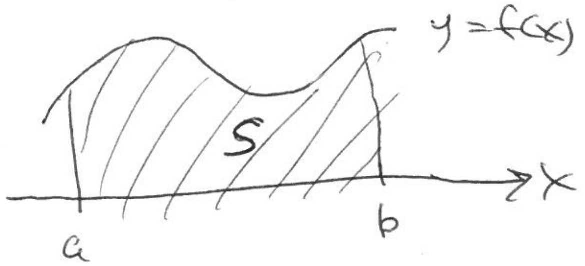
021 Sec 5.1 November

Section 5.1 Distance and area.

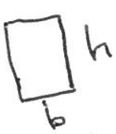
Recall that to find the slope of a tangent line, we looked at the limit, (limit being a calculus concept) of a sequence of slopes of secant lines, (non calculus concept).

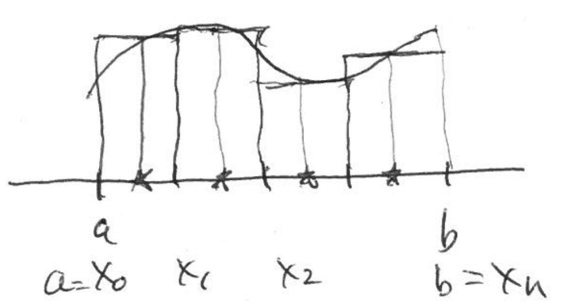


Let S be the region bounded by $y=f(x)$, $y=0$ (x-axis) and the vertical lines $x=a$, $x=b$.

What is the area of S?

Area of a rectangle is $A=bh$





Partition the interval $[a,b]$ into n subintervals of equal length.

$$\Delta x = \frac{b-a}{b}$$

Pick a sample pt x_i^* in each subinterval

Sum the areas of the rectangles with base $\frac{b-a}{n} = \Delta x$ and height $f(x_i^*)$

An approximation to the area of S is.

$$A \approx \sum_{i=1}^{n} f(x_i^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x$$

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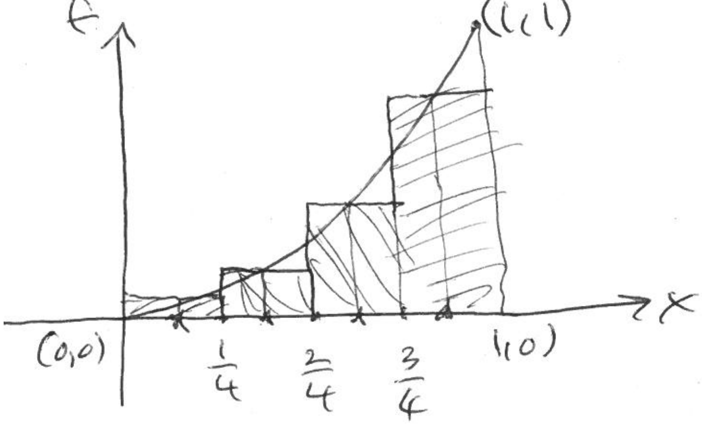
<u>Ex</u> Approximate the area between the curve $f(x) = x^2$, and $y = 0$ on the interval $[0,1]$ using four rectangles.

For the sample point, use the midpoint of each subinterval.



Solution: $\Delta x = \frac{1-0}{4} = \frac{1}{4}$

Test pts are: $\frac{1}{8}$

$$\frac{3}{8} = \frac{1}{8} + 1(\Delta x)$$

$$\frac{5}{8} = \frac{1}{8} + 2(\Delta x)$$

$$\frac{7}{8} = \frac{1}{8} + 3(\Delta x)$$

$$\text{Area} = \frac{1}{4}f\left(\frac{1}{8}\right) + \frac{1}{4}f\left(\frac{3}{8}\right) + \frac{1}{4}f\left(\frac{5}{8}\right) + \frac{1}{4}f\left(\frac{7}{8}\right)$$

$$= \frac{1}{4}\left[f\left(\frac{1}{8}\right) + f\left(\frac{3}{8}\right) + f\left(\frac{5}{8}\right) + f\left(\frac{7}{8}\right)\right]$$

$$= \frac{1}{4}\left[\left(\frac{1}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{5}{8}\right)^2 + \left(\frac{7}{8}\right)^2\right]$$

$$= \frac{1}{4}\left[\frac{1^2 + 3^2 + 5^2 + 7^2}{64}\right.$$

$$= \frac{1}{256}\left[1 + 9 + 25 + 49\right]$$

$$= \frac{84}{256} \approx 0.328$$

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We form US_4 (upper sum with 4 rectangles)

