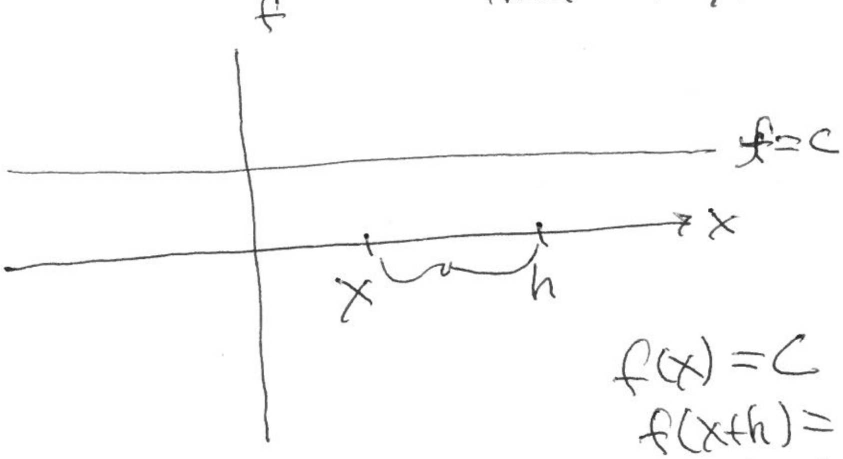
21 Sec 3.1

Section 3.1 Rules for finding derivatives

1. Let $f(x) = c$, c a constant

Then $f'(x) = 0$



$f(x) = c$

$f(x+h) = c$

so $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} =$

$= \lim_{h \to 0} \frac{0}{h} = 0.$

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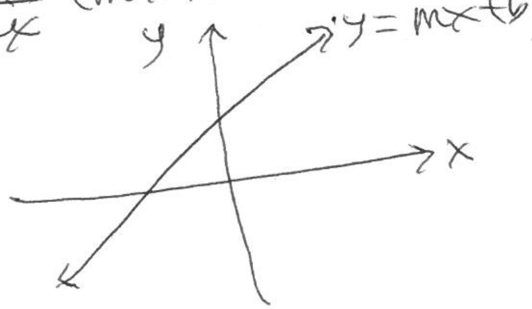
2. $\frac{d}{dx}(mx+b) = m$



$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{[m(x+h) + b] - [mx+b]}{h}$

$= \lim_{h \to 0} \frac{\cancel{mx} + mh + \cancel{b} - \cancel{mx} - \cancel{b}}{h}$

$= \lim_{h \to 0} \frac{mh}{h} = \lim_{h \to 0} m = m$

So, for a non-vertical straight line, the derivative is the slope of that line

③ The power rule

If $n \in \mathbb{Z}^+$, then $\frac{d}{dx} x^n = n x^{n-1}$

Ex $\frac{d}{dx} x^{2019} = 2019 x^{2019-1} = 2019 x^{2018}$

Pf $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim\limits_{h \to 0} \frac{(x+h)^n - x^n}{h}$, use the binomial thm

$= \lim\limits_{h \to 0} \frac{(\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + nxh^{n-1} + h^n) - \cancel{x^n}}{h}$

$= \lim\limits_{h \to 0} \frac{nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + nxh^{n-1} + h^n}{h}$

$= \lim\limits_{h \to 0} \cancel{h(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h^2} \quad \cdots + nxh^{n-2} + h^{n-1})$

$= \lim\limits_{h \to 0} \frac{\cancel{h}(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h^1 + \cdots + nxh^{n-2} + h^{n-1})}{\cancel{h}}$

$= \lim\limits_{h \to 0} (nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \cdots + nxh^{n-2} + h^{n-1})$

$= nx^{n-1}$

$\frac{d}{dx} x^3 = 3x^{3-1} = 3x^2$

The general power rule

Let r be any real number

Then $\frac{d}{dx}x^r = rx^{r-1}$

Ex $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}x^{-\frac{1}{2}} = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$

Ex $\frac{d}{dx}x^{3.74} = 3.74x^{2.74}$

$\frac{d}{dx}x^e = ex^{e-1}$

$\frac{d}{dx}\pi^3 = 0$, note π^3 is a constant by rule 1, ~~the~~ $\frac{d}{dx}c = 0$

$\frac{d}{dx}t^2 = 0$, when taking $\frac{d}{dx}$, the only variable is x, so t looks like and is treated as a constant.

