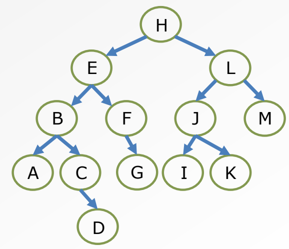
CE1107/CZ1107: DATA STRUCTURES AND ALGORITHMS

Binary Search Trees

College of Engineering
School of Computer Science and Engineering

OUTLINE

- **Binary Search Trees (BST)**
- BST Operations:
  - Traversal
  - Inserting a node
  - Removing a node

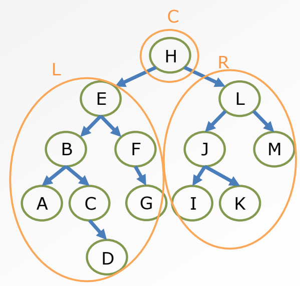


BINARY SEARCH TREE(BST)

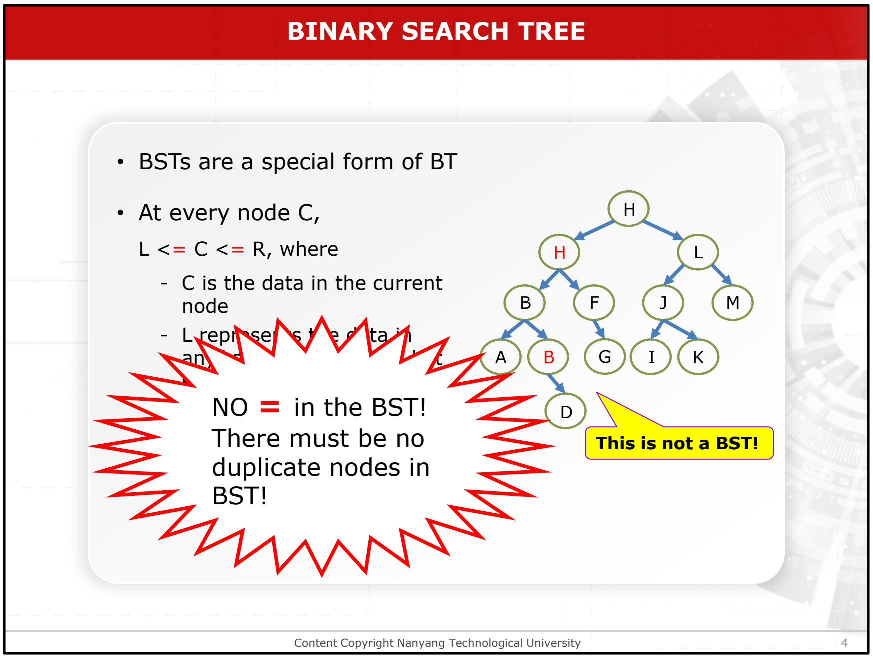
- BSTs are a special form of BT
- BST rule:

  At every node C,

  L < C < R, where
  - C is the data in the current node
  - L represents the data in any/ all nodes from C's left subtree
  - R represents the data in any/all nodes from C's right subtree



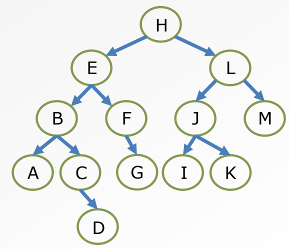
Binary Search Tree - L< C< R



Binary Search Tree cannot have duplicate nodes.

OUTLINE

- Binary Search Trees (BST)
- BST Operations:
  - **Traversal**
  - Inserting a node
  - Removing a node

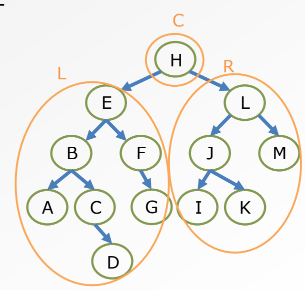


BINARY SEARCH TREE(BST)

- BSTs are a special form of BT
- BST rule:

