speed Torque Characteristic

Restricting ourselves to positive values of slip, we see that the curve has a peak point. This is the maximum torque that the machine can produce, and is called as stalling torque. If the load torque is more than this value, the machine stops rotating or stalls. It occurs at a slip \hat{s} , which for the machine of Fig: 3.19 is 0.38. At values of slip lower than \hat{s} , the curve falls steeply down to zero at $s = 0$. The torque at synchronous speed is therefore zero. At values of slip higher than $s = \hat{s}$, the curve falls slowly to a minimum value at $s = 1$. The torque at $s = 1$ (speed = 0) is called the starting torque. The value of the stalling torque may be obtained by differentiating the expression for torque with respect to zero and setting it to zero to find the value of \hat{s} . Using this method, we can write

$$\hat{s} = \frac{\pm R'_r}{\sqrt{R'^2 + (X_{ls} + X'_{lr})^2}}$$

Substituting \hat{s} into the expression for torque gives us the value of the stalling torque \hat{T}_e ,

$$\hat{T}_e = \frac{3V_s^2}{2\omega_s} \cdot \frac{1}{R_s \pm \sqrt{R_s^2 + (X_{ls} + X'_{lr})^2}}$$

- The negative sign being valid for negative slip.

The expression shows that \hat{T}_e is independent of $R'r$, while \hat{s} is directly proportional to $R'r$. This fact can be made use of conveniently to alter \hat{s} . If it is possible to change $R'r$, then we can get a whole series of torque-speed characteristics, the maximum torque remaining constant all the while.

We may note that if $R'r$ is chosen equal to =

$$\sqrt{R_s^2 + (X_{ls} + X'_{lr})^2}$$

The \hat{s} , becomes unity, which means that the maximum torque occurs at starting. Thus changing of $R'r$, wherever possible can serve as a means to control the starting torque Fig: 3.20.

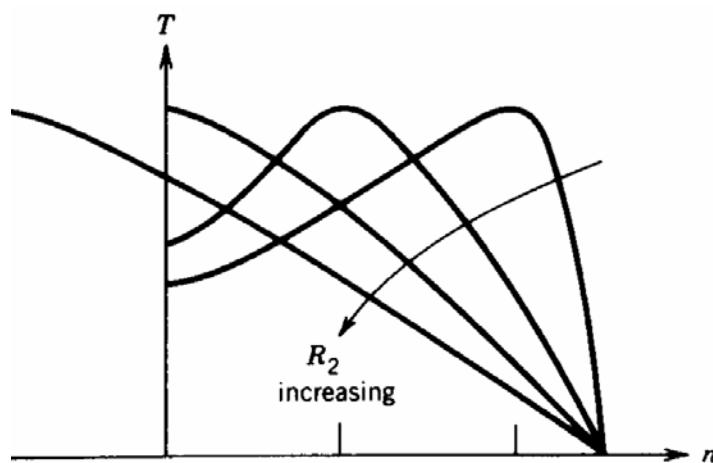


Fig: 3.20

While considering the negative slip range, (generator mode) we note that the maximum torque is higher than in the positive slip region (motoring mode).

3.13.3 Operating Point and Stable & Unstable region of Operation

Consider a speed torque characteristic shown in fig. 25 for an induction machine, having the load characteristic also superimposed on it. The load is a constant torque load i.e. the torque required for operation is fixed irrespective of speed.

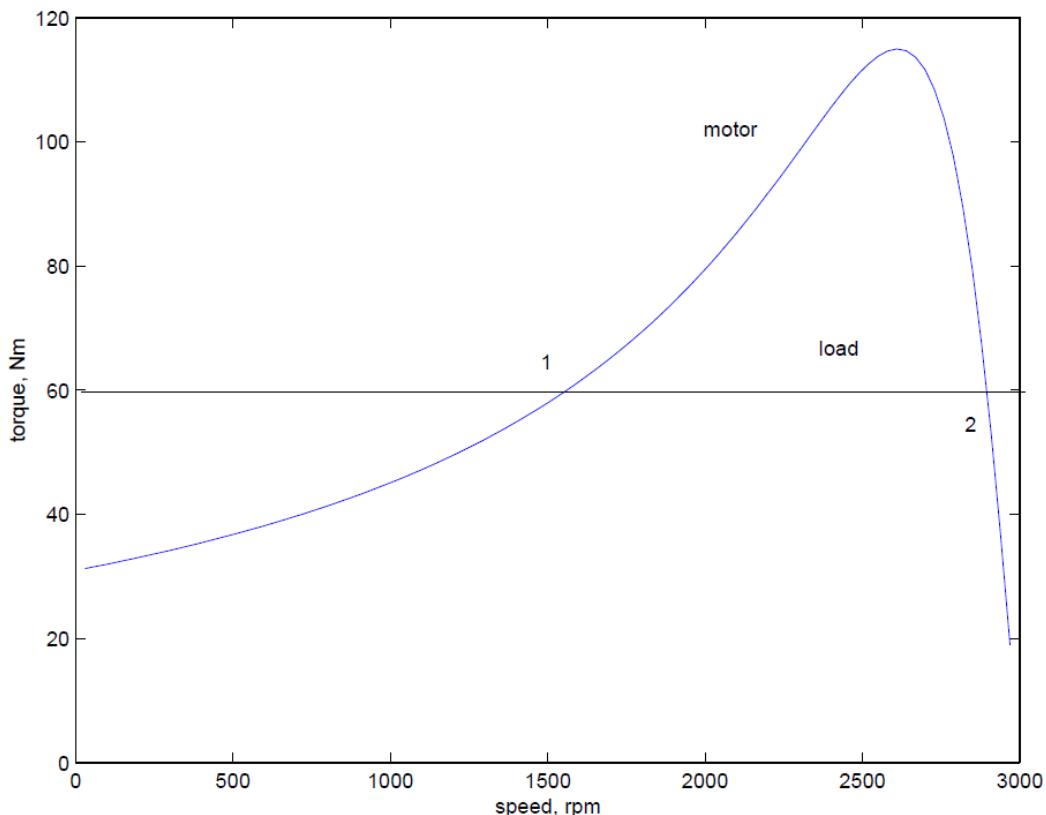


Fig: 3.21

The system consisting of the motor and load will operate at a point where the two characteristics meet. From the above plot, we note that there are two such points. We therefore need to find out which of these is the actual operating point. To answer this we must note that, in practice, the characteristics are never fixed; they change slightly with time. It would be appropriate to consider a small band around the curve drawn where the actual points of the characteristic will lie. This being the case let us consider that the system is operating at point 1, and the load torque demand increases slightly. This is shown in Fig: 3.22, where the change is exaggerated for clarity. This would shift the point of operation to a point 1' at which the slip would be less and the developed torque higher.

The difference in torque developed ΔT_e , being positive will accelerate the machine. Any overshoot in speed as it approaches the point 1' will cause it to further accelerate since the developed torque is increasing. Similar arguments may be used to show that if for some reason the developed torque becomes smaller the speed would drop and the effect is cumulative. Therefore we may conclude that 1 is not a stable operating point.

Let us consider the point 2. If this point shifts to 2', the slip is now higher (speed is lower) and the positive difference in torque will accelerate the machine. This behaviour will tend to bring the operating point towards 2 once again. In other words, disturbances at point 2 will not cause a

runaway effect. Similar arguments may be given for the case where the load characteristic shifts down. Therefore we conclude that point 2 is a stable operating point.

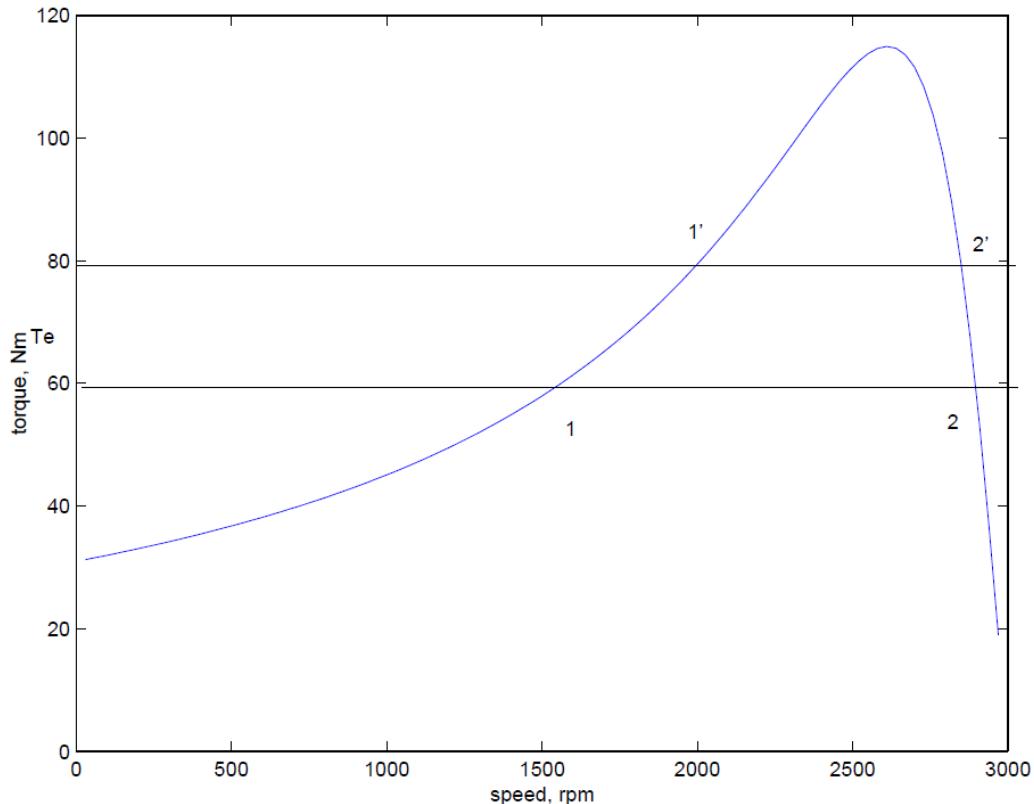


Fig: 3.22

From the above discussions, we can say that the entire region of the speed-torque characteristic from $s = 0$ to $s = \hat{s}$ is an unstable region, while the region from $s = \hat{s}$ to $s = 0$ is a stable region. Therefore the machine will always operate between $s = 0$ and $s = \hat{s}$.

3.14 Operation with Unbalanced Supply Voltage on Polyphase Induction Motors

Three phase induction motors are designed and manufactured such that all three phases of the winding are carefully balanced with respect to the number of turns, placement of the winding, and winding resistance. When line voltages applied to a polyphase induction motor are not exactly the same, unbalanced currents will flow in the stator winding, the magnitude depending upon the amount of unbalance. A small amount of voltage unbalance may increase the current an excessive amount. The effect on the motor can be severe and the motor may overheat to the point of burnout.

Unbalance Defined

The voltage unbalance (or negative sequence voltage) in percent may be defined as follows:

$$\text{Percent Voltage Unbalance} = 100 * (\text{Maximum Voltage Deviation} / \text{Average Voltage})$$

Example:

With voltages of 220, 215 and 210, in three phases respectively then the average is 215, the maximum deviation from the average is 5, and the percent unbalance = $100 \times 5/215 = 2.3$ percent.

Effect on performance-

General

The effect of unbalanced voltages on polyphase induction motors is equivalent to the introduction of a "negative sequence voltage" having a rotation opposite to that occurring with balanced voltages. This negative sequence voltage produces in the air gap a flux rotating against the rotation of the rotor, tending to produce high currents. A small negative sequence voltage may produce in the windings currents considerably in excess of those present under balanced voltage conditions.

Temperature rise and load carrying capacity

A relatively small unbalance in voltage will cause a considerable increase in temperature rise. In the phase with the highest current, the percentage increase in temperature rise will be approximately two times the square of the percentage voltage unbalance. The increase in losses and consequently, the increase in average heating of the whole winding will be slightly lower than the winding with the highest current.

To illustrate the severity of this condition, an approximate 3.5 percent voltage unbalance will cause an approximate 25 percent increase in temperature rise.

Torques

The locked-rotor torque and breakdown torque are decreased when the voltage is unbalanced. If the voltage unbalance should be extremely severe, the torque might not be adequate for the application.

Full-load speed

The full-load speed is reduced slightly when the motor operates at unbalanced voltages.

Currents

The locked-rotor current will be unbalanced to the same degree that the voltages are unbalanced but the locked-rotor KVA will increase only slightly. The currents at normal operating speed with unbalanced voltages will be greatly unbalanced in the order of approximately 6 to 10 times the voltage unbalance. This introduces a complex problem in selecting the proper overload protective devices, particularly since devices selected for one set of unbalanced conditions may be inadequate for a different set of unbalanced voltages. Increasing the size of the overload protective device is not the solution in as much as protection against heating from overload and from single phase operation is lost.

Thus the voltages should be evenly balanced as closely as can be read on the usually available commercial voltmeter.

3.15 Starting of Three Phase Induction Motor

The induction motor is fundamentally a transformer in which the stator is the primary and the rotor is short-circuited secondary. At starting, the voltage induced in the induction motor rotor is maximum ($s = 1$). Since the rotor impedance is low, the rotor current is excessively large. This large rotor current is reflected in the stator because of transformer action. This results in high starting current (4 to 10 times the full-load current) in the stator at low power factor and consequently the value of starting torque is low. Because of the short duration, this value of large current does not harm the motor if the motor accelerates normally.

However, this large starting current will produce large line-voltage drop. This will adversely affect the operation of other electrical equipment connected to the same lines. Therefore, it is desirable and necessary to reduce the magnitude of stator current at starting and several methods are available for this purpose.

3.15.1 Methods of Starting Three Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

- (i) Direct-on-line starting
- (ii) Stator resistance starting
- (iii) Autotransformer starting
- (iv) Star-delta starting
- (v) Rotor resistance starting

Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon, the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

(i) Direct-on-line starting

This method of starting is just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

Relation between starting and F.L. torques. We know that:

$$\text{Rotor input} = 2\pi N_s T = kT$$

$$\text{But} \quad \text{Rotor Cu loss} = s \times \text{Rotor input}$$

$$\therefore 3(I_2)^2 R_2 = s \times kT$$

$$\text{or} \quad T \propto (I_2)^2 / s$$

$$\text{or} \quad T \propto I_1^2 / s \quad (\because I_2 \propto I_1)$$

If I_{st} is the starting current, then starting torque (T_{st}) is

$$T \propto I_{st}^2 \quad (\because \text{at starting } s = 1)$$

If I_f is the full-load current and s_f is the full-load slip, then,

$$T_f \propto I_f^2 / s_f$$

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f} \right)^2 \times s_f$$

When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current I_{sc} .

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f$$

Let us illustrate the above relation with a numerical example. Suppose $I_{sc} = 5 I_f$ and full-load slip $s_f = 0.04$. Then,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f = \left(\frac{5 I_f}{I_f} \right)^2 \times 0.04 = (5)^2 \times 0.04 = 1$$

$$\therefore T_{st} = T_f$$

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

(ii) Stator resistance starting

In this method, external resistances are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor see Fig: 3.23.

This method suffers from two drawbacks. First, the reduced voltage applied to the motor during the starting period lowers the starting torque and hence increases the accelerating time. Secondly, a lot of power is wasted in the starting resistances.

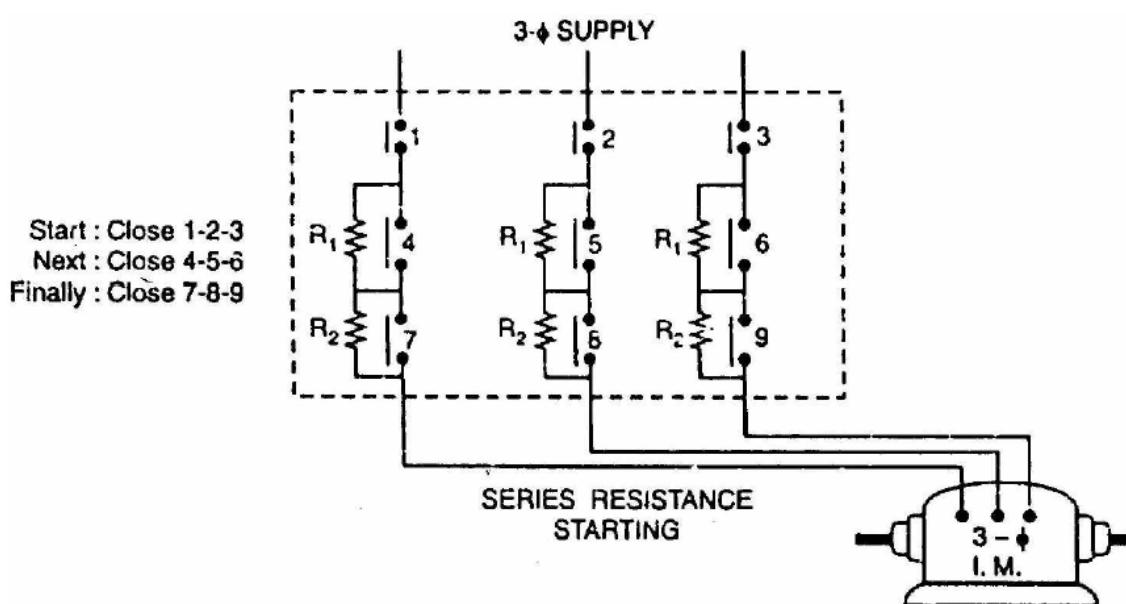


Fig: 3.23

Relation between starting and F.L. torques.

Let V be the rated voltage/phase. If the voltage is reduced by a fraction x by the insertion of resistors in the line, then voltage applied to the motor per phase will be xV .

So,

$$I_{st} = x I_{sc}$$

Now
$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f} \right)^2 \times s_f$$

or
$$\frac{T_{st}}{T_f} = x^2 \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f$$

Thus while the starting current reduces by a fraction x of the rated-voltage starting current (I_{sc}), the starting torque is reduced by a fraction x^2 of that obtained by direct switching. The reduced voltage applied to the motor during the starting period lowers the starting current but at the same time increases the accelerating time because of the reduced value of the starting torque. Therefore, this method is used for starting small motors only.

(iii) Autotransformer starting

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed. Fig: 3.24 shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

At the instant of starting, the change-over switch is thrown to “start” position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value. When the motor attains about 80% of normal speed, the changeover switch is thrown to “run” position. This takes out the autotransformer from the circuit and puts the motor to full line voltage. Autotransformer starting has several advantages viz low power loss, low starting current and less radiated heat. For large machines (over 25 H.P.), this method of starting is often used. This method can be used for both star and delta connected motors.

