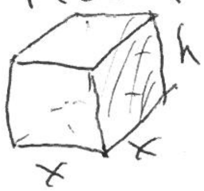
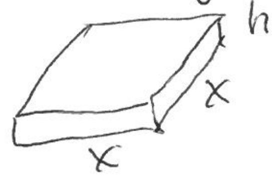
021 Sec 4.7

Section 4.7 Optimization

Ex An open top box must have a square base.
The total surface area of the box is 108 sq inches.
Find the dimensions of the box that maximize the volume





We seek to maximize: $V(x,h) = x^2 h$
objective function.

Such that $\text{Area} = 108 = x^2 + 4xh$
constraint

Solution From $108 = x^2 + 4xh \Rightarrow h = \dfrac{108 - x^2}{4x}$

so $V(x) = x^2\left(\dfrac{108-x^2}{4x}\right) = 27x - \dfrac{x^3}{4}$

$V(x) = 27x - \dfrac{x^3}{4}$

$27 = \frac{3}{4}x^2$

$\frac{4 \cdot 27}{3} = 36 = x^2$

we seek a max so we want $V' = 0$

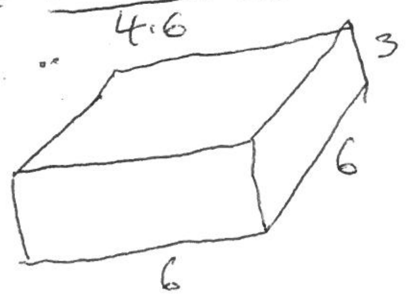
$V' = 27 - \frac{3}{4}x^2$, set $27 - \frac{3}{4}x^2 = 0$

To obtain $x = \pm 6$, we use only $x = +6$

We now use the second derivative test to see if $x = 6$ gives a max or a min.

$V''(x) = \frac{-6}{4}x$, $V''(6) = \frac{-6}{4}(6) < 0$, so $x = 6$ gives a max

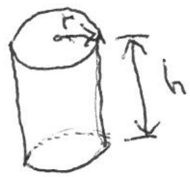
and $h = \dfrac{108 - 6^2}{4 \cdot 6} = 3$



Note max volume is $6^2 \cdot 3 = 108$
It is just a coincidence
that 108 is also
the surface area

021 Section 4.7

Ex A closed circular can is to hold 22 inch^3



Find the values of r and h which will minimize the surface area.

$$A(r,h) = 2\pi r^2 + 2\pi r h \text{ - the objective function}$$

s.t. $22 = \pi r^2 h$ - our constraint equation

so, $h = \dfrac{22}{\pi r^2}$

So, we seek to min: $A(r) = 2\pi r^2 + 2\pi r\left(\dfrac{22}{\pi r^2}\right)$

$$A(r) = 2\pi r^2 + \frac{44}{r}$$

So $\dfrac{dA}{dr} = 4\pi r - \dfrac{44}{r^2}$. Set $\dfrac{dA}{dr} = 0$, $4\pi r^3 = 44$, $r = \left(\dfrac{11}{\pi}\right)^{\frac{1}{3}}$

To check to see that we have a minimum, use the ~~second~~ first derivative test

$\dfrac{dA}{dr} < 0$ if $r < \left(\dfrac{11}{\pi}\right)^{\frac{1}{3}}$

$\dfrac{dA}{dr} > 0$ if $r > \left(\dfrac{11}{\pi}\right)^{\frac{1}{3}}$

, so we have a minimum at $r = \left(\dfrac{11}{\pi}\right)^{\frac{1}{3}}$

$$\text{So } h = \frac{22}{\pi r^2} = \frac{22}{\pi^1\left(\frac{11}{\pi}\right)^{2/3}} = \frac{2\cdot 11^1}{\pi^{1/3}\, 11^{2/3}} = \frac{2\cdot 11^{\frac{1}{3}}}{\pi^{1/3}} = 2\left(\frac{11}{\pi}\right)^{\frac{1}{3}}$$

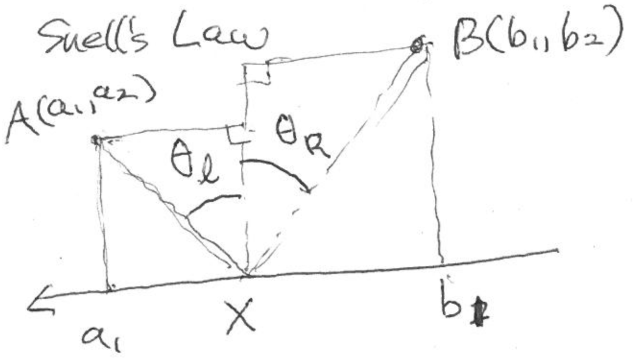
so $h = 2r$, so height equals the diameter

The can is as tall as it is wide

In general, to enclose a fixed volume inside a closed cylinder of minimum surface area, have the height of the cylinder equals its width.