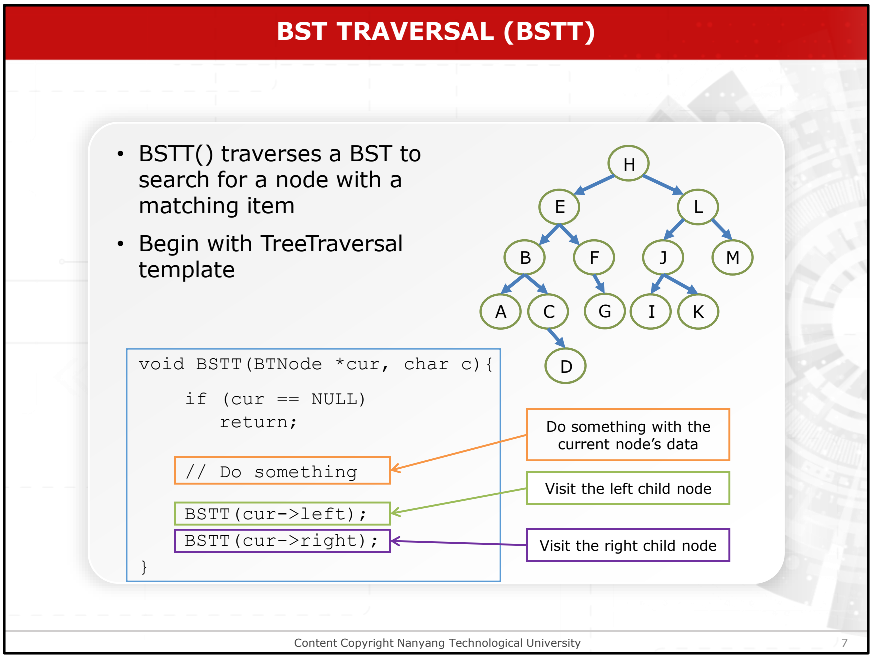
  At every node C,

  L < C < R, where
  - C is the data in the current node
  - L represents the data in any/ all nodes from C's left subtree
  - R represents the data in any/all nodes from C's right subtree

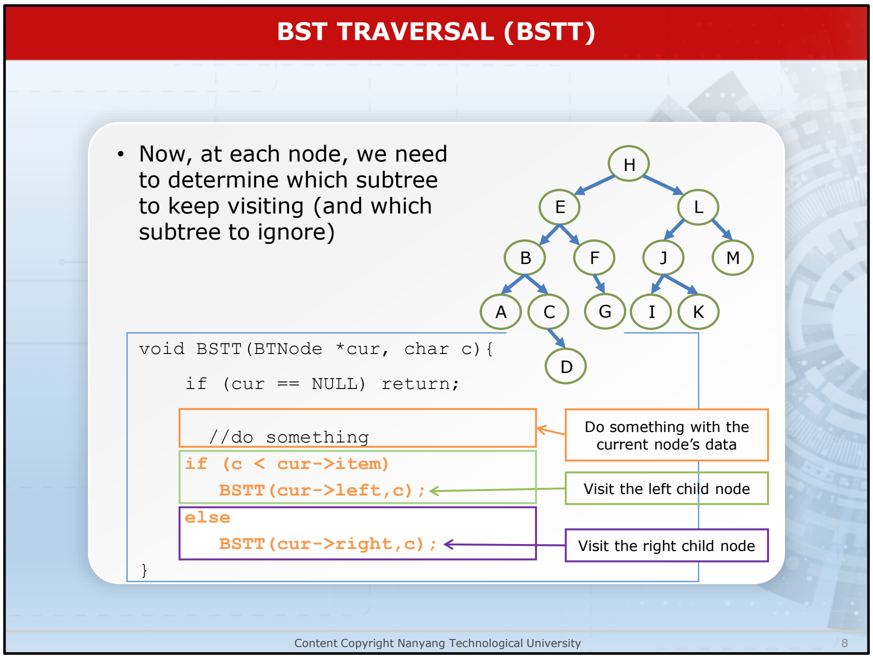


 6

Binary Search Tree - L< C< R



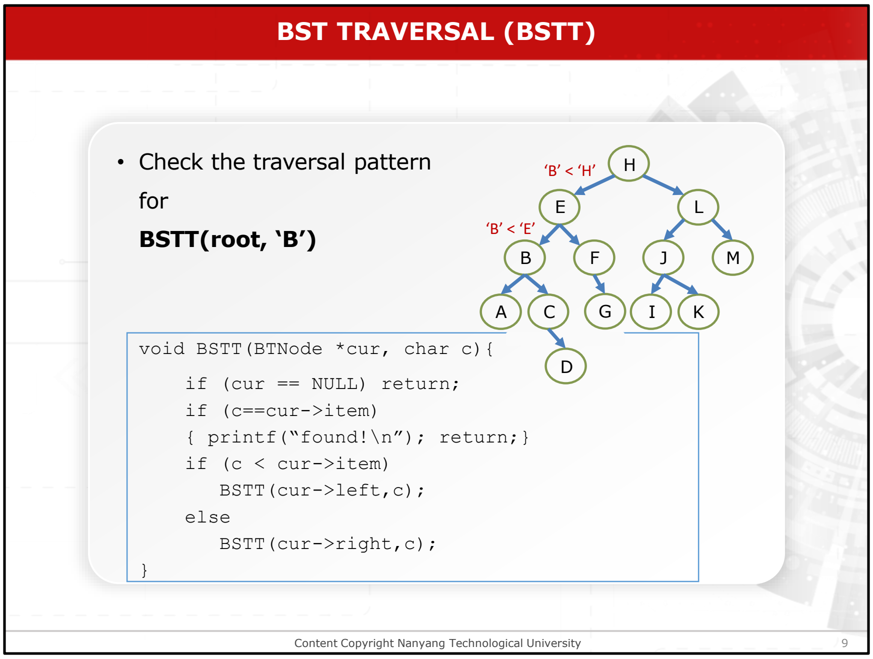
Binary Search Tree - L< C< R



Using the tree traversal template we can construct the code for BST Traversal.

- After defining the function 'BSTT' with a btnode typed pointer and a character typed variable we first check whether the current node is NULL or NOT NULL..

If the current node is not what we required we check whether C is less than the current item. If it is less than current item, it should be in current item's left subtree. Therefore we have to move to the left child node.

