Reduction Formulas

Use I of P to show that

$$I = \int \sin^n x\,dx = \frac{-1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$n \geq 2,\ n \in \mathbb{Z}^+$

Pf Let $u = \sin^{n-1} x$ $\quad dv = \sin x\,dx$

$du = (n-1)\sin^{n-2} x \cos x \quad v = -\cos x$

So $I = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x \cos^2 x\,dx$

Now use $\cos^2 x = 1 - \sin^2 x$

So $\int \sin^n x\,dx = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$

$+ (n-1)\int \sin^n x\,dx$ $\quad + (n-1)\int \sin^n x\,dx$

$$n\int \sin^n x\,dx = -\cos x \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,dx$$

i.e.

$$\int \sin^n x\,dx = \frac{-1}{n}\sin x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

The Tabular method for I. by P.

Ex Find $I = \int x^3 e^{-x}\,dx$

Derivatives		Integrals
		e^{-x}
x^3	$+$	$-e^{-x}$
$3x^2$	$+$	e^{-x}
$6x$	$+$	e^{-x}
6	$-$	e^{-x}
0		

$$I = -x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x} + C$$

$$I = \int x^2 \sin x \, dx$$

Der		Int
		$\sin x$
x^2	$+$	$-\cos x$
$2x$	$-$	$-\sin x$
2	$+$	$\cos x$
0		

$$I = -x^2 \cos x + 2x \sin x + 2\cos x + C$$

$$I = \int x^2 e^x \, dx$$

D		I
		e^x
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		

$$I = x^2 e^x - 2x e^x + 2e^x + C$$

Ex $I = \int x^2 \sqrt{x-1}\, dx$

Derivatives		Integrals
		$(x-1)^{1/2}$
x^2	$+$	$\frac{2}{3}(x-1)^{3/2}$
$2x$	$-$	$4/15\,(x-1)^{5/2}$
2	$+$	$\frac{8}{105}(x-1)^{7/2}$
0		

$$I = \frac{2}{3}x^2(x-1)^{3/2} - \frac{8}{15}x(x-1)^{5/2} + \frac{16}{105}(x-1)^{7/2} + C$$

Same Problem $I = \int x^2 \sqrt{x-1}\, dx$ by substitution

$u = x-1$

$x = u+1$

$x^2 = (u+1)^2 = u^2 + 2u + 1$

$du = dx$

$$I = \int (u^2+2u+1)\, u^{1/2}\, du$$

$$= \int \left[u^{5/2} + 2u^{3/2} + u^{1/2}\right] du$$

$$= \frac{2}{7}u^{7/2} + \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

Sec 7.2 Trigonometric Integrals

In a sense we extend the reduction formula

Say, we seek

$$I = \int \sin^3 x\,dx$$

<u>Solution</u> $I = \int \sin x \sin^2 x\,dx = \int \sin x\,(1-\cos^2 x)\,dx$

Let $u = \cos x$

$du = -\sin x$

$$I = -\int (-\sin x)\,(1-\cos^2 x)\,dx$$

$$= -\int (1-u^2)\,du = -u + \frac{u^3}{3} + C$$

$$I = -\cos x + \frac{\cos^3 x}{3} + C$$

Keep in mind: $\sin^2 x + \cos^2 x = 1$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

<u>Ex</u> $\int \cos^2 x\,dx = \frac{1}{2}\int (1+\cos 2x)\,dx$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)\,dx$$

use $u = 2x$

$du = 2dx$

$$= \frac{1}{2}x + \frac{1}{2}\left(\frac{1}{2}\right)\int (\cos(2x))\,2dx$$

$$= \frac{1}{2}x + \frac{1}{4}\int \cos u\,du = \frac{1}{2}x + \frac{\sin u}{4} + C$$

$$= \frac{1}{2}x + \frac{\sin(2x)}{4} + C$$

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Integrals involving sine and cosine

i) If the power of sine is odd and positive factor out one power of sin and convert the other, now even powers of sine to powers of $(1-\cos^2 x)$

EX $\int \sin^3 x \cos^4 x\,dx = \int \sin^2 x \cos^4 x \sin x\,dx$

$= \int (1-\cos^2 x)\cos^4 x \sin x\,dx$

$= \int (\cos^4 x - \cos^6 x)\sin x\,dx$

substitute: $u = \cos x$
$du = -\sin x$

$= -\int (\cos^4 x - \cos^6 x)(-\sin x)\,dx$

$= -\int (u^4 - u^6)\,du$

$= -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

If the power of cosine is odd and positive
break off a power of cosine

Ex $I = \int \sin^2 x \cos^7 x \, dx = \int \sin^2 x \cos^6 x \cos x \, dx$

$I = \int \sin^2 x (1 - \sin^2 x)^3 \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$I = \int u^2 (1-u^2)^3 \, du$

$= \int u^2 (1 - 3u^2 + 3u^4 - u^6) \, du$

$= \int (u^2 - 3u^4 + 3u^6 - u^8) \, du$

$= \frac{u^3}{3} - \frac{3u^5}{5} + \frac{3u^7}{7} - \frac{u^9}{9} + C$

$I = \frac{\sin^3 x}{3} - \frac{3}{5} \sin^5 x + \frac{3}{7} \sin^7 x + \frac{1}{9} \sin^9 x + C$