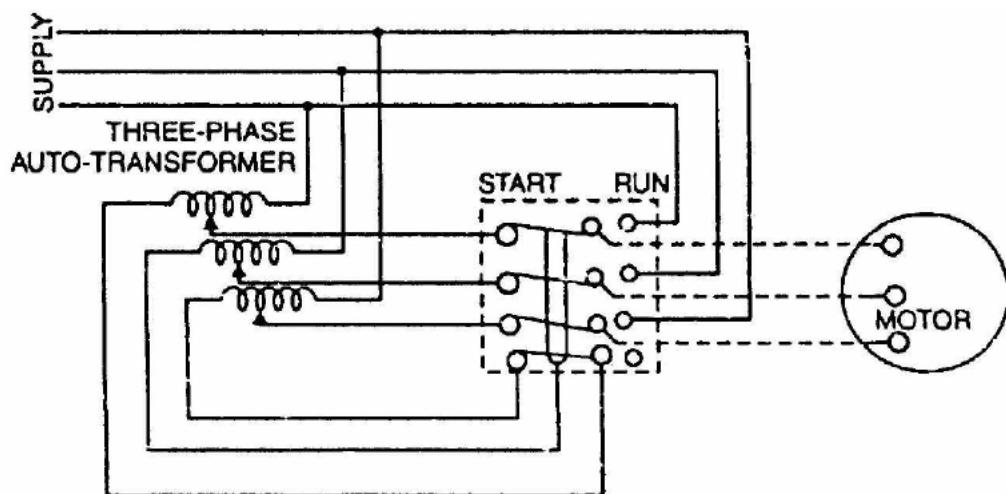


Fig: 3.24

Relation between starting And F.L. torques. Consider a star-connected squirrel-cage induction motor. If V is the line voltage, then voltage across motor phase on direct switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. In case of autotransformer, if a tapping of transformation ratio K (a fraction) is used, then phase voltage across motor is $KV/\sqrt{3}$ and $I_{st} = K I_{sc}$,

$$\text{Now } \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f} \right)^2 \times s_f = \left(\frac{K I_{sc}}{I_f} \right)^2 \times s_f = K^2 \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f$$

$$\therefore \frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f$$

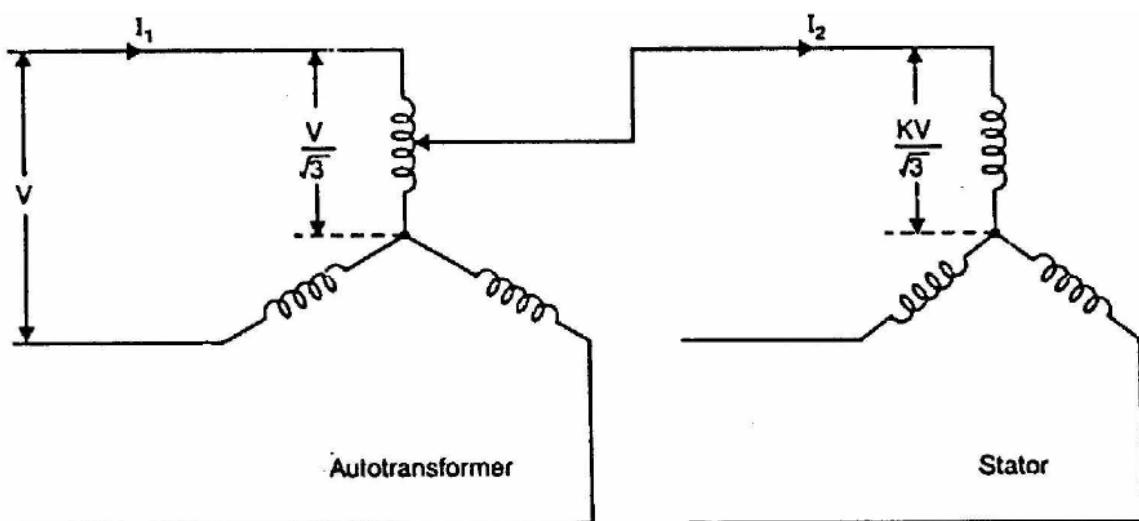


Fig: 3.25

The current taken from the supply or by autotransformer is $I_1 = KI_2 = K^2 I_{sc}$. Note that motor current is K times, the supply line current is K^2 times and the starting torque is K^2 times the value it would have been on direct-on-line starting.

(iv) Star-delta starting

The stator winding of the motor is designed for delta operation and is connected in star during the starting period. When the machine is up to speed, the connections are changed to delta. The circuit arrangement for star-delta starting is shown in Fig: 3.26.

The six leads of the stator windings are connected to the changeover switch as shown. At the instant of starting, the changeover switch is thrown to “Start” position which connects the stator windings in star. Therefore, each stator phase gets $V/\sqrt{3}$ volts where V is the line voltage. This reduces the starting current. When the motor picks up speed, the changeover switch is thrown to “Run” position which connects the stator windings in delta. Now each stator phase gets full line voltage V . The disadvantages of this method are:

- (a) With star-connection during starting, stator phase voltage is $1/\sqrt{3}$ times the line voltage. Consequently, starting torque is $(1/\sqrt{3})^2$ or $1/3$ times the value it would have with Δ -connection. This is rather a large reduction in starting torque.
- (b) The reduction in voltage is fixed.

This method of starting is used for medium-size machines (upto about 25 H.P.).

Relation between starting and F.L. torques. In direct delta starting,

Starting current/phase, $I_{sc} = V/Z_{sc}$ where V = line voltage

$$\text{Starting line current} = \sqrt{3} I_{sc}$$

In star starting, we have,

$$\text{Starting current/phase, } I_{st} = \frac{V/\sqrt{3}}{Z_{sc}} = \frac{1}{\sqrt{3}} I_{sc}$$

$$\text{Now } \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f} \right)^2 \times S_f = \left(\frac{I_{sc}}{\sqrt{3} \times I_f} \right)^2 \times S_f$$

$$\text{or} \quad \frac{T_{st}}{T_f} = \frac{1}{3} \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f$$

where I_{sc} = starting phase current (delta)
 I_f = F.L. phase current (delta)

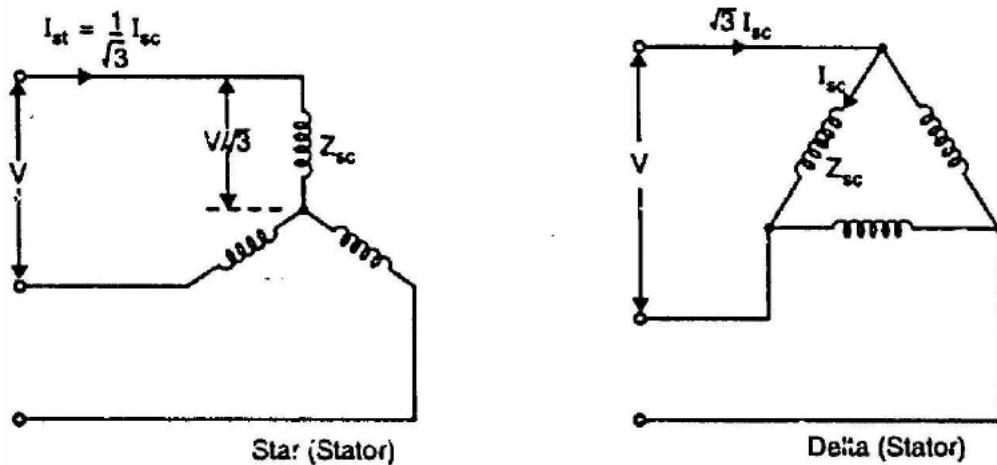


Fig: 3.26

Note that in star-delta starting, the starting line current is reduced to one-third as compared to starting with the winding delta connected. Further, starting torque is reduced to one-third of that obtainable by direct delta starting. This method is cheap but limited to applications where high starting torque is not necessary e.g., machine tools, pumps etc.

3.15.2 Starting of Slip-Ring Induction Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Fig: 3.27.

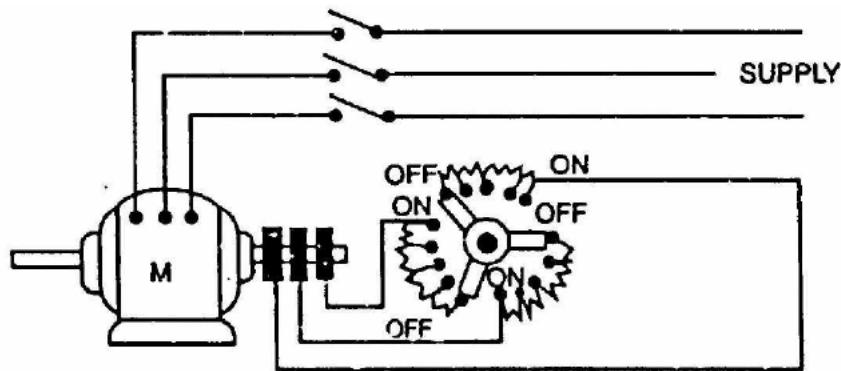


Fig: 3.27

- (i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.
- (ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

3.16 Speed control of Three Phase Induction Motors

The induction machine, when operating from mains is essentially a constant speed machine. Many industrial drives, typically for fan or pump applications, have typically constant speed requirements and hence the induction machine is ideally suited for these. However, the induction machine, especially the squirrel cage type, is quite rugged and has a simple construction. Therefore it is good candidate for variable speed applications if it can be achieved.

3.16.1 Speed control by changing applied voltage

From the torque equation of the induction machine we can see that the torque depends on the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in Fig: 3.28. These curves show that the slip at maximum torque \hat{s} remains same, while the value of stall torque comes down with decrease in applied voltage. The speed range for stable operation remains the same.

Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed.

Fig: 3.28 also shows a load torque characteristic — one that is typical of a fan type of load. In a fan (blower) type of load, the variation of torque with speed is such that $T \propto \omega^2$.

Here one can see that it may be possible to run the motor to lower speeds within the range n_s to $(1 - \hat{s}) n_s$. Further, since the load torque at zero speed is zero, the machine can start even at reduced voltages. This will not be possible with constant torque type of loads.

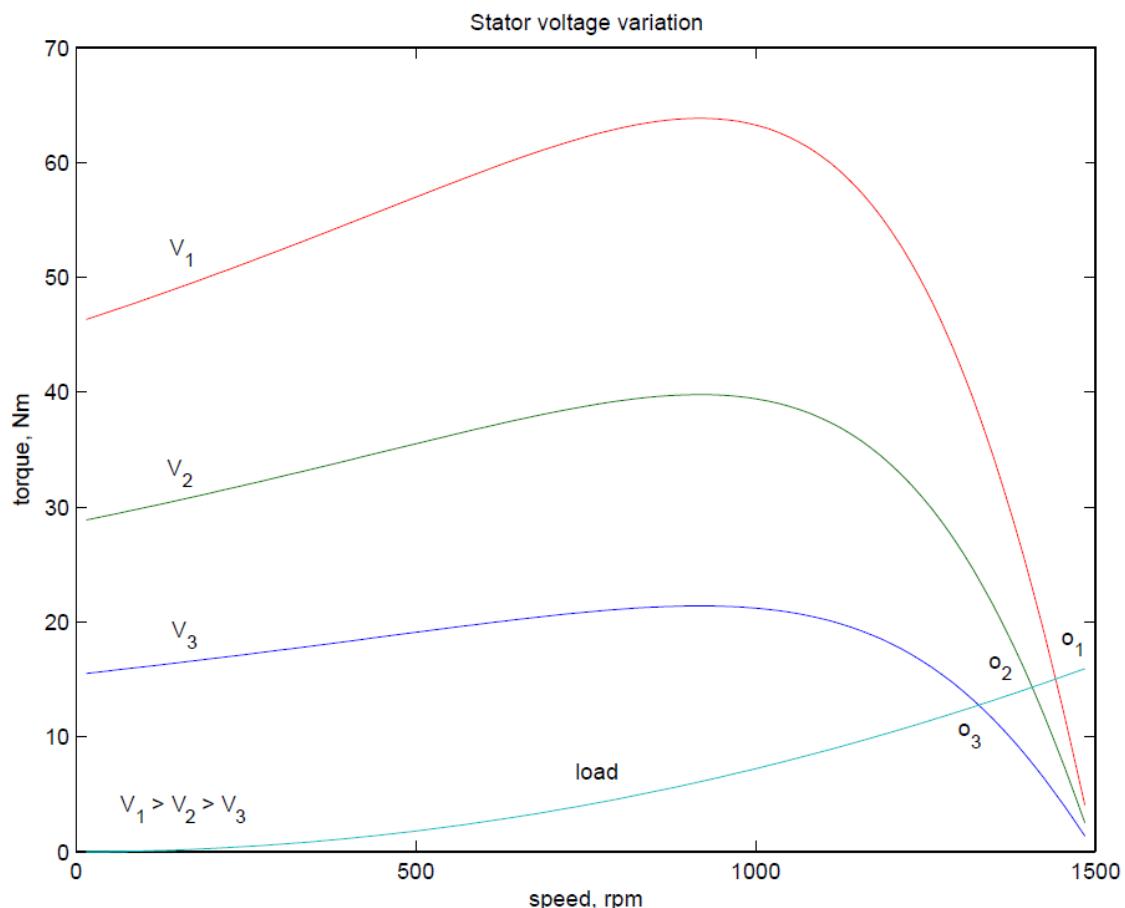


Fig: 3.28

One may note that if the applied voltage is reduced, the voltage across the magnetising branch also comes down. This in turn means that the magnetizing current and hence flux level are reduced. Reduction in the flux level in the machine impairs torque production which is primarily the explanation for Fig: 3.28. If, however, the machine is running under lightly loaded conditions, then operating under rated flux levels is not required. Under such conditions,

reduction in magnetizing current improves the power factor of operation. Some amount of energy saving may also be achieved.

Voltage control may be achieved by adding series resistors (a lossy, inefficient proposition), or a series inductor / autotransformer (a bulky solution) or a more modern solution using semiconductor devices. A typical solid state circuit used for this purpose is the AC voltage controller or AC chopper.

3.16.2 Rotor resistance control

The expression for the torque of the induction machine is dependent on the rotor resistance. Further the maximum value is independent of the rotor resistance. The slip at maximum torque is dependent on the rotor resistance. Therefore, we may expect that if the rotor resistance is changed, the maximum torque point shifts to higher slip values, while retaining a constant torque. Fig: 3.29 shows a family of torque-speed characteristic obtained by changing the rotor resistance.

Note that while the maximum torque and synchronous speed remain constant, the slip at which maximum torque occurs increases with increase in rotor resistance, and so does the starting torque. Whether the load is of constant torque type or fan-type, it is evident that the speed control range is more with this method. Further, rotor resistance control could also be used as a means of generating high starting torque.

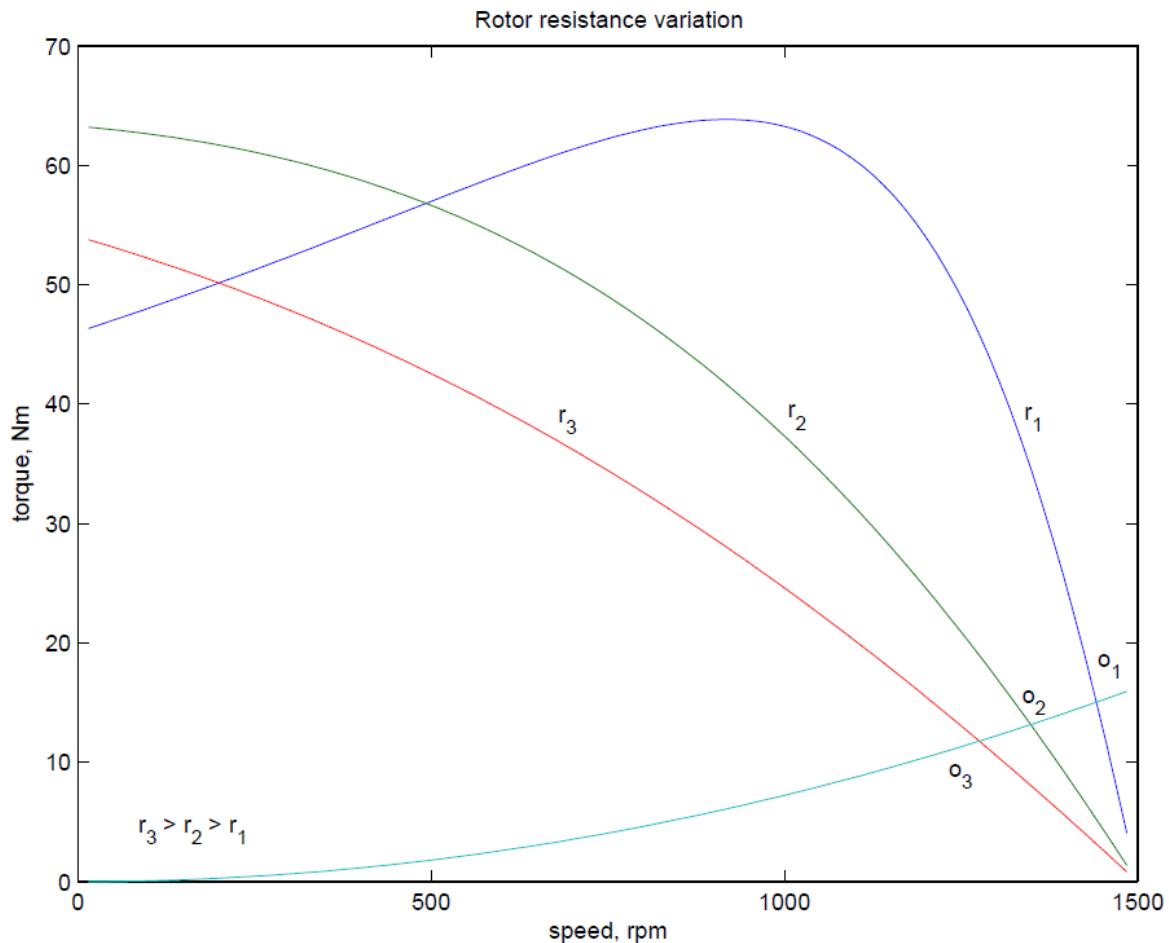


Fig: 3.29

For all its advantages, the scheme has two serious drawbacks. Firstly, in order to vary the rotor resistance, it is necessary to connect external variable resistors (winding resistance itself cannot be changed). This, therefore necessitates a slip-ring machine, since only in that case rotor terminals are available outside. For cage rotor machines, there are no rotor terminals. Secondly, the method is not very efficient since the additional resistance and operation at high slips entails dissipation.

The resistors connected to the slip-ring brushes should have good power dissipation capability. Water based rheostats may be used for this. A ‘solid-state’ alternative to a rheostat is a chopper controlled resistance where the duty ratio control of the chopper presents a variable resistance load to the rotor of the induction machine.

3.16.3 Cascade control

The power drawn from the rotor terminals could be spent more usefully. Apart from using the heat generated in meaningful ways, the slip ring output could be connected to another induction machine. The stator of the second machine would carry slip frequency currents of

the first machine which would generate some useful mechanical power. A still better option would be to mechanically couple the shafts of the two machines together. This sort of a connection is called cascade connection and it gives some measure of speed control.

Let the frequency of supply given to the first machine be f_1 , its number of poles be p_1 , and its slip of operation be s_1 . Let f_2 , p_2 and s_2 be the corresponding quantities for the second machine. The frequency of currents flowing in the rotor of the first machine and hence in the stator of the second machine is $s_1 f_1$. Therefore $f_2 = s_1 f_1$. Since the machines are coupled at the shaft, the speed of the rotor is common for both. Hence, if n is the speed of the rotor in radians,

$$n = \frac{f_1}{p_1}(1 - s_1) = \pm \frac{s_1 f_1}{p_2}(1 - s_2).$$

Note that while giving the rotor output of the first machine to the stator of the second, the resultant stator mmf of the second machine may set up an air-gap flux which rotates in the same direction as that of the rotor, or opposes it. This results in values for speed as –

$$n = \frac{f_1}{p_1 + p_2} \quad \text{or} \quad n = \frac{f_1}{p_1 - p_2} \quad (s_2 \text{ negligible})$$

The latter expression is for the case where the second machine is connected in opposite phase sequence to the first. The cascade connected system can therefore run at two possible speeds.