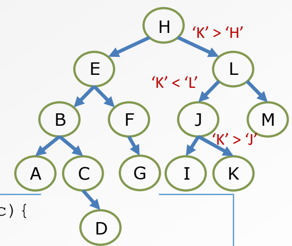


For an example, if you want search for 'B' in the given binary search tree;

- You start with 'H' which is the root node.
- Check whether 'H' is what we are searching for. Since we are searching for 'B', 'H' is not the correct node.
- Then we check whether 'B' is less than 'H'. 'B' comes before 'H' in the alphabet. Therefore 'B' < 'H'. Then we move to the left subtree of 'H', which is 'E'.

BST TRAVERSAL (BSTT)

- Check the traversal pattern for **BSTT(root, 'K')**



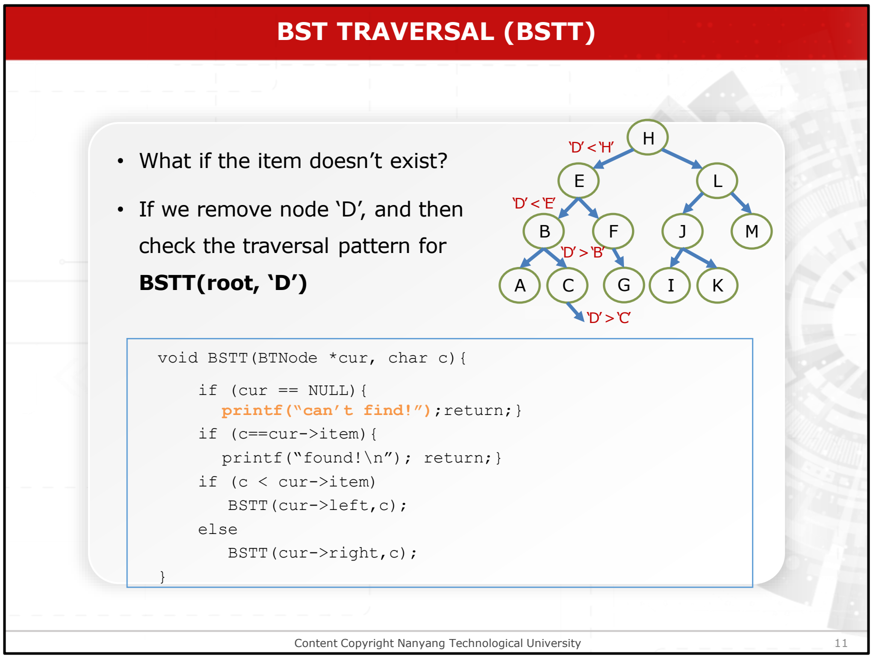

```
void BSTT(BTNode *cur, char c){
    if (cur == NULL) return;
    if (c==cur->item)
    { printf("found!\n"); return;}
    if (c < cur->item)
       BSTT(cur->left,c);
    else
       BSTT(cur->right,c);
}
```

 10

Same process should follow to search node ‘K’.

This time ‘K’ comes after ‘H’ in the alphabet ‘K’ is large than ‘H’.

Therefore we have to check with right subtree of ‘H’ this time.



What if we remove node 'D' and search for it?

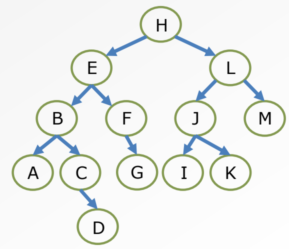
We start from the root node 'H' and follow the discussed process until 'C'.

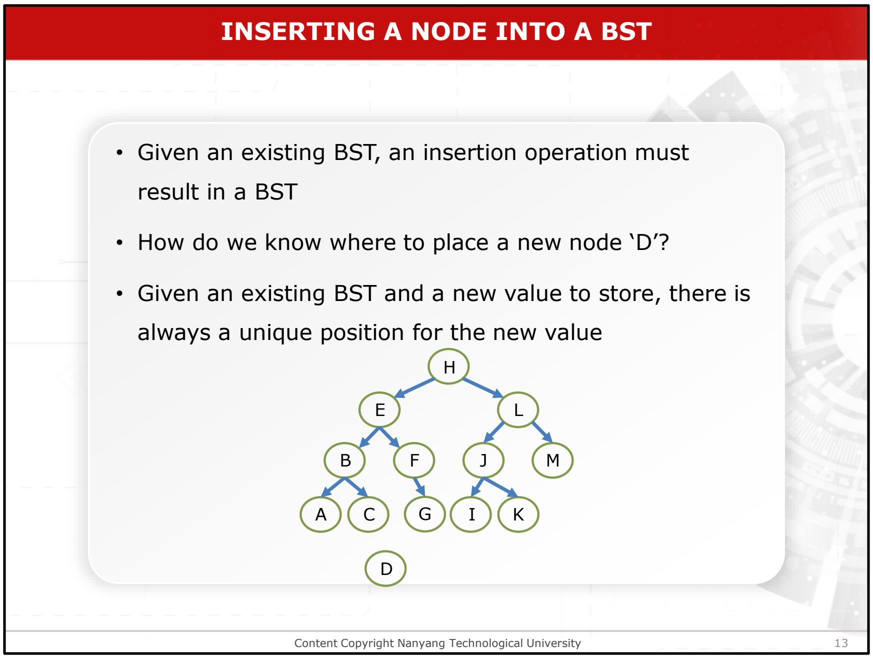
Now 'D' is larger than 'C'. Therefore we move from 'C' to its right child node.

