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1. a. Not a proposition
b. Not a proposition
c. Yes, a proposition. False, there are black flies in Maine.
d. Not a proposition because unknown truth value.
e. Yes, a proposition. False, the moon isn't made of green cheese.
f. Not a proposition because unknown truth value.

2. a. I did not buy a lottery ticket this weekend.
b. I bought a lottery ticket this weekend or I won the million dollar jackpot.
c. If I bought a lottery ticket this weekend, then I won the million dollar jackpot.
d. I bought a lottery ticket this weekend and I won the million dollar jacket.
e. I bought a lottery ticket this weekend if and only if I won the million dollar jackpot.
f. If I did not buy a lottery ticket this weekend, then I did not win the million dollar jackpot.
g. I did not buy a lottery ticket this weekend and I did not win the million dollar jackpot.
h. $\neg P \vee (P \wedge Q) \rightarrow (\neg P \vee P) \wedge (\neg P \vee Q) \rightarrow \neg P \vee Q$
I did not buy a lottery ticket this weekend or I won a million dollar jackpot.

3. a.

P	¬P	P → ¬P
T	F	F
F	T	T

b.

P	¬P	P ↔ ¬P
T	F	F
F	T	F

- Somedody loves everybody.
  - $(\exists x)[(\forall y)L(x,y)]$
  - $\neg[(\exists x)[(\forall y)L(x,y)]] \iff (\forall x)\neg[(\forall y)L(x,y)] \iff$
    $(\forall x)(\exists y)\neg L(x,y)$
  - Everybody do not love somebody.
- Socrates loves nobody.
  - $(\forall x)\neg L(s,x)$ (s: socrates)
  - $\neg\neg(\forall x)\neg L(s,x) \iff (\exists x)L(s,x)$
  - Socrates loves somebody.
- Somebody who isn't Socrates loves Socrates
  - $(\exists x)[NE(x,s) \rightarrow L(x,s)]$
  - $\neg(\exists x)[NE(x,s) \rightarrow L(x,s)] \iff (\forall x)\neg[NE(x,s) \rightarrow L(x,s)]$
    $\iff (\forall x)NE(x,s) \wedge \neg L(x,s)$
  - Nobody loves Socrates except himself.
- Everybody loves somebody who isn't themselves.
  - $(\forall x)[(\exists y)(NE(x,y) \rightarrow L(x,y))]$
  - $\neg[(\forall x)[(\exists y)(NE(x,y) \rightarrow L(x,y))]] \iff$
    $(\exists x)\neg[(\exists y)[NE(x,y) \rightarrow L(x,y)]] \iff$
    $(\exists x)(\forall y)NE(x,y) \wedge \neg L(x,y)$
  - Somebody loves nobody who isn't themselves.
- There are at least two people who love each other.
  - $(\exists x)[(\exists y)[NE(x,y) \wedge L(x,y)]]$
  - $\neg(\exists x)[[\exists y)[NE(x,y) \wedge L(x,y)]] \iff$
    $(\forall x)[(\forall y)\neg[NE(x,y) \wedge L(x,y)]] \iff$
    $(\forall x)[(\forall y)[\neg NE(x,y) \vee \neg L(x,y)]]$
  - Everybody loves nobody except himself
- There are at least two people who love each other and no one else.
  - $(\exists x)[(\exists y)[NE(x,y) \wedge L(x,y)]]$
  - $(\forall x)[(\forall y)[\neg NE(x,y) \vee \neg L(x,y)]]$
  - Everybody loves nobody, except himself.

7. Idk what it's asking.

8. Yes, theoretically it can. The boolean functions is given an answer as ⊤, ⊥, ∘. Boolean functions also give true or false values like ⊤, ⊥, ∘.

9. $(\neg P \wedge (P \to Q)) \to \neg Q$

P	Q	¬P	¬Q	P → Q	¬P ∧ (P → Q)	(¬P ∧ (P → Q)) → ¬Q
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Not a tautology: P = F, Q = T to make false

$(\neg Q \wedge (P \to Q)) \to \neg P$

P	Q	¬P	¬Q	P → Q	¬Q ∧ (P → Q)	(¬Q ∧ (P → Q)) → ¬P
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Tautology

First Order Predicate Logic

- Everybody loves somebody.
  - $(\forall x)[(\exists y) L(x,y)]$
  - $\neg[(\forall x)[(\exists y) L(x,y)]] \iff (\exists x) \neg[(\exists y) L(x,y)] \iff (\exists x)(\forall y) \neg L(x,y)$
  - Somebody do not love anybody.

c.

P	Q	P ∨ Q	P xor (P ∨ Q)
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d.

P	Q	P ∧ Q	P ∨ Q	(P ∧ Q) → (P ∨ Q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

e.

P	Q	¬P	Q → ¬P	P <=> Q	(Q → ¬P) <=> (P <=> Q)
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

f.

P	Q	¬Q	P <=> Q	P <=> ¬Q	(P <=> Q) xor (P <=> ¬Q)
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

4. a. ¬P b. F c. ¬P ∧ Q
   d. T e. ¬P ∧ (P <=> Q) f. T

5. & 6.

True	A ∨ ¬A	A NAND B	¬(A ∧ B)
False	A ∧ ¬A	A NOR B	¬(A ∨ B)
Identity(A)	A	A → B	¬A ∨ B
Identity(B)	B	Not(A → B)	¬(¬A ∨ B)
Flip(A)	¬A	B → A	¬B ∨ A
Flip(B)	¬B	Not(B → A)	¬(¬B ∨ A)
A AND B	A ∧ B	A <=> B	(A ∧ B) ∨ (¬A ∧ ¬B)
A OR B	A ∨ B	A XOR B	(¬A ∧ B) ∨ (A ∧ ¬B)