

Unit 12 - Composition, Recursion, and Exponential Functions*Topic: Composition of Functions***1**

If $f(x) = x^2 - 3x - 1$ and $g(x) = 1 - x$, what is the value of $f \circ g(-2)$?

- A) -3
- B) -1
- C) 1
- D) 3

2

If $f = \{(-4, 12), (-2, 4), (2, 0), (3, \frac{3}{2})\}$ and $g = \{(-2, 5), (0, 1), (4, -7), (5, -9)\}$, what is the value of $g \circ f(2)$?

- A) -9
- B) -7
- C) 1
- D) 5

3

A function f satisfies $f(-1) = 8$ and $f(1) = -2$.
A function g satisfies $g(2) = 5$ and $g(-1) = 1$.
What is the value of $f(g(-1))$?

- A) -2
- B) 1
- C) 5
- D) 8

4

If $f(x) = \frac{1-5x}{2}$ and $g(x) = 2 - x$, what is the value of $f(g(3))$?

- A) -7
- B) -2
- C) 2
- D) 3

Questions 5 and 6 refer to the following information.

x	$f(x)$	$g(x)$
-2	-5	0
0	6	4
3	0	-5

The table above gives values of f and g at selected values of x .

5

What is the value of $f(g(-2))$?

6

What is the value of $g(f(3))$?

Topic: Recursive Formula

1

A sequence is recursively defined by $a_n = \sqrt{(a_{n-1})^2 + 2}$. If $a_0 = \sqrt{2}$, what is the value of a_2 ?

- A) $\sqrt{5}$
- B) $\sqrt{6}$
- C) $\sqrt{8}$
- D) 3

2

A sequence is recursively defined by $a_{n+1} = a_n - \frac{f(a_n)}{g(a_n)}$. If $a_0 = 1$, $f(x) = x^2 - 3x$, and $g(x) = 2x - 3$, what is the value of a_2 ?

- A) -3
- B) $-\frac{1}{5}$
- C) 2
- D) $\frac{3}{2}$

3

If $f(x) = \sqrt{2x^2 - 1}$, what is the value of $f \circ f \circ f(2)$?

- A) $\sqrt{10}$
- B) $\sqrt{15}$
- C) $\sqrt{21}$
- D) 5

4

If A_0 is the initial amount deposited into a savings account that earns at a fixed rate of r percent per year, and a constant amount of $12b$ is added to the account each year, then amount A_n of the savings n years after the initial deposit is made is given by the equation $A_n = (1 + \frac{r}{100}) \cdot A_{n-1} + 12b$.

What is A_3 , the amount you have in the savings three years after you made the initial deposit, if $r = 5$, $A_0 = 12,000$, and $b = 400$?

- A) \$23,070.00
- B) \$26,048.00
- C) \$29,023.50
- D) \$35,274.68

5

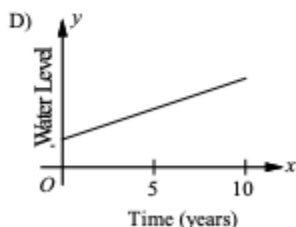
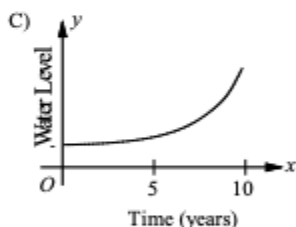
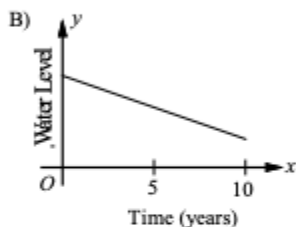
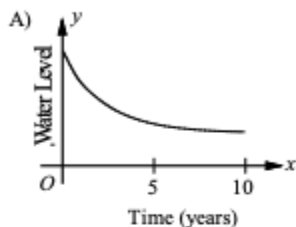
The number of gallons, P_n , of a pollutant in a lake at the end of each month is given by the recursively defined formula $P_n = 0.85P_{n-1} + 20$. If the initial amount P_0 of a pollutant in the lake is 400 gallons, what is P_3 , the amount of pollutant in the lake at the end of the third month, to the nearest gallon?

