022 Sec 10.3 Thursday, February 20, 2020

Section 10.3 Polar Coordinates

We need two coordinates to locate a point in the plane

Up till now, we have only used rectangular (cartesian) coordinates

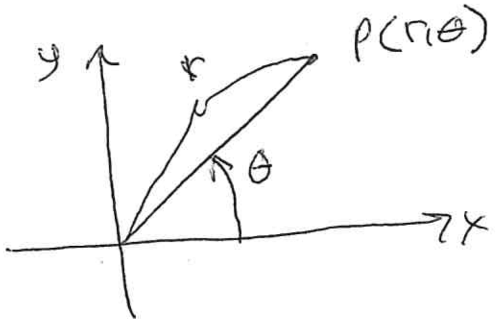
(x, y) : x - horizontal distance from the origin
distance along the x-axis $(y=0)$

y - vertical distance from the origin
distance along the y-axis $(x=0)$

We now introduce another coordinate system
Polar Coordinates (r, θ)

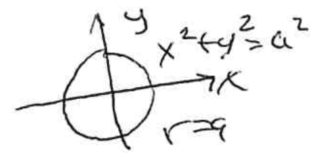
where r is the directed distance from the origin to the point

θ is the angle from the positive x-axis to the point with counterclockwise rotation

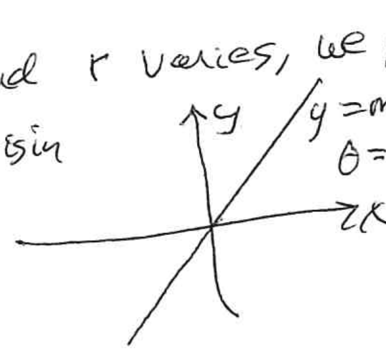


If r is constant and θ varies, the graph is a circle, centered at the origin, radius r

so, the rectangular equation: $x^2+y^2=a^2$
becomes the polar equation: $r=a$



If θ is constant and r varies, we get a straight line through the origin



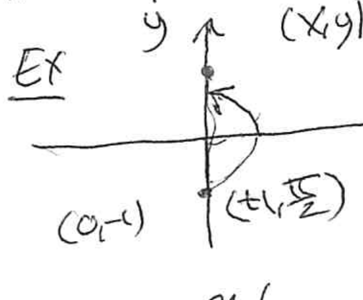
$y = mx$ (Rect)
$\theta = j$ (j fixed)

Drawback for polar coordinates

In rectangular coordinates, a point has only one set of coordinates

But in polar coordinates, a point has infinitely many representation

<u>Ex</u> $(x,y) = (0,1)_R$



$(0,1)_R = (r,\theta)_p = (1, \frac{\pi}{2}) = (1, \frac{\pi}{2} + 2\pi) = (1, \frac{\pi}{2} + 4\pi) + \ldots$

<u>Note</u> ~~for~~ to graph $(-1, -\frac{\pi}{2})$, go to $(1, -\frac{\pi}{2})$ and flip it over with respect to the origin

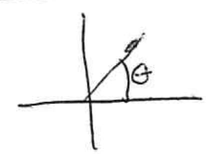
i.e. $-r$ gives the opposite direction as r

so $(1, \frac{\pi}{2}) = (-1, -\frac{\pi}{2}) = (-1, -\frac{\pi}{2} - 2\pi) = (-1, -\frac{\pi}{2} - 4\pi) \ldots$

So our pt is expressable as

$(1, \frac{\pi}{2} + 2n\pi)$ for any $n \in \mathbb{Z}$

$(-1, -\frac{\pi}{2} + 2n\pi)$ " " " " "



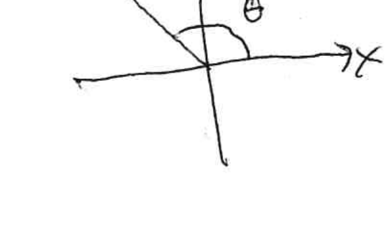
Conversions: Convert $(x,y)_R$ to $(r,\theta)_P$

$r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$

so $r = \pm\sqrt{x^2+y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

<u>Ex</u> Convert $(-2, 2\sqrt{3})_R$ to Polar

$r^2 = x^2 + y^2 = (-2)^2 + (2\sqrt{3})^2 = 4 + 12 = 16 \Rightarrow r = \sqrt{16} = 4$



Note that we are in the 2nd quadrant

and $\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

So $\theta = \tan^{-1}(-\sqrt{3})$, second quadrant

$\Longrightarrow \theta = \frac{2\pi}{3}$

Simplest answer is $(-2, 2\sqrt{3})_R = (4, \frac{2\pi}{3})_P$

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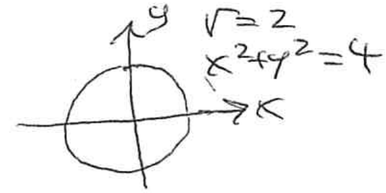
To go from $(r,\theta)_{polar}$ to $(x,y)\,R$

we $x = r\cos\theta$
$y = r\sin\theta$

Ex $\left(6, \frac{2\pi}{3}\right)_p = (\quad,\quad)\,R$

$x = 6\cos\left(\frac{2\pi}{3}\right) = 6\left(-\frac{1}{2}\right) = -3, \quad y = 6\sin\left(\frac{2\pi}{3}\right) = 6\left(\frac{\sqrt{3}}{2}\right) = 3\sqrt{3}$