More Rules

Constant Multiple Rule

$$\frac{d}{dx}\left[c f(x)\right] = c \frac{d}{dx} f(x) = c f'(x)$$

Ex $\frac{d}{dx} 5x^{10} = 5 \frac{d}{dx} x^{10} = 5(10x^9) = 50x^9$

pf Let $g(x) = c f(x)$

so $g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$

$= \lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h}$

$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c f'(x)$

Thm a) $\frac{d}{dx}\left[f(x)+g(x)\right] = f'(x)+g'(x)$

b) $\frac{d}{dx}\left[f(x)-g(x)\right] = f'(x)-g'(x)$

pf of b) Let $k(x) = f(x)-g(x)$

so $k'(x) = \lim_{h\to 0} \frac{k(x+h)-k(x)}{h}$

$= \lim_{h\to 0} \frac{[f(x+h)-g(x+h)]-[f(x)-g(x)]}{h}$

$= \lim_{h\to 0} \frac{f(x+h)-f(x)-g(x+h)+g(x)}{h}$

$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} - \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$= f'(x)-g'(x)$

Ex

$\frac{d}{dx}\left[4x^2-2x+5\right]$

$= \frac{d}{dx}4x^2 - \frac{d}{dx}2x + \frac{d}{dx}5$

$= 4\frac{d}{dx}x^2 - 2\frac{d}{dx}x + 0$

$= 4(2x)-2(1)$

$= 8x-2$

When does the graph of

$$f(x) = x^3 - 4x^2 - 7x - 2$$

have a horizontal tangent

i.e. when is $f' = 0$

$$f'(x) = 3x^2 - 8x - 7$$

so $3x^2 - 8x - 7 = 0$

when

$$x = \frac{8 \pm \sqrt{8^2 - 4(3)(-7)}}{2 \cdot 3}$$

$$x = \frac{8 \pm \sqrt{64 + 84}}{6} = \frac{8 \pm \sqrt{148}}{6}$$

$$x = \frac{8 \pm 2\sqrt{37}}{6} = \frac{4 \pm \sqrt{37}}{3}$$

So f has a horizontal tangent when

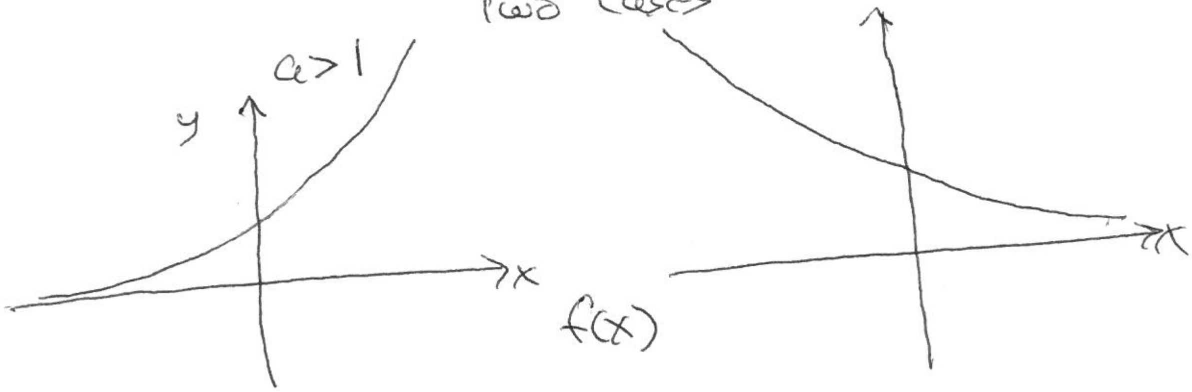
$$x = \frac{4 + \sqrt{37}}{3}, \quad x = \frac{4 - \sqrt{37}}{3}$$

