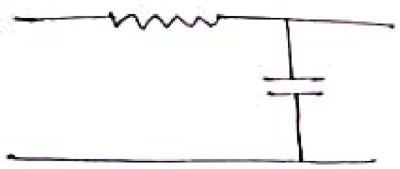
10/08/2022

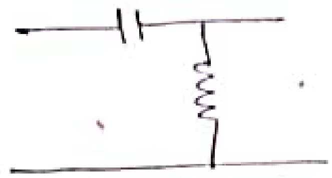
Analog Circuits (EC211)

- Wave shaping ckt
- Hybrid model (small signal analysis)



LPC

Low pass ckt



HPC

High pass ckt

RC

Book: Integrated Electronics. → Millman

i_c → Small signal ~~cor co~~ current.

i_C → Instantaneous current

I_C → ~~Quis~~ Quiscent value

$\Delta i_C = i_c$

$$\boxed{i_c = i_C - I_C}$$

Hybrid Model (h-parameter)

input current (i_1) and output voltage (v_2) are independent terms.

v_1 and i_2 are dependent terms.

$$V_1 = f_1(i_1, V_2) \quad \text{--- (I)}$$

$$i_2 = f_2(i_1, V_2) \quad \text{--- (II)}$$

$$\begin{bmatrix} V_1 \\ i_2 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{\text{h-parameter}} \begin{bmatrix} i_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} i_1 + h_{12} V_2 \quad \text{--- (III)}$$

$$i_2 = h_{21} i_1 + h_{22} V_2 \quad \text{--- (IV)}$$

Using eqn (III)

$h_{11} = \frac{V_1}{i_1}$ Ω (if $V_2 = 0$) — (V)

(V_2 is short circuited)

(reverse voltage amplification) $h_{12} = \frac{V_1}{V_2}$ (unit less) — (VI) (if $i_1 = 0$ open circuit)

Using eqn (IV)

(forward current Amplification) $h_{21} = \frac{i_2}{i_1}$ (unit less) — (VII) (if $V_2 = 0$, short circuited)

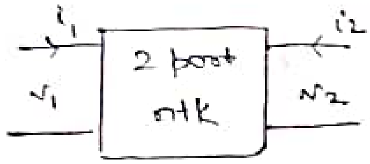
$h_{22} = \frac{i_2}{V_2}$ — (VIII) (if $i_1 = 0$, open circuit) (mho)

$h_{11} \rightarrow h_i$

$h_{12} \rightarrow h_r$

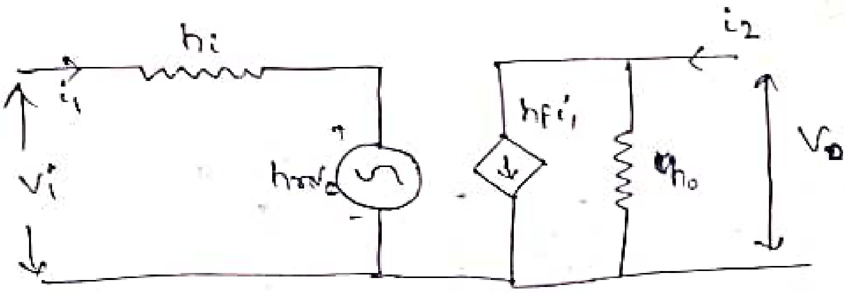
$h_{21} \rightarrow h_f$

$h_{22} \rightarrow h_o$



KVL, $v_1 = h_i i_1 + h_r v_2$

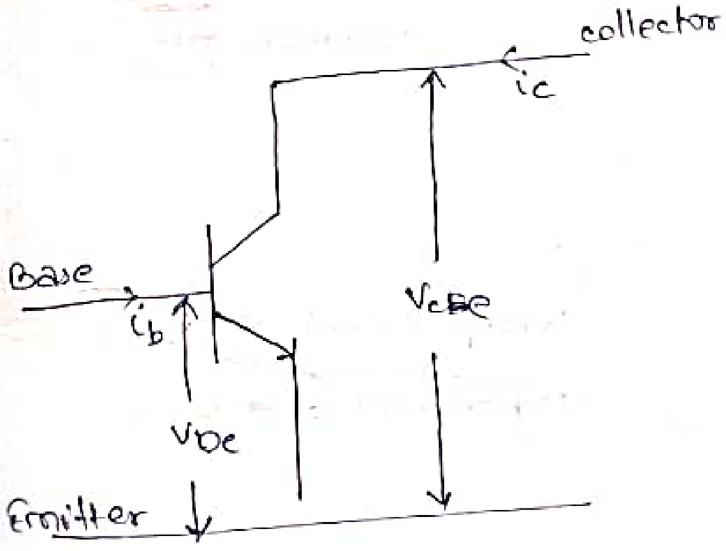
KCL, $i_2 = h_f i_1 + h_o v_2$



CE
CB
CC

Hybrid model of 2 port network ~~diagram~~

16/08/22



Independent → i_b, V_{cb}

Dependent → V_{be}, i_c

$V_{be} = f_1(i_b, V_{cb})$

$i_c = f_2(i_b, V_{cb})$

$$\begin{bmatrix} V_{be} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_b \\ V_{cb} \end{bmatrix}$$