so, $\left(6, \frac{2\pi}{3}\right)_p = (-3, 3\sqrt{3})\,R$



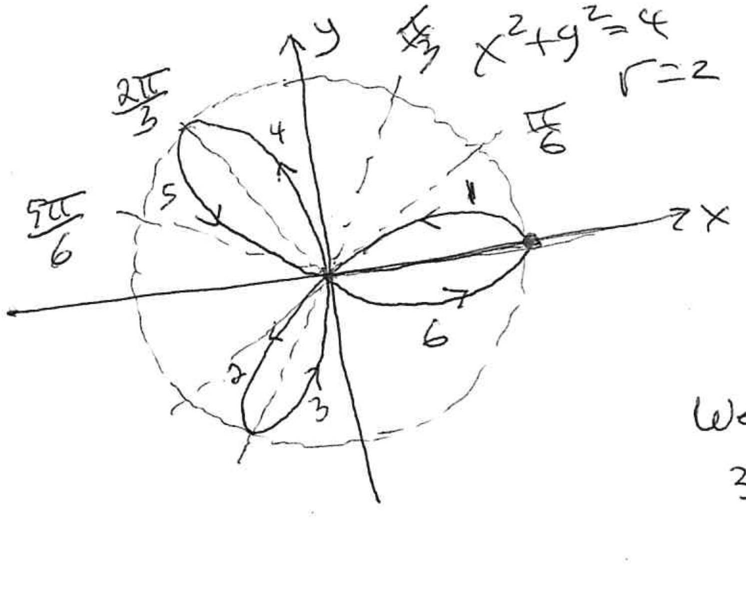
Graphing Polar Equations

Ex Graph $r = 2\cos(3\theta)$

First, note that $-1 \le \cos\alpha \le +1$, so
$-2 \le 2\cos(3\theta) \le +2$

Make a Table

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r $r=2\cos(3\theta)$	2	0	-2	0	2	0	-2

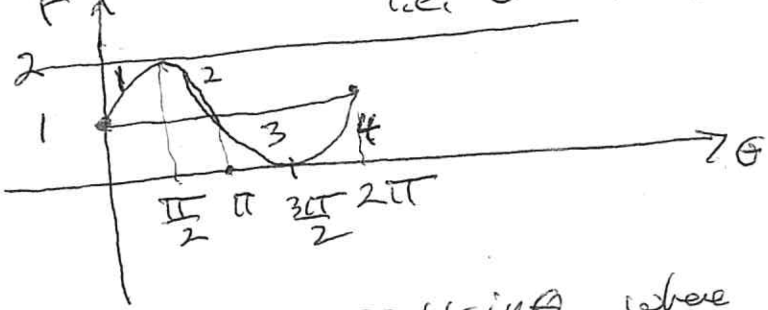


So, as θ varies from $\theta = 0$ to $\theta = \pi$, we get a complete graph.

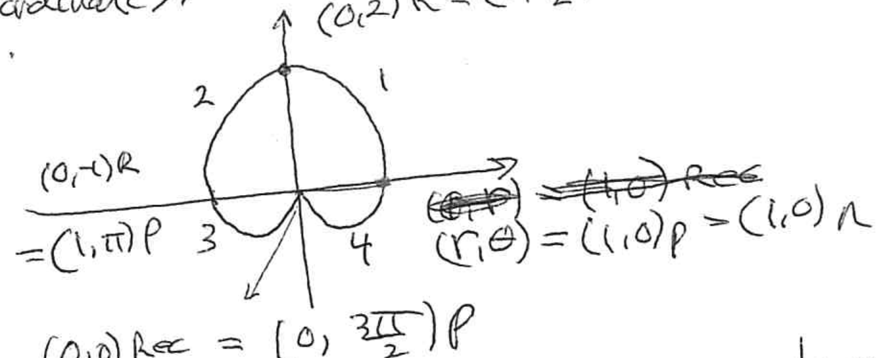
If we graph $r = 2\sin(3\theta)$ on ~~from~~ $0 \le \theta \le 2\pi$

We would trace out the 3 blade propellor twice

Ex Graph $r = 1+\sin\theta$ for $0 \le \theta \le 2\pi$
where (θ, r) are rectangular coordinates
i.e. θ acts like x, r acts like r



Now graph $r = 1+\sin\theta$ where (r, θ) are treated as polar coordinates.

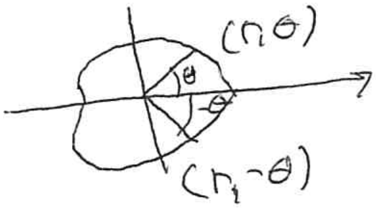


Notice that the graph of $r = 1+\sin\theta$ in rectangular coordinates is differentiable everywhere.
But in polar coordinates, the graph of $r = 1+\sin\theta$ has a cusp at $(0, \frac{3\pi}{2})$, where it is not differentiable.

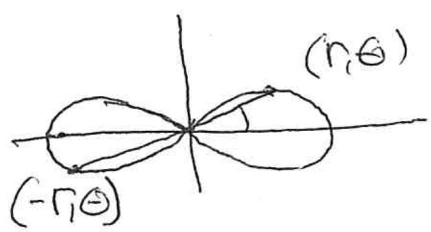
The properties of a graph of an equation can depend on the coordinate system used.

Symmetry of polar graphs

a) If a polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric wrt the polar axis



b) If the equation is unchanged when r is replaced by $-r$ or when θ is replaced by $\theta+\pi$, then the graph is symmetric wrt the pole (origin)



c) If the equation is unchanged when θ is replaced by $\pi-\theta$, the curve is symmetric about the line $\theta=\frac{\pi}{2}$ (the y-axis)