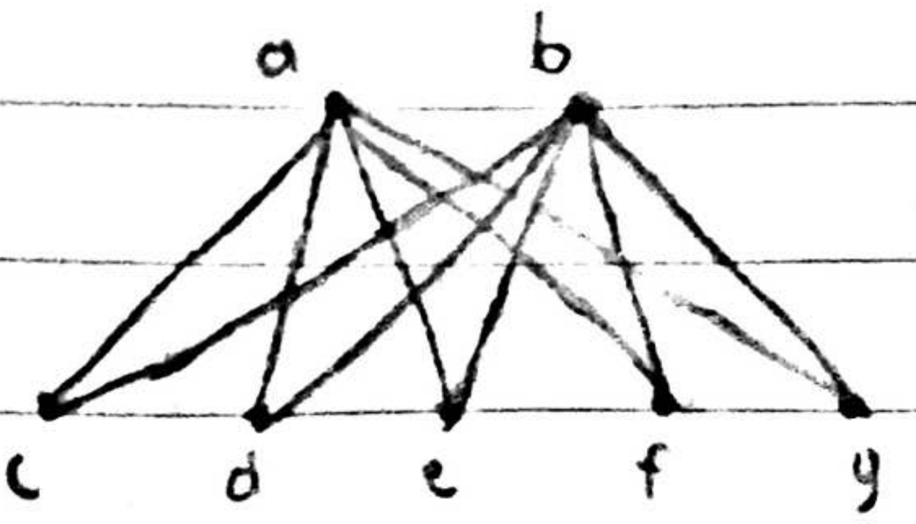
Assignment 9

1. a.



$A = \{a, b, c, d, e\}$ Yes
$B = \{f, g\}$

b. Has no Euler tour because degree of vertices is odd.

c. Eulerian path if one of m or n is odd.

d. Euler tour if both m and n are even.

2. G is simple so no vertex of G can be degree n. So if $V = \{u_1, u_2 \dots u_n\}$, then $0 \leq \deg(u_i) \leq n-1$ and $\deg(u_i) \leq n-2$ for all i if $\deg(u_i) = 0$. So, $0 \leq \deg(u_i) \leq n-2$ for $n \geq 2$ which means $|X| \leq n-1 < n$ if $\deg(u_i) \geq 1$. Then:

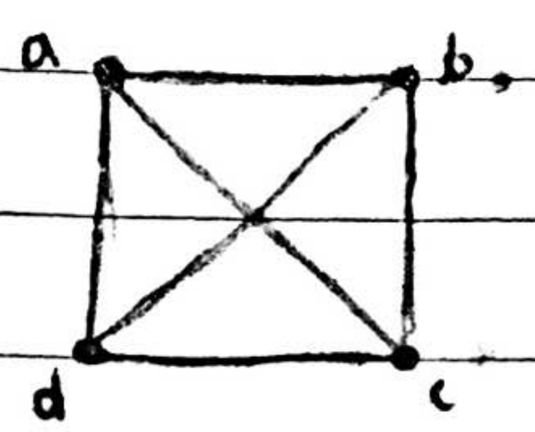
$1 \leq \deg(u_i) \leq n-1$, so $|X| \leq n-1 \leq n$ ✓

This means that $|X| \neq n$.

3. When all the vertices have their own distinct degree, we need n distinct numbers assigned to them. As stated in problem 2 where $0 \leq \deg(u_i) \leq n-2$ or $1 \leq \deg(u_i) \leq n-1$, we have $n-1$ numbers at max to assign a degree to n vertices. With pigeon-hole principle, that means we must have at least 2 vertices (u and v) where $u \neq v$ but $\deg(u) = \deg(v)$.

If we have self-loops, we can do

where $\deg(1) = 1$ and $\deg(2) = 2$ so each vertice has a distinct degree.