Now the pointer points at an empty node because we have already removed 'D.

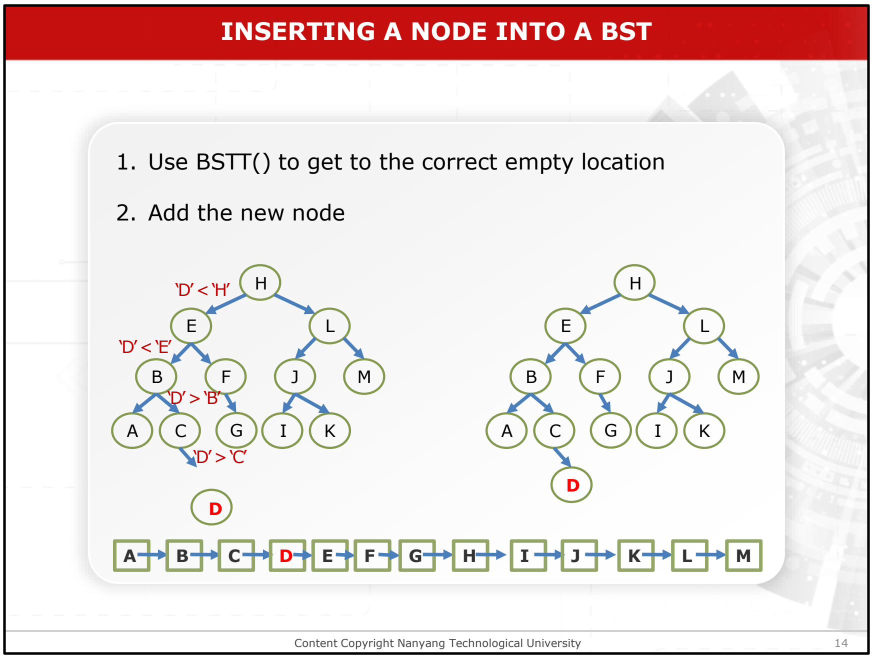
OUTLINE

- Binary Search Trees (BST)
- BST Operations:
  - Traversal
  - **Inserting a node**
  - Removing a node

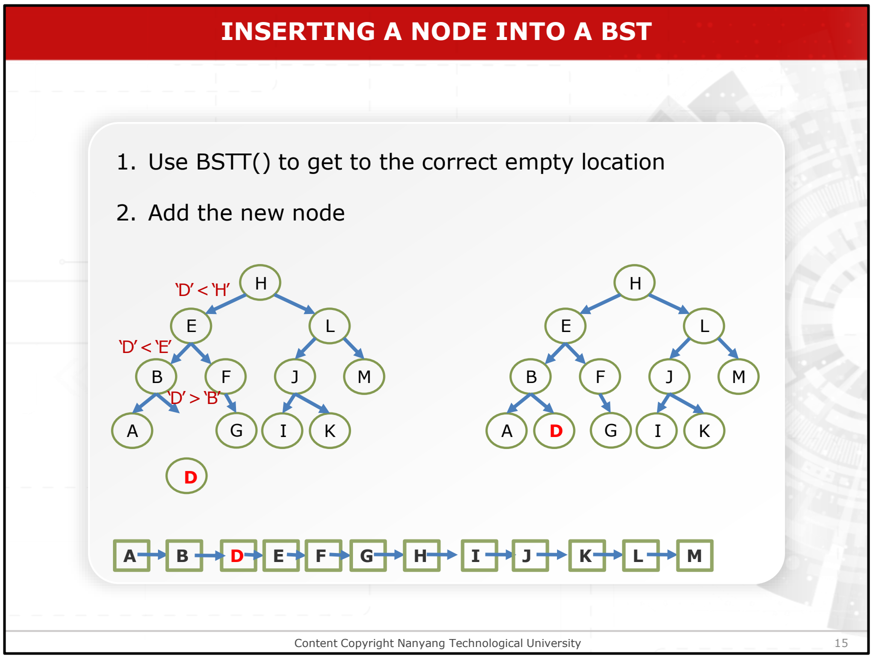




For the given binary search tree in the slide we try to insert node 'D' and still maintain the tree as a binary search tree.



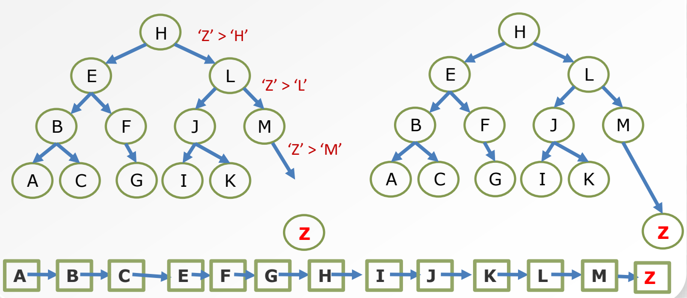
Before inserting the node we use Binary Tree Traversal Template to identify the correct empty location for the node.



Before inserting the node we use Binary Tree Traversal Template to identify the correct empty location for the node.

