Assignment 1

1. To prove by induction, we can start by evaluating when $n=1$. So:

$$\overline{\bigcup_{i=1}^{n} A_i} \to \overline{\bigcup_{i=1}^{1} A_i} \to \overline{A_i} \qquad \bigcap_{i=1}^{n} \overline{A_i} \to \bigcap_{i=1}^{1} \overline{A_i} \to \overline{A_i}$$

$$\overline{\bigcup_{i=1} A_i} = \overline{A_i} = \bigcap_{i=1} \overline{A_i}$$

With this, we've proved it's true when $n=1$. Now, suppose $n=k$. Then:

$$\overline{\bigcup_{i=1}^{k} A_i} = \bigcap_{i=1}^{k} \overline{A_i}$$

Now, we can prove by induction by showing it's true when $n=k+1$ as well. So:

$$\begin{aligned}\overline{\bigcup_{i=1}^{k+1} A_i} &\to \overline{\bigcup_{i=1}^{k} \cup A_{k+1}} \\ &= \overline{\bigcup_{i=1}^{k} A_i} \cap \overline{A_{k+1}} \\ &= \bigcap_{i=1}^{k} \overline{A_i} \cap \overline{A_{k+1}} \\ &= \bigcap_{i=1}^{k+1} \overline{A_i}\end{aligned}$$

We've shown $\overline{\bigcup_{i=1}^{k+1} A_i} = \bigcap_{i=1}^{k+1} \overline{A_i}$, which means it's true when $n=k+1$ as well. Therefore, by induction, it's true for all $n \in \mathbb{N}$.

2. a. $S = \{(x, y, z);\ x = 1, 2, 3, 4, 5, 6;\ y = 1, 2, 3, 4, 5, 6;\ z = 1, 2, 3, 4, 5, 6\}$

b. $S = \{$H, TH, TTH, TTTH, TTTTH, TTTTTH, TTTTTTH, TTTTTTTH, TTTTTTTT$\}$ "heads" = H "tails" = T

c. $S = \{$ it's $[\frac{1}{6}]^{\infty}$ because it technically never has to land on a 6 and end. Each role produces a $\frac{1}{6}$ of landing on 6 $\}$

d. $S = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

e. $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)\}$

3. a. Finite and countable. 3 die, each in range between 1-6, so $6 \times 6 \times 6 = 216$.

b. Finite and countable. Stopping point when flip H or after 8th turn regardless of result, so max is 9 combos since one H ends it.

c. Countable and infinite. We roll a die repeatedly until it lands on a 6. It's doesn't ever have to end so it's infinite, but each roll is countable by 1 so it's countable.

d. Finite and countable. Two picks taken at the same time out of 5, so 10 possible outcomes total.

e. Finite and countable. Two picks, both 5 options since we put it back. So $5 \times 5 = 25$ different potential outcomes.

4. a. $A \cup B \rightarrow$ At least 1 die must be 6 or 3 die add to odd total. $\frac{1}{6}$ chance of getting a 6 per die so $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. Odd total is $\frac{1}{2}$ chance. $\{1,1,6 \ldots\} \cup B$

$A \cap B \rightarrow$ Both at least 1 die is a 6 and the die add up to even must happen. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ for each die and $\frac{1}{2}$ chance for odd. $\{1, 1, 6 \dots\} \cap B$

b. $A \cup B \rightarrow$ Head occurs twice or tails occurs twice.
Heads occuring twice isn't possible, stops after first head.
Tails occurs twice in one scenerio if third is head.
A not possible so B is only, {TTH}.
$A \cap B \rightarrow$ Head occurs twice (A) isn't possible, so $A \cap B$ can't happen.

c. $A \cup B \rightarrow$ 5 occurs infinite amount or a 6 occurs
A 5 happening everytime is $[\frac{1}{5}]^{\infty}$, $\{\frac{1}{5}^{\infty}\} \cup B$
$A \cap B \rightarrow$ It's not physically possible for a die to roll a 5 every single time and eventually a 6, so not possible.

d. $A \cup B \rightarrow$ 5 is drawn at least once or 5 is drawn twice.
Since we don't put the ball back and it's 1, 2, 3, 4, 5, we can also draw 5 once so B not possible. So only A.
{(1, 5), (2, 5), (3, 5), (4, 5)}
$A \cap B \rightarrow$ 5 can't be drawn twice so not possible.

e. $A \cup B \rightarrow$ 5 is drawn at least once or 5 is drawn twice.
$A \rightarrow$ {(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), ...}
$B \rightarrow$ {(5, 5)}. B's sample space is covered in A.
$A \cap B \rightarrow$ For both to satisfy, and 5 be drawn at least once or drawn twice, it can only work if 5 is drawn twice. So {5, 5}, or just B being true is needed.