Page No. :
Date: / /

$$\phi_p(x) = D_p \frac{dp(x)}{dx} \quad \text{--- (v)}$$

→ Hence, current density is given by

$$J_n(\text{diffusion}) = -(-q) D_n \frac{dn(x)}{dx}$$

$$= q D_n \frac{dn(x)}{dx}$$

and,

$$J_p(\text{diffusion}) = -(+q) D_p \frac{dP(x)}{dx}$$

$$= -q D_p \frac{dP(x)}{dx}$$

--- (vi)

Electron and holes moves together in a carrier gradient but the resulting electric current are in opposite direction because of opposing charges of electrons and holes.

⇒ Drift Diffusion Carrier :-

→ Contribution of drift and diffusion components are responsible for current densities.

Electric field

$$J_n(x) = J_n(\text{drift}) + J_n(\text{diff.})$$

Gradient

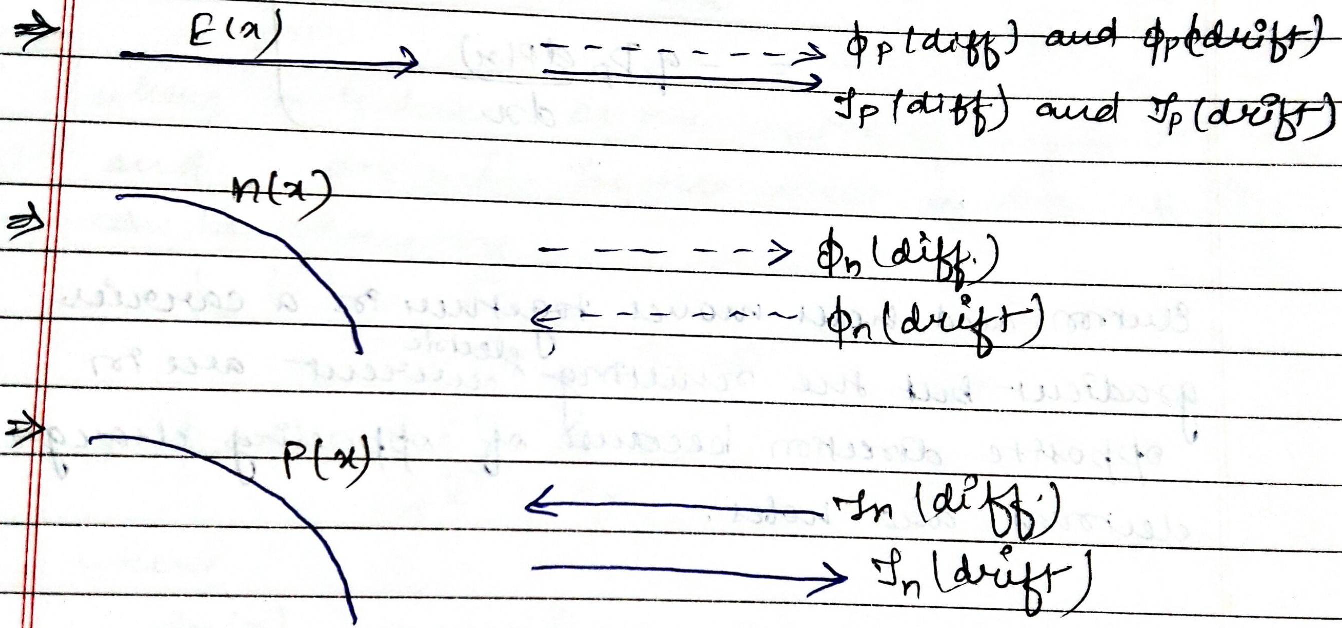
$$J_p(x) = J_p(\text{drift}) + J_p(\text{diff.})$$

$$J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx} \quad \text{— (a)}$$

$$J_p(x) = \underbrace{q\mu_p p(x)E(x)}_{\text{drift}} - qD_p \underbrace{\frac{dp(x)}{dx}}_{\text{Diffusion}} \quad \text{— (b)}$$

(VII)

$$J(x) = J_n(x) + J_p(x) \quad \text{— (VII)}$$



Drift and diffusion directions for electron and holes in a carrier gradient and an electric field.