$$US_4 = \text{Area}(R_1) + \text{Area}(R_2) + \text{Area}(R_3) + \text{Area}(R_4)$$

$$= \frac{1}{4} f\left(\frac{1}{4}\right) + \frac{1}{4} f\left(\frac{2}{4}\right) + \frac{1}{4} f\left(\frac{3}{4}\right) + \frac{1}{4} f\left(\frac{4}{4}\right)$$

$$= \frac{1}{4}\left[f\left(\frac{1}{4}\right) + f\left(\frac{2}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{4}{4}\right)\right]$$

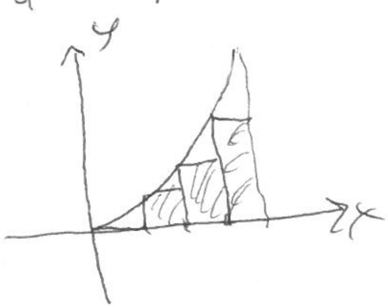
$$= \frac{1}{4}\left[\left(\frac{1}{4}\right)^2 + \left(\frac{2}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{4}{4}\right)^2\right]$$

$$= \frac{1}{4}\left[\frac{1^2+2^2+3^2+4^2}{16}\right]$$

$$= \frac{1}{64}\left[1^2+2^2+3^2+4^2\right] = \frac{30}{64} \approx 0.6875$$

We now do a lower sum by use the left hand endpts to compute the heights of the rectangles

$$LS_4 = \frac{1}{4} f\left(\frac{0}{4}\right) + \frac{1}{4} f\left(\frac{1}{4}\right) + \frac{1}{4} f\left(\frac{2}{4}\right) + \frac{1}{4} f\left(\frac{3}{4}\right)$$



$$= \frac{1}{4}\left[\frac{0^2}{4^2} + \frac{1^2}{4^2} + \frac{2^2}{4^2} + \frac{3^2}{4^2}\right]$$

$$= \frac{1}{64}\left[0^2+1^2+2^2+3^2\right] \approx 0.21875$$

This is an underestimate.

So, $0.21875 \leq$ exact area ≤ 0.46875

So, to get a better estimate, increase the number of rectangles so that the maximum of the rectanges $\to 0$.

We can approximate each strip by a rectangle that has the same base as the strip and whose height is the same as the right edge of the strip [see Figure 4(b)]. In other words, the heights of these rectangles are the values of the function $f(x) = x^2$ at the *right* endpoints of the subintervals $\left[0, \frac{1}{4}\right]$, $\left[\frac{1}{4}, \frac{1}{2}\right]$, $\left[\frac{1}{2}, \frac{3}{4}\right]$, and $\left[\frac{3}{4}, 1\right]$.

Each rectangle has width $\frac{1}{4}$ and the heights are $\left(\frac{1}{4}\right)^2$, $\left(\frac{1}{2}\right)^2$, $\left(\frac{3}{4}\right)^2$, and 1^2. If we let R_4 be the sum of the areas of these approximating rectangles, we get

$$R_4 = \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 + \frac{1}{4} \cdot 1^2 = \frac{15}{32} = 0.46875$$

From Figure 4(b) we see that the area A of S is less than R_4, so

$$A < 0.46875$$

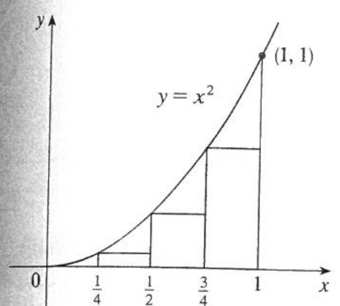


FIGURE 5

Instead of using the rectangles in Figure 4(b) we could use the smaller rectangles in Figure 5 whose heights are the values of f at the *left* endpoints of the subintervals. (The leftmost rectangle has collapsed because its height is 0.) The sum of the areas of these approximating rectangles is

$$L_4 = \frac{1}{4} \cdot 0^2 + \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 = \frac{7}{32} = 0.21875$$

We see that the area of S is larger than L_4, so we have lower and upper estimates for A:

$$0.21875 < A < 0.46875$$

We can repeat this procedure with a larger number of strips. Figure 6 shows what happens when we divide the region S into eight strips of equal width.