021 Sec 3.1

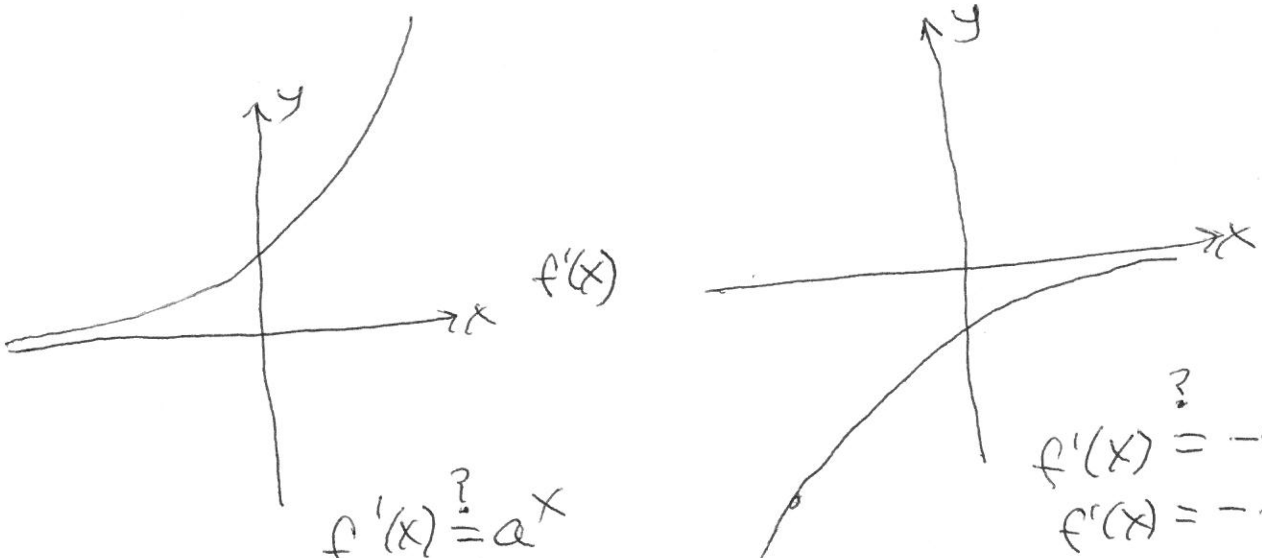
Derivatives of Exponential Functions

Graph $y = f(x) = a^x$ $\quad a > 0,\ a \neq 1$

Two Cases



$f(x)$



$f'(x)$

$f'(x) \stackrel{?}{=} a^x$

$f'(x) \stackrel{?}{=} f(x)$

$f'(x) \stackrel{?}{=} -a^x$

$f'(x) = -f(x)$

What is $\frac{d}{dx} a^x$, $a > 0,\ a \neq 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x a^h - a^x}{h}$$

$$f'(x) = \lim_{h \to 0} a^x \left(\frac{a^h - 1}{h} \right) = a^x \lim_{h \to 0} \left(\frac{a^h - 1}{h} \right)$$

<u>Note</u> $f'(0) = \lim_{h \to 0} \left(\frac{a^h - 1}{h} \right)$

So $f'(x) = a^x f'(0)$. Note $f'(0)$ is a constant and is the slope of the tangent line when $x = 0$.

$f'(0)$ will vary with the choice of a

We have $f'(x) = [f'(0)]\, a^x$

$f'(x) =$ (slope of the tangent line when $x=0$)(value of the function at x)

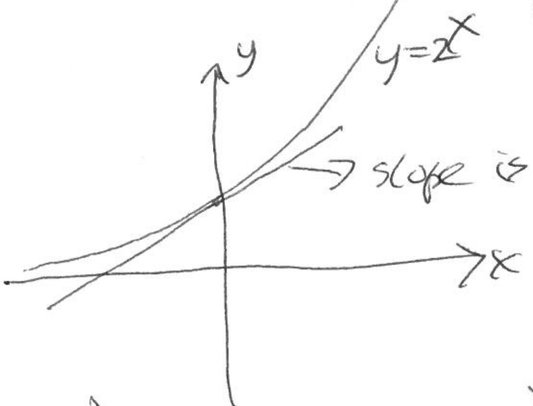
We now state, but do not prove, the important result

