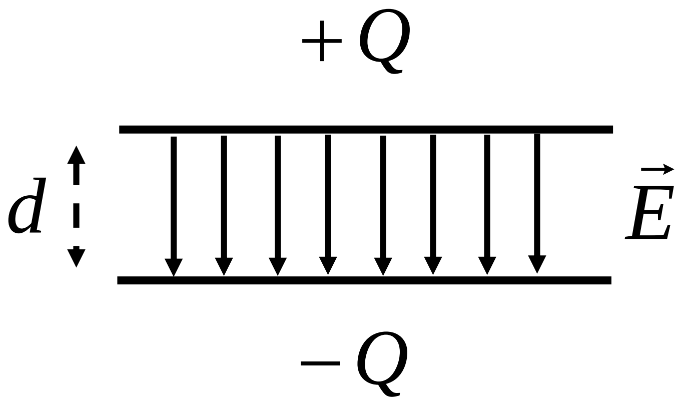
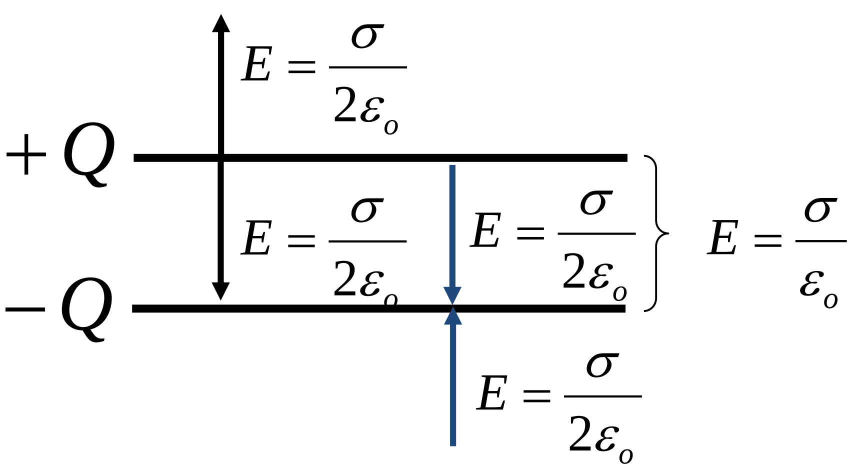
Parallel Plate Capacitor

$$C = Q/V$$





$$\Delta V = -\int \vec{E} \bullet d\vec{\ell}$$

$$= -\int \frac{\sigma}{\varepsilon_o} d\ell$$

$$= -\frac{\sigma}{\varepsilon_o} \int d\ell$$

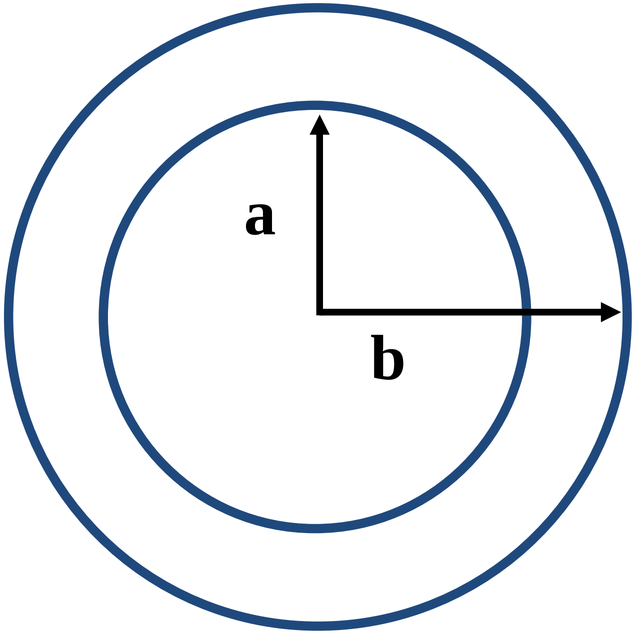
$$= -\frac{\sigma}{\varepsilon_o} d$$

$$= -\frac{Qd}{A\varepsilon_o}$$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_o A}{d}}$$