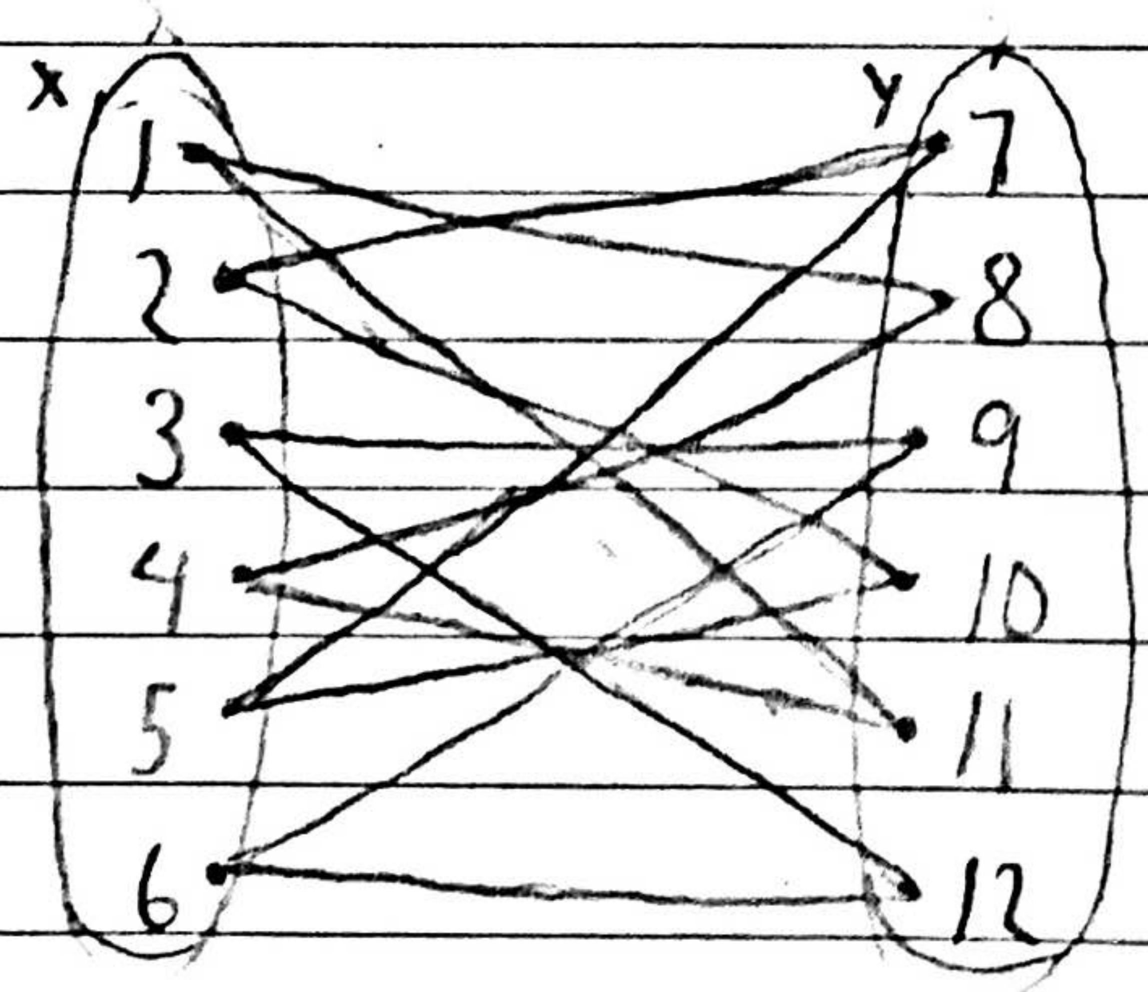
4. 

5. If there are no self-loops, then no it's not possible. If there are 37 vertices and 3 edges touching each, that 111 (37 × 3) points of connection. Since there needs to be an even amount for completion (because 2 points of connection on each edge), there will be one edge stranded on only one vertex so no.