021 Sec 4.7

Snell's Law



We will show that for the path from A to X and then from X to B of least distance, we have

$$\theta_l = \theta_r$$

the angle of incidence equals the angle of refraction.

i.e. the angle of incidence equals the angle of refraction.

$$L(x) = \sqrt{(x-a_1)^2 + a_2^2} + \sqrt{(x-b_1)^2 + b_2^2}$$

$$L'(x) = \frac{x-a_1}{\sqrt{(x-a_1)^2 + a_2^2}} + \frac{x-b_1}{\sqrt{(x-b_1)^2 + b_2^2}}$$

so $L'(x) = 0$ iff

$$\frac{x-a_1}{\sqrt{(x-a_1)^2 + a_2^2}} = \frac{b_1 - x}{\sqrt{(x-b_1)^2 + b_2^2}}$$

Note $\sin\theta_l = \frac{opp}{hyp} = \frac{x-a_1}{\sqrt{(x-a_1)^2 + a_2^2}}$

$\sin\theta_R = \frac{opp}{hyp} = \frac{b_1 - x}{\sqrt{(x-b_1)^2 + b_2^2}}$

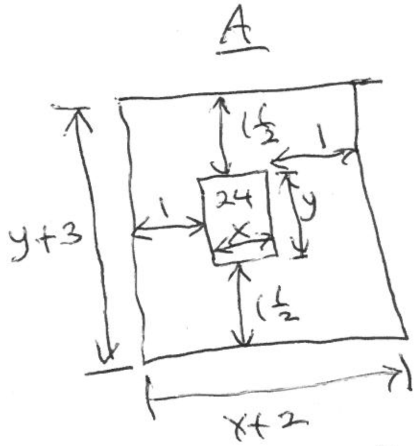
so $L'(x) = 0$, when $\sin\theta_l = \sin\theta_R$

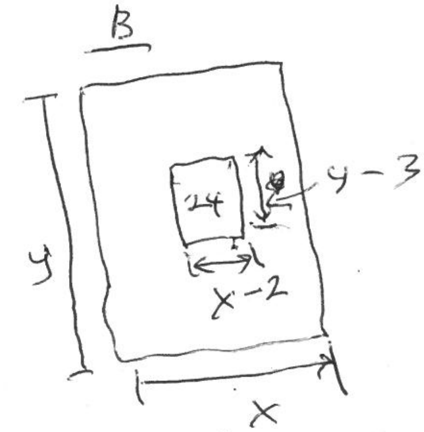
Together with $0 \le \theta \le \pi$

gives $\theta_l = \theta_R$ when the distance is a minimum

021 Sec 4.7

Ex A rectangular sheet of paper is to have 24 sq inches of print in a smaller inside rectangle. The top and bottom margins are $1\frac{1}{2}$ inches each. The left and right margins are 1 inch each.
Find the dimensions of the paper with least total area





We will solve A) We seek to minimize

$A(x,y) = (y+3)(x+2)$ our objective function

s.t. $xy = 24$ the constraint equation.

$x = \frac{24}{y}$, substitute into the objective function.

$A(y) = (y+3)\left(\frac{24}{y}+2\right) = 30 + 2y + \frac{72}{y}$

We seek to minimize $A(y)$, so set $A'(y) = 0$

$A'(y) = 2 - \frac{72}{y^2}$. Set $A'(y) = 0$ so $2 = \frac{72}{y^2}$, $2y^2 = 72$, $y^2 = 36$, $y = +6, -6$, For this problem $y = -6$ does not make sense, So use $y = 6$

We now use the second derivative test to ensure that we have a minimum. $A''(y) = \frac{144}{y^3}$, $A''(6) > 0$, so $A(6)$ gives a minimum

So $x = \frac{24}{y} = \frac{24}{6} = 4$. So the dimensions of the printed matter is 4×6

The dimensions of the whole sheet is $(4+2)$ by $(6+3) = 6$ by 9

To set up approach B) Want to minimize: $A(x,y) = xy$

s.t. $(x-2)(y-3) = 24$

021 Sec 4.7

<u>EX</u> Ticket Prices.

At a price of \$26 per ticket, 1000 tickets can be sold

For each one dollar decrease in price, 50 more tickets are sold.

Each ticket buyer will purchase an additional \$4 in snacks.

Assuming that the price of a ticket must be an integer.

What should the price of a ticket be in order to maximize revenue?