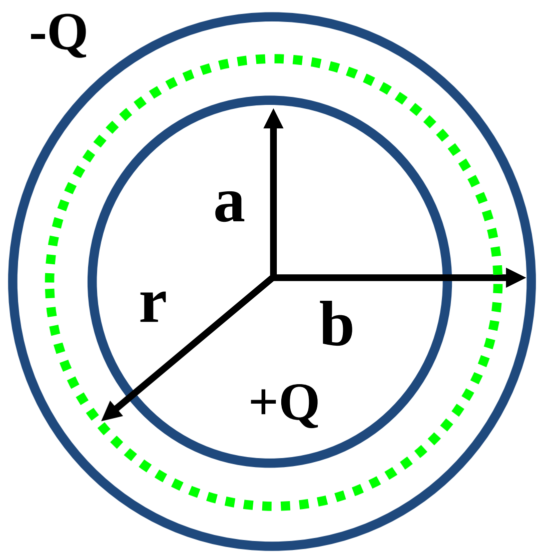
Capacitance depends on “geometry” & “material”

Capacitance of Concentric Spheres



1. C = Q / V
2. Assume a charge
3. Use Gauss's Law to find the E-field
4. Use $\Delta V = -\int \vec{E} \bullet d\vec{l}$ to find the potential
5. Solve for capacitance

Capacitance of Concentric Spheres



$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_o}$$

Since $\vec{E} \parallel d\vec{A}$ and $\vec{E}$ is constant everywhere on the Gaussian surface

$$\oint \vec{E} \bullet d\vec{A} = \oint E \cdot dA = E \oint dA = E\left(4\pi r^2\right)$$

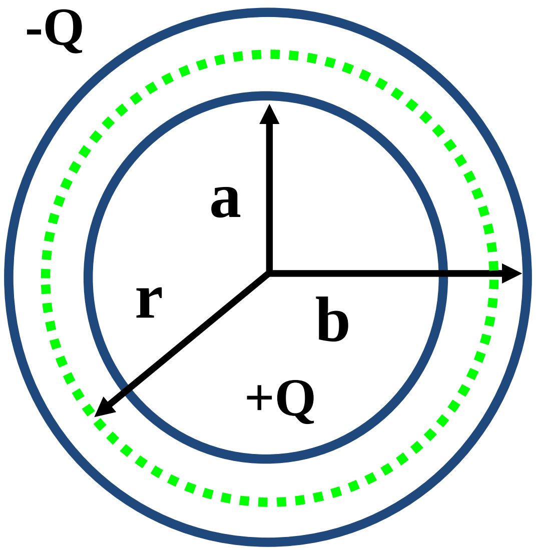
$$Q_{\text{encl}} = +Q$$

$$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{4\pi \varepsilon_o r^2}$$

1. **C = Q / V**
2. **Assume a charge**
3. **Use Gauss's Law to find the E-field**
4. **Use** $\Delta V = -\int \vec{E} \bullet d\vec{l}$ **to find the potential**
5. **Solve for capacitance**

Capacitance of Concentric Spheres



$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$|\Delta V| = \left| -\int_a^b \vec{E} \bullet d\vec{l} \right|$$

$$= \left| -\int_a^b \frac{Q}{4\pi\varepsilon_o r^2} dr \right|$$

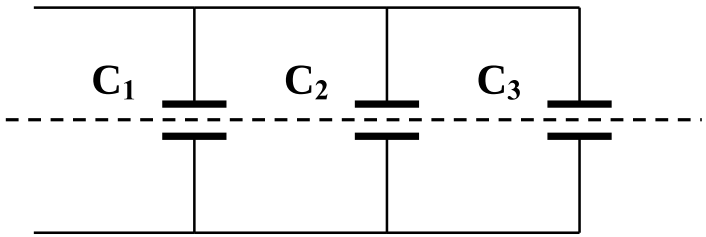
$$|\Delta V| = \left| \frac{Q}{4\pi\varepsilon_o r} \right|_a^b = \frac{Q}{4\pi\varepsilon_o} \left| \frac{1}{b} - \frac{1}{a} \right|$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_o}\left|\frac{1}{b} - \frac{1}{a}\right|} = \boxed{\frac{4\pi\varepsilon_o}{\left|\frac{1}{b} - \frac{1}{a}\right|}}$$

1. **C = Q / V**
2. **Assume a charge**
3. **Use Gauss's Law to find the E-field**
4. **Use** $\Delta V = -\int \vec{E} \bullet d\vec{l}$ **to find the potential**
5. **Solve for capacitance**

