Unit 18

Answer Key

Section 18-1

1. 4 2. 6 3. 112 4.68 5. 70 6. 24 7. 240

Section 18-2

1. 25 2. 7.5 3. 27 4. C 5. D

Section 18-3

1. 120 2. 5 3. 15 4. D 5. 108 6. 36 7. B

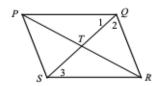
Chapter 18 Practice Test

1. C 2. B 3. 10.5 4. 174 5. D 6. 14 7. A 8. C 9. D 10. A

Answers and Explanations

Section 18-1

1. 4



 $PT = \frac{1}{2}PR$ Diagonals of \square bisect each other. $x + 2y = \frac{1}{2}(32) = 16$ Substitution ST = TQ Diagonals of \square bisect each other. 8x - y = 26 Substitution 2(8x - y) = 2(26) Multiply each side by 2.

16x-2y=52 Simplify. Add x+2y=16 and 16x-2y=52.

16x - 2y = 52 $+ \underbrace{x + 2y = 16}_{17x = 68}$ x = 4

2. (

Substitute 4 for x into the equation x+2y=16. 4+2y=16

$$2y = 12$$
$$y = 6$$

3. 112

 $m \angle 3 = m \angle 1$ If $\overline{PQ} \parallel \overline{RS}$, Alternate Interior $\angle s$ are \cong . $a^2 - 7 = 6a$ Substitution $a^2 - 6a - 7 = 0$ Make one side 0. (a - 7)(a + 1) = 0 Factor. a = 7 or a = -1

Discard a = -1, because the measure of angles in parallelogram are positive.

$$m\angle 1 = 6a = 6(7) = 42$$

 $m\angle 2 = 10a = 10(7) = 70$
 $m\angle PQR = m\angle 1 + m\angle 2$
 $= 42 + 70$
 $= 112$

4. 68

Since $\overline{PQ} \parallel \overline{RS}$, consecutive interior angles are supplementary. Thus, $m \angle PQR + m \angle QRS = 180$.

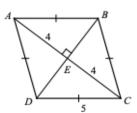
$$112 + m \angle QRS = 180 \qquad m \angle PQR = 112$$

$$m \angle QRS = 68$$

5. 70

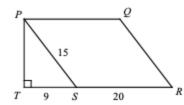
 $m\angle QTR = m\angle PRS + m\angle 3$ Exterior Angle Theorem $m\angle 3 = m\angle 1 = 42$ Given = 4(7) = 28 = 7 = 70 Substitution = 70

6. 24



 $CE^2 + DE^2 = CD^2$ Pythagorean Theorem $4^2 + DE^2 = 5^2$ $DE^2 = 9$ DE = 3Area of $ABCD = \frac{1}{2}AC \cdot BD = \frac{1}{2}(8)(6) = 24$

7. 240



$$PT^2 + ST^2 = PS^2$$

Pythagorean Theorem

$$PT^2 + 9^2 = 15^2$$

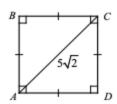
$$PT^2 = 15^2 - 9^2 = 144$$

$$PT = \sqrt{144} = 12$$

Area of $PQRS = SR \times PT = 20 \times 12 = 240$.

Section 18-2

1. 25



Let
$$AD = CD = s$$
.

$$AD^2 + CD^2 = (5\sqrt{2})^2$$

Pythagorean Theorem

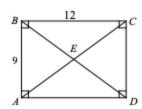
$$s^2 + s^2 = 50$$

$$2s^2 = 50$$

$$s^2 = 25$$

Area of $ABCD = s^2 = 25$.

2. 7.5



$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 9^2 + 12^2 = 225$$

Pythagorean Theorem Substitution

$$AC = \sqrt{225} = 15$$

$$AE = \frac{1}{2}AC$$

Diagonals of rectangle bisect each other.

$$=\frac{1}{2}(15)=7.5$$

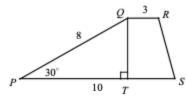
3. 27

Area of rectangle $ABCD = 12 \times 9 = 108$. In a rectangle, diagonals divide the rectangle into four triangles of equal area. Therefore,

Area of $\triangle CED = \frac{1}{4}$ the area of rectangle ABCD

$$=\frac{1}{4}(108)=27$$
.