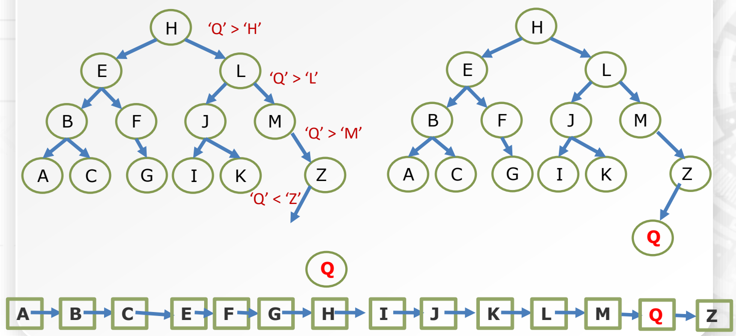
INSERTING A NODE INTO A BST

- Node insertion is relatively simple!
- Further exercise: Try Inserting 'Z'



INSERTING A NODE INTO A BST

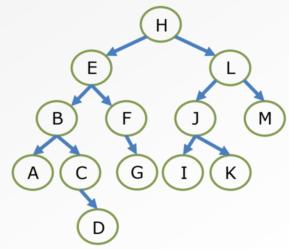
- Node insertion is relatively simple!
- Further exercise: Try Inserting ‘Q’



OUTLINE

- Binary Search Trees (BST)
- BST Operations:
  - Traversal
  - Inserting a node
  - **Removing a node**

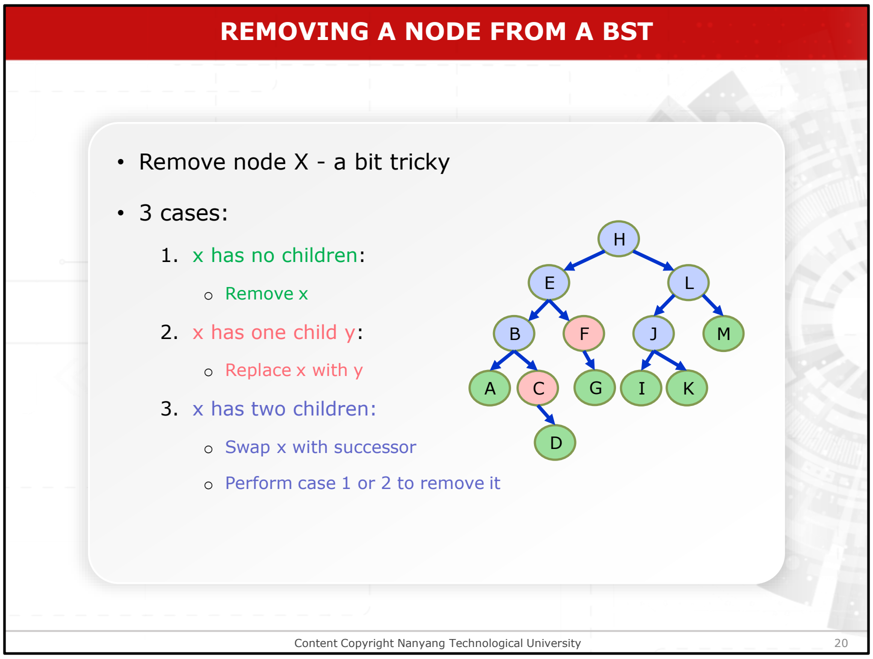
After removal, the tree is still a BST



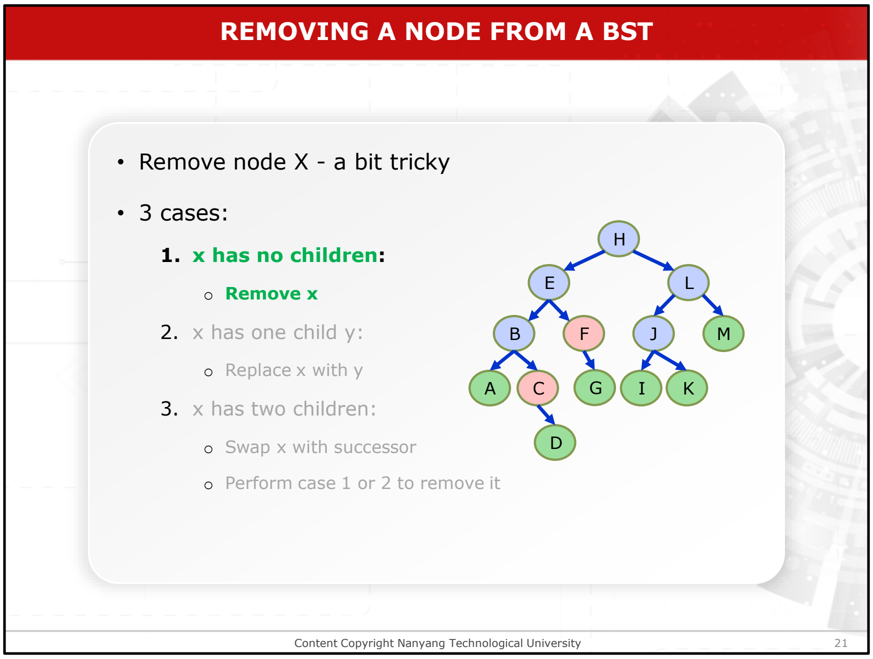
REMOVING A NODE FROM A BST

- Node removal is more complicated
- Beginning with a BST, the resulting tree after removing a node must still be a BST

