1.

A   M   B   P

$H)\ AM = MB$

$T)\ 2PM = PA + PB$
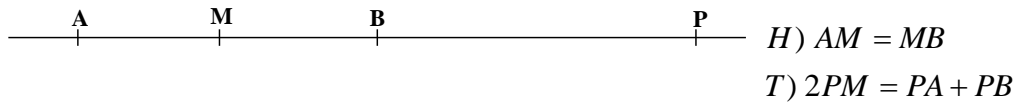
1) $PM = PA - AM$

2) $PM = PB + MB$

$2PM = PA + PB - AM + MB$ $(3)AM = MB$

$\Rightarrow 2PM = PA + PB - MB + MB$

$\quad 2PM = PA + PB$

2.

A   P   M   B

$H)\ 2PM = PB - PA$

$T)\ AM = MB$

1) $PM = AM - PA$

2) $PM = PB - MB$

$2PM = AM + PB - PA - MB$ $(H)2PM = PB - PA$

$\Rightarrow PB - PA = AM + PB - PA - MB$

$\quad PB - PB - PA + PA - AM = -MB$

$\quad -AM = -MB$

$\quad AM = MB$

3.

A   B   C

$H)\ AB = \dfrac{7}{5}BC$

$T)\ \dfrac{BC}{AC} = ?$
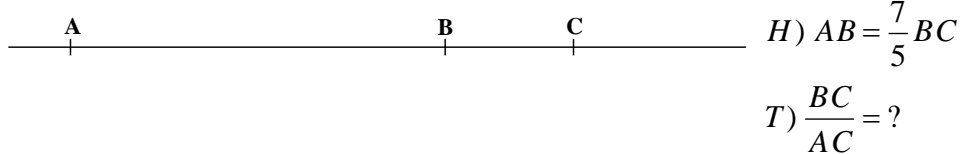
(1) $AC = AB + BC$

(2) $\dfrac{BC}{AC} = ?$

$\dfrac{BC}{AB + BC}$ (1) *en* (2)

$\dfrac{BC}{\dfrac{7}{5}BC + BC}$ (H) $AB = \dfrac{7}{5}BC$ *en* (2)

$\dfrac{BC}{\dfrac{7BC + 5BC}{5}}$

$\dfrac{BC}{\dfrac{12BC}{5}}$

$\dfrac{5}{12}$

4.
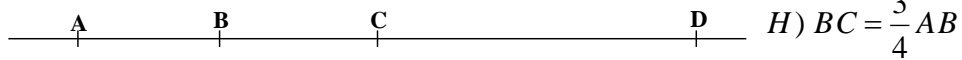
(H) $BC = \dfrac{3}{4}AB$

(T) $\dfrac{AC}{BC} = ?$

(1) $BC = \dfrac{3}{4}AB$

$\Rightarrow AB = \dfrac{4}{3}BC$

(2) $AC = AB + BC$

(3) $\dfrac{AC}{BC} = ?$

(1)*en*(2)

$AC = \dfrac{4}{3}BC + BC$

$AC = \dfrac{7}{3}BC$

(2)*en*(3)

$\dfrac{\dfrac{7BC}{3}}{BC}$

$\dfrac{7}{3}$

5. Dados los puntos colineales A, B y C. Si las longitudes AB y BC son proporcionales a los números 9 y 5 respectivamente, y AC=504u. Calcular AB.

A ———————————————————— B ———————————— C

$$H)\frac{AB}{BC}=\frac{9}{5}$$
$$AC=504u$$
$$T)\,AB=?$$

(1) $\dfrac{AB}{BC}=\dfrac{9}{5}$

$\Rightarrow BC=\dfrac{5}{9}AB$

(2) $AC=AB+AC$

(3) $AC=504u$

(1)en(2)

$AC=AB+\dfrac{5}{9}AB$
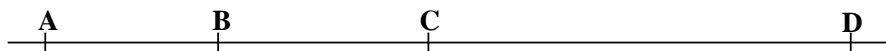
$AC=\dfrac{14}{9}AB$

(2)en(3)

$\dfrac{14}{9}AB=504u$

$AB=324u$

6. Dados los puntos colineales A, B, C y D. Si BD-AB=2BC. Demostrar que AC=CD

A ———— B ———— C ———————————— D

$$H)\,BD-AB=2BC$$
$$T)\,AC=CD$$

(1) $AC=AB+BC$

(2) $CD=BD-CD$

(3) $BD-AB=2BC$

$BD-AB=BC+BC$

$BD-BC=AB+BC$

(1) y (2) en (3)