<u>Solution</u> Total Revenue

R(x) = (revenue from the tickets) + (revenue from the snacks)

R(x) = (# of tickets sold)(price per ticket) + 4(# of tickets sold)

Let x be the number of \$1 decreases in the price of a ticket

so

$$R(x) = (1000 + 50x)(26 - x) + 4(1000 + 50x)$$

$$R(x) = -50x^2 + 500x + 30{,}000$$

$$R'(x) = -100x + 500$$

so $R'(x) = 0$ when $x = 5$

Also, $R''(x) = -100 < 0$, so any crit value will yield a maximum

To maximize revenue, set the price of a ticket

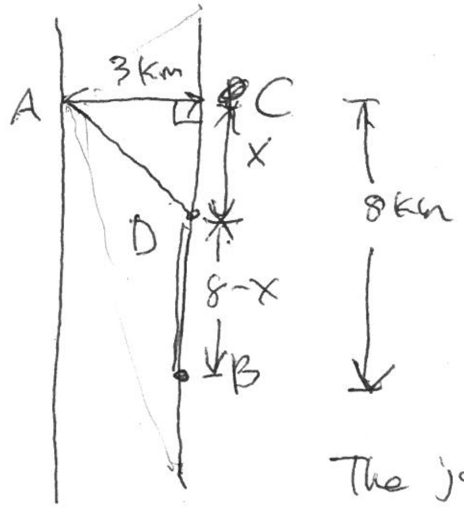
at ~~\$26 \$21~~
\$26 - \$5 = \$21

<u>Aside</u> if our critical value was $x = 4.9$

we would need to compute $R(4)$ and $R(5)$

and choose the value which gives the larger revenue.

021 Sec 4.7



You can row at 6 Km/hr.

" " jog along the bank at 8 Km/hr.

Find where D should be located to minimize the time need to go from A to B

The jogging distance is $8-x$

The rowing distance $= \sqrt{x^2+3^2}$

Now, distance is $D = (\text{time})(\text{speed})$

So the rowing time is $\dfrac{\sqrt{x^2+9}}{6}$

" jogging time is $\dfrac{8-x}{8}$

We seek to minimize the total time

$$T(x) = \frac{\sqrt{x^2+9}}{6} + \frac{8-x}{8}, \text{ with domain } [0,8]$$

Now, $T'(x) = \dfrac{x}{6\sqrt{x^2+9}} - \dfrac{1}{8}$, So $T'(x)=0$ when $\dfrac{x}{6\sqrt{x^2+9}} = \dfrac{1}{8}$

$8x = 6\sqrt{x^2+9}$, or $\frac{4}{3}x = \sqrt{x^2+9} \Rightarrow \frac{16}{9}x^2 = x^2+9$

$\frac{7}{9}x^2 = 9$, $7x^2 = 81$, $x = \pm\frac{9}{\sqrt{7}}$, we want $x = \frac{9}{\sqrt{7}}$

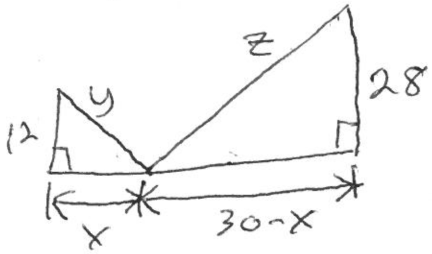
Also check endpts

$T(0) = 1.5$, $T\left(\frac{9}{\sqrt{7}}\right) = 1 + \frac{\sqrt{7}}{8} \approx 1.33$, $T(8) = \frac{\sqrt{73}}{6} \approx 1.42$

So, row to a point $\dfrac{9}{\sqrt{7}} \approx 3.4$ Km down from point C

021 Sec 4.7,

Ex Two posts: 12 and 28 ft are 30 ft apart on level ground. They are connected by two wires attached to a single ground stake



Seek to minimize $w = y + z$

Solution: Set up two right triangles

$x^2 + 12^2 = y^2$ and $(30-x)^2 + 28^2 = z^2$

so $y = \sqrt{x^2+12^2}$, $z = \sqrt{(30-x)^2+28^2}$

So $w = y + z = \sqrt{x^2+12^2} + \sqrt{(30-x)^2+28^2}$

$w = \sqrt{x^2+144} + \sqrt{x^2-60x+1684}$

$$\frac{dw}{dx} = \frac{x}{\sqrt{x^2+144}} + \frac{x-30}{\sqrt{x^2-60x+1684}}, \text{ with } 0 \le w \le 30$$

set $\frac{dw}{dx} = 0$, so $\frac{x}{\sqrt{x^2+144}} = \frac{30-x}{\sqrt{x^2-60x+1684}}$

square both sides

$$\frac{x^2}{x^2+144} = \frac{(30-x)^2}{x^2-60x+1684}$$

cross multiply

$x^2(x^2-60x+1684) = (30-x)^2(x^2+144)$

$x^2(x^2-60x+1684) = x^4-60x^3+1044x^2-8640x+129,600$

$300(x-9)(2x+45)=0$

so $x = 9, -22\frac{1}{2}$, use $x=9$

check endpts as well

$w(0) \approx 53$

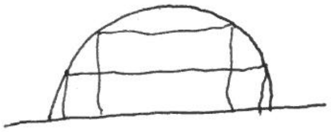
$w(9) = 50$

$w(30) \approx 60.31$

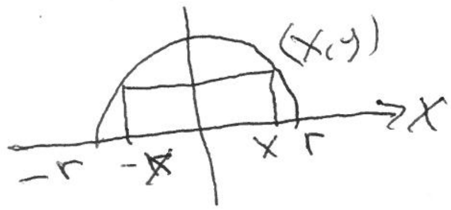
So, put the stake 9 ft away from the 12 ft. pole.