Speed control through rotor terminals can be considered in a much more general way. Consider the induction machine equivalent circuit of Fig: 3.30, where the rotor circuit has been terminated with a voltage source E_r .

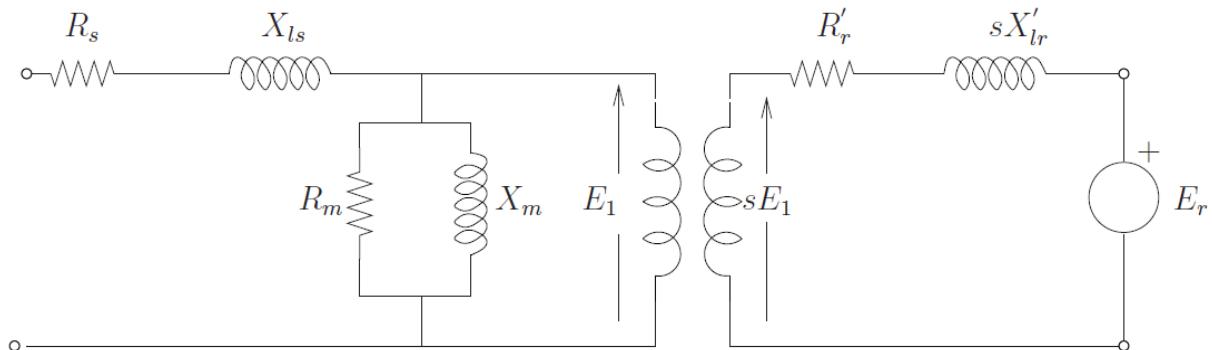


Fig: 3.30

If the rotor terminals are shorted, it behaves like a normal induction machine. This is equivalent to saying that across the rotor terminals a voltage source of zero magnitude is connected. Different situations could then be considered if this voltage source E_r had a non-zero magnitude. Let the power consumed by that source be P_r . Then considering the rotor side circuit power dissipation per phase

$$sE_1 I'_2 \cos \phi_2 = I'_2 R'_2 + P_r.$$

Clearly now, the value of s can be changed by the value of P_r . for $P_r = 0$, the machine is like a normal machine with a short circuited rotor. As P_r becomes positive, for all other circuit conditions remaining constant, s increases or in the other words, speed reduces. As P_r becomes negative, the right hand side of the equation and hence the slip decreases. The physical interpretation is that we now have an active source connected on the rotor side which is able to supply part of the rotor copper losses. When $P_r = -I'_2^2 R_2$ the entire copper loss is supplied by the external source. The RHS and hence the slip is zero. This corresponds to operation at synchronous speed. In general the circuitry connected to the rotor may not be a simple resistor or a machine but a power electronic circuit which can process this power requirement. This circuit may drive a machine or recover power back to the mains. Such circuits are called static Kramer drives.

3.16.4 Pole changing method

Sometimes induction machines have a special stator winding capable of being externally connected to form two different number of pole numbers. Since the synchronous speed of the induction machine is given by $n_s = f_s/p$ (in rev./s) where p is the number of pole pairs, this would correspond to changing the synchronous speed. With the slip now corresponding to the new synchronous speed, the operating speed is changed. This method of speed control is a stepped variation and generally restricted to two steps.

If the changes in stator winding connections are made so that the air gap flux remains constant, then at any winding connection, the same maximum torque is achievable. Such winding arrangements are therefore referred to as constant-torque connections. If however such connection changes result in air gap flux changes that are inversely proportional to the synchronous speeds, then such connections are called constant-horsepower type.

The following figure serves to illustrate the basic principle. Consider a magnetic pole structure consisting of four pole faces A, B, C, D as shown in Fig: 3.31.

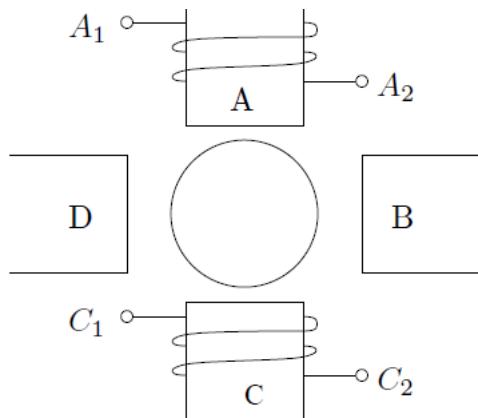


Fig: 3.31

Coils are wound on A & C in the directions shown. The two coils on A & C may be connected in series in two different ways — A₂ may be connected to C₁ or C₂. A₁ with the

Other terminal at C then form the terminals of the overall combination. Thus two connections result as shown in Fig: 3.32 (a) & (b).

Now, for a given direction of current flow at terminal A₁, say into terminal A₁, the flux directions within the poles are shown in the figures. In case (a), the flux lines are out of the pole A (seen from the rotor) for and into pole C, thus establishing a two-pole structure. In case (b) however, the flux lines are out of the poles in A & C. The flux lines will be then have to complete the circuit by flowing into the pole structures on the sides. If, when seen from the rotor, the pole emanating flux lines is considered as North Pole and the pole into which they enter is termed as south, then the pole configurations produced by these connections is a two-pole arrangement in Fig: 3.32(a) and a four-pole arrangement in Fig: 3.32 (b).

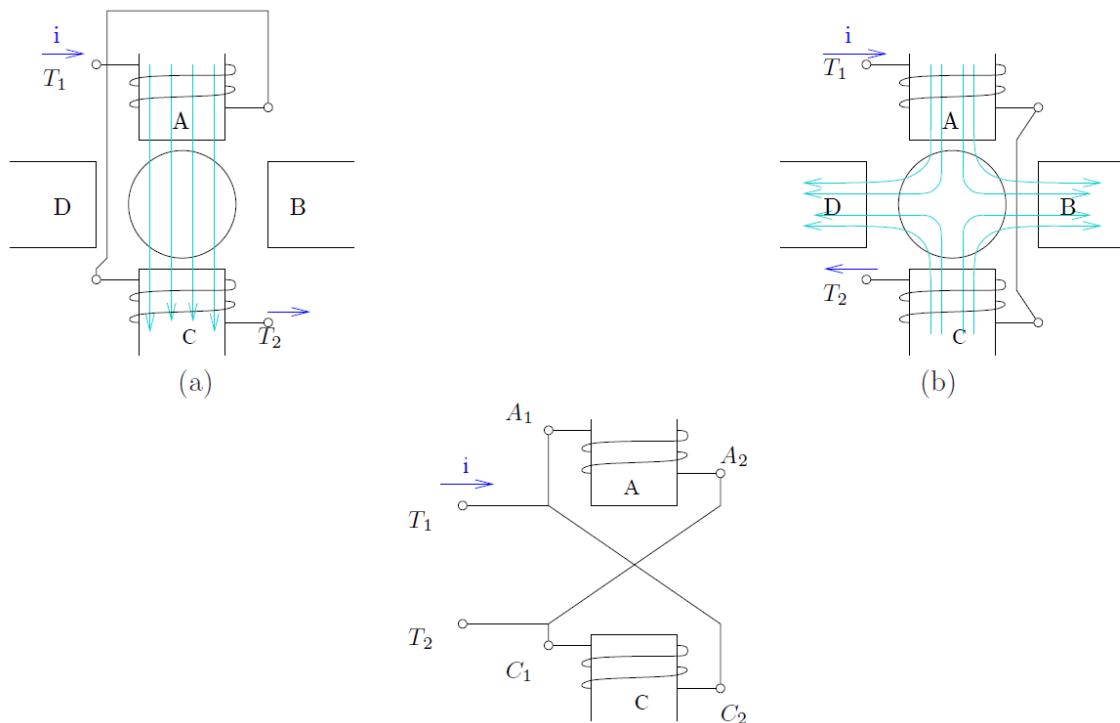


Fig: 3.32

Thus by changing the terminal connections we get either a two pole air-gap field or a four-pole field. In an induction machine this would correspond to a synchronous speed reduction

in half from case (a) to case (b). Further note that irrespective of the connection, the applied voltage is balanced by the series addition of induced emf's in two coils. Therefore the air-gap flux in both cases is the same. Cases (a) and (b) therefore form a pair of constant torque connections.

Consider, on the other hand a connection as shown in the Fig: 3.32 (c). The terminals T₁ and T₂ are where the input excitation is given. Note that current direction in the coils now resembles that of case (b), and hence this would result in a four-pole structure. However, in Fig: 3.32 (c), there is only one coil induced emf to balance the applied voltage. Therefore flux in case (c) would therefore be halved compared to that of case (b) or case (a), for that matter). Cases (a) and (c) therefore form a pair of constant horse-power connections.

It is important to note that in generating a different pole numbers, the current through one coil (out of two, coil C in this case) is reversed. In the case of a three phase machine, the following example serves to explain this. Let the machine have coils connected as shown [C₁ – C₆] as shown in Fig: 3.33.

The current directions shown in C1 & C2 correspond to the case where T1, T2, T3 are supplied with three phase excitation and Ta, Tb & Tc are shorted to each other (STAR point). The applied voltage must be balanced by induced emf in one coil only (C1 & C2 are parallel). If however the excitation is given to Ta, Tb & Tc with T1, T2, T3 open, then current through one of the coils (C1 & C2) would reverse. Thus the effective number of poles would increase, thereby bringing down the speed. The other coils also face similar conditions.

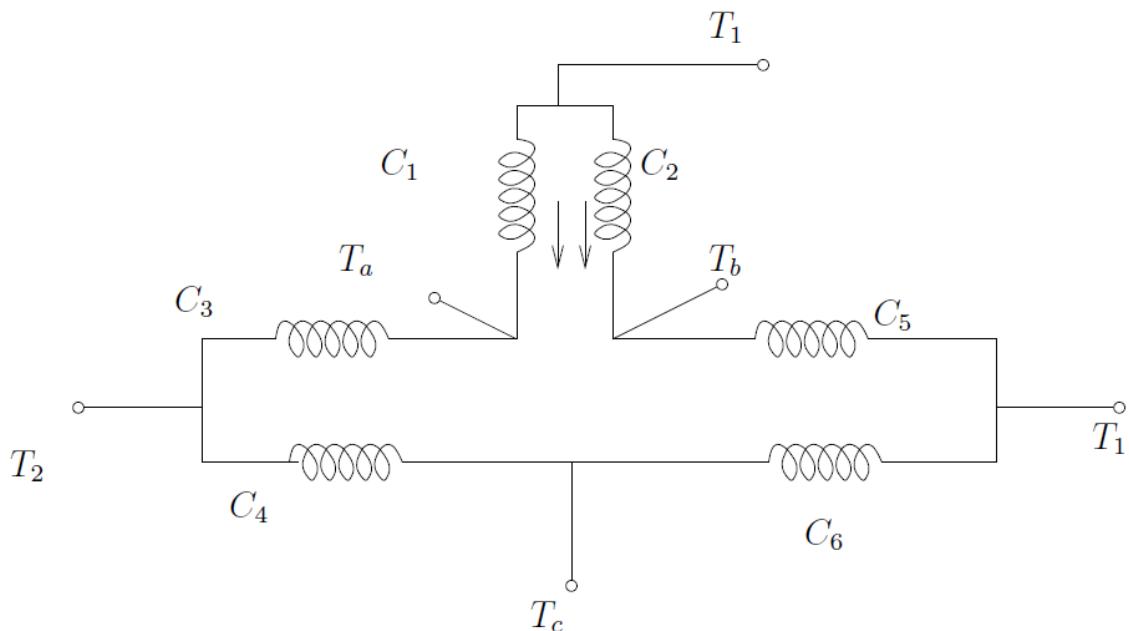


Fig: 3.33

3.16.5 Stator frequency control

The expression for the synchronous speed indicates that by changing the stator frequency also it can be changed. This can be achieved by using power electronic circuits called inverters which convert dc to ac of desired frequency. Depending on the type of control scheme of the inverter, the ac generated may be variable-frequency-fixed-amplitude or variable-frequency-variable-amplitude type. Power electronic control achieves smooth variation of voltage and frequency of the ac output. This when fed to the machine is capable of running at a controlled speed. However, consider the equation for the induced emf in the induction machine.

$$V = 4.44N\phi_m f$$

Where, N is the number of the turns per phase, ϕ_m is the peak flux in the air gap and f is the frequency.

Note that in order to reduce the speed, frequency has to be reduced. If the frequency is reduced while the voltage is kept constant, thereby requiring the amplitude of induced emf to remain the same, flux has to increase. This is not advisable since the machine likely to enter deep saturation. If this is to be avoided, then flux level must be maintained constant which implies that voltage must be reduced along with frequency. The ratio is held constant in order to maintain the flux level for maximum torque capability.

Actually, it is the voltage across the magnetizing branch of the exact equivalent circuit that must be maintained constant, for it is that which determines the induced emf. Under conditions where the stator voltage drop is negligible compared the applied voltage. In this mode of operation, the voltage across the magnetizing inductance in the 'exact' equivalent circuit reduces in amplitude with reduction in frequency and so does the inductive reactance. This implies that the current through the inductance and the flux in the machine remains constant. The speed torque characteristics at any frequency may be estimated as before. There is one curve for every excitation frequency considered corresponding to every

value of synchronous speed. The curves are shown below. It may be seen that the maximum torque remains constant.

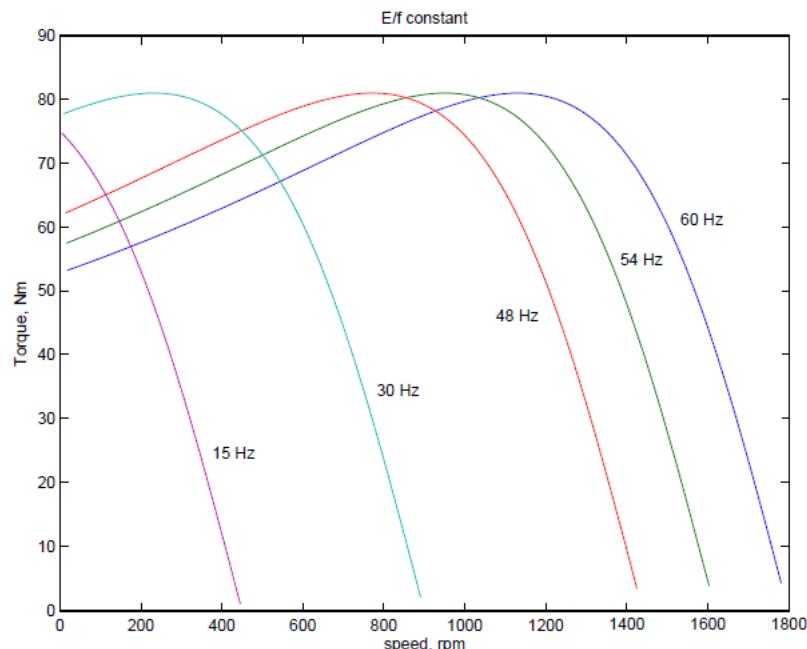


Fig: 3.34

This may be seen mathematically as follows. If E is the voltage across the magnetizing branch and f is the frequency of excitation, then $E = kf$, where k is the constant of proportionality. If $T = 2\pi f$, the developed torque is given by

$$T_{E/f} = \frac{k^2 f^2}{\left(\frac{R'_r}{s}\right)^2 + (\omega L'_{lr})^2} \frac{R'_r}{s\omega}$$

If this equation is differentiated with respect to s and equated to zero to find the slip at maximum torque \hat{s} , we get $\hat{s} = \pm R'_r / (\omega L'_{lr})$. The maximum torque is obtained by substituting this value into above equation,

$$\hat{T}_{E/f} = \frac{k^2}{8\pi^2 L'_{lr}}$$

It shows that this maximum value is independent of the frequency. Further $\hat{s}\omega$ is independent of frequency. This means that the maximum torque always occurs at a speed lower than synchronous speed by a fixed difference, independent of frequency. The overall effect is an apparent shift of the torque-speed characteristic as shown in Fig: 3.34.

Though this is the aim, E is an internal voltage which is not accessible. It is only the terminal voltage V which we have access to and can control. For a fixed V , E changes with operating slip (rotor branch impedance changes) and further due to the stator impedance drop. Thus if we approximate E/f as V/f , the resulting torque-speed characteristic shown in Fig: 3.35 is far from desirable.

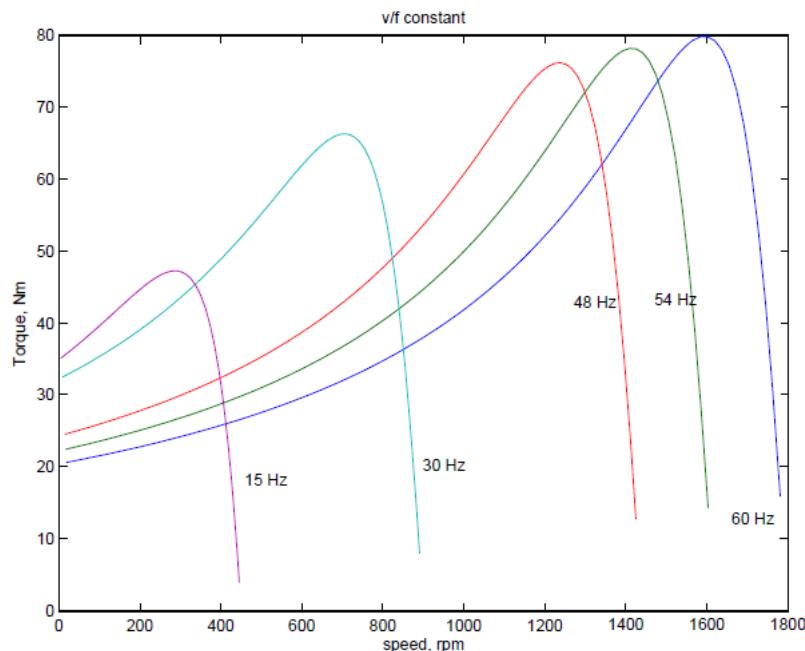


Fig: 3.35

At low frequencies and hence low voltages the curves show a considerable reduction in peak torque. At low frequencies (and hence at low voltages) the drop across the stator impedance prevents sufficient voltage availability. Therefore, in order to maintain sufficient torque at low frequencies, a voltage more than proportional needs to be given at low speeds.

Another component of compensation that needs to be given is due to operating slip. With these two components, therefore, the ratio of applied voltage to frequency is not a constant but is a curve such as that shown in Fig: 3.36

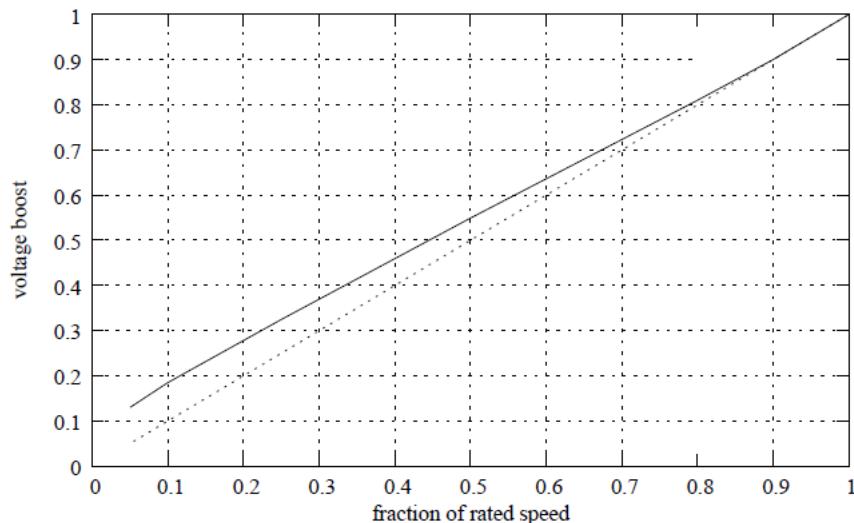


Fig: 3.36

With this kind of control, it is possible to get a good starting torque and steady state performance. However, under dynamic conditions, this control is insufficient. Advanced control techniques such as field- oriented control (vector control) or direct torque control (DTC) are necessary.