  Obey the BST rule: L < C < R

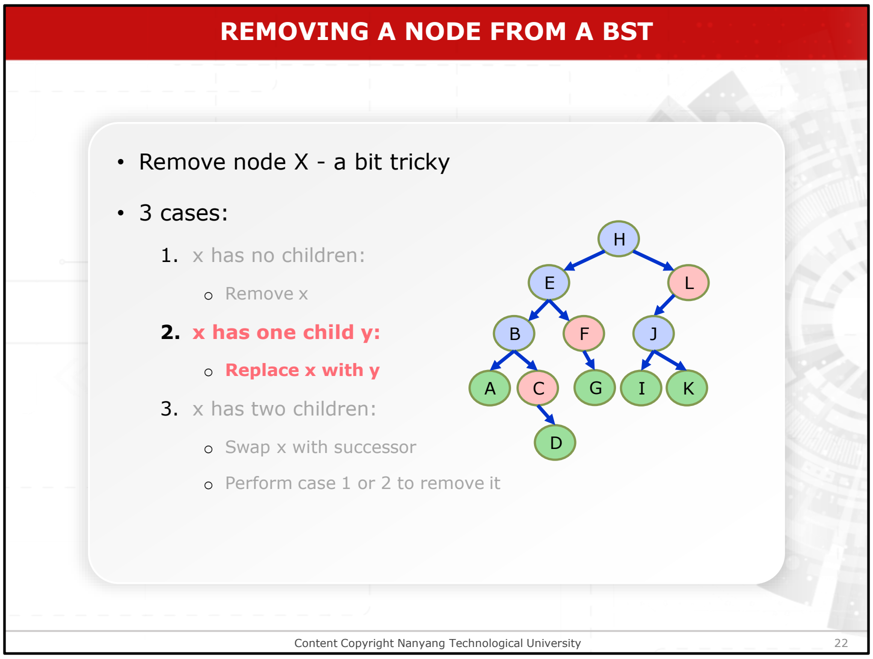


There are 3 cases we should consider when we try to remove a node from a BST.



<u>Case 1: X has no children</u>

If the node which we are trying to remove has no children, or in other world, if we are trying to remove a leaf node, we can just remove the node and the remaining tree is still a BST.

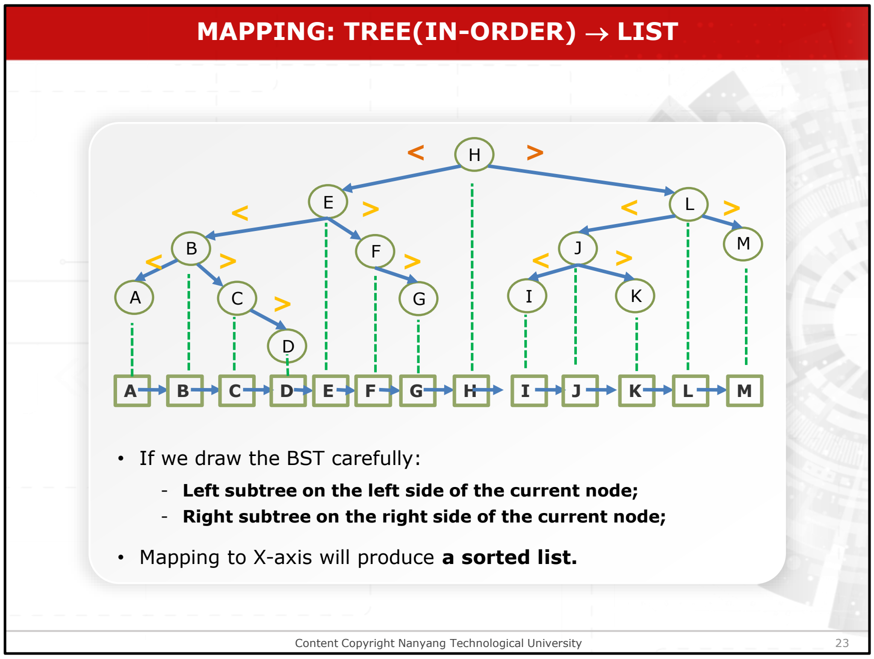


Case 2: If X has one child Y

The nodes coloured in 'pink' in the slide has only one child node.

For an example if we want to remove node 'F' from the BST, we can replace node 'F' with node 'G' because 'F' has only one child which is 'G'.

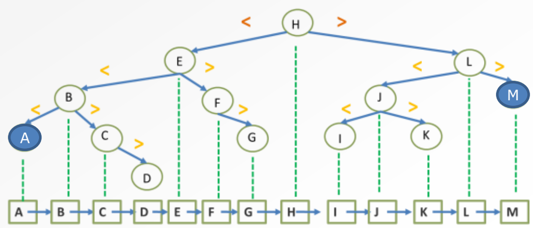
Same logic can be applied to node 'L' and node 'C'. 'L' can be replaced with 'J' and 'C' and be replaced with 'D'.

