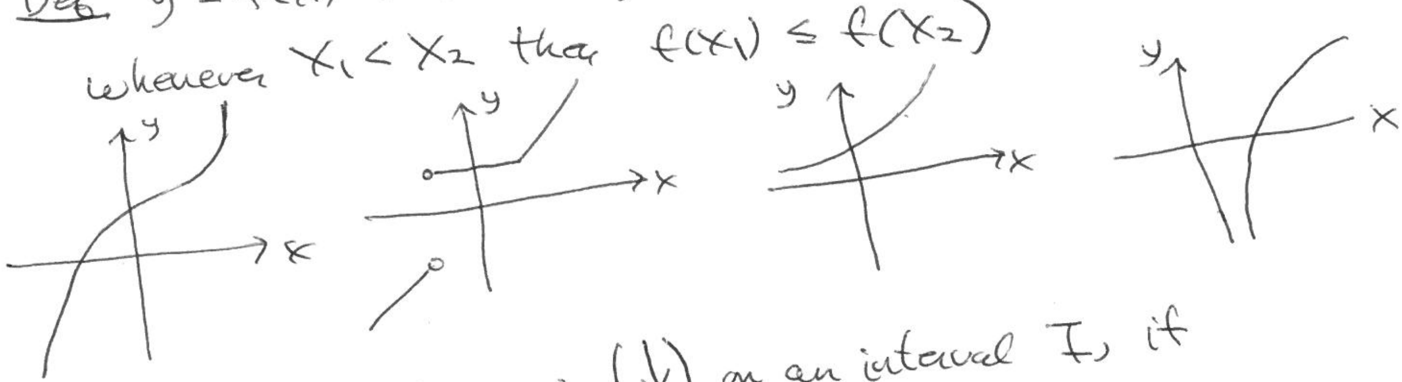
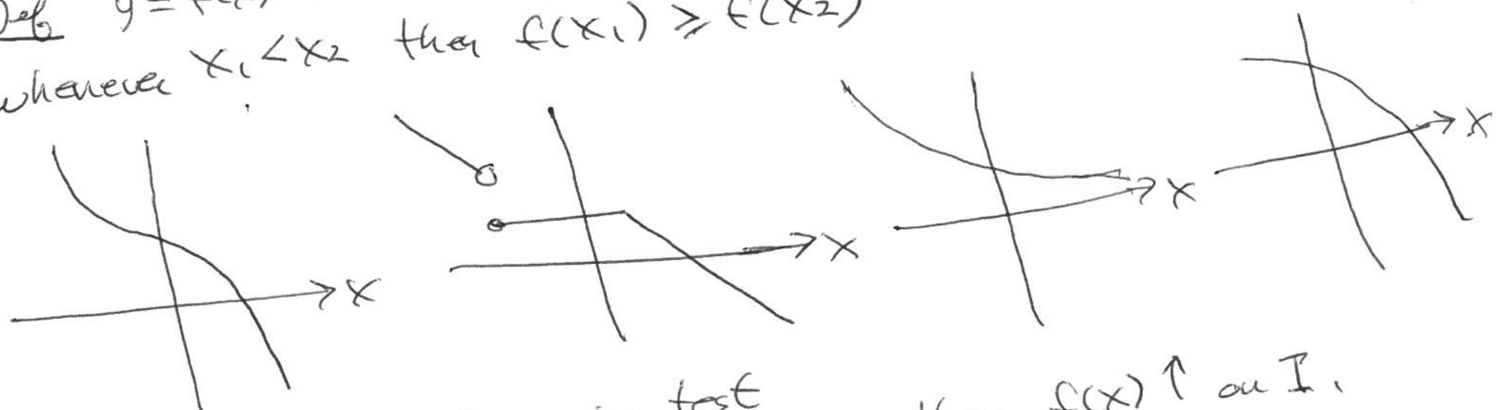
021 Sec 4.3

Section 4.3 How derivatives affect the shape of a graph.

Def. $y=f(x)$ is increasing ($\uparrow$) on an interval I, if whenever $x_1 < x_2$ then $f(x_1) \le f(x_2)$



Def $y=f(x)$ is decreasing ($\downarrow$) on an interval I, if whenever $x_1 < x_2$ then $f(x_1) \geq f(x_2)$



The increasing and decreasing test

a) If $f'(x) > 0$ on an open interval I, then $f(x) \uparrow$ on I.

b) If $f'(x) < 0$ " " " " " $f(x) \downarrow$ on I

Pf a) Let $x_1 < x_2$ on I. We want to show $f(x_1) < f(x_2)$

We are given that $f'(x) > 0$ on I so f' exists on I.

so f is differentiable on (x_1, x_2)

f is continuous " $[x_1, x_2]$

so by MVT, there is a c between x_1 and x_2 s.t.

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$$

We assumed $f'(c) > 0$, and $x_2 > x_1$ so $x_2 - x_1 > 0$

so RHS > 0

so LHS > 0

so $f(x_2) > f(x_1)$

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EX Find where $y = f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is ↑ and where ↓

Solution $f'(x) = 12x^3 - 12x^2 - 24x$
$= 12x(x+1)(x-2)$

In increasing order, the critical numbers are $x = -1, 0, 2$

Note f' can only change sign at a critical number

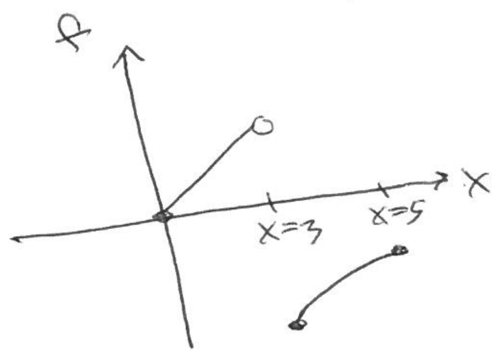
	12	x	x+1	x−2	f'	f
			−	−	−	↓
$(-\infty, -1)$	+	−	−	−	+	↑
$(-1, 0)$	+	−	+	−	−	↓
$(0, 2)$	+	+	+		+	↑
$(2, \infty)$	+	+	+	+		