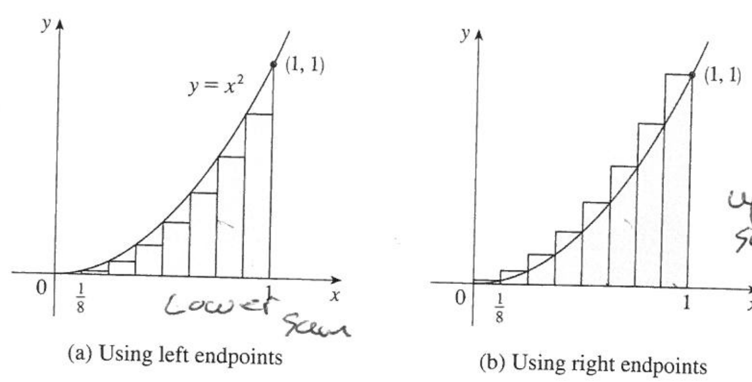


FIGURE 6
Approximating S with eight rectangles

10
upper 39
Sum 49

By computing the sum of the areas of the smaller rectangles (L_8) and the sum of the areas of the larger rectangles (R_8), we obtain better lower and upper estimates for A:

$$0.2734375 < A < 0.3984375$$

n	L_n	R_n
10	0.2850000	0.3850000
20	0.3087500	0.3587500
30	0.3168519	0.3501852
50	0.3234000	0.3434000
100	0.3283500	0.3383500
1000	0.3328335	0.3338335

So one possible answer to the question is to say that the true area of S lies somewhere between 0.2734375 and 0.3984375.

We could obtain better estimates by increasing the number of strips. The table at the left shows the results of similar calculations (with a computer) using n rectangles whose heights are found with left endpoints (L_n) or right endpoints (R_n). In particular, we see by using 50 strips that the area lies between 0.3234 and 0.3434. With 1000 strips we narrow it down even more: A lies between 0.3328335 and 0.3338335. A good estimate is obtained by averaging these numbers: $A \approx 0.3333335$. ■

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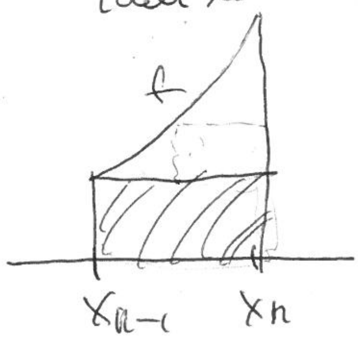
Why do lower sums increase and why do upper sums decrease.

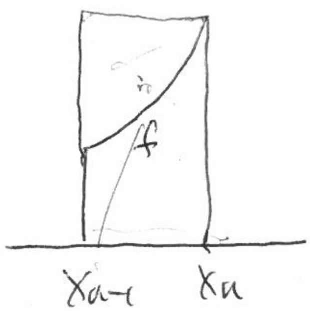
Lower Sum



Double the number of rectangles

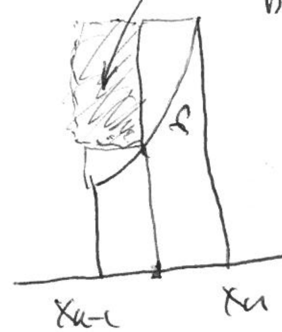
new and added on

Upper Sums



Double the number of rectangles

not included but was included before



021 Sec 5.1

<u>If</u> the area under the graph of a function exists, then

The upper sums $\xrightarrow{\text{decrease}}$ area

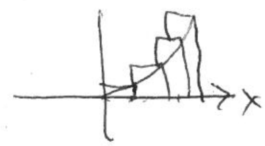
The lower sums $\xrightarrow{\text{increase}}$ area

<u>Thm</u> Let $f(x)$ be a bounded function on a closed interval I of finite length. Let $f(x)$ have a finite number of discontinuities.

<u>then</u> the area under $f(x)$ on I exists

i.e. $\lim_{n\to\infty} LS_n = \lim_{n\to\infty} US_n$

In our example: $f(x) = x^2$ on $[0,1]$. f is bounded on $[0,1]$ and $[0,1]$ is closed of finite length.