- A) 297
- B) 285
- C) 273
- D) 262

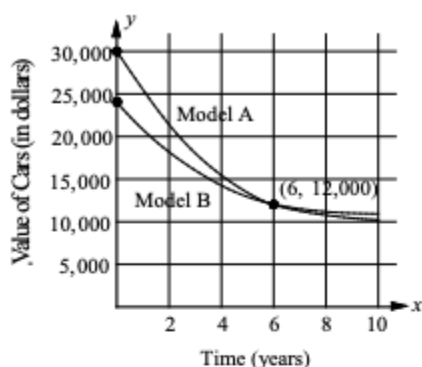
Topic: Exponential Functions and Graphs

1

During a decade of continuous drought, the water level of a lake has decreased by 10 percent each year. Which of the following graphs could model the water level of the lake as a function of time?



2



In the graph above, each exponential curve represents the values, in dollars, of two different cars as a function of time in years. At time $t = 0$, the price of model A was \$30,000 and the price of model B was \$24,000. At time $t = 6$, the price of both models were \$12,000.

Based on the graphs above, which of the following must be true?

- I. At time $t = 0$, the price of model A was 25% more than the price of model B .
- II. At time $t = 0$, the price of model B was 20% less than the price of model A .
- III. From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A was 1.5 times the average rate of decrease in the value of model B .

- A) I and II only
- B) I and III only
- C) II and III only
- D) I, II, and III

3

If $f(x) = 12,000(0.9)^x$ and $g(x) = 14,000(0.85)^x$, what is the value of $g(2) - f(2)$?

Topic: Exponential Growth and Decay

1

The number of rabbits in a certain population doubles every 40 days. If the population starts with 12 rabbits, which of the following gives the total number of rabbits in the population after t days?

- A) $12(2)(\frac{t}{40})$
- B) $12(2)(\frac{40}{t})$
- C) $12(2)^{\frac{40}{t}}$
- D) $12(2)^{\frac{t}{40}}$

2

Population P of a town is 80,000 this year. If the population of the town decreases at a rate of 4 percent each year, which of the following expressions gives population P after t years?

- A) $80,000(0.6)^t$
- B) $80,000(0.96)^t$
- C) $80,000(0.96t)$
- D) $80,000(1 - 0.04t)$

3

A house bought ten years ago for \$150,000 was sold for \$240,000 this year. Which of the following equations can be used to solve the annual growth rate r of the value of the house?

- A) $240,000 = 150,000(1 + \frac{r}{10})$
- B) $240,000 = 150,000(1 + 10r)$
- C) $240,000 = 150,000(1 + r)^{10}$
- D) $240,000 = 150,000(r)^{10}$

4

A certain radioactive substance has a half-life of 12 days. This means that every 12 days, half of the original amount of the substance decays. If there are 128 milligrams of the radioactive substance today, how many milligrams will be left after 48 days?

- A) 4
- B) 8
- C) 16
- D) 32

Questions 5 and 6 refer to the following information.

Evelyn deposited \$3,000 into her bank account, which earns 4 percent interest compounded annually. She uses the expression $\$3,000(x)^t$ to find the value of the account after t years.

5

What is the value of x in the expression?

6

Evelyn deposited the same amount into an account that earns 5 percent interest rate compounded annually. How much more money than her original deposit in the account with 4 percent interest rate compounded annually will she have earned in 10 years?
(Round your answer to the nearest dollar.)

Unit 12 Review Questions

1

If $f(x) = \sqrt{2x}$ and $g(x) = 2x^2$, what is the value of $f(g(1)) - g(f(1))$?

- A) -4
- B) -2
- C) 2
- D) 4

2

If $f(x) = \sqrt{625 - x^2}$ and $g(x) = \sqrt{225 - x^2}$, what is the value of $f(f(5)) - g(g(5))$?

- A) 0
- B) 5
- C) 10
- D) 20

3

The population of a certain town doubles every 25 years. If the population of the town was 51,200 in 1980, in what year was the population 6,400?

- A) 1855
- B) 1880
- C) 1905
- D) 1930

4

The half-life of a radioactive substance is the amount of time it takes for half of the substance to decay. The table below shows the time (in years) and the amount of substance left for a certain radioactive substance.

Time (years)	Amount (grams)
0	1,200
14	850
28	600
42	425
56	300

How much of the original amount of the substance, to the nearest whole gram, will remain after 140 years?

- A) 85
- B) 75
- C) 53
- D) 38

5

A radioactive substance decays at a rate of 18% per year. If the initial amount of the substance is 100 grams, which of the following functions models the remaining amount of the substance, in grams, after t years?