$V_{be} = h_{11} i_b + h_{12} V_{cb}$

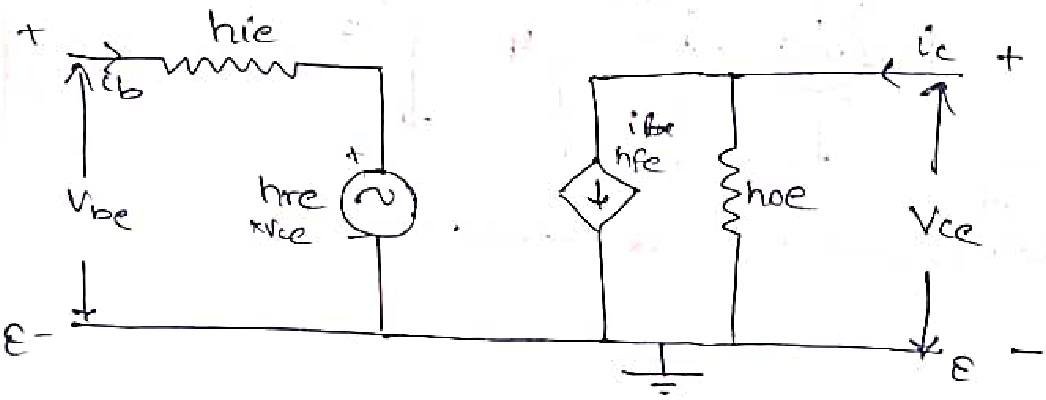
$i_c = h_{21} i_b + h_{22} V_{cb}$

$$h_{11} = \frac{V_{be}}{i_b} \quad (v_{ce} = 0) = h_{ie}$$

$$h_{12} = \frac{V_{be}}{V_{ce}} \quad (i_b = 0) = h_{re}$$

$$h_{21} = \frac{i_c}{i_b} \quad (v_{ce} = 0) = h_{fe}$$

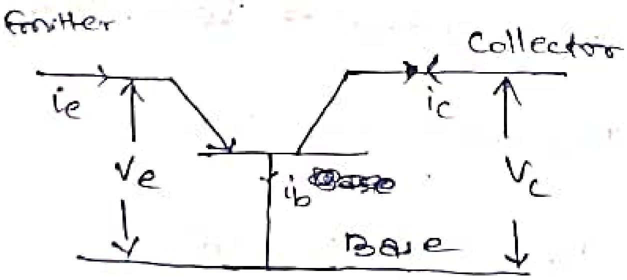
$$h_{22} = \frac{i_c}{V_{ce}} \quad (\text{if } i_b = 0) = \cancel{h_{f}} \cancel{h_{ie}}\ h_{oe}$$



$V_{be} \approx V_b$

$V_{ce} \approx V_c$

Common ~~Base~~ ~~Emitter~~ Emitter.



Independent → i_e, V_c

Dependent → i_c, V_e

$$\begin{bmatrix} V_e \\ i_c \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_e \\ V_c \end{bmatrix}$$

$v_e = f(i_e, v_c)$

$i_c = f(i_e, v_c)$

$$v_e = h_{11} i_e + h_{12} v_c$$

$$i_c = h_{21} i_e + h_{22} v_c$$

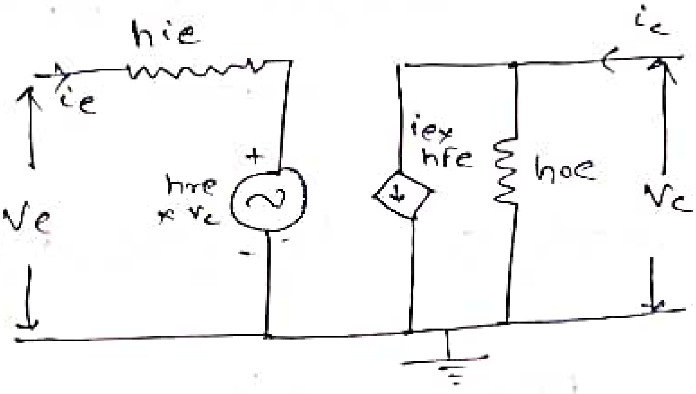
$$h_{11} = \frac{v_e}{i_e} = h_{ie}$$

$$h_{12} = \frac{v_e}{v_c} = h_{re}$$

$$h_{21} = \frac{i_c}{i_e} = h_{fe}$$

$$h_{22} = \frac{i_c}{v_c} = h_{oe}$$

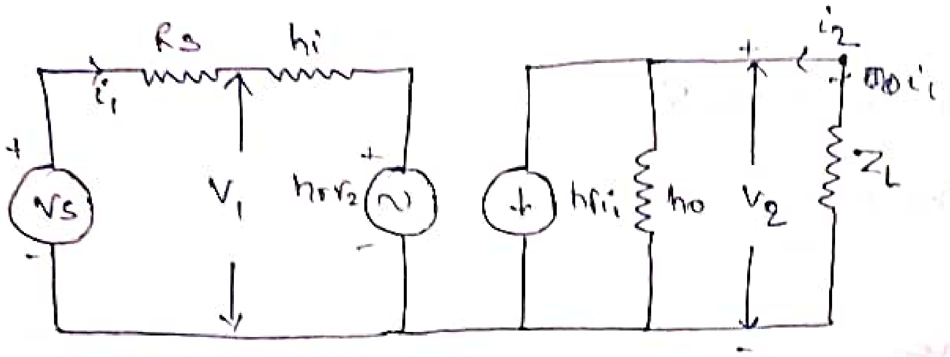
common - ~~Collector~~, Base



h-Parameter (I_e: 1.3mA)