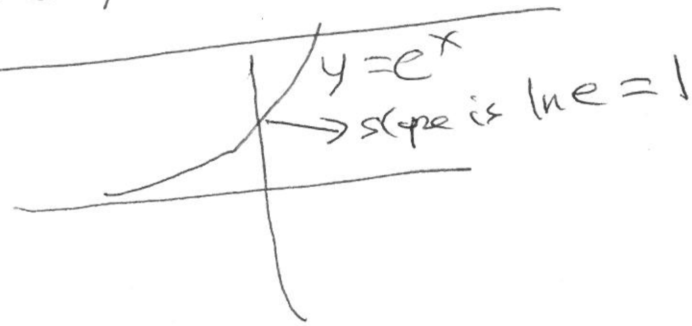
If $f(x) = a^x$ then $f'(0) = \ln a$

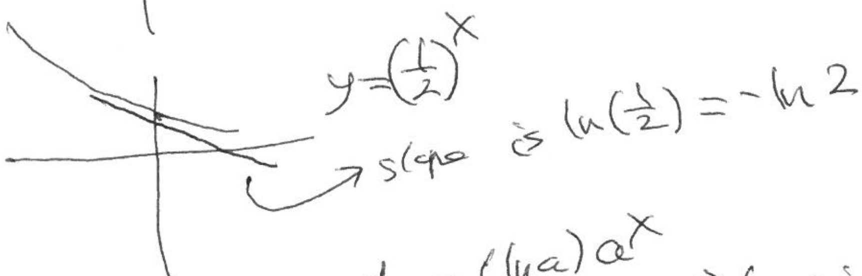
So $\frac{d}{dx} a^x = (\ln a) a^x$

<u>Special case</u> $\frac{d}{dx} e^x = (\ln e) e^x$

$\frac{d}{dx} e^x = e^x$, note $\ln e = 1$







<u>Note</u> $\frac{d}{dx} = (\ln a) a^x$

(a constant)(original function)

<u>Def</u> The number e is the number such that

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

The product rule

We have $(f \pm g)' = f' \pm g'$

However $(f \cdot g)' \neq f' \cdot g'$

Ex $\frac{d}{dx} x^{10} = 10x^9$

and $x^{10} = x^2 \cdot x^8$

$\frac{d}{dx} x^2 = 2x$, $\frac{d}{dx} x^8 = 8x^7$

so $(x^2)'(x^8)' = (2x)(8x^7) = 16x^8 \neq 10x^9$

Also $x^{10} = x^3 \cdot x^7$

$(x^3)' = 3x^2$, $(x^7)' = 7x^6$

so $(x^3)'(x^7)' = (3x^2)(7x^6) = 21x^8$

A different wrong answer

The correct formula, sometimes called Leibnitz's Formula is:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Ex $\frac{d}{dx}[x^2 x^8] = (2x)x^8 + (x^2)(8x^7) = 2x^9 + 8x^9 = 10x^9$

$\frac{d}{dx}[x^3 x^7] = (3x^2)x^7 + (x^3)(7x^6) = 3x^9 + 7x^9 = 10x^9$

Ex $\frac{d}{dx}[2^x \cdot x^4]$ recall $\frac{d}{dx} a^x = (\ln a) a^x$

$= (\ln 2) 2^x \cdot x^4 + 2^x \cdot 4x^3$

$= (\ln 2) 2^x x^4 + 4x^3 \cdot 2^x$

Pf of Product Rule

WTS $\frac{d}{ds}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$

$$\frac{d}{dx}\left[f(x)\,g(x)\right] = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x)}{h} + \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} f(x+h)\left[\frac{g(x+h)-g(x)}{h}\right] + \lim_{h\to 0} g(x)\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \left(\lim_{h\to 0} f(x+h)\right)\boxed{\lim_{h\to 0}\left[\frac{g(x+h)-g(x)}{h}\right]} + \lim_{h\to 0} g(x)\boxed{\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]}$$

$$= f(x)\,g'(x) + g(x)\,f'(x)$$

Extended ~~power~~ Product rule

$$\frac{d}{dx}\left[f\cdot g\cdot h\right] = \frac{d}{dx}\left[(f\cdot g)\cdot h\right] = (f\cdot g)'\cdot h + (f\cdot g)h'$$

$$= (f'\cdot g + f\cdot g')h + (f\cdot g)h'$$

so $\frac{d}{dx}(f\cdot g\cdot h) = f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h'$

use $\frac{d}{dx}a^x = (\ln a)a^x$

Ex $\frac{d}{dx}\left[x^3\cdot 5^x\cdot 6^x\right]$

$$= 3x^2\cdot 5^x\cdot 6^x + x^3(\ln 5)5^x 6^x + x^3 5^x(\ln 6)6^x$$

$$= 3x^2 5^x 6^x + (\ln 5)x^3\cdot 5^x 6^x + 5^x(\ln 6)6^x x^3$$

Find $\left[f\cdot g\cdot h\cdot k\cdot l\right]'$

$$= f'\cdot g\cdot h\cdot k\cdot l + f\cdot g'\cdot h\cdot k\cdot l + f\cdot g\cdot h'\cdot k\cdot l + f\cdot g\cdot h\cdot k'\cdot l + f\cdot g\cdot h\cdot k\cdot l'$$

12.1 The Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \cancel{g(x)f'(x)} \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}\left(\frac{hi}{ho}\right) = \frac{ho\, d hi - hi\, d ho}{ho\, ho}$$

EX Use the quotient rule to find the slope of tangent line to the curve $f(x) = \frac{e^x+1}{x^2}$, when $x=1$.