3.17 Power Stages in an Induction Motor

The input electric power fed to the stator of the motor is converted into mechanical power at the shaft of the motor. The various losses during the energy conversion are:

1. Fixed losses

- (i) Stator iron loss

- (ii) Friction and windage loss

The rotor iron loss is negligible because the frequency of rotor currents under normal running condition is small.

2. Variable losses

- (i) Stator copper loss
- (ii) Rotor copper loss

Fig: 3.37 shows how electric power fed to the stator of an induction motor suffers losses and finally converted into mechanical power.

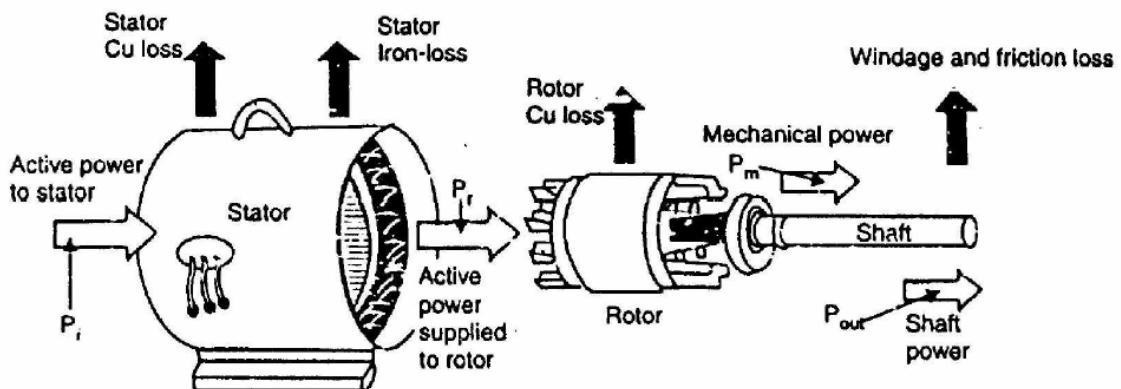


Fig: 3.37

The following points may be noted from the above diagram:

- (i) Stator input, $P_i = \text{Stator output} + \text{Stator losses}$
 $= \text{Stator output} + \text{Stator Iron loss} + \text{Stator Cu loss}$
- (ii) Rotor input, $P_r = \text{Stator output}$

It is because stator output is entirely transferred to the rotor through air-gap by electromagnetic induction.

(iii) Mechanical power available, $P_m = P_r - \text{Rotor Cu loss}$

This mechanical power available is the gross rotor output and will produce a gross torque T_g .

(iv) Mechanical power at shaft, $P_{out} = P_m - \text{Friction and windage loss}$

Mechanical power available at the shaft produces a shaft torque T_{sh} .

Clearly, $P_m - P_{out} = \text{Friction and windage loss}$.

3.18 Double Cage Induction Motor

One of the advantages of the slip-ring motor is that resistance may be inserted in the rotor circuit to obtain high starting torque (at low starting current) and then cut out to obtain optimum running conditions. However, such a procedure cannot be adopted for a squirrel cage motor because its cage is permanently short-circuited. In order to provide high starting torque at low starting current, double-cage construction is used.

Construction

As the name suggests, the rotor of this motor has two squirrel-cage windings located one above the other as shown in Fig: 3.38(i).

The outer winding consists of bars of smaller cross-section short-circuited by end rings. Therefore, the resistance of this winding is high. Since the outer winding has relatively open slots and a poorer flux path around its bars [See Fig: 3.38(ii)], it has a low inductance. Thus the resistance of the outer squirrel-cage winding is high and its inductance is low.

The inner winding consists of bars of greater cross-section short-circuited by end rings. Therefore, the resistance of this winding is low. Since the bars of the inner winding are thoroughly buried in iron, it has a high inductance [See Fig: 3.38(ii)]. Thus the resistance of the inner squirrel cage winding is low and its inductance is high.

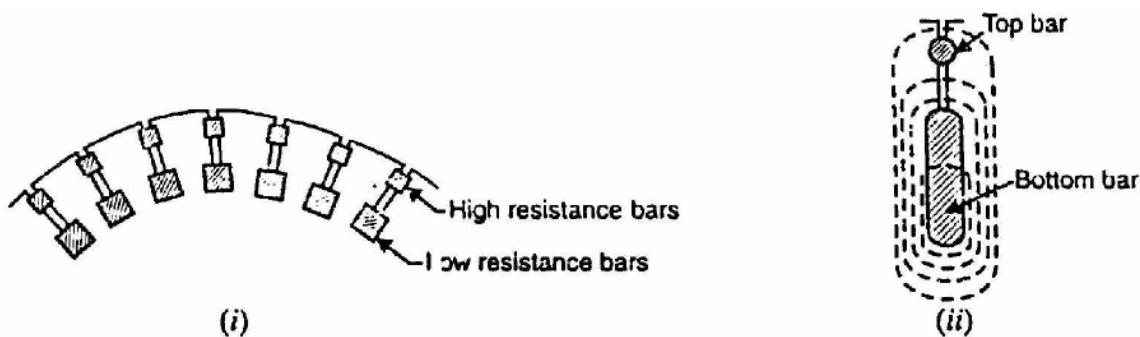


Fig: 3.38

Working

When a rotating magnetic field sweeps across the two windings, equal e.m.f.s are induced in each.

(i) At starting, the rotor frequency is the same as that of the line (i.e., 50 Hz), making the reactance of the lower winding much higher than that of the upper winding. Because of the high reactance of the lower winding, nearly all the rotor current flows in the high-resistance outer cage winding. This provides the good starting characteristics of a high-resistance cage winding. Thus the outer winding gives high starting torque at low starting current.

(ii) As the motor accelerates, the rotor frequency decreases, thereby lowering the reactance of the inner winding, allowing it to carry a larger proportion of the total rotor current. At the normal operating speed of the motor, the rotor frequency is so low (2 to 3 Hz) that nearly all the rotor current flows in the low-resistance inner cage winding. This results in good operating efficiency and speed regulation.

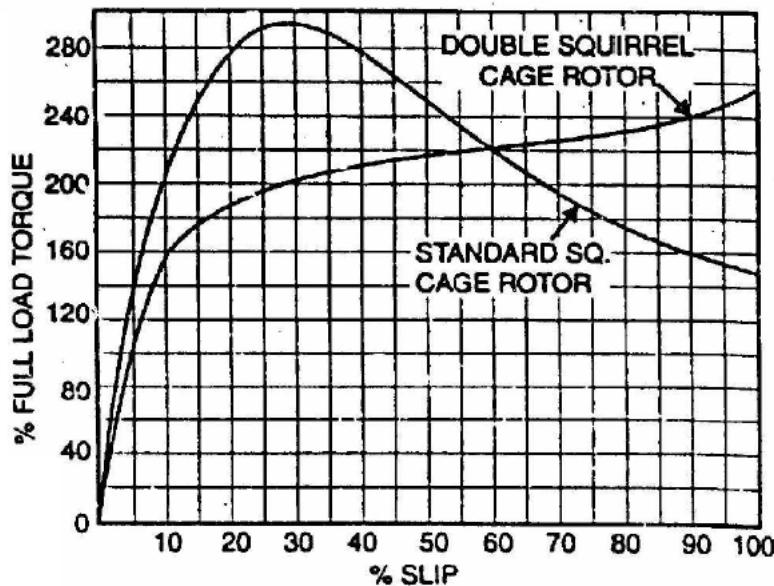


Fig: 3.39

Fig: 3.39 shows the operating characteristics of double squirrel-cage motor. The starting torque of this motor ranges from 200 to 250 percent of full-load torque with a starting current of 4 to 6 times the full-load value. It is classed as a high-torque, low starting current motor.

3.19 Cogging and Crawling of Induction Motor

Crawling of induction motor

Sometimes, squirrel cage induction motors exhibits a tendency to run at very slow speeds (as low as one-seventh of their synchronous speed). This phenomenon is called as crawling of an induction motor.

This action is due to the fact that, flux wave produced by a stator winding is not purely sine wave. Instead, it is a complex wave consisting a fundamental wave and odd harmonics like 3rd, 5th, 7th etc. The fundamental wave revolves synchronously at synchronous speed N_s whereas 3rd, 5th, 7th harmonics may rotate in forward or backward direction at $N_s/3$, $N_s/5$, $N_s/7$ speeds respectively. Hence, harmonic torques are also developed in addition with fundamental torque.

3rd harmonics are absent in a balanced 3-phase system. Hence 3rd harmonics do not produce rotating field and torque. The total motor torque now consist three components as: (i) the fundamental torque with synchronous speed N_s , (ii) 5th harmonic torque with synchronous speed

$N_s/5$, (iv) 7th harmonic torque with synchronous speed $N_s/7$ (provided that higher harmonics are neglected).

Now, 5th harmonic currents will have phase difference of

$$5 \times 120 = 600^\circ = 2 \times 360 - 120 = -120^\circ.$$

Hence the revolving speed set up will be in reverse direction with speed $N_s/5$. The small amount of 5th harmonic torque produces breaking action and can be neglected.

The 7th harmonic currents will have phase difference of

$$7 \times 120 = 840^\circ = 2 \times 360 + 120 = +120^\circ.$$

Hence they will set up rotating field in forward direction with synchronous speed equal to $N_s/7$. If we neglect all the higher harmonics, the resultant torque will be equal to sum of fundamental torque and 7th harmonic torque. 7th harmonic torque reaches its maximum positive value just before $1/7^{\text{th}}$ of N_s . If the mechanical load on the shaft involves constant load torque, the torque developed by the motor may fall below this load torque. In this case, motor will not accelerate up to its normal speed, but it will run at a speed which is nearly $1/7^{\text{th}}$ of its normal speed as shown in Fig: 3.40. This phenomenon is called as crawling of induction motors.

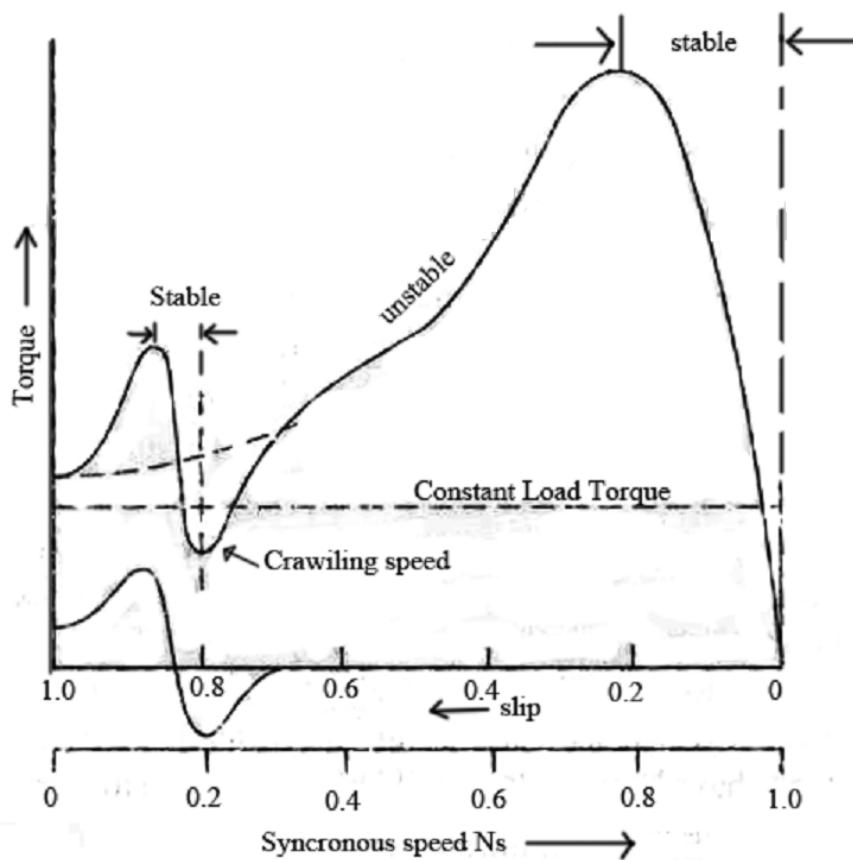


Fig: 3.40

Cogging (Magnetic Locking or Teeth Locking) of induction motor

Sometimes, the rotor of a squirrel cage induction motor refuses to start at all, particularly if the supply voltage is low. This happens especially when number of rotor teeth is equal to number of stator teeth, because of magnetic locking between the stator teeth and the rotor teeth. When the rotor teeth and stator teeth face each other, the reluctance of the magnetic path is minimum that is why the rotor tends to remain fixed. This phenomenon is called cogging or magnetic locking of induction motor.

3.20 Induction Generator

When a squirrel cage induction motor is energized from a three phase power system and is mechanically driven above its synchronous speed it will deliver power to the system. An induction generator receives its excitation (magnetizing current) from the system to which it is connected. It consumes rather than supplies reactive power (KVAR) and supplies only real power (KW) to the system. The KVAR required by the induction generator plus the KVAR requirements of all other loads on the system must be supplied from synchronous generators or static capacitors on the system.

Operating as a generator at a given percentage slip above synchronous speed, the torque, current, efficiency and power factor will not differ greatly from that when operating as a motor. The same slip below synchronous speed, the shaft torque and electric power flow is reversed. Typical speed torque characteristic of induction generator is shown in Fig: 3.41.

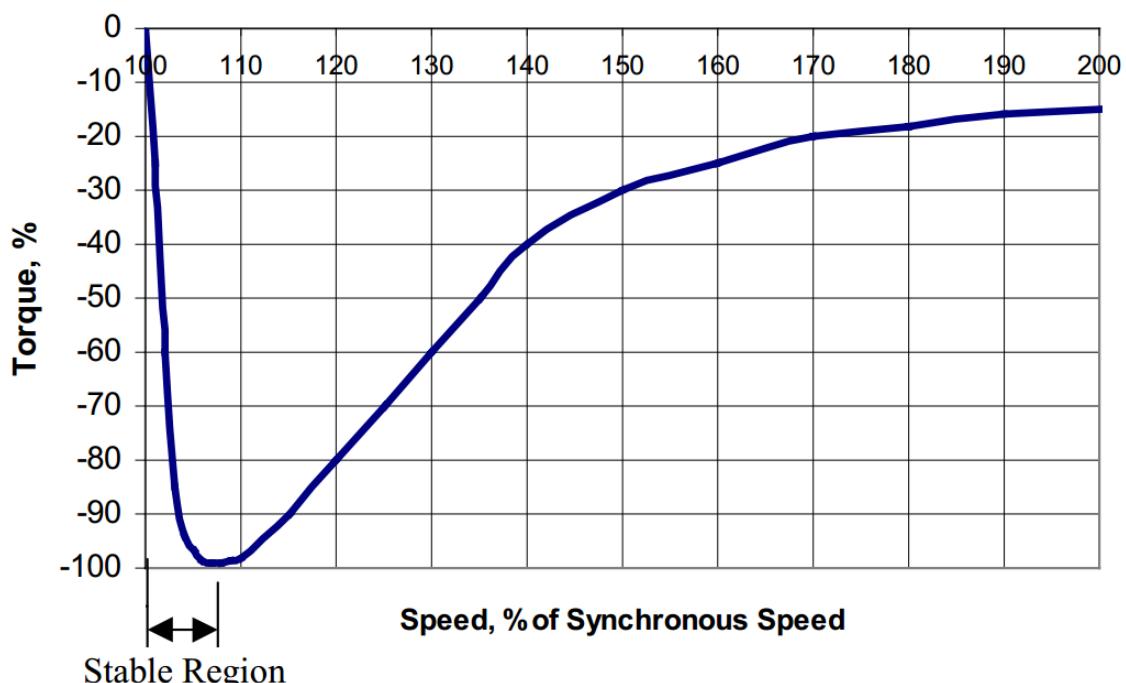


Fig: 3.41

Now for example, a 3600 RPM squirrel cage induction motor which delivers full load output at 3550 RPM as a motor will deliver full rated power as a generator at 3650 RPM. If the half-load motor speed is 3570 RPM, the output as a generator will be one-half of rated value when driven at 3630 RPM, etc. Since the induction generator is actually an induction motor being driven by a prime mover, it has several advantages.

1. It is less expensive and more readily available than a synchronous generator.
2. It does not require a DC field excitation voltage.
3. It automatically synchronizes with the power system, so its controls are simpler and less expensive.

The principal disadvantages of an induction generator are listed below

1. It is not suitable for separate, isolated operation
2. It consumes rather than supplies magnetizing KVAR
3. It cannot contribute to the maintenance of system voltage levels (this is left entirely to the synchronous generators or capacitors)
4. In general it has a lower efficiency.

Induction Generator Application

As energy costs so high, energy recovery became an important part of the economics of most industrial processes. The induction generator is ideal for such applications because it requires very little in the way of control system or maintenance.

Because of their simplicity and small size per kilowatt of output power, induction generators are also favoured very strongly for small windmills. Many commercial windmills are designed to operate in parallel with large power systems, supplying a fraction of the customer's total power needs. In such operation, the power system can be relied on for voltage & frequency control, and static capacitors can be used for power-factor correction.



MODULE-IV

SINGLE PHASE MOTORS

SYLLABUS/ TOPICS COVERED

Three Phase Induction Motors: Types, Construction and principle of operation, 3 phase Induction Motor, general phasor diagram, equivalent circuit, power and torque relations, condition for maximum torque, circle diagram, Performance characteristics, effect of rotor resistance on speed torque characteristics, stable & unstable region of operation, Operation with unbalanced supply voltage. Starting: Starting of 3 phase induction motors, high starting torque motors, speed control, rheostatic method, pole changing method cascade control of speed, Double cage induction motor, Cogging and Crawling of Induction motor, induction generator

[Topics are arranged as per above sequence]

Module -IV

4. Single Phase Induction Motors

Single phase Induction motors perform a great variety of useful services at home, office, farm, factory and in business establishments. Single phase motors are generally manufactured in fractional HP ratings below 1 HP for economic reasons. Hence, those motors are generally referred to as fractional horsepower motors with a rating of less than 1 HP. Most single phase motors fall into this category. Single phase Induction motors are also manufactured in the range of 1.5, 2, 3 and up to 10 HP as a special requirement.

