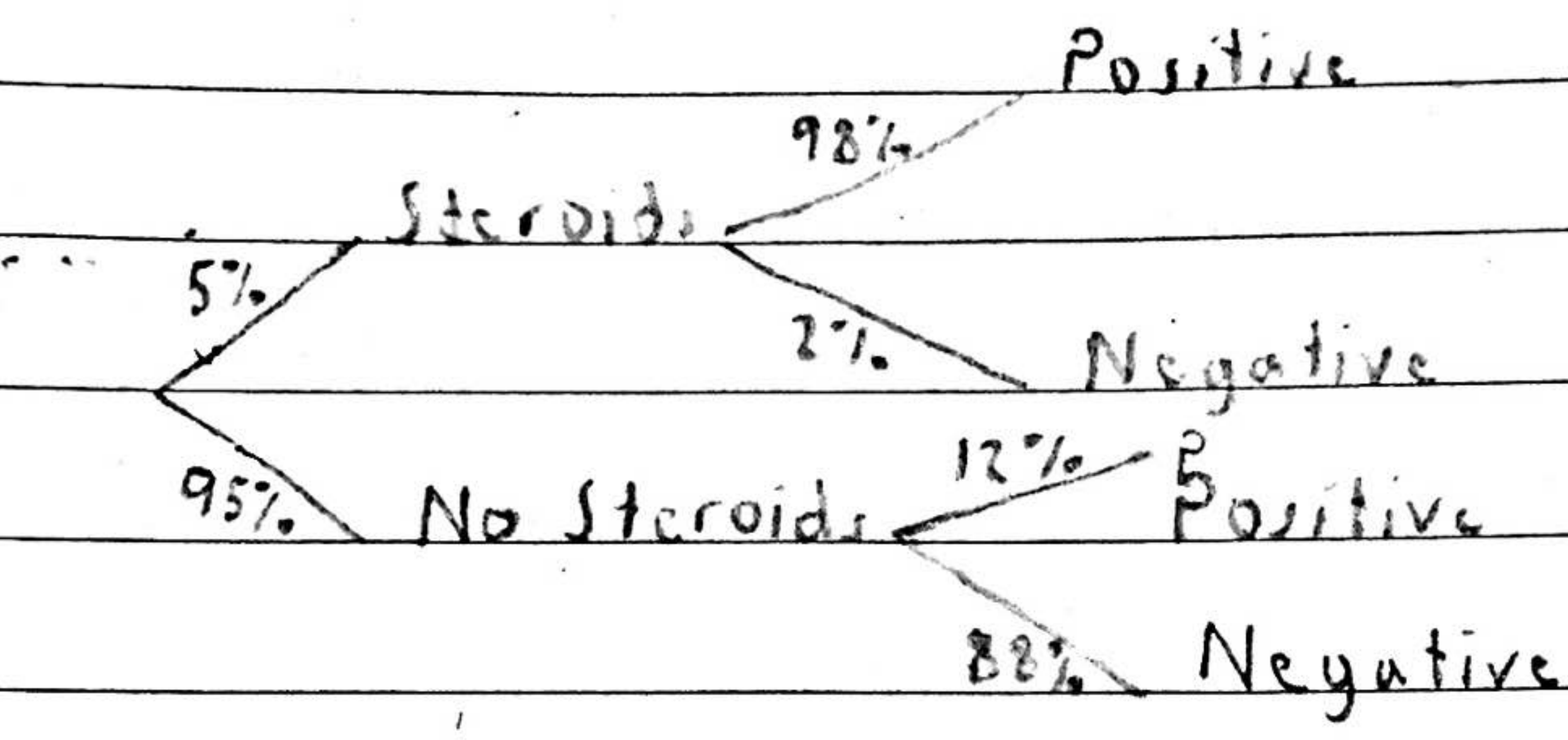
Assignment 3

1.



A: Joe tests positive
B: Joe is taking steroids
What is Pr[B | A]?

$$Pr[B \mid A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{(.98 \times .05)}{(.98 \times .05 + .12 \times .95)}$$

$$= \frac{.049}{.163} = .30 = 30\% \text{ chance}$$

2. $$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \rightarrow 2n! \approx \sqrt{4\pi n}\left(\frac{2n}{2e}\right)^n \rightarrow (n!)^2 \approx 2\pi n\left(\frac{n}{e}\right)^{2n} \rightarrow$$

$$\left[\frac{2n!}{(n!)^2}\right] \approx \frac{\sqrt{4\pi n}}{2\pi n}\left(\frac{2n^{2n}}{e^{2n}}\right)\left(\frac{e^{2n}}{n^{2n}}\right) \approx \binom{2n}{n} \approx e^{en} \cdot \frac{\sqrt{4\pi n}}{\sqrt{2\pi n}\cdot\sqrt{2\pi n}} \rightarrow$$

$$\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}} \rightarrow \binom{2n}{n} \approx \frac{4^n}{\sqrt{\pi n}}$$ so it's shown.

