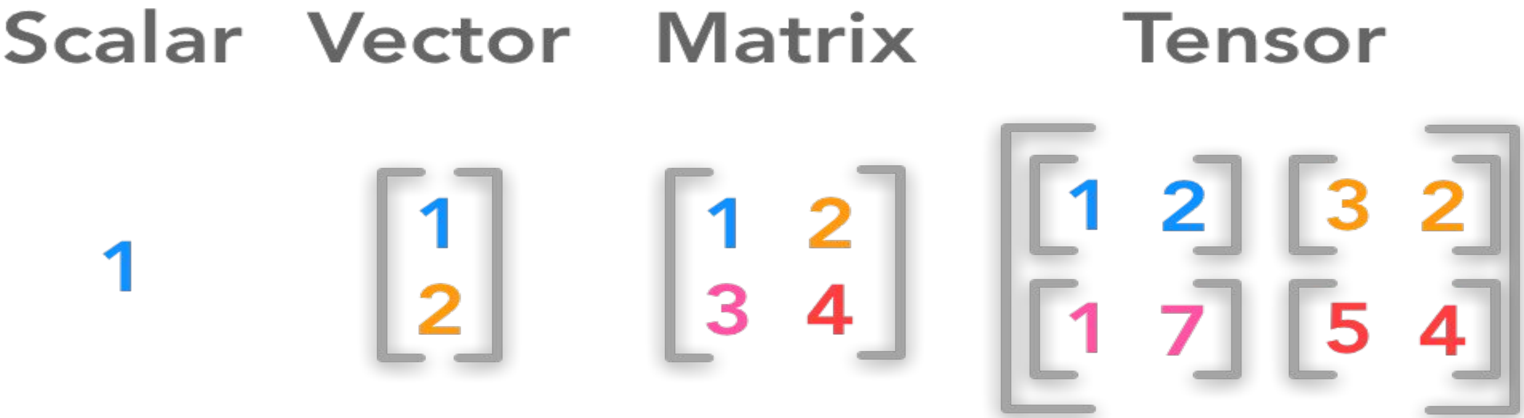
Linear Algebra Overview

Types of Structures



Dimensions

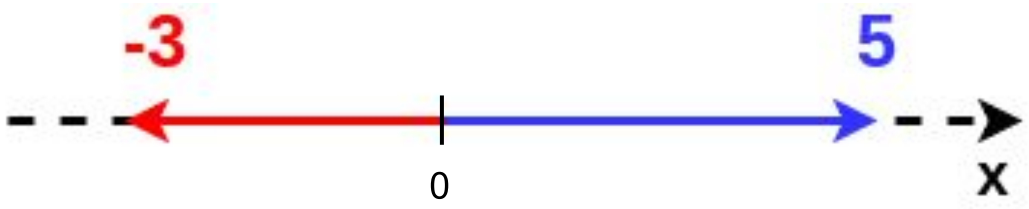
Number of values needed to uniquely specify a specific data point in a set

Address

{1,5}
{1,6,7}
{1,8,3,5,2}
{1,7,3,9……..n}

Scalars

Just Single Numbers (0-dimension)



Vectors

Ordered tuples of numbers (1-Dimensional)

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix} \begin{array}{l} \text{x-dimension} \\ \text{y-dimension} \\ \text{z-dimension} \\ . \\ . \\ . \\ n^{th}\text{-dimension} \end{array}$$