Parameter	CE	CC	CB
hi	1100Ω	1100Ω	21.6Ω
hr	2.5×10^{-4}	1	2.9×10^{-4}
hf	50	-51	-0.98
ho	24μA/V	25μA/V	0.49μA/V
1/ho	40KΩ	40KΩ	2.04MΩ

Analysis of a transistor amplifier circuit using h-parameter.



Transistor in h-parameter model.

Current gain / current amplification:

$$A_i = \frac{i_L}{i_1} = \frac{-i_2}{i_1} \quad \text{--- (i)}$$

$$i_2 = h_f i_1 + h_o V_2 \quad \text{--- (ii)}$$

$$V_2 = i_L \times Z_L$$

$$\Rightarrow V_2 = -i_2 \times Z_L \quad \text{--- (iii)}$$

from (ii) and (iii)

$$i_2 = h_f i_1 - h_o i_2 Z_L$$

$$i_2 (1 + h_o Z_L) = h_f i_1$$

$$\frac{i_2}{i_1} = \frac{h_f}{1 + h_o Z_L}$$

$$\Rightarrow \boxed{A_i = \frac{-h_f}{(1 + h_o Z_L)}} \quad \text{— (iv)}$$

Input Impedence (Z_i)

$$Z_i = \frac{V_1}{i_1}$$

$$V_1 = h_i i_1 + h_r V_2$$

$$Z_i = \frac{h_i i_1 + h_r V_2}{i_1} = h_i + \frac{h_r V_2}{i_1} \quad \text{— (v)}$$

$$V_2 = -i_2 Z_L = A_i i_1 Z_L \quad \text{— (vi)}$$

from (v) and (vi)

$$Z_i = h_i + h_r A_i Z_L$$

$$\boxed{Z_i = h_i - \frac{h_f h_r}{Y_L + h_o}} \quad \text{— (vii)}$$

Voltage Gain/ voltage Amplification:

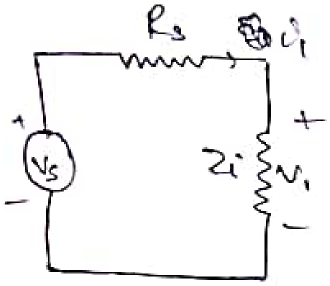
$$A_v = \frac{V_2}{V_1}$$

$$= \frac{A_i i_1 Z_L}{V_1} \quad \text{— (viii)}$$

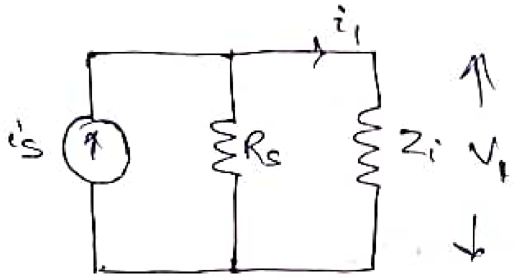
$$\boxed{A_v = \frac{A_i Z_L}{Z_i}} \quad \text{— (ix)} \quad \text{(Ideal value)}$$

Voltage Amplification Considering R_s as Source

$$A_{V_s} = \frac{V_2}{V_s} = \frac{V_2}{V_1}\frac{V_1}{V_s} = A_V\left(\frac{V_1}{V_s}\right) \quad \text{--- (x)}$$



Thevenin's equivalent of source.
(a)



Norton's eqv. of source.
(b)

From fig (a)

$$V_1 = \left(\frac{Z_i}{Z_i + R_s}\right) V_s \quad \text{--- (x1)}$$

$$\boxed{A_{V_s} = \frac{A_V Z_i}{Z_i + R_s} = \frac{A_i Z_L}{Z_i + R_s}} \quad \text{--- (x11)} \quad \text{(Real value)}$$