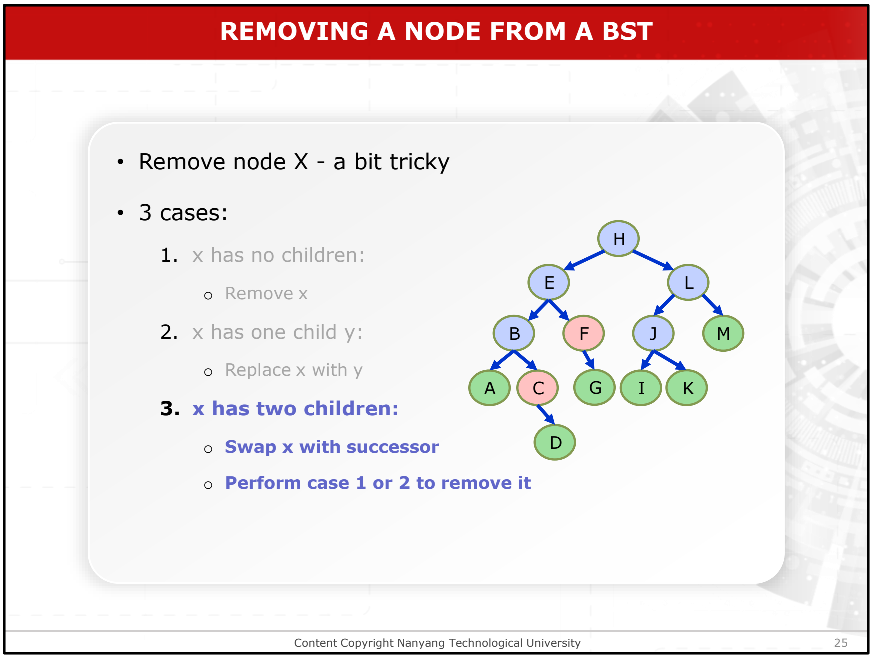


We get a sorted linkedlist if we flatten a binary search tree because we can map binary search tree into a linkedlist as shown in the slide.

FEATURES

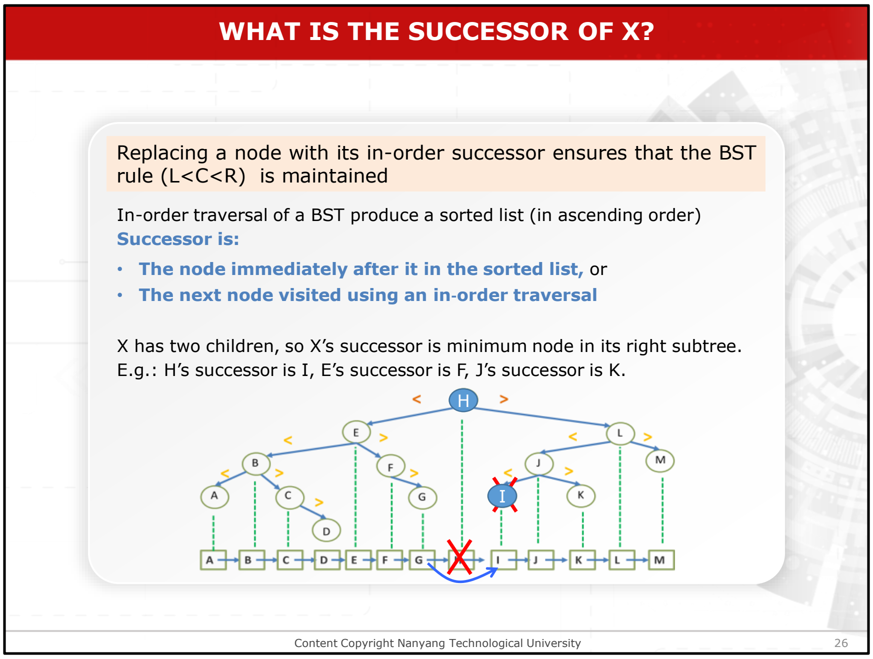
- BST's in-order traversal produces a sorted list!
  - **L < C < R** rule ensures sorted order
- The binary-search-tree property guarantees that:
  - The minimum is located at the left-most node
  - The maximum is located at the right-most node





Case 3: When X has two children

This case is little bit complicated.



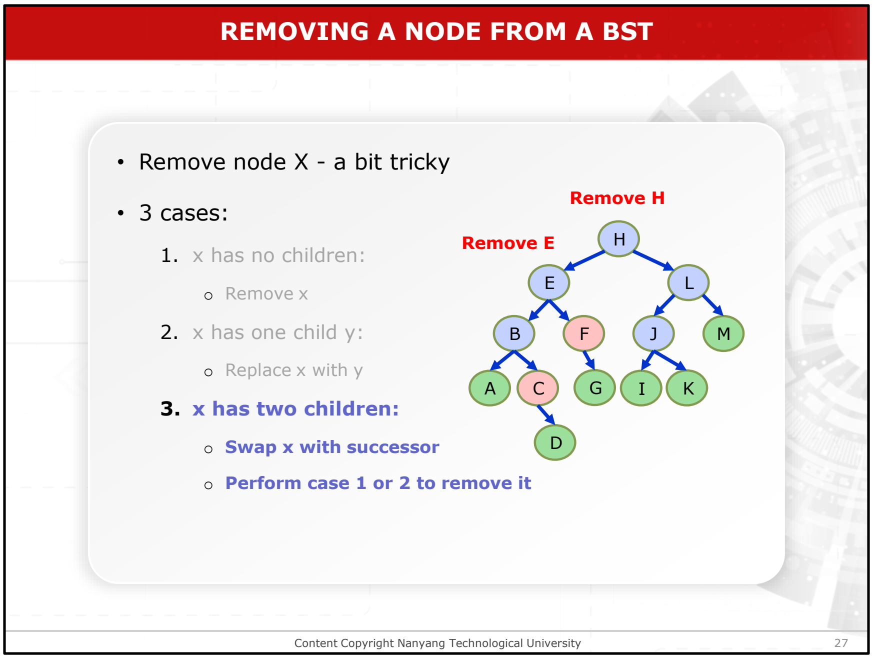
What is successor of X?

For an example, in the given tree, the successor of ‘H’ is ‘I’, because ‘I’ comes immediately after ‘H’ in the sorted list. ‘I’ is the node visited after ‘H’ using in-order traversal also.

Now we have to swap ‘H’ with ‘I’ if we want to remove ‘H’, because if we replace the node with its in-order successor, it ensures the BST rule (L < C < R) is maintained.

Therefore first we swap ‘H’ with ‘I’ and then ‘H’ become a leaf node. Now we can apply case 1 which we discussed and remove ‘H’.

Now the BST rule is maintained as before.



As discussed in the previous slide, if we want to remove node X which has two children;

- First we swap X with its successor
- Then perform case 1 or case 2 to remove X.

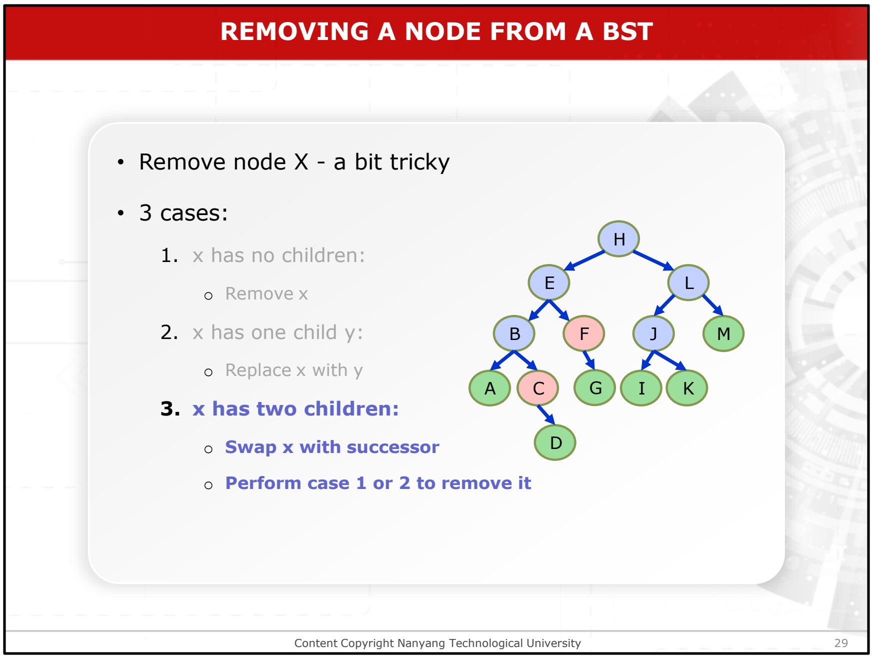
QUESTIONS

- **Why will case 3 always go to case 1 or case 2?**

  A: because when X has 2 children, its successor is the minimum in its right subtree, so the successor should not have left child.
  It might have no child(case 1) or one right child(case 2).

- **Could we swap x with predecessor instead of successor?**

  A: yes.



- The node immediately after it in the sorted list, or
- The next node visited using an in-order traversal

X has two children, so X's successor is minimum node in its right subtree.

WHAT IS THE SUCCESSOR OF X?

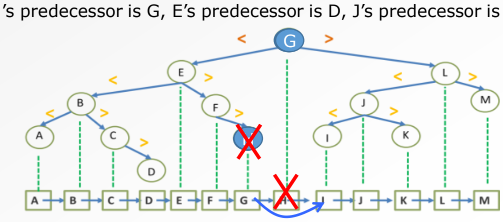
Replacing a node with its in-order **predecessor** ensures that the BST rule (L<C<R) is maintained

In-order traversal of a BST produce a sorted list (in ascending order)

Successor/predecessor:

- **The node immediately after/before it in the sorted list**
- **The next/previous node visited using an in-order traversal**

X has two children, so X's predecessor is maximum node in its left subtree.
E.g.: H's predecessor is G, E's predecessor is D, J's predecessor is I.



We can swap X with its predecessor instead of the successor as well.

TODAY YOU SHOULD BE ABLE TO

- Define a Binary Search Tree
- From a list, how do we construct a Binary Search Tree? Is it efficient?
- How do we traverse a BST to search a item?
- How do we insert/remove a node from a BST?