Column Vector

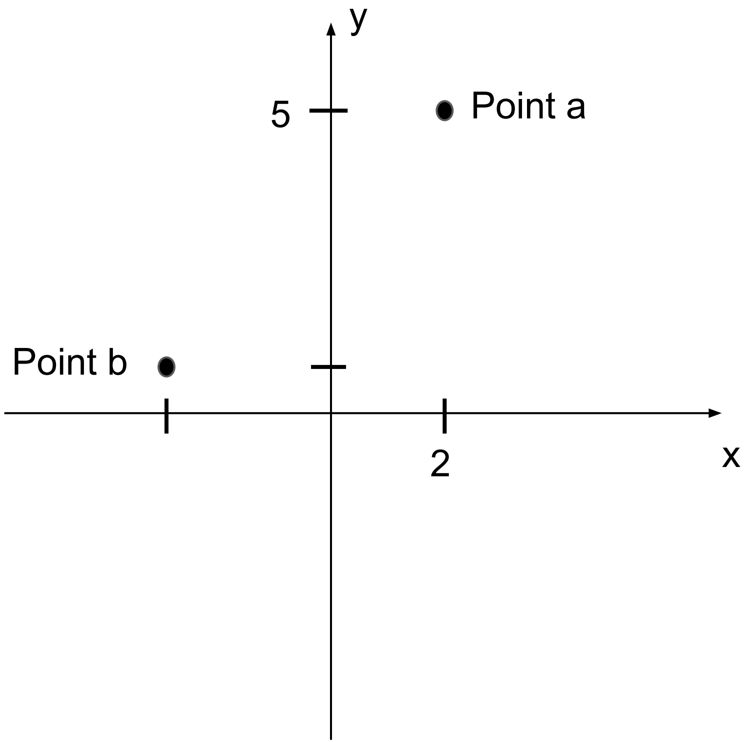
$$\mathbf{X} = \begin{bmatrix} x_0 & x_1 & x_2 & x_3 & \ldots & x_n \end{bmatrix}$$

Row Vector

Vector Spaces

$$\mathbf{a} = \begin{array}{cl} \begin{bmatrix} 2 \\ 5 \end{bmatrix} & \begin{array}{l} \leftarrow a_0 \\ \leftarrow a_1 \end{array} \end{array}$$

$$\mathbf{b} = \begin{array}{cl} \begin{bmatrix} -3 \\ 1 \end{bmatrix} & \begin{array}{l} \leftarrow b_0 \\ \leftarrow b_1 \end{array} \end{array}$$



Vector-Scalar Operations

$$\begin{bmatrix} 1 \\ 0 \\ 6 \\ 20 \\ -2 \\ 5 \end{bmatrix} + \mathbf{6} = \begin{bmatrix} 7 \\ 6 \\ 12 \\ 26 \\ 4 \\ 11 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 6 \\ 20 \\ -2 \\ 5 \end{bmatrix} - \mathbf{-1} = \begin{bmatrix} 0 \\ 1 \\ 7 \\ 21 \\ -1 \\ 6 \end{bmatrix}$$

Multiplication / Division works the same way.

Vector Addition / Subtraction

Addition

$$\begin{bmatrix} 1 \\ 0 \\ 6 \\ 20 \\ -2 \\ 5 \end{bmatrix} + \begin{bmatrix} -2 \\ 7 \\ 5 \\ 3 \\ 8 \\ 10 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 11 \\ 23 \\ 6 \\ 15 \end{bmatrix}$$

Subtraction

$$\begin{bmatrix} 1 \\ 0 \\ 6 \\ 2 \end{bmatrix} - \begin{bmatrix} -2 \\ 7 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \\ 1 \\ -1 \end{bmatrix}$$

Vectors must be same length!

Vector Multiplication / Dot Product

Multiplication

$$\begin{bmatrix} 3 \\ 12 \\ 9 \\ 7 \\ -1 \\ 8 \\ -5 \end{bmatrix} \times \begin{bmatrix} -2 \\ 7 \\ 5 \\ 3 \\ 8 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} -6 \\ 84 \\ 45 \\ 21 \\ -8 \\ 80 \\ -25 \end{bmatrix}$$

Dot Product

$$\begin{bmatrix} -1 \\ 9 \\ 2 \end{bmatrix} \bullet \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix} = 49$$

-1 x -2

\+

9 x 5

\+

2 x 1

Matrix

Array of numbers (2-Dimensional)

$$\mathbf{X} = \begin{bmatrix} x_{0,0} & x_{0,1} & x_{0,2} & x_{0,3} \\ x_{1,0} & x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,0} & x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,0} & x_{3,1} & x_{3,2} & x_{3,3} \end{bmatrix}$$

X is a 4x4 matrix
(4 rows and 4 columns)