$CD=AC$

7.
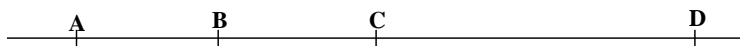
A ———— B ———— C ———————————— D

$$H)\,AB=BD-AC$$
$$T)\,CD=2AB$$

(1) $AB = BD - AC$

(2) $AB = AD - BD$

(3) $AB = AC - BC$

(4) $CD = AD - AB - BC$

(1) *en* (4)

   $CD = AD - (BD - AC) - BC$

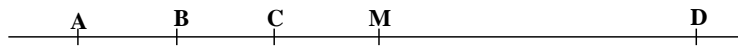 $CD = AD - BD + AC - BC$

(1) *y* (2) *en* (4)

   $CD = AB + AB$

   $CD = 2AB$

8.

$H) \, AM = AB + AC$

   $AB = BC$

   $AM = MD$

$T) \, CD = 2AC$

(1) $AM = AB + AC$

(2) $AB = BC$

(3) $AM = MD$

(4) $AM = AC + CM$

(5) $AC = AB + BC$

(6) $CD = CM + MD$

(3) *en* (6)

$CD = CM + AM$

(1) *en* (6)

$CD = CM + AB + AC$

$CD = AB + AC + CM$

(4) *en* (6)

$CD = AB + AM$

(1) *en* (5)

$CD = AB + AB + AC$

(2) *en* (6)
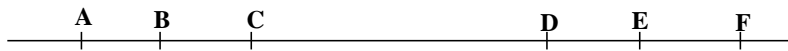
$CD = AB + BC + AC$

(5) *en* (6)

$CD = AC + AC$

$CD = 2AC$

9.

A   B   C   D   E   F

$$H) \; CD = 2AB$$
$$BF = \frac{3}{2}CE$$
$$T) \; DE = 2EF$$

$$DE = CE - CD$$
$$DE = \frac{2}{3}BF - CD$$
$$DE = \frac{2BF - 3CD}{3}$$
$$3DE = BF + BF - CD - CD - CD$$
$$3DE = BC + DE + EF + BC + DE + EF - CD$$
$$3DE = BC + DE + EF + BC + DE + EF - 2AB$$
$$3DE = 2DE + 2EF + 2BC - 2AB$$
$$3DE - 2DE = 2EF + 2AB - 2AB$$
$$DE = 2EF$$

10. Dados los puntos colineales A, R, C, P, y D tales que; AP=PD, AR=PC, Y RC=20. Calcular AD.

$$H) \; AP = PD$$
$$AR = PC$$
$$RC = 20$$
$$T) \; AD = ?$$
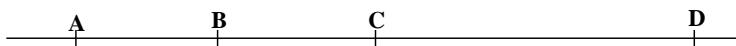$$RC = RP + PC$$
$$RC = RP + AR$$
$$RC = AP$$
$$AD = AP + PD$$
$$AD = RC + RC$$
$$AD = 20 + 20$$
$$AD = 40$$

11.

A   B   C   D

$$H) \; AC + BD = 14$$
$$AD = 11$$
$$T) \; BC = ?$$

(1) $AC + BD = 14$

$\Rightarrow BD = 14 - AC$

(2) $AD = 11$

(3) $AD = AC + CD$

(4) $BC = BD - CD$

(1) en (4)

$\quad BC = 14 - AC - CD$
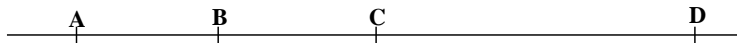
$\quad BC = 14 - (AC + CD)$

(3) en (4)

$\quad BC = 14 - AD$

(2) en (4)

$\quad BC = 14 - 11$

$\quad BC = 3$

12. Si en el gráfico: CD=2AB. Demostrar que: $AC = \dfrac{BD + BC}{2}$

(1) $CD = 2AB$

(2) $BD = AD - AB$

(3) $AC = AD - CD$

(4) $\underline{AC = AB + BC}$

$\quad 2AC = AD + AB - CD + BC$

(1) en (4)

$\quad 2AC = AD + AB - 2AB + BC$

$\quad 2AC = AD - AB + AB - AB + BC$

(2) en (4)

$\quad 2AC = BD + BC$

$\quad AC = \dfrac{BD + BC}{2}$

13. Dados los puntos colineales A, B, C, y D. Si AD=24u. CD=8u, y $\dfrac{AB}{BC} = 3$. Calcular BC.

$H) \; AD = 24u$

$\quad CD = 8u$

$\quad \dfrac{AB}{BC} = 3$

$T) \; BC = ?$

$$BC = AD - AB - CD$$
$$BC = AD - 3BC - CD$$
$$BC = 24 - 3BC - 8$$
$$4BC = 16$$
$$BC = \frac{16}{4}$$
$$BC = 4$$

14. Dados los puntos colineales consecutivos Q, A, B y P tales que: QA=20m, BP=40m, y QB y AP están en la razón 4/5. Calcular AB.

