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## 1 Basic

### 1.1 Increase Stack Size

```
//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        }
    }
}
```

### 1.2 Misc

```
編譯參數: -std=c++14 -Wall -Wshadow (-fsanitize=
undefined)
//check special cases for example (n==1)
//check size arrays

#include <random>
mt19937 gen(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

#define SECS ((double)clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;
__builtin_popcountll //換成二進位有幾個1
```

### 1.3 python-related

```
parser:
int(eval(num.replace("/", "/")))

from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

itwo = Decimal(0.5)
two = Decimal(2)

N = 200
def angle(cosT):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cosT = ((cosT + 1) / two) ** itwo
        sinT = (1 - cosT * cosT) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))
```

## 2 flow

### 2.1 ISAP

```
#define SZ(c) ((int)(c).size())
struct MaxFlow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
        }
    }
};
```

```

    iter[i] = d[i] = gap[i] = 0;
}
}
void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v])));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
}
int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
        Edge &e = G[p][i];
        if(e.c > 0 && d[p] == d[e.v]+1) {
            int f = dfs(e.v, min(flow, e.c));
            if(f) {
                e.c -= f;
                G[e.v][e.r].c += f;
                return f;
            }
        }
    }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
        d[p]++;
        iter[p] = 0;
        ++gap[d[p]];
    }
    return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
void reset() {
    for(int i=0; i<=tot; i++) {
        iter[i]=d[i]=gap[i]=0;
    }
}
} flow;

```

## 2.2 MinCostFlow

```

struct MinCostMaxFlow{
typedef int Tcost;
static const int MAXV = 20010;
static const int INFf = 1000000;
static const Tcost INFc = 1e9;
struct Edge{
    int v, cap;
    Tcost w;
    int rev;
    Edge(){}
    Edge(int t2, int t3, Tcost t4, int t5)
        : v(t2), cap(t3), w(t4), rev(t5) {}
};
int V, s, t;
vector<Edge> g[MAXV];
void init(int n, int _s, int _t){
    V = n; s = _s; t = _t;
    for(int i = 0; i <= V; i++) g[i].clear();
}
void addEdge(int a, int b, int cap, Tcost w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
}
Tcost d[MAXV];
int id[MAXV], mom[MAXV];
bool inqu[MAXV];
queue<int> q;
pair<int,Tcost> solve(){
    int mxf = 0; Tcost mnc = 0;
    while(1){
        fill(d, d+1+V, INFc);
        fill(inqu, inqu+1+V, 0);
        fill(mom, mom+1+V, -1);
        mom[s] = s;
        d[s] = 0;
        q.push(s); inqu[s] = 1;
        while(q.size()){
            int u = q.front(); q.pop();
            inqu[u] = 0;

```

```

                for(int i = 0; i < (int) g[u].size(); i++){
                    Edge &e = g[u][i];
                    int v = e.v;
                    if(e.cap > 0 && d[v] > d[u]+e.w){
                        d[v] = d[u]+e.w;
                        mom[v] = u;
                        id[v] = i;
                        if(!inqu[v]) q.push(v), inqu[v] = 1;
                    }
                }
            }
            if(mom[t] == -1) break ;
            int df = INFf;
            for(int u = t; u != s; u = mom[u])
                df = min(df, g[mom[u]][id[u]].cap);
            for(int u = t; u != s; u = mom[u]){
                Edge &e = g[mom[u]][id[u]];
                e.cap -= df;
                g[e.v][e.rev].cap += df;
            }
            mxf += df;
            mnc += df*d[t];
        }
        return {mxf,mnc};
    }
} flow;

```

## 2.3 Dinic

```

struct Dinic{
static const int MXN = 10000;
struct Edge{ int v,f,re; };
int n,s,t,level[MXN];
vector<Edge> E[MXN];
void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
}
void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
}
bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
        int u = que.front(); que.pop();
        for (auto it : E[u]){
            if (it.f > 0 && level[it.v] == -1){
                level[it.v] = level[u]+1;
                que.push(it.v);
            }
        }
    }
    return level[t] != -1;
}
int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == level[u]+1){
            int tf = DFS(it.v, min(nf,it.f));
            res += tf; nf -= tf; it.f -= tf;
            E[it.v][it.re].f += tf;
            if (nf == 0) return res;
        }
    }
    if (!res) level[u] = -1;
    return res;
}
int flow(int res=0){
    while ( BFS() )
        res += DFS(s,2147483647);
    return res;
}
} flow;

```

## 2.4 Kuhn Munkres 最大完美二分匹配

```

struct KM{ // max weight, for min negate the weights

```

```

static const int MXN = 2001; // 1-based
static const ll INF = 0x3f3f3f3f;
int n, mx[MXN], my[MXN], pa[MXN];
ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
bool vx[MXN], vy[MXN];
void init(int _n) {
    n = _n;
    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
}
void addEdge(int x, int y, ll w) {g[x][y] = w;}
void augment(int y) {
    for(int x, z; y; y = z)
        x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
}
void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
        while(q.size()) {
            int x=q.front(); q.pop(); vx[x]=1;
            for(int y=1; y<=n; ++y) if(!vy[y]){
                ll t = lx[x]+ly[y]-g[x][y];
                if(t==0){
                    pa[y]=x;
                    if(!my[y]){augment(y);return;}
                    vy[y]=1, q.push(my[y]);
                }else if(sy[y]>t) pa[y]=x, sy[y]=t;
            }
        }
        ll cut = INF;
        for(int y=1; y<=n; ++y)
            if(!vy[y]&&cut>sy[y]) cut=sy[y];
        for(int j=1; j<=n; ++j){
            if(vx[j]) lx[j] -= cut;
            if(vy[j]) ly[j] += cut;
            else sy[j] -= cut;
        }
        for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
        }
    }
}
ll solve(){
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
        lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
}
}graph;

```

## 2.5 Directed MST

```

/* Edmond's algorithm for Directed MST
 * runs in O(VE)
 */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
    int u, v, c;
    Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++V; }
inline void addEdge(int u, int v, int c)
{ edges[++E] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
    fill(con, con+V+1, 0);
    int r1 = 0, r2 = 0;
    while(1){
        fill(mnInW, mnInW+V+1, INF);
        fill(prv, prv+V+1, -1);
        REP(i, 1, E){
            int u=edges[i].u, v=edges[i].v, c=edges[i].c;

```

```

            if(u != v && v != root && c < mnInW[v])
                mnInW[v] = c, prv[v] = u;
        }
        fill(vis, vis+V+1, -1);
        fill(cyc, cyc+V+1, -1);
        r1 = 0;
        bool jf = 0;
        REP(i, 1, V){
            if(con[i]) continue;
            if(prv[i] == -1 && i != root) return -1;
            if(prv[i] > 0) r1 += mnInW[i];
            int s;
            for(s = i; s != -1 && vis[s] == -1; s = prv[s])
                vis[s] = i;
            if(s > 0 && vis[s] == i){
                // get a cycle
                jf = 1; int v = s;
                do{
                    cyc[v] = s, con[v] = 1;
                    r2 += mnInW[v]; v = prv[v];
                }while(v != s);
                con[s] = 0;
            }
        }
        if(!jf) break;
        REP(i, 1, E){
            int &u = edges[i].u;
            int &v = edges[i].v;
            if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
            if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
            if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
            if(u == v) edges[i--] = edges[E--];
        }
    }
    return r1+r2;
}

```

## 2.6 SW min-cut (不限 S-T 的 min-cut)

```

// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n; FZ(edge); FZ(del);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    void search(int &s, int &t){
        FZ(vst); FZ(wei);
        s = t = -1;
        while(true){
            int mx=-1, cur=0;
            for(int i=0; i<n; i++){
                if(!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            }
            if(mx == -1) break;
            vst[cur] = 1;
            s = t; t = cur;
            for(int i=0; i<n; i++){
                if(!vst[i] && !del[i]) wei[i] += edge[cur][i];
            }
        }
    }
    int solve(){
        int res = 2147483647;
        for(int i=0, x, y; i<n-1; i++){
            search(x, y);
            res = min(res, wei[y]);
            del[y] = 1;
            for(int j=0; j<n; j++){
                edge[x][j] = (edge[j][x] += edge[y][j]);
            }
        }
        return res;
    }
}graph;

```

## 2.7 Max flow with lower/upper bound

```

// flow use ISAP
// Max flow with lower/upper bound on edges

```

```

// source = 1 , sink = n
int in[ N ] , out[ N ];
int l[ M ] , r[ M ] , a[ M ] , b[ M ]; //0-base, a下界, b
上界
int solve(){
    flow.init( n ); //n為點的數量, m為邊的数量, 點是1-
    base
    for( int i = 0 ; i < m ; i ++ ){
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ] , r[ i ] , b[ i ] - a[ i ] );
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd = 0;
    for( int i = 1 ; i <= n ; i ++ ){
        if( in[ i ] < out[ i ] ){
            flow.addEdge( i , flow.t , out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
        if( out[ i ] < in[ i ] )
            flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
    }
    // original sink to source
    flow.addEdge( n , 1 , INF );
    if( flow.maxflow() != nd )
        // no solution
        return -1;
    int ans = flow.G[ 1 ].back().c; // source to sink
    flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
    // take out super source and super sink
    for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i
        ++ ){
        flow.G[ flow.s ][ i ].c = 0;
        Edge &e = flow.G[ flow.s ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i
        ++ ){
        flow.G[ flow.t ][ i ].c = 0;
        Edge &e = flow.G[ flow.t ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    flow.addEdge( flow.s , 1 , INF );
    flow.addEdge( n , flow.t , INF );
    flow.reset();
    return ans + flow.maxflow();
}