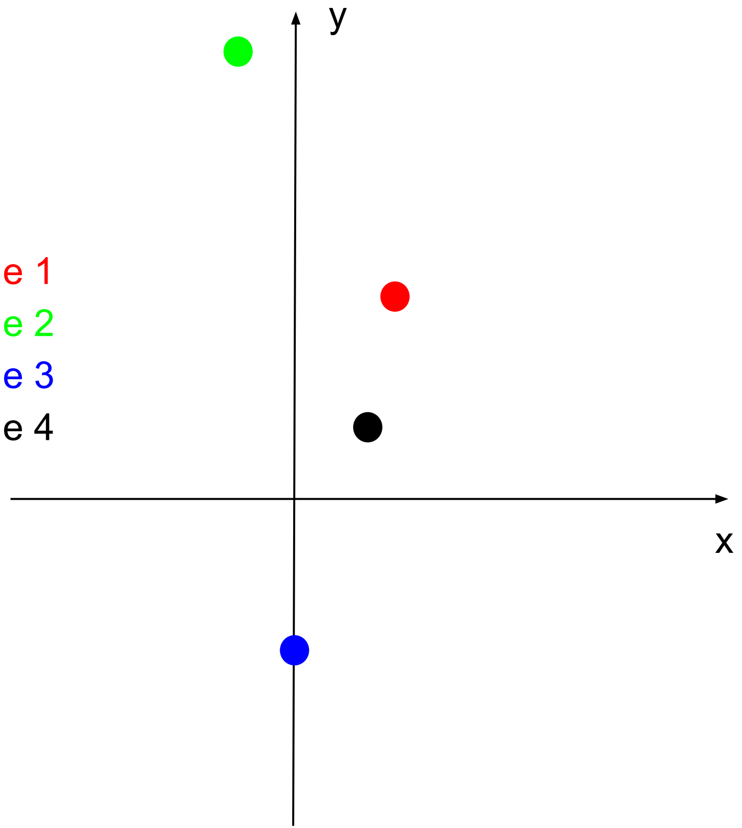
$$\mathbf{Y} = \begin{bmatrix} y_{0,0} & y_{0,1} \\ y_{1,0} & y_{1,1} \\ y_{2,0} & y_{2,1} \\ y_{3,0} & y_{3,1} \end{bmatrix}$$

Y is a 4x2 matrix
(4 rows and 2 columns)

Typically, in a dataset, each row represents a single sample and each column represents a feature of that sample.

Matrix Space

$$\mathbf{X} = \begin{array}{cc} \left[\begin{array}{cc} 2 & 3 \\ -1 & 8 \\ 0 & -3 \\ 1 & 1 \end{array}\right] & \begin{array}{l} \text{Sample 1} \\ \text{Sample 2} \\ \text{Sample 3} \\ \text{Sample 4} \end{array} \\ \begin{array}{cc} \text{Variable / Feature 1} & \text{Variable / Feature 2} \end{array} & \end{array}$$



Matrix-Scalar Operations

- Works the same way as in vectors
  - e.g. for addition between scalar and matrix, add scalar to every number in matrix

Matrix Addition / Subtraction

Addition

$$\begin{bmatrix} 2 & 10 \\ 9 & -5 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -7 & -1 \\ 6 & 11 \\ 2 & 9 \end{bmatrix} = \begin{bmatrix} -5 & 9 \\ 15 & 6 \\ 5 & 9 \end{bmatrix}$$

Subtraction

$$\begin{bmatrix} 2 & 10 & -1 & 0 \\ 2 & 13 & 5 & 2 \\ -10 & 2 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 7 & 10 & 3 & 6 \\ -2 & -1 & 9 & 1 \\ 5 & 3 & 3 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -4 & -6 \\ 4 & 14 & -4 & 1 \\ -15 & -1 & 0 & 1 \end{bmatrix}$$

Matrices must be same shape!

Matrix Transpose

3 x 2 matrix → 2 x 3 matrix

$$\begin{bmatrix} 2 & 10 \\ 9 & -5 \\ 3 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 9 & 3 \\ 10 & -5 & 0 \end{bmatrix}$$

3 x 3 matrix → 3 x 3 matrix

$$\begin{bmatrix} 2 & 10 & 6 \\ 9 & -5 & 2 \\ 3 & 0 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 9 & 3 \\ 10 & -5 & 0 \\ 6 & 2 & 1 \end{bmatrix}$$

Called a square matrix when # of rows and # of columns are the same.

