Assignment 7

1. We can use the equation $a_n = a_{n-1} + 2a_{n-2}$.
   First, multiply both sides by z^n and rearrange. So:
   $a_n z^n = a_{n-1} z^n + 2a_{n-2} z^n$
   $a_n z^n - a_{n-1} z^n - 2a_{n-2} z^n = 0$
   For $n \geq 2$, we can put it in sums as:
   $\sum_{n=2}^{\infty} a_n z^n - \sum_{n=2}^{\infty} a_{n-1} z^n - 2\sum_{n=2}^{\infty} a_{n-2} z^n = 0$

   1st term: $\sum_{n=2}^{\infty} a_n z^n = a_2 z^2 + a_3 z^3 + \dots$
   $\rightarrow A(z) - a_0 - a_1 z$
   2nd term: $\sum_{n=2}^{\infty} a_{n-1} z^n = a_1 z^2 + a_2 z^3 + a_3 z^4 + \dots$
   $\rightarrow (A(z) - a_0) z$
   3rd term: $\sum_{n=2}^{\infty} a_{n-2} z^n = a_0 z^2 + a_1 z^3 + \dots$
   $\rightarrow A(z) z^2$
   Now we can substitute these values into the equation with each respective term. So:
   $[A(z) - a_0 - a_1 z] - [(A(z) - a_0) z] - 2[A(z) z^2] = 0$
   $a_0 = a_1 = 1$, so by substituting this we get:
   $[A(z) - 1 - z] - [(A(z) - 1) z] - 2[A(z) z^2] = 0$
   $A(z) - 1 - z - A(z) z + z - 2A(z) z^2 = 0$
   $A(z) - A(z) z - 2A(z) z^2 = 1$
   $A(z) = \dfrac{1}{1 - z - 2z^2} = \dfrac{1}{(1-2z)(1+z)} = \dfrac{2}{3(1-2z)} + \dfrac{1}{3(1+z)}$
   $a_n = \frac{2}{3}(2)^n + \frac{1}{3}(-1)^n$

2. We can use the equation $c_n = c_{n-1} + 2c_{n-2} + 4$.
Like prob. 1, multiply by z^n and rearrange.

$c_n z^n = c_{n-1} z^n + 2c_{n-2} + 4z^n$

$c_n z^n - c_{n-1} z^n - 2c_{n-2} z^n - 4z^n = 0$

Breaking them into 4 terms as sums, we have:

1st term: $\sum_{n=2}^{\infty} c_n z^n = c_2 z^2 + c_3 z^3 + \ldots$
$\longrightarrow C(z) - c_0 - c_1 z$

2nd term: $\sum_{n=2}^{\infty} c_{n-1} z^n = c_1 z^2 + c_2 z^3 + \ldots$
$\longrightarrow (C(z) - c_0) z$

3rd term: $\sum_{n=2}^{\infty} c_{n-2} z^n = c_0 z^2 + c_1 z^3 + \ldots$
$\longrightarrow C(z) z^2$

4th term: $\sum_{n=2}^{\infty} z^n = z^2 + z^3 + \ldots$
$\longrightarrow C(z)$

Now we can substitute these values into the equation with each respective term. So:

$[C(z) - c_0 - c_1 z] - [(C(z) - c_0) z] - 2[C(z) z^2] - 4[C(z)] = 0$

$c_0 = c_1 = 1$, so by substituting this we get:

$[C(z) - 1 - z] - [(C(z) - 1) z] - 2[C(z) z^2] - 4[C(z)] = 0$

$C(z) - 1 - z - C(z) z + z - 2C(z) z^2 - 4C(z) = 0$

$C(z) - C(z) z - 2C(z) z^2 - 4C(z) = 1$

$C(z) = \dfrac{1}{1 - z - 2z^2 - 4} = \dfrac{1}{-2z^2 - z - 3} = \dfrac{1}{-1(2z^2 + z + 3)}$

$C(z) = -\dfrac{1}{(2z^2 + z + 3)}$

3. We can use the equation $b_n = 3b_{n-1} + 4^{n-1}$.
Once again, multiply by z^n and rearrange to get:
$b_n z^n - 3b_{n-1} z^n + 4^{n-1} z^n = 0$
For $n \geq 1$, we write the terms as sums so:

1st term: $\sum_{n=1}^{\infty} b_n z^n = b_1 z^1 + b_2 z^2 + \dots$
$\rightarrow B(z) - b_0 - z_1 =$

2nd term: $\sum_{n=1}^{\infty} b_{n-1} z^n = b_0 z^1 + b_1 z^2 + \dots$
$\rightarrow (B(z) - b_0) z$

3rd term: $\sum_{n=1}^{\infty} 1^{n-1} z^n = \underbrace{1^0 z^1 + 1^1 z^2}_{\text{always } = 1} + \dots$ $\rightarrow z$

Substitute into respective terms:
$[B(z) - b_0 - b_1 z] - 3[(B(z) - b_0) z] + 4[z] = 0$
$b_0 = 1$ so:
$[B(z) - 1 - b_1 z] - 3[(B(z) - 1) z] + 4[z] = 0$
$B(z) - 1 - z - 3B(z)z + 3z + 4z = 0$
$B(z) - 3zB(z) = 1 + 6z$
$B(z) = \frac{1 - 6z}{1 - 3z}$