Solution $f'(x) = \frac{x^2e^x - (2x)(e^x+1)}{x^4} = \frac{x^2e^x - 2xe^x - 2x}{x^4}$

$$= \frac{x(xe^x - 2e^x - 2)}{x^4} = \frac{xe^x - 2e^x - 2}{x^3}$$

$$f'(1) = \frac{1 \cdot e^1 - 2e^1 - 2}{1^3} = -2 - e$$

Ex $\frac{d}{dx} \frac{x^4 - x^2e^x + x^3}{x^2}$, we could use the quotient rule but easier to simplify first.

$$\frac{d}{dx}(x^2 - e^x + x) = 2x - e^x + 1$$

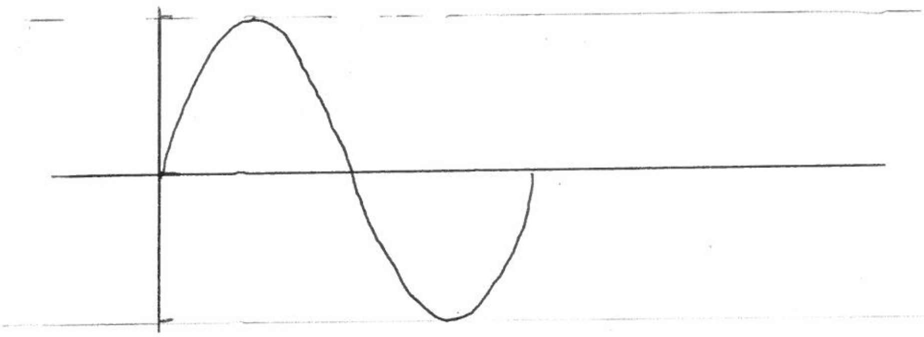
Be careful, $f(x) = \frac{x^4 - x^2e^x + x^3}{x^2}$, $f(0)$ is undefined so $f'(0)$ d.n.e. But $2x - e^x + 1$ exists for all values of x, including $x=0$. We can evaluate $2x - e^x + 1$ at $x=0$ to get $2(0) - e^{(0)} + 1 = -1+1 = 0$. But in the context of this problem, this answer is meaningless.

So $$\frac{d}{dx}\left(\frac{x^4 - x^2e^x + x^3}{x^2}\right) = \begin{cases} 2x - e^x + 1 & \text{for } x \neq 0 \\ \text{dne.} & \text{for } x = 0 \end{cases}$$