021 Sec 4.7

Ex Find the area of the largest rectangle that can be inscribed in a semicircle of radius r



Solution Use a coordinate system.
use the upper have circle
$$x^2+y^2=r^2$$



length is $2x$
height is y

We want to maximize $A = 2xy$ s.t. $y=\sqrt{r^2-x^2}$

with $0 \le x \le r$

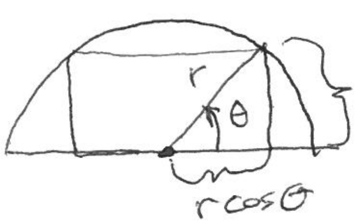
so $A(x) = 2x\sqrt{r^2-x^2}$,

$$A' = 2\sqrt{r^2-x^2} + 2x\frac{-2x}{2\sqrt{r^2-x^2}} = 2\sqrt{r^2-x^2} - \frac{2(r^2-2x^2)}{\sqrt{r^2-x^2}}$$

$A'(x)=0$ when $2x^2=r^2$ i.e. when $x=\frac{r}{\sqrt{2}}$,

$$A\left(\frac{r}{\sqrt{2}}\right) = 2\frac{r}{\sqrt{2}}\sqrt{r^2-\frac{r^2}{2}} = r^2$$

A solution without Calculus
θ is the variable



$A = b \cdot h$

$A = (2r\cos\theta)(r\sin\theta) = r^2\sin 2\theta$

Well $\sin 2\theta$ has a max of 1 and occurs when $2\theta = \frac{\pi}{2}$

so let $\theta = \frac{\pi}{4}$, $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$A = \cancel{\left(2r\frac{1}{\sqrt{2}}\right.}$ $A = r^2(1) = r^2$

Test 3 #10

$$g(x) + x\cos(g(x)) = x^8, \quad \text{find } g'(0)$$

$$g'(x) + \cos(g(x)) + x(-\sin(g(x))g'(x)) = 8x^7$$

$$g'(x)\left[1 - x\sin(g(x))\right] = 8x^7 - \cos(g(x))$$

$$g'(x) = \frac{\cancel{8x^7} - \cos(g(x))}{1 - \cancel{x\sin(g(x))}}$$

$$g'(0) = \frac{-\cos(g(0))}{1} = -\cos(g(0))$$

now, recall,

$$g(x) + \cancel{x\cos(g(x))} = x^8$$

$$g(0) = 0^8 = 0$$

$$g(0) = 0$$

so $g'(0) = -\cos(g(0)) = -\cos(0) = -1$

$$g'(0) = -1$$

Test 4 #2

$$\frac{dT}{dt} = k(T_i - T_s) = k(T-20)$$

Let $y = T - 20$

$y(0) = T(0) - 20 = 32.1 - 20 = 12.1$

$\frac{dy}{dt} = k(y)$ with $y(0) = 12.1$

$y(t) = y(0)e^{kt} = 12.1e^{kt}$

To find k, use $T(60) = 30.3$

so $y(60) = T(60) - 20 = 30.3 - 20 = 10.3$

so $12.1e^{60k} = 10.3$

$e^{60k} = \frac{10.3}{12.1} = 0.85124$

$60k = \ln(0.85124) = -0.16$

$k = -0.00268$

$y(t) = 12.1e^{-0.00268t}$

$T(t) = 20 + 12.1e^{-0.00268t}$

We seek t s.t. $T(t) = 37$

$37 = 20 + 12.1e^{-0.00268t}$

$1.404958 = e^{-0.00268t}$

take ln of both sides

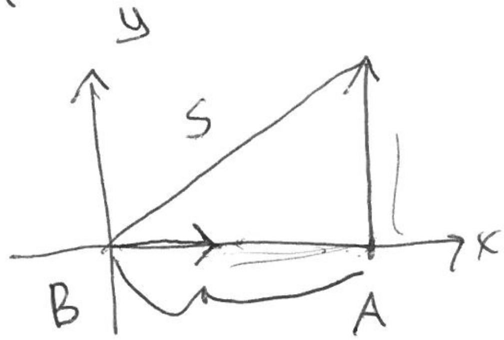
$.34 = -0.000268t$

$t \approx -127$

so 127 minutes before 1:30 PM

Test 4

3)