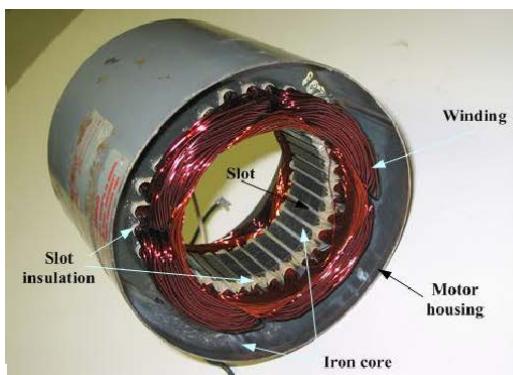


Fig: 4.1(a) Stator

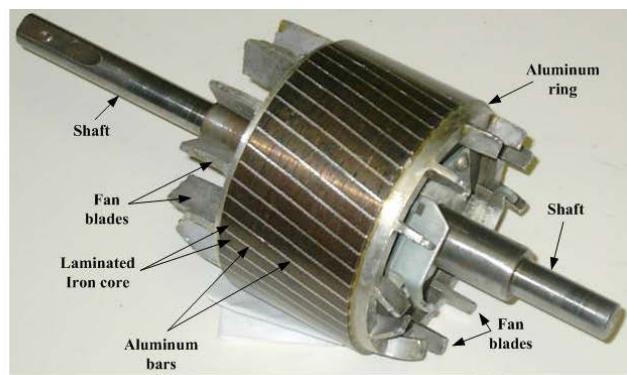


Fig: 4.1(b) Squirrel cage rotor

4.1 Theory of Operation

A single phase induction motor is similar in construction to that of a polyphase induction motor with difference that its stator has only one winding. If such a stator is supplied with single phase alternating current, the field produced by it changes in magnitude and direction sinusoidally. Thus the magnetic field produced in the air gap is alternating one but not rotating as a result these kind of motors are NOT SELF STARTING. Fig: 4.2 (a) shows the torque-speed characteristic of single phase induction motor.

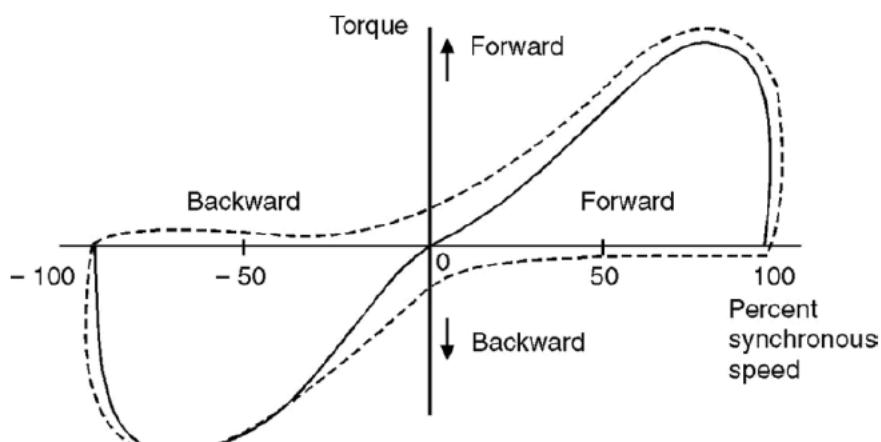


Fig: 4.2 (a)

Such an alternating field is equivalent to two fields of equal magnitude rotating in opposite directions at equal speed as explained below:

4.1.1 Double Revolving Field Theory of Single Phase Induction Motor

Consider two magnetic fields represented by quantities OA and OB of equal magnitude revolving in opposite directions as shown in fig: 4.1.

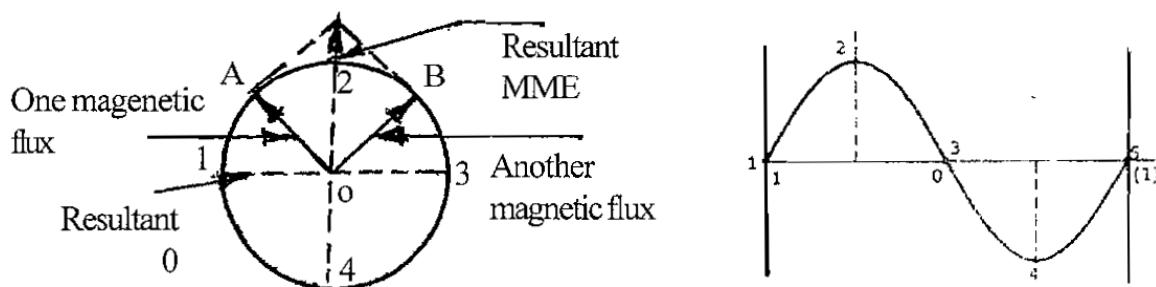


Fig: 4.2 (b)

The resultant of the two fields of equal magnitude rotating in opposite directions is alternating. Therefore an alternating current can be considered as having two components which are of equal in magnitude and rotating in opposite directions.

From the above, it is clear that when a single phase alternating current is supplied to the stator of a single phase motor, the field produced will be of alternating in nature which can be divided into two components of equal magnitude one revolving in clockwise and other in counter clockwise direction.

If a stationary squirrel cage rotor is kept in such a field equal forces in opposite direction will act and the rotor will simply vibrate and there will be no rotation.

But if the rotor is given a small jerk in any direction in this condition, it will go on revolving and will develop torque in that particular direction. It is clear from the above that a single phase induction motor when having only one winding is not a self-starting. To make it a self-starting anyone of the following can be adopted.

- (i) Split phase starting.
- (ii) Repulsion starting.
- (iii) Shaded pole starting.

4.2 EQUIVALENT CIRCUIT OF SINGLE PHASE INDUCTION MOTOR

The equivalent circuit of single phase induction motor is shown below (Fig: 4.3)

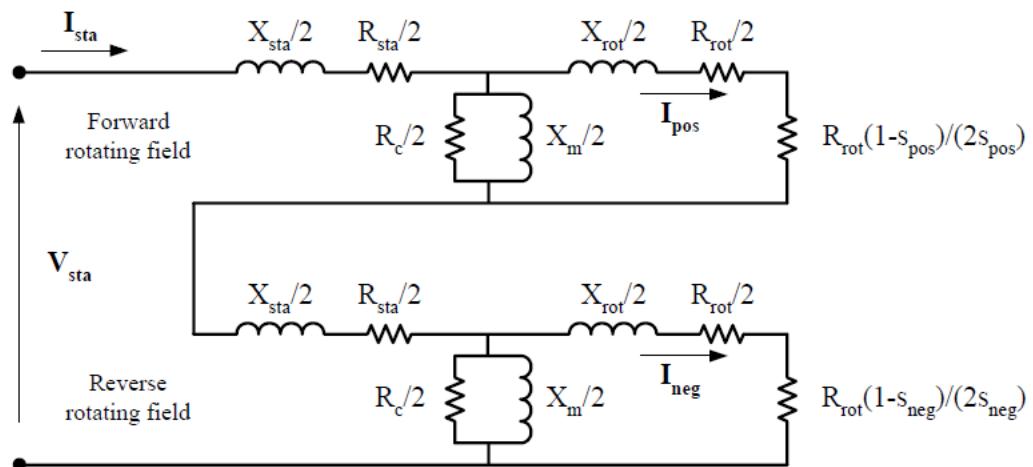


Fig: 4.3

4.2.1 Determination of Equivalent Circuit Parameters of Single Phase Induction motor

It is possible to find the parameters of the equivalent circuit of the single phase induction motor experimentally as shown in Fig.4.4. For this purpose, three tests should be conducted:

1- The DC Test:

The DC resistance of the stator can be measured by applying DC current to the terminals of the main winding and taking the reading of the voltage and the current (or using ohmmeter) and determine the DC resistance as follows:

$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

Then, the AC resistance is given by:

$$R_{AC} = 1.25 R_{DC}$$

2-The Blocked Rotor Test:

When the rotor is locked (i.e. prevented from running), $S_b = S_f = 1$. The secondary impedances become much less than the magnetizing branches and the corresponding equivalent circuit becomes that of Fig: 4.5.

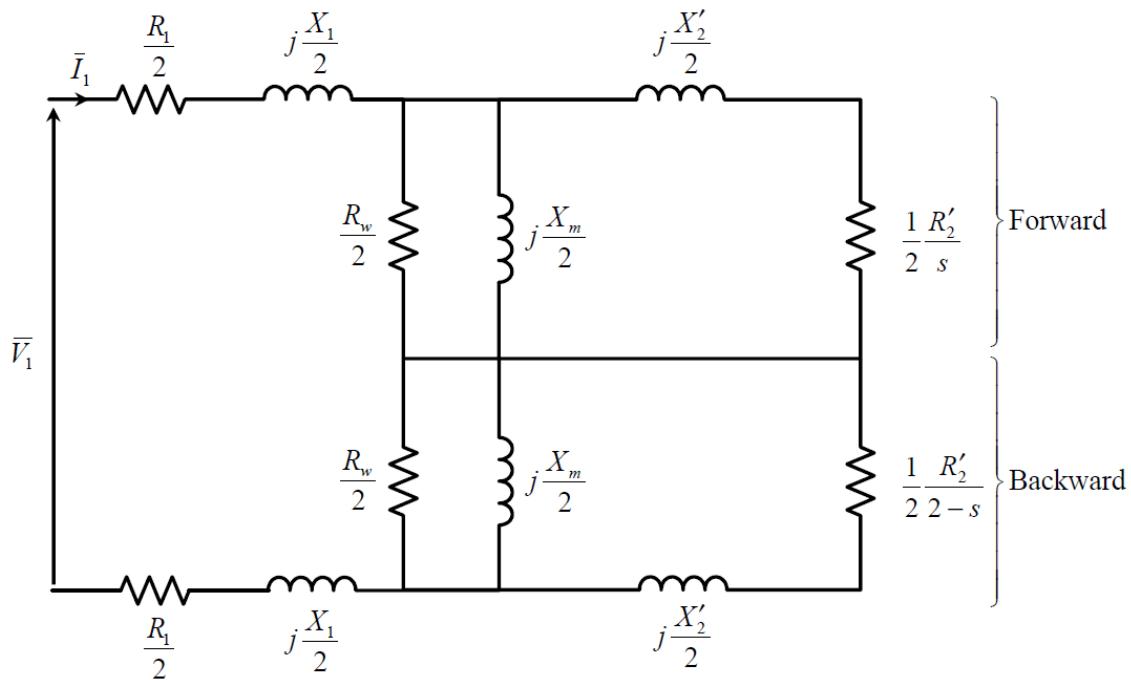


Fig: 4.4 Equivalent circuit of single phase induction motor.

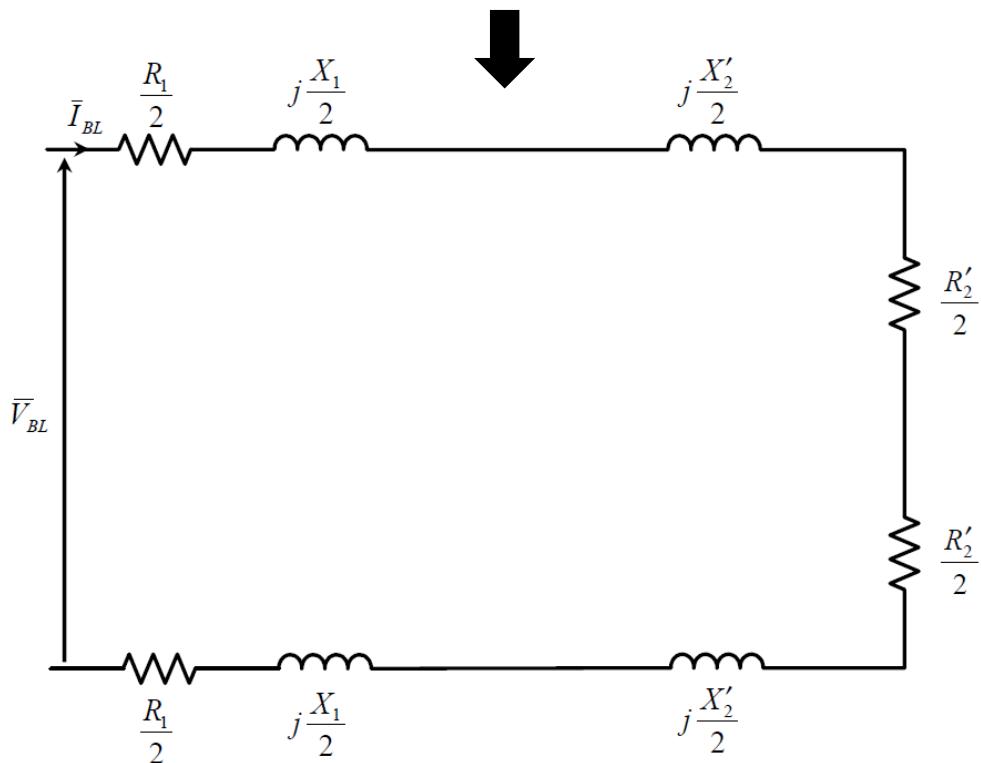


Fig: 4.5(a) Approximate equivalent circuit of the single phase induction motor at standstill.

The circuit in Fig: 4.5 (a) can be rearranged to the equivalent circuit that is shown in Fig: 4.5(b).

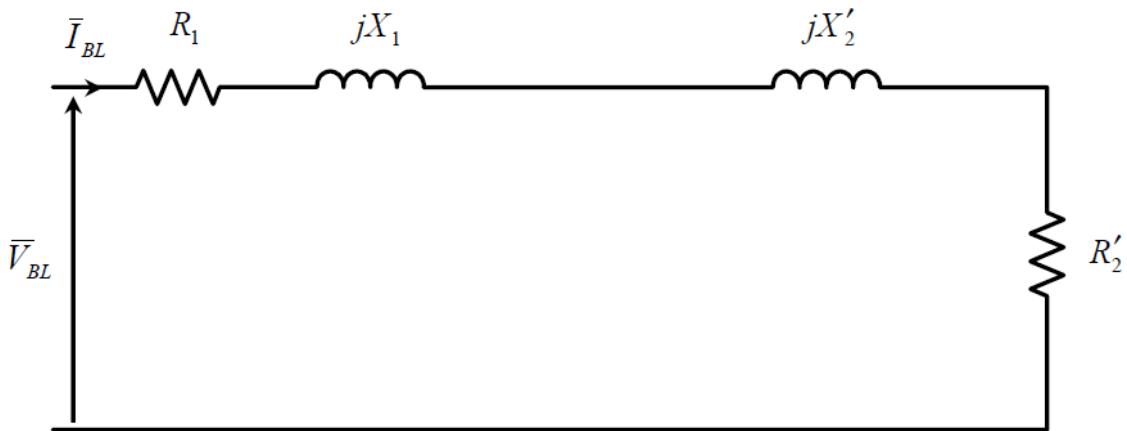


Fig: 4.5(b) Rearranged approximate equivalent circuit of the single phase induction motor at standstill.

The readings to be obtained from this test are:

- Single phase power P_{BL}
- Phase voltage V_{BL}
- Phase current I_{BL}

Then, R_{eq} , Z_{eq} , and X_{eq} can be obtained using the following equations:

$$R_{eq} = \frac{P_{BL}}{I_{BL}^2}$$

$$Z_{eq} = \frac{V_{BL}}{I_{BL}}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

Separation of X_1 , X'_2 , R_1 , and R'_2 can be done as follows:

$$X_1 = X'_2 = \frac{1}{2} X_{eq}$$

$$R'_2 = R_{eq} - R_1$$

3-The No Load Test:

When the induction motor is allowed to run freely at no load, the forward slip S_f approaches zero and the backward slip S_b approaches 2 ($S_f = s$, $S_b = 2-s$). The secondary forward impedance becomes very large with respect to the magnetizing branch, while the secondary backward impedance becomes very small if compared with the magnetizing branch. Accordingly, the equivalent circuit corresponding to these operating conditions can be approximated by that of Fig: 4.6.

The readings to be obtained from this test are:

- d) Single phase power P_{NL}
- e) Phase voltage V_{NL}
- f) Phase current I_{NL}

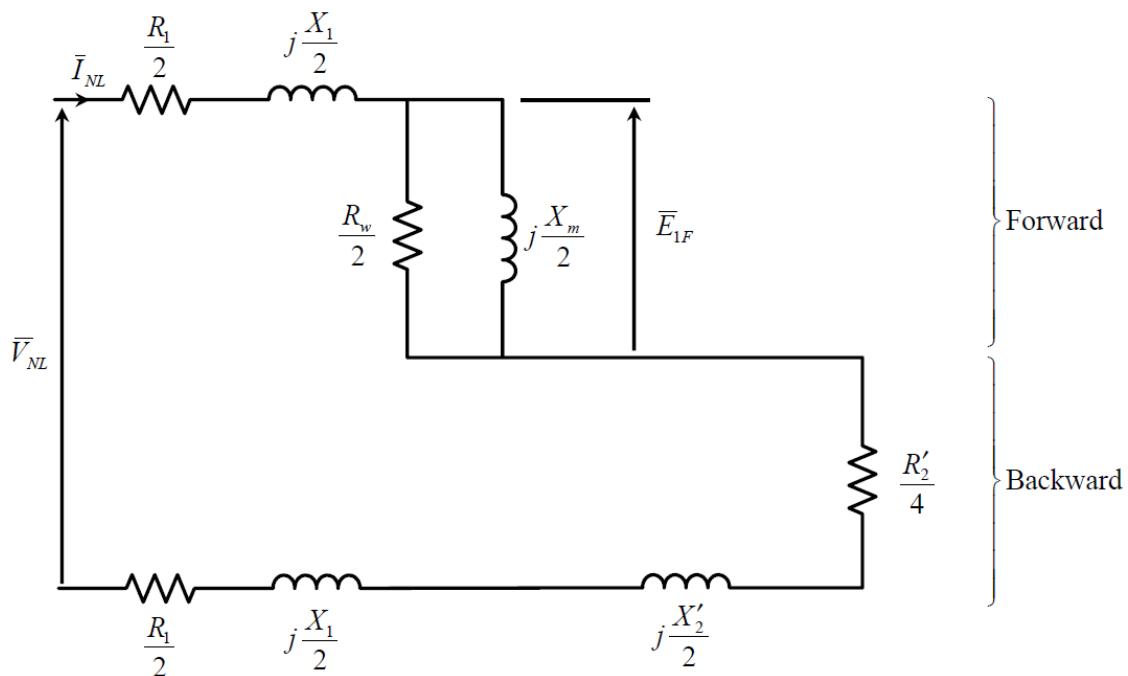
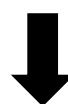


Fig: 4.6 (a) Approximate equivalent circuit of the single phase induction motor at no load.



