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1. We know that an odd integer n can be represented in the form of $n = 4k+1$ or $n = 4k+3$ for some integer k. Since we're looking at n^2-1, we can use substitution in 2 cases.

Case 1:

$$
\begin{aligned}
n^2-1 &= (4k+1)^2-1 \\
&= (16k^2+8k+1)-1 \\
&= 16k^2+8k+1 \\
&= 8k(2k+1), \text{ which is divisible by } 8.
\end{aligned}
$$

Case 2:

$$
\begin{aligned}
n^2-1 &= (4k+3)^2-1 \\
&= (16k^2+24k+9)-1 \\
&= 16k^2+24k+8 \\
&= 8(2k^2+3k+1), \text{ which is divisible by } 8.
\end{aligned}
$$

Both odd integer forms substituted into n^2-1 are divisible by 8. Therefore, when n is odd, n^2-1 is divisible by 8.

2. Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in \{0, 1, 2, \dots, M-1\}$. We can write them out as:

$f(x_1) = Ax_1 + B \pmod M$ $\quad$ $f(x_2) = Ax_2 + B \pmod M$

Since they're equal to each other, we write:

$$
\begin{aligned}
Ax_1 + B &\equiv Ax_2 + B \pmod M \\
Ax_1 &\equiv Ax_2 \pmod M \qquad [GCD(A, M) = 1] \\
x_1 &\equiv x_2 \pmod M
\end{aligned}
$$

M divides $x_1 - x_2$

Since $x_1, x_2 \in \{0, 1, 2, \dots, M-1\}$, and M is the largest value, then $x_1 - x_2 < M$

Since $x_1 - x_2$ is smaller than M and $M \mid x_1 - x_2$, then $x_1 - x_2$ must be equal to 0, which means x_1 and x_2 are always equal.

$f(x_1) = f(x_2)$ and $x_1 = x_2$

Therefore, f is injective.

3. Let $f(x_1) = f(x_2)$, where $x_1, x_2 \leftarrow \{0, 1, 2, \dots, M-1\}$

$f(x_1) = Ax_1 + B \pmod M$ $\quad f(x_2) = Ax_2 + B \pmod M$

Since $f(x_1)$ and $f(x_2)$ are equal, then

$Ax_1 + B \equiv Ax_2 + B \pmod M$

$Ax_1 \equiv Ax_2 \pmod M$ $\quad [GCD(A, M) \neq 1]$

M divides $Ax_1 - Ax_2$

M divides $A(x_1 - x_2)$

Separated as A and $(x_1 - x_2)$, we can have 3 results:

1. $M \mid A$ and $M \mid (x_1 - x_2)$, where $x_1 - x_2 = 0$
2. $M \nmid A$ and $M \mid (x_1 - x_2)$
3. $M \mid A$ and $M \nmid (x_1 - x_2)$

With the third one, $M \mid A$ works because $GCD(A, M) \neq 1$. However, for the second part, we've shown how $M \mid x_1 - x_2$ only if $x_1 - x_2 = 0$. Since it's doesn't, then

$x_1 - x_2 \neq 0$ which means x_1 and x_2 aren't always the

same. Then there must be some in $x_1, x_2 \leftarrow \{0, 1, 2, \dots, M-1\}$ where $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.

Therefore, f is not injective.

4. The powerset is the set of all subsets of S.

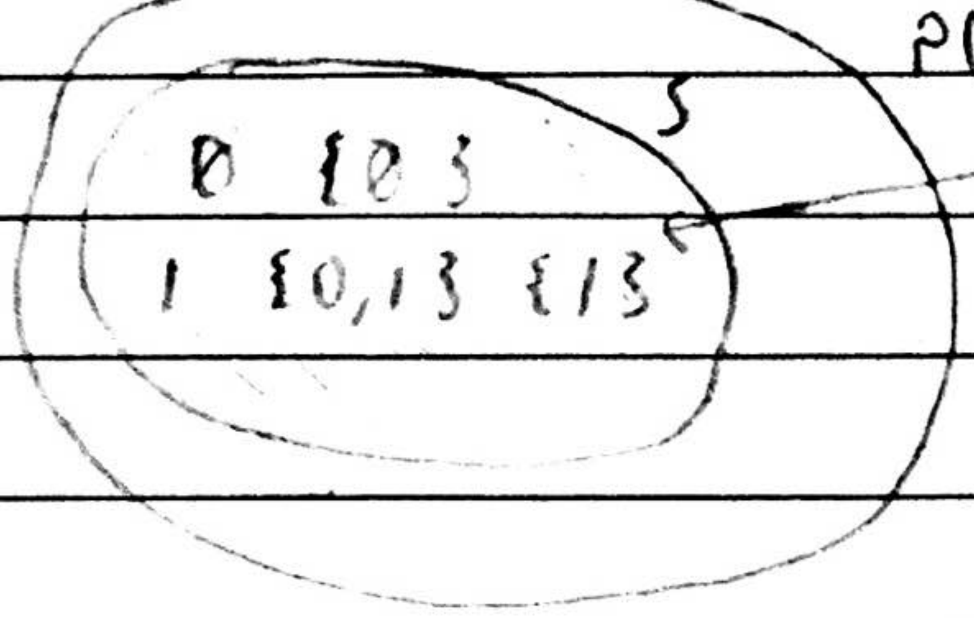
For the subsets within S of $\{\emptyset\}$, $\{1\}$, and $\{0, 1\}$, they should have all 2^n elements. $\quad 2^1 = 2$. $\quad 2^2 = 4$