3. Between 10^6 (1,000,000) and 10^7 (10,000,000) there are 7 digits to be fills. The first digit can't be 0, so 1-9 or 9 possibilities. The second digits can't be last one chosen, but now 0 is a possibility, so still 9. Then 3rd slot 8 choices, 4th slot 7 and so on:

$$\underset{1}{9} \times \underset{2}{9} \times \underset{3}{8} \times \underset{4}{7} \times \underset{5}{6} \times \underset{6}{5} \times \underset{7}{4}$$

= 544,320 elements

4. Total numbers in range is $10^7 - 10^6 = 9{,}000{,}000$ (or 9×10^6).
Two consecutive digits can't be the same, so:
  1st slot can be 1-9 (9 digits)
  2nd slot can be 0-9 minus whatever prev. num (9)
  3rd slot 0-9 minus whatever prev. num (9)
  4th slot 0-9 minus whatever prev. num (9)

  ⋮

$= 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^7$

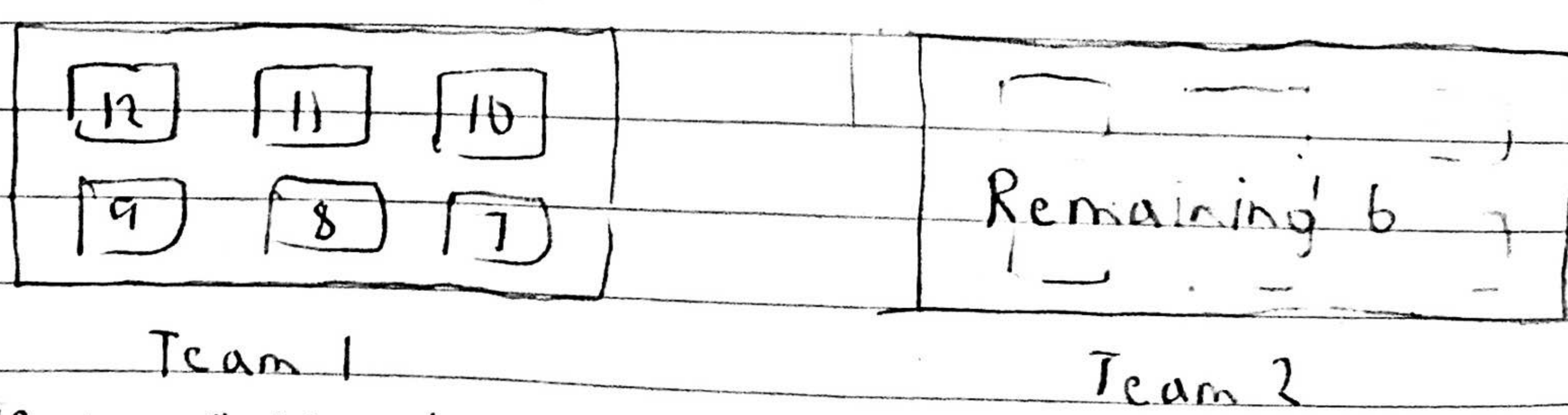
$P[A] = \frac{\text{num of } A}{\text{total}} = \frac{9^7}{9 \cdot 10^6} = .9^6 \approx .531 \approx 53\%$

5.i. Heterosexual pairings is n^2
ii. Alternating sexes is n^3

$M_1 \quad F_1 \quad n=2 \longrightarrow M_1 F_1 M_2 F_2$
$M_2 \quad F_2 \qquad\qquad M_1 F_2 M_2 F_1 \quad \times 2$ start with Fs
$M_2 F_1 M_1 F_2$ so 8
$M_2 F_2 M_1 F_1 \quad n^3 \to (2)^3 = 8$

Arranged around a table is n.

6.



12 available players, 6 spots on team 1. Order doesn't matter. Team 2 falls into place after getting 6 for team 1.

$\binom{n}{k} = \frac{n!}{k!(n-k)!} \longrightarrow \binom{12}{6} = \frac{12!}{6!(6!)} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{6!}}$

$\frac{665{,}280}{720} = 924$ combos

7. $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$ $n=50$ $x^{18} \rightarrow k=18$ $y^{32} \rightarrow n-k=32$

This means $\binom{n}{k} = \binom{50}{18}$ so:

$$\frac{50!}{18! \cdot (50-18)!} = \frac{50!}{18! \cdot 32!} = 1.805 \times 10^{13}$$

8. $\left.\begin{matrix}(x+y+z)^n \\ x^p y^q z^r\end{matrix}\right] \longrightarrow \frac{n!}{p! \cdot q! \cdot r!}$ $n=75$ $p=18$ $q=32$ $r=25$

$$\frac{75!}{18! \cdot 32! \cdot 25!} = 9.4941 \times 10^{32}$$

For monomials, we have $(x+y+z)^n = \binom{n+r-1}{n}$ where $n=75$ and $r=3$ from 3 terms. So:

$$\binom{75+3-1}{75} = \binom{77}{75}$$

$$\frac{77!}{75!(77-75)!} = \frac{77!}{75! \cdot 2!} = \frac{77 \cdot 76 \cdot 75!}{75! \cdot 2!} = \frac{77 \cdot 76}{2} = 2{,}926$$

9. 10 left gloves (L), 10 right gloves (R)

Only way this wouldn't satisfy if pick all Ls or Rs

Alls Ls probability: All Rs probability:

$\frac{10}{20} \cdot \frac{9}{19} \cdot \frac{8}{18} \cdot \frac{7}{17} \cdot \frac{6}{16}$ $\frac{10}{20} \cdot \frac{9}{19} \cdot \frac{8}{18} \cdot \frac{7}{17} \cdot \frac{6}{16}$

$\approx .0163$ $\approx .0163$

So, $1 - .0163 - .0163 = .9674 = 96.74\%$

10.i. First try $= \frac{1}{10}$

Second try $= \frac{9}{10} \cdot \frac{1}{9} = \frac{9}{90} = \frac{1}{10}$

J^{th} try $= \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{7}{8} \cdot \frac{6}{7} \cdot \frac{5}{6} \ldots = \frac{1}{10}$

ii. With replacement

Second try $= \frac{9}{10} \cdot \frac{1}{10} = \frac{9}{100}$

Third try $= \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{1}{10} = \frac{81}{1{,}000}$