$x(0) = 170$

$\frac{dx}{dt} = -30.$

$\frac{dy}{dt} = 20$

Seek $\frac{ds}{dt}$ when $t=4$

$$s = (x^2+y^2)^{\frac{1}{2}}$$

$$\frac{ds}{dt} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\left[2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right]$$

$$\frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

When $t=4$, $x = 170-30(4) = 50$
$y = 20(4) = 80$

$$s = \sqrt{x^2+y^2} = \sqrt{50^2+80^2} = \sqrt{8900}$$

So $$\frac{ds}{dt} = \frac{50(-30) + 80(20)}{\sqrt{8900}}$$

$$= \frac{100}{\sqrt{8900}} = \frac{10}{\sqrt{89}}$$

Test 4 #7

$\tanh(x) = \frac{4}{5}$

$1 - \tanh^2 x = \operatorname{sech}^2(x)$

$1 - \left(\frac{4}{5}\right)^2 = \operatorname{sech}^2 x$

$\frac{9}{25} = \operatorname{sech}^2 x$

$\operatorname{sech}(x) = \frac{3}{5}$

$\cosh x = \frac{5}{3}$

$\sinh x = \frac{4}{3}$

$\tanh x = \frac{4}{5}$

$\operatorname{sech} x = \frac{3}{5}$

$\operatorname{csch} x = \frac{3}{4}$

$\coth x = \frac{5}{3}$

Now use

$\cosh^2 x - \sinh^2 x = 1$

$\left(\frac{5}{3}\right)^2 - \sinh^2(x) = 1$

$\frac{25}{9} - 1 = \sinh^2 x$

$\frac{16}{9} = \sinh^2 x$

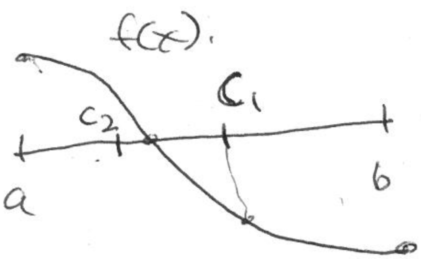
$\sinh x = \sqrt{\frac{16}{9}} = \frac{4}{3}$

021 Section 4.8

~~Mathematica Lab is due the Monday after Thanksgiving break, at the start of class. I encourage you to work with each other.~~
~~Each student needs to turn in their own paper copy.~~
~~To find the lab, search for UVM mathematica labs.~~
~~Go to the labs for math 021. Do the lab for Newton's method.~~
~~Before working on the lab, go through the tutorial videos that Prof Read made.~~

Sec 4.8 Newton's Method

We first briefly discuss the bisection method for finding roots,



Let $f(x)$ be continuous on $[a,b]$ with $f(a)$ and $f(b)$ having opposite signs.
By the intermediate value thm, there is a c in $[a,b)$ s.t. $f(c)=0$

In the bisection method. Let $c_1 = \frac{a+b}{2}$

If $f(c_1)=0$, you are done, $c_1 = c$

If $f(c_1) \neq 0$, then $f(c_1)$ must have a sign opposite to either $f(a)$ or $f(b)$. In our example, $f(c_1) < 0$, $f(a) > 0$

So set $c_2 = \frac{a+c_1}{2}$

Repeat this process until you get either $f(c_n)=0$
or $f(c_n)$ is within the desired accuracy.
$f(c_n)$ is as close to 0 as you want.

The nice feature is that nothing can go wrong.
The bad feature is that the bisection method is slow.
It can take many iterations to obtain the desired accuracy.

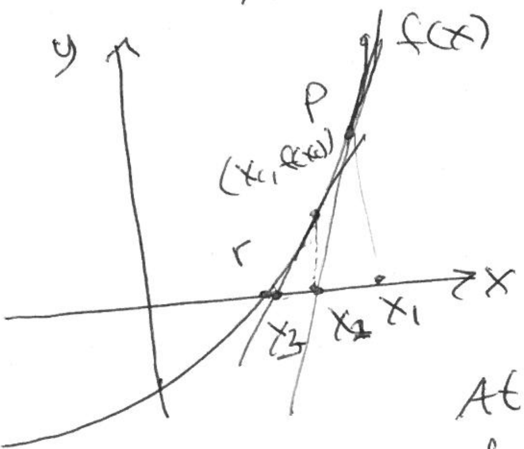
021 Section 4.8

Newtons method

when it works, it work fast

drawbacks 1) We need the function to be differentiable as well as continuous

2) Even for differen function, it might not work.



We seek the value of r

let $x_1 \neq r$

Let $P_1 = (x_1, f(x_1))$

Construct the tangent line at P_1 call it T_1

where T_1 intersects the x-axis is x_2

At, $P_2 = (x_2, f(x_2))$ construct the tangent line.

Repeat until you have the accuracy you want.

