Assignment 4

1. 20 gloves total, 10 matching pairs mixed. When I pull 1 glove, there's only one left out of all the remaining that matches. So do reverse and check probability after picking first that next chosen 'doesn't' match, so:

$$1 \cdot \frac{18}{19} \cdot \frac{17}{18} \cdot \frac{16}{17} \cdot \frac{15}{16} = .7895$$ chance no matches, so

$1 - .7895 = .2105$ or 21.05% probability of a match

2. 2 match in 56 from 5 tries

1 match in 46 from 1 tries

$\binom{56}{5}$ but only 2 needed so:

$$\frac{56 \cdot 55}{5!} = \frac{56 \cdot 55}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{1}{25\frac{2}{3}} = \frac{3}{77}$$ for 2 matches in 56 pool

$$\frac{3}{77} \cdot \frac{1}{46} \text{ (1 match in 46)} = \frac{3}{3,542}$$

There is a $\frac{3}{3,542}$ (or .085%) chance of winning the \$10, which is a slightly worse probability of $\frac{1}{844}$ (or .19%) they tell you. The odds against winning this way is $\frac{3,539}{3,542}$ (or 99.925%).

3. No. Mutually exclusive means that A and B can't happen at the same time. Independent means the probability of A happening doesn't affect the probability of B and vice versa, them being independents means there's no ties between them. However, for A and B to be mutually exclusive, when one event is happening the other can't be happening, meaning these events are tied to each other. So no, A and B can't be both mutually exclusive and independent.

4. $A \subseteq B$ means that B is a subset of A, where every element in the set of B is present in the set of A. So, if anything in set B is triggered, then A is triggered as well so B isn't independent. However, since B is just a subset of A, there may be elements in A that aren't present in B, making A partially independent.

5. i. $P((A \cap B) \cap C) \rightarrow P(A \cap B \cap C) \rightarrow P(A)P(B)P(C) \rightarrow$
$P(A \cap B)P(C)$, so the probabilities are independent.

ii. $P((A \cup B) \cap C) \rightarrow P((A \cap C) \cup (B \cap C)) \rightarrow$
$P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \rightarrow$
$P(A)P(C) + P(B)P(C) - P(A)P(B)P(C)$
$P(C)[P(A) + P(B) - P(A \cap B)]$
$P(A \cup B)P(C)$, so the probabilities are independent.

iii. If A, B, C are mutually independent instead of pairwise-independent, they would still be independent results. because pairwise-independent means the probability of an event in each pair has no bearing on the probability of the other.

6. i. $P(\text{male} \mid \text{colorblind}) = \dfrac{P(M \cap CB)}{P(CB)} \rightarrow$ Bayes. Rule $\rightarrow$
(M) (CB) $\bar{M} = F$

$$\frac{P(CB \mid M)P(M)}{P(CB \mid M)P(M) + P(CB \mid F)P(F)} = \frac{.08 \cdot .5}{(.08)(.5) + (.005)(.5)} =$$

$\dfrac{.04}{.0425} = .941$ or 94.1% prob. of male

ii. If twice as many males than females, then $P(M) = .667$ and $P(F) = .333$. Same formula, so:

$$\frac{P(CB|M)P(M)}{P(CB|M)P(M) + P(CB|F)P(F)} = \frac{.08 \cdot .667}{(.08)(.667) + (.005)(.333)} =$$

$$\frac{.05336}{.055025} = .9697 \approx 96.97\%$$ prob of male [illegible]