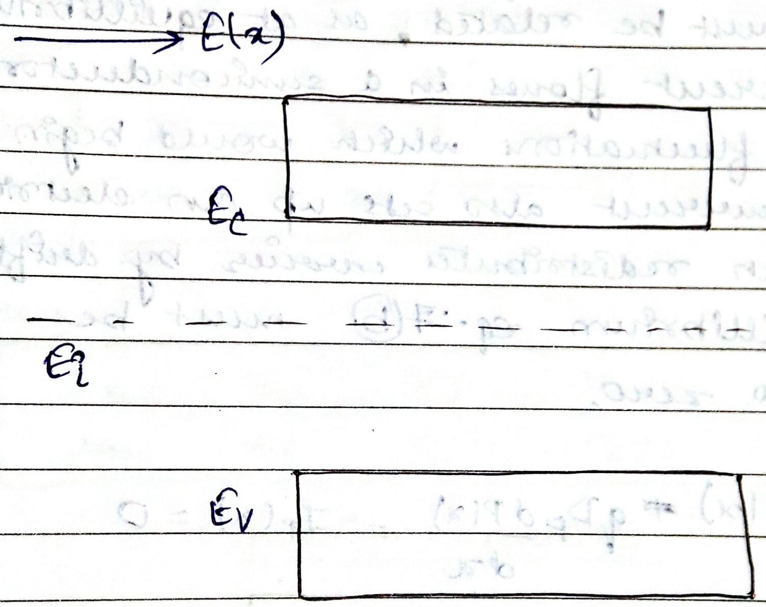
→ Particle flow denoted by dotted line and current flow is denoted by dark line.

→ Drift terms are proportional to carrier concentration, minority carriers provide much drift current.

→ Diffusion current is proportional to gradient of concentration.

Eg :-

In n-type of semiconductor, holes may be many order of magnitude smaller than n but gradient dp/dx may be significant.



Energy band diagram of a semiconductor in an electric field $\mathcal{E}(x)$

→ Electrostatic potential $V(x)$ varies in opposite directions and can be related to the electric field by

$$V(x) = \frac{E(x)}{-q} \quad \text{(ix)}$$

Page No. :
Date: / /

Electric field can be written as

$$E(x) = -\frac{dV(x)}{dx} \quad \text{—— (X)}$$

$$= -\frac{d}{dx}\left[\frac{E_i}{q}\right]$$

$$\Rightarrow \quad E(x) = \frac{-1}{q}\frac{dE_i}{dx} \quad \text{—— (XI)}$$

Imp → At the equilibrium, diffusion coefficient and mobility must be related, as at equilibrium no net current flows in a semiconductor. Thus any fluctuations which would begin a diffusion current also sets up an electric field which redistributes carriers by drift. So, at equilibrium eq. 7(b) must be equated to zero.

As,

$$q\mu_p P(x) E(x) - qD_p \frac{dP(x)}{dx} = J_p(x) = 0$$

$$\Rightarrow \quad \boxed{E(x) = \frac{D_p}{\mu_p} \frac{1}{P(x)} \cdot \frac{dP(x)}{dx}}$$

Again,

$$E(x) = \frac{D_p}{\mu_p}\frac{1}{KT}\left(\frac{dE_i}{dx} - \frac{dE_f}{dx}\right) \quad \text{—— (XII)}$$

The equilibrium fermi level doesn't vary with x, the derivative of E_i is given by eq^n (XI) is (XII) reduces to

Page No. :
Date: / /

As, we have

$$E(x) = \frac{1}{q}\left(\frac{dE_i}{dx}\right)$$

$$\frac{dE_i}{dx} = qE(x)$$

So

from eq^n (XII)

we have

$$E(x) = \frac{D_p}{\mu_p}\cdot\frac{1}{kT}\left(\frac{dE_i}{dx} - \cancel{\frac{dE_F}{dx}}^{\,0}\right) \quad \left\{\text{As, at eq}^m,\ dE_F/dx = 0\right\}.$$