Matrix Multiplication

3 x 4 matrix **A** · 4 x 2 matrix **B** = 3 x 2 matrix **AB**

$$\begin{bmatrix} 2 & 1 & 6 & -3 \\ 3 & -5 & -1 & 0 \\ 3 & 0 & 1 & 9 \end{bmatrix} \cdot \begin{bmatrix} -1 & 6 \\ 2 & -2 \\ 0 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -6 & -14 \\ -13 & 31 \\ 15 & 33 \end{bmatrix}$$

Note: # of **columns** in matrix 1 must be the same as the # of **rows** in matrix 2!

Not Commutative: AB ≠ BA

Matrix Multiplication

$$\underset{\text{3 x 4 matrix } \mathbf{A}}{\begin{bmatrix} 2 & 1 & 6 & -3 \\ 3 & -5 & -1 & 0 \\ 3 & 0 & 1 & 9 \end{bmatrix}} \cdot \underset{\text{4 x 2 matrix } \mathbf{B}}{\begin{bmatrix} -1 & 6 \\ 2 & -2 \\ 0 & -3 \\ 2 & 2 \end{bmatrix}} = \underset{\text{3 x 2 matrix } \mathbf{AB}}{\begin{bmatrix} -6 & -14 \\ -13 & 31 \\ 15 & 33 \end{bmatrix}}$$

$$(2 * -1) + (1 * 2) + (6 * 0) + (-3 * 2) = -6$$

Matrix Multiplication

3 x 4 matrix **A**

$$\begin{bmatrix} 2 & 1 & 6 & -3 \\ 3 & -5 & -1 & 0 \\ 3 & 0 & 1 & 9 \end{bmatrix}$$

4 x 2 matrix **B**

$$\begin{bmatrix} -1 & 6 \\ 2 & -2 \\ 0 & -3 \\ 2 & 2 \end{bmatrix}$$

3 x 2 matrix **AB**

$$\begin{bmatrix} 2 & 1 & 6 & -3 \\ 3 & -5 & -1 & 0 \\ 3 & 0 & 1 & 9 \end{bmatrix} \cdot \begin{bmatrix} -1 & 6 \\ 2 & -2 \\ 0 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -6 & -14 \\ -13 & 31 \\ 15 & 33 \end{bmatrix}$$

(3*1) + (-5*2) +(-1*0) +(0*2) =-13

Matrix Multiplication

$$\underset{\text{3 x 4 matrix } \mathbf{A}}{\begin{bmatrix} 2 & 1 & 6 & -3 \\ 3 & -5 & -1 & 0 \\ 3 & 0 & 1 & 9 \end{bmatrix}} \cdot \underset{\text{4 x 2 matrix } \mathbf{B}}{\begin{bmatrix} -1 & 6 \\ 2 & -2 \\ 0 & -3 \\ 2 & 2 \end{bmatrix}} = \underset{\text{3 x 2 matrix } \mathbf{AB}}{\begin{bmatrix} -6 & -14 \\ -13 & 31 \\ 15 & 33 \end{bmatrix}}$$

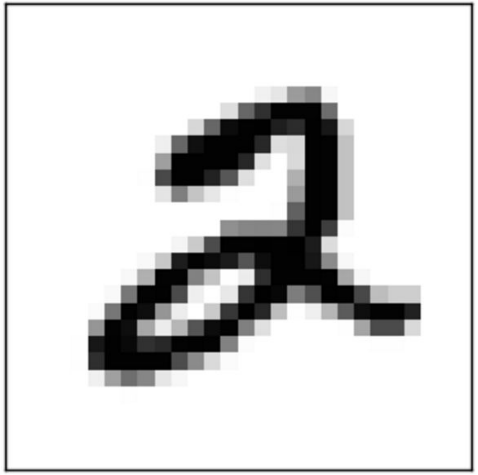
$$(2*6) + (1*-2) + (6*-3) + (-3*2) = -14$$

Note: matrix **AB** will have the same number of **rows** as matrix **A** and the same number of **columns** as matrix **B**.

