Universitat Politècnica de Catalunya

Escola Superior d'Enginyeries Industrial, Aerospacial i Audiovisual de Terrassa (ESEIAAT)

UNIVERSITAT POLITÈCNICA DE CATALUNYA
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VEHICLES AEROESPACIALS

TASK 1 Matrix structural analysis of an optical mount

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1 Part 1

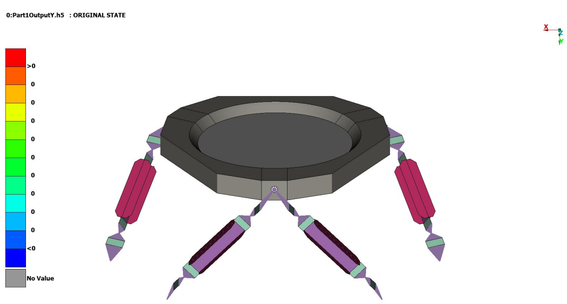
1.1 Displacements and rotations due to gravity

The resulting displacements and rotations due to gravity can be seen in Table 1. These are overall quite small, at the order of micrometers, as the structure weighs little and is quite stiff. Displacements are only considerable on the solicited directions, as everywhere else the values are much smaller.

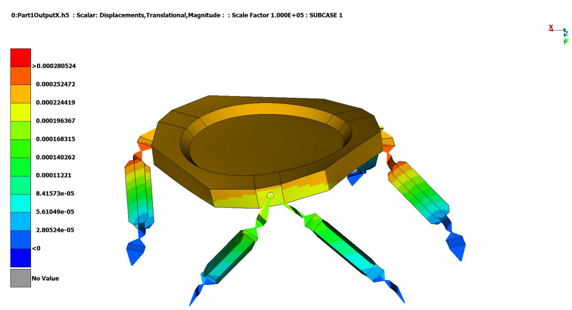
Table 1. Displacements and rotations at the reference node due to unit gravity accelerations in the three directions

	X (μm)	**Y** (μm)	**Z** (μm)	**R1** (μrad)	**R2** (μrad)	**R3** (μrad)
gX	0.2307	-1.73e-06	3.76e-07	2.25e-08	1.05e-07	4.85e-04
gY	-8.48e-07	0.3873	-1.08e-07	4.86e-08	-1.91e-11	4.94e-07
gZ	7.07e-07	1.03e-06	0.2369	4.85e-04	-1.17e-07	-1.58e-08

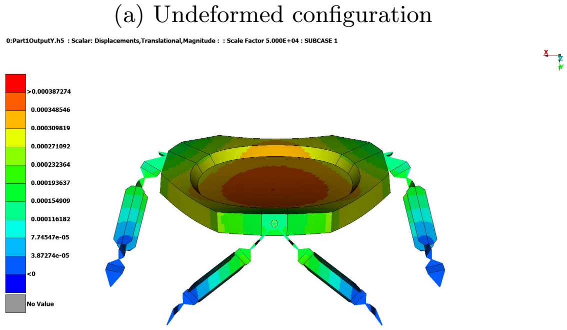
And the results can be visualized with META as in Figure 1.



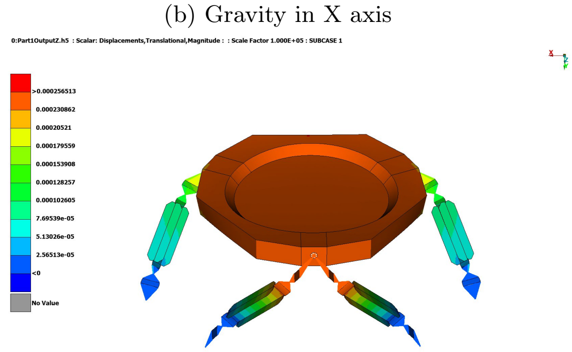
(a) Undeformed configuration



(b) Gravity in X axis



(c) Gravity in Y axis



(d) Gravity in Z axis

Figure 1. Deformation due to gravity load in each axis, and the undeformed configuration.

1.2 Reaction forces due to gravity

The reactions at each support caused by gravity forces in all the 3 directions have also been computed in Tables 2, 3 and 4

gX	**Fx** (N)	**Fy** (N)	**Fz** (N)	**M1** (Nm)	**M2** (Nm)	**M3** (Nm)
Suport 1	-1.66	3.31	-2.87	17.07	-0.10	-9.90
Suport 2	-1.66	-3.31	2.87	-17.07	-0.10	-9.90
Suport 3	-1.77	3.54	3.06	-18.15	0.13	-10.72
Suport 4	-6.85	6.85	0.00	-0.17	-0.03	-40.80
Suport 5	-6.85	-6.85	-0.00	0.17	-0.03	-40.80
Suport 6	-1.77	-3.54	-3.06	18.15	0.13	-10.72
Sum	-20.55	0	0	0	0	-122.83