So

$$\cancel{E(x)} = \frac{D_p}{\mu_p}\,\frac{q\cancel{E(x)}}{kT}$$

$$\Rightarrow \boxed{\frac{D_p}{\mu_p} = \frac{kT}{q}} \quad \text{—— (XIII)}$$

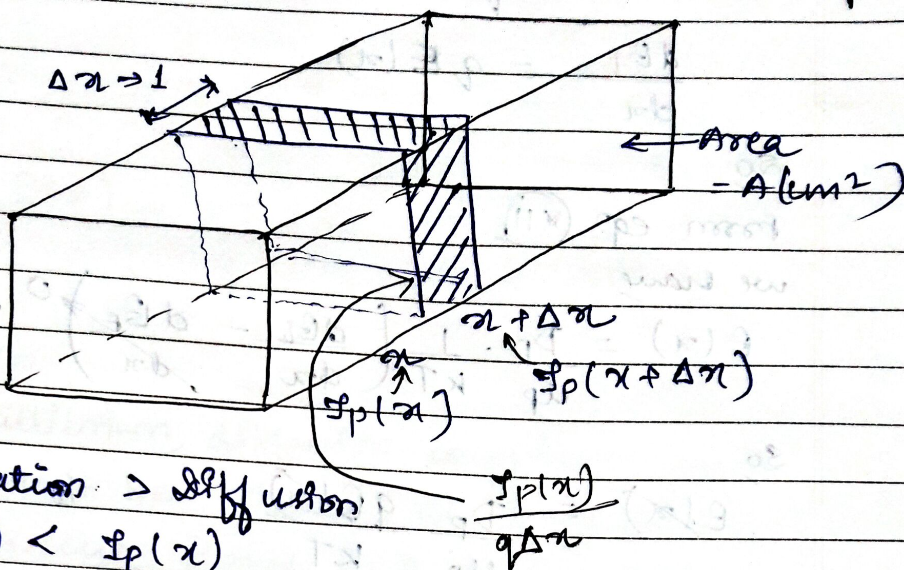
Thus

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q} = \frac{V_T}{11600} = 0.25\,V} \quad \text{—— (XIV)}$$

↳ Einstein's Relation.

Imp:

→ Diffusion and Recombination :- The continuity Equation:-



If;

→ Recombination > Diffusion

→ $I_p(x+\Delta x) < I_p(x)$

If;

→ Recombination < Diffusion

→ $I_p(x+\Delta x) > I_p(x)$.

→ $$\phi_p(x) = \frac{I_p(x)}{q\Delta x}$$

→ The hole current density leaving the volume $I_p(x+\Delta x)$ can be larger or smaller than ~~$I_p(x)$~~ the current density entering the volume $I_p(x)$, depending upon the generation & recombination of carriers taking place within the volume.

The net change in hole concentration $\frac{\partial p}{\partial t}$ is the difference between the hole flux per unit volume entering and leaving, minus the recombination rate.

→ Hole current density can be converted to hole particle flux density by dividing $\frac{I_p(x)}{q}$.

Again, dividing $(I_p(x)/q)$ by Δx gives the number of carriers per unit volume entering $\Delta x A$ per unit time and $\frac{I_p(x+\Delta x)}{q \cdot \Delta x}$

is a number leaving per unit volume and time.

$$\underbrace{\left.\frac{\partial P}{\partial t}\right|_{x \to x+\Delta x}}_{\substack{\text{Rate of} \\ \text{hole build up}}} = \frac{1}{q} \underbrace{\frac{I_p(x) - I_p(x+\Delta x)}{\Delta x}}_{\substack{\text{Increase of hole} \\ \text{concentration in} \\ \Delta x A \text{ per unit time.} \\ \text{(Diffusion factor)}}} - \underbrace{\frac{\delta P}{\tau_p}}_{\substack{\text{Recombination} \\ \text{factor (rate).}}} \quad \text{—(i)}$$

as $\Delta x \to 0$, we can write the current change in derivative form as

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial \delta P}{\partial t} = -\frac{1}{q}\frac{\partial I_p}{\partial x} - \frac{\delta P}{\tau_p} \quad \text{—(ii)}$$

Eqn (ii) is called continuity eqns for holes, similarly continuity eqn for electron is given by,

$$\frac{\partial \delta n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{T_n} \quad \text{--- (III)}$$

When current is carrying out strictly by diffusion then neglecting the drift component, eqn (II) and (III) becomes,

$$J_n(\text{diff}) = qD_n \frac{\partial \delta n}{\partial x} \quad \text{--- (IV)}$$

