IMM - JPDA - UKF

N_r : number of models , M_J is J'th model   k : k'th step (in terms of measurements)

T : number of existing tracks.

m(k) : number of received measurements.

- What we assume to have before start to filtering

→ Each track has u_{k-1}^{J} (mode probability for each model) , $\hat{x}_{k-1}^{J}$, P_{k-1}^{J} for each model in the track

- Now we should mix models for each model in track.

(This step is independent between tracks)

MIXING

$$u_{k-1|k-1}^{Ji} \triangleq P(r_{k-1} = J \mid r_k = i) = \frac{\pi_{Ji}\, \mu_{k-1}^{J}}{\sum_{L=1}^{N_r} \pi_{Li}\, \mu_{k-1}^{L}}$$

$$\hat{x}_{k-1|k-1}^{[i]} = \sum_{J=1}^{N_r} \mu_{k-1|k-1}^{Ji}\, \hat{x}_{k-1|k-1}^{J}$$

$$P_{k-1|k-1}^{[i]} = \sum_{J=1}^{N_r} \mu_{k-1|k-1}^{Ji} \left[P_{k-1|k-1}^{J} + (\hat{x}_{k-1|k-1}^{J} - \hat{x}_{k-1|k-1}^{[i]})(\hat{x}_{k-1|k-1}^{J} - \hat{x}_{k-1|k-1}^{[i]})^T \right]$$

→ Each model has it's own new mixed states for each track.

- Now each model will predict the next state using UKF

(This step is independent between tracks and models)

→

Predictions

the state mean and covariance for model r ; track t. Since independent between tracks and models, dropping those.

unscented transform

$$X^{0}_{k-1|k-1} = \hat{X}_{k-1|k-1}$$

$$X^{i}_{k-1|k-1} = \hat{X}_{k-1|k-1} + \left(\sqrt{(L+\lambda)P_{k-1|k-1}}\right)_i , \quad i \in 1,2,\dots L$$

denotes the i'th row of square matrix

$$X^{i}_{k-1|k-1} = \hat{X}_{k-1|k-1} - \left(\sqrt{(L+\lambda)P_{k-1|k-1}}\right)_{i-L} , \quad i \in L+1, L+2 \dots 2L$$

$\downarrow$

$$X^{*,i}_{k|k-1} = f(X^{i}_{k-1|k-1}) \quad i \in (0,1,2,\dots 2L)$$

process noise covariance

$$\hat{X}^{-}_{k|k-1} = \sum_{i=0}^{2L} W_s^i X^{*,i}_{k|k-1} , \quad P^{-}_{k|k-1} = \sum_{i=0}^{2L} W_c^i \left[X^{*,i}_{k|k-1} - \hat{X}^{-}_{k|k-1}\right]\left[X^{*,i}_{k|k-1} - \hat{X}^{-}_{k|k-1}\right]^T + Q$$

where $W_s^0 = \frac{\lambda}{L+\lambda}$, $W_c^0 = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta)$ | $W_s^i = W_c^i = \frac{1}{2(L+\lambda)}$, $\lambda = \alpha^2(L+K) - L$

$$\rightarrow \; Z_k^i = h(X^{i}_{k|k-1}) , \quad \hat{Z}^{-}_k = \sum_{i=0}^{2L} W_s^i Z_k^i , \quad P_{Z_k,Z_k} = \sum_{i=0}^{2L} W_c^i [Z_k^i - \hat{Z}^{-}_k][Z_k^i - \hat{Z}^{-}_k]^T + R$$

Measurement noise covariance

$$P_{X_k,Z_k} = \sum_{i=0}^{2L} W_c^i \left[X^{i}_{k|k-1} - \hat{X}^{-}_{k|k-1}\right]\left[Z_k^i - \hat{Z}^{-}_k\right]^T$$

$$\boxed{K_k = P_{X_k,Z_k} P_{Z_k,Z_k}^{-1}} \rightarrow \text{kalman gain}$$

Now it is time to update each models state, But before that we need to associate measurements with tracks.

$\rightarrow$ First calculate the data association probabilities : B_{Jt}

$\longrightarrow$

let's assume that M number of measurements are received and we have N_r number of existing tracks.