Matrix-Vector Multiplication

$$\underset{\text{3 x 4 matrix}}{\begin{bmatrix} 2 & 1 & 6 & -3 \\ 3 & -5 & -1 & 0 \\ 3 & 0 & 1 & 9 \end{bmatrix}} \cdot \underset{\text{4-D Column vector}}{\begin{bmatrix} 0 \\ 3 \\ 1 \\ 6 \end{bmatrix}} = \underset{\text{3-D column vector}}{\begin{bmatrix} -9 \\ -16 \\ 55 \end{bmatrix}}$$

Examples



28 pixels x 28 pixels

$$=$$

$$\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & \mathbf{0} & \mathbf{0} & 1 & 1 & \mathbf{0} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \mathbf{0} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 \\
1 & 1 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

28 x 28 matrix

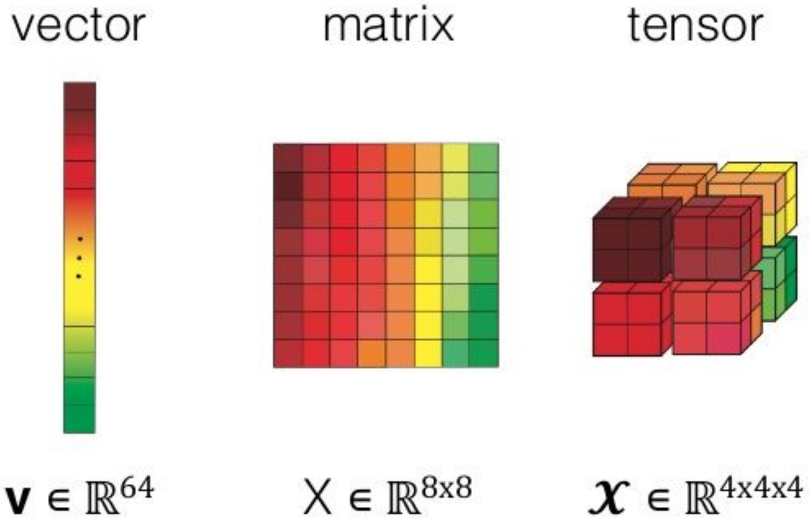
$$=$$

$$\begin{pmatrix}
1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ . \\ . \\ .
\end{pmatrix}$$

784-D vector

Tensors

N-Dimensional Arrays



Tensors

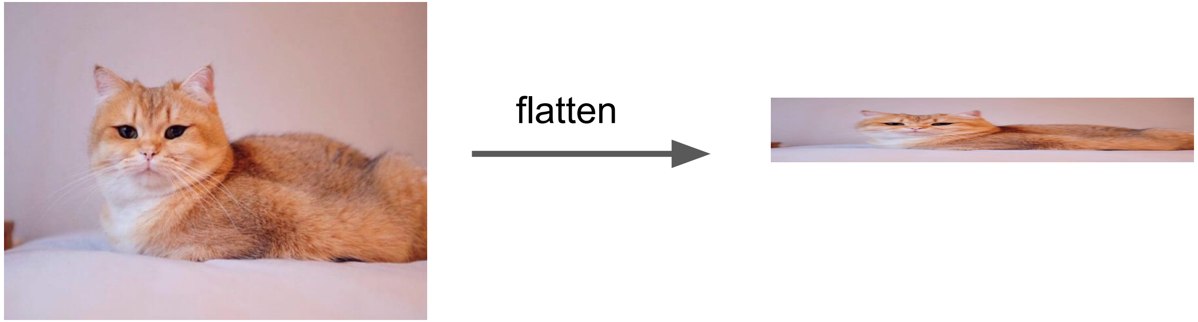
Example: in this class, we will be stacking images on top of each other and storing them as a tensor.