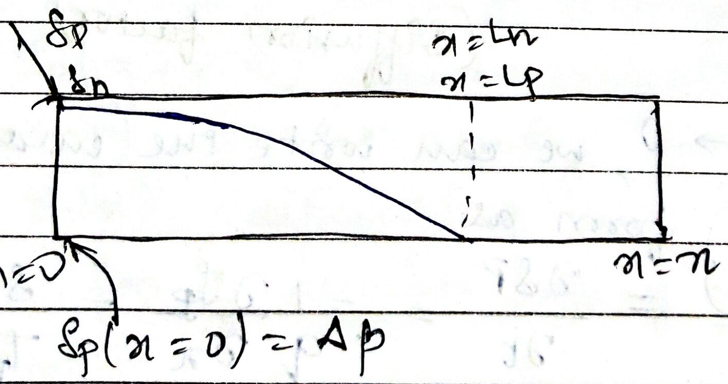
Now,

$$\frac{\partial \delta n}{\partial t} = D_n \cdot \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{T_n} \quad \text{--- (V)}$$

Similarly,

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{T_p} \quad \text{--- (VI)}$$

→ Steady-state carrier Injection, Diffusion Length :-



→ for steady state equate eqn (V) and (VI) with with 0, then we have,

Page No. :
Date: / /

$$\frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n T_n} = \frac{\delta n}{L_n^2} \quad \text{--- (VII)}$$

Similarly,

$$\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p T_p} = \frac{\delta p}{L_p^2} \quad \text{--- (VIII)}$$

where,

$L_n = \sqrt{D_n T_n}$, electron diffusion length.

and $L_p = \sqrt{D_p T_p}$, hole diffusion length.

As time variation is zero, for steady state consideration, hence no longer partial derivatives are required in the case, hence the partial terms of eqn (V) and (VI) are converted to differential term in eqn (VII) and (VIII).

→ Let us consider that excess hole injected in a semiconductor bar at $x = 0$ and at steady state hole injection maintains a constant excess hole concentration at injection point

$$\delta p(x=0) = \Delta p$$

→ Now distribution of excess hole decay to 0, after a prolonge distance x.

Imp.

→ considering the solution of diffusion eqn of holes from eqn (VIII), the solution to this equation is given by the following form:-

$$\delta P(x) = c_1 e^{x/L_p} + c_2 e^{-x/L_p} \quad \text{--- (IX)}$$

$\Rightarrow \quad \delta p(x=0) = \Delta p$

$$\left.\begin{array}{l} \rightarrow x=0, \quad \delta p(x) = \Delta p \text{ (max value)} \\ \rightarrow x=\infty, \quad \delta p(x) = 0 \end{array}\right\} \text{Boundary conditions}$$