$x+1 > 0$
$x > -1$

$x-2 > 0$
$x > 2$

So f is increasing on the open intervals $(-1, 0)$ and $(2, \infty)$
f is decreasing " " " " $(-\infty, -1)$ and $(0, 2)$

Caution ~~We have~~ We can have
f ↑ on $[0, 3)$
f ↑ on $[3, 5]$
but f is not increasing on $[0, 5]$



Note f is discontinuous at $x = 3$

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First Derivative Test

Let $x=c$ be a crit number for a continuous function f

Going Left to Right

1) If f' goes from positive to neg at $x=c$

   f " " $\uparrow$ to $\downarrow$

   Then $(c, f(c))$ is a local maximum

2) If f' goes from negative to positive at $x=c$

   f " " $\downarrow$ to $\uparrow$

   Then $(c, f(c))$ is a local minimum

<u>Note</u> The first derivative test does not give the value of the function at the local min or max, only where the extrema occurs. To find the value of the function, evaluate the function at the critical pt.

<u>Ex</u> Using the first derivative test find where $f(x)$ has local maximums and minimums if

$$f'(x) = -3x(x-4)(x+2)(x-3)^2$$

	-3	x	$x-4$	$x+2$	$(x-3)^2$	f'	f
$(-\infty,-2)$	$-$	$-$	$-$	$-$	$+$	$+$	$\uparrow$
$(-2,0)$	$-$	$-$	$-$	$+$	$+$	$-$	$\downarrow$
$(0,3)$	$-$	$+$	$-$	$+$	$+$	$+$	$\uparrow$
$(3,4)$	$-$	$+$	$-$	$+$	$+$	$+$	$\uparrow$
$(4,\infty)$	$-$	$+$	$+$	$+$	$+$	$-$	$\downarrow$

local max at $x=-2$

local min at $x=0$

local max at $x=4$

f is increasing on $(-\infty,-2)$ and on $(0,4)$

f is decreasing on $(-2,0)$ and on $(4,\infty)$

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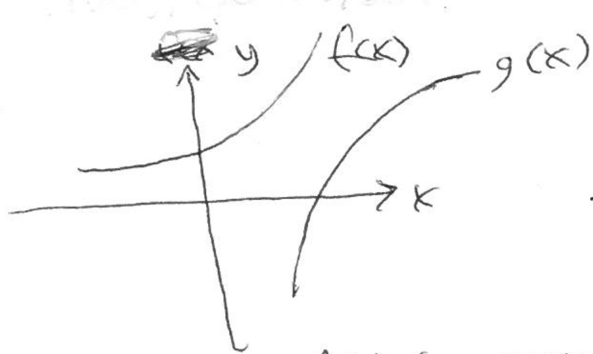
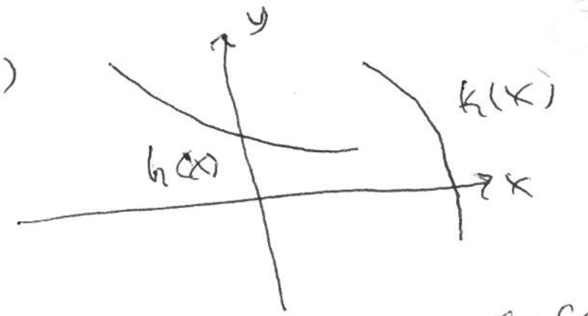
Concavity





Concavity A function $f(x)$ is concave down, $\cap$ at a pt $(c, f(c))$ if, near p, the graph of f lies below the tangent line at $(c, f(c))$

A function $f(x)$ is concave up, $\cup$, at a pt $p = (c, f(c))$ if near p, The graph of f lies above the tangent line at $(c, f(c))$

A function f is concave up, (resp, concave down) on an open interval I, if the function is concave up, (res, concave down) for all pts in I

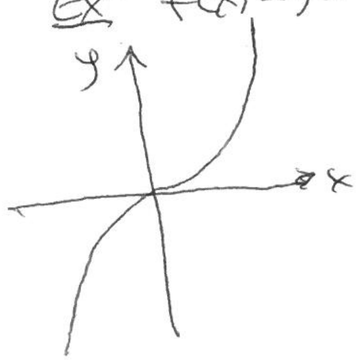
Note $f \uparrow \cup$, $g \uparrow \cap$, $h \downarrow \cup$, $k \downarrow \cap$

So, no connection between increasing/decreasing and concave up/concave down

Now f'' measures the rate of change of the rate of change, So if $f'' > 0$, the rate of change of the first derivative is positive so $f'' > 0 \Rightarrow f$ is concave up,

Likewise $f'' < 0 \Rightarrow f$ is concave down,

Ex $f(x) = y = x^3$



$f'(x) = 3x^2$, so $f \uparrow$ everywhere

$f''(x) = 6x$

	6	x	f''	
$(-\infty, 0)$	+	−	−	$\cap$
$(0, \infty)$	+	+	+	$\cup$

So f is concave down on $(-\infty, 0)$

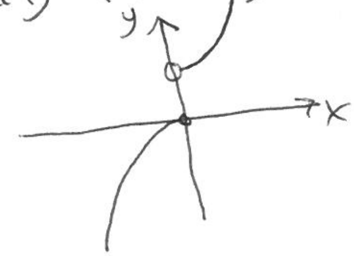
f " " up on $(0, \infty)$

The pt $(0,0)$ is an inflection pt.

