022 Sec 12.2 Monday, June 16, 2019 ①

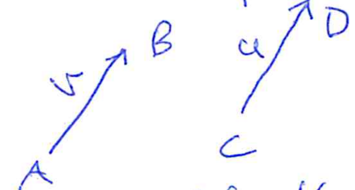
Section 12.2 Vectors

scalar - magnitude, temp, pressure, speed

vector - magnitude, direction
velocity

For a vector, we have an initial pt A and a terminal pt B
tail, heel tip

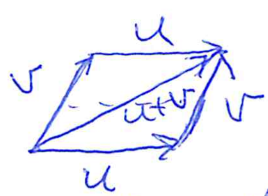
Two vectors are equal iff they have the same magnitude and direction, even if they are in different position.



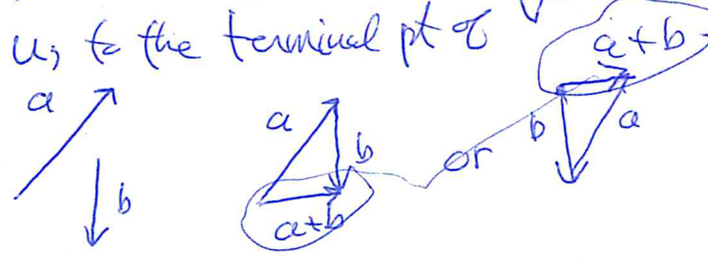
The zero vector $\vec{0}$ is a vector of length. The zero vector is the only vector without a direction.

Combining Vectors

We add u and v, to form the vector u+v by placing the initial pt of v at the terminal pt of ~~u~~ u. This is the vector from the initial pt of u, to the terminal pt of v



Ex



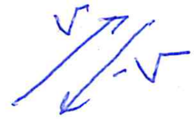
Scalar Multiplication

Let c be a scalar and v a vector, $c > 0$

Then the scalar multiple cv is a vector in the direction of v and whose length is $|cv| = |c||v|$

<u>Note</u> if $c < 0$, the direction of cv is opposite of the direction of v

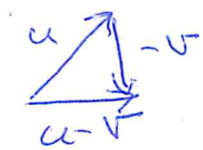
Now $(-1)v = -v$ is the vector whose length is the same as the length of v, but the direction is opposite.



We have $u + (-v) = u - v$

ex 

so this  is $-v$

so $u - v =$ 

Vectors in terms of components

The components of a vector in a coordinate system, are the coordinates of the tip when the initial pt of the vector has been moved to $\langle 0,0,0 \rangle$

When the initial pt of the vector is $\langle 0,0,0 \rangle$, we have the position vector

Ex If the tail, (initial point) of a vector is $\langle 3,-4,2 \rangle$ and the terminal point, (tip) is $\langle 0,14,-5 \rangle$

The position vector is $\langle 0-3, 14-(-4), -5, -2 \rangle = \langle -3, 18, -7 \rangle$

The magnitude of ~~a~~ position vector, also called the norm or length is $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \|a\| = |a|$

The magnitude of a general vector is the magnitude of its position vector.

Operations Let $a = \langle a_1, a_2, a_3 \rangle$, $b = \langle b_1, b_2, b_3 \rangle$

$$a + b = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

and $$ca = \langle ca_1, ca_2, ca_3 \rangle$$

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Properties of Vectors

Let a, b, c be vectors and let d, e be scalars.

Thm 1) $a+b=b+a$ (commutativity)

2) $a+(b+c)=(a+b)+c$ (associativity)

3) $a+\vec{0}=a$

4) $a+(-a)=\vec{0}$

5) $e(a+b)=ea+eb$

6) $(a+b)d=ad+bd$

7) $(ed)a=e(da)$

8) $1\cdot a=a$

prove these by using components

Ex Proof of 5) Let $a=\langle a_1,a_2,a_3\rangle$

$b=\langle b_1,b_2,b_3\rangle$

$$\begin{aligned}
e(a+b) &= e(\langle a_1,a_2,a_3\rangle+\langle b_1,b_2,b_3\rangle)\\
&= e\langle a_1+b_1, a_2+b_2, a_3+b_3\rangle\\
&= \langle e(a_1+b_1), e(a_2+b_2), e(a_3+b_3)\rangle\\
&= \langle ea_1+eb_1, ea_2+eb_2, ea_3+eb_3\rangle\\
&= \langle ea_1, ea_2, ea_3\rangle+\langle eb_1, eb_2, eb_3\rangle\\
&= e\langle a_1,a_2,a_3\rangle+e\langle b_1,b_2,b_3\rangle\\
&= ea+eb
\end{aligned}$$

Standard Basis,

In $\mathbb{R}^3$, their are three very important vectors

The Standard basis vectors, also called

" " unit " ,

$i = \langle 1,0,0 \rangle$, $j = \langle 0,1,0 \rangle$, $k = \langle 0,0,1 \rangle$

<u>Note</u> $|i| = \sqrt{1^2+0^2+0^2} = \sqrt{1^2} = 1$

$|j| = 1$, $|k| = 1$.