Now taking the boundary conditions take

(i) $x=\infty$, then $\delta p(x) = 0$

$$0 = c_1 e^{x/L_p} + \underbrace{c_2 e^{-x/L_p}}_{0}$$

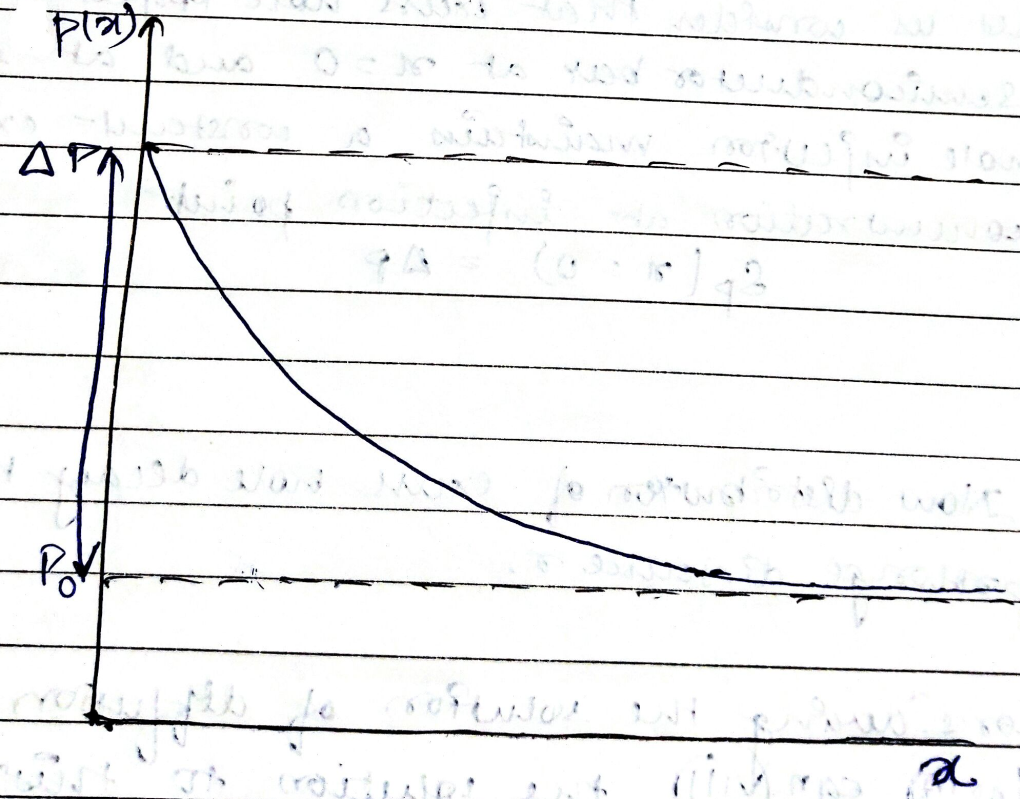
So,

$$c_1 = 0$$

(ii) $x=0$, then $\delta p(x) = \Delta p$

$$\Delta P = c_2$$

$$\boxed{\delta p(x) = \Delta P e^{-x/L_p}} \quad \text{—(x)}$$



Injection of holes at $x=0$ giving a steady state hole distribution $P(x)$ and resulting diffusion current $I_p(x)$.

Date: / /

$$\boxed{P(x) = P_0 + \Delta p e^{-x/L_p}} \quad \text{— (XI)}$$

$$J_p(x) = -qD_p \frac{dP(x)}{dx}$$

$$\Rightarrow \boxed{J_p(x) = \frac{qD_p}{L_p} \delta P(x)} \quad \text{— (XII)}$$

$$\delta p(x) = \Delta p e^{-x/L_p}$$

→ The injected hole concentration dies out exponentially in x due to recombination and diffusion length L_p represent the distance at which the excess hole distribution reduce to $1/e$ of its value at the point of injection. Hence L_p is the average distance of hole diffused before recombination.

→ On Probability that a hole injected at $x=0$ survives to x without recombination is given by

$$\frac{\delta p(x)}{\Delta p} = e^{-x/L_p} \quad \text{— (XIII)}$$

→ Whereas the probability that a hole at x will recombine in subsequent interval $d(x)$ is given by

$$\frac{\delta P(x) - \delta P(x+dx)}{\delta P(x)} = \frac{-\left(d\delta P(x)/dx\right)dx}{\delta P(x)}$$

$$\Rightarrow \quad = \frac{1}{L_p} dx \quad \text{— (XIV)}$$

Thus, the total probability that a hole injected at $x=0$, will recombine in a given $d(x)$ is the product of eq.^{n} (XIII) and (XIV).

$$\boxed{\left(e^{-x/L_p}\right)\left(\frac{1}{L_p}dx\right) = \frac{1}{L_p}e^{-x/L_p}\,dx} \quad \text{(XV)}$$

→ Now, through the technique of averaging the average distance of hole diffused before recombining is given by

$$\langle x\rangle = \int_0^\infty \frac{x e^{-x/L_p}}{L_p}\,dx$$

$$= L_p \quad \text{(XVI)}$$

Hence, hole current arises due to diffusion is given by

$$J_p(x) = -qD_p\frac{dP}{dx}$$

$$= -qD_p\frac{d\delta P}{dx}$$

$$= \frac{qD_p}{L_p}\Delta P e^{-x/L_p}$$

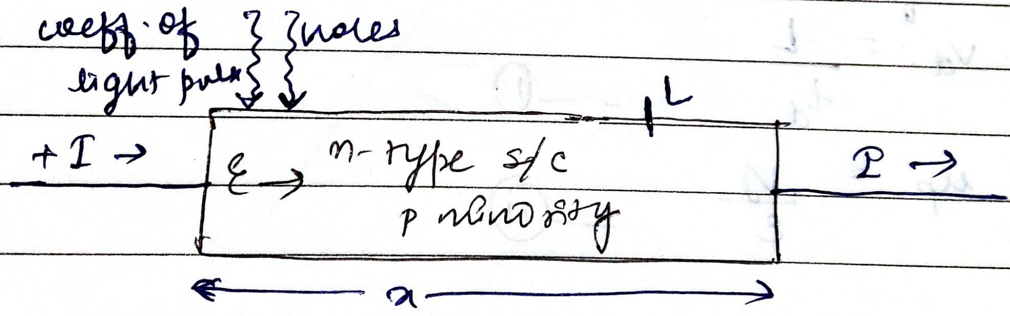
$$\Rightarrow \boxed{J_p(x) = \frac{qD_p}{L_p}\,\delta p(x)} \quad \text{(XVII)}$$