Def A pt $(c, f(c))$ is an inflection pt for the graph of $f(x)$ if

1) The concavity of f changes at pt $(c, f(c))$
2) The pt $(c, f(c))$ is a point of continuity for $f(x)$

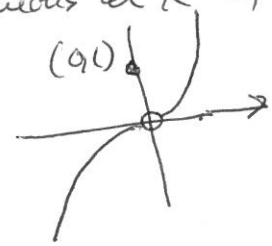
Non Ex $y = f(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ x^3+1 & \text{if } x > 0 \end{cases}$



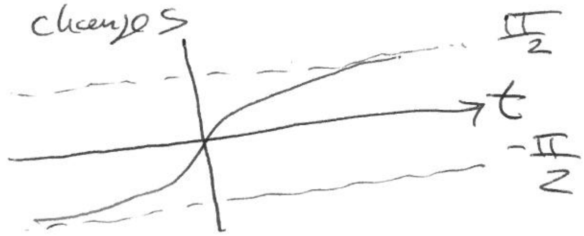
Note for $\begin{cases} x<0 & f \text{ is cc down} \\ x>0 & f \text{ is cc up} \end{cases}$

So, the concavity changes, but f is discontinuous at $x=0$, so no inflection pt.

Non Ex $f(x) = \begin{cases} x^3 & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$



No inflection pt at $x=0$ even though concavity changes



Ex $g(t) = \tan^{-1}(t)$

$g'(t) = \frac{1}{1+t^2}$

$g''(t) = \frac{(1+t^2)0 - 2t(1)}{(1+t^2)^2}$

$g''(t) = \frac{-2t}{(1+t^2)^2}$. **Note** $g''(t) > 0$ if $t<0$, so g cc up on $(-\infty, 0)$
$g''(t) < 0$ if $t > 0$, so g cc down on $(0, \infty)$

So, concavity changes at $t=0$
~~And~~, $(0,0)$ is a pt of continuity for $g(t)$
So $(0,0)$ is an inflection pt

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The second derivative test

Let $f(x)$ have a critical value at $x=c$, then

a) $f''(c)>0$ implies a local minimum at $x=c$

b) $f''(c)<0$ " " " maximum at $x=c$

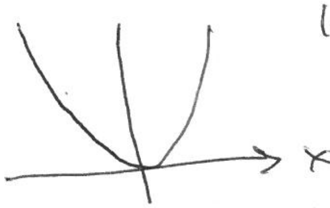
c) $f''(c)=0$, no conclusion can be drawn.

Investigate c)

i) $f(x)=x^4$, $f'=4x^3$, $f'=0$ when $x=0$

$f''=12x^2$, $f''(0)=0$

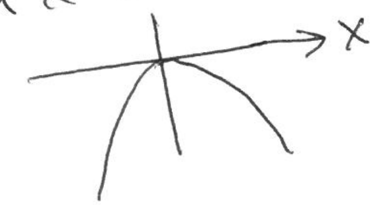
local minimum at $x=0$



ii) $f(x)=-x^4$, $f'=-4x^3$, $f'(x)=0$ at $x=0$

$f''(x)=-12x^2$, $f''(0)=0$

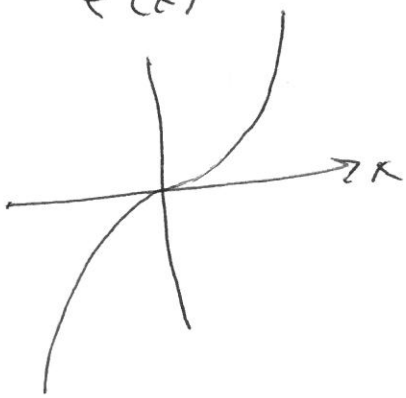
Local maximum at $(0,0)$



iii) $f(x)=x^3$, $f'(x)=3x^2$, $f'(0)$ at $x=0$

$f''(x)=6x$ $f''(0)=0$

no min, nor max, but an inflection pt at $(0,0)$,



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Ex Use the second derivative test to find extrema for

$$f(x) = -3x^5 + 5x^3$$

Solution $f'(x) = -15x^4 + 15x^2 = -15x^2(x^2-1) = -15x^2(x-1)(x+1)$

critical values at $x = -1, 0, 1$

Now $f''(x) = -60x^3 + 30x$

And

For crit value $x=-1$, $f''(-1) = 30 > 0$, so a local min at $x=-1$

" " " $x=+1$, $f''(1) = -30 < 0$ " " local max at $x=1$

For crit value $x=0$, $f''(0) = 0$. So, other methods must be used.

Use the first derivative test:

	-15	x^2	$x-1$	$x+1$	f'	f
$(-\varepsilon, 0)$	$-$	$+$	$-$	$+$	$+$	$\uparrow$
$(0, \varepsilon)$	$-$	$+$	$-$	$+$	$+$	$\uparrow$

$\rightarrow$ no max nor min at $x=0$

We now look at f''

	-30	x	$x-1$	$x+1$	f''	f
$(-\varepsilon, 0)$	$-$	$-$	$-$	$+$	$-$	$\frown$
$(0, \varepsilon)$	$-$	$+$	$-$	$+$	$+$	$\smile$

concavity changes at $x=0$

and $(0,0)$ is a pt of continuity

so the $(0,0)$ is an inflection pt.

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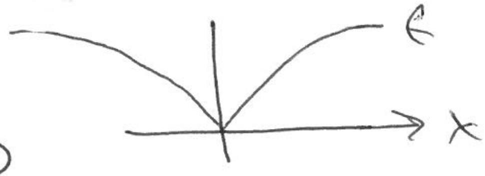
Ex $f(x)\ \sqrt{|x|}$



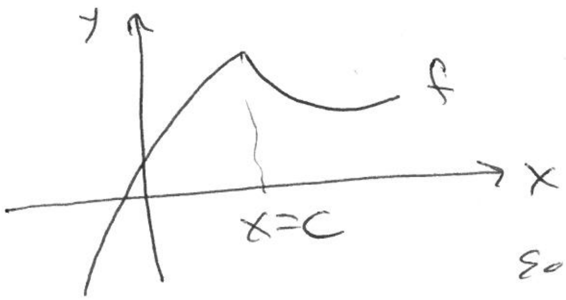
Note f is cc down on $(-\infty,0)$

f is cc down on $(0,\infty)$

f is contin at $x=0$

since the concavity does not change at $x=0$, no inflection pt

Ex



f is cc down on $(-\infty,c)$

f is cc up on (c,∞)