Table 2. Reactions when applying a gravity load in the X direction

gY	**Fx** (N)	**Fy** (N)	**Fz** (N)	**M1** (Nm)	**M2** (Nm)	**M3** (Nm)
Suport 1	1.71	-3.42	2.97	-17.67	-0.26	10.14
Suport 2	-1.71	-3.42	2.97	-17.67	0.26	-10.14
Suport 3	1.72	-3.42	-2.96	17.62	-0.26	10.23
Suport 4	3.42	-3.42	-0.00	0.05	0.26	20.37
Suport 5	-3.42	-3.42	-0.00	0.05	-0.26	-20.37
Suport 6	-1.72	-3.42	-2.96	17.62	0.26	-10.23
Sum	0	-20.55	0	0	0	0

Table 3. Reactions when applying a gravity load in the Y direction

gZ	**Fx** (N)	**Fy** (N)	**Fz** (N)	**M1** (Nm)	**M2** (Nm)	**M3** (Nm)
Suport 1	-2.99	5.99	-5.19	31.04	0.09	-17.67
Suport 2	2.99	5.99	-5.19	31.04	-0.09	17.67
Suport 3	2.93	-5.86	-5.08	30.23	0.04	17.54
Suport 4	0.13	-0.13	-0.00	0.14	0.14	0.78
Suport 5	-0.13	-0.13	-0.00	0.14	-0.14	-0.78
Suport 6	-2.93	-5.86	-5.08	30.23	-0.04	-17.54
Sum	0	0	-20.55	122.83	0	0

Table 4. Reactions when applying a gravity load in the Z direction

1.3 Reaction forces check

The mass of the model as obtained by the mass matrix is 2.1 kg.

As can be seen in tables 2, 3 and 4 the sum of the forces equals 0 except in the case where the reaction is aligned with the direction of the load where it equals the mass times the acceleration. However nonzero values are seen in the moments in the X and Z direction. This is because the external force due to gravity is applying a moment that will compensate this. If we had the position of the center of gravity we could check it by doing the following operation:

$$\overrightarrow{M} = \overrightarrow{r_{CG}} \times \overrightarrow{g} m \tag{1}$$

2 Part 2

2.1 Displacements and rotations due to enforced unit displacements

Enforcing a Y movement on the suport nodes causes the displacements seen in Table 5 to appear at the reference node.

Table 5. Movements at the reference node due to imposed unit displacements in y at the support points

Reference node movements	**X** (mm)	**Y** (mm)	**Z** (mm)	**R1** ($mrad$)	**R2** ($mrad$)	**R3** ($mrad$)
gY	-0.164133	0.167054	-0.289957	0.001558	0.001557	0.002697

2.2 Shims used to compensate the deviation

Likewise, enforcing two rotations of values $-500\mu rad$ around X and $+200\mu rad$ around Z the reference nodes can cause displacements to appear in the Y direction at the support nodes. These values can be seen in Table 6

Table 6. Shims used to compensate for the imposed deviation at the reference node

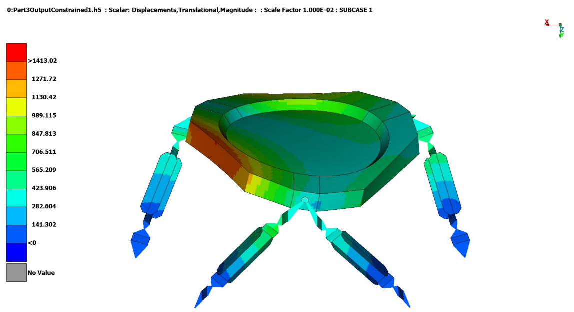
New shims	**Shim 1** (mm)	**Shim 2** (mm)	**Shim 3** (mm)	**Shim 4** (mm)	**Shim 5** (mm)	**Shim 6** (mm)
	1.0074	1.0432	1.0438	0.9485	0.9488	1.0083

To find the values of these shims, we have freed the 2nd DOF at each support and imposed the displacement at the reference node. Next, in the DOFs 1, 2, 3 and 5 we have imposed a value of 0, at the 4th DOF a value of $+500\mu rad$ and in the 6th DOF a value of -200μrad to compensate for the rotation found in the optics.