$$H)\ QA = 20$$
$$BP = 40$$
$$\frac{QB}{AP} = \frac{4}{5}$$
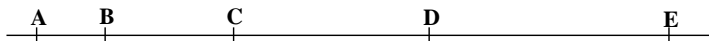$$T)\ AB = ?$$
$$\frac{QA + AB}{AB + BP} = \frac{4}{5}$$
$$5(QA + AB) = 4(AB + BP)$$
$$5(20 + AB) = 4(AB + 40)$$
$$100 + 5AB = 4AB + 160$$
$$AB = 60$$

15.

$$H)\ AB = \frac{BC}{2} = \frac{CD}{3} = \frac{DE}{4}$$
$$DE - BC = 2.$$
$$T)\ CD = ?$$

(1) $\dfrac{CD}{3} = \dfrac{DE}{4}$

$\Rightarrow DE = \dfrac{4}{3}CD$

(2) $\dfrac{BC}{2} = \dfrac{CD}{3}$

$\Rightarrow BC = \dfrac{2}{3}CD$

(3) $DE - BC = 2$

(1) y (2) en (3)

$\dfrac{4}{3}CD - \dfrac{2}{3}CD = 2$
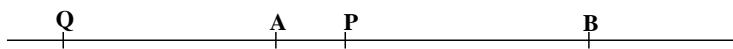
$4CD - 2CD = 6$

$2CD = 6$

$CD = \dfrac{6}{2}$

$CD = 3$

16. Dados los puntos colineales A, B, C, D, E y F. Si AB=BD, BC=CE, DE=EF y BD-EF=6u. Calcular CD.

17. Dados los puntos colineales A, B, C, D y E. Si BC=3AB, $\dfrac{CD}{BC} = \dfrac{1}{3}$ y $\dfrac{AC}{CE} = \dfrac{4}{9}$. Encontrar $\dfrac{DE}{BC}$

18. Dados los puntos colineales A, B, C, D y E. Si $\dfrac{BD}{CE} = \dfrac{2}{5}$, $DE - AB = 6$, $AE = 40$ y $BD = 10$. Calcular CD.
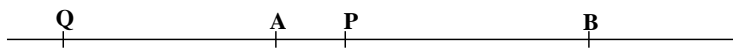
19.

$H)\ \dfrac{AP}{PB} = \dfrac{AQ}{QB}$
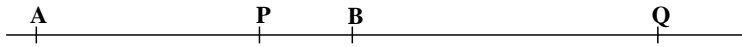
$PB = 3420u$

$BQ = 16074u$

$T)\ AP = ?$

20.

$H)\ \dfrac{AP}{PB} = \dfrac{AQ}{QB}$

$AB = 792u$

$PQ = 247u$

$T)\ AQ = ?$

21.

A —— P B ———— Q

$$H) \; \frac{AP}{PB} = \frac{AQ}{QB}$$

$$PB \times BQ = 28u$$

$$BQ - PB = 7u$$

$$T) \; AB = ?$$

$(1) \; PB \times BQ = 28u$

$(2) \; BQ - PB = 7u$

$\Rightarrow PB = BQ - 7$

$(2) \; en \; (1)$

$(BQ - 7) \times BQ = 28u$

$BQ^2 - 7BQ - 28$

$$BQ = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$BQ = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-28)}}{2(1)}$$

$BQ = 9,84u$

$PB = BQ - 7$

$\Rightarrow PB = 9,84 - 7$

$\Rightarrow PB = 2,84$

$$\frac{AP}{PB} = \frac{AQ}{QB}$$

$$\frac{AB - BP}{PB} = \frac{AB + BQ}{BQ}$$

$$\Rightarrow \frac{AB - 2,84}{2,84} = \frac{AB + 9,84}{9,84}$$

$9,84(AB - 2,84) = 2,84(AB + 9,84)$
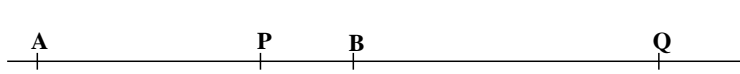
$9,84AB - 27,95 = 2,84AB + 27,95$

$7AB = 55,9$

$AB = 8u$

22.
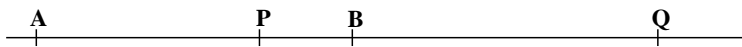
A —— P B ———— Q

$$H) \; \frac{AP}{PB} = \frac{AQ}{QB}$$
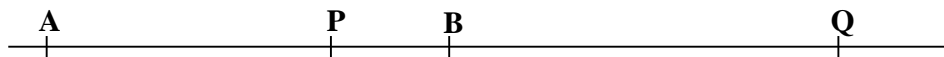
$$T) \; AB = \frac{2AQ \times PB}{PQ}$$

23.