Current Amplification A_{Is} taking source resistance R_s:

$$A_{is} = \frac{-i_2}{i_s}$$

$$= \frac{-i_2}{i_1} \frac{i_1}{i_s}$$

$$A_{is} = A_i \left(\frac{i_1}{i_s}\right) \quad \text{—— (xiii)}$$

from fig (b) $\quad i_1 = \left(\frac{R_s}{R_s + Z_i}\right) i_s$

$$\boxed{A_{is} = A_i \frac{R_s}{R_s + Z_i}} \quad \text{—— (xiv)}$$

considering $R_s = \infty$

$$\boxed{A_{is} = A_i}$$

Dividing Eqn (xii) by (xiv)

$$\boxed{A_{vs} = A_{is} \frac{Z_L}{R_s}} \quad \text{—— (xv)}$$

Output Admittance

$$Z_o = \frac{1}{Y_o}$$

$$Y_o = \frac{i_2}{V_2} \quad \text{with } v_s = 0 \text{ and } R_L = \infty \quad \text{— (xvi)}$$

$$Y_o = h_f \frac{i_1}{V_2} + h_o \quad \text{— (xvii)}$$

Considering h-model and $V_s = 0$

$$R_s i_1 + h_r V_2 + h_i i_1 = 0$$

$$\frac{i_1}{V_2} = \frac{-h_r}{h_i + R_s} \quad \text{— (xviii)}$$

Putting i_1 from (xviii) to (xvii)

$$\boxed{Y_o = h_o - \frac{h_f h_r}{h_i + R_s}} \quad \text{— (xix)}$$

Hence output admittance is a function of source resistance (R_s).

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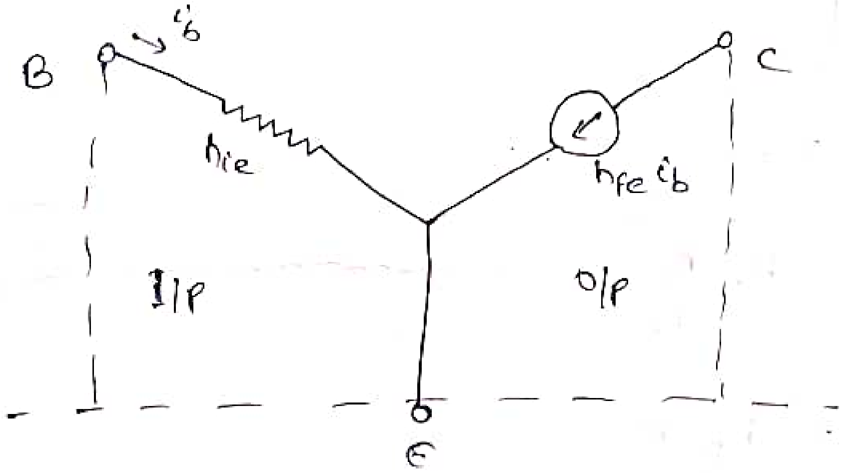
Small Signal Analysis of a Transistor Amplifier:

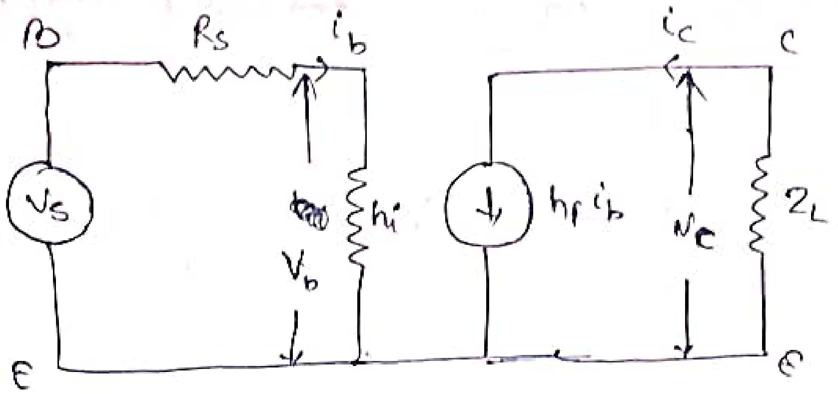
$A_i = \dfrac{-h_f}{1+h_o Z_L}$	$Y_o = h_o - \dfrac{h_f h_r}{h_i + R_s} = \dfrac{1}{Z_o}$
$Z_i = h_i + h_r A_i Z_L$	$A_{Vo} = \dfrac{A_V Z_i}{Z_i + R_s} = \dfrac{A_i Z_L}{Z_i + R_s}$
$A_V = \dfrac{A_i Z_L}{Z_i}$	$A_{is} = \dfrac{A_i R_s}{Z_i + R_s} = A_{Vs} \dfrac{R_s}{Z_L}$

Simplified Hybrid CE Model:

$\boxed{\begin{matrix} h_i \\ h_f \end{matrix}}$ $\left.\begin{matrix} h_r \\ h_o \end{matrix}\right\}$ neglected   5 to 10% error.

$h_o R_L < 0.1$





Current Gain:

$$A_i = \frac{-h_{fe}}{1+h_{oe}R_L} = -h_{fe} \quad (h_{oe}R_L < 0.1)$$

$$\boxed{A_i = -h_{fe}}$$

Input Impedance:

$$R_i = h_{ie} + h_{re}A_iR_L$$

$$= h_{ie}\left[1 - \underbrace{\frac{h_{re}h_{fe}}{h_{ie}h_{oe}}}_{<0.5} \frac{|A_i|}{h_{fe}} \underbrace{h_{oe}R_L}_{<0.1}\right]$$