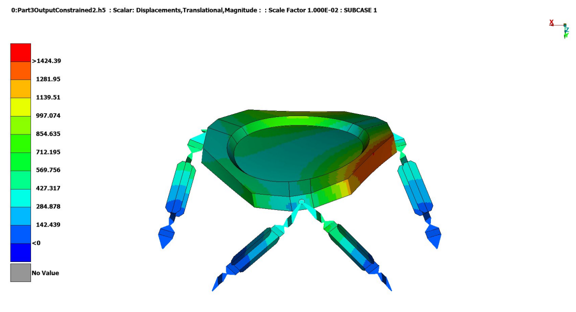
3 Part 3

3.1 Modal response under constrained conditions

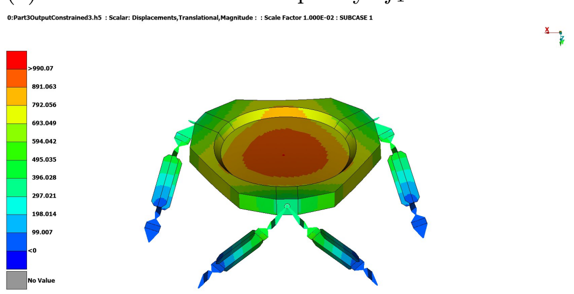
The modes of vibration, in the case of constrained conditions at the support nodes, result in the shapes and frequencies seen in Figure 2.



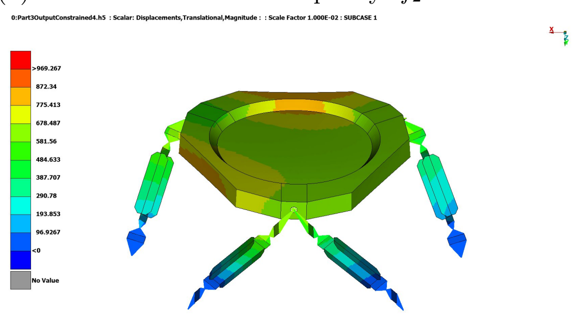
(a) First elastic mode. Frequency: $f_1 = 904.42Hz$



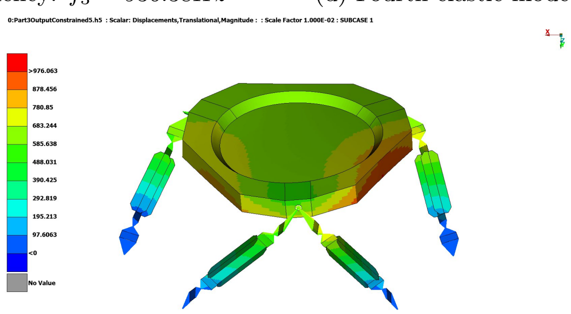
(b) Second elastic mode. Frequency: $f_2 = 904.43Hz$



(c) Third elastic mode. Frequency: $f_3 = 936.38Hz$



(d) Fourth elastic mode. Frequency: $f_4 = 1069.2Hz$

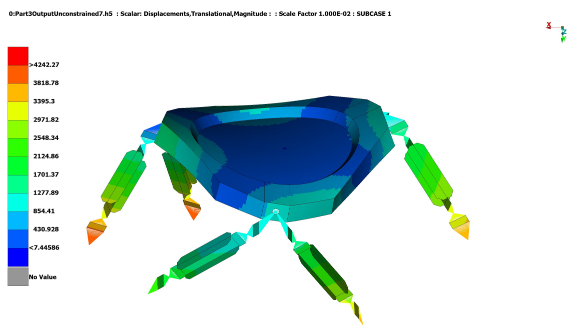


(e) Fifth elastic mode. Frequency: $f_5 = 1069.2Hz$

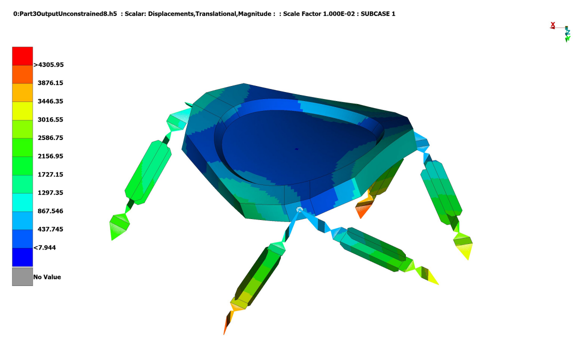
Figure 2. Modes of vibration with constrained support nodes.

3.2 Modal response under unconstrained conditions

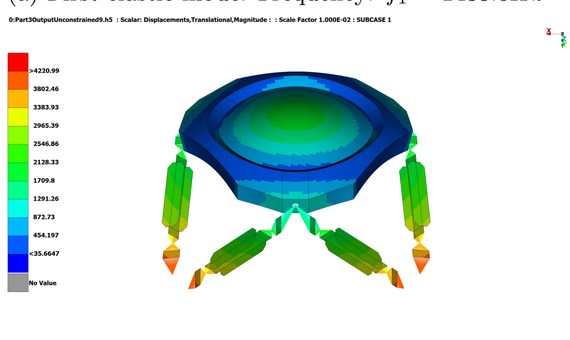
The modes of vibration, in the case of constrained conditions at the support nodes, result in the shapes and frequencies seen in Figure 3.