```

## 2.8 HLPPA (稠密圖 flow)

```

template <int MAXN, class T = int>
struct HLPP {
    const T INF = numeric_limits<T>::max();
    struct Edge {
        int to, rev; T f;
    };
    int n, s, t;
    vector<Edge> adj[MAXN];
    deque<int> lst[MAXN];
    vector<int> gap[MAXN];
    int ptr[MAXN];
    T ef[MAXN];
    int h[MAXN], cnt[MAXN], work, hst=0/*highest*/;
    void init(int _n, int _s, int _t) {
        n=_n+1; s=_s; t=_t;
        for(int i=0;i<n;i++) adj[i].clear();
    }
    void addEdge(int u,int v,T f,bool isDir = true){
        adj[u].push_back({v,adj[v].size(),f});
        adj[v].push_back({u,adj[u].size()-1,isDir?f:0});
    }
    void updHeight(int v, int nh) {
        work++;
        if(h[v] != n) cnt[h[v]]--;
        h[v] = nh;
        if(nh == n) return;
        cnt[nh]++, hst = nh; gap[nh].push_back(v);
        if(ef[v]>0) lst[nh].push_back(v), ptr[nh]++;
    }
    void globalRelabel() {
        work = 0;
        fill(h, h+n, n);
    }
};

```

```

fill(cnt, cnt+n, 0);
for(int i=0; i<=hst; i++){
    lst[i].clear(), gap[i].clear(), ptr[i] = 0;
    queue<int> q({t}); h[t] = 0;
    while(!q.empty()) {
        int v = q.front(); q.pop();
        for(auto &e : adj[v])
            if(h[e.to] == n && adj[e.to][e.rev].f > 0)
                q.push(e.to), updHeight(e.to, h[v] + 1);
        hst = h[v];
    }
}
void push(int v, Edge &e) {
    if(ef[e.to] == 0)
        lst[h[e.to]].push_back(e.to), ptr[h[e.to]]++;
    T df = min(ef[v], e.f);
    e.f -= df, adj[e.to][e.rev].f += df;
    ef[v] -= df, ef[e.to] += df;
}
void discharge(int v) {
    int nh = n;
    for(auto &e : adj[v]) {
        if(e.f > 0) {
            if(h[v] == h[e.to] + 1) {
                push(v, e);
                if(ef[v] <= 0) return;
            }
            else nh = min(nh, h[e.to] + 1);
        }
    }
    if(cnt[h[v]] > 1) updHeight(v, nh);
    else {
        for(int i = h[v]; i < n; i++) {
            for(auto j : gap[i]) updHeight(j, n);
            gap[i].clear(), ptr[i] = 0;
        }
    }
}
T solve() {
    fill(ef, ef+n, 0);
    ef[s] = INF, ef[t] = -INF;
    globalRelabel();
    for(auto &e : adj[s]) push(s, e);
    for(; hst >= 0; hst--) {
        while(!lst[hst].empty()) {
            int v=lst[hst].back(); lst[hst].pop_back();
            discharge(v);
            if(work > 4 * n) globalRelabel();
        }
    }
    return ef[t] + INF;
}
};

```

## 2.9 Flow Method

Maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ ;  
with the corresponding symmetric dual problem,  
Minimize  $b^T y$  subject to  $A^T y \geq c, y \geq 0$ .

Maximize  $c^T x$  subject to  $Ax \leq b$ ;  
with the corresponding asymmetric dual problem,  
Minimize  $b^T y$  subject to  $A^T y = c, y \geq 0$ .

Minimum vertex cover on bipartite graph =  
Maximum matching on bipartite graph =  
Max flow with source to one side, other side to sink

To reconstruct the minimum vertex cover, dfs from each unmatched vertex on the left side **and** with unused edges only. Equivalently, dfs from source with unused edges only **and** without visiting sink. Then, a vertex is chosen  
iff. it is on the left side **and** without visited **or** on the right side **and** visited through dfs.

Maximum density subgraph  $(\sum W_e + \sum W_v) / |V|$

Binary search on answer:

For a fixed D, construct a Max flow model as follow:

Let  $S$  be Sum of all weight( or inf)  
 1. from source to each node with  $cap = S$   
 2. For each  $(u,v,w)$  in  $E$ ,  $(u \rightarrow v, cap=w)$ ,  $(v \rightarrow u, cap=w)$   
 3. For each node  $v$ , from  $v$  to sink with  $cap = S + 2 * D - deg[v] - 2 * (W \text{ of } v)$   
 where  $deg[v] = \text{sum weight of edge associated with } v$   
 If  $maxflow < S * |V|$ ,  $D$  is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose  $cap > 0$ .

## 3 Math

### 3.1 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n, ll a[], int _m, ll b[], ll ans[])
{
    int n=1, sum=_n+_m-1;
    while(n<sum)
        n<<=1;
    for(int i=0; i<n; i++)
    {
        double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y, x-y);
    }
    fft(n, arr);
    for(int i=0; i<n; i++)
        arr[i]=arr[i]*arr[i];
    fft(n, arr, true);
    for(int i=0; i<sum; i++)
        ans[i]=(long long int)(arr[i].real()/4+0.5);
}
```

### 3.2 NTT

```
typedef long long LL;
// Remember coefficient are mod P
/* p=a*2^n+1
   n   2^n      p      a      root
   16   65536    65537    1      3
   20  1048576  7340033    7     3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
```

```
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b >= 1; bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1)return 1;
        return (((LL)(a-inv(b*a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta%MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if(a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^ k); k >= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n,P);
            reverse(a+1, a+n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

### 3.3 Fast Walsh Transform

```
/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * =>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z'
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL pw( LL x , LL k ) {
    LL res = 1;
    for( LL bs = x ; k ; k >= 1, bs = (bs * bs)%MOD )
        if( k&1 ) res = ( res * bs ) % MOD;
    return res;
}
inline LL inv( LL x ) {
    return pw( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
```

```

    for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
        LL ta = x[ i ] , tb = x[ j ];
        x[ i ] = ta+tb;
        x[ j ] = ta-tb;
        if( x[ i ] >= MOD ) x[ i ] -= MOD;
        if( x[ j ] < 0 ) x[ j ] += MOD;
    }
}
if( inv )
for( int i = 0 ; i < N ; i++ ) {
    x[ i ] *= inv( N );
    x[ i ] %= MOD;
}
}

```

### 3.4 Poly operator

```

struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
    NTT<P, root, MAXN> ntt;
    static int nxt2k(int x) {
        int i = 1; for (; i < x; i <= 1); return i;
    }
    void Mul(int n, LL a[], int m, LL b[], LL c[]) {
        static LL aa[MAXN], bb[MAXN];
        int N = nxt2k(n+m);
        copy(a, a+n, aa); fill(aa+n, aa+N, 0);
        copy(b, b+m, bb); fill(bb+m, bb+N, 0);
        ntt(N, aa); ntt(N, bb);
        FOR(i, N) c[i] = aa[i] * bb[i] % P;
        ntt(N, c, 1);
    }
    void Inv(int n, LL a[], LL b[]) {
        // ab = aa^-1 = 1 mod x^(n/2)
        // (b - a^-1)^2 = 0 mod x^n
        // bb - a^-2 + 2 ba^-1 = 0
        // bba - a^-1 + 2b = 0
        // bba + 2b = a^-1
        static LL tmp[MAXN];
        if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
        Inv((n+1)/2, a, b);
        int N = nxt2k(n*2);
        copy(a, a+n, tmp);
        fill(tmp+n, tmp+N, 0);
        fill(b+n, b+N, 0);
        ntt(N, tmp); ntt(N, b);
        FOR(i, N) {
            LL t1 = (2 - b[i] * tmp[i]) % P;
            if (t1 < 0) t1 += P;
            b[i] = b[i] * t1 % P;
        }
        ntt(N, b, 1);
        fill(b+n, b+N, 0);
    }
    void Div(int n, LL a[], int m, LL b[], LL d[], LL r
        []) {
        // Ra = Rb * Rd mod x^(n-m+1)
        // Rd = Ra * Rb^-1 mod
        static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
        if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);
            return;}
        // d: n-1 - (m-1) = n-m (n-m+1 terms)
        copy(a, a+n, aa); copy(b, b+m, bb);
        reverse(aa, aa+n); reverse(bb, bb+m);
        Inv(n-m+1, bb, tb);
        Mul(n-m+1, ta, n-m+1, tb, d);
        fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
        // r: m-1 - 1 = m-2 (m-1 terms)
        Mul(m, b, n-m+1, d, ta);
        FOR(i, n) { r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
            += P; }
    }
    void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i
        -1] = i * a[i] % P; }
    void Sx(int n, LL a[], LL b[]) {
        b[0] = 0;
        FOR(i, n) b[i+1] = a[i] * ntt.iv[i+1] % P;
    }
    void Ln(int n, LL a[], LL b[]) {
        // Integral a' a^-1 dx
        static LL a1[MAXN], a2[MAXN], b1[MAXN];
        int N = nxt2k(n*2);

```

```

        dx(n, a, a1); Inv(n, a, a2);
        Mul(n-1, a1, n, a2, b1);
        Sx(n+n-1-1, b1, b);
        fill(b+n, b+N, 0);
    }
    void Exp(int n, LL a[], LL b[]) {
        // Newton method to solve g(a(x)) = ln b(x) - a(x)
        // = 0
        // b' = b - g(b(x)) / g'(b(x))
        // b' = b (1 - lnb + a)
        static LL lnb[MAXN], c[MAXN], tmp[MAXN];
        assert(a[0] == 0); // dont know exp(a[0]) mod P
        if (n == 1) {b[0] = 1; return;}
        Exp((n+1)/2, a, b);
        fill(b+(n+1)/2, b+n, 0);
        Ln(n, b, lnb);
        fill(c, c+n, 0); c[0] = 1;
        FOR(i, n) {
            c[i] += a[i] - lnb[i];
            if (c[i] < 0) c[i] += P;
            if (c[i] >= P) c[i] -= P;
        }
        Mul(n, b, n, c, tmp);
        copy(tmp, tmp+n, b);
    }
} polyop;