$(c,f(c))$ is a pt of continuity

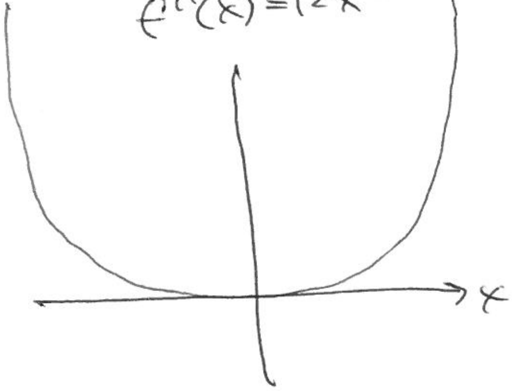
so $(c,f(c))$ is an inflection pt,

even though $f''(c)$ dne,

Ex $f(x)=x^4$

$f'(x)=4x^3$

$f''(x)=12x^2$



$f''(x)>0$ if $x\neq 0$, f cc up for $x\neq 0$

$(0,0)$ is a pt of continuity

$f''(0)=0$

but $(0,0)$ is not an inflection pt

because the concavity does not change.

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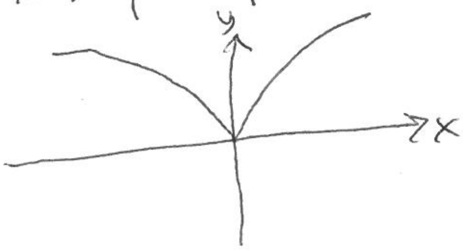
Ex $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

The only critical pt is at $x=0$, where f' d.n.e.

	$2/3$	$x^{-1/3}$	f'	f
$(-\infty, 0)$	+	−	−	↘
$(0, \infty)$	+	+	+	↗

} local min at $x=0$ $(0,0)$



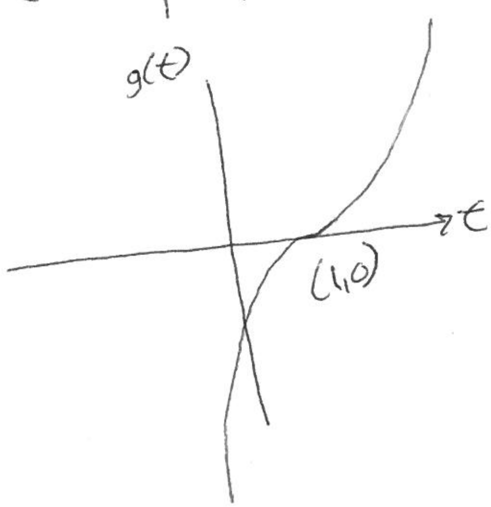
Caution $f'(c) = 0$ does not mean that we must have a local max or a local min at $x=c$

Ex $g(t) = (t-1)^3$

$g'(t) = 3(t-1)^2 \cdot 1 = 3(t-1)^2$

$g'(t) = 0$ at $t=1$

	3	$(t-1)^2$	g'	g
$(-\infty, 1)$	+	+	+	↗
$(1, \infty)$	+	+	+	↗



$g'(0) = 0$ at $(1,0)$ but the point $(1,0)$ is not a maximum nor a minimum

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Sec 4.4 Indeterminate Forms and L'Hospital's Rule

Recall that $\frac{d}{dx} \frac{f(x)}{g(x)} \neq \frac{f'(x)}{g'(x)}$

However there is a use for $\frac{f'(x)}{g'(x)}$

Consider $\lim_{x\to\infty} \frac{e^x}{x}$. This is an indeterminate form of type $\frac{\infty}{\infty}$

One part of L'Hospitals rule is:

If $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is an indeterminate form of type $\frac{\infty}{\infty}$

then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$

In our example: $\lim_{x\to\infty} \frac{e^x}{x} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{d}{dx} e^x}{\frac{d}{dx} x} = \lim_{x\to\infty} \frac{e^x}{1} = \lim_{x\to\infty} e^x = \infty$

Ex $\lim_{x\to\infty} \frac{e^x}{x^2} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{2x} = \frac{1}{2} \lim_{x\to\infty} \frac{e^x}{x} \overset{H}{=} \frac{1}{2} \lim_{x\to\infty} \frac{e^x}{1} = \frac{1}{2} \lim_{x\to\infty} e^x$

$= \frac{1}{2}(\infty) = \infty$

In general: $\lim_{x\to\infty} \frac{e^x}{\text{any polynomial}} = \pm\infty$

$\lim_{x\to\infty} \frac{\text{any polynomial}}{e^x} = 0$

Ex $\lim_{x\to 0} \frac{\sin x}{x}$, this is an indeterminate form of type $\frac{0}{0}$

$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\frac{d}{dx} \sin x}{\frac{d}{dx} x} = \lim_{x\to 0} \frac{\cos x}{1} = \cos(0) = 1$

L' Hospital's Rule

Let f and g be differentiable functions on same open interval I, which contains a, (Note f, g need not be diff at a itself)

Suppose that

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0$$

or

$$\lim_{x \to \infty} f(x) = \pm\infty, \quad \lim_{x \to \infty} g(x) = \pm\infty$$

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Provided that $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists or equals $+\infty$ or $-\infty$.

Ex $\lim_{x \to 0} \frac{e^x - 1}{\sin x} \overset{H}{=} \lim_{x \to 0} \frac{e^x}{\cos x} = \frac{e^0}{\cos 0} = \frac{1}{1} = 1$

Ex $\lim_{x \to 0} \frac{\sin x}{x^2 + x} \overset{H}{=} \lim_{x \to 0} \frac{\cos x}{2x+1} = \frac{\cos 0}{2 \cdot 0 + 1} = \frac{1}{1} = 1$

note this is not an indeterminate form, So, we can not use L'Hospital's rule

If we did use L'Hospital's rule.

$$\lim_{x \to 0} \frac{\frac{d}{dx} \cos x}{\frac{d}{dx}(2x+1)} = \lim_{x \to 0} \frac{-\sin x}{2} = \frac{-\sin(0)}{2} = 0$$

a wrong answer.

