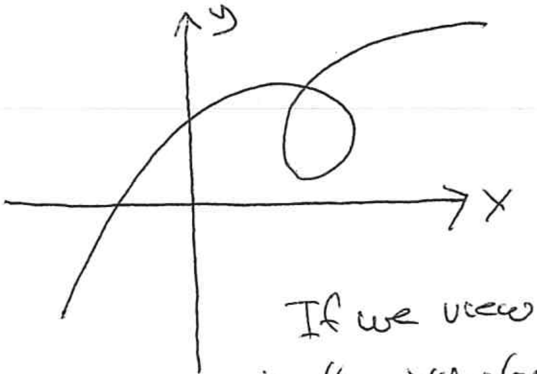
022 Sec 10.1 Weds Feb 12, 2020 ①

Sec 10.1 Curves defined by parametric equations



This path is not the graph of a function in x, (it does pass the vertical line test)

The path also is not the graph of a function in y, (it does not pass the horizontal line test,

If we view the curve as the path of a particle traveling in the xy-plane, then the curve is a function in time t.

Note at any time t, the particle is in a unique plane in the plane

The point in the plane where the particle is located at time t is $P(x,y)$

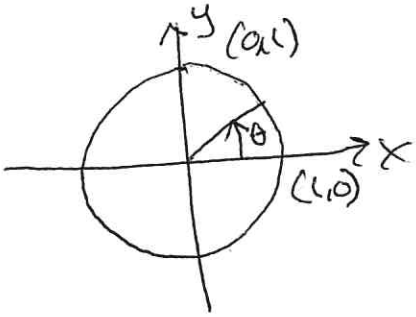
The x coordinate of P varies with t, and is a function in t, $x(t)$

Similarly, The y-coordinate of P " " ", $y(t)$

Our book might use, $\left.\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}\right\}$ these two equations are a pair of parametric equations for the curve.

Comment A curve defined by a pair of parametric equations usually will have many different sets of parametric equations that define the curve.

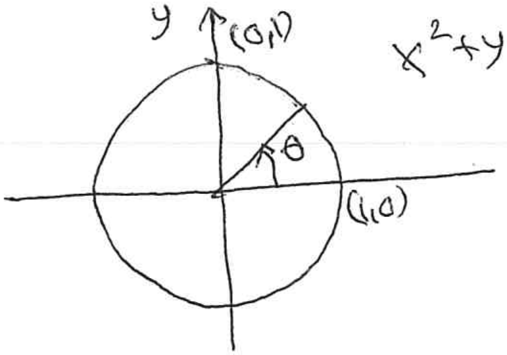
Ex Graph: $x^2+y^2=1$



Let θ be the angle from the positive x-axis, rotated counterclockwise

One set of parametric equations is

$$S_1: \left.\begin{aligned} x(t) &= \cos\theta \\ y(t) &= \sin\theta \end{aligned}\right\} \quad 0 \le \theta \le 2\pi$$



$x^2+y^2=1$

Another set of parametric equations is:

$$S_2: \begin{array}{l} x=\cos(2\theta) \\ y=\sin(2\theta) \end{array} \quad 0\le\theta\le\pi$$

As sets, the graphs of the two pair of parametric equation are the same. But you are at different points on circle for the same value θ, depending on which pair of parametric equations you use

Ex $S_1(\frac{\pi}{2})$ gives $x=\cos\frac{\pi}{2}=0$, $y=\sin(\frac{\pi}{2})=1$, $S_1(\frac{\pi}{2})=(0,1)$

$S_2(\frac{\pi}{2})$ gives $x=\cos(2\frac{\pi}{2})=\cos\pi=-1$

$y=\sin(2\frac{\pi}{2})=\sin\pi=0$

so $S_2(\frac{\pi}{2})=(-1,0)$

Parametric Equations generalizes the idea of an explicit function $y=f(x)$

$y=f(t)$ can be viewed ~~as~~ parametrically as

$$x=t$$
$$y=f(t)$$

Ex If $y=f(x)=x^2$

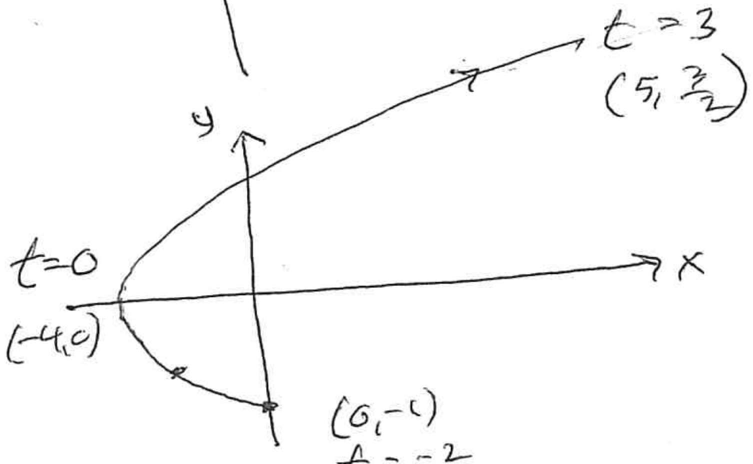
set ~~$x=t$~~ $\left.\begin{array}{l} x(t)=t \\ y(t)=(x(t))^2=t^2 \end{array}\right\}$

parametric equations for the original function.

<u>Ex</u> Sketch the curve described by

$$x = t^2 - 4, \quad y = \frac{t}{2} \quad \text{for } -2 \le t \le 3$$

t	-2	-1	0	1	2	3
x	0	-3	-4	-3	0	5
y	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$



<u>Note</u> the arrowhead in the direction of increasing t

We now show that the curve is a parabola

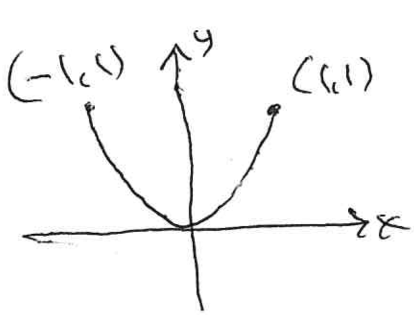
a) $x = t^2 - 4$

$y = \frac{t}{2} \Rightarrow 2y = t$, substitute into a)

$x = (2y)^2 - 4 = 4y^2 - 4 = 4(y^2 - 1)$

i.e. $x = 4(y^2 - 1)$

Sketch the curve $x = \sin t$, $y = \sin^2 t$ for any t



<u>Note</u> for any t: $-1 \le \sin t \le +1$

$0 \le \sin^2 t \le +1$

since $y = x^2$, we have a parabola

Note $-1 \le x \le 1$

$0 \le y \le 1$