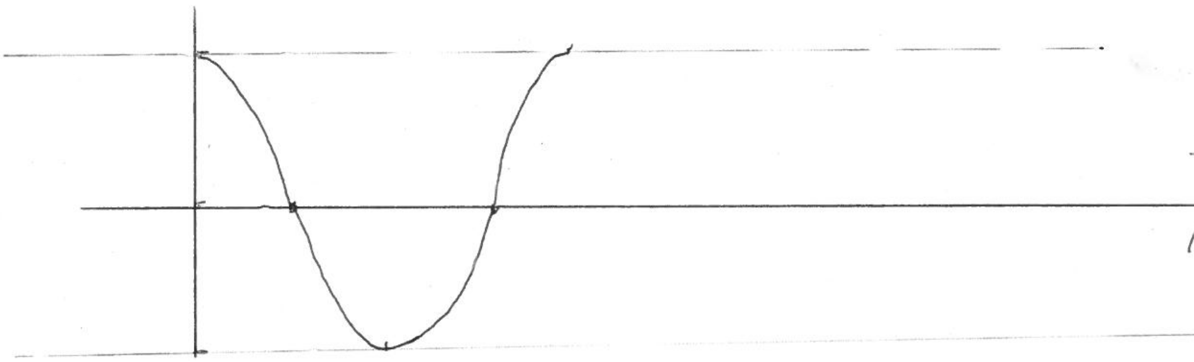
021 Sec 3.3

Derivatives of Trig Functions

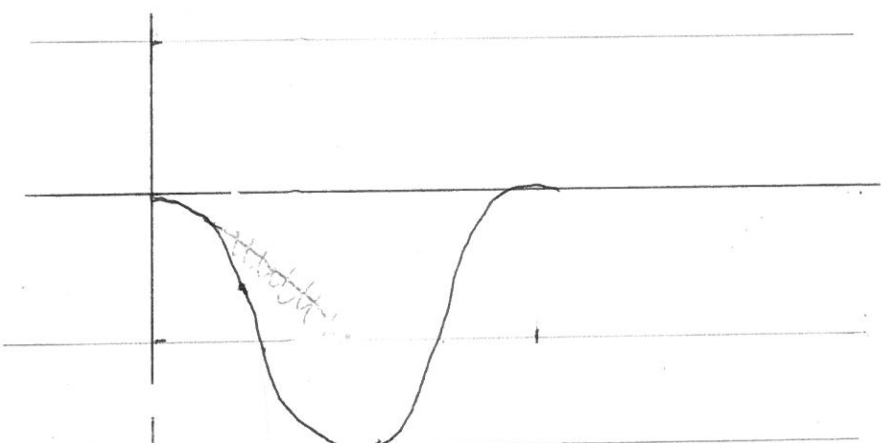
For motivation we will graph $\sin x$ and below it graph the derivative of $\sin x$. Likewise for $\cos x$



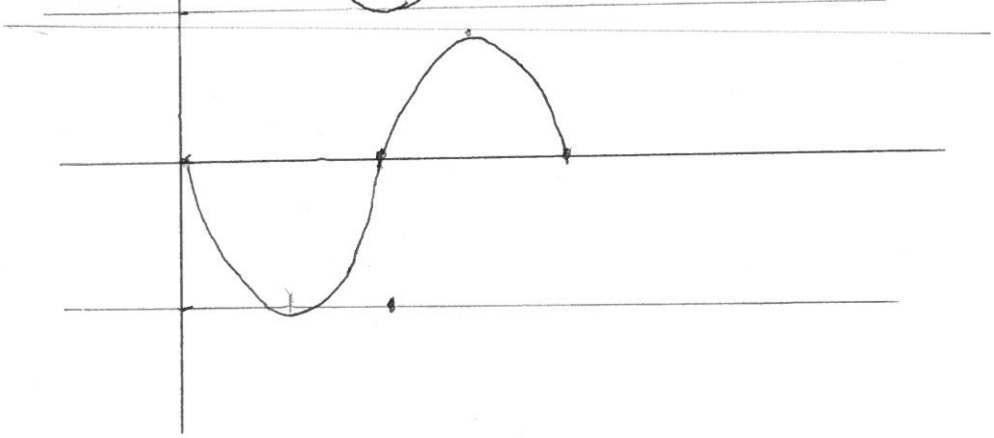
$f(x) = \sin x$



$f'(x)$ looks like $\cos x$



$g(x) = \cos x$



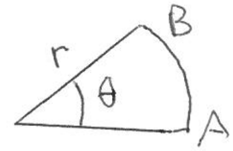
$g'(x)$ looks like $-\sin x$

To rigorously show that $\frac{d}{dx}\sin x = \cos x$, we need

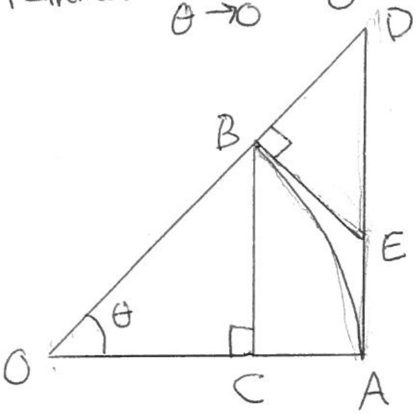
① $\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ and ② $\lim\limits_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$, all angles in radians

Once we have these limits, the rest is easy

lemma $\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$



, recall arc $\widehat{AB} = r\theta$



Assume $|OB| = |OA| = 1$

note: $|BC| = |OB|\sin\theta = 1 \sin\theta = \sin\theta$

So $\sin\theta < \theta$

$\frac{\sin\theta}{\theta} < 1$

Now, arc $\widehat{AB} < |AE| + |BE|$. This comes from the fact that the circumference of a circle is less than the perimeter of a polygon that encloses it.