A ——— P — B ———————— Q ———

$$H) \frac{AP}{PB} = \frac{AQ}{QB}$$

$$T) AB = \frac{2AP \times AQ}{2AP + PQ}$$

24.

A ————————— P — B ————————— Q ———

$H) AP \times BQ = PB \times AQ \qquad T)PB = ?$

$\quad AP = PQ = 20u$

$AP = PQ = 20$

$PB = PQ - PB$

$PB = 20 - PB$

$AQ = AP + PQ$

$AQ = AP + AP$

$AQ = 20 + 20$

$AQ = 40$

$AP \times BQ = PB \times AQ$

$20(20 - PB) = 40PB$

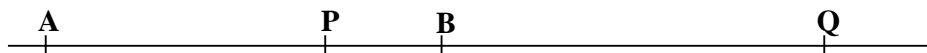$400 - 20PB = 40PB$
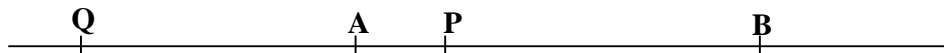
$60PB = 400$

$PB = \dfrac{400}{60}$

$PB = 6,66u$

25.

A ————————— P — B ————————— Q ———

$H) AP \times BQ = PB \times AQ \qquad T)PB = ?$

$\quad AP - BQ = 8u$
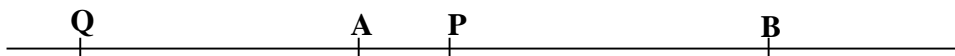
26.

——— Q ————————— A — P ————— B ———

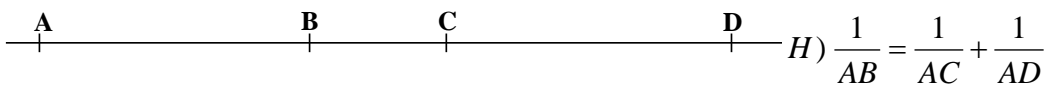$H)\ AP \times BQ = PB \times AQ$ $T)PB = \dfrac{2}{3}AB$

$AQ = AB$

27. En una recta se toman los puntos $A,\, B,\, C$ y D de manera que: $\dfrac{AB}{AD} = \dfrac{BC}{BD}$. Demostrar que
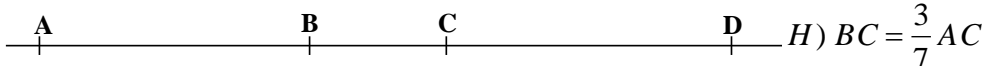
$BC = \dfrac{AB \times BD}{AB + BD}$

Q ————— A — P ————————— B

$H)\ AP \times BQ = PB \times AQ$ $T)AB = \dfrac{2PB \times QB}{BQ + PB}$

A ——————— B ——— C ——————————— D

$H)\ \dfrac{1}{AB} = \dfrac{1}{AC} + \dfrac{1}{AD}$

$T)\ AB^2 = BC \times BD$

A ——————————— B ——— C ———————————————— D

$H)\ BC = \dfrac{3}{7}AC$

$T)\ BD = \dfrac{4CD + 3AD}{7}$

A ————————— B — C ———————————— D ————— E

$H)\ \dfrac{AB}{AD} = \dfrac{DE}{EB}$

$T)\ AD = EB$

P ——————————— Q ——————————— R ——————————— S
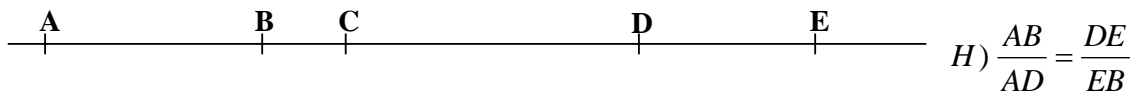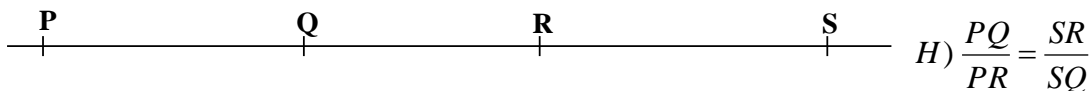
$H)\ \dfrac{PQ}{PR} = \dfrac{SR}{SQ}$