First we should eleminate the measurements which are unlikely to be associated with any track, How to implement this part is unclear, not given in [BYL95]

Each model will calculate the Mahalanobis distance with each measurement. Ones at longer distances will be eleminated.

$$\Rightarrow \text{Validated Measurements} = \bigcup_{J}^{N_r} \{ z : [z - \hat{z}_J(k|k-1)]^T P^{-1}_{z_1,z_2} [z - \hat{z}_J(k|k-1)] \leq \gamma \}$$

γ: gate threshold

<u>Note:</u> The measurement that are not validated might be used to initiate new tracks

let $Q = \bigcap_{J=1}^{M} Q_{Jt_J}$, Q_{Jt} is the event that measurement J is originated from target t

$\Rightarrow$ If using parametric JPDA

$$P(Q \mid z^k) = \frac{1}{C_2} \prod_{J}^{m(k)} (\lambda^{-1} f_{t_J}[z_J(k)])^{\tau_J} \cdot \prod_{t}^{N_r} (P_D^t)^{\delta_t} (1-P_D^t)^{1-\delta_t}$$

$\Rightarrow$ If using nonparametric JPDA

$$P(Q \mid z^k) = \frac{1}{C_4} \phi! \cdot \prod_{J}^{m(k)} (V \cdot f_{t_J}[z_J(k)])^{\tau_J} \cdot \prod_{t}^{N_r} (P_D^t)^{\delta_t} (1-P_D)^{1-\delta_t}$$

where <u>τ_J</u>: Just a binary number showing that measurement J is associated with a target in event Q
$\equiv \tau_J(Q)$

<u>t_J</u>: is index of the target to which measurement J is associated in the event consideration.

<u>δ_t</u>: if the target t is associated with a measurement (a binary number)

<u>P_D^t</u>: target t detection probability

V: Surveillance region

<u>λ</u>: spatial density λ of the false measurements.

$\longrightarrow$

Note:

V: surveillance region: → not clear how to implement this in [B4L95]

one solution is to take the volume of the track with highest volume

↳ page 211 [B4L95]

↳ actually it is argued for one track to use the biggest volume it has among it's models, not using it for all tracks (each track has it's own Volume)

or maybe summing the max volumes of all tracks could be a solution.

This is why one might choose to use parametric JPDA instead / down side is λ has to be estimated.

Now normalize event probabilities ⇒ $P\{Q|z^k\} = \dfrac{P\{Q|z^k\}}{\sum_Q P\{Q|z^k\}}$

$$\Rightarrow \beta_{jt} \triangleq P\{Q_{jt}|z^k\} = \sum_{Q:\, Q_{jt} \in Q} P\{Q|z^k\}$$: probability that measurement j is associated with target t.

$$\beta_{0t} = 1 - \sum_j \beta_{jt}$$: probability that target t is not associated with any measurements.

After this part the dependence between targets is over, β_{jt}'s are calculated.

Now calculate for each target and for their models independently next updated states

PDA: (This part is independent between tracks and models)

kalman gain calculated at page 2

$$\boxed{\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\, \nu(k)}$$

$$\nu(k) = \sum_{j=1}^{m(k)} \beta_{jt}(k)\, \nu_j(k) \quad , \quad \nu_j(k) = z_j(k) - \hat{z}(k|k-1)$$

$$\boxed{P(k|k) = \beta_{0t}(k)\, P(k|k-1) + [1-\beta_{0t}(k)]\, P^c(k|k) + \tilde{P}(k|k)}$$

↳ $P^c(k|k) = P(k|k-1) - K(k)\, P_{z_k z_k}\, K(k)^T \quad , \quad \tilde{P}(k) = K(k)\left[\left(\sum_{j=1}^{m(k)} \beta_{jt}(k)\, \nu_j(k)\, \nu_j(k)^T\right) - \nu(k)\, \nu(k)^T\right] K(k)^T$

Now each track and their models have updated their states

it is time to output an unified estimate for each track

$\longrightarrow$ Calculate the updated mode probability

(This part is independent between tracks)

Update Mode Probability: Let $m_t(k)$ be measurements which are in the validation for target t

$$\Lambda_r [z(k)] = \frac{1 - P_D}{V^{m(k)}} + \frac{P_D}{m(k)\, V^{m(k)-1}} \sum_{j=1}^{m(k)} \mathcal{N}[z_j(k) ; \hat{z}^r(k|k-1) , S^r(k)]$$

$$\Rightarrow \boxed{\mu_k^r = \frac{\Lambda_r[z(k)] \sum_{J=1}^{N_r} \pi_{Ji}\, \mu_{k-1}^J}{\sum_{l=1}^{N_r} \Lambda_l [z(k)] \sum_{J=1}^{N_r} \pi_{Jl}\, \mu_{k-1}^J}}$$

Note: Here the highest volume among the models can be used as V

$m_t(k)$ & $m(k)$ are not necessarily the same measurements, $m_t(k)$ might have less measurements compared to $m(k)$, since track t might be far away from (in mahalanobis) some measurements.

$\Rightarrow$

For each track now estimate the outputs

$$\boxed{\hat{X}_{t\,k|k} = \sum_{i=1}^{N_r} \mu_{t\,k}^i\, \hat{X}_{t\,k|k}^i} \quad , \quad \boxed{P_{t\,k|k} = \sum_{i=1}^{N_r} \mu_{t\,k}^i \left[P_{t\,k|k}^i + (\hat{X}_{t\,k|k}^i - \hat{X}_{t\,k|k})(\hat{X}_{t\,k|k}^i - \hat{X}_{t\,k|k})^T \right]}$$

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