So the area under $f(x) = x^2$ on $[0,1]$ exists

$$\text{Area} = \lim_{n\to\infty} LS_n = \lim_{n\to\infty} US_n$$

To find what this area equals, lets compute $\lim_{n\to\infty} US_n$

<u>note</u> $f\uparrow$ on $[0,1]$ so $US_n = RHS$

$$US_n = \frac{1}{n} f\left(\frac{1}{n}\right) + \frac{1}{n} f\left(\frac{2}{n}\right) + \cdots + \frac{1}{n} f\left(\frac{n}{n}\right)$$

$$= \frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f\left(\frac{n}{n}\right)\right]$$

$$= \frac{1}{n}\left[\frac{1^2}{n^2} + \frac{2^2}{n^2} + \frac{3^2}{n^2} + \cdots + \frac{n^2}{n^2}\right]$$

$$= \frac{1}{n^3}\left[1^2 + 2^2 + 3^2 + \cdots + n^2\right]$$

we now use: $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

$$= \frac{1}{n^3}\left[\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right] = \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

$$\text{Area} = \lim_{n\to\infty} US_n = \lim_{n\to\infty}\left(\frac{1}{3} + \cancel{\frac{1}{2n}} + \cancel{\frac{1}{6n^2}}\right) = \frac{1}{3}$$

Area of $\frac{1}{3}$

021 Sec 5.1

Area under $f(x) = x^2$ on $[0,1] = \frac{1}{3}$

Preview of the fundamental theorem of Calculus

Find an antiderivative of $f(x) = x^2$

$$F(x) = \frac{x^3}{3} + C$$

Now Find $F(x)\Big|_0^1 = \left(\frac{1^3}{3}+c\right) - \left(\frac{0^3}{3}+c\right) = \frac{1}{3}$

Show that:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof by induction

1) first show the formula is true for $n=1$

$$1^2 \stackrel{?}{=} \frac{1(1+1)(2\cdot1+1)}{6} = \frac{1\cdot2\cdot3}{6} \checkmark$$

2) The inductive step – show that if the formula is true for n then the formula must be true for $n+1$

$$\left(1^2+2^2+\cdots n^2\right) + (n+1)^2 \stackrel{?}{=} \frac{(n+1)(n+2)\,[2(n+1)+1]}{6}$$

$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2 \stackrel{?}{=} \frac{(n+1)(n+2)(2n+3)}{6}$$

$$n(n+1)(2n+1) + 6(n+1)^2 \stackrel{?}{=} (n+1)(n+2)(2n+3)$$

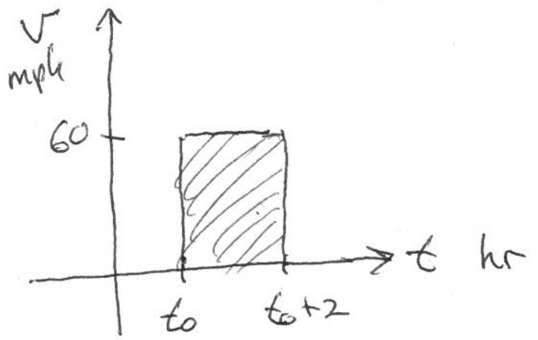
$$(n^2+n)(2n+1) + (6n^2+12n+6) \stackrel{?}{=} (n^2+3n+2)(2n+3)$$

$$2n^3+3n^2+n+6n^2+12n+6 = 2n^3+3n^2+6n^2+4n+6 \checkmark$$

$$2n^3+9n^2+13n+6 = 2n^3+9n^2+13n+6$$

021 Sec 5.1

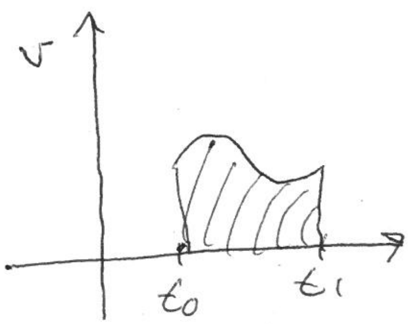
Distance = (velocity)(time)



Here $t = 2$ hr

$v = 60$ mph

$D = \left(\frac{60\,m}{hr}\right)(2\,hr) = 120$ miles



Distance traveled is the area under the curve

$$\int_{t_0}^{t_1} v(t)\,dt$$

Ex If $v(t) = t^2$ on the time interval $[0,1]$

The distance traveled is

$$\int_0^1 t^2\,dt = \frac{1}{3}$$

O21 Sec 5.1/5.2

Sigma Notation

$\sum$ - upper case greek letter sigma

$$\sum_{i=1}^{5} i = 1+2+3+4+5 = 15$$

$$\sum_{i=11}^{13} i^2 = 11^2 + 12^2 + 13^2 = 435$$

$$\sum_{i=3}^{6} (i^2 - i) = (3^2-3) + (4^2-4) + (5^2-5) + (6^2-6) = 68$$

$$\sum_{i=4}^{5} \frac{1}{i^2} = \frac{1}{4^2} + \frac{1}{5^2} = \frac{1}{16} + \frac{1}{25}$$