- A) $f(t) = 100(0.18)^t$
- B) $f(t) = 100(0.82)^t$
- C) $f(t) = 100 - 100(0.18)^t$
- D) $f(t) = 100 - 100(0.82)^t$

6

$$5,000\left(1 + \frac{r}{100}\right)^t$$

The expression above gives the value of an investment, in dollars, that pays an annual interest rate of $r\%$ compounded yearly. 5,000 is the initial amount and t is the number of years after the initial amount was deposited. Which of the following expressions shows the difference between the value of a 15 year investment at 6% annual compound interest and a 12 year investment at 6% annual compound interest?

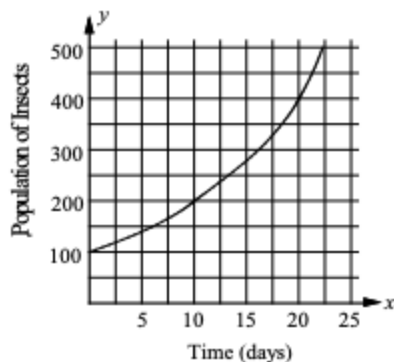
- A) $5,000 \left[(1.06)^{15} \right]$
 B) $5,000 \left[\frac{(1.06)^{15}}{(1.06)^{12}} \right]$
 C) $5,000 \left[(1.06)^{15} - (1.06)^{12} \right]$
 D) $5,000 \left[(1.06)^{15-12} \right]$

7

The price P , in dollars, of a truck t years after it was purchased is given by the function

$P(t) = 24,000\left(\frac{1}{2}\right)^{\frac{t}{6}}$. To the nearest dollar, what is the price of the truck 9 years after it was purchased?

Questions 8 and 9 refer to the following information.



The graph above shows the size of a certain insect population over 25 days. The population at time $t = 0$ was 100. A biologist used the equation

$$f(t) = 100(2)^{\frac{t}{d}}$$

to model the population.

8

What is the value of d in the equation?

9

What was the population of the insect after 15 days, to the nearest whole number?

Unit 13 - Polynomial and Radical Functions

Topic: Polynomial Functions and their Graphs

1

The graph of $f(x) = ax^3 + x^2 - 18x - 9$ intersects the x -axis at $(3, 0)$. What is the value of a ?

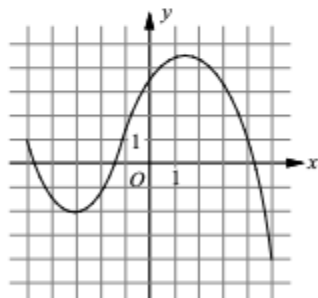
- A) -1
- B) 0
- C) 1
- D) 2

2

In the xy -plane, the graph of function f has x -intercepts at -7 , -5 , and 5 . Which of the following could define f ?

- A) $f(x) = (x-7)(x^2-25)$
- B) $f(x) = (x-7)(x^2+25)$
- C) $f(x) = (x+7)(x^2-25)$
- D) $f(x) = (x+7)(x^2+25)$

3

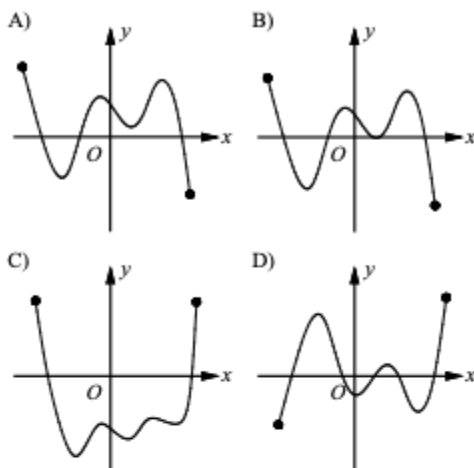


What is the minimum value of the function graphed on the xy -plane above, for $-5 \leq x \leq 5$?

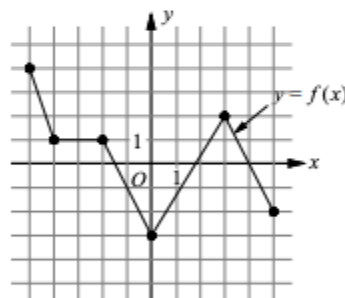
- A) -4
- B) -3
- C) -2
- D) $-\infty$

4

If function f has four distinct zeros, which of the following could represent the complete graph of f in the xy -plane?



5



The complete graph of function f is shown on the xy -plane above, for $-5 \leq x \leq 5$. Which of the following is/are true?