To find x_2 in terms of x_1, use the fact that the slope of L is $f'(x_1)$, so $y - f(x_1) = f'(x_1)(x - x_1)$

Letting the x-intercept of L being x_2, we have that the point $(x_2, 0)$ is on L,

so, $0 - f(x_1) = f'(x_1)(x - x_1)$

If $f'(x_1) \neq 0$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general, you have a recursive relation

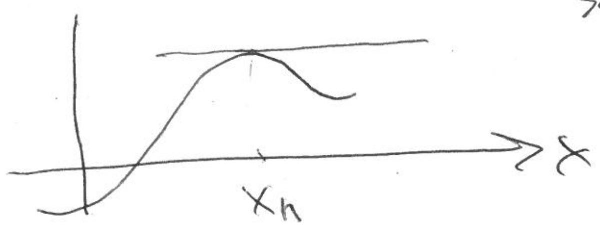
$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

021 Section 4.8

Famous Last Words

"what can possibly go wrong"

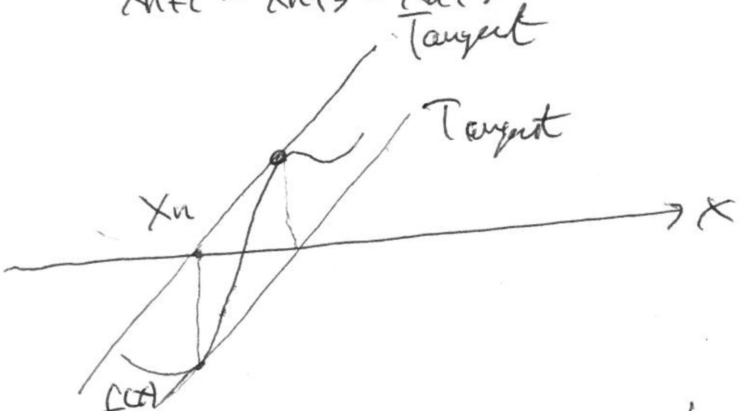
1) $f'(x_n)=0$. So the tangent line is horizontal so the tangent line will not intersect the x-axis so no x_{n+1}



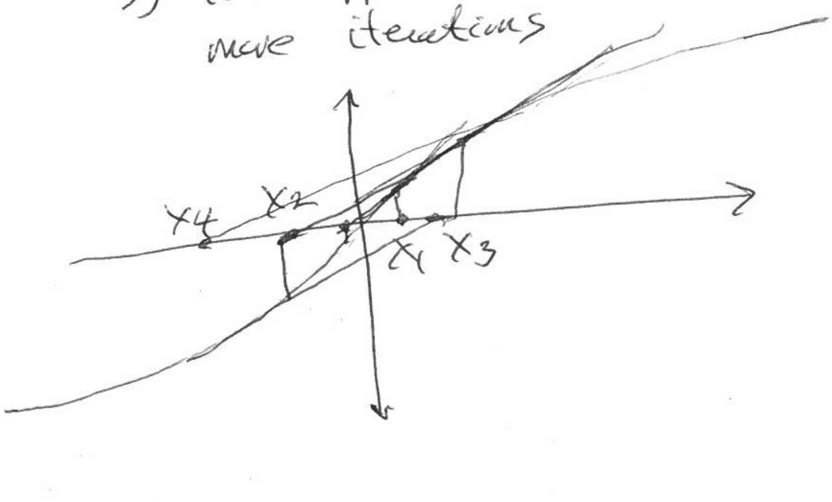
2) You can get into a loop so that

$x_n = x_{n+2} = x_{n+4} = \cdots$

$x_{n+1} = x_{n+3} = x_{n+5} = \cdots$



3) Your approximations might become worse as you do more iterations



021 Sec 4.9 Antiderivatives

To find velocity from position, take the derivative of position.

To find position from velocity we will take the antiderivative of the position.

Given $f(x)$, find $F(x)$ s.t. $F'(x) = f(x)$

in this case, $F(x)$ is called <u>an</u> antiderivative of $f(x)$.

$f(x) = x^2$, Find $F(x)$ s.t. $F'(x) = x^2$

$$F(x) = \frac{x^3}{3}; \text{ check } F'(x) = \frac{3x^2}{3} = x^2$$

$$F(x) = \frac{x^3}{3} + 12$$

$$F'(x) = x^2$$

If $F(x)$ is an antiderivative of $f(x)$

then $F(x) + C$ is also an antiderivative of $f(x)$

Derivatives and antiderivatives undo each other.

$$f(x) \xrightarrow{\int f(x)dx} F(x) \xrightarrow{\frac{d}{dx}F(x)} f(x)$$

Ex $f(x) = x^2 \xrightarrow{\int x^2 dx} \frac{x^3}{3} + C \xrightarrow{\frac{d}{dx}(x^3/3 + C)} x^2$

$$F(x) = x^2 \xrightarrow{\frac{d}{dx}x^2} 2x \xrightarrow{\int 2x\,dx} x^2 + C$$

$$F(x) \xrightarrow{\frac{d}{dx}F(x)} f(x) \xrightarrow{\int f(x)dx} F(x) + C$$

$\frac{d}{dx}\tan x = \sec^2 x$, so $\int \sec^2 x\,dx = \tan x + C$

Rules for finding antiderivatives.

