021 Sec 11.1 Monday, Feb 24 2020 ①

Questions for quiz on Thursday

① Derivative of a parametric curve 10.2?

② Area ~~of a~~ enclosed by curves given parametrically 10.2

③ Describe a particular polar curve 10.3

④ Find area enclosed by a curve given in polar form 10.4

Section 11.1 Sequences

A sequence is a list of numbers in a well-defined order.

We denote a sequence a_n as $\{a_n\}$, $\{a_n\}_{n=1}^{\infty}$

Ex $a_n = n$, here, $a_1 = 1, a_2 = 2, a_3 = 3, \ldots$

Ex $\{a_n\}$ with $a_n = 3 + (-1)^n$

so, $a_1 = 3 + (-1)^1 = 2$

$a_2 = 3 + (-1)^2 = 4$

$a_3 = 3 + (-1)^3 = 2$

$a_4 = 3 + (-1)^4 = 4$

We can a sequence as a function, whose domain is the set of positive integers.

In the above ex. $f(n) = 3 + (-1)^n$

So $f(1) = 2$, $f(2) = 4$, $f(3) = 2$, $f(4) = 4$, ...

Ex $b_n = \frac{n}{1-2n}$. Here $b_1 = \frac{1}{1-2(1)} = -1$, $b_2 = \frac{2}{1-2\cdot 2} = -\frac{2}{3}$

$b_3 = \frac{3}{1-2(3)} = -\frac{3}{5}$, $b_4 = \frac{4}{1-2\cdot 4} = \frac{-4}{7}$

In the above examples, we have a formula for a general term. It is easy to go from the general term to the specific terms.

It can be very difficult, if not impossible to obtain a formula for the general term from specific terms.

Ex $a_1 = \frac{3}{3}$, $a_2 = \frac{-5}{4}$, $a_3 = \frac{9}{5}$, $a_4 = \frac{-17}{6}$

Solution is, ~~n+1~~. Denominator is $n+2$

The alternating sign indicates a factor of $(-1)^n$ or $(-1)^{n+1} = (-1)^{n-1}$

$$a_n = \frac{2^n + 1}{n+2}(-1)^{n+1}$$

EX Given a formula for a_n, there is only one sequence for that formula

However, given the terms of a sequence there might be many formulas for that sequence

Recursively Defined Sequences. A sequence where the n'th term is defined by previously occuring terms

EX Fibonacci

Let $a_0 = 1,\ a_1 = 1, \ldots\ a_n = a_{n-1} + a_{n-2}$

$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$

Ex $a_0 = 2,\quad a_n = 2^{a_{n-1}}$

$a_1 = 2^{a_1} = 2^2 = 4$

$a_2 = 2^{a_1} = 2^4 = 16$

$a_3 = 2^{a_2} = 2^{16} = 65526$

$a_4 = 2^{a_3} = 2^{65526}$

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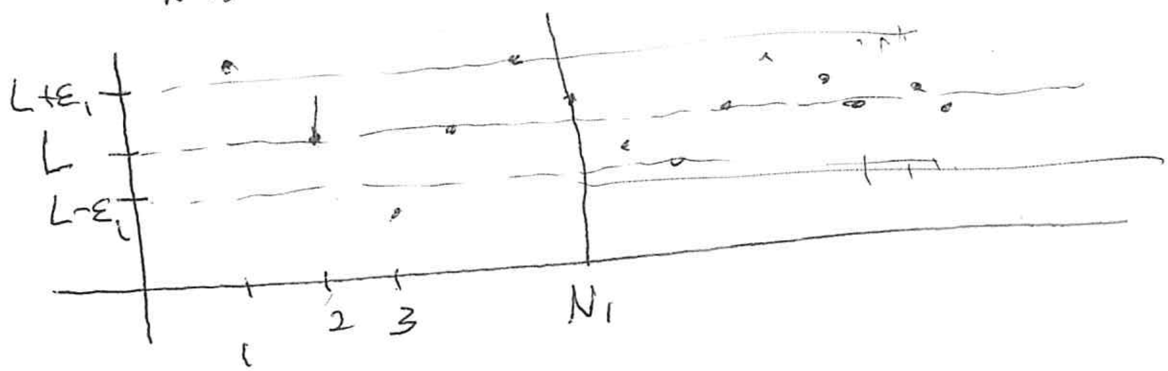
The limit of a sequence.