- I. f is strictly decreasing for $-5 < x < 0$.
- II. $f(-3) = 1$
- III. f is minimum at $x = 5$.

- A) I only
- B) II only
- C) III only
- D) I and II only

Topic: Remainder Theorem and Factor Theorem

1

If -1 and 1 are two real roots of the polynomial function $f(x) = ax^3 + bx^2 + cx + d$ and $(0,3)$ is the y -intercept of graph of f , what is the value of b ?

- A) -3
- B) -1
- C) 2
- D) 4

2

What is the remainder of polynomial $p(x) = 81x^5 - 121x^3 - 36$ divided by $x+1$?

- A) -76
- B) -36
- C) 4
- D) 6

3

If $x-2$ is a factor of polynomial $p(x) = a(x^3 - 2x) + b(x^2 - 5)$, which of the following must be true?

- A) $a+b=0$
- B) $2a-b=0$
- C) $2a+b=0$
- D) $4a-b=0$

4

x	$f(x)$
-4	-10
-3	0
-1	-4
2	20

The function f is defined by a polynomial. Some values of x and $f(x)$ are shown in the table above. Which of the following must be a factor of $f(x)$?

- A) $x+4$
- B) $x+3$
- C) $x+1$
- D) $x-2$

5

$$x^3 - 8x^2 + 3x - 24 = 0$$

For what real value of x is the equation above true?

6

If $x > 0$, what is the solution to the equation $x^4 - 8x^2 = 9$?

Topic: Radical Expressions

1

Which of the following is equal to $a^{-\frac{1}{2}}$?

- A) $-\sqrt{a}$
- B) $\frac{1}{\sqrt{a}}$
- C) $-\frac{1}{\sqrt{a}}$
- D) $\frac{1}{a^2}$

2

Which of the following is equal to $\frac{1}{3-2\sqrt{2}}$?

- A) $3-\sqrt{2}$
- B) $3+\sqrt{2}$
- C) $3+2\sqrt{2}$
- D) $3+4\sqrt{2}$

3

If $(x+1)^3 = -64$, what is the value of x ?

- A) -6
- B) -5
- C) -4
- D) -3

4

Which of the following is equal to $\sqrt{8} + \sqrt{18} - \sqrt{32}$?

- A) $\sqrt{2}$
- B) $2\sqrt{2}$
- C) $3\sqrt{2}$
- D) $\sqrt{3}$

5

Which of the following is equal to $(1+\sqrt{3})(2-\sqrt{3})$?

- A) $1-\sqrt{3}$
- B) $1+\sqrt{3}$
- C) $-1-\sqrt{3}$
- D) $-1+\sqrt{3}$

6

Which of the following is equal to $b^{\frac{5}{3}}$?

- A) $b \cdot \sqrt{b}$
- B) $b \cdot \sqrt[3]{b^2}$
- C) $b \cdot \sqrt[3]{b}$
- D) $b \cdot \sqrt[3]{b^2}$

Topic: Solving Radical Expressions

1

$$11 - \sqrt{2x+3} = 8$$

What is the solution set of the equation above?

- A) 0
- B) 3
- C) 6
- D) 9

2

$$\sqrt{-3x+4} = 7$$

What is the solution set of the equation above?

- A) -15
- B) -12
- C) -8
- D) -6

3

$$\sqrt{x+18} = x-2$$

What is the solution set of the equation above?

- A) $\{-2\}$
- B) $\{7\}$
- C) $\{-2, 7\}$
- D) $\{2, -7\}$

4

$$\sqrt{5x-12} = 3\sqrt{2}$$

What is the solution set of the equation above?

- A) 2
- B) 4
- C) 6
- D) 8

5

If $a = \sqrt{3}$ and $\sqrt{2-3x} = \frac{1}{3}a$, what is the value of x ?

6

If $k = 8 - \sqrt{2}$ and $\sqrt[3]{x-k} = -2$, what is the value of x^2 ?

Topic: Complex Numbers

1

Which of the following is equal to $\sqrt{-1} - \sqrt{-4} + \sqrt{-9}$?

- A) i
- B) $2i$
- C) $3i$
- D) $4i$

2

Which of the following is equal to $\sqrt{-2} \cdot \sqrt{-8}$?

- A) $-4i$
- B) $4i$
- C) -4
- D) 4

3

Which of the following complex numbers is equal to $\frac{3-i}{3+i}$?

