Assignment 2

1.a. Pr[A]:

Prob of not getting 6 each role is $\frac{5}{6}$

$\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{125}{216}$

We want prob of getting at least one 6 so

$1 - \frac{125}{216} = \frac{216}{216} - \frac{125}{216} = \frac{91}{126}$

$Pr[A] = \frac{91}{216}$

Pr[B]:

The total can only be odd or even and there's an equal chance for both so $\frac{1}{2}$.

$Pr[B] = \frac{1}{2}$

Pr[B|A]:

Probability of "odd total" given "at least one 6"

A 6 is an even number. So, if we were to get at least one 6, the other 2 dice could be:

2 evens (6s or not) → 3 evens so even X

2 odds (no other 6s) → 2 odds = even + even is even X

1 odd and 1 even (6 or not) → 2 evens + 1 odd is odd. ✓

$Pr[B|A] = \frac{1}{3}$

Pr[A|B]:

$$\frac{Pr[A \cap B]}{Pr[B]} = \frac{\frac{1}{9}}{\frac{1}{2}} \rightarrow \frac{1}{9} \cdot \frac{2}{1} = \frac{2}{9}$$

$Pr[A|B] = \frac{2}{9}$

c. Pr[A]:

5 occurs infinite amount of time is $[5]^{\infty}$, basically 0%.

$Pr[A] = 0$

Pr[B]:

6 eventually occuring or never stops is basically 100%

$Pr[B] = 1$

d. Pr[A]:
5 balls total, 2 out of 5 balls will be picked so 40% chance
$Pr[A] = \frac{2}{5}$
Pr[B]:
Ball isn't put back when picked so impossible to pick twice.
$Pr[B] = 0$

e. Pr[A]:
Probability of not picking 5 is $\frac{4}{5} \cdot \frac{4}{5} = \frac{16}{25}$, so picking 5 at least once is $1 - \frac{16}{25} = \frac{9}{25}$
$Pr[A] = \frac{9}{25}$
Pr[B]:
5 being drawn both times is simply $\frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$
$Pr[B] = \frac{1}{25}$
Pr[B|A]:
Probability of "5 being drawn twice" given "5 is drawn at least once." We know 1 of the 2 turns has to pull a 5, so the other chance is 1 out of the 5 balls.
$Pr[B|A] = \frac{1}{5}$
Pr[A|B]:
Probability of "5 being drawn at least once" given "5 being drawn twice is 100%. Can't be drawn twice without being drawn at least once.
$Pr[A|B] = 1$

2.i. Prob. 6 occurs on 2nd throw. (Missed 6 is $\frac{5}{6}$, hit 6 is $\frac{1}{6}$)
$\frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$

ii. Prob. 6 occurs on 3rd throw.
$\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{25}{216}$

iii. Prob. 6 occurs within the first three throws.

$$\frac{1}{6} + \left(\frac{5}{6}\cdot\frac{1}{6}\right) + \left(\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}\right)$$

first, second, third throw

$$\frac{1}{6} + \frac{5}{35} + \frac{25}{216} = \frac{91}{216}$$

iv. Only be able to land on an odd and skipping over evens makes this an infinite geometric series. If we were to call the probability of getting 6 "a", and getting anything 1-5 "b", we can take the series sum to find the probability. So, with $a = \frac{1}{6}$ and $\underbrace{b = \left(\frac{5}{6}\right)^2}_{\text{(for skip evens)}}$, using $\frac{a}{1-b}$, we get $\frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}}$

$$\frac{1}{6}\cdot\frac{36}{11} = \frac{36}{66} = \frac{6}{11}$$

3. $n=1$, is 0 because there's only one person rolling a die

$n=2$, is $\frac{1}{6}$.

$$\left\{\begin{array}{cccccc} 11 & 12 & 13 & 14 & 15 & 16 \\ 21 & 22 & 23 & 24 & 25 & 26 \\ 31 & 32 & 33 & 34 & 35 & 36 \\ 41 & 42 & 43 & 44 & 45 & 46 \\ 51 & 52 & 53 & 54 & 55 & 56 \\ 61 & 62 & 63 & 64 & 65 & 66 \end{array}\right\} \quad \frac{6}{36} = \frac{1}{6}$$

$n=3$, we start to work backwards. First roll can be anything. Next roll has $\frac{5}{6}$ of not being same. Third roll has $\frac{4}{6}$ of not being same to either roll. So $1\cdot\frac{5}{6}\cdot\frac{4}{6} = \frac{20}{36}$ that all are different. $1 - \frac{20}{36} = \frac{16}{36} = \frac{4}{9}$

$n=3$, is $\frac{4}{9}$.

For $n=4$, we continue where roll number 4 has $\frac{3}{6}$ of being different than them all. So $1\cdot\frac{5}{6}\cdot\frac{4}{6}\cdot\frac{3}{6} = \frac{60}{216}$ that all are different. $1-\frac{60}{216} = \frac{156}{216}$

$n=4$, is $\frac{156}{216}$.

$n=5$, roll number 5 has $\frac{2}{6}$ of being completely different. So $1\cdot\frac{5}{6}\cdot\frac{4}{6}\cdot\frac{3}{6}\cdot\frac{2}{6} = \frac{120}{1{,}296}$, which means $1-\frac{120}{1{,}296} = \frac{1{,}176}{1{,}296}$

$n=5$, is $\frac{1{,}176}{1{,}296}$.