```

### 3.5 O(1)mul

```

LL mul(LL x, LL y, LL mod){
    LL ret=x*y-(LL)((long double)x/mod*y)*mod;
    return ret<0?ret+mod:ret;
}

```

### 3.6 BigInt

```

struct BigInt{
    static const int LEN = 60;
    static const int BIGMOD = 10000;
    int s;
    int vl, v[LEN];
    // vector<int> v;
    BigInt() : s(1) { vl = 0; }
    BigInt(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while (a) {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    BigInt(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-') {
            stPos = 1;
            s = -1;
        }
        for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD) {
                push_back(num);
                num = 0; q = 1;
            }
        }
        if (num) push_back(num);
        n();
    }
    int len() const {
        return vl;
        // return SZ(v);
    }
    bool empty() const { return len() == 0; }
    void push_back(int x) {
        v[vl++] = x;
        // v.PB(x);
    }
    void pop_back() {
        vl--;
        // v.pop_back();
    }
    int back() const {

```



```

    return v[vl-1];
    // return v.back();
}
void n() {
    while (!empty() && !back()) pop_back();
}
void resize(int nl) {
    vl = nl;
    fill(v, v+vl, 0);
    // v.resize(nl);
    // fill(ALL(v), 0);
}
void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d", v[i]);
}
friend ostream& operator << (ostream& out,
    const Bigint &a) {
    if (a.empty()) { out << "0"; return out; }
    if (a.s == -1) out << "-";
    out << a.back();
    for (int i=a.len()-2; i>=0; i--) {
        char str[10];
        snprintf(str, 5, "%.4d", a.v[i]);
        out << str;
    }
    return out;
}
int cp3(const Bigint &b) const {
    if (s != b.s) return s - b.s;
    if (s == -1) return -(*this).cp3(-b);
    if (len() != b.len()) return len() - b.len(); //int
    for (int i=len()-1; i>=0; i--)
        if (v[i] != b.v[i]) return v[i] - b.v[i];
    return 0;
}
bool operator<(const Bigint &b) const
{ return cp3(b)<0; }
bool operator<=(const Bigint &b) const
{ return cp3(b)<=0; }
bool operator==(const Bigint &b) const
{ return cp3(b)==0; }
bool operator!=(const Bigint &b) const
{ return cp3(b)!=0; }
bool operator>(const Bigint &b) const
{ return cp3(b)>0; }
bool operator>=(const Bigint &b) const
{ return cp3(b)>=0; }
Bigint operator - () const {
    Bigint r = (*this);
    r.s = -r.s;
    return r;
}
Bigint operator + (const Bigint &b) const {
    if (s == -1) return -(*this)+(-b);
    if (b.s == -1) return (*this)-(-b);
    Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
    for (int i=0; i<nl; i++) {
        if (i < len()) r.v[i] += v[i];
        if (i < b.len()) r.v[i] += b.v[i];
        if (r.v[i] >= BIGMOD) {
            r.v[i+1] += r.v[i] / BIGMOD;
            r.v[i] %= BIGMOD;
        }
    }
    r.n();
    return r;
}
Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(*this)-(-b);
    if (b.s == -1) return (*this)+(-b);
    if ((*this) < b) return -(b-(*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
        r.v[i] += v[i];
        if (i < b.len()) r.v[i] -= b.v[i];
        if (r.v[i] < 0) {

```

```

            r.v[i] += BIGMOD;
            r.v[i+1]--;
        }
    }
    r.n();
    return r;
}
Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
        for (int j=0; j<b.len(); j++) {
            r.v[i+j] += v[i] * b.v[j];
            if (r.v[i+j] >= BIGMOD) {
                r.v[i+j+1] += r.v[i+j] / BIGMOD;
                r.v[i+j] %= BIGMOD;
            }
        }
    }
    r.n();
    return r;
}
Bigint operator / (const Bigint &b) {
    Bigint r;
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
        int d=0, u=BIGMOD-1;
        while(d<u) {
            int m = (d+u+1)>>1;
            r.v[i] = m;
            if ((r*b2) > (*this)) u = m-1;
            else d = m;
        }
        r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
}
Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
}
};

```

### 3.7 Linear Recurrence

```

// Usage: linearRec({0, 1}, {1, 1}, k) //k'th fib
typedef vector<ll> Poly;
//S: 前i項的值, tr: 遞迴係數, k: 求第k項
ll linearRec(Poly& S, Poly& tr, ll k) {
    int n = tr.size();
    auto combine = [&](Poly& a, Poly& b) {
        Poly res(n * 2 + 1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            res[i+j] = (res[i+j] + a[i]*b[j])%mod;
        for (int i = 2*n; i > n; --i) rep(j, 0, n)
            res[i-1-j] = (res[i-1-j] + res[i]*tr[j])%mod;
        res.resize(n + 1);
        return res;
    };
    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;
    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }
    ll res = 0;
    rep(i, 0, n) res = (res + pol[i+1]*S[i])%mod;
    return res;
}

```

### 3.8 Stirling's approximation

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$

### 3.9 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61

```

```

// n < 1,122,004,669,633    4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383    6 : pimes <= 13
// n < 2^64                7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[]={}
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x=myspow(a,u,n);
    for(int i=0; i<t; i++){
        LL nx=(x*x)%n;
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(LL n) {
    int s=(magic number size)
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=magic[s]%n;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

### 3.10 Faulhaber ( $\sum_{i=1}^n i^p$ )

```

/* faulhaber' s formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 4600000000000000000133;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q,t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for(int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2; i<=MAXK; i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0; j<i; j++)
            b[i]=sub(b[i], mul(cm[i][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1,j)*Bj*n^(p-j+1)}
    for(int i=1; i<=MAXK; i++) {
        co[i][0]=0;
        for(int j=0; j<=i; j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}

```

```

/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
    int sol=0, m=n;
    for(int i=1; i<=p+1; i++) {
        sol=add(sol, mul(co[p][i], m));
        m = mul(m, n);
    }
    return sol;
}

```

### 3.11 Chinese Remainder

```

LL x[N], m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
LL solve(int n) { // n>=2, be careful with no solution
    LL res=CRT(x[0], m[0], x[1], m[1]), p=m[0]/__gcd(m[0], m[1])*m[1];
    for(int i=2; i<n; i++){
        res=CRT(res, p, x[i], m[i]);
        p=p/__gcd(p, m[i])*m[i];
    }
    return res;
}

```

### 3.12 Pollard Rho 找因數

```

// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

### 3.13 Josephus Problem

```

int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}

```

### 3.14 ax+by=gcd

```

PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

### 3.15 Discrete sqrt

```

void calch(LL &t, LL &h, const LL p) {
    LL tmp=p-1; for(t=0; (tmp&1)==0; tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(LL a, LL p, LL &x, LL &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=myspow(a, (p+1)/4, p); y=p-x; return true;
    } else {
        LL t, h, b, pb; calch(t, h, p);
    }
}

```



```

if (t >= 2) {
    do {b = rand() % (p - 2) + 2;
        } while (mypow(b, p / 2, p) != p - 1);
    pb = mypow(b, h, p);
    int s = mypow(a, h / 2, p);
    for (int step = 2; step <= t; step++) {
        int ss = (((LL)(s * s) % p) * a) % p;
        for (int i=0; i<t-step; i++) ss=mul(ss,ss,p);
        if (ss + 1 == p) s = (s * pb) % p;
        pb = ((LL)pb * pb) % p;
    } x = ((LL)s * a) % p; y = p - x;
} return true;
}

```

### 3.16 Romberg 定積分

```

// Estimates the definite integral of
// \int_a^b f(x) dx
template<class T>
double romberg( T& f, double a, double b, double eps=1e
-8){
    vector<double>t; double h=b-a,last,curr; int k=1,i=1;
    t.push_back(h*(f(a)+f(b))/2);
    do{ last=t.back(); curr=0; double x=a+h/2;
        for(int j=0;j<k;j++) curr+=f(x), x+=h;
        curr=(t[0] + h*curr)/2; double k1=4.0/3.0,k2
        =1.0/3.0;
        for(int j=0;j<i;j++){ double temp=k1*curr-k2*t[j];
            t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
        } t.push_back(curr); k*=2; h/=2; i++;
    }while( fabs(last-curr) > eps);
    return t.back();
}

```

### 3.17 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```

### 3.18 Roots of Polynomial 找多項式的根

```

const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ];
int n;
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++){
        sum=sum+a[i]*tmp; tmp=tmp*x; }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){

```

```

        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
}
int main() {
    scanf("%d",&n);
    for(int i=n;i>=0;i--) scanf("%lf",&a[i]);
    int nx;
    solve(n,a,x,nx);
    for(int i=1;i<=nx;i++) printf("%.6f\n",x[i]);
}

```

### 3.19 inverse

```

f[0]=1; //f[x]=x!
for(ll i=1;i<MAXN;i++)
    f[i]=(f[i-1]*i)%mod;
inv[MAXN-1]=ppow(f[MAXN-1],mod-2);
ll c(ll x,ll y){ //c(x,y)
    return f[x]*inv[y]%mod*inv[x-y]%mod;
}

```

### 3.20 Primes

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[ N ] , p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}

```

### 3.21 Result

- Lucas' Theorem :  
For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $C(m, n) \bmod P = \prod (C(m_i, n_i))$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \Pi_{i=0}^{n-1} (x+i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick' s Theorem :  $A = i + b/2 - 1$
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$

## 4 Geometry

### 4.1 Intersection of 2 lines

```
Pt interPnt( Pt p1, Pt p2, Pt q1, Pt q2){
    double f1 = ( p2 - p1 ) ^ ( q1 - p1 );
    double f2 = ( p2 - p1 ) ^ ( p1 - q2 );
    double f = ( f1 + f2 );
    if( fabs( f ) < eps ) return Pt( nan(""), nan("") );
    return q1 * ( f2 / f ) + q2 * ( f1 / f );
}
```

### 4.2 halfPlaneIntersection