- A) $\frac{9}{10} - \frac{3i}{5}$
- B) $\frac{9}{10} + \frac{3i}{5}$
- C) $\frac{3}{5} - \frac{3i}{5}$
- D) $\frac{4}{5} - \frac{3i}{5}$

4

Which of the following is equal to $\frac{1}{2}(5i-3) - \frac{1}{3}(4i+5)$?

- A) $\frac{3}{2}i - \frac{5}{2}$
- B) $\frac{7}{6}i - \frac{7}{3}$
- C) $\frac{7}{6}i - \frac{19}{6}$
- D) $\frac{5}{6}i - \frac{17}{6}$

5

If $(4+i)^2 = a+bi$, what is the value of $a+b$?

6

If the expression $\frac{3-i}{1-2i}$ is rewritten in the form $a+bi$, in which a and b are real numbers, what is the value of $a+b$?

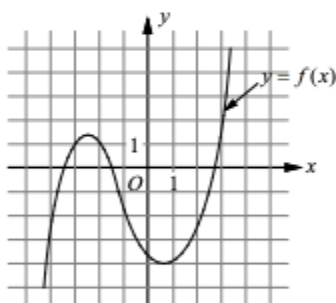
Unit 13 Review Questions

1

If the graph of $f(x) = 2x^3 + bx^2 + 4x - 4$ intersects the x -axis at $(\frac{1}{2}, 0)$, and $(-2, k)$ lies on the graph of f , what is the value of k ?

- A) -4
- B) -2
- C) 0
- D) 2

2



The function $y = f(x)$ is graphed on the xy -plane above. If k is a constant such that the equation $f(x) = k$ has one real solution, which of the following could be the value of k ?

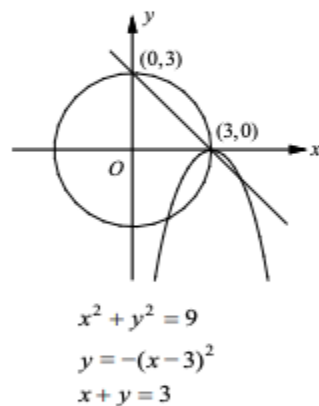
- A) -3
- B) -1
- C) 1
- D) 3

3

What is the value of a if $x + 2$ is a factor of $f(x) = -(x^3 + 3x^2) - 4(x - a)$?

- A) -2
- B) -1
- C) 0
- D) 1

4



A system of three equations and their graphs on the xy -plane are shown above. How many solutions does the system have?

- A) 1
- B) 2
- C) 3
- D) 4

5

Which of the following complex numbers is equivalent to $\frac{(1-i)^2}{1+i}$?

- A) $-\frac{i}{2} - \frac{1}{2}$
 B) $-\frac{i}{2} + \frac{1}{2}$
 C) $-i - 1$
 D) $-i + 1$

6

Which of the following is equal to $a\sqrt[3]{a}$?

- A) $a^{\frac{2}{3}}$
 B) $a^{\frac{4}{3}}$
 C) $a^{\frac{5}{3}}$
 D) $a^{\frac{7}{3}}$

7

$$p(x) = -2x^3 + 4x^2 - 10x$$

$$q(x) = x^2 - 2x + 5$$

The polynomials $p(x)$ and $q(x)$ are defined above. Which of the following polynomials is divisible by $x-1$?

- A) $f(x) = p(x) - \frac{1}{2}q(x)$
 B) $g(x) = -\frac{1}{2}p(x) - q(x)$
 C) $h(x) = -p(x) + \frac{1}{2}q(x)$
 D) $k(x) = \frac{1}{2}p(x) + q(x)$

8

$$\sqrt{2x+6} = x+3$$

What is the solution set of the equation above?

- A) $\{-3\}$
 B) $\{-1\}$
 C) $\{-3, 2\}$
 D) $\{-3, -1\}$

9

What is the remainder when polynomial

$$p(x) = 24x^3 - 36x^2 + 14 \text{ is divided by } x - \frac{1}{2}?$$

- A) 4
 B) 6
 C) 8
 D) 10

10

The function f is defined by a polynomial. If $x+2$, $x+1$, and $x-1$ are factors of f , which of the following table could define f ?

A)

x	$f(x)$
-2	4
-1	0
1	0
2	0

B)

x	$f(x)$
-2	0
-1	4
1	0
2	0

C)

x	$f(x)$
-2	0
-1	0
1	4
2	0

D)

x	$f(x)$
-2	0
-1	0
1	0
2	4