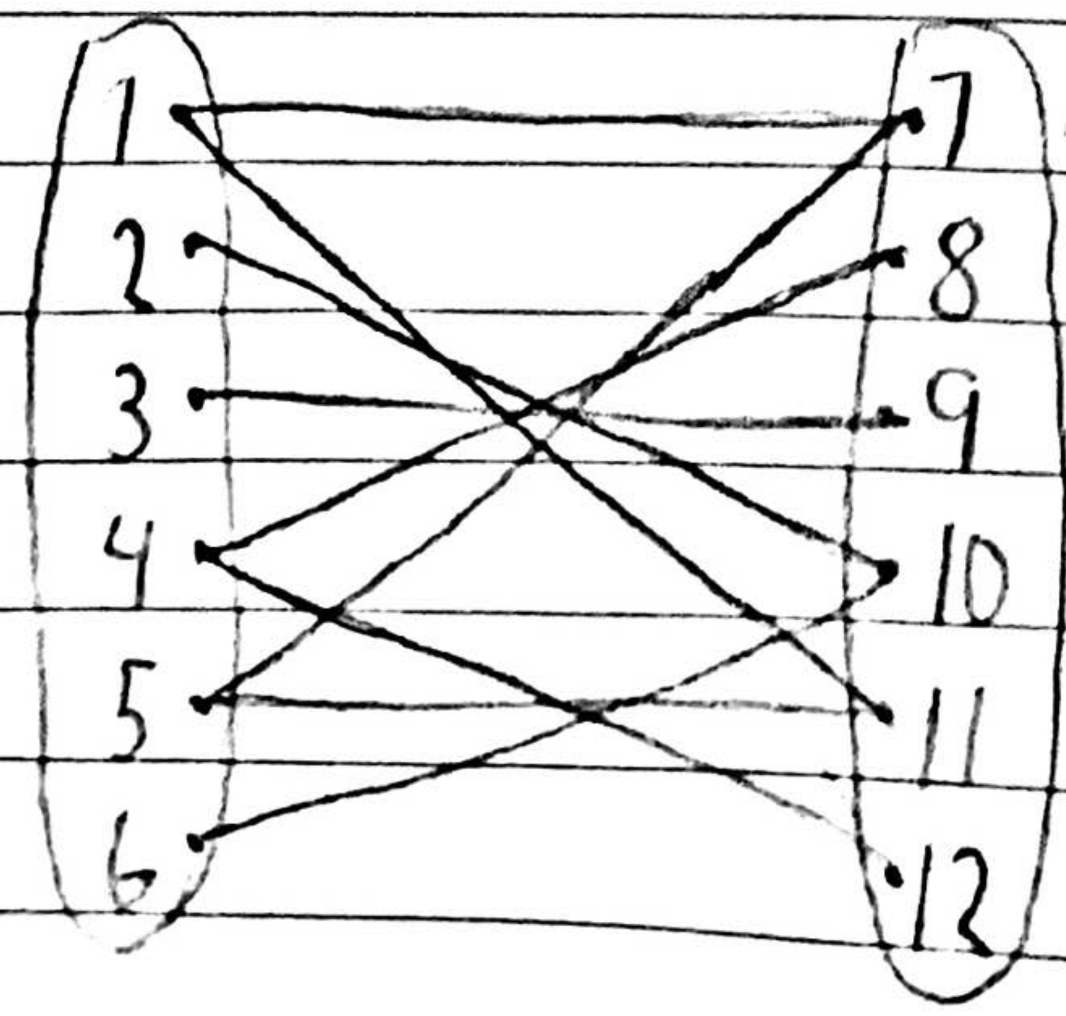
6. 55%

1 2 3 4 5 ... n

Greater chance of odd because it starts at 1.

7. 

a. N({2}) = {7, 10} = 2
b. N({2, 5}) = {7, 10} = 2
c. N({2, 6}) = {7, 10, 9, 12} = 4
d. Yes

8. 

a. N({2}) = {10} = 1
b. N({2, 5}) = {10, 7, 11} = 3
c. N({2, 6}) = {10} = 1
d. Yes