Let $x_1 = 7$, $x_2 = 4$, $x_3 = 0$, $x_4 = 7$, $x_5 = -6$

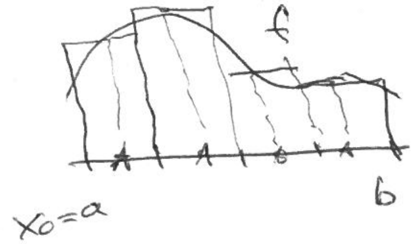
Find $\sum_{i=1}^{5} (2x_i - 1) = (2\cdot 7-1) + (2\cdot 4-1) + (2\cdot 0-1) + (2\cdot 7-1) + (2(-6)-1)$
$= 19$

We will need

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

021 Sec 5.2



<u>Def</u> of a definite integral

Let f be defined on the interval $a \le x \le b$

Divide [a,b] into n subintervals of equal length

$\Delta x = \frac{b-a}{n}$

Let $a=x_0<x_1<x_2<\dots<x_n=b$

Let x_i^* be a sample point in the i'th subinterval

The definite integral of f from a to b is

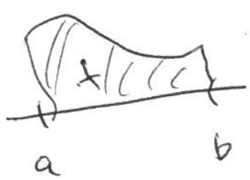
$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i^*)}_{\text{height}} \underbrace{\Delta x}_{\text{width}}$, provided that this limit exists,

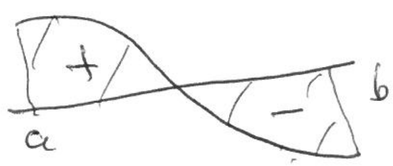
If this limit exists, we say that f is integrable on [a,b]

If f is integrable, the value of $\int_a^b f(x)dx$ is independent of the choice of the x_i's.

<u>Note</u> f(x) is called the integrand

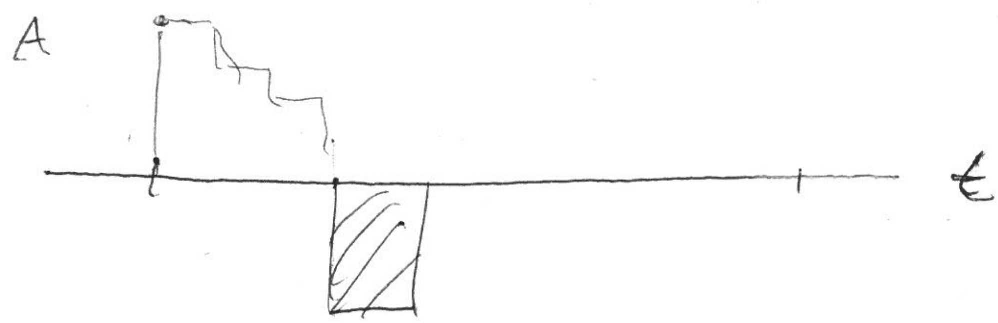
<u>Note</u> $I=\int_a^b f(x)dx$ gives the signed or net area between $y=f(x)$, the x-axis on the interval [a,b]





Saying that a function is integrable on an interval [a,b] is equivalent to the function having an area between the graph of f(x) and the y-axis on the interval [a,b]

A = amount in a checking account



<u>Aside</u> A function $f(x)$ having a derivative is pretty sensitive.
Change the function a little bit so that it has a small corner at a point, then the function is no longer differentiable because of the corner.

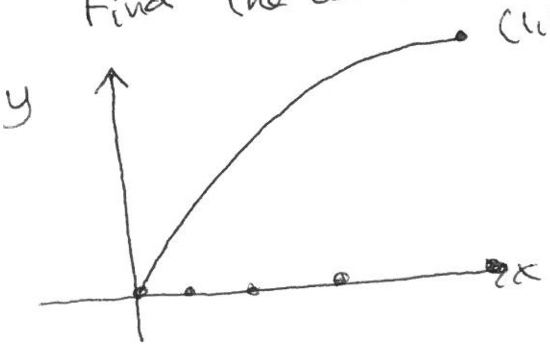
A function $f(x)$ being integrable is very robust.
You can put in corners, put in pts of discontinuities,
You will change the value of the integral
But most likely the function will still be integrable.