$$\int \left[f(x) \pm g(x)\right]dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int a\, f(x)\,dx = a\int f(x)\,dx$$

Ex $\int \left[3x^5 - 7x^3 + 4\right]dx$

$= \int 3x^5 dx + \int (-7)x^3 dx + \int 4\,dx$

$= 3\int x^5 dx - 7\int x^3 dx + 4\int dx$

$= \left(\frac{3x^6}{6} + C_1\right) - 7\frac{x^4}{4} + C_2 + 4x + C_3$

$= \frac{x^6}{2} - \frac{7x^4}{4} + 4x + C$, where $C = C_1 + C_2 + C_3$

Ex $\int \left[3x^2 + \cos x - e^{5x} + \frac{1}{x}\right]dx$

$\frac{\cancel{3}x^3}{\cancel{3}} + \sin x - \frac{e^{5x}}{5} + \ln|x| + C$

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EX $\int \frac{x+1}{\sqrt{x}}\,dx = \int \left[\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right]dx = \int \left[x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right]dx$

$= \left(\frac{x^{3/2}}{\frac{3}{2}} + C_1\right) + \left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C_2\right) = \frac{2}{3}x^{3/2} + 2x^{\frac{1}{2}} + C$

EX $\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x \cos x}\,dx = \int \left(\frac{\sin x}{\cos x}\right)\frac{1}{\cos x}\,dx$

$= \int \tan x \sec x\,dx = \sec x + C$

(since $\frac{d}{dx}\sec x = \tan x \sec x$)

EX Find $F(x)$ if $F'(x) = \frac{x^2}{3} + \frac{3x}{2} + 7$ and $F(1) = 2$

$F(x) = \frac{x^3}{9} + \frac{3x^2}{4} + 7x + C$

To find the actual value of C, use $F(1) = 2$

$2 = \frac{1^3}{9} + \frac{3 \cdot 1^2}{4} + 7(1) + C$

$2 = \frac{283}{36} + C \Rightarrow C = \frac{-211}{36}$

$F(x) = \frac{x^3}{9} + \frac{3x^2}{4} + 7x - \frac{211}{36}$

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$$\int x^2\,dx = \frac{x^3}{3} + C$$

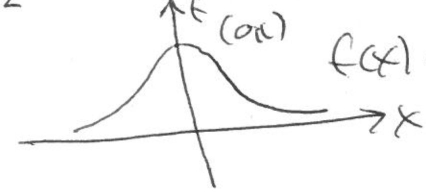
Any rule or formula for finding a derivative give a rule or formula for finding an antiderivative

$f(x)$	$F(x)$ (antiderivative)
$\cos x$	$\sin x + C$
$\sin x$	$-\cos x + C$
e^{ax}	$\frac{e^{ax}}{a} + C$
$x^n,\ n \neq -1$	$\frac{x^{n+1}}{n+1} + C$

If $n=-1$, $x^n = x^{-1} = \frac{1}{x}$ $\qquad \ln|x| + C$

Note Not all differentiable $f(x)$ have an antiderivative.

Ex $f(x) = e^{-x^2}$ i.e. There is no (elementary) function $F(x)$ s.t. $F'(x) = e^{-x^2}$



$f(x) = e^{-x^2}$

Note $F(x)$ is often written as:

$\int f(x)\,dx$. This is the indefinite integral of $f(x)$

In $\int f(x)\,dx$, the dx means that the only letter treated as a variable is x

Ex $\int xy^3\,dx = \frac{x^2}{2}y^3 + C$

$\int xy^3\,dy = \frac{xy^4}{4} + C$

Find $f(x)$ if $f''(x) = 12x^2 + 6x - 4$

and $f(0) = 4$ and $f(1) = 1$

$f'(x) = \int f''(x)dx = \int (12x^2 + 6x - 4)dx = 4x^3 + 3x^2 - 4x + C$

now $f(x) = \int f'(x)dx = \int (4x^3 + 3x^2 - 4x + C)$

$= x^4 + x^3 - 2x^2 + Cx + D$

Now use $f(1) = 1$, $1 = 1^4 + 1^3 - 2\cdot 1^2 + C\cdot 1 + D$

$1 = C + D$

also $f(0) = 4$, $4 = D$

so $1 = C + D = C + 4$

$C = -3$

$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$

Ex Find $f(x)$ if $f''(x) = 12x^2 + 6x - 4$ and $f(0) = 4$, $f'(0) = 22$

$f'(x) = \int f''(x)dx = \int (12x^2 + 6x - 4)dx = 4x^3 + 3x^2 - 4x + C$

now use, $f'(0) = 22$, so, $22 = 4\cdot 0^3 + 3\cdot 0^2 - 4(0) + C \Rightarrow C = 22$

so $f'(x) = 4x^3 + 3x^2 - 4x + 22$

now $f(x) = \int f'(x)dx = \int (4x^3 + 3x^2 - 4x + 22)dx$

$f(x) = x^4 + x^3 - 2x^2 + 22x + D$,

now $4 = f(0) = 0^4 + 0^3 - 2\cdot 0^2 + 22\cdot 0 + D$

$4 = D$

$f(x) = x^4 + x^3 - 2x^2 + 22x + 4$

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Rectilinear Motion - back and forth motion along a straight line