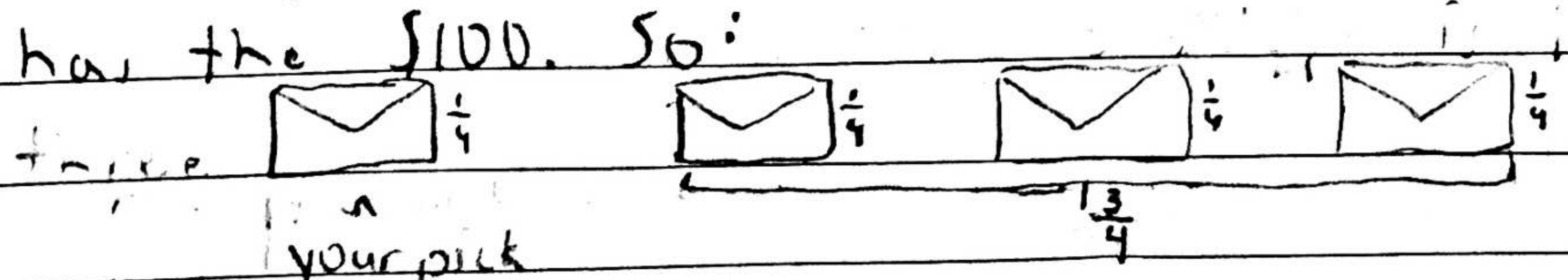
$n = 6$, roll 6 has $\frac{1}{6}$ of being completely different. So $1 \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} = \frac{120}{7{,}776}$ of all 6 being different, $1 - \frac{120}{7{,}776} = \frac{7{,}656}{7{,}776}$
$n = 6$, is $\frac{7{,}656}{7{,}776}$.

$n = 7$ is 1, there are no ways 7 people can roll 7 different numbers on a 6-sided die so at least 2 will be the same.

4. We know $Pr[A \cap B] \geq Pr[A] + Pr[B] - 1$ because the maximum probability of any event happening is 1 (100%). We can take the extreme sides of the case. So if $Pr[A]$ and $Pr[B]$ where both guarenteed to happen, so 1, then $Pr[A \cap B]$ is also 1 so $1 \geq 1 + 1 - 1 = 1 \geq 1$. If both had 0 chance of happening, then $0 \geq 0 + 0 - 1 = 0 \geq -1$. Even if $Pr[A] = 1$ and $Pr[B] = 0$ making $Pr[A \cap B] = 0$, still $0 \geq 1 + 0 - 1 = 0 \geq 0$ so this statement works.

For second part side on the left starts with $i=1$ for A_i while right side is a sequence starting with $i=1$ at A_i and subtracting $(n-1)$ when $n \geq 2$

5. You should return the envelope and select a new one. There is a $\frac{1}{4}$ chance the envelope you pick has the \$100, meaning there's a $\frac{3}{4}$ one of the three remaining envelopes has the \$100. So:



When you remove an envelope, the chances the non-selected contain the envelope is still $\frac{3}{4}$, just condensed into the 2 left. It's 50/50 on the switch, meaning you have a $\frac{3}{8}$ chance when changing to select the right one when switching.

The probability of switching strategy is $\frac{3}{8}$ to get the $100 compared to $\frac{1}{4}$ of just keeping the original so you should always switch for best probability.

6. • $Pr[A \cap B] = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

• $Pr[\bar{A}] = \frac{2}{3}$ $Pr[B] = \frac{1}{2} \rightarrow Pr[\bar{A}] + Pr[B] - Pr[\bar{A} \cap B]$

$Pr[\bar{A} \cup B] = \frac{2}{3} + \frac{1}{2} - \frac{1}{3} = \frac{4}{6} + \frac{3}{6} - \frac{2}{6} = \frac{5}{6}$

• $Pr[\bar{A}] = \frac{2}{3}$ $Pr[B] = \frac{1}{2} \rightarrow Pr[\bar{A} \cap B] = \frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

$Pr[A \cap B] = \frac{1}{6}$ $Pr[\bar{A} \cup B] = \frac{5}{6}$ $Pr[\bar{A} \cap B] = \frac{1}{3}$

Ec.

a. $Pr[A] = \frac{1}{6}$

$Pr[A | B_1] = \frac{1}{6}$

$$\left\{ \begin{matrix} 11 & 12 & 13 & 14 & 15 & 16 \\ 21 & 22 & 23 & 24 & 25 & 26 \\ 31 & 32 & 33 & 34 & 35 & 36 \\ 41 & 42 & 43 & 44 & 45 & 46 \\ 51 & 52 & 53 & 54 & 55 & 56 \\ 61 & 62 & 63 & 64 & 65 & 66 \end{matrix} \right\} \frac{6}{36} = \frac{1}{6}$$

b. We can show $Pr[A | B_i] = Pr[A | B_j]$ based on the necessary pairing to make A true. The only pairs that can make 7 total true is 16, 25, and 34. If i is 4, A only happens if the other is 3. And if j is 3, A only happens if the other is 4 where both others have a $\frac{1}{6}$ chance of happening.