022 Sec 7.4 Thursday, Jan 23, 2020 (

Section 7.4 Partial Fraction Decomposition

We ask $I = \int \frac{1}{x^2+5x+6}\,dx = ?$

Case 1 Distinct Linear terms

The key is to decompose

$$\frac{1}{x^2+5x+6} = \frac{1}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2} = \frac{A(x+2)+B(x+3)}{(x+3)(x+2)}$$

So $1 = A(x+2) + B(x+3)$

We seek the values of A and B

→ this is an identity, so it must be true for all values of X

If $x=-2$, we have

$1 = A(-2+2) + B(-2+3)$

$1 = B$

If $x=-3$, we have: $1 = A(-3+2) + B(-3+3)$

$1 = -A$

$A = -1$ so $\frac{1}{x^2+5x+6} = \frac{-1}{x+3} + \frac{1}{x+2}$

So $I = \int \frac{1}{x^2+5x+6}\,dx = \int \frac{-1}{x+3}\,dx + \int \frac{1}{x+2}\,dx$

$I = -\ln|x+3| + \ln|x+2| + C$

$I = \ln\left|\frac{x+2}{x+3}\right| + C$

Ex of a repeated linear factor using partial fractions

$$I = \int \frac{4}{(x-5)^3}\,dx$$

Solution

$$\frac{4}{(x-5)^3} = \frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3}$$

$$\frac{4}{(x-5)^3} = \frac{A(x-5)^2 + B(x-5) + C}{(x-5)^3}$$

so $4 = A(x-5)^2 + B(x-5) + C$

Let $x = 5$

$4 = A(5-5)^2 + B(5-5) + C$, or $C = 4$

so, $\cancel{4} = A(x-5)^2 + B(x-5) + \cancel{4}$

$0 = A(x-5)^2 + B(x-5)$

$0 = A(x-5)^2 + Bx - 5B$

$0 = A(x^2 - 10x + 25) + Bx - 5B$

$0 = Ax^2 - 10Ax + 25A + Bx - 5B$

$0 = Ax^2 + (-10A + B)x + (25A - 5B)$

$0 = \cancel{Ax^2} + (\cancel{-10A} + B)$

Note no x^2's on the LHS so no x^2's on RHS, so $A = 0$

$0 = Bx - 5B$

Likewise no x's on LHS so no x's on RHS so $B = 0$

so $\frac{4}{(x-5)^3} = \frac{0}{x-5} + \frac{0}{(x-5)^2} + \frac{4}{(x-5)^3}$

so we want $\int \frac{4}{(x-5)^3}\,dx$

Easily solved with a substitution

$u = x - 5$

$du = dx$

Case 3 For an irreducible (can't be factored) quadratic factor

$$\frac{x}{(x-2)(x^2+1)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{x^2+4}$$

note x^2+1 and x^2+4 are irreducible, (can't be factored over the real numbers

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Caution

$$I = \int \frac{2x+4}{x^3-2x^2}\,dx$$

solution $\frac{2x+4}{x^2(x-2)}$ x^2 is not an irreducible quadratic since x^2 is factored as $x^2 = x \cdot x$

so $$\frac{2x+4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$