4. C



Draw \overline{QT} , which is perpendicular to \overline{PS} , to

make triangle PQT, a 30°-60°-90° triangle.

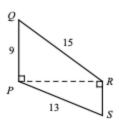
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$$QT = \frac{1}{2}PQ = \frac{1}{2}(8) = 4$$
.

Area of trapezoid $PQRS = \frac{1}{2}(PS + QR) \cdot QT$

$$=\frac{1}{2}(10+3)\cdot 4=26$$

5. D



$$PR^2 + PQ^2 = QR^2$$

Pythagorean Theorem

$$PR^2 + 9^2 = 15^2$$

Substitution

$$PR^2 = 15^2 - 9^2 = 144$$

$$PR = \sqrt{144} = 12$$

$$12^2 + RS^2 = 13^2$$

Pythagorean Theorem

$$RS^2 = 13^2 - 12^2 = 25$$

$$RS = \sqrt{25} = 5$$

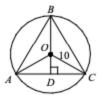
Area of trapezoid PQRS

$$= \frac{1}{2}(PQ + RS) \cdot PR = \frac{1}{2}(9+5) \cdot 12$$

= 84

Section 18-3

1. 120



$$m\angle AOB = m\angle BOC = m\angle AOC = \frac{1}{3}(360) = 120$$

2. 5

$$m\angle COD = \frac{1}{2} m\angle AOC = \frac{1}{2}(120) = 60$$

Since triangle COD is a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg.

Therefore,
$$OD = \frac{1}{2}CO = \frac{1}{2}(10) = 5$$
.

3. 15

In a circle all radii are equal in measure.

Therefore,
$$AO = BO = CO = 10$$
.
 $BD = BO + OD = 10 + 5 = 15$

4 T

In a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times as long as the shorter leg. Therefore,

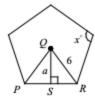
$$CD = \sqrt{3}OD = 5\sqrt{3}$$

$$AC = 2CD = 10\sqrt{3}$$

Area of \(\Delta ABC\)

$$=\frac{1}{2}(AC)(BD)=\frac{1}{2}(10\sqrt{3})(15)=75\sqrt{3}$$

5. 108



The measure of each interior angle of a regular n-sided polygon is $\frac{(n-2)180}{n}$. Therefore,

$$x = \frac{(5-2)180}{5} = 108$$
.

6. 36

$$m\angle PQR = \frac{360}{5} = 72$$

 $m\angle RQS = \frac{1}{2}m\angle PQR = \frac{1}{2}(72) = 36$

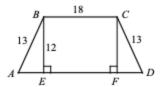
7. 1

In triangle RQS, QR is the hypotenuse and QS is adjacent to $\angle RQS$. Therefore the cosine ratio can be used to find the value of a.

$$\cos \angle RQS = \frac{\text{adjacent to } \angle RQS}{\text{hypotenuse}} = \frac{a}{6}$$

Chapter 18 Practice Test

1. C



$$AE^2 + BE^2 = AB^2$$

 $AE^2 + 12^2 = 13^2$

Pythagorean Theorem

$$AE^2 = 13^2 - 12^2 = 25$$

$$AE = \sqrt{25} = 5$$

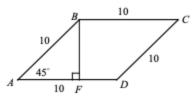
Also DF = 5.

$$AD = AE + EF + DF = 5 + 18 + 5 = 28$$

Area of trapezoid = $\frac{1}{2}(AD + BC) \cdot BF$

$$=\frac{1}{2}(28+18)\cdot 12=276$$

2. B



Draw \overline{BF} perpendicular to \overline{AD} to form a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle.

In a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg. Therefore, $\sqrt{2}BF = AB$.

$$\sqrt{2}BF = 10$$
 Substitution
$$BF = \frac{10}{\sqrt{2}} = \frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$
Area of rhombus $ABCD$

$$= \frac{1}{2}AD \cdot BF = \frac{1}{2}(10)(5\sqrt{2}) = 25\sqrt{2}$$

3. 10.5

The length of the midsegment of a trapezoid is the average of the lengths of the bases. Therefore,

$$EO = \frac{1}{2}(TP + RA)$$
.
 $18 = \frac{1}{2}(TP + 15)$ Substitution
 $2 \times 18 = 2 \times \frac{1}{2}(TP + 15)$
 $36 = TP + 15$
 $21 = TP$
In ΔTRP , $EZ = \frac{1}{2}TP = \frac{1}{2}(21) = 10.5$.

4. 174

Let w = the width of the rectangle in meters, then 2w+6 = the length of the rectangle in meters.

Area of rectangle = length \times width

$$=(2w+6)\times w=2w^2+6w$$
.

Since the area of the rectangle is 1,620 square meters, you can set up the following equation.

$$2w^2 + 6w = 1620$$

$$2w^2 + 6w - 1620 = 0$$
 Make

Make one side 0.

$$2(w^2 + 3w - 810) = 0$$

Common factor is 2.

Use the quadratic formula to solve the equation, $w^2 + 3w - 810 = 0$.

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-810)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{3249}}{2} = \frac{-3 \pm 57}{2}$$

Since the width is positive, $w = \frac{-3+57}{2} = 27$.

The length is 2w+6=2(27)+6=60.

The perimeter of the rectangle is 2(length + width) = 2(60 + 27) = 174

5. I

Area of an equilateral triangle with side length of $a = \frac{\sqrt{3}}{4}a^2$. Since the area of the equilateral

triangle is given as $25\sqrt{3}$, you can set up the following equation.

$$\frac{\sqrt{3}}{4}a^2 = 25\sqrt{3}$$
$$a^2 = 25\sqrt{3} \cdot \frac{4}{\sqrt{3}} = 100$$

The area of each square is a^2 , or 100, so the sum of the areas of the three squares is 3×100 , or 300.

6. 14

Let w = the width of the rectangle. The perimeter of the rectangle is given as 5x. Perimeter of rectangle = 2(length + width)

$$5x = 2(\frac{3}{2}x + w)$$
$$5x = 3x + 2w$$
$$2x = 2w$$

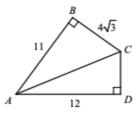
Area of rectangle = length \times width = 294

$$\frac{3}{2}x \cdot x = 294$$

$$x^2 = 294 \cdot \frac{2}{3} = 196$$

$$x = \sqrt{196} = 14$$

7. A



$$AC^2 = AB^2 + BC^2$$
 Pythagorean Theorem
 $AC^2 = 11^2 + (4\sqrt{3})^2$ Substitution
 $AC^2 = 121 + 48 = 169$
 $AC = \sqrt{169} = 13$
 $AC^2 = AD^2 + CD^2$ Pythagorean Theorem
 $169 = 12^2 + CD^2$ Substitution

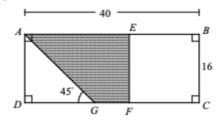
$$25 = CD^2$$
$$5 = CD$$

The area of region ABCD is the sum of the area of $\triangle ABC$ and the area of $\triangle ADC$.

Area of the region ABCD

$$= \frac{1}{2}(11)(4\sqrt{3}) + \frac{1}{2}(12)(5)$$
$$= 22\sqrt{3} + 30$$

8. C



Since BCFE is a square,

$$BC = BE = CF = EF = 16$$
.

$$AE = AB - BE$$
$$= 40 - 16 = 24$$

Triangle AGD is a 45°-45°-90° triangle.

In a 45°-45°-90° triangle, the length of the two legs are equal in measure. Therefore,

$$AD = DG = 16$$
.

$$FG = DC - DG - CF$$

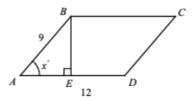
= $40 - 16 - 16 = 8$

Area of the shaded region

$$=\frac{1}{2}(AE+FG)\cdot EF$$

$$=\frac{1}{2}(24+8)\cdot 16=256$$

9. D



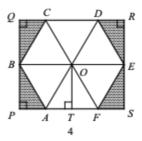
Draw \overline{BE} perpendicular to \overline{AD} .

In
$$\triangle ABE$$
, $\sin x^{\circ} = \frac{BE}{9}$

Therefore, $BE = 9 \sin x^{\circ}$. Area of parallelogram *ABCD*

$$= AD \times BE = 12 \times 9 \sin x^{\circ}$$

10. A



Draw the diagonals of a regular hexagon, \overline{AD} , \overline{BE} , and \overline{CF} .

$$BE = BO + OE = 8$$
 and $QR = BE = 8$

Since ABCDEF is a regular hexagon, the diagonals intersect at the center of the hexagon. Let the point of intersection be O. The diagonals divide the hexagon into 6 equilateral triangles with side lengths of 4. Area of each equilateral triangle

with side lengths of 4 is
$$\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$$
.

Draw \overline{OT} perpendicular to \overline{PS} .

Triangle AOT is a 30°-60°-90° triangle.

Therefore,
$$AT = \frac{1}{2}AO = \frac{1}{2}(4) = 2$$
 and

$$OT = \sqrt{3}AT = 2\sqrt{3}$$
.

In rectangle PQRS, $RS = 2OT = 2(2\sqrt{3}) = 4\sqrt{3}$.

Area of rectangle $PORS = OR \times RS$

$$= 8 \times 4\sqrt{3} = 32\sqrt{3}$$
.

Area of regular hexagon ABCDEF

= 6×area of the equilateral triangle

$$= 6 \times 4\sqrt{3} = 24\sqrt{3}$$

Area of shaded region

= area of rectangle - area of hexagon

$$=32\sqrt{3}-24\sqrt{3}=8\sqrt{3}$$
.

Answer Key

Section 20-1

1. D

4.4.5 3. C

Section 20-2

1. B

3. D

Section 20-3

1. B

4. B 3. D

4. C

Chapter 20 Practice Test

2. C

2. C

1. D

2. A 7.27

4. D 5. C

6. C

8.85

3. B

9. $\frac{16}{3}$ or 5.33

Answers and Explanations

Section 20-1

1. D

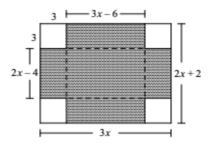
Volume of the block is $36 \times 20 \times 9 - 2(10 \times 8 \times 9)$,

Weight of the cement block = density × volume $= 1.7 \text{ gram } / \text{ cm}^3 \times 5,040 \text{ cm}^3 = 8,568 \text{ gram}$.

2. B

Volume of the aluminum block is $10 \times 8 \times 12 - 8 \times 6 \times 12$, or 384 in³. Weight of the aluminum block = $density \times volume$ $= 0.098 \text{ lb} / \text{in}^3 \times 384 \text{ in}^3 = 37.632 \text{ lb}$.

3. C



The dimension of the box is $(3x-6)\times(2x-4)\times3$. Since the volume of the box is given as 162 in3, set up the following equation:

$$(3x-6)\times(2x-4)\times3=162$$

$$(3x-6)\times(2x-4)=54$$

$$6x^2 - 24x + 24 = 54$$

$$6x^2 - 24x - 30 = 0$$

$$6(x^2-4x-5)=0$$

$$6(x^{-}-4x-5)=0$$

$$6(x-5)(x+1)=0$$

$$x = 5$$
 or $x = -1$

The length of the side is positive, so x = 5. Therefore, the dimension of the original cardboard is (3×5) in $\times(2\times5+2)$ in , or 15 in $\times12$ in .

4. 4.5

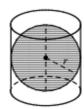
Let h be the height of water when 2,400 cubic inches of water is added into the empty tank. Then $20 \times 16 \times h = 2,400$. Solving for h yields

$$h = \frac{2,400}{20 \times 16} = 7.5 \text{ in}.$$

Since the aquarium tank is 12 inches high, the surface of water will be 12-7.5, or 4.5 inches, from the top of the tank.

Section 20-2

1. B



Volume of the sphere = $\frac{4}{3}\pi r^3$

Volume of the cylinder = πr^2 · height

$$= \pi r^2 \cdot 2r$$
$$= 2\pi r^3$$

Volume of the cylinder

2. C

Volume of the mechanical part

$$=\pi(3)^2 \cdot 8 - \pi(1.5)^2 \cdot 8 = 54\pi \text{ in}^3$$

Since 1 ft = 12 in, 1 ft³ =
$$(12 \text{ in})^3 = 1,728 \text{ in}^3$$
.

Thus
$$1 \text{ in}^3 = \frac{1}{1.728} \text{ ft}^3$$
.

Mass of the mechanical part = density × volume = 490 lb / $\hat{t}t^3 \times 54\pi$ in³ $\cdot \frac{1 \hat{t}t^3}{1,728 \text{ in}^3} \approx 48.1 \text{ lb}$.

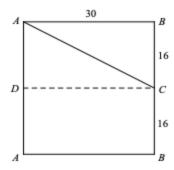
3. D

Volume of cylinder I = $\pi(r)^2 \cdot 2h = 2\pi r^2 h$ Volume of cylinder II = $\pi(2r)^2 \cdot h = 4\pi r^2 h$ volume of cylinder II = $\frac{2\pi r^2 h}{4\pi r^2 h} = \frac{1}{2}$

Thus if the volume of cylinder I is 45π in³, the volume of cylinder II is 90π in³.

4. C

The figure below shows the rectangle, which was laid flat from a cut along \overline{AB} on the cylinder.



$$AC^{2} = AB^{2} + BC^{2}$$

$$AC^{2} = 30^{2} + 16^{2}$$

$$= 900 + 256$$

$$= 1,156$$

$$AC = \sqrt{1,156} = 34$$

Pythagorean Theorem Substitution

Section 20-3

1. B



The regular hexagon consists of 6 equilateral triangles. So the area of the regular hexagon is the sum of the areas of 6 equilateral triangles. Since the area of the equilateral triangle with side

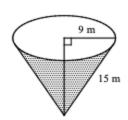
length of a is $\frac{\sqrt{3}}{4}a^2$, the area of the equilateral

triangle with side length of 4 is $\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$.

The area of the hexagon is $6 \times 4\sqrt{3} = 24\sqrt{3}$. Volume of the pyramid

$$=\frac{1}{3}Bh=\frac{1}{3}(24\sqrt{3})(7)=56\sqrt{3}$$

2. C



Let h = the height of the cone.

$$9^2 + h^2 = 15^2$$

Pythagorean Theorem

$$h^2 = 15^2 - 9^2 = 144$$

$$h = \sqrt{144} = 12$$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$=\frac{1}{3}\pi(9)^2(12)=324\pi \text{ m}^3$$

Number of minutes it takes to fill up the reservoir = total volume ÷ rate of filling

$$= 324\pi \text{ m}^3 \div 2.4 \text{ m}^3 / \text{min}$$

3. D



Let r = the radius of the smaller cone. Since the bases are parallel, the proportion

$$\frac{8}{r} = \frac{16}{6}$$
 can be used to find the radius of the

smaller cone.
$$\frac{8}{r} = \frac{16}{6} \implies r = 3$$

Volume of the remaining bottom part =volume of the cone - volume of the top part

$$=\frac{1}{3}\pi(6)^2(16)-\frac{1}{3}\pi(3)^2(8)=168\pi$$

4. B

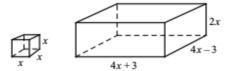
$$C = 2\pi r = 10 \implies r = \frac{10}{2\pi} = \frac{5}{\pi}$$

Volume of the cone

$$=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (\frac{5}{\pi})^2 (8) = \frac{200}{3\pi} \approx 21.2$$

Chapter 20 Practice Test

1. D



Volume of the rectangular prism

$$=(4x+3)(4x-3)(2x)=(16x^2-9)(2x)$$

Volume of the cube = x^3

Since the volume of the rectangular prisms is 30 times the volume of the cube, the equation $(16x^2 - 9)(2x) = 30x^3$ can be used to find the value of x.

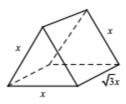
$$(16x^2 - 9)(2x) - 30x^3 = 0$$
 Make one side 0.
 $2x[(16x^2 - 9) - 15x^2] = 0$ GCF is $2x$.
 $2x(x^2 - 9) = 0$ Simplify.

$$2x(x^2-9)=0$$

$$2x(x+3)(x-3) = 0$$
 Factor.
 $x = 0$, $x = -3$, and $x = 3$

Since the dimension has to be positive, x = 3is the correct answer.

2. A



Area of the equilateral triangle with side length

of x is
$$\frac{\sqrt{3}}{4}x^2$$
.

Volume of the triangular prism

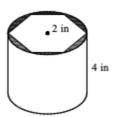
$$= B \cdot h = \frac{\sqrt{3}}{4} x^2 \cdot \sqrt{3} x = \frac{3}{4} x^3$$

Since the volume of the prism is given as $\frac{81}{4}$,

the equation $\frac{3}{4}x^3 = \frac{81}{4}$ can be used to find the value of x.

$$\frac{3}{4}x^3 = \frac{81}{4} \implies 3x^3 = 81 \implies x^3 = 27$$

3. B



Area of the equilateral triangle with side length

of
$$2 = \frac{\sqrt{3}}{4}(2)^2 = \sqrt{3}$$
.

Area of the regular hexagon = $6\sqrt{3}$.

Volume of the hexagonal prism = $6\sqrt{3} \cdot 4 = 24\sqrt{3}$

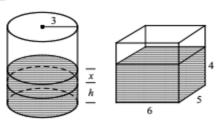
Volume of the cylinder = $\pi(2)^2 \cdot 4 = 16\pi$

The volume of the waste generated by creating the hexagonal prism from the cylinder can be found by subtracting the volume of the hexagonal prism from the volume of the cylinder.

$$16\pi - 24\sqrt{3} \approx 8.69$$

The volume of the waste is about 9 cubic inches.

4. D



The volume of the cylinder with a radius of 3 and a height of x is $\pi(3)^2 x$, or $9\pi x$.

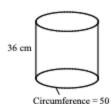
Volume of the water in the rectangular container is 6×5×4, or 120.

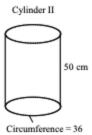
To solve for x, let $9\pi x = 120$.

$$x = \frac{120}{9\pi} \approx 4.24$$

5. C

Cylinder I





Let r_1 = the radius of cylinder I and let r_2 = the radius of cylinder II.

$$2\pi r_1 = 50 \implies r_1 = \frac{50}{2\pi} = \frac{25}{\pi}$$

$$2\pi r_2 = 36 \implies r_2 = \frac{36}{2\pi} = \frac{18}{\pi}$$

Volume of cylinder I

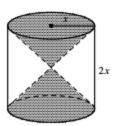
$$=\pi(r_1)^2 h = \pi(\frac{25}{\pi})^2 (36) = \frac{22,500}{\pi}$$

Volume of cylinder II

$$=\pi(r_2)^2h=\pi(\frac{18}{\pi})^2(50)=\frac{16,200}{\pi}$$

The difference of volume between the two cylinders is $\frac{22,500}{\pi} - \frac{16,200}{\pi} \approx 2,005.3$. Choice C is correct.

6. C



Volume of the space inside the cylinder but outside the double cone = volume of the cylinder - volume of the two cones.

$$\pi(x)^{2}(2x) - 2\left[\frac{1}{3}\pi(x)^{2}(x)\right]$$
$$= 2\pi x^{3} - \frac{2}{3}\pi x^{3} = \frac{4}{3}\pi x^{3}$$

7. 27

Surface area of the cube = $6s^2$ Since the surface area of the cube is given as 54 cm^2 , $6s^2 = 54$. $6s^2 = 54$ is simplified to $s^2 = 9$. Solving for s gives s = 3.

Volume of the cube = $s^3 = (3)^3 = 27$

8. 8

Volume of the cone = $\frac{1}{3}\pi(3)^2(10) = 30\pi$

Since the cone is 90 percent filled with shaved ice, the volume of the shaved ice is $30\pi \times 0.9$, or 27π cubic centimeters.

$$27\pi \text{ cm}^3 \approx 84.8 \text{ cm}^3$$

Therefore, to the nearest cubic centimeter, the volume of the shaved is 85 cm³.

9.
$$\frac{16}{3}$$
 or 5.33

Let h = the height of the square pyramid. Volume of the square pyramid

$$=\frac{1}{3}Bh=\frac{1}{3}(6)^2h=12h$$

Volume of the cube = $s^3 = (4)^3 = 64$

Since the square pyramid and the cube have equal volumes, 12h = 64.

Solving for *h* gives $h = \frac{64}{12} = \frac{16}{3}$, or 5.33.