~~021 Sec 5.2 Tuesday, June 18, 2019~~

~~Next quiz is on Friday, June 21, 2019~~

Ex of finding an integral by using Riemann sums where the rectangles have unequal widths,

Find the area under $f(x) = y = \sqrt{x} = x^{\frac{1}{2}}$ on $[0,1]$



Partition $[0,1]$ with partition pts

$$x_i = \frac{i^2}{n^2}$$

Ex for $n=4$, the partition points are $\frac{0^2}{4^2}, \frac{1^2}{4^2}, \frac{2^2}{4^2}, \frac{3^2}{4^2}, \frac{4^2}{4^2}$

$$= 0, \frac{1}{16}, \frac{4}{16}, \frac{9}{16}, \frac{16}{16}$$

The width of the i'th subinterval is $\Delta x_i = x_i - x_{i-1} = \frac{i^2}{n^2} - \frac{(i-1)^2}{n^2} = \frac{2i-1}{n^2}$

Note that the widths vary with i,

We want, using the right-hand rule

$\cancel{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n} (}$ $\sum_{n=0}^{\infty} f(c_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\sqrt{\frac{i^2}{n^2}}}_{\text{heights}} \underbrace{\left(\frac{2i-1}{n^2}\right)}_{\text{widths}}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n} \left(\frac{2i-1}{n^2}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3} \left(\frac{2i^2-i}{1}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^3} \left[2 \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} \right]$$

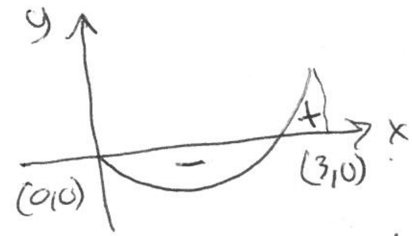
$$= \lim_{n\to\infty} \frac{4n^3+3n^2-n}{6n^3} = \frac{2}{3}$$

So $\int_0^1 x^{\frac{1}{2}} dx = \frac{2}{3}$. Note $\int x^{\frac{1}{2}} dx = \frac{2}{3} x^{3/2}$

and $\frac{2}{3} x^{3/2} \Big|_{x=0}^{x=1} = \frac{2}{3}$

021 Sec 5.2 Weds Nov 13, 2019

Ex Find $I = \int_0^3 (x^3-6x)\,dx$



Divide $[0,3]$ into n subintervals of equal width

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

The partition pts are: $x_0 = 0,\ x_1 = 0+\frac{3}{n},\ x_2 = 0+2\left(\frac{3}{n}\right), \dots, x_i = 0 + i\left(\frac{3}{n}\right)$

Now $I = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\frac{27i^3}{n^3} - \frac{18i}{n}\right] = \lim_{n\to\infty} \left[\frac{81}{n^4} \sum i^3 - \frac{54}{n^2} \sum_{i=1}^{n} i\right]$$

Now use $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$= \lim_{n\to\infty} \left[\frac{81}{n^4}\left(\frac{n(n+1)}{2}\right)^2 - \frac{54}{n^2}\left(\frac{n(n+1)}{2}\right)\right]$$

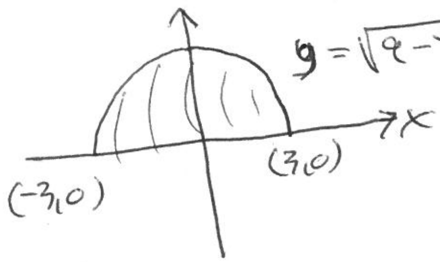
$$= \lim_{n\to\infty} \left[\frac{81}{4}\left(1+\frac{1}{n}\right)^2 - 27\left(1+\frac{1}{n}\right)\right] = \frac{81}{4} - 27 = \frac{-27}{4}$$

Note $\int_0^3 (x^3-6x)\,dx = \left(\frac{x^4}{4} - 3x^2\right)\Big|_{x=0}^{x=3} = \left(\frac{3^4}{4} - 3\cdot 3^2\right) - (0)$

$$= \frac{-27}{4}$$

021 Sec 5.2

Ex Find $I = \int_{-3}^{3} \sqrt{9-x^2}\,dx$. To find I using calculus would be very difficult. But don't forget what you already know.



I represents the area under the curve.

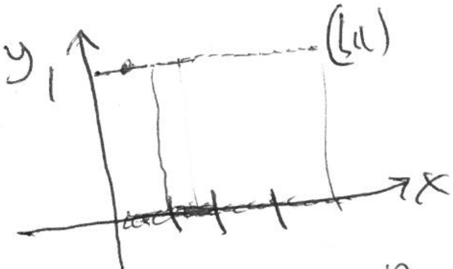
$$I = A = \frac{\pi r^2}{2}\Big|_3 = \frac{9\pi}{2}$$

Ex of a non integrable function. A function without an area.