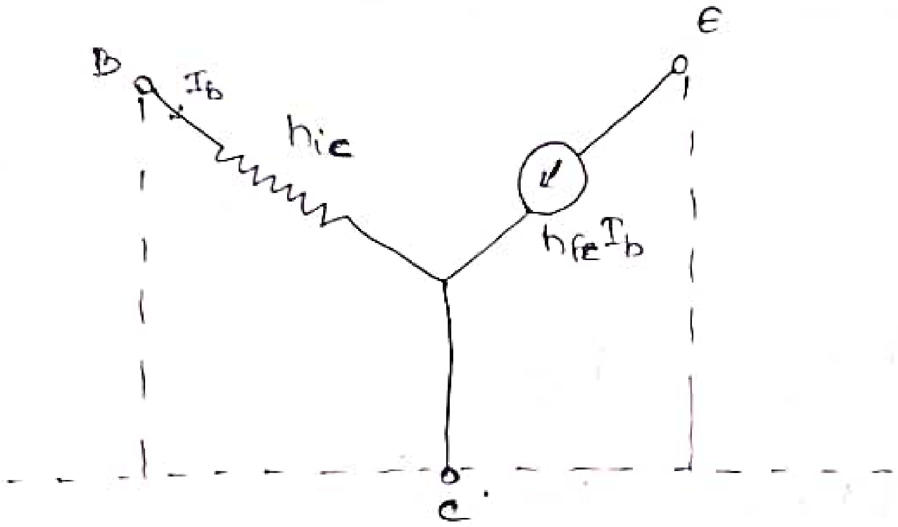
$$\boxed{R_i = h_{ie}}$$

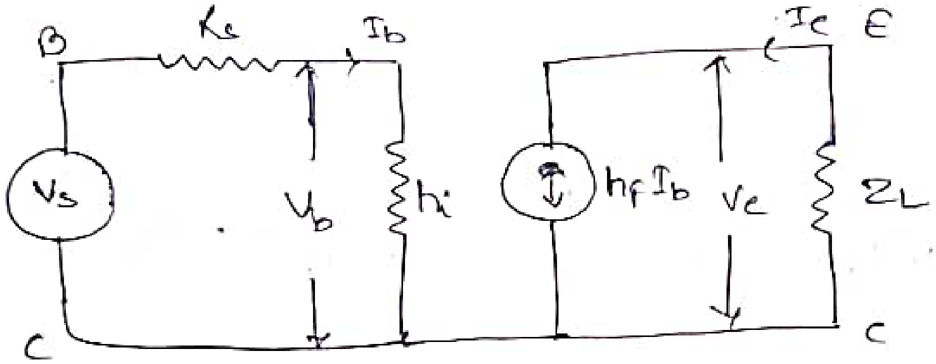
Voltage Gain:

$$A_v = A_i \frac{R_L}{R_i}$$

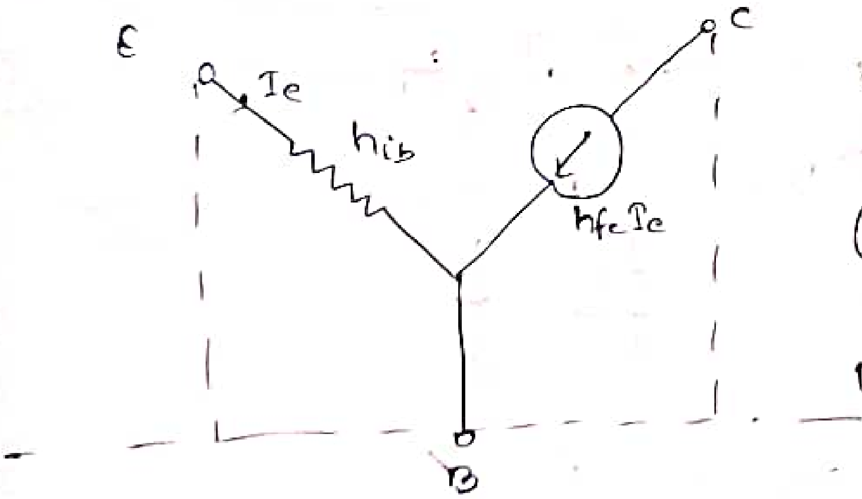
$$= \frac{-h_{fe}R_L}{h_{ie}}$$

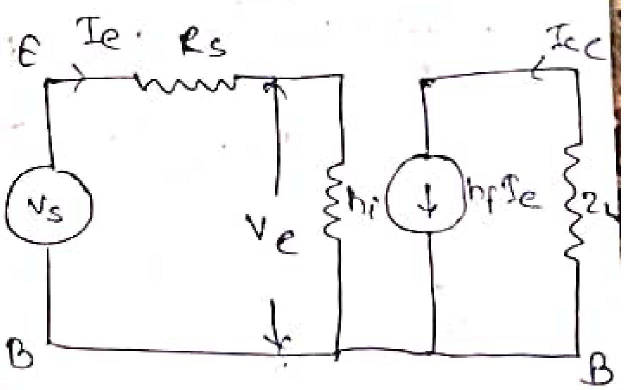
Simplified Hybrid CC Model:





Simplified Hybrid CB Model:





Conversion formula:

h_{ie} h_{ic}

CE $\longrightarrow$ CC

$h_{ic} = h_{ie}$ $h_{rc} = 1$

$h_{fc} = -(1+h_{fe})$ $h_{oc} = h_{oe}$

CB $\rightarrow$ CE

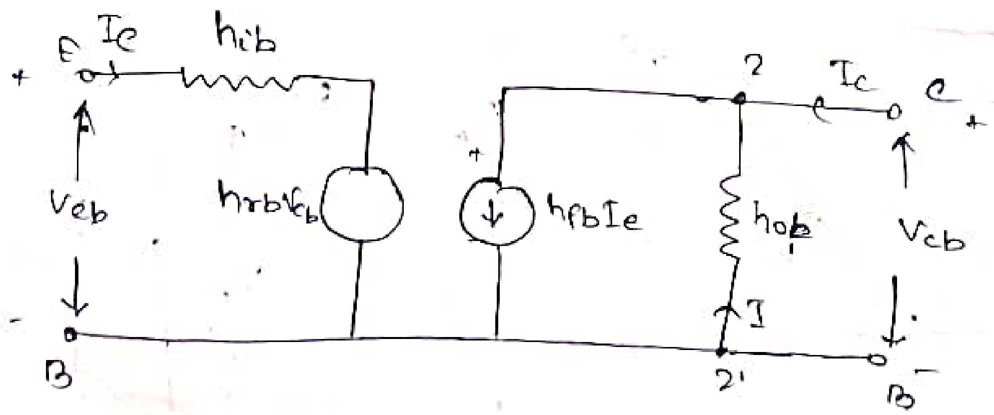
CE $\rightarrow$ CB
(b $\rightarrow$ e)

$$h_{ib} = \frac{h_{ie}}{1+h_{fe}} \qquad h_{rb} = \frac{h_{ie}\,h_{oe}}{1+h_{fe}} - h_{re}$$

$$h_{fb} = \frac{-h_{fe}}{1+h_{fe}} \qquad h_{ob} = \frac{h_{oe}}{1+h_{fe}}$$

Q. Find h_{re} in terms of CB h-parameters:

Ans:



$$h_{re} = \frac{V_{be}}{V_{ce}} \quad (I_b = 0)$$

$$= \left.\frac{V_{bc} + V_{ce}}{V_{ce}}\right|_{I_b=0}$$

$$h_{re} = \left.\left(1 + \frac{V_{bc}}{V_{ce}}\right)\right|_{I_b=0}$$

At node

$$I_e + I_b + I_c = 0$$

$$I_e + I_c = 0$$

$$I_e = -I_c$$

Applying KCL at 2

$$h_{fb} I_e + I_e = I$$

$$\Rightarrow I_e (1 + h_{fb}) = I$$

$$= h_{ob} V_{bc}$$

$$\Rightarrow \boxed{I = I_e (1 + h_{fb}) = h_{ob} V_{bc}}$$

Applying KVL E-B-C-E :

$$I_e h_{ib} + h_{rb} V_{cb} + V_{bc} + V_{ce} = 0$$

$$I_e = \frac{h_{ob} V_{bc}}{1 + h_{fb}}$$

$$\Rightarrow \frac{h_{ob} V_{bc}}{1 + h_{fb}} h_{ib} - h_{rb} V_{bc} + V_{bc} + V_{ce} = 0$$

$$\Rightarrow V_{bc} \left(\frac{h_{ob} h_{ib}}{1 + h_{fb}} - h_{rb} + 1 \right) + \frac{V_{ce}}{V_{bc}}) = 0$$

$$V_{ce} = -V_{bc}\left(\frac{h_{ob}h_{ib}}{1+h_{fb}} - h_{rb} + 1\right)$$

$$\frac{V_{ce}}{V_{bc}} + 1 = -\frac{h_{ob}h_{ib}}{1+h_{fb}} + h_{rb}$$

$$h_{re} = h_{rb} - \frac{h_{ob}h_{ib}}{1+h_{fb}}$$

$$h_{re} = \frac{h_{rb}(1+h_{fb}) - h_{ob}h_{ib}}{(1+h_{fb})}$$

$$\frac{V_{bc}}{V_{ce}} = \frac{-(1+h_{fb})}{h_{ib}h_{ob} + (1-h_{rb})(1+h_{fb})}$$

$$\boxed{h_{re} = 1 + \frac{V_{bc}}{V_{ce}} = \frac{h_{ib}h_{ob} - (1+h_{fb})h_{rb}}{h_{ib}h_{ob} + (1-h_{rb})(1+h_{fb})}}$$ exact expression

$$h_{rb} << 1$$

$$h_{ob}h_{ib} << 1 + h_{fb}$$

$$\boxed{h_{re} \approx \frac{h_{ib}h_{ob}}{1+h_{fb}} - h_{rb}}$$ Approximate expression.

Q. h_{rb} in terms of CE.

H.W.