Ex A particle moves along a straight line with acceleration $a(t) = 6t - 4$

Find the position function $s(t)$ given that $v(1) = 3$, $s(2) = -5$

Solution

$$V(t) = \int a(t)\,dt = \int (6t-4)\,dt$$

$$= \frac{6t^2}{2} - 4t + C_1 = 3t^2 - 4t + C_1$$

$$v(1) = 3 = 3\cdot 1^2 - 4(1) + C_1 \Rightarrow C_1 = 4$$

so $V(t) = 3t^2 - 4t + 4$

$$s(t) = \int v(t)\,dt = \int (3t^2 - 4t + 4)\,dt$$

$$= \frac{3t^3}{3} - \frac{4t^2}{2} + 4t + C_2$$

$$s(t) = t^3 - 2t^2 + 4t + C_2$$

$$s(2) = -5 = \cancel{2^3 - 2\cdot 2^2} + 4(2) + C_2$$

$$-5 = 8 + C_2 \Rightarrow C_2 = -5 - 8 = -13$$

$$s(t) = t^3 - 2t^2 + 4t - 13$$

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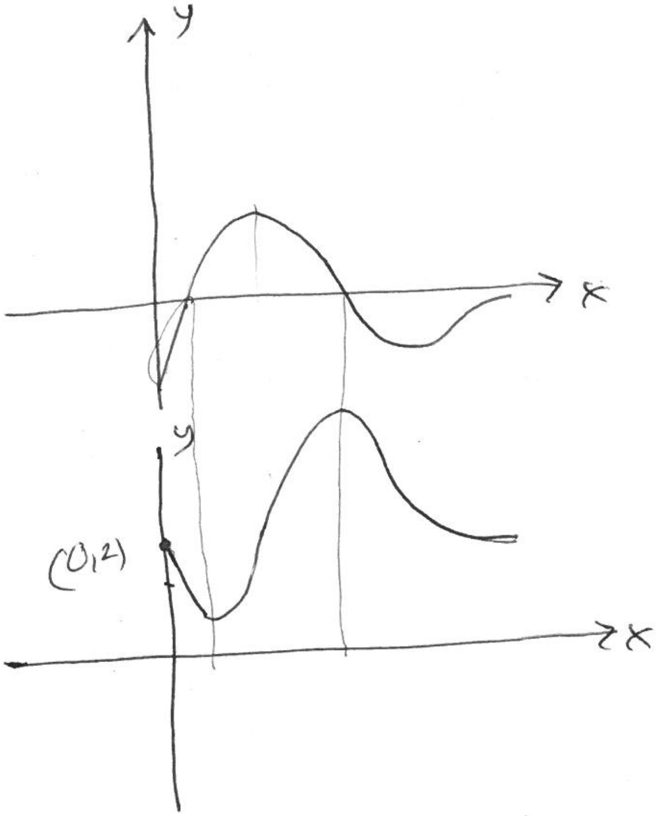
Graphing an antiderivative:

Let $F'(x) = f(x)$. Given the graph of $f(x)$, find the graph of $F(x)$ and $F(0) = 2$



$y = f(x) = F'(x)$

$F(x)$

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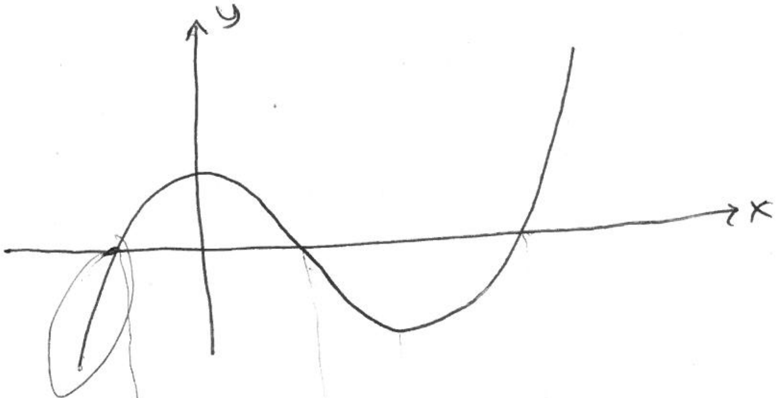

$y = f(x)$

Graph $F(x)$

if $F'(x) = f(x)$

and $F(1) = -2$