The circuit in Fig: 4.6 (a) can be rearranged to the equivalent circuit that is shown in Fig: 4.6 (b)

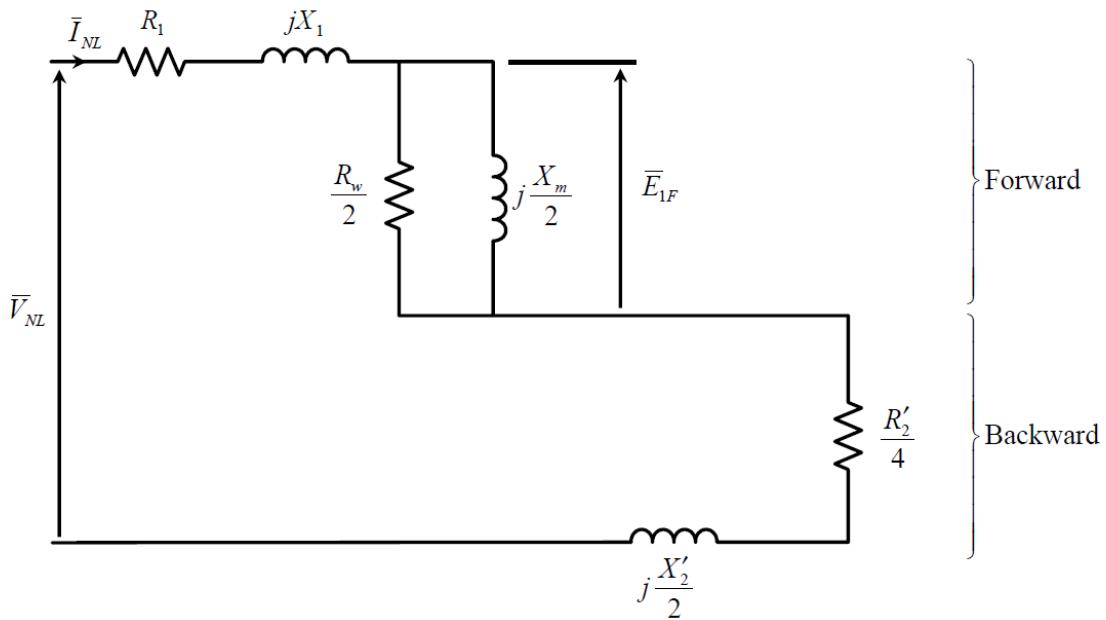


Fig: 4.6 (b) Rearranged approximate equivalent circuit of the single phase induction motor at no load

Then, R_w , and X_m , can be obtained as follows:

$$P_{core+mechanical} = P_{NL} - I_{NL}^2 \left(R_1 + \frac{R'_2}{4} \right)$$

$$\bar{E}_{1F} = \bar{V}_{NL} - \bar{I}_{NL} \left(\left(R_1 + \frac{R'_2}{4} \right) + j \left(X_1 + \frac{X'_2}{2} \right) \right)$$

Note: $(\bar{I}_{NL} = I_{NL} \angle -\theta, \quad \theta = \cos^{-1} \frac{P_{NL}}{V_{NL} I_{NL}})$

$$R_w = 2 \frac{|E_{1F}|^2}{P_{core+mechanical}}$$

$$I_w = \frac{|E_{1F}|}{\left(\frac{R_w}{2} \right)} = 2 \frac{|E_{1F}|}{R_w}$$

$$I_m = \sqrt{I_{NL}^2 - I_w^2}$$

$$X_m = 2 \frac{|E_{1F}|}{I_m}$$

4.3 Methods of Starting

It is clear from previous discussion that a single phase induction motor when having only one winding and it is not self-starting. To make it a self-starting anyone of the following can be adopted.

- (1) Split phase starting.
- (2) Repulsion starting.
- (3) Shaded pole starting.

4.3.1 PRINCIPLE OF SPLIT PHASE INDUCTION MOTOR

The basic principle of operation of a split phase induction motor is similar to that of a polyphase induction motor. The main difference is that the single phase motor does not produce a rotating magnetic field but produces only a pulsating field.

Hence, to produce the rotating magnetic field for self-starting, phase splitting is to be done to make the motor to work as a two phase motor for starting.

4.3.1 *Working of Split Phase Motor*

In split phase motor two windings named as main winding and starting winding are provided. At the time of starting, both the main and starting windings should be connected across the supply to produce the rotating magnetic field.

The rotor is of a squirrel cage type and the revolving magnetic field sweeps part the stationary rotor, inducing emf in the rotor. As the rotor bars are short-circuited, a current flows through them producing a magnetic field.

This magnetic field opposes the revolving magnetic field and will combine with the main field to produce a revolving field. By this action, the rotor starts revolving in the same direction of the rotating magnetic field as in the case of a squirrel cage induction motor.

Hence, once the rotor starts rotating, the starting winding can be disconnected from the supply by some mechanical means as the rotor and stator fields from a revolving magnetic field. There are several types of split phase motors.

4.3.2 TYPES OF SPLIT-PHASE INDUCTION MOTORS

1. Resistance-start, induction-run motors
2. Capacitor-start, induction-run motors
3. Capacitor-start, capacitor-run motors
4. Shaded pole motors.

1. RESISTANCE-START, INDUCTION-RUN MOTORS

As the starting torque of this type of motor is relatively small and its starting current is high, these motors are most commonly used for rating up to 0.5 HP where the load could be started easily. The essential parts are shown in Fig: 4.7.

- Main winding or running winding.
- Auxiliary winding or starting winding
- Squirrel cage type rotor.
- Centrifugal switch.

CONSTRUCTION AND WORKING

The starting winding is designed to have a higher resistance and lower reactance than the main winding. This is achieved by using small conductors in the auxiliary winding than in the main winding. The main winding will have higher inductance when surrounded by more iron, which could be made possible by placing it deeper into the stator slots, it is obvious that the current would split as shown in Fig: 4.7(b).

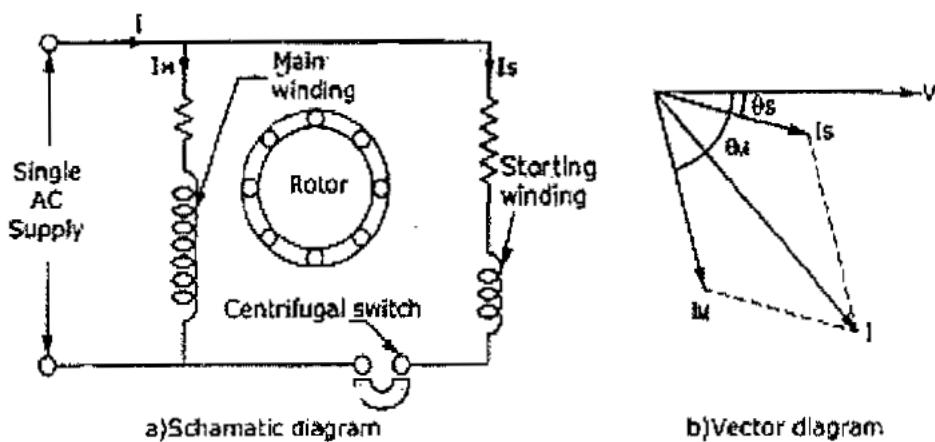


Fig: 4.7

The starting current "I" start will lag the main supply voltage "V" line by 15 degree and the main winding current "I" main lags the main voltage by about 80 degree. Therefore, these currents will differ in time phase and their magnetic fields will combine to produce a rotating magnetic field.

When the motor has come upto about 75 to 80% of synchronous speed, the starting winding is opened by a centrifugal switch and the motor will continue to operate as a single phase motor.

CHARACTERISTICS

At the point where the starting winding is disconnected, the motor develops nearly as much torque with the main winding alone as with both windings connected. This can be observed from, the typical torque-speed characteristics of this motor, as shown in Fig: 4.8.

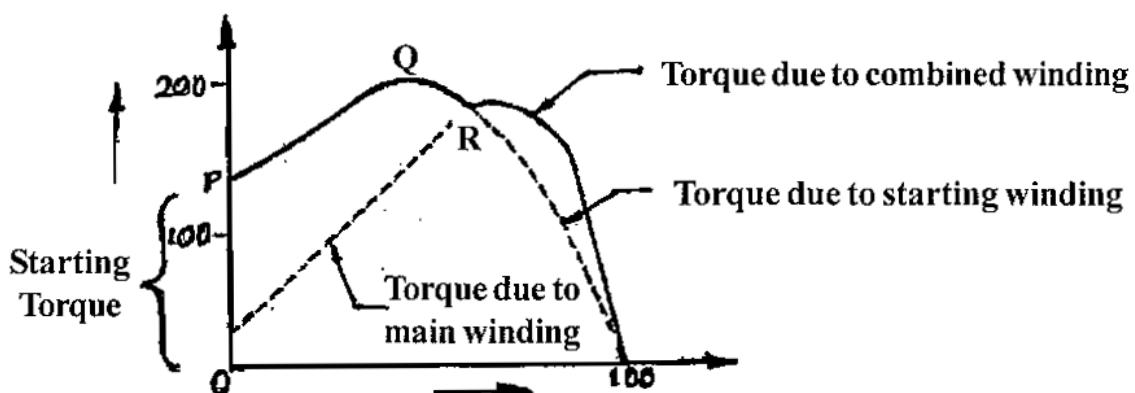


Fig: 4.8

The direction of rotating of a split-phase motor is determined by the way the main and auxiliary windings are connected. Hence, either by changing the main winding terminals or by changing the starting winding terminals, the reversal of direction of rotating could be obtained.

APPLICATIONS

These motors are used for driving fans, grinders, washing machines.

2. CAPACITOR-START, INDUCTION-RUN MOTOR

A drive which requires a large starting torque may be fitted with a capacitor-start, induction-run motor as it has excellent starting torque as compared to the resistance-start, induction-run motor.

CONSTRUCTION AND WORKING

Fig: 4.9(a) shows the schematic diagram of a capacitor-start, induction-run motor. As shown, the main winding is directly connected across the main supply whereas the starting winding is connected across the main supply through a capacitor and centrifugal switch.

Both these windings are placed in a stator slot at 90 degree electrical apart, and a squirrel cage type rotor is used.

As shown in Fig: 4.9(b), at the time of starting the current in the main winding lags the supply voltages by 90 degrees, depending upon its inductance and resistance. On the other hand, the current in the starting winding due to its capacitor will lead the applied voltage, by say 20 degrees.

Hence, the phase difference between the main and starting winding becomes near to 90 degrees. This in turn makes the line current to be more or less in phase with its applied voltage, making the power factor to be high, thereby creating an excellent starting torque.

However, after attaining 75% of the rated speed, the centrifugal switch operates opening the starting winding and the motor then operates as an induction motor, with only the main winding connected to the supply.

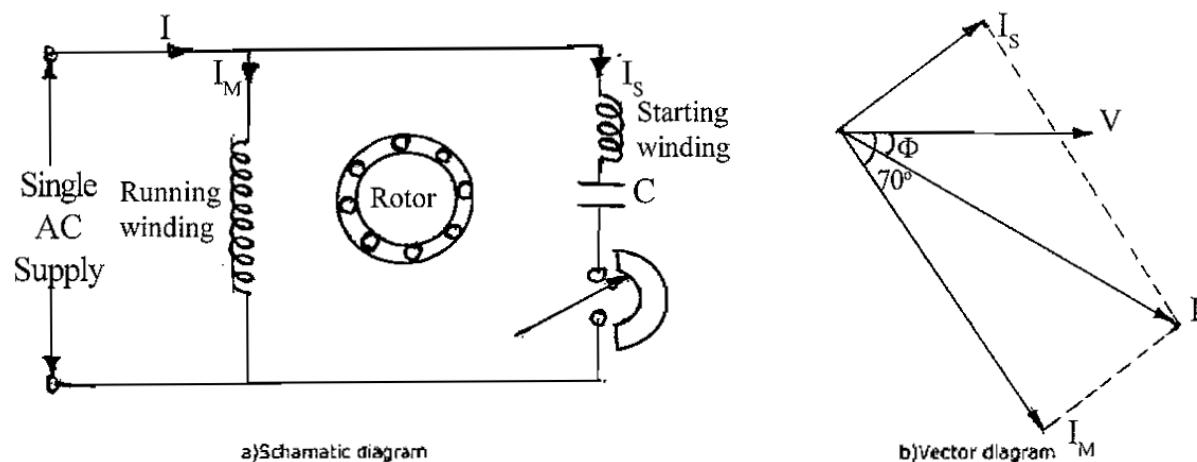


Fig: 4.9

As shown in Fig: 4.9(b), the displacement of current in the main and starting winding is about 80/90 degrees, and the power factor angle between the applied voltage and line current is very small. This results in producing a high power factor and an excellent starting torque, several times higher than the normal running torque as shown in Fig: 4.10.

CHARACTERISTICS

The torque-speed characteristics of this motor is shown in Fig: 4.10.

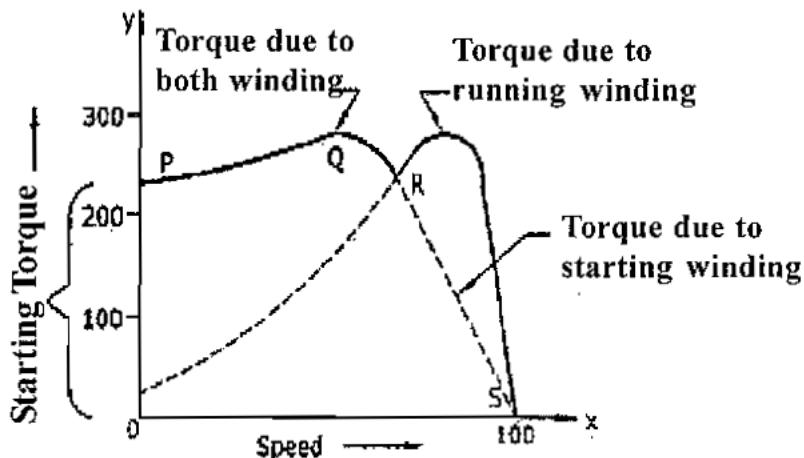


Fig: 4.10

In order to reverse the direction of rotation of the capacitor-start, induction-run motor, either the starting or the main winding terminals should be changed.

This is due to the fact that the direction of rotation depends upon the instantaneous polarities of the main field flux and the flux produced by the starting winding. Therefore, reversing the polarity of one of the field will reverse the torque.

APPLICATIONS

Due to the excellent starting torque and easy direction-reversal characteristics,

- Used in belted fans,
- Used in blowers dryers,
- Used in washing machines,
- Used in pumps and compressors.

3. CAPACITOR-START, CAPACITOR-RUN MOTORS

As discussed earlier, one capacitor-start, induction-run motors have excellent starting torque, say about 300% of the full load torque and their power factor during starting is high.

However, their running torque is not good, and their power factor, while running is low. They also have lesser efficiency and cannot take overloads.

CONSTRUCTION AND WORKING

The aforementioned problems are eliminated by the use of a two valve capacitor motor in which one large capacitor of electrolytic (short duty) type is used for starting whereas a smaller

capacitor of oil filled (continuous duty) type is used for running, by connecting them with the starting winding as shown in Fig:4.11. A general view of such a two valve capacitor motor is shown in Fig: 4.11.

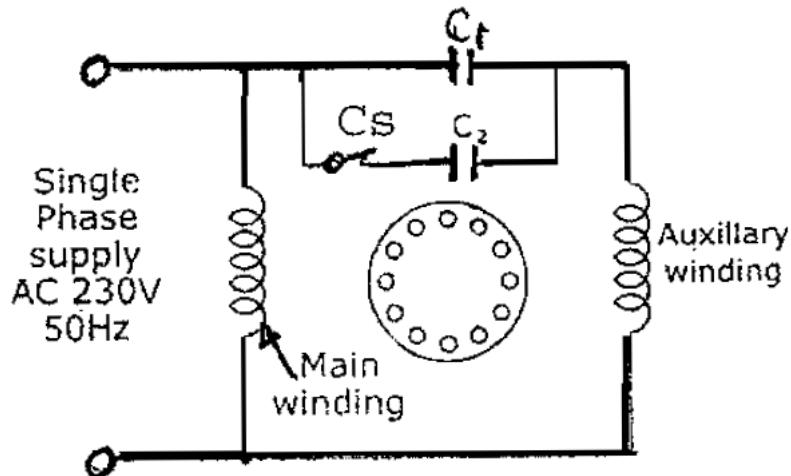


Fig: 4.11

This motor also works in the same way as a capacitor-start, induction-run motor, with exception, that the capacitor C_1 is always in the circuit, altering the running performance to a great extent.

The starting capacitor which is of short duty rating will be disconnected from the starting winding with the help of a centrifugal switch, when the starting speed attains about 75% of the rated speed.

CHARACTERISTICS

The torque-speed characteristics of this motor is shown in Fig: 4.12.

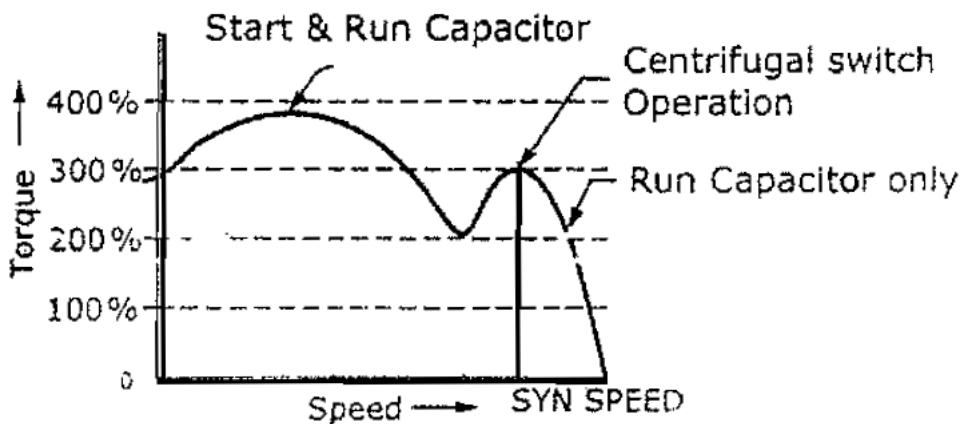


Fig: 4.12

This motor has the following advantages:

- The starting torque is 300% of the full load torque
- The starting current is low, say 2 to 3 times of the running current.
- Starting and running power factor are good.
- Highly efficient running.
- Extremely noiseless operation.
- Can be loaded upto 125% of the full load capacity.

APPLICATIONS

- Used for compressors, refrigerators, air-conditioners, etc.
- Higher starting torque.
- High efficiency, higher power factor and overloading.
- Costlier than the capacitor-start — Induction run motors of the same capacity.

4.3.2 REPULSION STARTING

This type of starting need a wound rotor with brush and commutator arrangement like a dc armature Fig 4.13(a). The starting operation is based on the principle of repulsion and hence the name.

CONSTRUCTION AND WORKING

Repulsion starting, though complicated in construction and higher in cost, are still used in certain industries due to their excellent starting torque, low starting current, ability to withstand long spell of starting currents to drive heavy loads and their easy method of reversal of direction.

Now there is a condition that the rotor north pole will be repelled by the main north pole and the rotor south pole is repelled by the main south pole, so that a torque could be developed in the rotor. Now due to the repulsion action between the stator and the rotor poles, the rotor will start rotating in a clockwise direction. As the motor torque is due to repulsion action, this starting method is named as repulsion starting.

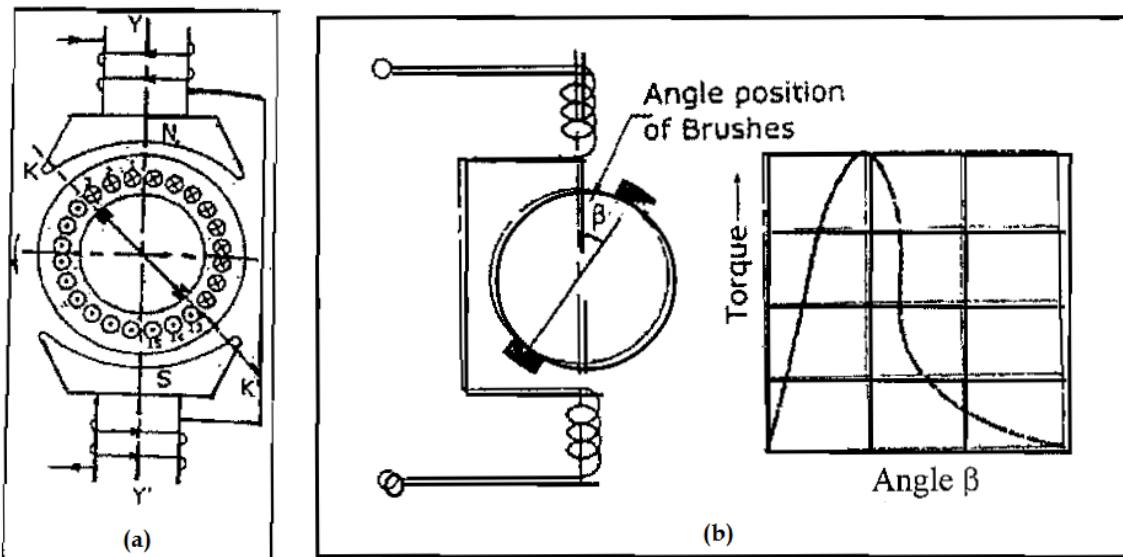


Fig: 4.13

To change the direction of rotation of this motor, the brush axis needs to be shifted from the right side as shown in Fig:4.13(b) to the left side of the main axis in a counter clockwise direction as shown in Fig:4.13(b).

