Joint Probability Data Association

- t number of trackers, m number of valid measurements (that are in the gating area)

- θ_{Jt} is the event that measurement J is originated from target t.

- θ is one of the joint event of $\prod \theta_{Jt}$ for in t trackers and m measurements.
  ↳ one of the possibilities the trackers can be associated with measurements.

→ If using Parametric JPDA

$$\Rightarrow P\{\theta \mid Z^k\} = \frac{1}{c_2} \prod_J \left(\lambda^{-1} f_{t_J}[z_J(k)]\right)^{\tau_J} \prod_t (P_D^t)^{\delta_t} (1-P_D^t)^{1-\delta_t}$$

τ_J : Just a binary number showing that measurement J is associated with a target in event θ
↳ in fact one should show it like this → $\tau_J(\theta)$

t_J : is the index of the target to which measurement J is associated in the event consideration.

δ_t : if target t is associated with a measurement (a binary number)

P_D^t : target t detection probability

λ : spatial density λ of the false measurements

$$f_{t_J}[z_J(k)] = \mathcal{N}[z_J(k);\ \hat{z}^{t_J}(k|k-1),\ S^{t_J}(k)]$$

! before using the values $P\{\theta, Z^k\}$, normalize them, since they all have $\frac{1}{c_2}$

$$\Rightarrow P\{\theta' \mid Z^k\} = \frac{P\{\theta \mid Z^k\}}{\sum_\theta P\{\theta \mid Z^k\}}$$

$$\Rightarrow \boxed{\beta_{Jt}} \triangleq P\{\theta_{Jt} \mid Z^k\} = \sum_{\theta:\, \theta_{Jt} \in \theta} P\{\theta \mid Z^k\}$$

↳ the outcome of the "jointly association"

use it in the probability data association filter (PDAF) using B_{Jt} as B_i for target t, where $i = J$

$B_{Jt} : B_i$ = Probability (Association) that measurement i is associated with the target t.

Note that β_0 : Probability that target t is not associated with any measurements.

$$\Rightarrow \quad \beta_0 = 1 - \sum_J B_{Jt}$$

⇒ update equations

- $\boxed{\hat{X}(k|k) = \hat{X}(k|k-1) + W(k)\, V(k)}$, $\underline{W(k)} = P(k|k-1)\, H(k)^T\, S(k)^{-1}$ Kalman gain

$$V(k) = \sum_{i=1}^{m(k)} \beta_i(k)\, V_i(k) \quad , \; V_i(k) = z_i(k) - \hat{z}(k|k-1)$$

- $\boxed{P(k|k) = \beta_0(k)\, P(k|k-1) + [1 - \beta_0(k)]\, P^c(k|k) + \tilde{P}(k)}$

↳ $P^c(k|k) = P(k|k-1) - W(k)\, S(k)\, W(k)^T$

↳ $\tilde{P}(k) = W(k) \left[\sum_{i=1}^{m(k)} \beta_i(k)\, V_i(k)\, V_i(k)^T - V(k)\, V(k)^T \right] W(k)^T$

— END —