We need the following fact.

Every open interval on the real line contains rationals numbers and irrational numbers.

Dirichlet function $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$ on $[0,1]$



Partition $[0,1]$ into n subintervals of equal width

So $LS_n = 0\left(\frac{1}{n}\right) + 0\left(\frac{1}{n}\right) + \cdots + 0\left(\frac{1}{n}\right) = 0$

So, for every n, $LS_n = 0$, So $\lim_{n\to\infty} LS_n = 0$

~~Moreover~~, Now, $US_n = \underbrace{1\left(\frac{1}{n}\right) + 1\left(\frac{1}{n}\right) + \cdots + 1\left(\frac{1}{n}\right)}_{n \text{ of these}} = n\left(\frac{1}{n}\right) = 1$

So $US_n = 0$ for any n, so $\lim_{n\to\infty} US_n = 1$

So $\lim_{n\to\infty} LS_n \neq \lim_{n\to\infty} US_n$

So f is not integrable

021 Sec 5.2

Properties of the definite Integral

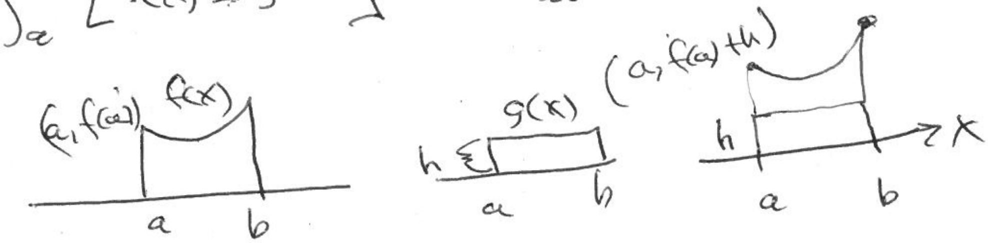
What is $\int_3^3 \sinh^5(\cos(4x^3))\,dx = 0$

1) $\int_a^a f(x)\,dx = 0$

2) $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$

3), 4) $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

3)



5) Let c be a constant

$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

Ex $\int_0^1 [3x^2 - 7x + 5]\,dx$

$= \int_0^1 3x^2\,dx - \int_0^1 7x\,dx + \int_0^1 5\,dx$

$= 3\int_0^1 x^2\,dx - 7\int_0^1 x\,dx + 5\int_0^1 dx$

$= 3\left(\frac{x^3}{3}\right)\Big|_0^1 - 7\left(\frac{x^2}{2}\right)\Big|_0^1 + 5x\Big|_0^1$

$= x^3\Big|_0^1 - \frac{7}{2}x^2\Big|_0^1 + 5x\Big|_0^1$

$= (1^3 - 0^3) - \frac{7}{2}(1^2 - 0^2) + 5(1-0)$

$= \frac{5}{2}$

021 Sec 5.2

Caution on rule 5), $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

only if c does not involve x

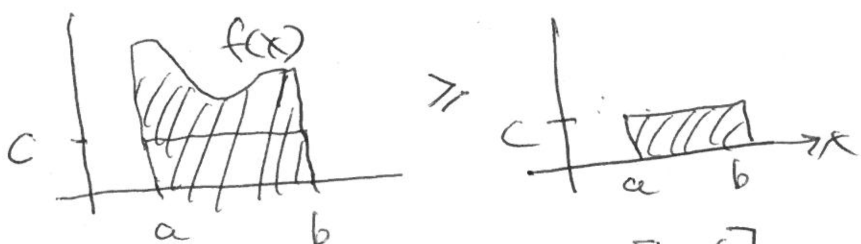
In general $\int x f(x)\,dx \neq x\int f(x)\,dx$

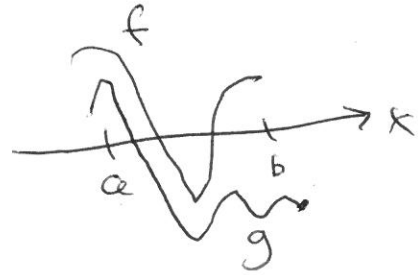
Ex Let $f(x)=x^2$

Compare

$\int x f(x)\,dx$	$x\int f(x)\,dx$
$=\int x\cdot x^2\,dx$	$=x\int x^2\,dx$
$=\int x^3\,dx$	$=x\left(\frac{x^3}{3}+C\right)$
$=\frac{x^4}{4}+C$	$=\frac{x^4}{3}+CX$