Any vector that has a length of 1, is called a unit vector.

Important property

Any vector $a = \langle a_1, a_2, a_3 \rangle$

can be written as ~~ai~~

$a = a_1 i + a_2 j + a_3 k$

<u>Pf</u> $a_1 i + a_2 j + a_3 k$

$= a_1 \langle 1,0,0 \rangle + a_2 \langle 0,1,0 \rangle + a_3 \langle 0,0,1 \rangle$

$= \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0,0,a_3 \rangle$

$= \langle a_1+0+0, \; 0+a_2+0, \; 0+0+a_3 \rangle$

$= \langle a_1, a_2, a_3 \rangle$

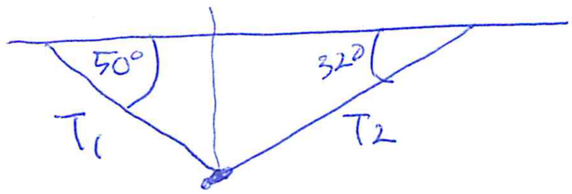
Find a unit vector (length 1) in the direction of

$v = 3i - 4j + k$. Idea set $u = v \left| \frac{1}{v} \right|$

<u>Solution</u> $|v| = \sqrt{3^2 + (-4)^2 + 1^2} = \sqrt{26}$

set $u = \frac{1}{\sqrt{26}} \langle 3, -4, 1 \rangle = \left\langle \frac{3}{\sqrt{26}}, \frac{-4}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right\rangle$

Ex. A 100 ~~lb~~ lb weight hangs from 2 wires as shown



Find the tension in each wire and their directions.

use $i = \langle 1, 0 \rangle$, $j = \langle 0, 1 \rangle$

Solution

First note that: $T_1 = -|T_1| \cos 50° i + |T_1| \sin 50° j$,

$T_2 = |T_2| \cos 32° i + |T_2| \sin 32° j$

The weight is not moving, so the sum of the forces on the weight adds to 100

so $T_1 + T_2 = -w = 100j$, Hence

$(-|T_1| \cos 50° + |T_2| \cos 32°) i + (|T_1| \sin 50° + |T_2| \sin 32°) j = 100 j$

so a) $-|T_1| \cos 50° + |T_2| \cos 32° = 0$

b) $|T_1| \sin 50° + |T_2| \sin 32° = 100$

Now, in a) solve for T_2 in terms of T_1 and substitute into b)

$$|T_1| \sin 50° + \underbrace{\frac{|T_1| \cos 50°}{\cos 32°}}_{T_2} \sin 32° = 100$$

$|T_1| \approx 85.64$ lbs

$|T_2| \approx 64.91$ lbs

$T_1 \approx -55.05 i + 65.60 j$

$T_2 \approx 55.05 i + 34.40 j$

Sec 12.3 The Dot Product

Let $a = \langle a_1, a_2, a_3 \rangle$, $b = \langle b_1, b_2, b_3 \rangle$

The dot product of a and b is

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Ex $a = \langle 3, -1, 0 \rangle$, $b = \langle 2, -6, -4 \rangle$

$$a \cdot b = 3 \cdot 2 + (-1)(-6) + 0(-4)$$
$$= 6 + 6 + 0 = 12$$

Properties of the dot product

Let a, b, c be vectors in $\mathbb{R}^3$, let d be a scalar, then

1) $a \cdot a = |a|^2$
2) $a \cdot b = b \cdot a$
3) $a \cdot (b + c) = a \cdot b + a \cdot c$
4) $(da) \cdot b = d(a \cdot b) = a \cdot (db)$
5) $\vec{0} \cdot \vec{a} = 0$

vector 0 → number 0

Let $a = \langle a_1, a_2 a_3 \rangle$

1) ~~a·a = a~~

$$a \cdot a = a_1 \cdot a_1 + a_2 \cdot a_2 + a_3 \cdot a_3$$
$$= a_1^2 + a_2^2 + a_3^2$$

$$a \cdot a = |a|^2$$

<u>Thm</u> If θ is the acute angle between non-zero vectors a and b,

Then $a \cdot b = |a||b| \cos\theta$, so $\cos\theta = \dfrac{a \cdot b}{|a||b|}$, $\theta = \cos^{-1} \dfrac{a \cdot b}{|a||b|}$