```
bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    bool res; Pt p = interPnt(l1, l2, res);
    return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F) ^ (p - l.F) > 0
 */
/* --- Line.FI --- Line.SE --- */
vector<Line> halfPlaneInter( vector<Line> lines ){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for( int i=0; i<sz; i++) {
        ord[i] = i;
        Pt d = lines[i].SE - lines[i].FI;
        ata[i] = atan2(d.Y, d.X);
    }
    sort( ord.begin(), ord.end(), [&](int i, int j) {
        if( fabs(ata[i] - ata[j]) < eps )
            return ( (lines[i].SE - lines[i].FI) ^
                    (lines[j].SE - lines[i].FI) ) < 0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for( int i=0; i<sz; i++)
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.PB(lines[ord[i]]);
    deque<Line> dq;
    for( int i=0; i<(int)(fin.size()); i++) {
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[(int)(dq.size()-2)],
                    dq[(int)(dq.size()-1)]))
            dq.pop_back();
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while( (int)(dq.size()) >= 3 and
        not isin(dq[0], dq[(int)(dq.size()-2)],
                dq[(int)(dq.size()-1)]))
        dq.pop_back();
    while( (int)(dq.size()) >= 3 and
        not isin(dq[(int)(dq.size()-1)], dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(dq.begin(), dq.end());
    return res;
}
```

### 4.3 Convex Hull

```
double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for( int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2], stk[top-1], pt[i])
            ) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for( int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2], stk[top-1], pt[i])
            ) <= 0)
            top--;
        stk[top++] = pt[i];
    }
}
```

```
top--;
stk[top++] = pt[i];
}
stk.resize(top-1);
return stk;
}
```

### 4.4 Intersection of 2 segments

```
int ori( const Pt& o , const Pt& a , const Pt& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
              const Pt& q1 , const Pt& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1 , p2 , q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
            ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
            ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
            ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
    }
    return (ori( p1, p2, q1 ) * ori( p1, p2, q2 )<=0) &&
        (ori( q1, q2, p1 ) * ori( q1, q2, p2 )<=0);
}
```

### 4.5 Intersection of circle and segment

```
bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
    Pt dp = p2 - p1;
    double a = dp * dp;
    double b = 2 * ( dp * ( p1 - cc.O ) );
    double c = cc.O * cc.O + p1 * p1 - 2 * ( cc.O * p1 )
        - cc.R * cc.R;
    double bb4ac = b * b - 4 * a * c;
    return !( fabs( a ) < eps or bb4ac < 0 );
}
```

### 4.6 Intersection of 2 circles

### 4.7 Circle cover

```
#define N 1021
#define D double
struct CircleCover{
    int C; Circ c[ N ]; //填入C(圓數量),c(圓陣列)
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.O , o2 = b.O;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return false;
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return true;
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
    struct Teve {
        Pt p; D ang; int add;
        Teve() {}
        Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang;
        }
    }eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
    bool disjunct( Circ& a, Circ &b, int x )
    {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
    bool contain( Circ& a, Circ &b, int x )
    {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
    bool contain(int i, int j){
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 ||

```

```

        (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
        contain(c[i], c[j], -1);
    }
    void solve(){
        for( int i = 0 ; i <= C + 1 ; i ++ )
            Area[ i ] = 0;
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                overlap[i][j] = contain(i, j);
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
        for( int i = 0 ; i < C ; i ++ ){
            int E = 0, cnt = 1;
            for( int j = 0 ; j < C ; j ++ )
                if( j != i && overlap[j][i] )
                    cnt ++;
            for( int j = 0 ; j < C ; j ++ )
                if( i != j && g[i][j] ){
                    Pt aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                    D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                    eve[E ++] = Teve(bb, B, 1);
                    eve[E ++] = Teve(aa, A, -1);
                    if(B > A) cnt ++;
                }
            if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
            else{
                sort( eve , eve + E );
                eve[E] = eve[0];
                for( int j = 0 ; j < E ; j ++ ){
                    cnt += eve[j].add;
                    Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
                    D theta = eve[j + 1].ang - eve[j].ang;
                    if (theta < 0) theta += 2.0 * pi;
                    Area[cnt] +=
                        (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
                }
            }
        }
    }
};

```

#### 4.8 Convex Hull trick

```

/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;

```

```

            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));
        for( ; l + 1 < r; ) {
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt
            (p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
        return 1;
    }
    // 2. Find 2 tang pts on CH of a given outside point
    // return true with i0, i1 as index of tangent points
    // return false if inside CH
    bool get_tang(Pt p, int &i0, int &i1) {
        if (contain(p)) return false;
        i0 = i1 = 0;
        int id = lower_bound(lower.begin(), lower.end(), p)
            - lower.begin();
        bi_search(0, id, p, i0, i1);
        bi_search(id, (int)lower.size(), p, i0, i1);
        id = lower_bound(upper.begin(), upper.end(), p,
            greater<Pt>()) - upper.begin();
        bi_search((int)lower.size() - 1, (int)lower.size()
            - 1 + id, p, i0, i1);
        bi_search((int)lower.size() - 1 + id, (int)lower.
            size() - 1 + (int)upper.size(), p, i0, i1);
        return true;
    }
    // 3. Find tangent points of a given vector
    // ret the idx of vertex has max cross value with vec
    int get_tang(Pt vec){
        pair<LL, int> ret = get_tang(upper, vec);
        ret.second = (ret.second+(int)lower.size()-1)%n;
        ret = max(ret, get_tang(lower, vec));
        return ret.second;
    }
    // 4. Find intersection point of a given line
    // return 1 and intersection is on edge (i, next(i))
    // return 0 if no strictly intersection
    bool get_intersection(Pt u, Pt v, int &i0, int &i1){
        int p0 = get_tang(u - v), p1 = get_tang(v - u);
        if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
            if (p0 > p1) swap(p0, p1);
            i0 = bi_search(u, v, p0, p1);
            i1 = bi_search(u, v, p1, p0 + n);
            return 1;
        }
        return 0;
    }
};

```

#### 4.9 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );

```

```

Pt v = ( c2.0 - c1.0 ) / d;
double c = ( c1.R - sign1 * c2.R ) / d;
if( c * c > 1 ) return ret;
double h = sqrt( max( 0.0 , 1.0 - c * c ) );
for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt p1 = { v.X * c - sign2 * h * v.Y ,
              v.Y * c + sign2 * h * v.X };
    Pt p2 = c1.0 + n * c1.R;
    Pt p2 = c2.0 + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
        p2 = p1 + perp( c2.0 - c1.0 );
    ret.push_back( { p1 , p2 } );
}
return ret;
}

```

#### 4.10 KD Tree

```

const int MXN=100005;
const int MXK=10;
struct KDTree{
    struct Nd{
        LL x[MXK],mn[MXK],mx[MXK];
        int id,f;
        Nd *l,*r;
    }tree[MXN],*root;
    int n,k;
    LL dis(LL a,LL b){return (a-b)*(a-b);}
    LL dis(LL a[MXK],LL b[MXK]){
        LL ret=0;
        for(int i=0;i<k;i++) ret+=dis(a[i],b[i]);
        return ret;
    }
    void init(vector<vector<LL>> &ip,int _n,int _k){
        n=_n,k=_k;
        for(int i=0;i<n;i++){
            tree[i].id=i;
            copy(ip[i].begin(),ip[i].end(),tree[i].x);
        }
        root=build(0,n-1,0);
    }
    Nd* build(int l,int r,int d){
        if(l>r) return NULL;
        if(d==k) d=0;
        int m=(l+r)>>1;
        nth_element(tree+l,tree+m,tree+r+1,[&](const Nd &a,
            const Nd &b){return a.x[d]<b.x[d];});
        tree[m].f=d;
        copy(tree[m].x,tree[m].x+k,tree[m].mn);
        copy(tree[m].x,tree[m].x+k,tree[m].mx);
        tree[m].l=build(l,m-1,d+1);
        if(tree[m].l){
            for(int i=0;i<k;i++){
                tree[m].mn[i]=min(tree[m].mn[i],tree[m].l->mn[i]);
                tree[m].mx[i]=max(tree[m].mx[i],tree[m].l->mx[i]);
            }
        }
        tree[m].r=build(m+1,r,d+1);
        if(tree[m].r){
            for(int i=0;i<k;i++){
                tree[m].mn[i]=min(tree[m].mn[i],tree[m].r->mn[i]);
                tree[m].mx[i]=max(tree[m].mx[i],tree[m].r->mx[i]);
            }
        }
        return tree+m;
    }
    LL pt[MXK],md;
    int mID;
    bool touch(Nd *r){
        LL d=0;
        for(int i=0;i<k;i++){
            if(pt[i]<=r->mn[i]) d+=dis(pt[i],r->mn[i]);
            else if(pt[i]>=r->mx[i]) d+=dis(pt[i],r->mx[i]);
        }
        return d<md;
    }
}

```

```

void nearest(Nd *r){
    if(!r||!touch(r)) return;
    LL td=dis(r->x,pt);
    if(td<md) md=td,mID=r->id;
    nearest(pt[r->f]<r->x[r->f]?r->l:r->r);
    nearest(pt[r->f]<r->x[r->f]?r->r:r->l);
}
pair<LL,int> query(vector<LL> &_pt,LL _md=1LL<<57){
    mID=-1,md=_md;
    copy(_pt.begin(),_pt.end(),pt);
    nearest(root);
    return {md,mID};
}
}tree;

```

#### 4.11 Lower Concave Hull

```

const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        return s ? b - s->b < (s->m - m) * rhs.m : 0;
    }
}; // maintain upper hull for maximum
struct HullDynamic : public multiset<Line> {
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z==end())return y->m==x->m&&y->b<=x->b;
        return (x->b-y->b)*(z->m-y->m)>=
            (y->b-z->b)*(y->m-x->m);
    }
    void insert_line(ll m, ll b) {
        auto y = insert({m, b});
        y->succ = [=]{return next(y)==end()?0:&*next(y);};
        if(bad(y)) {erase(y); return; }
        while(next(y)!=end()&&bad(next(y)))erase(next(y));
        while(y!=begin()&&bad(prev(y)))erase(prev(y));
    }
    ll eval(ll x) {
        auto l = *lower_bound((Line) {x, is_query});
        return l.m * x + l.b;
    }
};