CE with resistance Re : Stabilization

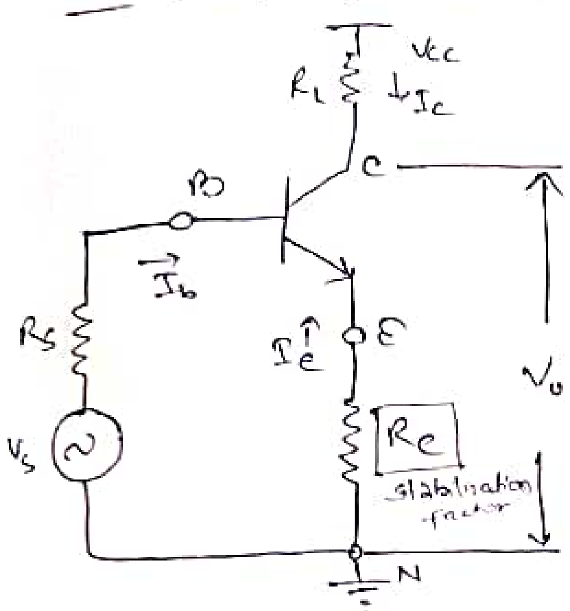


fig. CE Amplifier with emitter resistance

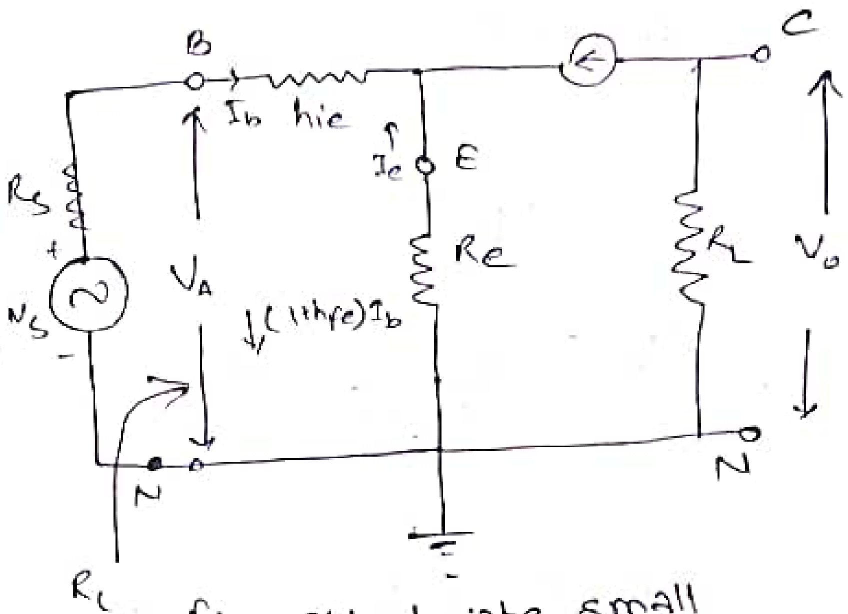


fig. Appropriate small signal equivalent ckt.

Transistor affected by:

→ Aging

→ Temp. of operation

→ ~~Q factor~~ Q point.

$$A_V = -\frac{R_L}{R_e}$$

Approximate Solution

$$A_i = \frac{-I_c}{I_b}$$

$$= \frac{-h_{fe} I_b}{I_b}$$

$$\Rightarrow \boxed{A_i = -h_{fe}} \quad \text{— (i)}$$

$$V_i = h_{ie} I_b + (1+h_{fe}) I_b R_e$$

$$R_i = \frac{V_i}{I_b}$$

$$\boxed{R_i = h_{ie} + (1+h_{fe}) R_e} \quad \text{— (ii)}$$

$$(1+h_{fe}) R_e = 51\,K \ggg h_{ie} \; (1k) \quad \text{— (iii)}$$

$R_e = 1\,k\Omega$

$h_{fe} = 50$

$$\boxed{R_i = f(R_e)} \quad \text{— (iv)}$$

Higher emitter resistance, higher input impedance.

$$\cancel{A_v = \frac{A_i R_L}{R_e} \quad \frac{A_i R_L}{R_i}}$$

$$= \cancel{\frac{R_L \cdot h_{fe}}{h_{ie} + (1+h_{fe})R_e}}$$

$$\cancel{\frac{-h_{fe}}{h_{ie} + 1 + h_{fe}}}$$

$$A_v = A_i \frac{R_L}{R_i}$$

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (1+h_{fe})R_e}$$

$$\approx \frac{-h_{fe} R_L}{(1+h_{fe})R_e}$$

$$\boxed{A_v \approx \frac{-R_L}{R_e}} \quad \text{— } (V)$$

$\Rightarrow$ $A_v \propto R_L$

$\propto 1/R_e$

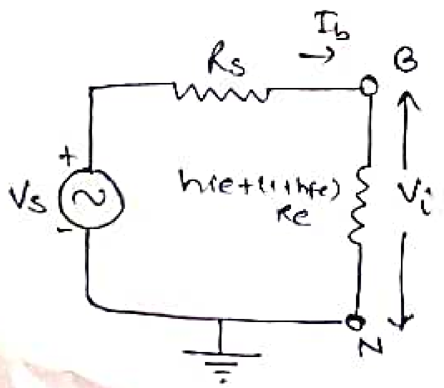
$\not\propto$ $h_{ie}, h_{oe}, h_{re}, h_{fe}$.

