# **Assignment 5: Quicksort Algorithm: Implementation, Analysis, and Randomization**

## **Quicksort Implementation and Analysis**

### **1. Implementation**

**A screenshot of a computer program

Description automatically generated**

### **2. Performance Analysis**

Time Complexity Analysis

Best Case:  
The best-case scenario occurs when the pivot divides the array into two equal (or nearly equal) halves at each recursive step. This ensures the depth of recursion is minimized.

The recurrence relation is T(n)=2T(n/2)+O(n) , this recurrence gives T(n)=O(n log n)

This efficiency arises because splitting into equal halves ensures minimal levels of recursion.

Average Case:  
The average-case occurs when the pivot divides the array into two reasonably balanced subarrays. Each element has the same chance of being chosen as the pivot.

The recurrence remains approximately T(n)=2T(n/2)+O(n) leading to O(n log n)

On average, Quicksort maintains balance, making it highly efficient for most datasets.

Worst Case:  
The worst case arises when the pivot divides the array into unbalanced partitions (e.g., one subarray contains n−1n-1n−1 elements, and the other contains 0).

The recurrence relation is T(n)=T(n−1)+O(n), this gives T(n)=O(n^2)

This happens when the input is already sorted (ascending or descending) and the pivot is poorly chosen like for example the first or last element always

Space Complexity

Quicksort is often implemented in-place, meaning it requires O(1) additional memory for partitioning. However, recursion adds overhead for storing call stack frames (Cormen et al, 2009).

### **3. Randomized Quicksort**

**A computer screen shot of a program

Description automatically generated**

Analysis

Randomization improves Quicksort performance by reducing the likelihood of encountering the worst-case scenario. By choosing the pivot randomly, the chance of consistently selecting poor pivots that lead to unbalanced partitions is minimized. This results in the following

Worst-Case:  
Although it is still m theory O(n^2), the probability of consistently encountering unbalanced partitions is extremely low in reality.

Average-Case:  
Randomized Quicksort retains the O(n log n) average-case complexity, as the random selection of the pivot ensures balanced partitions on average.

### **4. Empirical Analysis**

Sorted array

QuickSort Time: 0.01932 seconds

0.01178 seconds

0.01221 seconds

Randomized QuickSort Time: 0.00200 seconds

0.00200 seconds

0.00200 seconds

Random array

QuickSort Time: 0.00981 seconds

0.00500 seconds

0.00516 seconds

Randomized QuickSort Time: 0.00200 seconds

0.00100 seconds

0.00200 seconds

Reverse sorted array

QuickSort Time: 0.02106 seconds

0.01318 seconds

0.01963 seconds

Randomized QuickSort Time: 0.00100 seconds

0.00100 seconds

0.00201 seconds

Analysis:

Sorted Arrays:

Deterministic Quicksort shows higher execution times (~0.01–0.02 seconds) due to its vulnerability to already sorted inputs if no optimized pivot selection (like median-of-three) is applied.

Randomized Quicksort consistently achieves the lowest execution time (~0.002 seconds), as the random pivot avoids the worst-case scenario of sorted data.

Random Arrays:

Deterministic Quicksort performs well (~0.005–0.01 seconds) due to more balanced partitions in random input, but it's slightly slower than randomized Quicksort.

Randomized Quicksort maintains very low execution times (~0.001–0.002 seconds), emphasizing its robustness for random inputs.

Reverse-Sorted Arrays:

Deterministic Quicksort faces its worst-case performance (~0.02 seconds) due to unbalanced partitions.

Randomized Quicksort avoids this inefficiency with consistently low execution times (~0.001–0.002 seconds).

In conclusion Randomized Quicksort outperforms deterministic Quicksort in all scenarios, especially for sorted and reverse-sorted data, due to its ability to mitigate the worst-case scenario through random pivot selection.

References

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms*