Homework Packet 9 - Answer Key

Unit 15

Answer Key

Section 15-1

1. B	2. C	3. B	4. D	5. C

Section 15-2

1. B	2. C	3. D	4. A

Section 15-3

1. A	2. C	3. B	4. D

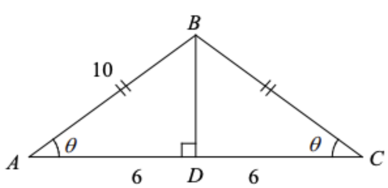
Chapter 15 Practice Test

1. D	2. C	3.C	4. B	5. D
6. D	7. $\frac{5}{13}$	8. 9	9. 10.5	

Answers and Explanations

Section 15-1

1. B



Draw a perpendicular segment from B to the opposite side AC. Let the perpendicular segment intersect side AC at D. Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(12) = 6$.

In right ΔABD,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{6}{10} = 0.6.$$

2. C

$AB^2 = BD^2 + AD^2$ Pythagorean Theorem

$10^2 = BD^2 + 6^2$

$100 = BD^2 + 36$

$64 = BD^2$

$8 = BD$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{8}{10} = 0.8$$

3. B

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BD}{AD} = \frac{8}{6} = \frac{4}{3}$$

4 . D

If x and y are acute angles and $\cos x^\circ = \sin y^\circ$, $x + y = 90$ by the complementary angle theorem.

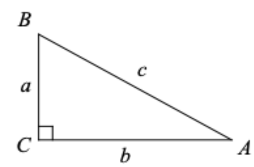
$(3a-14)+(50-a)=90$ $x = 3a-14$, $y = 50-a$

$2a+36=90$ Simplify.

$2a=54$

$a=27$

5. C



I. $\sin A = \frac{\text{opposite of } \angle A}{\text{hypotenuse}} = \frac{a}{c}$

Roman numeral I is true.

II. $\cos B = \frac{\text{adjacent of } \angle B}{\text{hypotenuse}} = \frac{a}{c}$

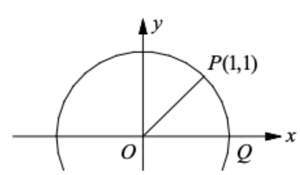
Roman numeral II is true.

III. $\tan A = \frac{\text{opposite of } \angle A}{\text{adjacent of } \angle A} = \frac{a}{b}$

Roman numeral III is false.

Section 15-2

1. B



The graph shows $P(x,y) = P(1,1)$. Thus, $x = 1$ and $y = 1$. Use the distance formula to find the length of radius OA.

$$OA = \sqrt{x^2+y^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \text{ or } \sin\theta = \frac{\sqrt{2}}{2}$$

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Therefore, the measure of $\angle POQ$ is 45°,

which is equal to $45(\frac{\pi}{180}) = \frac{\pi}{4}$ radians.

Thus, $k = \frac{1}{4}$.

2. C

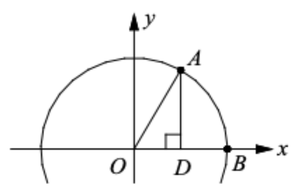
Use the complementary angle theorem.

$\cos(\theta) = \sin(90^\circ - \theta)$, or $\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$

Therefore, $\cos(\frac{\pi}{8}) = \sin(\frac{\pi}{2} - \frac{\pi}{8}) = \sin(\frac{3\pi}{8})$.

All the other answer choices have values different from $\cos(\frac{\pi}{8})$.

3. D

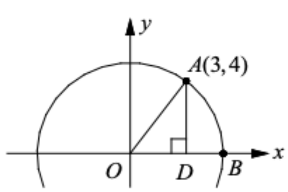


In ΔOAD, $\sin\frac{\pi}{3} = \sin 60^\circ = \frac{AD}{OA} = \frac{AD}{6}$.

Since $\sin 60^\circ = \frac{\sqrt{3}}{2}$, you get $\frac{AD}{6} = \frac{\sqrt{3}}{2}$.

Therefore, $2AD = 6\sqrt{3}$ and $AD = 3\sqrt{3}$.

4. A



Use the distance formula to find the length of OA.

$OA = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\cos\angle AOD = \frac{OD}{OA} = \frac{3}{5}$

Section 15-3

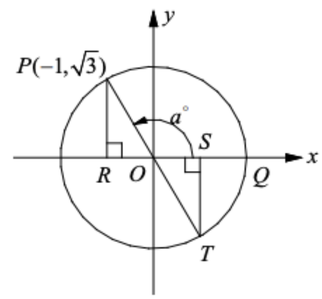
1. A

Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -1$ and $y = \sqrt{3}$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (\sqrt{3})^2 + (-1)^2 = 4$

Which gives $OP = 2$.

$\cos a^\circ = \frac{x}{OP} = \frac{-1}{2}$

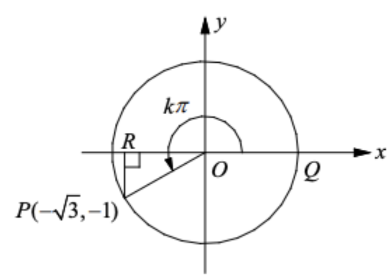


2. C

Since the terminal side of $(a+180)^\circ$ is OT, the value of $\cos(a+180)^\circ$ is equal to $\frac{OS}{OT}$.

$\frac{OS}{OT} = \frac{1}{2}$

3. B



Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -\sqrt{3}$ and $y = -1$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (-1)^2 + (\sqrt{3})^2 = 4$

Which gives $OP = 2$.

Since $\sin\angle POR = \frac{y}{OP} = \frac{-1}{2}$, the measure of $\angle POR$ is equal to 30°, or $\frac{\pi}{6}$ radian.

$k\pi = \pi + \frac{\pi}{6} = \frac{7}{6}\pi$

Therefore, $k = \frac{7}{6}$

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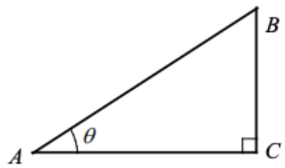
4. D

$$\tan(k\pi) = \tan(\frac{7}{6}\pi) = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Chapter 15 practice Test

1. D



Note: Figure not drawn to scale.

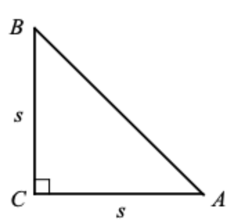
In ΔABC, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC}$.

If $\tan\theta = \frac{3}{4}$, then $BC = 3$ and $AC = 4$.

By the Pythagorean theorem,
$AB^2 = AC^2 + BC^2 = 4^2 + 3^2 = 25$, thus
$AB = \sqrt{25} = 5$.

$$\sin\theta = \frac{BC}{AB} = \frac{3}{5}$$

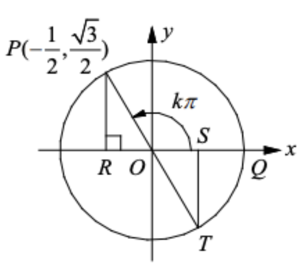
2. C



$$\tan\angle A = \frac{\text{opposite side of } \angle A}{\text{adjacent side of } \angle A} = \frac{s}{s} = 1$$

$$= \frac{s}{s} = 1$$

3. C



Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. To find the length of OP, use the Pythagorean theorem.

$$OP^2 = PR^2 + OR^2 = (\frac{\sqrt{3}}{2})^2 + (\frac{-1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$$

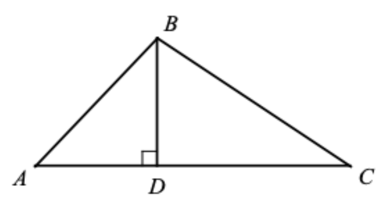
Which gives $OP = 1$. Thus, triangle OPR is 30°-60°-90° triangle and the measure of $\angle POR$ is 60°, which is $\frac{\pi}{3}$ radian. Therefore, the measure of $\angle POQ$ is $\pi - \frac{\pi}{3}$, or $\frac{2\pi}{3}$ radian. If $\angle POQ$ is $k\pi$ radians then k is equal to $\frac{2}{3}$.

4. B

Since the terminal side of $(k+1)\pi$ is OT, the value of $\cos(k+1)\pi$ is equal to $\frac{OS}{OT}$.

$$\frac{OS}{OT} = \frac{1}{2}$$

5. D



Area of triangle $ABC = \frac{1}{2}(AC)(BD)$

Check each answer choice.

A) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(AB\cos\angle ABD)$

$$= \frac{1}{2}(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC})(AB \cdot \frac{BD}{AB})$$

$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

B) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(BC\sin\angle C)$

$$= \frac{1}{2}(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC})(BC \cdot \frac{BD}{BC})$$

$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

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C) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(AB\sin\angle A)$

$= \frac{1}{2}(AB\cdot\frac{AD}{AB} + BC\cdot\frac{CD}{BC})(AB\cdot\frac{BD}{AB})$

$= \frac{1}{2}(AD+CD)(BD) = \frac{1}{2}(AC)(BD)$

D) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(BC\cos\angle C)$

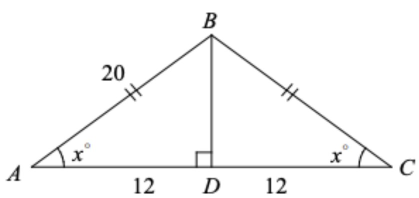
$= \frac{1}{2}(AB\cdot\frac{AD}{AB} + BC\frac{CD}{BC})(BC\cdot\frac{CD}{BC})$

$= \frac{1}{2}(AD+CD)(CD) = \frac{1}{2}(AC)(CD)$

Which does not represent the area of triangle ABC.

Choice D is correct.

6. D



Draw segment BD, which is perpendicular to side AC. Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(24) = 12$.

By the Pythagorean theorem, $AB^2 = BD^2 + AD^2$

Thus, $20^2 = BD^2 + 12^2$.

$BD^2 = 20^2 - 12^2 = 256$

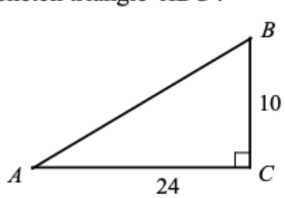
$BD = \sqrt{256} = 16$

In right ΔABD,

$\sin x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{16}{20} = \frac{4}{5}$.

7. $\frac{5}{13}$

Sketch triangle ABC.



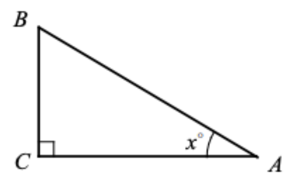
$AB^2 = BC^2 + AC^2$

$AB^2 = 10^2 + 24^2 = 676$

$AB = \sqrt{676} = 26$

$\sin A = \frac{10}{26} = \frac{5}{13}$

8. 9



$\cos x^\circ = \frac{AC}{AB} = \frac{3}{5}$

Let $AC = 3x$ and $AB = 5x$.

$AB^2 = BC^2 + AC^2$ Pythagorean Theorem

$(5x)^2 = 12^2 + (3x)^2$ $BC = 12$

$25x^2 = 144 + 9x^2$

$16x^2 = 144$

$x^2 = 9$

$x = \sqrt{9} = 3$

Therefore, $AC = 3x = 3(3) = 9$

9. 10.5

According to the complementary angle theorem, $\sin\theta = \cos(90-\theta)$.

If $\sin(5x-10)^\circ = \cos(3x+16)^\circ$,

$3x+16 = 90-(5x-10)$.

$3x+16 = 90-5x+10$

$3x+16 = 100-5x$

$8x = 84$

$x = 10.5$

Homework Packet 9 - Answer Key

Answer Key

Section 16-1

1. D   2. C   3. B   4. D

Section 16-2

1. D   2. A   3. B   4. C

Section 16-3

1. A   2. C   3. D   4. B

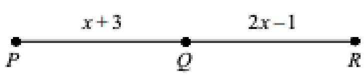
Chapter 16 Practice Test

1. C   2. B   3. A   4. C   5. A
6. D   7. 540   8. 105

Answers and Explanations

Section 16-1

1. D



$PQ = QR$	Definition of Midpoint
$x+3 = 2x-1$	Substitution
$x+3-x = 2x-1-x$	Subtract x from each side.
$3 = x-1$	Simplify.
$4 = x$	
$PR = PQ + QR$	Segment Addition Postulate
$= x+3+2x-1$	Substitution
$= 3x+2$	
$= 3(4)+2 = 14$	$x = 4$

2. C

Note: Figure not drawn to scale.

Let $PS = x$, then $QR = \frac{1}{3}PS = \frac{1}{3}x$.

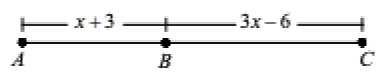
$PR = PQ + QR$	Segment Addition Postulate
$12 = PQ + \frac{1}{3}x$	$PR = 12$ and $QR = \frac{1}{3}x$
$PQ = 12 - \frac{1}{3}x$	Solve for PQ.
$QS = QR + RS$	Segment Addition Postulate
$16 = \frac{1}{3}x + RS$	$QS = 16$ and $QR = \frac{1}{3}x$
$RS = 16 - \frac{1}{3}x$	Solve for RS.
$PS = PQ + QR + RS$	Segment Addition Postulate
$x = (12 - \frac{1}{3}x) + \frac{1}{3}x + (16 - \frac{1}{3}x)$	Substitution
$x = 28 - \frac{1}{3}x$	Simplify.
$\frac{4}{3}x = 28$	Add $\frac{1}{3}x$ to each side.
$\frac{3}{4}\cdot\frac{4}{3}x = \frac{3}{4}\cdot 28$	Multiply $\frac{3}{4}$ by each side.
$x = 21$	

Therefore, $PS = x = 21$.

3. B

Ray CA and Ray CD are opposite rays, because points A, C, and D are collinear and C is between A and D.

4. D



Note: Figure not drwan to scale.

$AB = \frac{2}{3}BC$	Given
$x+3 = \frac{2}{3}(3x-6)$	Substitution
$x+3 = 2x-4$	Simplify.
$7 = x$	Solve for x.
$AC = AB + BC$	Segment Addition Postulate
$= x+3+3x-6$	Substitution
$= 4x-3$	Simplify.
$= 4(7)-3$	$x = 7$
$= 25$	

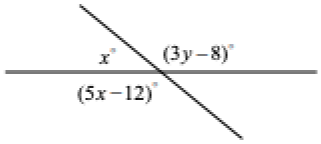
Section 16-2

1. D

$40 + x - 90 = 180$	Straight $\angle$ measures 180.
$x - 50 = 180$	Simplify.
$x - 50 + 50 = 180 + 50$	Add 50 to each side.
$x = 230$	

Homework Packet 9 - Answer Key

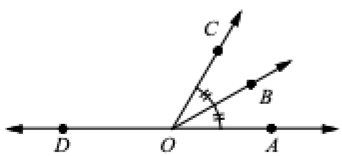
2. A



Note: Figure not drawn to scale.

$x+5x-12=180$	Straight $\angle$ measures 180.
$6x-12=180$	
$6x=192$	
$x=32$	
$x+3y-8=180$	Straight $\angle$ measures 180.
$32+3y-8=180$	$x=32$
$24+3y=180$	Simplify.
$24+3y-24=180-24$	
$3y=156$	
$y=52$	

3. B



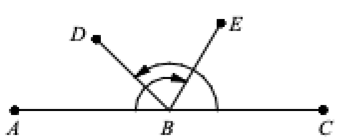
Note: Figure not drawn to scale.

$m\angle BOA=\frac{1}{2}m\angle COA$	Definition of $\angle$ bisector
$m\angle BOA=\frac{1}{2}(8x-12)$	Substitution
$m\angle BOA=4x-6$	Simplify.
$m\angle DOB+m\angle BOA=180$	Straight $\angle$ measures 180.
$11x+6+4x-6=180$	Substitution
$15x=180$	Simplify.
$x=12$	

Thus, $m\angle COA=8x-12=8(12)-12=84$.

$m\angle DOC+m\angle COA=180$	Straight $\angle$ measures 180.
$m\angle DOC+84=180$	$m\angle COA=84$
$m\angle DOC=96$	

4. C



Note: Figure not drawn to scale.

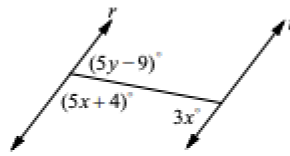
Let $m\angle DBE=x$

$m\angle ABE$	
$=m\angle ABD+m\angle DBE$	Angle Addition Postulate
$120=m\angle ABD+x$	Substitution
$120-x=m\angle ABD$	
$m\angle ABD+m\angle CBD=180$	Straight $\angle$ measures 180.
$120-x+135=180$	Substitution
$255-x=180$	Simplify.
$x=75$	

Therefore, $m\angle DBE=x=75$.

Section 16-3

1. A

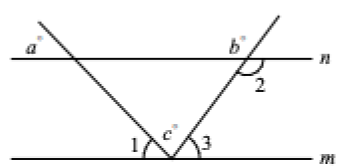


Note: Figure not drawn to scale

$5x+4+3x=180$	If $r \parallel t$, consecutive interior $\angle s$ are supplementary.
$8x+4=180$	Simplify.
$8x=176$	
$x=22$	
$5x+4+5y-9=180$	Straight $\angle$ measures 180.
$5x-5+5y=180$	Simplify.
$5(22)-5+5y=180$	$x=22$
$110-5+5y=180$	Simplify.
$105+5y=180$	Simplify.
$5y=75$	Simplify.
$y=15$	

Therefore, $x+y=22+15=37$.

2. C



$m\angle 1=a$	If $m \parallel n$, corresponding $\angle s$ are $\cong$.
$m\angle 1=50$	$a=50$
$m\angle 2=b$	Vertical $\angle s$ are $\cong$.
$m\angle 2=120$	$b=120$

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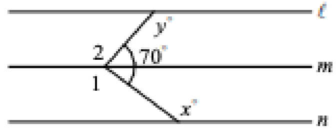
$m\angle 2 + m\angle 3 = 180$	If $m \parallel n$, consecutive interior $\angle s$ are supplementary.
$120 + m\angle 3 = 180$	$m\angle 2 = 120$
$m\angle 3 = 60$	
$m\angle 1 + c + m\angle 3 = 180$	Straight $\angle$ measures 180.
$50 + c + 60 = 180$	$m\angle 1 = 50$ and $m\angle 3 = 60$
$c + 110 = 180$	Simplify.
$c = 70$	

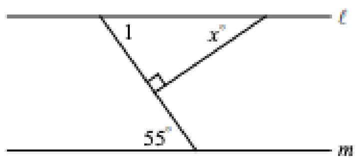
3. D



Note: Figure not drawn to scale.

$m\angle 1 = x$	If $m \parallel n$, alternate interior $\angle s$ are $\cong$.
$m\angle 2 = y$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$m\angle 1 + m\angle 2 + 70 = 360$	There are 360° in a circle.
$x + y + 70 = 360$	$m\angle 1 = x$ and $m\angle 2 = y$
$x + y = 290$	

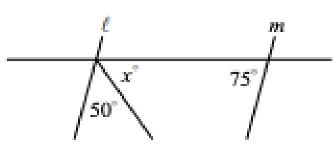
4. B



$m\angle 1 = 55$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$m\angle 1 + x = 90$	The acute $\angle s$ of a right triangle are complementary.
$55 + x = 90$	$m\angle 1 = 55$
$x = 35$	

Chapter 16 Practice Test

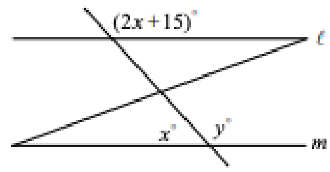
1. C



Note: Figure not drawn to scale.

$50 + x + 75 = 180$	If $\ell \parallel m$, consecutive interior $\angle s$ are supplementary.
$125 + x = 180$	Simplify.
$x = 55$	

2. B

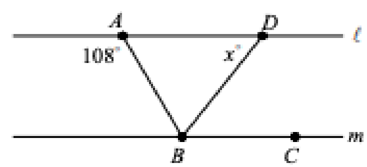


Note: Figure not drwan to scale.

$y = 2x + 15$	If $\ell \parallel m$, consecutive interior $\angle s$ are supplementary.
$x + y = 180$	Straight $\angle$ measures 180.
$x + (2x + 15) = 180$	$y = 2x + 15$
$3x + 15 = 180$	Simplify.
$3x = 165$	
$x = 55$	

Therefore, $y = 2x + 15 = 2(55) + 15 = 125$.

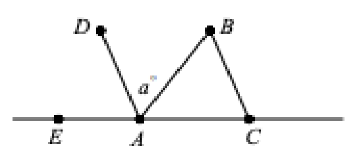
3. A



Note: Figure not drawn to scale.

$m\angle ABC = 108$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$m\angle DBC = \frac{1}{2} m\angle ABC$	Definition of $\angle$ bisector
$m\angle DBC = \frac{1}{2}(108)$	$m\angle ABC = 108$
$m\angle DBC = 54$	Simplify.
$x = m\angle DBC$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$x = 54$	$m\angle DBC = 54$

4. C



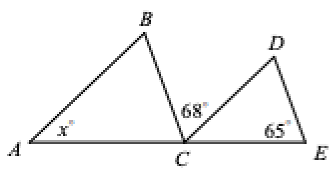
Homework Packet 9 - Answer Key

$m\angle BAC = m\angle DAB$	Definition of $\angle$ bisector
$m\angle BAC = a$	$m\angle DAB = a$

Since straight angles measure 180,
$m\angle DAE + m\angle DAB + m\angle BAC = 180$.

$m\angle DAE + a + a = 180$	$m\angle DAB = m\angle BAC = a$
$m\angle DAE = 180 - 2a$	Subtract $2a$.
$m\angle BCA = m\angle DAE$	If $DA \parallel BC$, corresponding $\angle s$ are $\cong$.
$m\angle BCA = 180 - 2a$	$m\angle DAE = 180 - 2a$

5. A



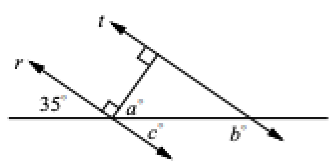
Note: Figure not drawn to scale.

$m\angle BCA = m\angle DEC$	If $DE \parallel BC$, corresponding $\angle s$ are $\cong$.
$m\angle BCA = 65$	$m\angle DEC = 65$
$m\angle DCE = x$	If $AB \parallel CD$, corresponding $\angle s$ are $\cong$.

Since straight angles measure 180,
$m\angle BCA + m\angle BCD + m\angle DCE = 180$.

$65 + 68 + x = 180$	Substitution
$133 + x = 180$	Simplify.
$x = 47$	

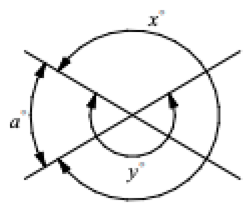
6. D



$c = 35$	Vertical $\angle s$ are $\cong$.
$a + c = 90$	$\angle a$ and $\angle c$ are complementary.
$a + 35 = 90$	$c = 35$
$a = 55$	
$b + c = 180$	If $r \parallel t$, consecutive interior $\angle s$ are supplementary.
$b + 35 = 180$	$c = 35$
$b = 145$	

Therefore, $a + b = 55 + 145 = 200$.

7. 540

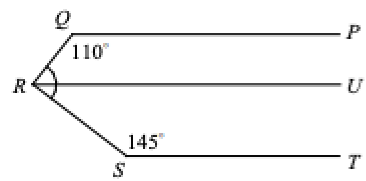


Draw $\angle a$.

$x + a = 360$	360° in a circle.
$x = 360 - a$	Subtract a from each side.
$y - a = 180$	Straight $\angle$ measures 180.
$y = 180 + a$	Add a to each side.

Therefore, $x + y = (360 - a) + (180 + a) = 540$.

8. 105



Note: Figure not drawn to scale.

Draw $\overline{RU}$, which is parallel to $\overline{PQ}$ and $\overline{ST}$.

If two lines are parallel, then the consecutive interior angles are supplementary. Therefore,
$m\angle PQR + m\angle QRU = 180$ and
$m\angle RST + m\angle URS = 180$.

$110 + m\angle QRU = 180$	$m\angle PQR = 110$
$m\angle QRU = 70$	Subtract 110.
$145 + m\angle URS = 180$	$m\angle RST = 145$
$m\angle URS = 35$	Subtract 145.

By the Angle Addition Postulate,
$m\angle QRS = m\angle QRU + m\angle URS$.
Substituting 70 for $m\angle QRU$ and 35 for $m\angle QRU$ gives $m\angle QRS = 70 + 35 = 105$.