$S = \{0, \{\emptyset\}, 1, \{0, 1\}, \{1\}\}$

$\{\emptyset, \emptyset, \{\emptyset\}, 1, \emptyset, 1, \{\emptyset\}, \{1\}, 1, \{1\}$ are all the individuals that would than be put in sets together. So, the elements of $P(S)$ are $\emptyset$, 1, $\{\emptyset\}$, and $\{1\}$

↓

For $S \cap P(S)$, the $\cap$ means only the elements that would fit in both S and $P(S)$. The elements of S $(\emptyset, \{\emptyset\}, 1, \{\emptyset, 1\}, \{1\})$ all belong in $P(S)$, so $S \cap P(S)$ is $\{\emptyset, \{\emptyset\}, 1, \{\emptyset, 1\}, \{1\}\}$.



those S are in both circles

5. There is no satisfying assignment. We know A must be true as the first proposition is just A. In the next one, $\neg A \vee B$, either $\neg A$ or B must be true. Since A is true, $\neg A$ is false, meaning B must also be true. For $B \wedge C$, both B and C must be true so C has to be true for the proposition to be true. For $\neg C \vee \neg D$, it's an "or" and $\neg C$ is false so $\neg D$ must be true, meaning D is false. D xor A means 1 true and 1 false, and A is true and D is false so this is true. $C \rightarrow A$, both C and A are true so problem is true. Finally, $\neg B \vee D$, where B is true so $\neg B$ is false and D is false so it doesn't satisfy the "or" where one must be true. Therefore, it doesn't work.

A	$\neg A \vee B$	$B \wedge C$	$\neg C \vee \neg D$	D xor A
A must = T	F	T	F	F T
	B must = T	C must = T	$\neg D$ must = T	✓
			so D is F	

$C \rightarrow A$	$\neg B \vee D$
T T	F F
✓	X

$n \le n^2$ for all $n \ge 1$, $c = 1$

6. a. True

b. True $n(n^2+3n+2)-n^3 = 3n^2+2n \le n^3$ $n \ge 4$ $c = 1$

$3(4)^2+2(4) \to 56 \le 64$

c. True $n(n^2+3n+2)-n^3 = 3n^2+2n \le 3n^2+2n^2$ $n \ge 1$

$3n^2+2n \le 5n^2$ $c = 5$

d. True $n \ln n \le n^2$ $n \ge 0$ $c = 1$

e. False $n^2 \le n \ln n$ $n_0 > n$ n^2 always bigger.

f. True $n \ge 1$ $c = 1$ $\frac{1}{n} \le 1$ $\frac{1}{n}$ shrinks

g. False $1{,}000{,}000 n \le n$ $n_0 > n$

h. True $2^n \le 3^n$ $n \ge 0$ $c = 1$ always bigger after 0

i. False $3^n \le 2^n$ $n_0 > n$

j. True $i = 1$ $(1)(2)(3) = 6$ $i = 2$ $(2)(3)(4) = 24$

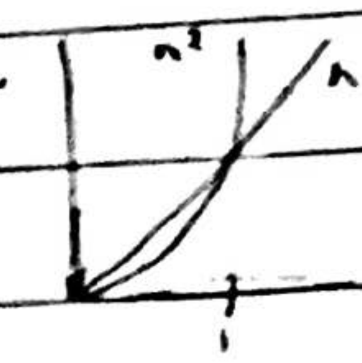
$i = 3$ $(3)(4)(5) = 60$ beneath n^4

Bonus: I should be worried because $O(\ln \sqrt{N + \ln N})$ as function is beneath $O(\ln N)$ slightly. Both big Os are equal when $n = 1$ because $\ln(1) = 0$ and $\ln(\sqrt{1 + \ln(1)}) = \ln(\sqrt{1}) = 0$. Once $n > 1$, my rival's is a bit smaller and therefore more efficient. When $n = 1000$, my rival's is already about twice as quick.

$\ln(n) \le \ln(\sqrt{n + \ln(n)})$ $n \ge 1$ $c = 1$

$\ln(\sqrt{n + \ln n})$ is always smaller when $n \ge 1$.

$n>1$ for all
6. a. True $n \le n^2$ $n>1$
$c=1$



b. True $n(n^2+3n+2)-n^3 = 3n^2+2n \le n^3$ $n>4$
$3(4)^2+2(4) \to 56 < 64$ $c=1$

c. True $n(n^2+3n+2)-n^3 = 3n^2+2n \le 3n^2+2n^2$ $n>1$
$3n^2+2n \le 5n^2$ $c=5$

d. True $n\lg n \le n^2$ $n>0$ $c=1$

e. False $n^2 \le n\lg n$ no $>n$ n^2 always bigger.

f. True $n>1$ $c=1$ $\frac{1}{n} \le 1$ $\frac{1}{n}$ shrinks

g. False $1{,}000{,}000n \le n$ no $>n$

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$\ln(n) \le \ln(\sqrt{n+\ln(n)})$ $n>1$ $c=1$
$\ln(\sqrt{n+\ln(n)})$ is always smaller when $n=1$.