So, $0 < \text{arc}\,\widehat{AB} < |AE| + |EB| \le |AE| + |ED|$

To see this, consider the right triangle $\triangle BED$

Also, $|AD| = |OA|\tan\theta = \tan\theta$

note $|OA| = 1$

So $\theta < \tan\theta \le \frac{\sin\theta}{\cos\theta}$. Now divide by θ and multiply by $\cos\theta$

$\theta < \frac{\sin\theta}{\cos\theta}$

So $\cos\theta < \frac{\sin\theta}{\theta} < 1$. Now take limits as $\theta \to 0$

$\lim\limits_{\theta \to 0} \cos\theta \le \lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} \le \lim\limits_{\theta \to 0} 1$

$1 \le \lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} \le 1$

Now use the squeeze thm to get

$\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

<u>lemma</u> $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$

<u>Pf</u> $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}\left(\frac{\cos\theta + 1}{\cos\theta + 1}\right) = \lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta(\cos\theta + 1)} = \lim_{\theta \to 0} \frac{-\sin^2\theta}{\theta(\cos\theta + 1)}$

$= -\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{(\cos\theta + 1)} = -\boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} \cdot \lim_{\theta \to 0} \frac{\sin\theta}{(\cos\theta + 1)}$

$= (-1)\left(\frac{0}{1+1}\right) = 0$

We now use

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$, $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$

and the trig identity: $\sin(x+h) = \sin x \cosh + \cos x \sinh$

To prove

<u>Thm</u> $\frac{d}{dx} \sin x = \cos x$

<u>Pf</u> $\frac{d}{dx} \sin x = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x \cosh - \sin x + \cos x \sinh}{h}$

$= \lim_{h \to 0} \left(\frac{\sin x \cosh - \sin x}{h}\right) + \lim_{h \to 0} \frac{\cos x \sinh}{h}$

$= \lim_{h \to 0} \sin x \lim_{h \to 0} \frac{\cosh - 1}{h} + \lim_{h \to 0} \cos x \lim_{h \to 0} \frac{\sinh}{h}$

$= (\sin x)(0) + \cos(x)(1) = \cos x$

So $\frac{d}{dx} \sin x = \cos x$

Thm $\frac{d}{dx}\tan x = \sec^2 x$

Pf $\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$, use the quotient rule

$$= \frac{\cos x \frac{d}{dx}\sin x - \sin x \frac{d}{dx}\cos x}{(\cos x)^2}$$

$\cos^2 x + \sin^2 x = 1$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\frac{1}{\tan x} = \frac{d}{dx}\frac{\cos x}{\sin x} = \frac{(\sin x)(-\sin x) - \cos x \cos x}{(\sin^2 x)}$$

$$= \frac{-1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = = \frac{\cos x \frac{d}{dx}(1) - (1)\frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{(\cos x)0 - (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)$$

$$= \tan x \sec x$$

$$\frac{d}{dx}\csc x = \frac{d}{dx}\frac{1}{\sin x} =$$

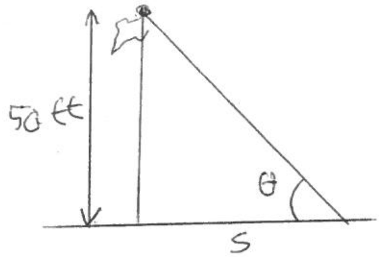
$$= \frac{(\sin x)(0) - 1(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x}$$

$$= \left(\frac{-\cos x}{\sin x}\right)\left(\frac{1}{\sin x}\right) = -\cos(x)\csc x$$

The derivative of a trig function has a minus sign iff it begins with a "c"

021 Sec 3.3

On a sunny day, a 50 ft flagpole casts a shadow that changes with the elevation of the sun. Let s be the length of the shadow and θ the angle of elevation of the sun.



Find the rate at which the length of the shadow is changing wrt θ when $\theta = 45°$.