CHARACTERISTICS

The torque developed in a repulsion motor will depend upon the amount of brush shaft as shown in Fig: 4.13 (b), whereas the direction of shift decides the direction of rotation.

Further, the speed depends upon the amount of brush shift and the magnitude of the load also on the relationship between the torque and brush-position angle.

Though the starting torque from 250 to 400% of the full load torque, the speed will be dangerously high during light loads. This is due to the fact that the speed of the repulsion motor start does not depend on frequency or number of poles but depends upon the repulsion principle.

Further, there is a tendency of sparking in the brushes at heavy loads, and the PF will be poor at low speeds. Hence the conventional repulsion motor start is not much popular.

4.3.3 SHAPED POLE STARTING

The motor consists of a yoke to which salient poles are fitted as shown in Fig: 4.14(a) and it has a squirrel cage type rotor.

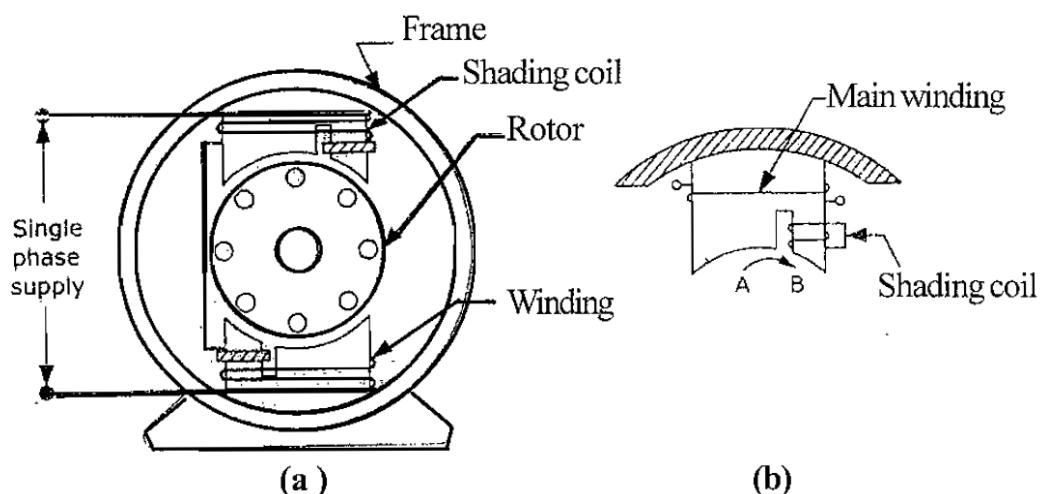


Fig: 4.14

A shaded pole made of laminated sheets has a slot cut across the lamination at about one third the distance from the edge of the pole.

Around the smaller portion of the pole, a short-circuited copper ring is placed which is called the shading coil, and this part of the pole is known as the shaded part of the pole. The remaining part of the pole is called the unshaded part which is clearly shown in Fig: 4.14(b).

Around the poles, exciting coils are placed to which an AC supply is connected. When AC supply is effected to the exciting coil, the magnetic axis shifts from the unshaded part of the pole to the shaded part as will be explained in details in the next paragraph. This shifting of axis is equivalent to the physical movement of the pole.

This magnetic axis, which is moving, cuts the rotor conductors and hence, a rotational torque is developed in the rotor.

By this torque the rotor starts rotating in the direction of the shifting of the magnetic axis that is from the unshaded part to the shaded part.

THE MAGNETIC FLUX SHIFTING

As the shaded coil is of thick copper, it will have very low resistance but as it is embedded in the iron case, it will have high inductance. When the exciting winding is connected to an AC supply, a sine wave current passes through it.

Let us consider the positive half cycle of the AC current as shown in Fig: 4.15.

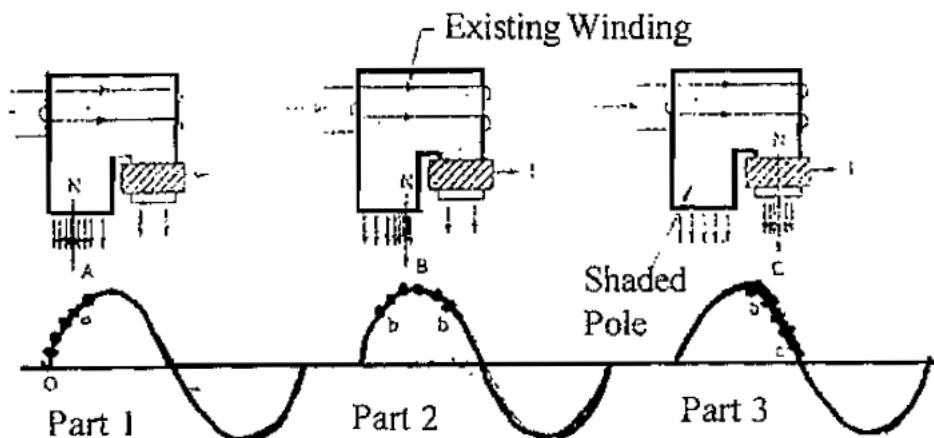


Fig: 4.15 Shifting of magnetic flux

When the current raises from "Zero" Value of point "0" to a point "a" the change in current is very rapid (Fast). Hence, it reduces an emf in the shaded coil on the basis of Faraday's law of electromagnetic induction.

The induced emf in the shaded coil produces a current which, in turn, produces a flux in accordance with Lenz Law. This induced flux opposes the main flux in the shaded portion and reduces the main flux in that area to a minimum value as shown in Fig: 4.15.

This makes the magnetic axis to be in the centre of the unshaded portion as shown by the arrow in part of Fig: 4.15. On the other hand as shown in part 2 of 3 when the current raises from point "a" to point "b" the change in current is slow the induced emf and resulting current in the shading coil is minimum and the main flux is able to pass through the shade portion.

This makes the magnetic axis to be shifted to the centre of the whole pole as shown in by the arrow in part 2 of Fig: 4.15.

In the next instant, as shown in part 3 of Fig: 4.15. When the current falls from "b" to "c" the change in current is fast but the change of current is from maximum to minimum.

Hence a large current is induced in the shading ring which opposes the diminishing main flux, thereby increasing the flux density in the area of the shaded part. This makes the magnetic axis to shift to the right portion of the shaded part as shown by the arrow in part.

From the above explanation it is clear the magnetic axis shifts from the unshaded part to the shaded part which is more or less a physical rotary movement of the poles.

Simple motors of this type cannot be reversed. Specially designed shaded pole motors have been constructed for reversing operations. Two such types:

- a. The double set of shading coils method
- b. The double set of exciting winding method.

Shaded pole motors are built commercially in very small sizes, varying approximately from 1/250 HP to 1/6 HP. Although such motors are simple in construction and cheap, there are certain disadvantages with these motor as stated below:

- Low starting torque.
- Very little overload capacity.
- Low efficiency.

APPLICATIONS

- Record players
- Fans
- Hair dryers.

4.4 Single Phase Series Motor

The single-phase series motor is a commutator-type motor. If the polarity of the line terminals of a dc series motor is reversed, the motor will continue to run in the same direction. Thus, it might be expected that a dc series motor would operate on alternating current also. The direction of the torque developed in a dc series motor is determined by both field polarity and the direction of current through the armature [$T \propto \phi I_a$].

4.4.1 Operation

Let a dc series motor be connected across a single-phase ac supply. Since the same current flows through the field winding and the armature, it follows that ac reversals from positive to negative, or from negative to positive, will simultaneously affect both the field flux polarity and the current direction through the armature. This means that the direction of the developed torque will remain positive, and rotation will continue in the same direction. Thus, a series motor can run both on dc and ac.

However, a series motor which is specifically designed for dc operation suffers from the following drawbacks when it is used on single-phase ac supply:

1. Its efficiency is low due to hysteresis and eddy-current losses.
2. The power factor is low due to the large reactance of the field and the armature winding.
3. The sparking at the brushes is excessive.

In order to overcome these difficulties, the following modifications are made in a D.C. series motor that is to operate satisfactorily on alternating current:

1. The field core is constructed of a material having low hysteresis loss. It is laminated to reduce eddy-current loss.
2. The field winding is provided with small number of turns. The field-pole areas is increased so that the flux density is reduced. This reduces the iron loss and the reactive voltage drop.
3. The number of armature conductors is increased in order to get the required torque with the low flux.
4. In order to reduce the effect of armature reaction, thereby improving commutation and reducing armature reactance, a compensating winding is used.

The compensating winding is put in the stator slots. The axis of the compensating winding is 90 (electrical) with the main field axis. It may be connected in series with both the armature and field as shown in Fig: 4.16. In such a case the motor is conductively compensated.

The compensating winding may be short circuited on itself, in which case the motor is said to be inductively compensated shown in Fig: 4.17.

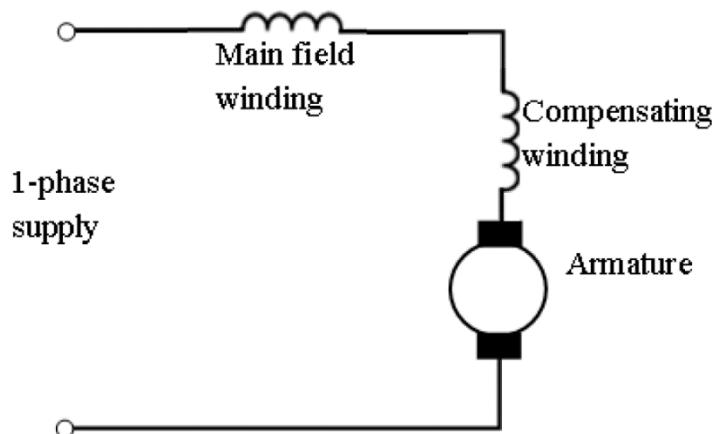


Fig: 4.16

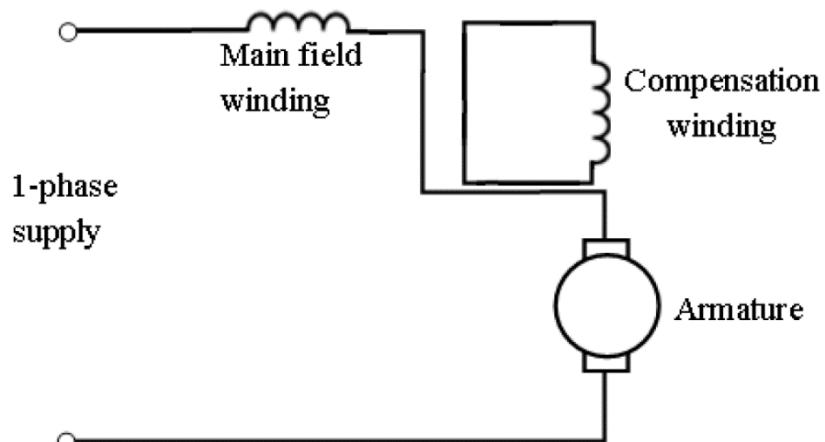


Fig: 4.17

The characteristics of single-phase series motor are very much similar to those of D.C. series motors, but the series motor develops less torque when operating from an a.c. supply than when working from an equivalent D.C. supply [Fig: 4.18]. The direction of rotation can be changed by interchanging connections to the field with respect to the armature as in D.C. series motor.

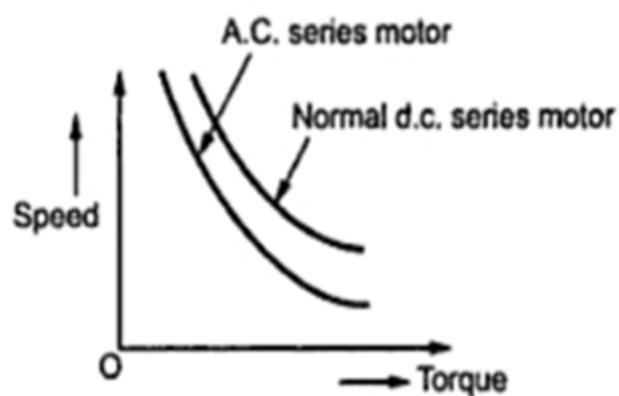


Fig: 4.18

Speed control of universal motors is best obtained by solid-state devices. Since the speed of these is not limited by the supply frequency and may be as high as 20,000 r.p.m. (greater than the maximum synchronous speed of 3000 r.p.m. at 50 Hz), they are most suitable for applications requiring high speeds.

4.4.2 Phasor Diagram of A.C Series Motor

The schematic diagram and phasor diagram for the conductively coupled single-phase ac series motor are shown in Fig: 4.19 and Fig: 4.20 respectively.

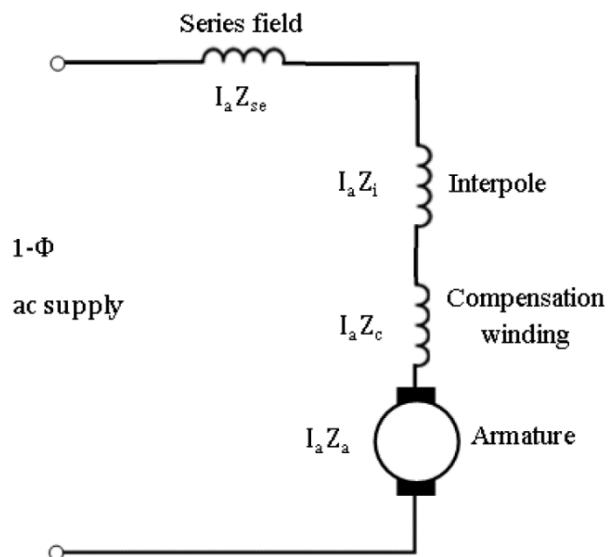


Fig: 4.19

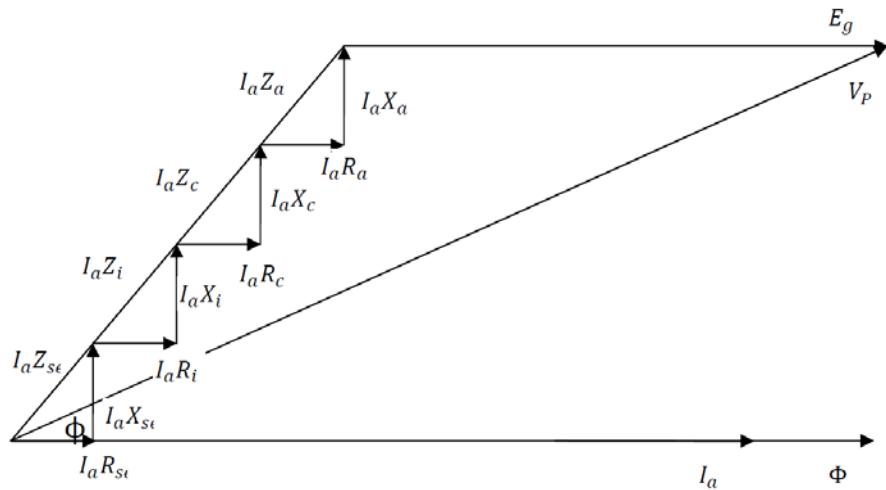


Fig: 4.20

The resistance $I_a R_{se}$, $I_a R_i$, $I_a R_c$ and $I_a R_a$ drops are due to resistances of series field, interpole winding, compensating winding and of armature respectively are in phase with armature current I_a . The reactance drops $I_a X_{se}$, $I_a X_i$, $I_a X_c$ and $I_a X_a$ are due to reactance of series field, interpole winding, compensating winding and of armature respectively lead current I_a by 90° . The generated armature counter emf is E_g . The terminal phase voltage V_p is equal to the phasor sum of E_g and all the impedance drops in series.

$$V_p = E_g + I_a Z_{se} + I_a Z_i + I_a Z_c + I_a Z_a$$

The power factor angle between V_p and I_a is .

4.4.3 Applications

There are numerous applications where single-phase ac series motors are used, such as hair dryers, grinders, table-fans, blowers, polishers, kitchen appliances etc. They are also used for many other purposes where speed control and high values of speed are necessary.

4.5 Schrage Motor

Schrage motor is basically an inverted polyphase induction motor, with primary winding on the rotor and secondary winding on the stator. The primary winding on the rotor is fed through three slip rings and brushes at line frequency; secondary winding on the stator has slip frequency voltages induced in it.

The speed and power factor of slip ring induction motor can be controlled by injecting slip frequency voltage in the rotor circuit. If resultant rotor voltage increases, current increases, torque increases and speed increases. Depending on the phase angle of injected voltage, power factor can be improved. In 1911, K. H. Schrage of Sweden combined elegantly a SRIM (WRIM) and a frequency converter into a single unit.

4.5.1 Construction and Operation

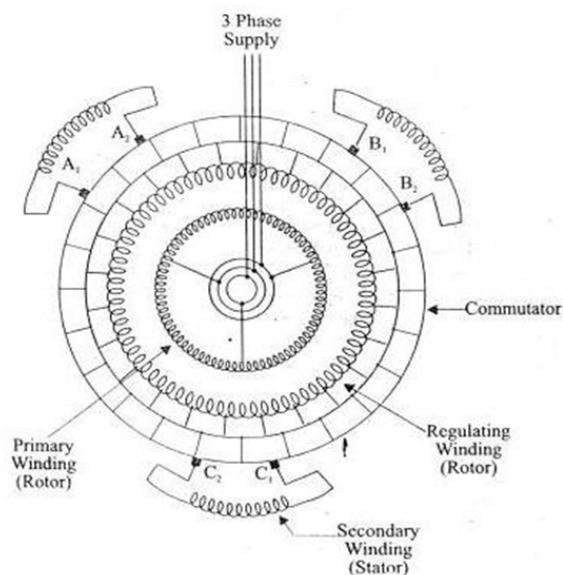


Fig: 4.21

Schrage motor has three windings- Two in Rotor and One in Stator.

Primary winding: Placed on the lower part of the slots of the Rotor. Three phase supply at line frequency is fed through slip rings and brushes which generates working flux in the machine.

Regulating winding: Placed on the upper part of the slots of the Rotor. These are connected to commutator segments in a manner similar to that of D.C. machine. Regulating windings are also known as *tertiary winding / auxiliary winding / commutator winding*.

Secondary winding: Same is phase wound & located on stator. Each winding is connected to a pair of brushes arranged on the commutator. Brushes are mounted on brush rockers. These are designed to move in opposite directions, relative to the centre line of its stator phase.

Brushes A₁, B₁ & C₁ move together and are 120° apart.