```

#### 4.12 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[ N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
        }
    }
}

```

```

for (int j=0; j<i; j++){
    if (norm2(cen-p[j]) <= r2) continue;
    cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
    r2 = norm2(cen-p[j]);
    for (int k=0; k<j; k++){
        if (norm2(cen-p[k]) <= r2) continue;
        cen = center(p[i],p[j],p[k]);
        r2 = norm2(cen-p[k]);
    }
}
return {cen,sqrt(r2)};
}
} mec;

```

#### 4.13 Min Enclosing Ball

```

// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
    Pt q[3]; double m[3][3], sol[3], L[3], det;
    int i,j; res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2; radius=norm2(res, outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q[i] * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
            if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps) return;
            L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
            L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
            res=outer[0]+q[0]*L[0]+q[1]*L[1];
            radius=norm2(res, outer[0]);
            break;
        case 4:
            for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=(q[i] * q[i]);
            for (i=0; i<3; ++i) for(j=0; j<3; ++j) m[i][j]=(q[i] * q[j])*2;
            det= m[0][0]*m[1][1]*m[2][2]
                + m[0][1]*m[1][2]*m[2][0]
                + m[0][2]*m[1][0]*m[2][1]
                - m[0][2]*m[1][1]*m[2][0]
                - m[0][1]*m[1][0]*m[2][2]
                - m[0][0]*m[1][2]*m[2][1];
            if ( fabs(det)<eps ) return;
            for (j=0; j<3; ++j) {
                for (i=0; i<3; ++i) m[i][j]=sol[i];
                L[j]=( m[0][0]*m[1][1]*m[2][2]
                    + m[0][1]*m[1][2]*m[2][0]
                    + m[0][2]*m[1][0]*m[2][1]
                    - m[0][2]*m[1][1]*m[2][0]
                    - m[0][1]*m[1][0]*m[2][2]
                    - m[0][0]*m[1][2]*m[2][1] ) / det;
                for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
            } res=outer[0];
            for (i=0; i<3; ++i) res = res + q[i] * L[i];
            radius=norm2(res, outer[0]);
    }
}
void minball(int n){ ball();
    if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
        if( norm2(res, pt[i]) - radius > eps ){
            outer[ nouter ++ ] = pt[ i ]; minball(i); -- nouter;
            if(i>0){ Pt Tt = pt[i];
                memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt;
            }
        }
}
double solve(){
    // n points in pt
    random_shuffle(pt, pt+n); radius=-1;
    for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
        nouter=1, outer[0]=pt[i], minball(i);
    return sqrt(radius);
}

```

#### 4.14 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
    if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                          x0+L, y0, H, W, L);
    if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                          x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                          x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                          x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
        else swap(x1,z1), swap(x2,z2), swap(L,H);
    }
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}

```

#### 4.15 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // 内心
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a, Pt &b, Pt &c) { // 垂心
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
           A = ca.X * ba.Y - ba.X * ca.Y,
           x0 = (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
           y0 = -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}

```

### 5 Graph

#### 5.1 HeavyLightDecomp

```

#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruli
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfsz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfsz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfshl(int u){
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
        sort(ALL(g[u]),

```



```

    [&](int a, int b){return sz[a] > sz[b];});
    bool flag = 1;
    for(int& v:g[u]) if(v != prt[u][0]){
        if(flag) head[v] = head[u], flag = 0;
        dfs1(v);
        tr[u] = tr[v];
    }
}
inline int lca(int a, int b){
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
        b = prt[b][k];
    }
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
        a = prt[a][k]; b = prt[b][k];
    }
    return prt[a][0];
}
void init( int _n ){
    n = _n; REP( i , 1 , n ) g[ i ].clear();
}
void addEdge( int u , int v ){
    g[ u ].push_back( v );
    g[ v ].push_back( u );
}
void yutruli(){ //build function
    dfssz(1, 0);
    ts = 0;
    dfs1(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
        prt[i][k] = prt[prt[i][k-1]][k-1];
}
vector< PII > getPath( int u , int v ){
    vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ){
        res.push_back( PII(tid[ head[ v ] ], tid[ v ]) );
        v = prt[ head[ v ] ][ 0 ];
    }
    res.push_back( PII( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res;
}
/* res : list of intervals from u to v
 * u must be ancestor of v
 * usage :
 * vector< PII >& path = tree.getPath( u , v )
 * for( PII tp : path ) {
 *     int l , r;tie( l , r ) = tp;
 *     upd( l , r );
 *     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
 *     uu ~> vv is a heavy path on tree
 * }
 */
}
} tree;

```

## 5.2 DominatorTree

```

const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n , m , s;
    vector< int > g[ MAXN ] , pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
    int par[ MAXN ]; //idom[u] s到u的最後一個必經點
    int sdom[ MAXN ] , idom[ MAXN ];
    int mom[ MAXN ] , mn[ MAXN ];
    inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ]; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if(cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ))
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n , int _m , int _s ){
        ts = 0; n = _n; m = _m; s = _s;

```

```

        REP( i , 1 , n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i , 1 , n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i , n , 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i , 2 , n ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            if( idom[ u ] != sdom[ u ] )
                idom[ u ] = idom[ idom[ u ] ];
        }
    }
} domT;

```

## 5.3 MaxClique 最大團

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset< N > Int;
    Int linkto[ N ] , v[ N ];
    int n;
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            linkto[ i ].reset();
            v[ i ].reset();
        }
    }
    void addEdge( int a , int b ){
        v[ a ][ b ] = v[ b ][ a ] = 1;
    }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans , stk[ N ];
    int id[ N ] , di[ N ] , deg[ N ];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num;
            cans.reset();
            for( int i = 0 ; i < elem_num ; i ++ )
                cans[ id[ stk[ i ] ] ] = 1;
            //potential,smaller_candi
            int potential = elem_num + popcount(candi);
            if(potential <= ans) return;

```



```

int pivot = lowbit(candi);
Int smaller_candi = candi & (~linkto[pivot]);
while(smaller_candi.count() && potential > ans){
    int next = lowbit(smaller_candi);
    candi[next] = !candi[next];
    smaller_candi[ next ] = !smaller_candi[ next ];
    potential --;
    if(next == pivot || (smaller_candi & linkto[next
    ]).count() ){
        stk[elem_num] = next;
        maxclique(elem_num + 1, candi & linkto[next]);
    }
}
int solve(){
    for( int i = 0 ; i < n ; i ++ ){
        id[ i ] = i;
        deg[ i ] = v[ i ].count();
    }
    sort( id , id + n , [&](int id1, int id2){
        return deg[id1] > deg[id2]; } );
    for( int i = 0 ; i < n ; i ++ )
        di[ id[ i ] ] = i;
    for( int i = 0 ; i < n ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            if( v[ i ][ j ] )
                linkto[ di[ i ] ][ di[ j ] ] = 1;
    Int cand; cand.reset();
    for( int i = 0 ; i < n ; i ++ )
        cand[ i ] = 1;
    ans = 1;
    cans.reset(); cans[ 0 ] = 1;
    maxclique(0, cand);
    return ans;
}
} solver;

```

## 5.4 Strongly Connected Component

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for( int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for( auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for( auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for( int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(),vec.end());
        FZ(vst);
        for( auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

## 5.5 Dynamic MST

```

/* Dynamic MST  $O(Q \lg^2 Q)$ 
(qx[i], qy[i]) -> chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infty)
add an edge: change from \infty to specific value
*/
const int SZ=M+3*MXQ;

```

```

int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)
        vd[i]=++n2;
    for(int i=1;i<=n;i++) if(a[i])
        vd[i]=vd[find(i)];
    int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
        Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
        app[qx[i]]=m2; m2++;
    }
    for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
        if(ri!=rj){
            a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
            Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
        }
    }
    int mid=Q/2;
    solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
    solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }

```

## 5.6 Maximum General graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
    int to[E],bro[E],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init( int _n ){
        stp = 0; e = 1; n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            lnk[i] = vis[i] = 0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v,lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)){
                    return true;
                }
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;
        for(int i=1;i<=n;i++)
            if(!lnk[i]){
                stp++; ans += dfs(i);
            }
        return ans;
    }
} graph;
```

## 5.7 Minimum General Weighted Matching

```
struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
```

```
for (int i=0; i<n; i+=2){
    match[i] = i+1;
    match[i+1] = i;
}
while (true){
    int found = 0;
    for( int i = 0 ; i < n ; i ++ )
        onstk[ i ] = dis[ i ] = 0;
    for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
            found = 1;
            while (SZ(stk)>=2){
                int u = stk.back(); stk.pop_back();
                int v = stk.back(); stk.pop_back();
                match[u] = v;
                match[v] = u;
            }
        }
        if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
        ret += edge[i][match[i]];
    ret /= 2;
    return ret;
}
}graph;
```

## 5.8 Maximum General Weighted Matching

```
struct WeightGraph {
    static const int INF = INT_MAX;
    static const int N = 514;
    struct edge{
        int u,v,w; edge(){}
        edge(int ui,int vi,int wi)
            :u(ui),v(vi),w(wi){}
    };
    int n,n_x;
    edge g[N*2][N*2];
    int lab[N*2];
    int match[N*2],slack[N*2],st[N*2],pa[N*2];
    int flo_from[N*2][N+1],S[N*2],vis[N*2];
    vector<int> flo[N*2];
    queue<int> q;
    int e_delta(const edge &e){
        return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
    }
    void update_slack(int u,int x){
        if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][x]))slack[x]=u;
    }
    void set_slack(int x){
        slack[x]=0;
        for(int u=1;u<=n;++u)
            if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                update_slack(u,x);
    }
    void q_push(int x){
        if(x<=n)q.push(x);
        else for(size_t i=0;i<flo[x].size();i++)
            q_push(flo[x][i]);
    }
    void set_st(int x,int b){
        st[x]=b;
        if(x>n)for(size_t i=0;i<flo[x].size();++i)
            set_st(flo[x][i],b);
    }
    int get_pr(int b,int xr){
        int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].begin();
        if(pr%2==1){
            reverse(flo[b].begin()+1,flo[b].end());
            return (int)flo[b].size()-pr;
        }else return pr;
    }
    void set_match(int u,int v){
        match[u]=g[u][v].v;
        if(u<=n) return;
        edge e=g[u][v];
```