EX $\lim_{x\to 1^+}\left[\frac{1}{\ln x}-\frac{1}{x-1}\right]$, indeterminate form of type $\infty-\infty$

$= \lim_{x\to 1^+}\left[\frac{(x-1)-\ln x}{(\ln x)(x-1)}\right]$, type $\frac{0}{0}$, we can now use "the standard" L'Hospital's Rule

$= \lim_{x\to 1^+}\frac{1-\frac{1}{x}}{(\ln x)\cdot 1+\frac{1}{x}(x-1)}$

$\overset{\text{algebra}}{=} \lim_{x\to 1^+}\frac{1}{1+\ln x+x\left(\frac{1}{x}\right)} = \lim_{x\to 1^+}\frac{1}{2+\ln x} = \frac{1}{2+0} = \frac{1}{2}$

Indeterminate Products of type $(0\cdot\infty)$ or $(0\cdot(-\infty))$

EX $\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}}$, type $\left(\frac{-\infty}{\infty}\right)$ we use L'Hospital's Rule

$\overset{H}{=} \frac{\frac{d}{dx}\ln x}{\frac{d}{dx}\left(\frac{1}{x}\right)} = \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-x) = 0$

So $\lim_{x\to 0^+} x\ln x = 0$

Indeterminate Powers

We have already seen that: $\lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right) = e$

This is an example of an indeterminate power of type 1^∞

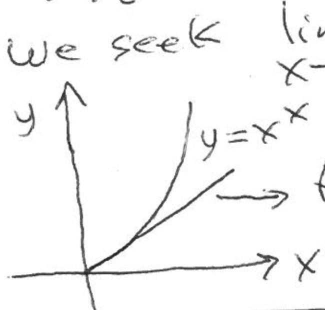
Other indeterminate powers are

$0^0,\ \infty^0,\ 0^\infty$

Ex $\lim_{x\to 0^+} x^x$: Solution: write $x^x = (e^{\ln x})^x$

We seek $\lim_{x\to 0^+} e^{x\ln x} = e^{\boxed{\lim_{x\to 0^+} x\ln x}} = e^0 = 1$

↓ is our previous example



→ tangent line at ~~x~~=0, has slope of 1

Ex $\lim_{x\to 0^+} \left(1+\sin 4x\right)^{\cot x}$ type 1^∞ $\cot x = \dfrac{\cos x}{\sin x}$

Solution Let $y = (1+\sin 4x)^{\cot x}$

Take ln of both sides

$\ln y = \ln(1+\sin 4x)^{\cot x}$

$\ln y = \cot x \ln(1+\sin 4x)$

$\ln y = \dfrac{\ln(1+\sin 4x)}{\tan x}$, type $\frac{0}{0}$, use L'Hospitals

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\frac{d}{dx}\ln(1+\sin 4x)}{\frac{d}{dx}\tan x} = \lim_{x\to 0^+} \frac{\frac{d}{dx}\ln(1+\sin 4x)}{\frac{d}{dx}\tan x}$$

$$= \lim_{x\to 0^+} \frac{\frac{4\cos 4x}{1+\sin 4x}}{\sec^2 x} = \frac{\frac{4\cos(4\cdot 0)}{1+\sin(4\cdot 0)}}{\sec^2(0)} = \frac{\frac{4\cdot 1}{1+0}}{1^2} = 4$$

So $\lim_{x\to 0} \ln y = 4$. Now, exponentiate

$e^{\lim_{x\to 0} \ln y} = e^4$

$y = e^4$, recalling our definition of y

$\lim_{x\to 0^+} (1+\sin 4x)^{\cot x} = e^4$

Section 4.5 – Summary of curve sketching

<u>Ex</u> Graph: $f(x) = x^4 - 12x^3 + 48x^2 - 64x$

$$f(x) = x(x-4)^3$$

$$f'(x) = 4(x-1)(x-4)^2$$

$$f''(x) = 12(x-4)(x-2)$$

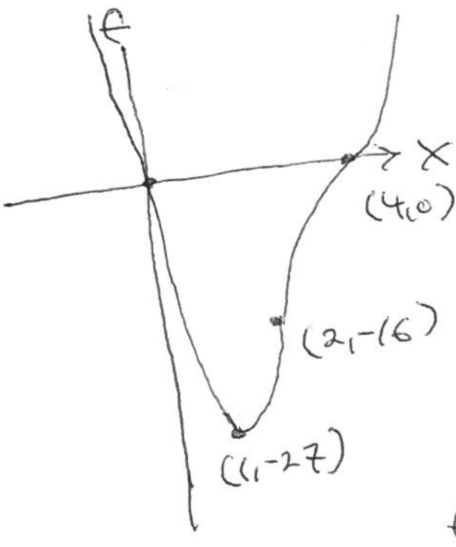
$f(1) = -27$, $f(2) = -16$, $f(4) = 0$, $f(0) = 0$

Critical values in order are 1, 2, 4

	f	4	$x-1$	$(x-4)^2$	f'	12	$x-4$	$x-2$	f''
$(-\infty, 1)$	↓ ∪	+	−	+	−	+	−	−	+
$(1,2)$	↑ ∪	+	+	+	+	+	−	−	+
$(2,4)$	↑ ∩	+	+	+	+	+	−	+	−
$(4,\infty)$	↑ ∪	+	+	+	+	+	+	+	+

$x-1>0$
$x>1$

$x-1<0$
$x<1$



f is increasing on $(1,\infty)$

f is decreasing on $(-\infty,1)$

f is concave up on $(-\infty,2)$
and on $(4,\infty)$

f is concave down on $(2,4)$

f has a local and global minimum at $(1,-27)$

f does not have any global or local maximums

f has an inflection pt at $(2,-16)$
and also at $(4,0)$

$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} f(x) = \infty$$

021 Sec 4.5

Graph $f(x) = \dfrac{2(x^2-9)}{x^2-4}$

given that $f'(x) = \dfrac{20x}{(x^2-4)^2}$, $f''(x) = \dfrac{-20(3x^2+4)}{(x^2-4)^3}$