Capacitors in Parallel

Capacitors in parallel each have the same potential difference



$$Q_{\text{total}} = \sum_i Q_i = C_1V_1 + C_2V_2 + C_3V$$

$$\sum_i Q_i = V\left(C_1 + C_2 + C_3\right)$$

$$C_1 = \frac{Q_1}{V_1},\ C_2 = \frac{Q_2}{V_2},\ C_3 = \frac{Q_3}{V_3}$$

$$\sum_i Q_i = V\sum_i C_i = VC_{\text{eq}}$$

$$\boxed{C_{\text{eq, parallel}} = \sum_i C_i}$$

Conductors are *equipotential surfaces.*

$$V_1 = V_2 = V_3$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

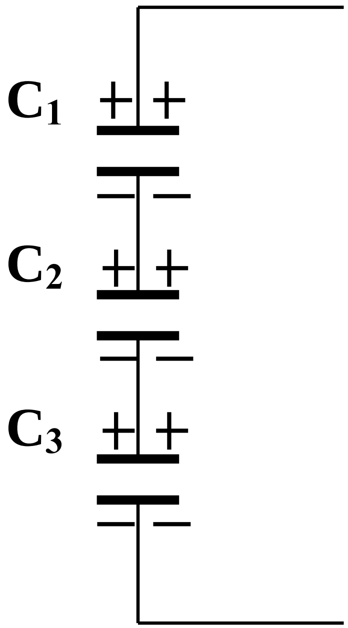
Why put capacitors in parallel?

$$U = \frac{1}{2}CV^2$$

When *C* increases, *U* also increases (for a given *V*)

Capacitors in Series

Capacitors in series each store the same charge.



$$C_1 = \frac{Q_1}{V_1}$$

$$C_2 = \frac{Q_2}{V_2}$$

$$C_3 = \frac{Q_3}{V_3}$$

$$V_{\text{total}} = \sum_i V_i = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

***Q*'s in series are the same**

$$V_{\text{total}} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = Q\sum_i \frac{1}{C_i}$$

$$V_{\text{total}} = \frac{Q}{C_{\text{eq}}}$$

$$\boxed{\frac{1}{C_{\text{eq, series}}} = \sum_i \frac{1}{C_i}}$$

Energy Stored in Capacitors

$$C = Q/V \qquad dU = V\,dq = \frac{q}{C}\,dq$$

$$U = \int dU = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

Consider the Parallel-Plate Capacitor:

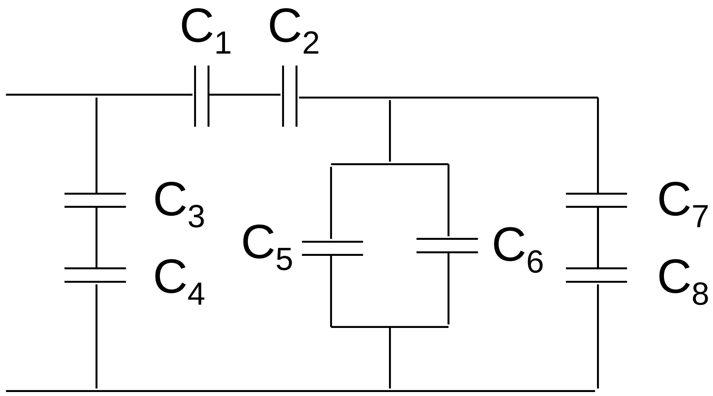
$$\Delta V = -\int \vec{E} \bullet d\vec{\ell}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_o A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_o E^2 (Ad)$$

$$u_E = \frac{\text{energy}}{\text{volume}} = \frac{1}{2}\varepsilon_o E^2$$

Energy Density: True for any electric field

Arbitrary Combinations of Capacitors

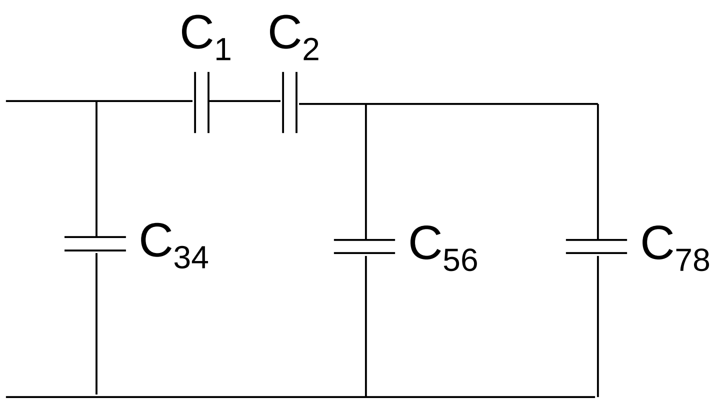


Find a convenient starting point and start making combinations.