```

    int xr=flo_from[u][e.u],pr=get_pr(u,xr);
    for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i
        ^1]);
    set_match(xr,v);
    rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end
        ());
}
void augment(int u,int v){
    for(;;){
        int xnv=st[match[u]];
        set_match(u,v);
        if(!xnv)return;
        set_match(xnv,st[pa[xnv]]);
        u=st[pa[xnv]],v=xnv;
    }
}
int get_lca(int u,int v){
    static int t=0;
    for(++t;u!=v;swap(u,v)){
        if(u==0)continue;
        if(vis[u]==t)return u;
        vis[u]=t;
        u=st[match[u]];
        if(u==st[pa[u]]);
    }
    return 0;
}
void add_blossom(int u,int lca,int v){
    int b=n+1;
    while(b<=n_x&&st[b]++b;
    if(b>n_x)++n_x;
    lab[b]=0,S[b]=0;
    match[b]=match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for(int x=u,y; x!=lca; x=st[pa[y]])
        flo[b].push_back(x),flo[b].push_back(y=st[match[x
            ]]),q_push(y);
    reverse(flo[b].begin()+1,flo[b].end());
    for(int x=v,y; x!=lca; x=st[pa[y]])
        flo[b].push_back(x),flo[b].push_back(y=st[match[x
            ]]),q_push(y);
    set_st(b,b);
    for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
    for(int x=1;x<=n;++x)flo_from[b][x]=0;
    for(size_t i=0;i<flo[b].size();++i){
        int xs=flo[b][i];
        for(int x=1;x<=n_x;++x)
            if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b
                ][x]))
                g[b][x]=g[xs][x],g[x][b]=g[x][xs];
        for(int x=1;x<=n;++x)
            if(flo_from[xs][x])flo_from[b][x]=xs;
    }
    set_slack(b);
}
void expand_blossom(int b){
    for(size_t i=0;i<flo[b].size();++i)
        set_st(flo[b][i],flo[b][i]);
    int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
    for(int i=0;i<pr;i+=2){
        int xs=flo[b][i],xns=flo[b][i+1];
        pa[xs]=g[xns][xs].u;
        S[xs]=1,S[xns]=0;
        slack[xs]=0,set_slack(xns);
        q_push(xns);
    }
    S[xr]=1,pa[xr]=pa[b];
    for(size_t i=pr+1;i<flo[b].size();++i){
        int xs=flo[b][i];
        S[xs]=-1,set_slack(xs);
    }
    st[b]=0;
}
bool on_found_edge(const edge &e){
    int u=st[e.u],v=st[e.v];
    if(S[v]==-1){
        pa[v]=e.u,S[v]=1;
        int nu=st[match[v]];
        slack[v]=slack[nu]=0;
        S[nu]=0,q_push(nu);
    }else if(S[v]==0){

```

```

        int lca=get_lca(u,v);
        if(!lca)return augment(u,v),augment(v,u),true;
        else add_blossom(u,lca,v);
    }
    return false;
}
bool matching(){
    memset(S+1,-1,sizeof(int)*n_x);
    memset(slack+1,0,sizeof(int)*n_x);
    q=queue<int>();
    for(int x=1;x<=n_x;++x)
        if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q_push(x);
    if(q.empty())return false;
    for(;;){
        while(q.size()){
            int u=q.front();q.pop();
            if(S[st[u]]==1)continue;
            for(int v=1;v<=n;++v)
                if(g[u][v].w>0&&st[u]!=st[v]){
                    if(e_delta(g[u][v])==0){
                        if(on_found_edge(g[u][v]))return true;
                    }else update_slack(u,st[v]);
                }
        }
        int d=INF;
        for(int b=n+1;b<=n_x;++b)
            if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
        for(int x=1;x<=n_x;++x)
            if(st[x]==x&&slack[x]){
                if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
                else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x
                    ])/2);
            }
        for(int u=1;u<=n;++u){
            if(S[st[u]]==0){
                if(lab[u]<=d)return 0;
                lab[u]-=d;
            }else if(S[st[u]]==1)lab[u]+=d;
        }
        for(int b=n+1;b<=n_x;++b)
            if(st[b]==b){
                if(S[st[b]]==0)lab[b]+=d*2;
                else if(S[st[b]]==1)lab[b]-=d*2;
            }
        q=queue<int>();
        for(int x=1;x<=n_x;++x)
            if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta
                (g[slack[x]][x])==0)
                if(on_found_edge(g[slack[x]][x]))return true;
        for(int b=n+1;b<=n_x;++b)
            if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(
                b);
    }
    return false;
}
pair<long long,int> solve(){
    memset(match+1,0,sizeof(int)*n);
    n_x=n;
    int n_matches=0;
    long long tot_weight=0;
    for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
    int w_max=0;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v){
            flo_from[u][v]=(u==v?0);
            w_max=max(w_max,g[u][v].w);
        }
    for(int u=1;u<=n;++u)lab[u]=w_max;
    while(matching())++n_matches;
    for(int u=1;u<=n;++u)
        if(match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
    g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
    n = _n;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v)
            g[u][v]=edge(u,v,0);
}

```

```

}
} graph;

```

## 5.9 Minimum Steiner Tree

```

// Minimum Steiner Tree 重要點的mst
//  $O(V^3AT + V^2 2^AT)$ 
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n , dst[V][V] , dp[1 << T][V] , tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui , int vi , int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
        dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
    }
    void shortest_path(){
        for( int k = 0 ; k < n ; k ++ )
            for( int i = 0 ; i < n ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dst[ i ][ j ] = min( dst[ i ][ j ] ,
                        dst[ i ][ k ] + dst[ k ][ j ] );
    }
    int solve( const vector<int>& ter ){
        int t = (int)ter.size();
        for( int i = 0 ; i < ( 1 << t ) ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                dp[ i ][ j ] = INF;
        for( int i = 0 ; i < n ; i ++ )
            dp[ 0 ][ i ] = 0;
        for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
            if( msk == ( msk & (-msk) ) ){
                int who = __lg( msk );
                for( int i = 0 ; i < n ; i ++ )
                    dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                continue;
            }
            for( int i = 0 ; i < n ; i ++ )
                for( int submsk = ( msk - 1 ) & msk ; submsk ;
                    submsk = ( submsk - 1 ) & msk )
                    dp[ msk ][ i ] = min( dp[ msk ][ i ] ,
                        dp[ submsk ][ i ] +
                        dp[ msk ^ submsk ][ i ] );
            for( int i = 0 ; i < n ; i ++ ){
                tdst[ i ] = INF;
                for( int j = 0 ; j < n ; j ++ )
                    tdst[ i ] = min( tdst[ i ] ,
                        dp[ msk ][ j ] + dst[ j ][ i ] );
            }
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = tdst[ i ];
        }
        int ans = INF;
        for( int i = 0 ; i < n ; i ++ )
            ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
        return ans;
    }
} solver;

```

## 5.10 BCC based on vertex

```

struct BccVertex {
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;

```

```

        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z; //進到if裡面u為關節點
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u],dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i,i);
            }
        REP(i,nScc) res.PB(sccv[i]);
        return res;
    }
}graph;

```

## 5.11 Min Mean Cycle

```

/* minimum mean cycle  $O(VE)$  */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        fill(vst,0); edgeID.clear(); cycle.clear(); rho.
            clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;

```

```

    edgeID.PB(prve[i][st]);
    rho.PB(st);
}
while (vst[st] != 2) {
    if(rho.empty()) return inf;
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
}
reverse(ALL(edgeID));
edgeID.resize(SZ(cycle));
return mmc;
}
} mmc;

```

## 5.12 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[++bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i]+1, dp[i]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] = min(dp[i][g[j][k].to],
                                            dp[i-1][j]+g[j][k].w);
                }
            }
        }
        mu=INF; LL bunbo=1;
        for(int i=1; i<=n; i++) if(dp[n][i] < INF){
            LL a=-INF, b=1;
            for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                    a = dp[n][i]-dp[j][i];
                    b = n-j;
                }
            }
            if(mu*b > bunbo*a)
                mu = a, bunbo = b;
        }
        if(mu < 0) return -1; // negative cycle
        if(mu == INF) return INF; // no cycle
        if(mu == 0) return 0;
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w *= bunbo;
            }
            memset(p, 0, sizeof(p));
            queue<int> q;
            for(int i=1; i<=n; i++){
                q.push(i);
                inq[i] = true;
            }
            while(!q.empty()){

```

```

                int i=q.front(); q.pop(); inq[i]=false;
                for(int j=0; j<(int)g[i].size(); j++){
                    if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                        p[g[i][j].to] = p[i]+g[i][j].w-mu;
                        if(!inq[g[i][j].to]){
                            q.push(g[i][j].to);
                            inq[g[i][j].to] = true;
                        }
                    }
                }
            }
        }
        for(int i=1; i<=n; i++) grev[i].clear();
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w += p[i]-p[g[i][j].to];
                grev[g[i][j].to].push_back(edge(i, g[i][j].w));
            }
        }
        LL mlcd = n*mu;
        for(int i=1; i<=n; i++){
            bn=mlcd/mu, bsz=0;
            memset(hd, 0, sizeof(hd));
            fill(d+i+1, d+n+1, INF);
            b_insert(d[i]=0, i);
            for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
                b[k].next){
                int u = b[k].u;
                LL du = b[k].d;
                if(du > d[u]) continue;
                for(int l=0; l<(int)g[u].size(); l++) if(g[u][l]
                    ].to > i){
                    if(d[g[u][l].to] > du + g[u][l].w){
                        d[g[u][l].to] = du + g[u][l].w;
                        b_insert(d[g[u][l].to], g[u][l].to);
                    }
                }
            }
        }
        for(int j=0; j<(int)grev[i].size(); j++) if(grev[
            i][j].to > i)
            mlcd=min(mlcd, d[grev[i][j].to] + grev[i][j].w);
        }
        return mlcd / bunbo;
    }
} graph;

