COMPUTATIONAL ENGINEERING

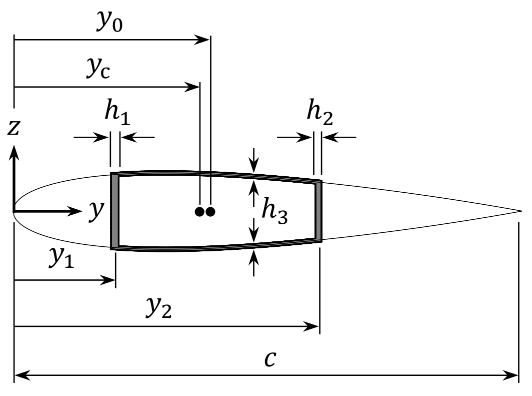
Project – Wing modelling

Part I – Wingbox modelling

The objective of this part is to validate the beams and shells models studying the bending and torsion response of the wingbox structure depicted in Figure 1. The material considered has a Young's modulus of $E = 110$ GPa, a Poisson's ratio of $\nu = 0.33$, and a density of $\rho = 3200$ kg/m^3. The geometric and section properties of the wingbox are given in Table 1 below. To do so, two tests will be performed on the wing fixed at the root:

1) Bending: Applying a unit shear load on the shear center of the wing tip.
2) Torsion: Applying a unit torque on the wing tip.

Table 1. Geometric properties of the wingbox.

NACA0015 profile	Data	Value	Property	Value
	c (m)	2	Area A (m^2)	0.0247
	b (m)	12	Inertia $I_{y'}$ (m^4)	0.234 x 10^{-3}
	y_0 (m)	0.725	Inertia $I_{z'}$ (m^4)	3.131 x 10^{-3}
	y_1 (m)	0.4	Polar inertia J (m^4)	3.365 x 10^{-3}
	y_2 (m)	1.2	Shear correction $k_{y'}$	0.2621
	h_1 (mm)	40	Shear correction $k_{z'}$	0.2417
	h_2 (mm)	30	Torsion correction k_t	0.149
	h_3 (mm)	4	Shear center y_c (m)	0.684

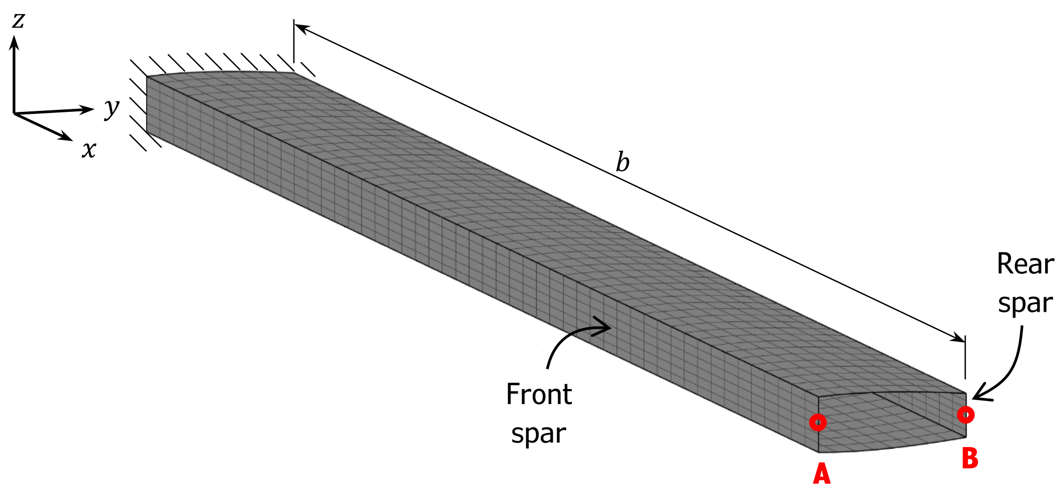


Figure 1. Wingbox structure.

Procedure:

1) Model the wingbox structure with beam elements using the data given in **beam.mat** file and compute:
   i. Vertical deflection distribution along the spanwise direction, u_z vs. x, resulting from the bending (1) and torsion (2) tests.
   ii. Twist angle distribution along the spanwise direction, θ_x vs. x, resulting from the bending (1) and torsion (2) tests.
   iii. Deflection and twist angle components (u_y, u_z, θ_x) for the first 6 natural modes along with their corresponding frequencies.
2) Model the wingbox structure with flat shell elements using the data given in **shell.mat** file and compute:
   i. Average vertical deflection distribution along the spanwise direction, $\bar{u}_z$ vs. x, resulting from the bending (1) and torsion (2) tests.
   ii. Average twist angle distribution along the spanwise direction, $\bar{\theta}_x$ vs. x, resulting from the bending (1) and torsion (2) tests.
   iii. Average deflection and twist angle components $(\bar{u}_y, \bar{u}_z, \bar{\theta}_x)$ for the first 6 natural modes along with their corresponding frequencies.