We also have $f(0) = \frac{9}{2}$

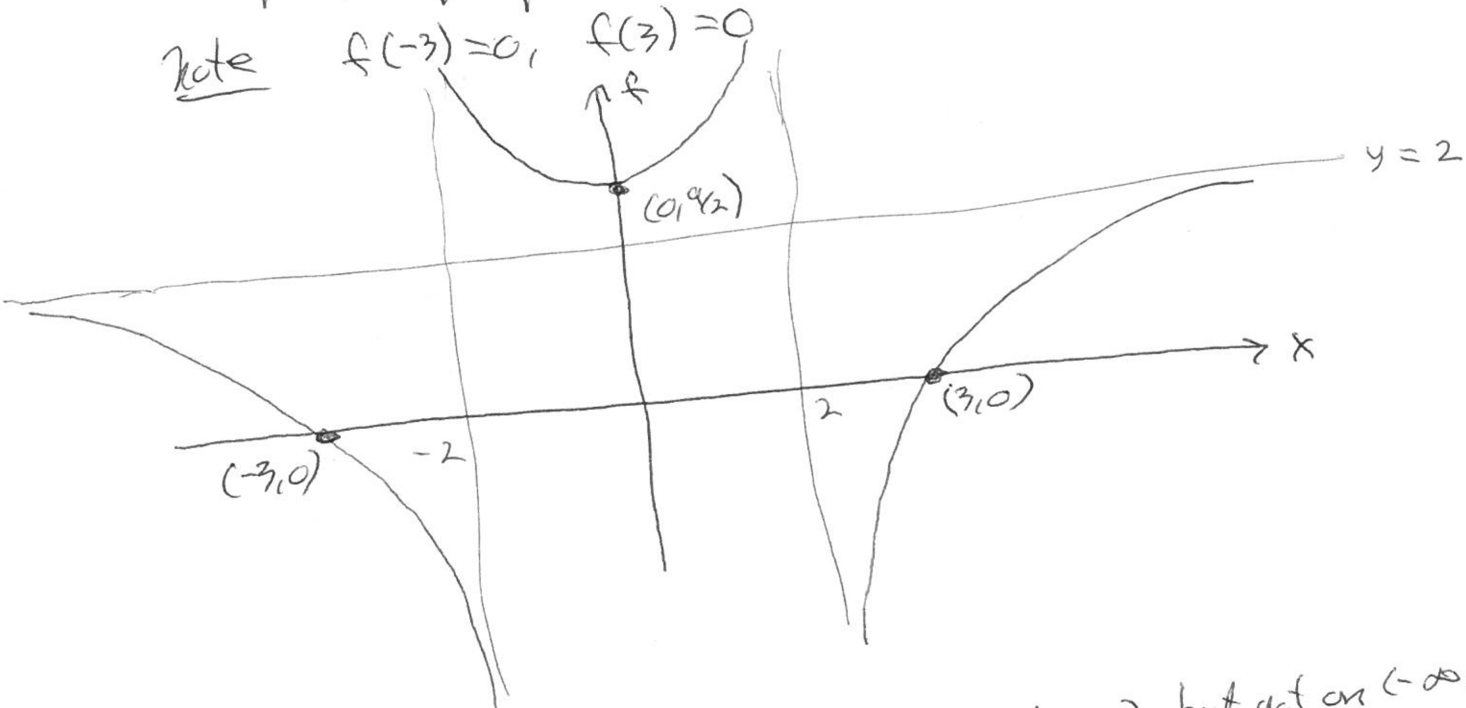
Vertical asymptotes at $x = -2, 2$

Horizontal asymptotes $\lim\limits_{x\to\infty} f(x) = 2$, $\lim\limits_{x\to-\infty} f(x) = 2$

Critical values are $x = -2, 0, 2$

	f	$2c$	x	$(x^2-4)^2$	f'	-20	$3x^2+4$	$(x^2-4)^3$	f''
$(-\infty,-2)$	↓∩	+	−	+	−	−	+	+	−
$(-2,0)$	↓∪	+	−	+	−	−	+	−	+
$(0,2)$	↑∪	+	+	+	+	−	+	−	+
$(2,\infty)$	↑∩	+	+	+	+	−	+	+	−

Note $f(-3)=0$, $f(3)=0$



$f(x)$ is decreasing on $(-\infty,-2)$ and on $(-2,0)$, but not on $(-\infty,0)$

$f(x)$ is increasing on $(0,2)$ and on $(2,\infty)$ but not on $(0,\infty)$

f is concave up on $(-2,2)$, f is concave down on $(-\infty,-2)$ and $(2,\infty)$

local minimum at $(0, \frac{9}{2})$. No local maximums, No global extremaa

No inflection pts. Note concavity changes at $x = \pm 2$ but when $x = 2$ we do not have a continuity pt.

$\lim\limits_{x\to-\infty} f(x) = 2$, $\lim\limits_{x\to\infty} f(x) = 2$

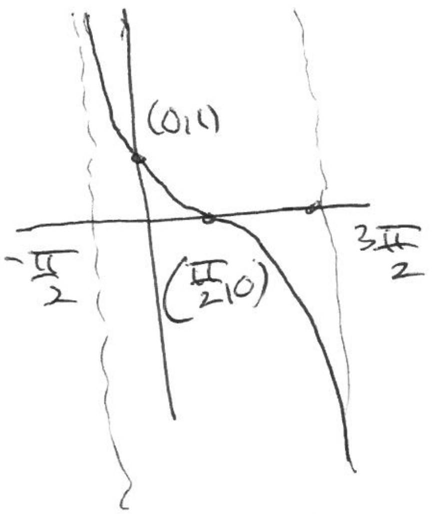
021 Sec 4.5

Ex graph $f(x) = \frac{\cos x}{1+\sin x}$ given $f' = \frac{-1}{1+\sin x}$, $f'' = \frac{\cos x}{(1+\sin x)^2}$

Solution The period of the function is 2π. Let graph f on the widest interval without asymptotes.