Tensor

Rotate each image on its back and stack them up vertically.

flatten

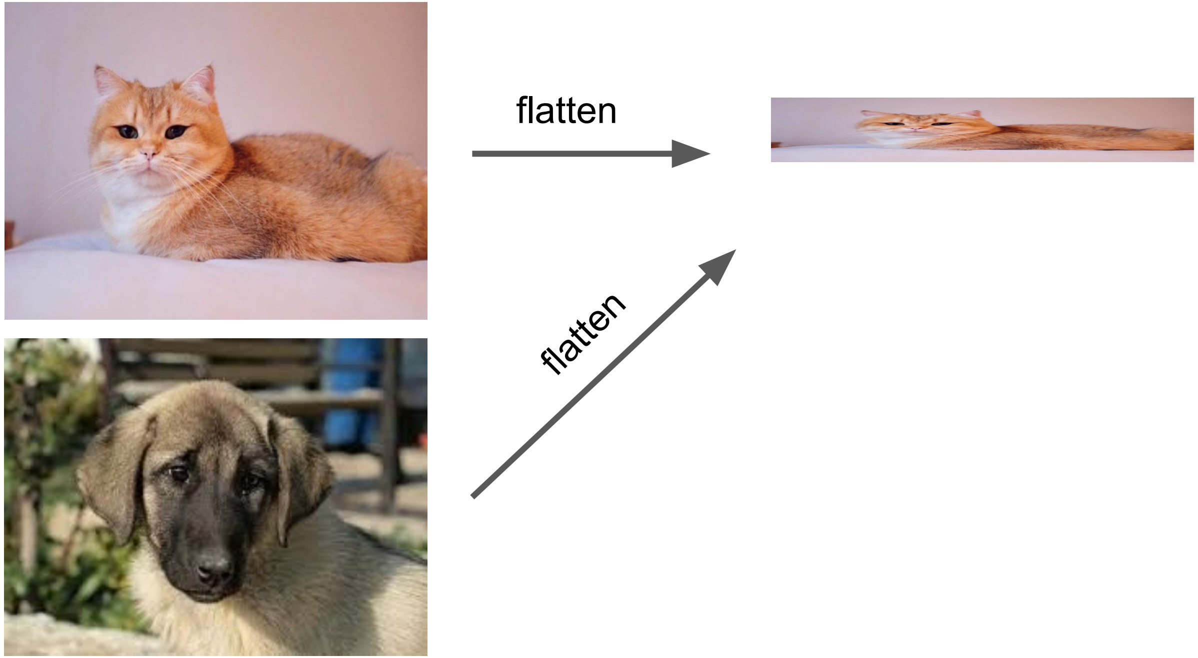
Tensor



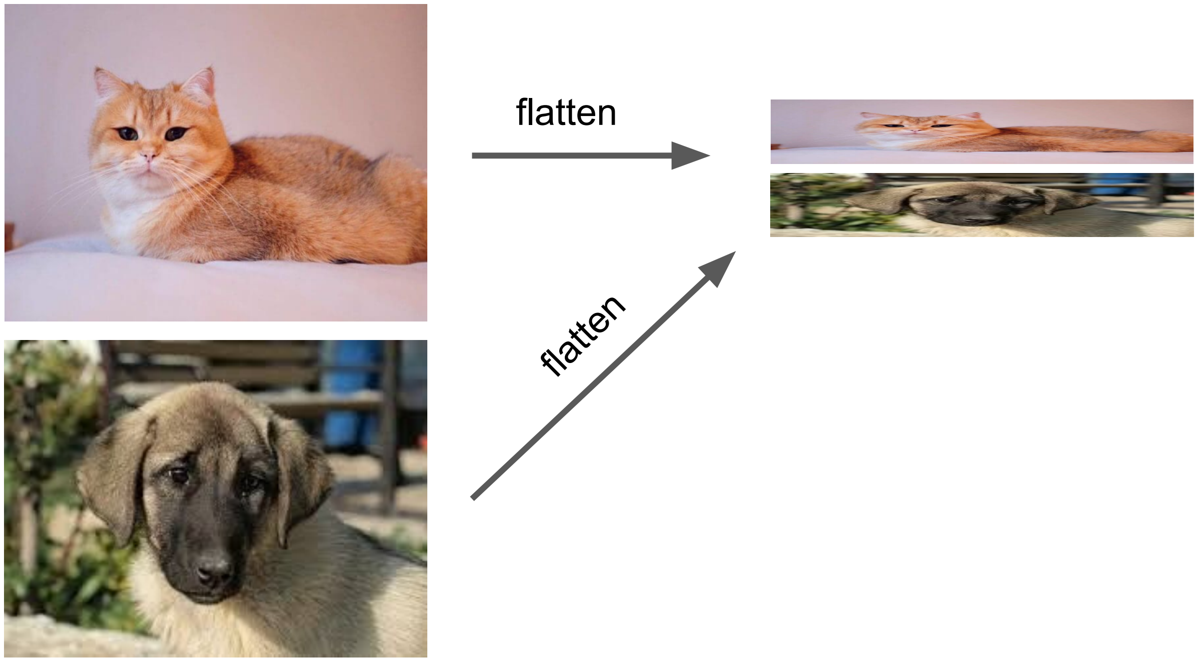
Tensor