   Note 1: To help apply the boundary condition (clamped root), the indices of the nodes contained in the wingbox root are stored in the **indRoot** array.

   Note 2: To model the application of a shear load F at the shear center in the bending test, apply the following equivalent vertical forces on points A and B of the wing tip:

$$F_{\mathrm{A}} = F\frac{y_2 - y_{\mathrm{c}}}{y_2 - y_1}; \quad F_{\mathrm{B}} = F\frac{y_{\mathrm{c}} - y_1}{y_2 - y_1}$$

   Similarly, to model the application of a torque T in the torsion test:

$$F_{\mathrm{A}} = -\frac{T}{y_2 - y_1}; \quad F_{\mathrm{B}} = \frac{T}{y_2 - y_1}$$

   The nodal index corresponding to points A and B in the mesh are stored in **indPointA** and **indPointB** variables, respectively.

   Note 3: The average deflection and twist angles will be derived from the corresponding displacements u_y and u_z obtained in the front spar (1) and rear spar (2) centerlines:

$$\bar{\theta}_x = \frac{u_z^{(2)} - u_z^{(1)}}{y_2 - y_1}; \quad \bar{u}_z = u_z^{(1)} + \bar{\theta}_x (y_{\mathrm{c}} - y_1); \quad \bar{u}_y = \frac{u_y^{(1)} + u_y^{(2)}}{2}$$

   The values for $u_z^{(1)}$ and $u_z^{(2)}$ can be obtained from the resulting global displacements vector using the nodal indices stored in **indSpar1** and **indSpar2** arrays, respectively.

The mesh variables contained in **beam.mat** and **shell.mat** files are: **xn** (nodal coordinates matrix), **Tn** (nodal connectivities matrix), and **Tm** (material connectivities matrix). For the shell case, material index 1 refers to the front spar, 2 refers to the rear spar, and 3 to the upper and lower surfaces of the wingbox.

The script files **MAIN_beam.m**, **MAIN_shell.m** can be used as templates to develop the code to get the requested results.

Part II – Whole wing structure

The same wingbox structure is now reinforced with ribs and stringers and covered by a skin to offer aerodynamic capabilities to the wing.

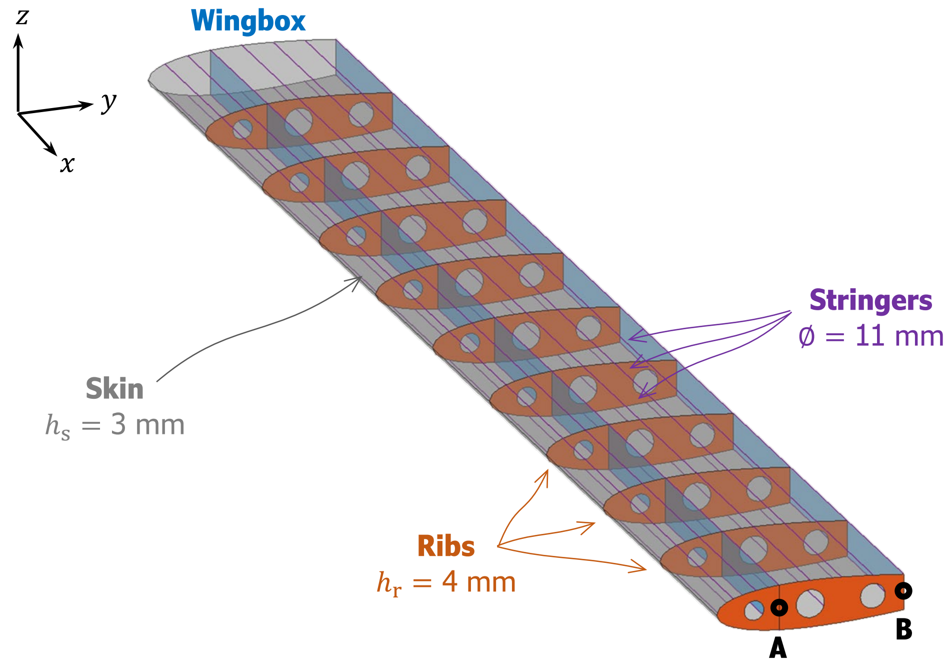
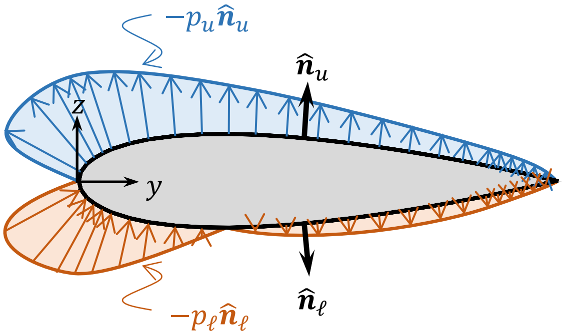


Figure 2. Wing structure.