We will use $x \in (-\frac{\pi}{2}, \frac{3\pi}{2})$

x-intercept is $(\frac{\pi}{2}, 0)$, y intercept is the pt $(0,1)$



no critical values $f' < 0$ on $(-\frac{\pi}{2}, \frac{3\pi}{2})$

$f' = \frac{-}{+}$ " "

so f is decreasing on $(-\frac{\pi}{2}, \frac{3\pi}{2})$

$f'' = \frac{\cos x}{(1+\sin x)^2} = \frac{}{+}$

The denom is always > 0

The numer is neg for $x > \frac{\pi}{2}$

" numerator is pos for $(-\frac{\pi}{2}, \frac{\pi}{2})$

So f is cc up on $(-\frac{\pi}{2}, \frac{\pi}{2})$

f is cc down on $(\frac{\pi}{2}, \frac{3\pi}{2})$

Inflection pt at $(\frac{\pi}{2}, 0)$

No max's or min.

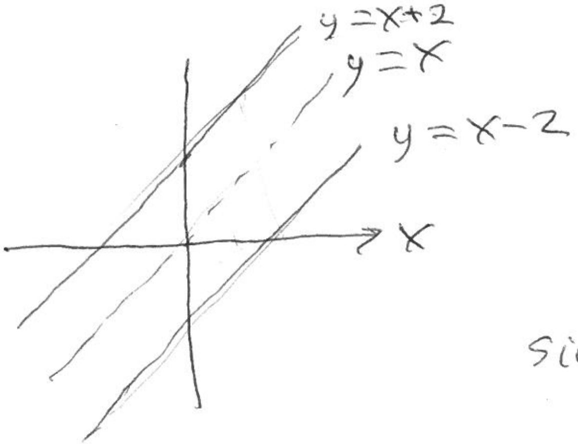
vertical asymptotes at $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

021 Section 4.5

EX Graph $f(x) = x + 2\sin x$

Note $-1 \le \sin x \le 1$

So $-2 \le 2\sin x \le 2$



$f' = 1 + 2\cos x$

since $-1 \le \cos x \le 1$

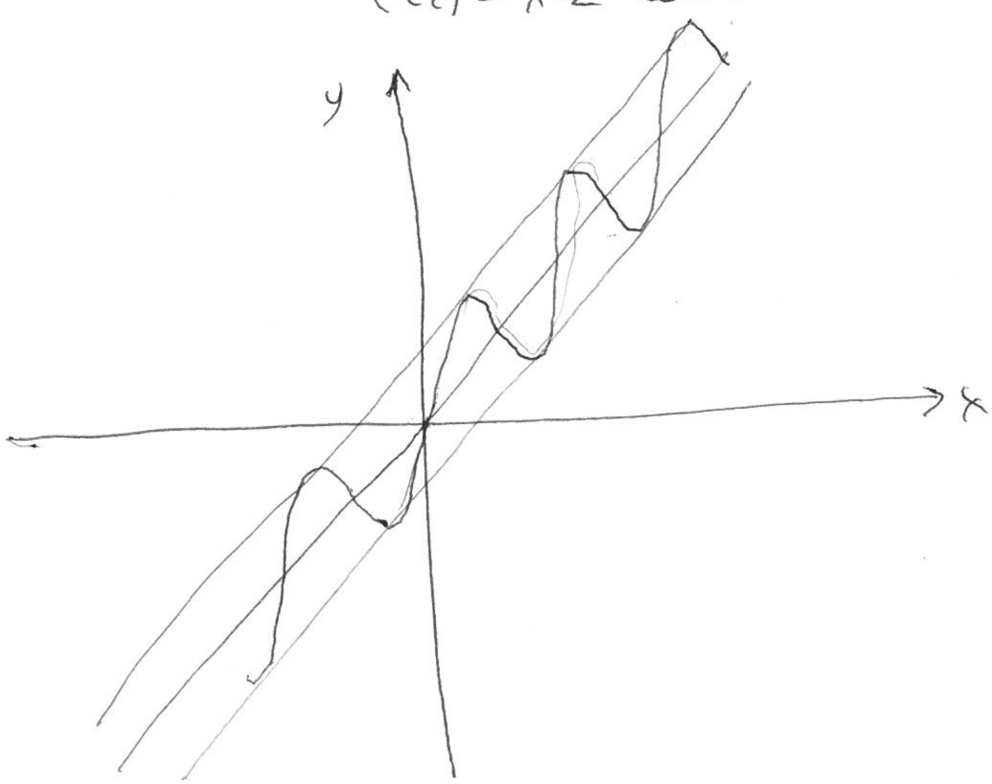
$-2 \le 2\cos x \le 2$

So f' is always in the interval $[-1, 3]$

Also $f(x) = x$ when $\sin x = 0$, $x = 0, \pi, 2\pi$

$f(x) = x+2$ when $\sin x = 1$, $x = \frac{\pi}{2} \cdots$

$f(x) = x-2$ when $\sin x = -1$, $x = \frac{3\pi}{2}$



021 Sec 4.5

Qu

do th

EX Graph $f(x) = \frac{x}{\sqrt{x^2-1}}$ given that

$f'(x) = \frac{-1}{(x^2-1)^{3/2}}$, $f''(x) = \frac{3x}{(x^2-1)^{5/2}}$

Solution 1) Domain of f we need $x^2-1>0$, we need $x<-1$, or $x>+1$
$(x-1)(x+1)>0$,

Note $f>0$ if $x>1$, $f<0$ if $x<-1$

2) No roots: $f(x)$ is never zero, because $x=0$ is not in the domain.