```

## 5.13 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v, d;
        nd(int ui = 0, int vi = 0, int di = INF)
        { u = ui; v = vi; d = di; }
    };
    struct heap{
        nd* edge; int dep; heap* chd[4];
    };
    static int cmp(heap* a, heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; LL d; heap* H; nd* E;
        node(){
            node(LL _d, int _v, nd* _E)
            { d = _d; v = _v; E = _E; }
            node(heap* _H, LL _d)
            { H = _H; d = _d; }
            friend bool operator<(node a, node b)
            { return a.d > b.d; }
        };
        int n, k, s, t, dst[ N ];
        nd *nxt[ N ];
        vector<nd*> g[ N ], rg[ N ];
        heap *nullNd, *head[ N ];
        void init( int _n , int _k , int _s , int _t ){
            n = _n; k = _k; s = _s; t = _t;
            for( int i = 1 ; i <= n ; i ++ ){
                g[ i ].clear(); rg[ i ].clear();
                nxt[ i ] = head[ i ] = NULL;
                dst[ i ] = -1;
            }
        }
        void addEdge( int ui , int vi , int di ){

```

```

    nd* e = new nd(ui, vi, di);
    g[ ui ].push_back( e );
    rg[ vi ].push_back( e );
}
queue<int> dfsQ;
void dijkstra(){
    while(dfsQ.size()) dfsQ.pop();
    priority_queue<node> Q;
    Q.push(node(0, t, NULL));
    while (!Q.empty()){
        node p = Q.top(); Q.pop();
        if(dst[p.v] != -1) continue;
        dst[ p.v ] = p.d;
        nxt[ p.v ] = p.E;
        dfsQ.push( p.v );
        for(auto e: rg[ p.v ])
            Q.push(node(p.d + e->d, e->u, e));
    }
}
heap* merge(heap* curNd, heap* newNd){
    if(curNd == nullNd) return newNd;
    heap* root = new heap;
    memcpy(root, curNd, sizeof(heap));
    if(newNd->edge->d < curNd->edge->d){
        root->edge = newNd->edge;
        root->chd[2] = newNd->chd[2];
        root->chd[3] = newNd->chd[3];
        newNd->edge = curNd->edge;
        newNd->chd[2] = curNd->chd[2];
        newNd->chd[3] = curNd->chd[3];
    }
    if(root->chd[0]->dep < root->chd[1]->dep)
        root->chd[0] = merge(root->chd[0], newNd);
    else
        root->chd[1] = merge(root->chd[1], newNd);
    root->dep = max(root->chd[0]->dep, root->chd[1]->
        dep) + 1;
    return root;
}
vector<heap*> V;
void build(){
    nullNd = new heap;
    nullNd->dep = 0;
    nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd+4, nullNd);
    while(not dfsQ.empty()){
        int u = dfsQ.front(); dfsQ.pop();
        if(!nxt[ u ]) head[ u ] = nullNd;
        else head[ u ] = head[nxt[ u ]->v];
        V.clear();
        for( auto& e : g[ u ] ){
            int v = e->v;
            if( dst[ v ] == -1 ) continue;
            e->d += dst[ v ] - dst[ u ];
            if( nxt[ u ] != e ){
                heap* p = new heap;
                fill(p->chd, p->chd+4, nullNd);
                p->dep = 1;
                p->edge = e;
                V.push_back(p);
            }
        }
        if(V.empty()) continue;
        make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
        for( size_t i = 0 ; i < V.size() ; i ++ ){
            if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
            else V[i]->chd[2]=nullNd;
            if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
            else V[i]->chd[3]=nullNd;
        }
        head[u] = merge(head[u], V.front());
    }
}
vector<LL> ans;
void first_K(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;
    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )

```

```

        Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
        node p = Q.top(); q; Q.pop();
        ans.push_back( p.d );
        if(head[ p.H->edge->v ] != nullNd){
            q.H = head[ p.H->edge->v ];
            q.d = p.d + q.H->edge->d;
            Q.push(q);
        }
        for( int i = 0 ; i < 4 ; i ++ )
            if( p.H->chd[ i ] != nullNd ){
                q.H = p.H->chd[ i ];
                q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
                    edge->d;
                Q.push( q );
            }
    }
}
void solve(){
    dijkstra();
    build();
    first_K();
}
} solver;

```

## 6 String

### 6.1 PalTree

```

const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    char s[MXN]={'-1'};
    int newNode(int l, int f){
        len[tot]=l, fail[tot]=f, cnt[tot]=num[tot]=0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int push(){
        int c=s[n]-'a', np=getfail(lst);
        if(!(lst=nxt[np][c])){
            lst=newNode(len[np]+2, nxt[getfail(fail[np])][c]);
            nxt[np][c]=lst;
            num[lst]=num[fail[lst]]+1;
        }
        return ++cnt[lst], lst;
    }
    void init(const char *_s){
        tot=lst=n=0;
        newNode(0, 1), newNode(-1, 0);
        for(; _s[n];) s[n+1]=_s[n], ++n, state[n-1]=push();
        for(int i=tot-1; i>1; i--) cnt[fail[i]]+=cnt[i];
    }
}palt;

```

### 6.2 KMP

```

int failure[MXN];
void KMP(string& t, string& p)
{
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i)
    {
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i)
    {
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1)
        {
            cout << i - p.size() + 1<<" ";
            j = failure[j];
        }
    }
}

```



```

    }
}
}

```

### 6.3 SAIS

```

const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i+1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]]] = p[q[i]=nn++] = i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmzx+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[p[nsa[i]]]] = p[nsa[i]]);
    }
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

### 6.4 SuffixAutomata

```

// any path start from root forms a substring of S
// occurrence of P : iff SAM can run on input word P
// number of different substring : ds[1]-1
// total length of all different substring : dsl[1]

```

```

// max/min length of state i : mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P : cnt[i]
// first occurrence position of P : fp[i]-lpl+1
// all position of P : fp of "dfs from i through rmom"
const int MXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MXM], mx[MXM]; //ind[MXM]
    int nxt[MXM][33]; //cnt[MXM],ds[MXM],dsl[MXM],fp[MXM]
    // bool v[MXM]
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = 0; //cnt=ds=dsl=fp=v=0
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode(); //cnt[np]=1
        mx[np] = mx[p]+1; //fp[np]=mx[np]-1
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode(); //fp[nq]=fp[q]
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for(; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void calc(){
        calc(root);
        iota(ind,ind+tot,1);
        sort(ind,ind+tot,[&](int i,int j){return mx[i]<mx[j];});
        for(int i=tot-1;i>=0;i--)
            cnt[mom[ind[i]]]+=cnt[ind[i]];
    }
    void calc(int x){
        v[x]=ds[x]=1;dsl[x]=0; //rmom[mom[x]].push_back(x);
        for(int i=1;i<=26;i++){
            if(nxt[x][i]){
                if(!v[nxt[x][i]]) calc(nxt[x][i]);
                ds[x]+=ds[nxt[x][i]];
                dsl[x]+=ds[nxt[x][i]]+dsl[nxt[x][i]];
            }
        }
    }
    void push(char *str){
        for(int i = 0; str[i]; i++)
            push(str[i]-'a'+1);
    }
} sam;

```

### 6.5 Aho-Corasick

```

struct ACautomata{
    struct Node{
        int cnt,i;
        Node *go[26], *fail, *dic;
        Node (){
            cnt = 0; fail = 0; dic=0;
            memset(go,0,sizeof(go));
        }
    }pool[1048576],*root;
    int nMem,n_pattern;
    Node* new_Node(){

```

```

    pool[nMem] = Node();
    return &pool[nMem++];
}
void init() {nMem=0;root=new_Node();n_pattern=0;}
void add(const string &str) { insert(root,str,0); }
void insert(Node *cur, const string &str, int pos){
    for(int i=pos;i<str.size();i++){
        if(!cur->go[str[i]-'a'])
            cur->go[str[i]-'a'] = new_Node();
        cur=cur->go[str[i]-'a'];
    }
    cur->cnt++; cur->i=n_pattern++;
}
void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
        Node* fr=que.front(); que.pop();
        for (int i=0; i<26; i++){
            if (fr->go[i]){
                Node *ptr = fr->fail;
                while (ptr && !ptr->go[i]) ptr = ptr->fail;
                fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
                fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
                que.push(fr->go[i]);
            }
        }
    }
}
void query(Node* root,string s){
    Node *cur=root;
    for(int i=0;i<(int)s.size();i++){
        while(cur&&!cur->nxt[s[i]-'a']) cur=cur->fail;
        cur=(cur?cur->nxt[s[i]-'a']:root);
        if(cur->i>=0) ans[cur->i]++;
        for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
            ans[tmp->i]++;
    }
} // ans[i] : number of occurrence of pattern i
}AC;

```

## 6.6 Z Value

```

char s[MAXN];
int len,z[MAXN];
void Z_value() {
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++){
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

## 6.7 BWT

```

struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
        for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
            res[ i ] = ori[ a[ ptr ] ];
            ptr = a[ ptr ];
        }
    }
}

```