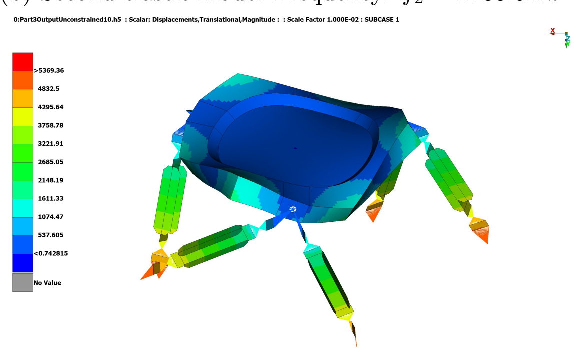
(a) First elastic mode. Frequency: $f_1 = 1433.0Hz$



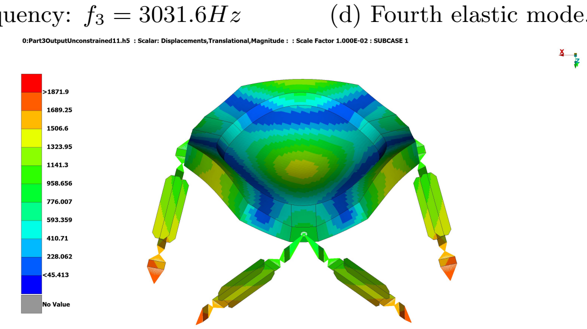
(b) Second elastic mode. Frequency: $f_2 = 1433.0Hz$



(c) Third elastic mode. Frequency: $f_3 = 3031.6Hz$



(d) Fourth elastic mode. Frequency: $f_4 = 3393.8Hz$



(e) Fifth elastic mode. Frequency: $f_5 = 4603.1Hz$

Figure 3. Modes of vibration with unconstrained support nodes.

In this configuration, the first 6 nodes of vibration are rigid body modes of vibration, as seen in Table 7

Table 7. Modes of vibration with unconstrained support nodes.

Mode	**Frequency (Hz)**
Rigid 1	0.0000 + 0.2985i
Rigid 2	0.5784
Rigid 3	0.0000 + 0.2985i
Rigid 4	0.749
Rigid 5	0.929
Rigid 6	1.686
Elastic 1-5	1430 - 4630

Considering that their frequency values are much smaller than the elastic modes and they can sometimes have complex components (because of MATLAB's eigs implementation, in theory they sould all be 0), the supposition that these are rigid body modes can be confirmed.

4 Part 4

4.1 X displacement frequency response due to a unit acceleration at 2000Hz

Finally, the frequency response of the structure can be found. In this case, an enforced oscillation in the X direction at values between 0-2000Hz will be considered. Also, a critical response ratio of 0.02 has been considered for this calculation. Then, by applying the modal method, Figure 4 has been obtained.

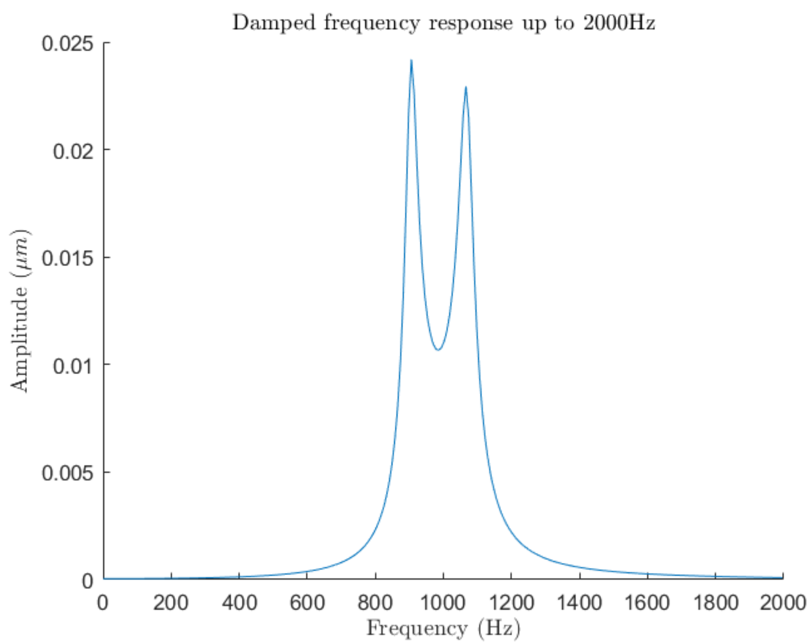


Figure 4. X displacement frequency response due to a unit acceleration at 2000Hz in the X direction.

References

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