Solution s and θ are related by $\tan\theta = \frac{50}{s}$

i.e. $s = 50\cot\theta$

If θ is in radians,

$$\frac{ds}{d\theta} = -50\csc^2\theta$$

when $\theta = \frac{\pi}{4}$

$$\left.\frac{ds}{d\theta}\right|_{\theta=\frac{\pi}{4}} = -50\csc^2\left(\frac{\pi}{4}\right) = -100 \text{ ft/rad}$$

or, converting radians to degrees

$$-100\ \frac{\text{ft}}{\text{rad}} \cdot \frac{\pi}{180}\ \frac{\text{rad}}{\text{deg}} = -\frac{5}{9}\pi\ \frac{\text{ft}}{\text{deg}} \approx -1.75 \text{ ft/deg}$$

So, when $\theta = 45°$, the shadow length is decreasing (note the minus sign) at an approximate rate of 1.75 ft/rad increase in the angle of elevation.

Ex Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1+\cos x}$

Solution We use the quotient rule

$$\frac{dy}{dx} = \frac{(1+\cos x)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{\cos x + 1}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

Ex Find the 83rd derivative of $\cos x$

Solution Note that with $f(x) = \cos x$

$f^{(0)}(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^{iv}(x) = \cos x$

So, the cycle of derivatives repeats every four derivatives.

And $4 \overline{)83}$ 20 r. 3

We count the remainder 3

so $f^{(83)}(x) = \sin x$

Ex Find $\lim_{x\to 0} \frac{\sin 3x}{4x}$. We want to use $\lim_{x\to 0} \frac{\sin x}{x} = 0$

So, to put our expression in proper form,

Write $\lim_{x\to 0} \frac{\sin 3x}{4x} = \lim_{x\to 0} \frac{3}{4} \frac{\sin 3x}{3x}$

Note, we multiplied by $\frac{3}{3}$

$= \frac{3}{4} \lim_{x\to 0} \frac{\sin 3x}{3x}$

Now let $h = 3x$, note as $x \to 0$, $h \to 0$

$= \frac{3}{4} \lim_{x\to 0} \frac{\sin h}{h} = \frac{3}{4}(1) = \frac{3}{4}$

Ex Find $\lim_{x\to 0} x\cot x$.

Solution $\lim_{x\to 0} x\cot x = \lim_{x\to 0} \frac{x\cos x}{\sin x}$

$= \lim_{x\to 0} \frac{\cos x}{\frac{\sin x}{x}} = \frac{\lim_{x\to 0} \cos x}{\lim_{x\to 0} \frac{\sin x}{x}}$

$= \frac{\cos 0}{1} = 1$

Trig functions are useful in ~~[illegible]~~ working with vibrations and waves

Suppose an object is bouncing up and down with height at time t given by

$$s(t) = 3\sin t$$

Find the velocity and acceleration at time t.

$$v = \frac{ds}{dt} \qquad = 3\cos t$$

acceleratn $a =$ $\qquad a = -3\sin t$

So the speed is $|v| = |3\cos t|$

and speed is at a max when $\cos t = 1$, so when $t = 0, \pi$

The speed is at a min when $\cos t = 0$ i.e. when $t = \frac{\pi}{2}, \frac{3\pi}{2}$

Acceleration is at a max when $|-3\sin t|$ is a max. i.e. when $\sin t = 1$, so when $t = 0, \pi$

Acceleration is a minimum when $|-3\sin t|$ is a min. i.e. when $\sin t = 0$, so, when $t = \frac{\pi}{2}, \frac{3\pi}{2}$

Ex Find where the slope of the tangent line to the graph of $f(x) = \frac{\sin x}{1+\cos x}$, equal 1

Solution: $f'(x) = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$

$$f'(x) = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}, \quad \text{use } \cos^2 x + \sin^2 x = 1$$

$$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x},$$

Note $f'(x)$ never equals 0 so $f(x)$ never has a horizontal tangent

set $f'(x) = 1$, $\frac{1}{1+\cos x} = 1$

so $\cos x = 0$, $x = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{3\pi}{2},$

$x = n\pi + \frac{\pi}{2}$, $n \in \mathbb{Z}$

$$\frac{d}{dx}(\sin x)(1+\cos x)^{-1} = (\sin x)(-1)((1+\cos x)^{-2}(-\sin x) + (\cos x)((1+\cos x)^{-1}$$

$$= \sin^2 x(1+\cos x)^{-2} + (\cos x)((1+\cos x)^{-1}$$

$$= \frac{\sin^2 x}{(1+\cos x)^2} + \frac{\cos x}{1+\cos x}$$

$$= \frac{\sin^2 x + \cos x(1+\cos x)}{(1+\cos x)^2} = \frac{\sin^2 x + \cos x + \cos^2 x}{(1+\cos x)^2}$$

$$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$