$\lim_{n\to\infty} a_n = L$ roughly means as n gettes larger and larger a_n gets closer and closer to L

A little more precise:
As n becomes arbitrarly large, a_n becomes arbitrarly close to L.

Def $\lim_{n\to\infty} a_n = L$ if $\forall \varepsilon > 0, \exists N > 0$ st. if $n > N$ then $|a_n - L| < \varepsilon$



Ex $a_n = \frac{1}{n}$, $\lim_{n\to\infty} a_n = 0$

$n > N$, $\frac{1}{n} < \frac{1}{4} < \frac{1}{3}$ ✓

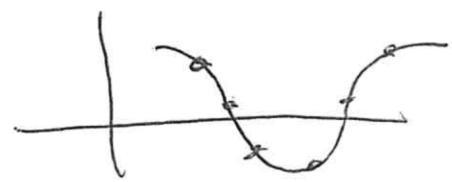
If $\varepsilon = \frac{1}{3}$, $N = 4$ so if

If $\varepsilon = \frac{1}{10}$, need $N = 11$,
so, if $n > 11$, $\frac{1}{n} < \frac{1}{11} < \frac{1}{10}$

From a function $f(x)$ defined on $x > 0$

we can define a sequence

$\{a_n\}$ by $a_n = f(n)$



Ex $f(x) = x+3$

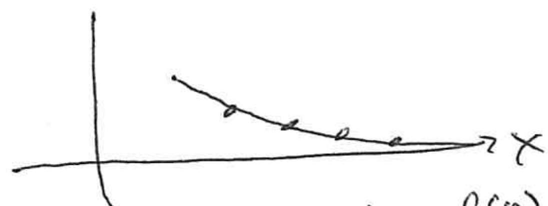
$a_n = n+3$ for $n \in \mathbb{Z}^+$

Thm If $\lim_{n\to\infty} f(x) = L$ and we define a sequence

by $a_n = f(n)$, then $\lim_{n\to\infty} a_n = L$

Ex ~~lim~~ Let $f(x) = \frac{1}{x}$ for $x \geq 1$

$\lim_{x\to\infty} f(x) = 0$

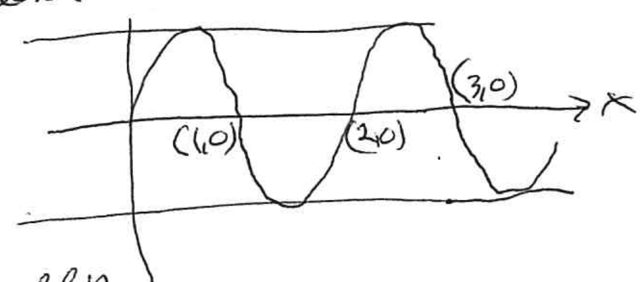


so $\lim_{n\to\infty} a_n = \lim_{n\to\infty} f(n) = 0$

Caution the converse is not true

Just because $\lim_{n\to\infty} a_n = L$, we don't have to have $\lim_{x\to\infty} f(x) = L$

Ex Let $f(x) = \cancel{\frac{\sin \pi}{x}}$ $\sin \frac{x}{\pi}$



Let $a_n = f(n)$

So $a_n = \sin \frac{n}{\pi} = 0$, for all n

So $\lim_{n\to\infty} \{a_n\} = 0$

But $\lim_{n\to\infty} f(x)$ dne.

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<u>Def</u> $\lim_{n\to\infty} a_n = \infty$ if $\forall M > 0, \exists N$ s.t. if $n > N$, $a_n > M$

i.e. as n increases without bounds
a_n " " "