3) $f(-x) = \frac{-x}{\sqrt{(-x)^2-1}} = \frac{-x}{\sqrt{x^2-1}} = -f(x)$. So f is an odd function

So the graph of f is symmetric wrt the origin

4) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2-1}} = 1$, $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-1}} = -1$

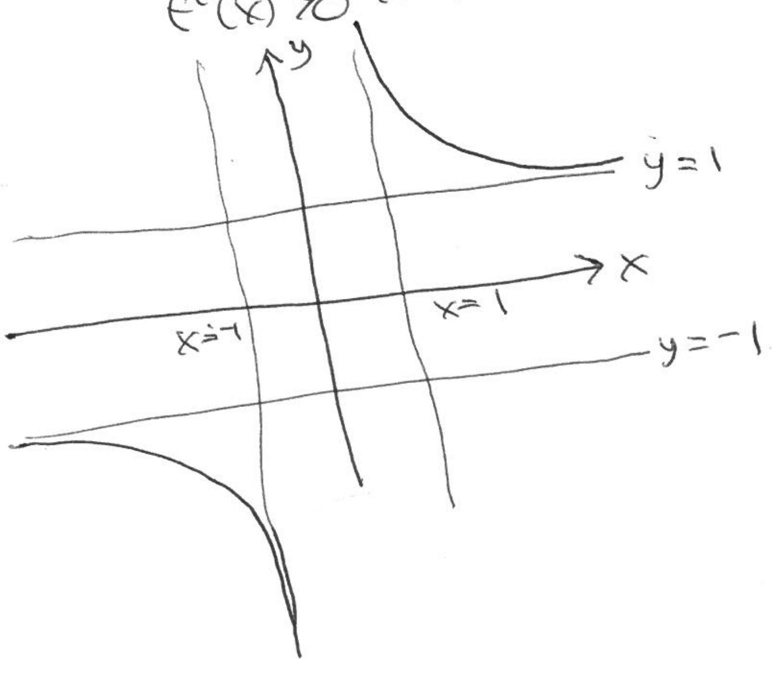
So horizontal asymptotes of $y=1$; $y=-1$

5) Vertical asymptotes of $x=-1$, $x=1$

6) $f'(x)<0$ on the domain of f, so $f \downarrow$ on its domain.

7) $f''(x)<0$ if x is <-1, so f is cc down on $(-\infty,-1)$

$f''(x)>0$ if x is >1, so f is cc up on $(1,\infty)$



No maximums nor minimums

No inflection pts.

021 Sec 4.5

<u>Ex</u> Graph: $f(x) = 8x^5 - 5x^4 - 20x^3$

Given that $f'(x) = 20x^2(x+1)(2x-3)$

f is critical at $x = -1, 0, 3/2$

$f''(x) = 160x^3 - 60x^2 - 120x$

$= 20x(8x^2 - 3x - 6) = 20x(x - r_1)(x - r_2)$

where r_1, r_2 are roots of $8x^2 - 3x - 6$

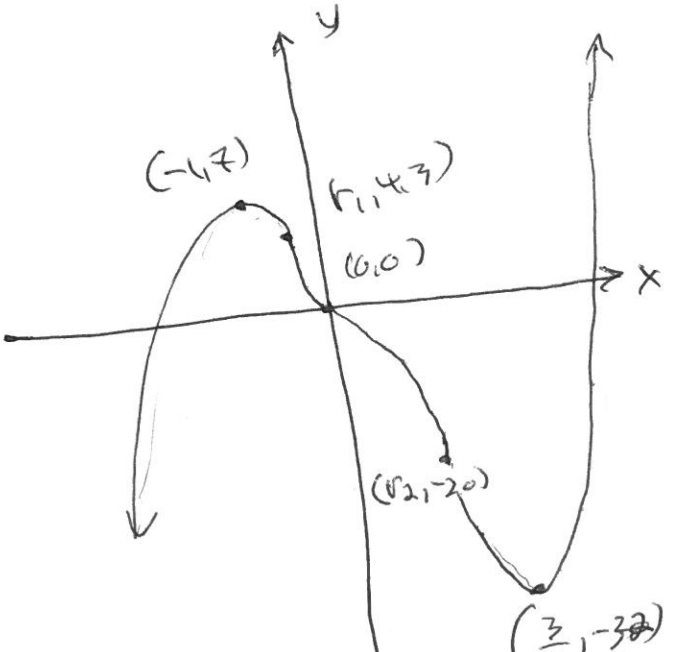
$r_1 = \frac{1}{16}(3 - \sqrt{201}) \approx -0.70$

$r_2 = \frac{1}{16}(3 + \sqrt{201}) \approx 1.07$

All the critical values are: $-1, r_1, 0, r_2, 3/2$

$f(-1) = 7$, $f(r_1) \approx 4.3$, $f(0) = 0$, $f(r_2) \approx -20$, $f(3/2) \approx -32$

	f	20	x^2	$x+1$	$2x-3$	f'	20	x	$x-r_1$	$x-r_2$	f''
$(-\infty,-1)$	↑ ∩	+	+	−	−	+	+	−	−	−	−
$(-1,r_1)$	↓ ∩	+	+	+	−	−	+	−	−	−	−
$(r_1,0)$	↓ ∪	+	+	+	−	−	+	−	+	−	+
$(0,r_2)$	↓ ∩	+	+	+	−	−	+	+	+	−	−
$(r_2,3/2)$	↓ ∪	+	+	+	−	−	+	+	+	+	+
$(3/2,\infty)$	↑ ∪	+	+	+	+	+	+	+	+	+	+



$\lim_{x\to\infty} f(x) = +\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

f is
increasing on $(-\infty,-1)$ and on $(3/2,\infty)$
decreasing on $(-1, 3/2)$

By the first derivative test
local max at $(-1, 7)$
local min at $(3/2, -32)$

f is concave down on $(-\infty, r_1)$
and on $(0, r_2)$

f is concave up on $(r_1, 0)$ and on (r_2, ∞)

f is continuous everywhere.
So f has inflection pts at $(r_1, 4.3)$
and at $(r_2, -20)$, and $(0,0)$

No global max nor min.