1) The first goal is to analyze the effects of adding these different elements to the overall structure. To do so:
   i. Build the stiffness and mass matrices for the **wingbox** modelled with shell elements (use the `Tn_wb` and `Tm_wb` variables as nodal and material connectivities matrices).
   ii. Build the stiffness and mass matrices for the **stringers** modelled with beam elements of circular cross-section with 11 mm diameter, considering $k_{y'} = k_{z'} = 5/6$, and $k_t = 1$ (use the `Tn_st` and `Tm_st` variables as nodal and material connectivities matrices).
   iii. Build the stiffness and mass matrices for the **ribs** modelled with shell elements (use the `Tn_rb` and `Tm_rb` variables as nodal and material connectivities matrices).
   iv. Build the stiffness and mass matrices for the **skin** modelled with shell elements (use the `Tn_sk` and `Tm_sk` variables as nodal and material connectivities matrices).
   v. Repeat the bending and torsion tests of Part I, but considering all or some part of the structural elements combined, and get the same results (assume the Notes for the shells analysis also apply to this case).

   Hint: To study the effects of each element separately, the corresponding mass and sitffness matrices can be switched on/off. Example: multiplying by zero (or a very small constant) the stiffness and mass matrices for the skin elements virtually removes them from the structure.

2) A wing tunnel test is now performed so the wing is subjected to its own weight and a pressure load distributed over the upper and lower skin surfaces, according to:



$$p_u(x,y) = -\alpha P(x)\left[\left(1-\frac{y}{c}\right)^4 + \sqrt{1-\frac{y}{c}}\right]$$

$$p_\ell(x,y) = \alpha P(x)\left[\left(1-\frac{y}{c}\right)^4 - \frac{1}{4}\sqrt{1-\frac{y}{c}}\right]$$

$$P(y) = \begin{cases} p_\infty\left(0.5 + 4.8\dfrac{x}{b} - 11.2\left(\dfrac{x}{b}\right)^2\right) & x \in [0, 0.25b] \\ p_\infty\left(1.2 - 0.8\dfrac{x}{b}\right) & x \in [0.25b, 0.75b] \\ p_\infty\left(-2.4 + 8.8\dfrac{x}{b} - 6.4\left(\dfrac{x}{b}\right)^2\right) & x \in [0.75b, b] \end{cases}$$

with $\alpha = 10°$ being the angle of attack (expressed in radians), and $p_\infty = 0.75 \times 10^5$ Pa.

In this scenario:

i. Solve the static problem and obtain the average vertical deflection $\bar{u}_z$ and twist angle $\bar{\theta}_x$ distributions along the wingspan (use formulas in page 2).
ii. Obtain and plot the Von Mises stress distribution in the wingbox, ribs and skin elements separately.
iii. Develop and apply a model-order reduction strategy based on a modal projection to solve the problem with a reduced set of natural modes. Justify which modes are more relevant for this case and compare the results.

Note: To define the external pressure force distribution, matrices stored in the **n_u** and **n_l** variables contain the components of the unit normal vectors for the upper and lower skin surfaces, respectively, on the first three columns, and the indices for the corresponding nodes on the fourth column.

The mesh variables contained in **wing.mat** file are: **xn** (nodal coordinates matrix), **Tn_st** and **Tm_st** (nodal and material connectivities matrices for the stringers), **Tn_sk** and **Tm_sk** (nodal and material connectivities matrices for the skin), **Tn_rb** and **Tm_rb** (nodal and material connectivities matrices for the ribs), and **Tn_wb** and **Tm_wb** (nodal and material connectivities matrices for the winbgox). In the latter case, material index 1 refers to the front spar, 2 refers to the rear spar, and 3 to the upper and lower surfaces of the wingbox. In addition, the **indRoot**, **indPointA**, **indPointB**, **indSpar1**, and **indSpar2** variables are also included in this case (see Part I for details). The script file **MAIN_wing.m** can be used as template to develop the code to get the requested results.

<u>**Important**</u>: Initialize the global mass and stiffness matrices using the **sparse** function instead of the **zeros** function. This will allow you to use the **eigs** function for the modal analysis obtaining just the needed number of modes (and speeding up the computation).