$T)\ PR = QS$
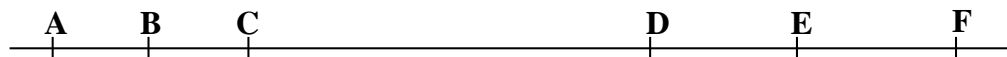
A — B — C ————————————————— D ———— E ———— F

$H)\ AB = BC$ $T)BE = ?$

$DE = EF$

$AD = 10u$

$CF = 8u$

A ————— P — B ———————————————— C ————————— D
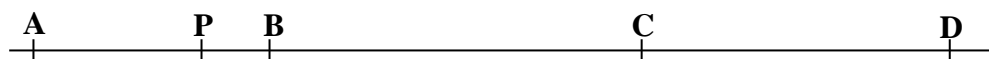
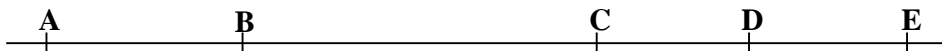$H)\ AB = BC = CD$ $T)\ PB = \dfrac{PD - 2AP}{3}$
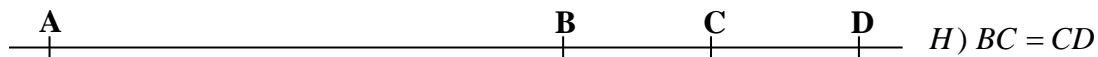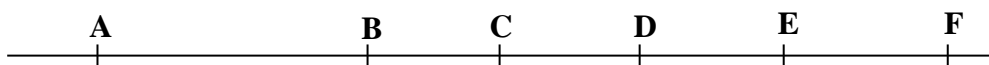
A————————————————B———C———D $H) BC = CD$

$$T) AC^2 = AB \times AD + \frac{BD^2}{4}$$

37. En una recta se ubican los puntos colineales $A, B, C, D, E,$ y F. Si: $AB = BC, CE = EF,$ y $AD = DF,$ Demostrar que $CD = EF - BC.$

A————B————————C————D————E

$H) \dfrac{AC}{CE} = \dfrac{BE}{AD}$ $T) AC^2 - CE^2 = BC \times DE - AB \times CD$

A————B————C————D————E————F

$H) AB = BD$ $T) BE = ?$
$CE = EF$
$AC = 10u$
$DF = 14u$
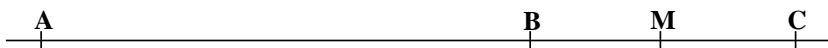
A————————————B———M———C $H) BM = MC$

$$T) AB^2 + AC^2 = 2(AM^2 + BM^2)$$
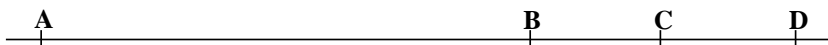
$H) BC = DC$

A————————————B———C———D $AB = a$
$AC = m$
$AD = b$

$$T) m = \sqrt{a \times b + \frac{(b-a)^2}{4}}$$
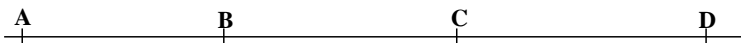
A————————————————B———C———D

$H_1) AB \times CD = 2AD \times BC$ $H_2 = \dfrac{1}{AD} + \dfrac{7}{AB} = \dfrac{8}{AC}$
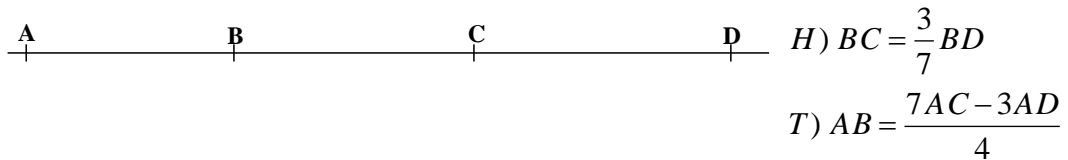
$T_1) \dfrac{2}{AB} + \dfrac{1}{AD} = \dfrac{3}{AC}$ $T_2 = AB \times CD = 7BC \times AD$

A————B————C————D

$H) \dfrac{1}{AB} + \dfrac{1}{BD} = \dfrac{1}{BC}$ $T) AB \times BD = BC \times AD$

$$H)\ BC = \frac{3}{7}BD$$

$$T)\ AB = \frac{7AC - 3AD}{4}$$

45. En una recta se toman los puntos consecutivos $A, B, C, D, E,$ y F, de modo que: $BE = \frac{5}{8}AF$. Calcular sabiendo que $AC + BD + CE + DF = 39u$.