Brushes A₂, B₂ & C₂ also move together and are 120° apart.

Now the primary energized with line frequency voltage. Transformer action occurs between primary and regulating winding. Induction motor action occurs between primary and secondary windings. Commutator acting as a frequency converter converts line frequency voltage of regulating winding to slip frequency voltage and feeds the same to secondary winding on the stator.

Voltage across the brush pairs A₁ - A₂, B₁ - B₂ & C₁ - C₂ increases as brushes are separated.

Magnitude of voltage injected into the secondary winding depends on the angle of separation ‘θ’ of the brushes A₁ & A₂, B₁ & B₂, C₁ & C₂. (‘θ’ – Brush separation angle).

When primary is energized synchronously rotating field in clockwise direction is set up in the rotor core. Assume that the brushes are short circuited through commutator segment i.e. the secondary is short circuited. Rotor still at rest, the rotating field cuts the stationary secondary winding, induces an e.m.f. The stator current produce its own field. This stator field reacts with the rotor field thus a clockwise torque produced in the stator. Since the stator cannot rotate, as a reaction, it makes the rotor rotate in the counter clockwise direction.

Suppose that the rotor speed is N_r rpm. Rotor flux is rotating with N_s relative to primary & regulating winding. Thus the rotor flux will rotate at slip speed (N_s - N_r) relative to secondary winding in stator with reference to space.

4.5.2 Speed Control

Speed of Schrage Motor can be obtained above and below Synchronous speed by changing the Brush position i.e. changing “ θ ” (θ – Brush separation angle).

In Fig: 4.22 (a) Brush pair on the same commutator segment i.e. the secondary winding short circuited. Thus the Injected voltage $E_j = 0$ and the machine operates as an Inverted Induction Motor so here $N_r < N_s$.

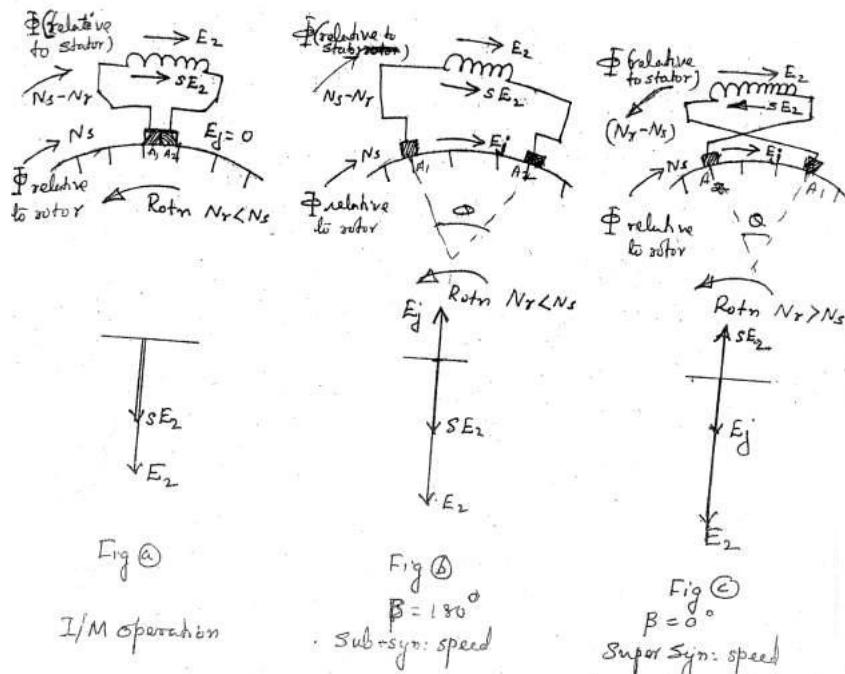


Fig: 4.22 (a) ,(b) & (c)

In Fig: 4.22 (b) Brushes parted in one direction which produces sub-synchronous speed. Injected voltage E_j , is obtained from the section of the regulating winding between them. If the centre line of this group of conductors is coincident with the centre line of the corresponding secondary phase, then E_2 and E_j are in phase opposition.

Neglecting impedance drop, sE_2 must be equal and opposite of E_j .

“ β ” is the angle between E_2 and E_j . $\beta=180^\circ$ and so here also $N_r < N_s$.

In Fig: 4.22 (c) Brushes parted in opposite direction which produces super-synchronous speed. Here E_j is reversed relative to E_2 i.e. $\beta=0^\circ$ & sE_2 must also be reversed.

This is occurring only because ‘ s ’ becoming negative i.e. The speed is thus above synchronous speed so $N_r > N_s$.

The commutator provides maximum voltage when the brushes are separated by one pole pitch. i.e. ‘ θ ’ = 180° .

4.5.3 Power Factor Improvement

This can be obtained by changing the phase angle of the injected voltage into the secondary winding. In this case one set of brushes is advanced more rapidly than the other set. Now the two centre lines do not coincide, have an angle ' ρ ' between them. (" ρ " – Brush shift angle).

In Fig: 4.22 (d) Brush set is moved against the direction of rotation of rotor. In this case Speed decreases and the p.f. is improved.

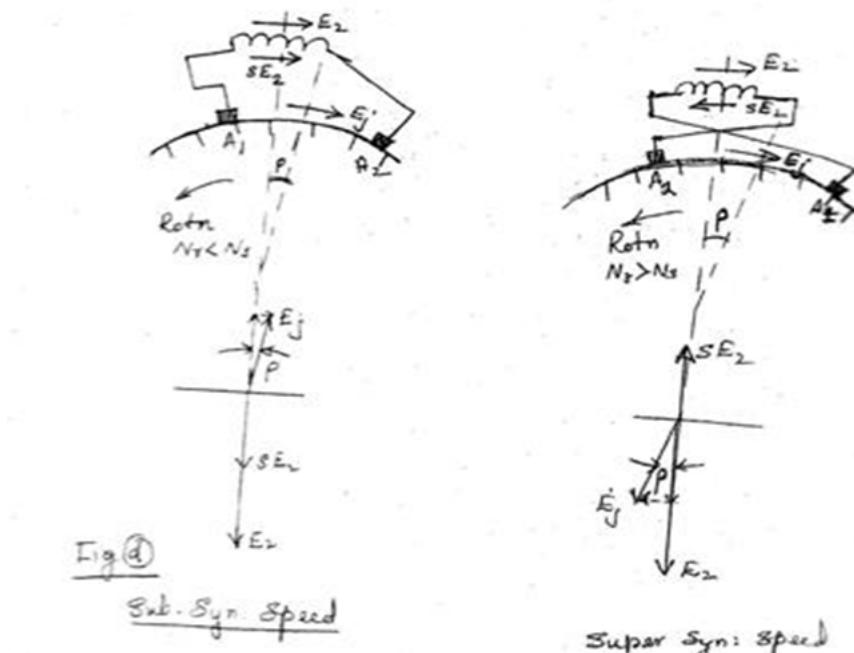


Fig: 4.22 (d) & (e)

In Fig: 4.22 (e) Brush set is moved in the same direction of rotation of rotor. In this case Speed increases, the p.f. is also improved.

Both p.f. and speed can be controlled by varying ' θ ' & ' ρ '.

Thus ' $E_J \cos \rho$ ' and ' $E_J \sin \rho$ ' effect the speed and p.f. respectively. Fig: 4.23 show Variation of no load speed with Brush Separation.

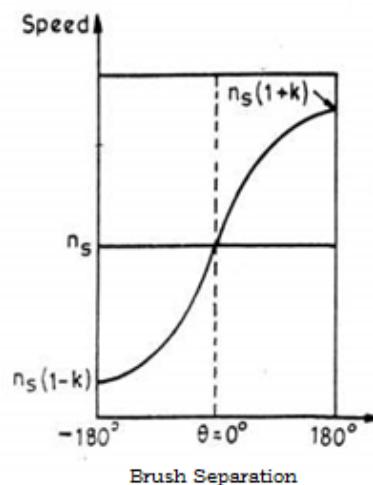


Fig: 4.23

4.5.4 Speed Torque Characteristics

Above discussion reveals that the Schrage Motor is almost a constant speed motor i.e. it has D.C Shunt motor characteristics. Figure 4.23 shows the typical speed-torque characteristics of Schrage motor.

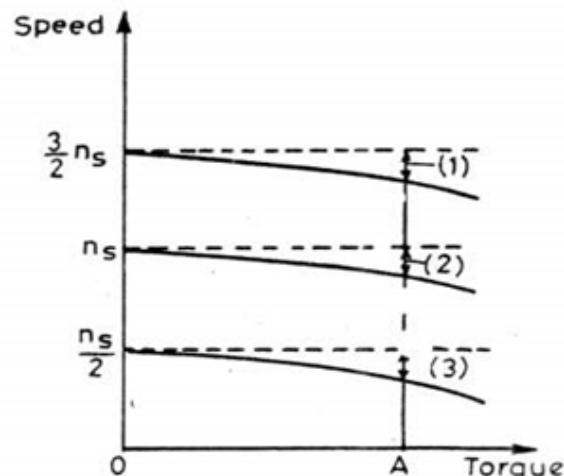


Fig: 4.23

4.5.5 Advantages & Shortcomings

Advantages:

- (i) Good Speed Regulation.
- (ii) High p.f. for high speed setting.

- (iii) High efficiency at all speeds except N_s

Shortcomings:

- (i) Operating voltage has to be limited to 700V because the power is to be supplied through slip rings.
- (ii) Low p.f. at low speed settings.
- (iii) Poor commutation.
- (iv) High Cost.

4.5.6 Applications

Can be applied to any individual drive requiring variable speed, especially in knitting & Ring spinning applications, Cranes & Hoists Fans & Centrifugal Pumps, printing Machinery Conveyors, Packing machinery & Paper Mills etc.

4.6 Universal Motors

It is also commutator type motor. A universal motor is one which operates both on AC and DC supplies. It develops more horsepower per Kg. weight than any other AC motor mainly due to its high speed.

The principle of operation is the same as that of a DC motor. Though a universal motor resembles a DC series motor, it required suitable modification in the construction, winding and brush grade to achieve sparkless commutation and reduced heating when operated on AC supply, due to increased inductance and armature reaction.

A universal motor could therefore be defined as a series or a compensated series motor [Fig: 4.24 & Fig: 4.25 (a), (b)]designed to operate at approximately the same speed and output at either direct current or single phase alternating current of a frequency not greater than 50Hz, and of approximately the same RMS voltage. Universal motor is also named as AC single phase series motor.



Fig: 4.24

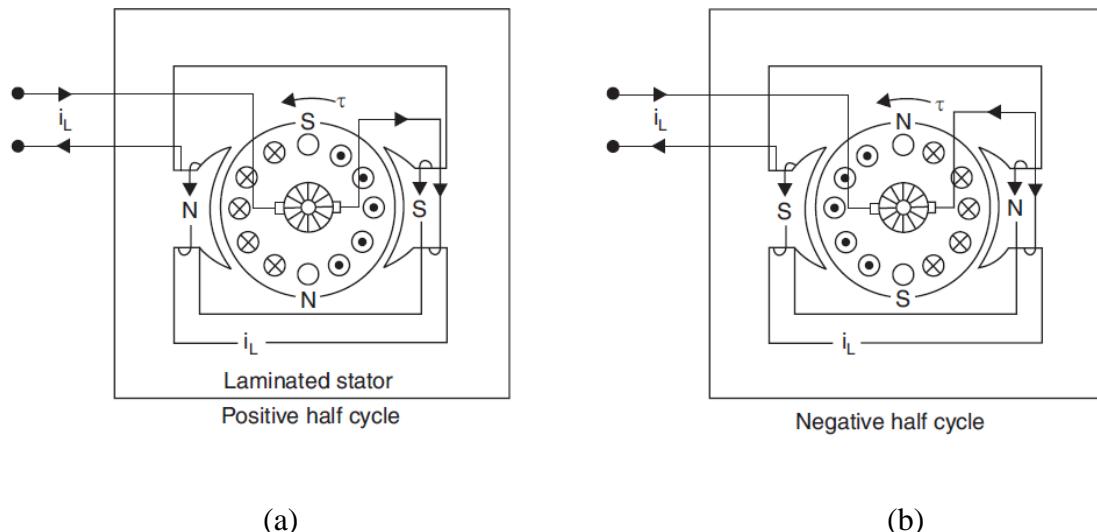


Fig: 4.25

The main parts of a universal motor are an armature, field winding, stator stampings, frame and plates and brushed. The increased sparking at the brush position in AC operation is reduced by the following means:

Providing commutating inter poles in the stator and connecting the interpole winding in series with the armature winding. Providing high contact resistance brushed to reduce sparking at brush positions.

4.6.1 Operation

A universal motor works on the same principles as a DC motor i.e. force is created on the armature conductors due to the interaction between the main field flux and the flux created by the current carrying armature conductors. A universal motor develops unidirectional torque regardless of whether it operated on AC or DC supply.

Fig: 4.25 (a),(b) & Fig: 4.26 shows the operation of a universal motor on AC supply. In AC operation, both field and armature currents change their polarities, at the same time resulting in unidirectional torque.

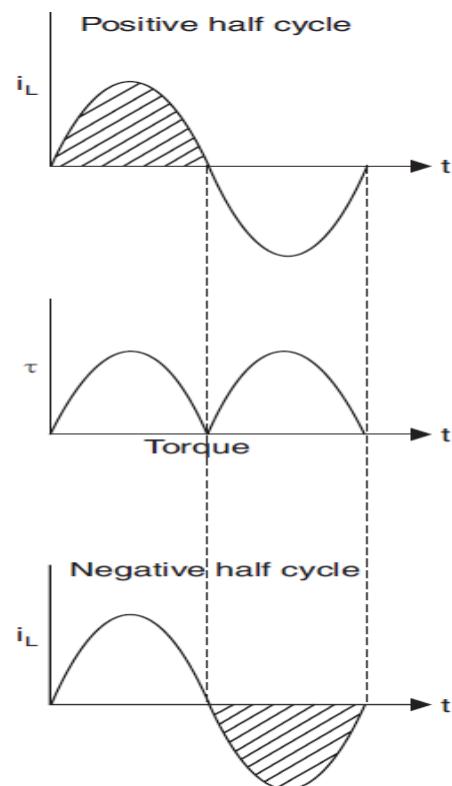


Fig: 4.26

4.6.2 Characteristic

The speed of a universal motor inversely proportional to the load i.e. speed is low at full load and high, on no load.

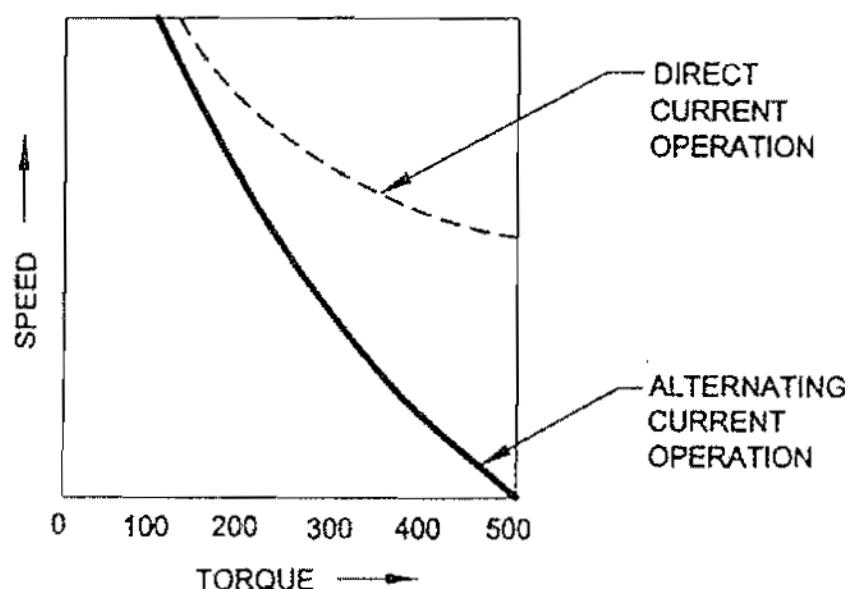


Fig: 4.27

The speed reaches a dangerously high value due to low field flux at no loads in fact the no load speed is limited only by its own friction and windage losses. As such these motors are connected with permanent loads or gear trains to avoid running at no load thereby avoiding high speeds.

Fig: 4.27 shows the typical torque-speed relation of a universal motor, both for AC and DC operations. This motor develops about 450 % of full load torque at starting, as such higher than any other type of single phase motor.

4.6.3 Applications

There are numerous applications where universal motors are used, such as hand drills, hair dryers, grinders, blowers, polishers, and kitchen appliances etc. They are also used for many other purposes where speed control and high values of speed are necessary like in vacuum cleaners, food mixers, portable drills and domestic sewage machines. Universal motors of a given horse power rating are significantly smaller than other kinds of a.c. motors operating at the same frequency.

Acknowledgement

The committee members gratefully acknowledge google, scribd, NPTEL, openoffice, sumatra pdf, scilab for myriad suggestions and help for preparing this lecture note. The committee members also wants to express their gratitude to the persons out there who think knowledge should be free and be accessible and sharable without any restrictions so that every single person on this planet has the same opportunity to explore, expand and become enlightened by the collective gifts of humankind.

However apart from this lecture note students/readers are strongly recommended to follow the below mentioned books and above all confer with the concern faculty for thorough knowledge of this authoritative subject of electrical engineering.

Text / Reference Books

1. *Electrical Machinery* [7th Ed.]
by P. S. Bimbhra

Publisher- Khanna Publisher

2. *Generalized Theory of Electrical Machines* [2nd Ed.]
by P.S. Bimbhra

Publisher- Khanna Publisher

3. *The Performance and Design of Alternating Current Machines* [3rd Ed.] by M. G. Say

Publisher- CBS Publisher

4. *Electric Machinery* [6th Ed.]
by A. E. Fitzgerald, Stephen D. Umans, Charles Kingsley Jr.

Publisher- McGraw Hill Education (India)
Private Limited

5. *Electric Machinery Fundamentals* [5th Ed.]
by Stephen J. Chapman

Publisher- McGraw Hill Education (India)
Private Limited

6. *Theory of Alternating Current Machinery* [2nd Ed.]
by Alexander S Langsdorf

Publisher- McGraw Hill Education (India)
Private Limited

7. *Alternating- Current Machines* [3rd Ed.]
by T.C. Lloyd and A.G. Conrad Puchstein A.F.

Publisher- John Wiley & Sons

8. *Electric Machinery and Transformers* [2nd Ed.]
by Irving Kosow

Publisher- Pearson India

9. *Analysis of Electric Machine* [2nd Ed.]
by Kraus, P.C.

Publisher- McGraw Hill Education (India)
Private Limited

10. *A Course in Electrical Engineering Vol.-I & Vol.-II*
by Chester L. Dawes, S. B.

Publisher- McGraw Hill Book Company Inc.

11. *Principles of Alternating-Current Machinery* [4th Ed.]
by R R Lawrence & H.E Richards

Publisher- McGraw Hill Book Company Inc.

12. *Electrical Machines* [2nd Ed.]
by P. K. Mukherjee & S. Chakravorty

Publisher- Dhanpat Rai Publications

13. *Electrical Machines* [4th Ed.]
by I.J Nagrath & D.P Kothari

Publisher- McGraw Hill Education (India)
Private Limited

14. *Problems in Electrical Engineering* [9th Ed.]
by N. N. Parker Smith

Publisher- CBS Publisher

Wish you all the best