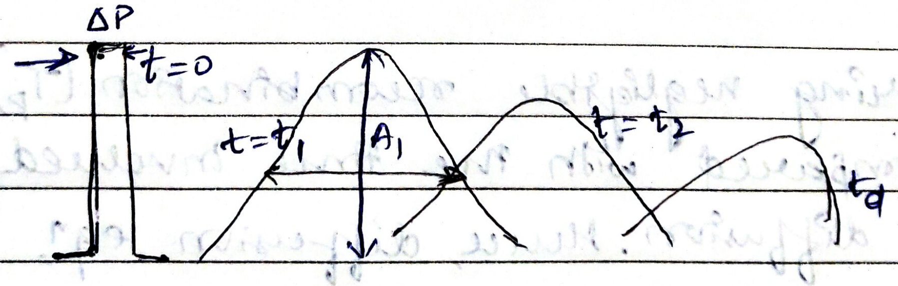
→ Since $P(x) = P_0 + \delta P(x)$, the space derivative involves only the excess concentration.

→ Diffusion current at any point x is proportional to the excess concentration $\delta P(x)$ at that point.

⇒ The Haynes - Shockley Experiment :-

→ This experiment was demonstrated by J.R. Haynes and W. Shockley for drift and diffusion of minority carriers. Through this experiment minority carrier mobility, μ and diffusion coefficient, D can be calculated.





Drift and diffusion of a hole pulse in n-type semiconductor.

→ A pulse of holes created in n-type bar that contains an electric field $\mathcal{E}$, as the pulse drift in the field and spread out by diffusion, the excess hole concentration is monitored at some point down the bar. The time required for the holes to drift in a given distance in the field gives a major of the mobility, the spreading of the pulse during given time is used to calculate the diffusion coefficient.

→ The excess hole drift in in the direction of electric field and eventually reach at the point $x = L$ by measuring the drift time t_d, drift velocity and hole mobility can be calculated.

Drift time $= t_d$

$$V_d = \frac{L}{t_d} \quad \text{---(I)}$$

$$\mu_p = \frac{V_d}{\mathcal{E}} \quad \text{---(II)}$$

→ Considering negligible recombination (T_p) is long compared with the time involved in the diffusion. Hence, diffusion eqn can be given by

$$\frac{\partial \delta P(x)}{\partial t} = D_p \frac{\partial^2 \delta P}{\partial x^2} - \left(\frac{\delta p}{T_p}\right)$$

≈ 0

$$\frac{\partial \delta p(x,t)}{\partial t} = D_p \frac{\partial^2 \delta p(x,t)}{\partial x^2} \quad \text{—— (III)}$$

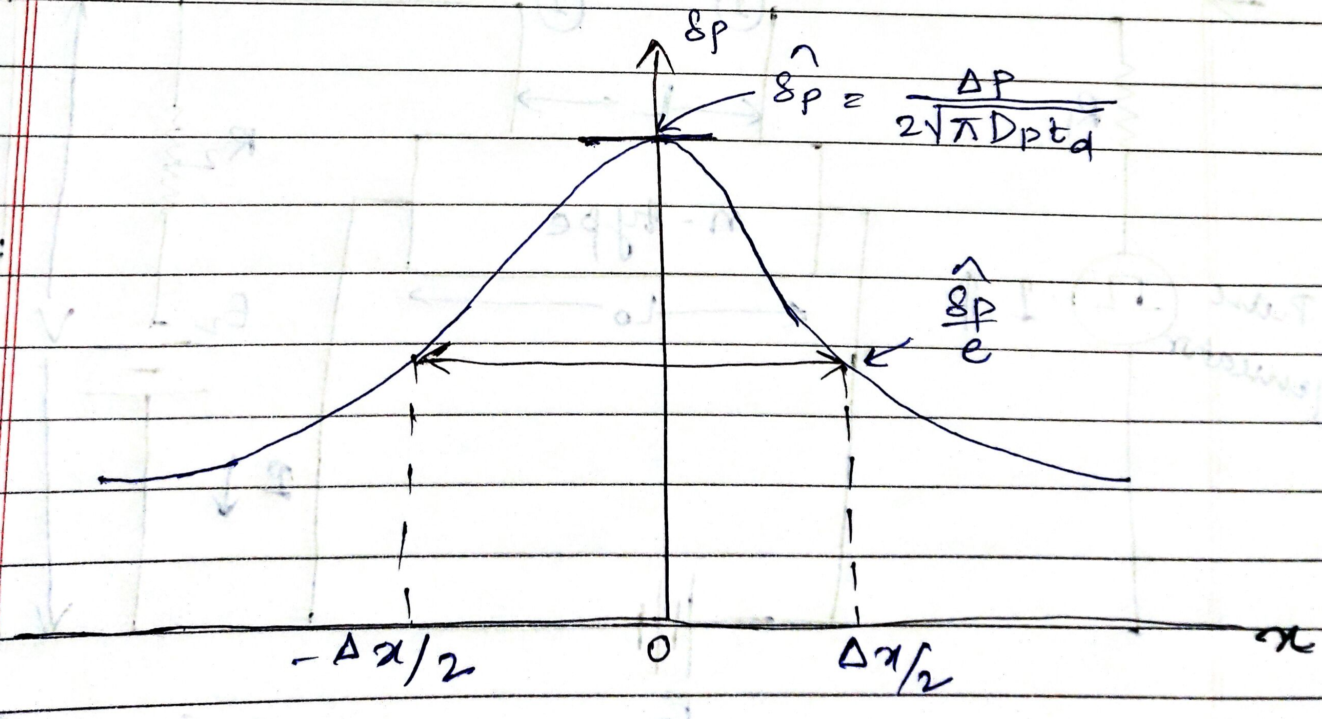
→ The function which satisfy the eqn (III) is gaussian distribution can be given by

