TALLER ECUACIONES DIFERENCIALES NO EXACTAS Y HOMOGÉNEAS

ALUMNOS

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CURSO

ECUACIONES DIFERENCIALES

PROGRAMA

INGENIERÍA DE SISTEMAS

SEMESTRE V

QUIBDÓ-CHOCÓ

Taller

Curso: Ecuaciones Diferenciales

1. **Resolver las siguientes ecuaciones diferenciales**

$$\cos x dx + \left(1 + \frac{2}{y}\right) \operatorname{sen} x dy = 0.$$
$$(y + x)dy = (y - x)dx$$

2. **Comprobar que las siguientes ecuaciones diferenciales son homogéneas y hallar la solución general:**

$$y' = \frac{y+x}{x}.$$
$$y' = \frac{y}{x+y}.$$

DESARROLLO

1.Resolver las siguientes ecuaciones diferenciales:

a)

$$\cos x dx + \left(1 + \frac{2}{y}\right) \operatorname{sen} x dy = 0.$$

$$\cos x\, dx + \left(1 + \frac{2}{y}\right) sen\ x\ dy\ = 0$$

$$M = \cos x\, dx \qquad N = \left(1 + \frac{2}{y}\right) sen\ x$$

$$M_y = 0 \qquad N_x = \left(1 + \frac{2}{y}\right) cos\ x$$

$M_y \neq N_x$ **NO EXACTA**

$$F(x) = \frac{M_y - N_x}{N} = \frac{0 - \left(1 + \frac{2}{y}\right) cos\ x}{\left(1 + \frac{2}{y}\right) sen\ x} = -\frac{cos\ x}{sen\ x}$$

$$\mu = e^{\int f\,(x)^{dx}} = e^{-\int cot\ x\ dx} = e^{-ln\ sen\ x} = e^{ln\ (sen\ x)^{-1}}$$

$$\mu = \frac{1}{sen\ x} = csc\ x$$

$$\left[cos\ x\ dx + \left(1 + \frac{2}{y}\right) sen\ x\ dy = 0\right] * \frac{1}{sen\ x}$$

$$\frac{cos\ x}{sen\ x}\ dx + \left(1 + \frac{2}{y}\right) \frac{sen\ x}{sen\ x}\ dy\ = 0$$

$$\cot x\, dx + \left(1 + \frac{2}{y}\right) dy = 0$$

$$M = \cot x \qquad N = \left(1 + \frac{2}{y}\right)$$

$$M_y = 0 \qquad N_x = 0$$

$M_y = N_x$ **EXACTA**

$$f(x, y) = \int \cot x dx + c_y = ln(sen\ x) + c_y$$

$$F_y = 0 + c'_y = 1 + \frac{2}{y} \rightarrow c_y = \int \left(1 + \frac{2}{y}\right) dy$$

$$c_y = \int dy + 2 \int \frac{1}{y} dy = y + 2\ ln\ y + c$$

$$c_y = y + 2ln\ y + c$$

$$f = ln(sen\ x) + y + 2\ ln\ y = c$$

b)

$$(y+x)dy = (y-x)dx$$

$$(y+x)dy = (y-x)dx$$

$$u = \frac{y}{x} \qquad y = ux \qquad dy = udx + xdu$$

$$(ux+x)(udx+xdu) = (ux-x)dx$$

$$u^2x\,dx + ux^2du + ux\,dx + x^2du = ux\,dx - x\,dx$$

$$ux^2du + x^2du = ux\,dx - x\,dx - u^2x\,dx - uxdx$$

$$x^2du\,(u+1) = -xdx\,(1+u^2)$$

$$\frac{(u+1)}{1+u^2} = \frac{xdx}{x^2}$$

$$\int \frac{udu}{1+u^2} + \int \frac{du}{1+u^2} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\,ln\,(1+u^2) + Arctan\,U = -ln\,x + c$$

$$ln\left(1+\frac{y^2}{x^2}\right) + 2Arctan\left(\frac{y}{x}\right) = -2lnx + 2c$$

$$ln\left(1+\frac{y^2}{x^2}\right) + ln\,x^2 = -2\,Arctan\left(\frac{y}{x}\right) + 2c$$

$$\boldsymbol{ln\left(1+\frac{y^2}{x^2}\right)+(\,x^2) = \;-2\;Arctan\left(\frac{y}{x}\right)+2c}$$

$$\boldsymbol{ln(x^2+\;y^2) = \;-2\;Arctan\left(\frac{y}{x}\right)+2c}$$

2.Comprobar que las siguientes ecuaciones diferenciales son homogéneas y hallar la solución general:

a)

$$y' = \frac{y+x}{x}.$$

$$xdy = (y + x)dx$$

$$y = ux \qquad dy = udx + xdu \qquad u = \frac{y}{x}$$

$$x(u\,dx + x\,du) = (ux + x)\,dx$$

$$uxdx + \; x^2\,du = uxdx + xdx$$

$$x^2\,du = uxdx + xdx - uxdx$$

$$x^2du = xdx$$

$$du = \frac{dy}{x}$$

$$\int du = \int \frac{dy}{x}$$

$$u = \ln x + c$$

$$\frac{y}{x} = ln\, x + c$$

b)

$$y' = \frac{y}{x+y}.$$

$$(x+y)dy = y\,dx$$

$$y = ux \qquad dy = udx + xdu \qquad u = \frac{y}{x}$$

$$(x+ux)(udx+xdu\,) = ux\,dx$$

$$uxdx + \ x^2\,du + u^2xdx + ux^2du = uxdx$$

$$x^2du + ux^2du = uxdx - uxdx - u^2xdx$$

$$x^2\,du(1+u) = xdx\,(u-u-u^2)$$

$$\frac{du(1+u)}{-u^2} = \frac{xdx}{x^2}$$

$$\frac{du(1+u)}{-u^2} = \frac{dx}{x}$$

$$\int\left(\frac{1}{-u^2}+\frac{1}{-u}\right)du = \int\frac{dx}{x}$$

$$\frac{1}{u} - ln\,u = ln\,x + c$$

$$\frac{x}{y} - ln\left(\frac{y}{x}\right) = ln(x) + c$$