```

    }
    res[ len ] = 0;
}
} bwt;

```

## 6.8 ZValue Palindrome

```

void z_value_pal(char *s,int len,int *z){
    len=(len<1)+1;
    for(int i=len-1;i>=0;i--){
        s[i]=i&1?s[i>1]:'@';
        z[0]=1;
        for(int i=1,l=0,r=0;i<len;i++){
            z[i]=i<r?min(z[l+l-i],r-i):1;
            while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
                ++z[i];
            if(i+z[i]>r) l=i,r=i+z[i];
        }
    }
}

```

## 6.9 Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

## 6.10 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
    }
}

```

```

    strcpy(tmp,a);
    strcpy(a,b);
    strcpy(b,tmp);
}
strcpy(tmp,a);
strcat(a,tmp);
// basic lcs
for(int i=0;i<=2*al;i++) {
    dp[i][0]=0;
    pred[i][0]=U;
}
for(int j=0;j<=bl;j++) {
    dp[0][j]=0;
    pred[0][j]=L;
}
for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
        if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
        else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
        if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
        else if(a[i-1]==b[j-1]) pred[i][j]=LU;
        else pred[i][j]=U;
    }
}
// do cyclic lcs
int clcs=0;
for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
}
// recover a
a[al]='\0';
return clcs;
}

```

## 7 Data Structure

### 7.1 Segment tree

```

struct seg_tree{
    static const int MXN=1e5+5,NO_TAG=0; // to be set
    ll a[MXN],val[MXN*4],tag[MXN*4],v;
    int n,ql,qr;
    void push(int i,int l,int r){
        if(tag[i]!=NO_TAG){
            val[i]+=tag[i]; // update by tag
            if(l!=r){
                tag[cl(i)]+=tag[i]; // push
                tag[cr(i)]+=tag[i]; // push
            }
            tag[i]=NO_TAG;
        }
    }
    void pull(int i,int l,int r){
        int mid=(l+r)>>1;
        push(cl(i),l,mid);push(cr(i),mid+1,r);
        val[i]=max(val[cl(i)],val[cr(i)]); // pull
    }
    void build(int i,int l,int r){
        if(l==r){
            val[i]=a[l]; // set value
            return;
        }
        int mid=(l+r)>>1;
        build(cl(i),l,mid);build(cr(i),mid+1,r);
        pull(i,l,r);
    }
    void update(int i,int l,int r){
        push(i,l,r);
        if(ql<=l&&r<=qr){
            tag[i]+=v; // update tag
            return;
        }
        int mid=(l+r)>>1;
        if(ql<=mid) update(cl(i),l,mid);
        if(qr>mid) update(cr(i),mid+1,r);
        pull(i,l,r);
    }
    void query(int i,int l,int r){
        push(i,l,r);
        if(ql<=l&&r<=qr){
            v=max(v,val[i]); // update answer

```

```

        return;
    }
    int mid=(l+r)>>1;
    if(ql<=mid) query(cl(i),l,mid);
    if(qr>mid) query(cr(i),mid+1,r);
}
}tree;

```

### 7.2 Treap

```

struct Treap{
    int sz , val , pri , tag;
    Treap *l , *r;
    Treap( int _val ){
        val = _val; sz = 1;
        pri = rand(); l = r = NULL; tag = 0;
    }
};
void push( Treap * a ){
    if( a->tag ){
        Treap *swp = a->l; a->l = a->r; a->r = swp;
        int swp2;
        if( a->l ) a->l->tag ^= 1;
        if( a->r ) a->r->tag ^= 1;
        a->tag = 0;
    }
}
inline int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
    a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a , Treap *b ){
    if( !a || !b ) return a ? a : b;
    if( a->pri > b->pri ){
        push( a );
        a->r = merge( a->r , b );
        pull( a );
        return a;
    }else{
        push( b );
        b->l = merge( a , b->l );
        pull( b );
        return b;
    }
}
void split_kth( Treap *t , int k , Treap*&a , Treap*&b )
){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( Size( t->l ) + 1 <= k ){
        a = t;
        split_kth( t->r , k - Size( t->l ) - 1 , a->r , b );
        pull( a );
    }else{
        b = t;
        split_kth( t->l , k , a , b->l );
        pull( b );
    }
}
void split_key(Treap *t, int k,Treap*&a ,Treap*&b){
    if(!t){ a = b = NULL; return; }
    push(t);
    if(k<=t->val){
        b = t;
        split_key(t->l,k,a,b->l);
        pull(b);
    }
    else{
        a = t;
        split_key(t->r,k,a->r,b);
        pull(a);
    }
}
}

```

### 7.3 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;

```

```

int val, rev, size;
Splay (int _val=-1) : val(_val), rev(0), size(1)
{ f = ch[0] = ch[1] = &nil; }
bool isr()
{ return f->ch[0] != this && f->ch[1] != this; }
int dir()
{ return f->ch[0] == this ? 0 : 1; }
void setCh(Splay *c, int d){
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
}
void push(){
    if (!rev) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev=0;
}
void pull(){
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
}
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);

```

```

    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}

```

## 7.4 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
    r[ 1 ].erase( 1 , 1 );
    cout << r[ 1 ].substr( 0 , 2 );
}

```

## 8 Others

### 8.1 Find max tangent(x,y is increasing)

```

typedef long long LL;
const int MAXN = 100010;
struct Coord{
    LL x, y;
    Coord operator - (Coord ag) const{
        Coord res;
        res.x = x - ag.x;
        res.y = y - ag.y;
        return res;
    }
}sum[MAXN], pnt[MAXN], ans, calc;

inline bool cross(Coord a, Coord b, Coord c){
    return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
}

int main(){
    int n, l, np, st, ed, now;

```

```

scanf("%d %d\n", &n, &l);
sum[0].x = sum[0].y = np = st = ed = 0;
for (int i = 1, v; i <= n; i++){
    scanf("%d", &v);
    sum[i].y = sum[i - 1].y + v;
    sum[i].x = i;
}
ans.x = now = 1;
ans.y = -1;
for (int i = 0; i <= n - 1; i++){
    while (np > 1 &&
        cross(pnt[np - 2], pnt[np - 1], sum[i]))
        np--;
    if (np < now && np != 0) now = np;
    pnt[np++] = sum[i];
    while (now < np &&
        !cross(pnt[now - 1], pnt[now], sum[i + 1]))
        now++;
    calc = sum[i + 1] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y){
        ans = calc;
        st = pnt[now - 1].x;
        ed = i + 1;
    }
}
double res = (sum[ed].y - sum[st].y) /
    (sum[ed].x - sum[st].x);
printf("%f\n", res);
return 0;
}

```

## 8.2 Exact Cover Set

```

// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM], R[NM], D[NM], U[NM], C[NM], S[NM], ROW[NM];
void remove(int c){
    L[R[c]] = L[c]; R[L[c]] = R[c];
    for (int i = D[c]; i != c; i = D[i]){
        for (int j = R[i]; j != i; j = R[j]){
            U[D[j]] = U[j]; D[U[j]] = D[j]; S[C[j]]--;
        }
    }
}
void resume(int c){
    for (int i = D[c]; i != c; i = D[i]){
        for (int j = L[i]; j != i; j = L[j]){
            U[D[j]] = D[U[j]] = j; S[C[j]]++;
        }
    }
    L[R[c]] = R[L[c]] = c;
}
int dfs(){
    if (R[0] == 0) return 1;
    int md = 1000000000, c;
    for (int i = R[0]; i != 0; i = R[i]){
        if (S[i] < md){ md = S[i]; c = i; }
    }
    if (md == 0) return 0;
    remove(c);
    for (int i = D[c]; i != c; i = D[i]){
        used[ROW[i]] = 1;
        for (int j = R[i]; j != i; j = R[j]) remove(C[j]);
        if (dfs()) return 1;
        for (int j = L[i]; j != i; j = L[j]) resume(C[j]);
        used[ROW[i]] = 0;
    }
    resume(c);
    return 0;
}
int exact_cover(int n, int m){
    for (int i = 0; i <= m; i++){
        R[i] = i + 1; L[i] = i - 1; U[i] = D[i] = i;
        S[i] = 0; C[i] = i;
    }
    R[m] = 0; L[0] = m;
    int t = m + 1;
    for (int i = 0; i < n; i++){

```

```

int k = -1;
for (int j = 0; j < m; j++){
    if (!A[i][j]) continue;
    if (k == -1) L[t] = R[t] = t;
    else{ L[t] = k; R[t] = R[k]; }
    k = t; D[t] = j + 1; U[t] = U[j + 1];
    L[R[t]] = R[L[t]] = U[D[t]] = D[U[t]] = t;
    C[t] = j + 1; S[C[t]]++; ROW[t] = i; id[i][j] = t++;
}
}
for (int i = 0; i < n; i++) used[i] = 0;
return dfs();
}

```

```

.....gg.....
.....g1a1111111g.....
.....gg1111g.....gaaa11111111aaa1
`g1aaaaa111111g`.....gaa1ggg11111111a1aa
gaa1g1111111111aa1`.....ga1ggg111111111111aag`
`ga1gg1111111111111aa`.....aaa1aaaa111111111111a`
`ag`g111111111111111ag`.....aaaaaa1g111111111111a`
`11`g111111111111111aa`.....gaag`g111111111111gg
gagg111111111111111aaaa`.....1ag`1111111111111gg
gagg1111NASTYA1aaaaaaa`.....1a1gg1RUDENK011111g1`
ga1g111111111111111ggggg`.....aaaaaa111111111111ga`
`1a1111111111111111aaaaag`.....aaaa111111111111a1`
`aa1111111111111111aaa`.....aaa111111111111a1
aaa1111111HATE1a1aaa`.....1aa111g111LOVEa11a1
1aaa11111111111gggg111aaaa11111111a11aag`
`gaaaaa11111111111a111g`.....aaa111a1aag`
`gaaaaa111111111111a`aaa111aaa1`.....
`1aaaaa11111111ga`gaaa111aa`.....
`1aaaaa1111aaa1aaag`.....
`g1aaa1aag`.....
`11g`.....

```