flatten

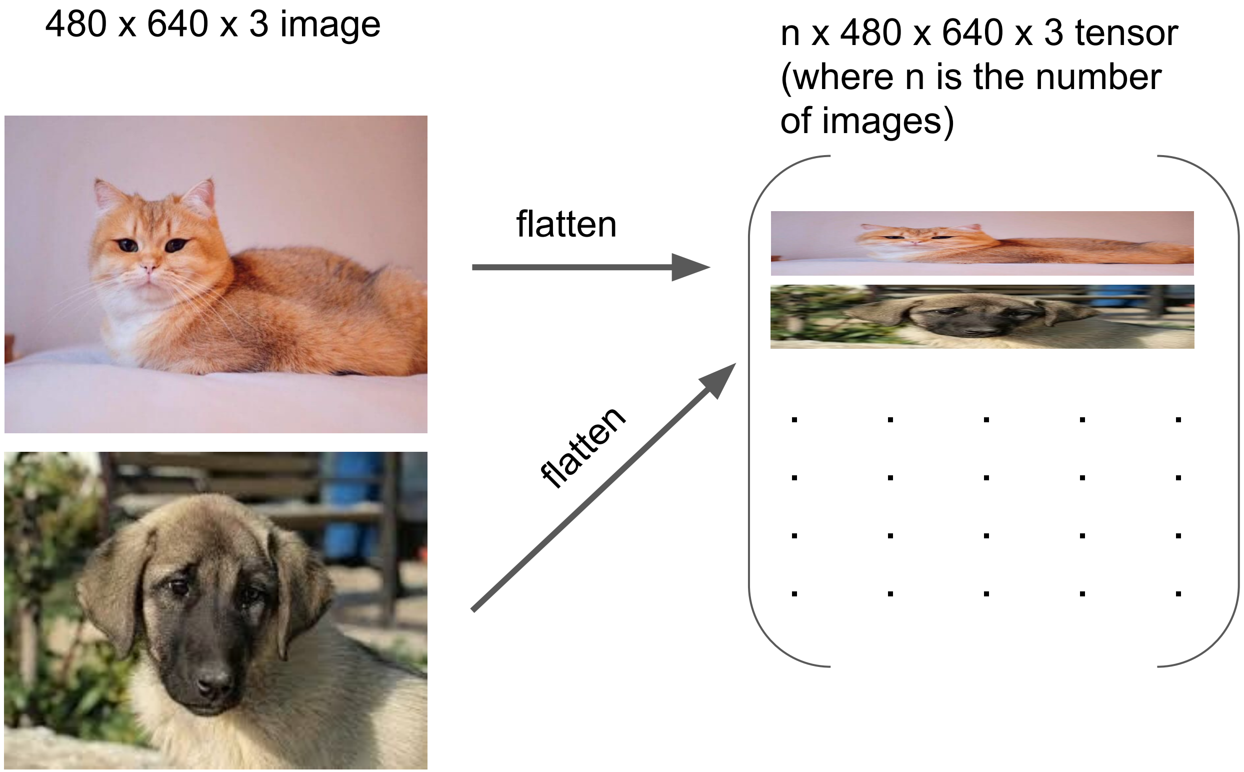
Tensor



Tensor



Tensor


480 x 640 x 3 image
n x 480 x 640 x 3 tensor
(where n is the number
of images)
flatten
flatten