$$C_5 \parallel C_6 \Rightarrow C_{56} = C_5 + C_6$$

$$C_3 \text{ series } C_4 \Rightarrow C_{34} = \left(\frac{1}{C_3} + \frac{1}{C_4} \right)^{-1}$$

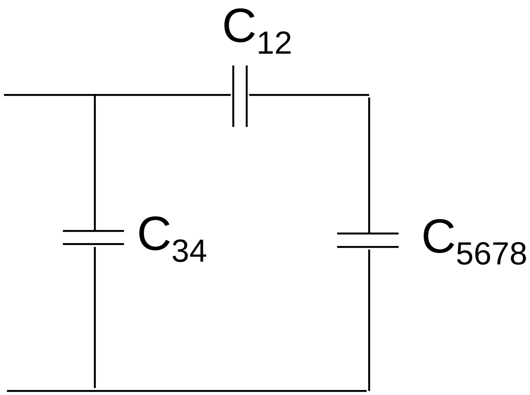
$$C_7 \text{ series } C_8 \Rightarrow C_{78} = \left(\frac{1}{C_7} + \frac{1}{C_8} \right)^{-1}$$



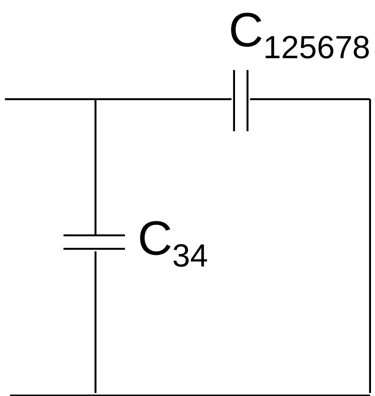
$$C_1 \text{ series } C_2 \Rightarrow C_{12} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$$

$$C_{56} \parallel C_{78} \Rightarrow C_{5678} = C_{56} + C_{78}$$

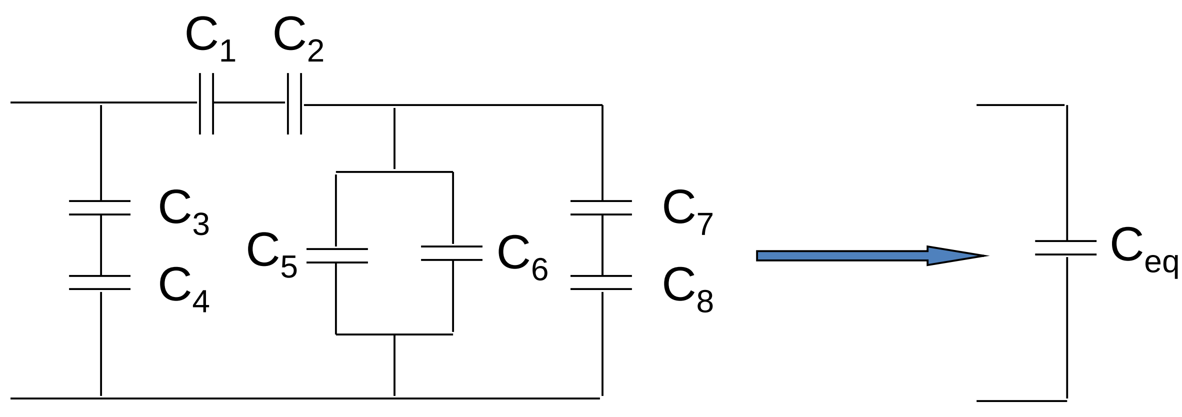
Arbitrary Combinations of Capacitors



$$C_{12} \text{ series } C_{5678} \Rightarrow C_{125678} = \left(\frac{1}{C_{12}} + \frac{1}{C_{5678}} \right)^{-1}$$



$$C_{34} \parallel C_{125678} \Rightarrow C_{12345678} = C_{34} + C_{125678} = C_{\text{eq}}$$



Plugging in numbers early is wise here