Ryan Pollack

1. Given $(P \vee Q) \wedge (\neg P \vee R) \rightarrow (Q \vee R)$

$\equiv \neg[(P \vee Q) \wedge (\neg P \vee R)] \vee (Q \vee R)$ Law of Implies

$\equiv \neg[Q \vee (P \wedge \neg P) \vee R] \vee (Q \vee R)$ Commutative Law

$\equiv \neg[Q \vee (F) \vee R] \vee (Q \vee R)$ $A \wedge \neg A = F$

$\equiv \neg[Q \vee R] \vee (Q \vee R)$ $A \vee F = A$

$\equiv \neg Q \wedge (\neg R \vee R) \vee Q$ DeMorgan's Law

$\equiv (\neg Q \wedge Q) \vee (\neg R \vee R)$ Commutative Law

$\equiv F \vee T$

$\equiv T$

Tautology

2. $P \wedge (P \rightarrow Q)$

$\equiv P \wedge (\neg P \vee Q)$ Law of Implies

$\equiv (P \wedge \neg P) \vee Q$ Associative Law

$\equiv F \vee Q$ $A \vee F = A$

$\equiv Q$

3. Because the more tools/rules we know, the easier it will be to solve the problem. It may be true the resolution inference rule can be utilized for all of these, but some other rules will be quicker and easier to use depending on the problem.

4. i. Not a valid argument

If a^2 is positive where a is real number, then a could be positive or negative.

If $a = 5 \rightarrow (5)^2 = 25$

If $a = -5 \rightarrow (-5)^2 = 25$

$(5)^2$ is positive while 5 is positive, but $(-5)^2$ is also positive while -5 is negative.

ii. A valid argument
Let a be real number. If $a^2 \neq 0$, then $a \neq 0$ because
$a^2 = a \cdot a$
If a isn't 0 then multiplying it by itself can't be 0.
a^2 only equals 0 if a equals 0.

5. $\forall x[P(x) \to Q(x)]$ Premise 1
$\forall x[Q(x) \to R(x)]$ Premise 2
$P(c) \to Q(c)$ Universal instantitation from 1
$Q(c) \to R(c)$ Universal instantiation from 2
$P(c) \to R(c)$ Hypothetical syllogism from 3 and 4
$\forall x[P(x) \to R(x)]$ Universal generalization from 5
Therefore, if $\forall x: P(x) \to Q(x)$ and $\forall x: Q(x) \to R(x)$ are true, then $\forall x: P(x) \to R(x)$ is true.

6. We need to flip cards A and 7.
- Card A must have an even number for claim to be true so we check that.
- Card 7 must not have a vowel for claim to be true so we check that.
- Card B can be even or odd so always true.
- Card 4 can be vowel or consonant so always true.

1. We solve this by using contradiction because I'm the most comfortable with it and disproving both being odd is easiest.

Suppose m is odd and n is odd

Then $m = 2x+1$ and $n = 2y+1$ for some integers x and y.

Then $mn = (2x+1)(2x+1) = 4xy + 2x + 2y + 1 = 2(2xy + x + y) + 1$

If we have $z = 2xy$, then z is even

$mn = 2(z + x + y) + 1$

Let $p = x + y + z$, then p is even.

$mn = p + 1$

So, $p+1$ is an odd integer, but we started with the assumption that mn is even. This is a contradiction and means either m is even and n is odd, m is odd and n is even, or both m and n are even.

Therefore, either m is even or n is even.

2. We solve by contradiction b/c disproving n doesn't always divide by 3, if n^2 does is easier.

Suppose n doesn't divide by 3.

Then $n = 3x - 1$ or $n = 3x - 2$ for some integer x.

Case 1: $n = 3x - 1$

Then $n^2 = (3x-1)(3x-1) = 9x^2 - 6x + 1$

Then $(9x^2 - 6x + 1)$ isn't divisible by 3 because it's result of $3x^2 - 2x + \frac{1}{3}$ has 2 integers and a fraction, which isn't an integer.

Case 2: $n = 3x - 2$

Then $n^2 = (3x-2)(3x-2) = 9x^2 - 12x + 4$

$9x^2 - 12x + 4$ also isn't divisible by 3 because it's result of $3x^2 - 4x + \frac{4}{3}$ has a fraction as well.

But we do know n^2 is divisible by 3 so we have our contradiction.

Therefore, n must also be divisible by 3.

For the case involving 4, if $n = 2$, then $2^2 = 4$ which is divisible by 4 but 2 isn't so it would be false.

3. We solve by contradiction b/c disproving $\sqrt{3}$ being rational is easier.
Suppose $\sqrt{3}$ is rational.
Then $\sqrt{3} = \frac{a}{b}$ or $a^2 = 3b^2$, where a and b are integers.
If b is even, then a is even but $\frac{a}{b}$ wouldn't be in simplest form. However, if b is odd, then a is odd. So:
$a = 2m+1$, for some integer m.
$b = 2n+1$, for some integer n.
$(2m+1)^2 = 3(2n+1)^2$
$4m^2+4m+1 = 12n^2+12n+3$
$2m^2+2m = 6n^2+6n+1$
$2(m^2+m) = 2(3n^2+3n)+1$
Since (m^2+m) is an integer, left side is even.
Since $(3n^2+3n)$ is an integer, right side is odd.
We have our contradiction, therefore $\sqrt{3}$ has to be irrational.

4. We solve by contradiction b/c I'm comfortable with it.
Assume there is no hole that holds more than 1 pigeon.
Then, every hole holds at most 1 pigeon.
Let $P = \{P_1, P_2, P_3, ..., P_{101}\}$ be the set representing all the pigeons, where P_n is the nth pigeon.
Let $H = \{H_1, H_2, H_3, ..., H_{100}\}$ be the set representing all the holes, where H_n is the nth hole.
Each H can be paired with 1 P.
So $P \leq H$. However, P is greater than H so this is a contradiction. Therefore, there must be a hole with at least 2 pigeons.

5. If you have N amount of pigeons and M holes for them, but there are more pigeons than holes, at least 1 hole will contain more than 1. If decimals are workable than 1 hole will have at $\frac{N}{M}$, but when working with integers like pigeons 1 hole must have at least 2 since $N > M$.