IMM Filter

$$\mu^{Ji}_{k-1|k-1} \triangleq P(r_{k-1} = J \mid r_k = i, y_{0:k-1})$$

Transition Markov Probability

- **Mixing** $\mu^{Ji}_{k-1|k-1} = \dfrac{\pi_{Ji}\, \mu^{J}_{k-1}}{\sum_{L=1}^{Nr} \pi_{Li}\, \mu^{L}_{k-1}}$ $\qquad \pi_{Ji} =$ (J) ⟶ (i)

$$\hat{x}^{0i}_{k-1|k-1} = \sum_{J=1}^{Nr} \mu^{Ji}_{k-1|k-1}\, \hat{x}^{J}_{k-1|k-1}$$

$$\Sigma^{0i}_{k-1|k-1} = \sum_{J=1}^{Nr} \mu^{Ji}_{k-1|k-1} \left[\Sigma^{J}_{k-1|k-1} + \left(\hat{x}^{J}_{k-1|k-1} - \hat{x}^{0i}_{k-1|k-1} \right)\left(\hat{x}^{J}_{k-1|k-1} - \hat{x}^{0i}_{k-1|k-1} \right)^{T} \right]$$

- **Mode Matched Prediction Update:**

$$\hat{x}^{i}_{k|k-1} = A(i)\, \hat{x}^{0i}_{k-1|k-1}$$

$$\Sigma^{i}_{k|k-1} = A(i)\, \Sigma^{0i}_{k-1|k-1} A^{T}(i) + B(i)\, Q\, B^{T}(i)$$

- **Mode Matched Measurement Update:** For ith model, $i = 1, \ldots, Nr$

$$\hat{x}^{i}_{k|k} = \hat{x}^{i}_{k|k-1} + k^{i}_{k}\left(y_k - \hat{y}^{i}_{k|k-1}\right), \quad \Sigma^{i}_{k|k} = \Sigma^{i}_{k|k-1} - k_k^{i}\, S_k^{i}\, k_k^{iT}$$

⟶ Calculate the updated mode probability $\mu_k^{i} = \dfrac{N(y_k;\, \hat{y}^{i}_{k|k-1},\, S^{i}_{k})\, \sum_{J=1}^{Nr} \pi_{Ji}\, \mu^{J}_{k-1}}{\sum_{L=1}^{Nr} N(y_k;\, \hat{y}^{L}_{k|k-1},\, S^{L}_{k}) \sum_{J=1}^{Nr} \pi_{JL}\, \mu^{J}_{k-1}}$

- **Output Estimate Calculation:**

$$\hat{x}_{k|k} = \sum_{i=1}^{Nr} \mu^{i}_{k}\, \hat{x}^{i}_{k|k} \qquad \Sigma_{k|k} = \sum_{i=1}^{Nr} \mu_k^{i} \left[\Sigma^{i}_{k|k} + \left(\hat{x}^{i}_{k|k} - \hat{x}_{k|k} \right)\left(\hat{x}^{i}_{k|k} - \hat{x}_{k|k} \right)^{T} \right]$$