⑥ If $y=f(x)=C$, $\forall x$ in $[a,b]$, C is a constant

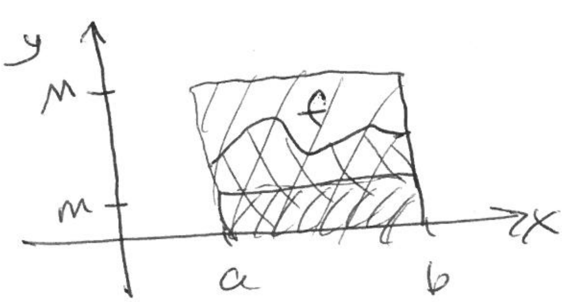




7) If $f(x) \geq g(x)$, $\forall x$ in $[a,b]$

Then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

⑧ If $m \leq f(x) \leq M$, for all x in $[a,b]$, with m, M constants



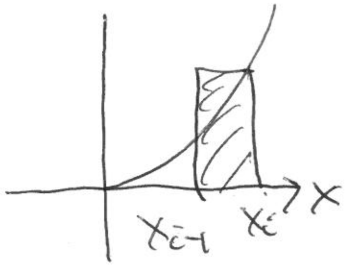
$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

021 Sec 5.2

We know that $\int_0^1 x^2 dx = \frac{1}{3}$

We will now see why this forces $\int_0^1 -(x^2)\, dx = -\frac{1}{3}$

First consider an approximating rectangle for $\int_0^1 x^2 dx$



The area of the rectangle is

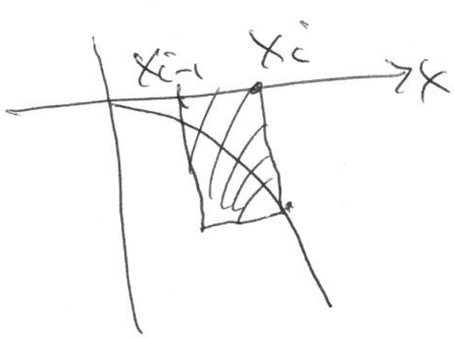
$A = $ (base)(height)

$= (x_i - x_{i-1})\, f(x_i)$

$= (+)(+) = +$

Sum up positive numbers, you get a positive number

Now, for $\int_0^1 (-x^2)\, dx$



The area of an approximating rectangle is (base)(height)

$= (x_i - x_{i-1})\, f(x_i)$

$(+)(-) = -$

So sum up negative numbers, you get a negative number.

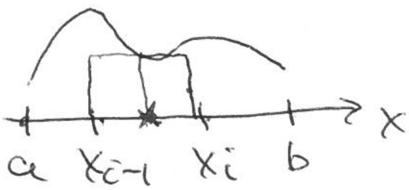
In general: $\int_a^b (-f(x))\, dx = -\int_a^b f(x)\, dx$

021 Sec 5.2

(9) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

For now, assume $a<b$, $f(x)>0$

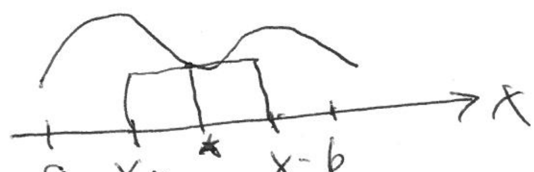
In $\int_a^b f(x)\,dx$



The area of an approximating rectangle is

$$A = bh = (x_i - x_{i-1})\, f(x_i^*)$$
$$(+)(+) = +$$

In $\int_b^a f(x)\,dx$



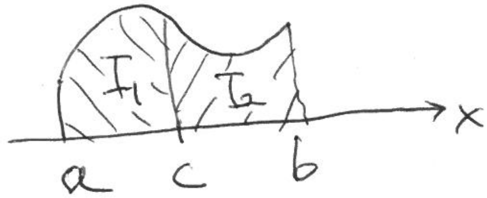
The area of an approximating rectangle is

$$A = bh = (x_{i-1} - x_i)\, f(x_i^*)$$
$$(-)(+) = -$$

so the approximating rectangle has negative area

(10) If $a<c<b$ then $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

$$I_1 + I_2 = I$$



In general, no matter what the relationship of a, b, c are, we have $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

Ex