$$\delta p(x,t) = \underbrace{\left[\frac{\Delta P}{2\sqrt{\pi D_p t}}\right]}_{\text{Amplitude term}} \underbrace{e^{-x^2/4D_p t}}_{\text{spreading term}} \quad \text{—— (IV)}$$

(Amplitude term) indicate peak value of the pulse at $x = 0$ decrease with time.

(spreading term) spread of the pulse in +ve or −ve direction.

Considering peak value of the pulse as $\hat{\delta p}$ at time t_d, eqn (IV) can be used to calculate D_p from the value of δp at same point t_d.



→ The most convenient choice is the point $\Delta x/2$ at which δp is reduced by $1/e$ of its peak value $\hat{\delta p}$. At this point we can write

$$\frac{\delta p}{e} = \hat{\delta p}\, e^{-(\Delta x/2)^2/4D_p t_d} \quad \text{—(V)}$$

$$\boxed{D_p = \frac{(\Delta x)^2}{16 t_d}} \quad \text{—(VI)}$$

→ To calculate Δx experimental setup is required where Δt can be observed through oscilloscope with time Δt. Again Δx can be given by

$$\Delta x = \Delta t V_d$$

$$\Rightarrow \Delta x = \Delta t \cdot \frac{L}{t_d} \quad \text{—(VII)}$$

→

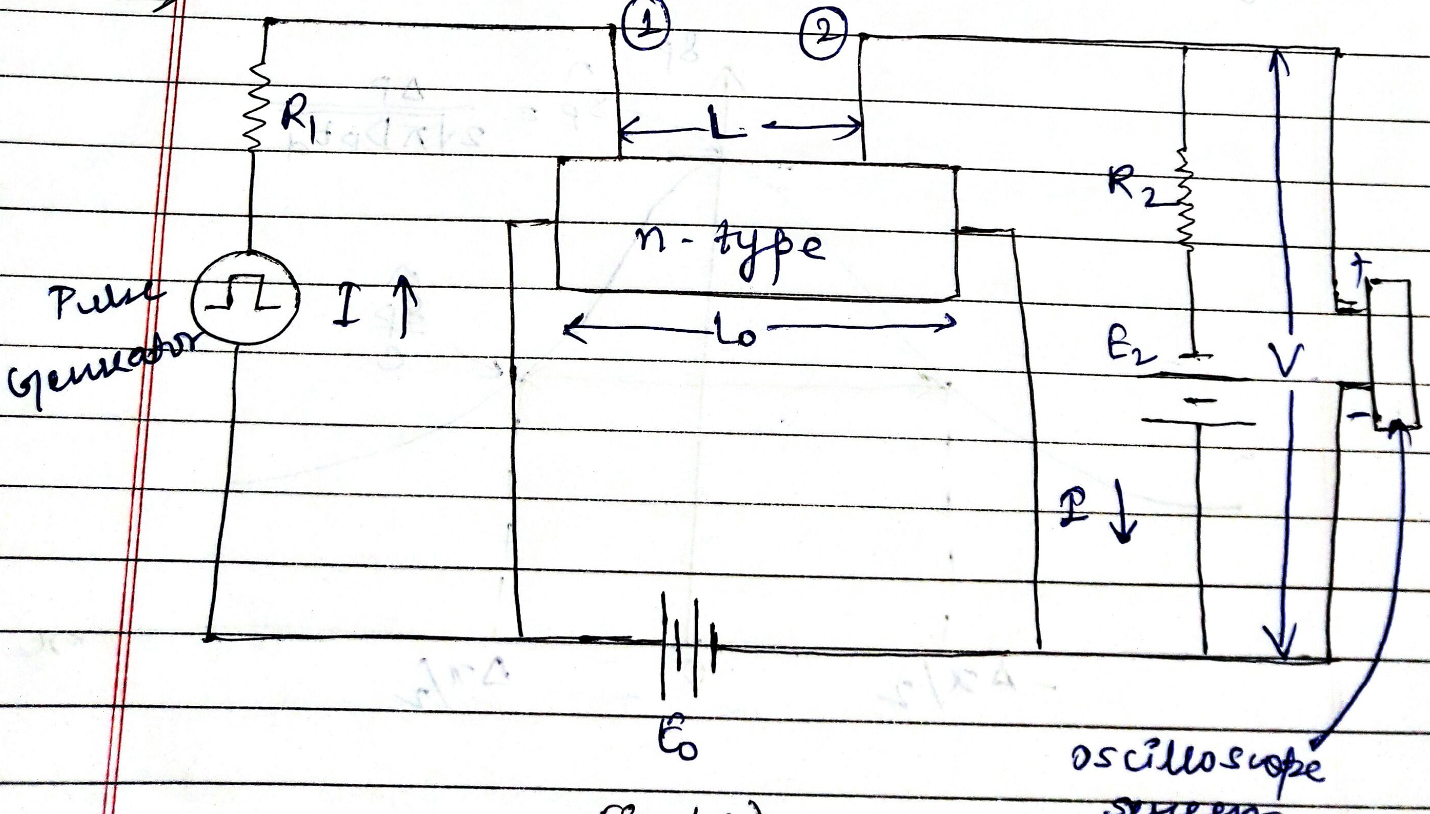


Fig. (A)

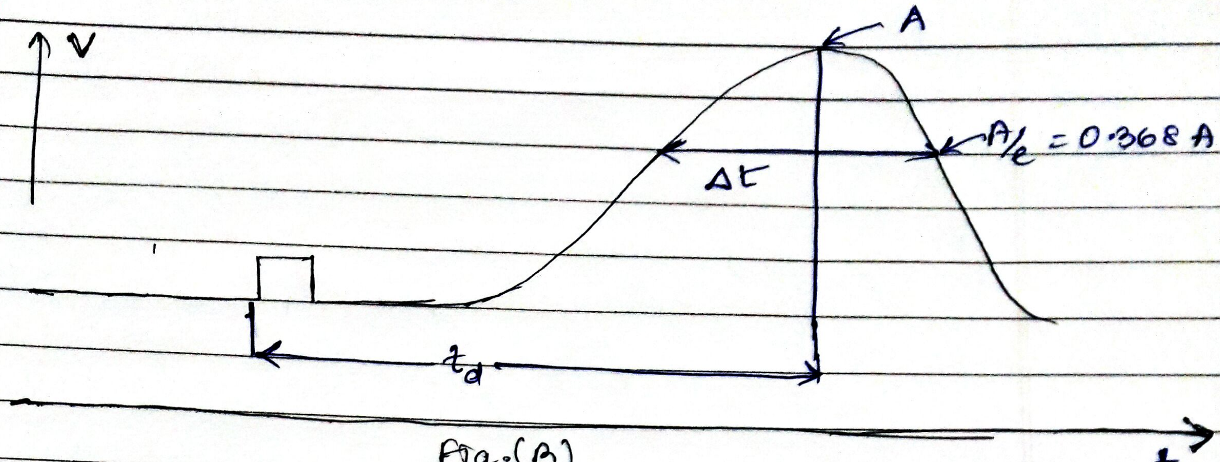


Fig.(B)

The Haynes Shockley experiment:-
(A) circuit systematic
(B) typical trace on oscilloscope screen.