Summary Table

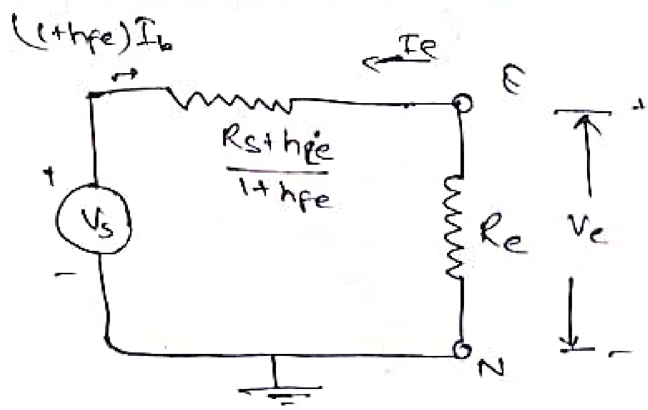
$h_{oe}(R_e + R_L) \le 0.1$

	CE	CE with R_e	CC	CB
A_i	$-h_{fe}$	$-h_{fe}$	$1+h_{fe}$	$-h_{fe} = \frac{h_{fe}}{1+h_{fe}}$
R_i	h_{ie}	$h_{ie} + (1+h_{fe})R_e$	$h_{ie} + (1+h_{fe})R_L$	$h_{ib} = \frac{h_{ie}}{1+h_{ie}}$
A_v	$\frac{-h_{fe}R_L}{h_{ie}}$	$\frac{-h_{fe}R_L}{R_i} = \frac{-R_L}{R_e}$	$1 - \frac{h_{ie}}{R_i}$	$h_{fe}\frac{R_L}{h_{ie}}$
R_o	∞	∞	$\frac{R_o + h_{ie}}{1+h_{fe}}$	∞
R_o'	R_L	R_L	$R_o \parallel R_L$	R_L

Looking into Base and Emitter of Transistor:



(a) Equivalent ckt looking into base



(b) Looking into emitter.

$$\boxed{I_b = \frac{V_s}{R_s + h_{ie} + (1+h_{fe})R_e}} \quad \text{—— (vi)}$$

$$V_e = (1+h_{fe})I_b \times R_e$$

$$= \frac{(1+h_{fe}) \times V_s}{R_s + h_{ie} + (1+h_{fe})R_e} \times R_e$$

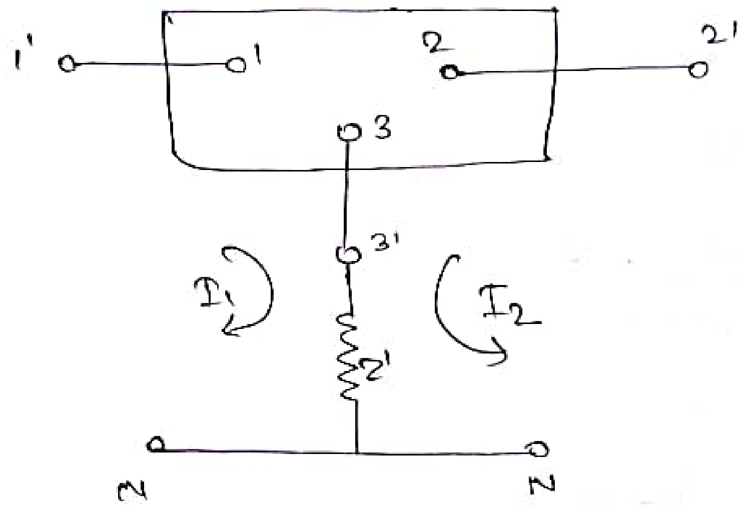
$$\boxed{V_e = \frac{V_s R_e}{\frac{R_e + h_{ie}}{(1+h_{fe})} + R_e}} \quad \text{—— (vii)}$$

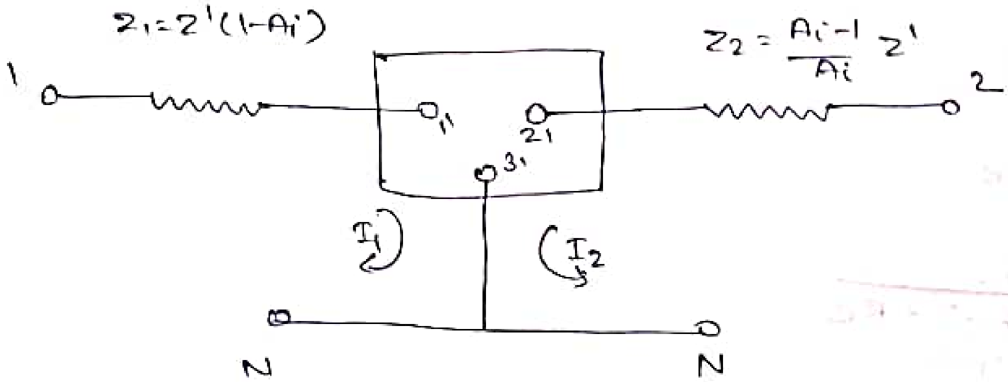
$$I_e = \frac{-V_e}{R_e}$$

$$I_e = \frac{-V_s}{\frac{(R_e + h_{ie})}{(1+h_{fe})} + R_e}$$

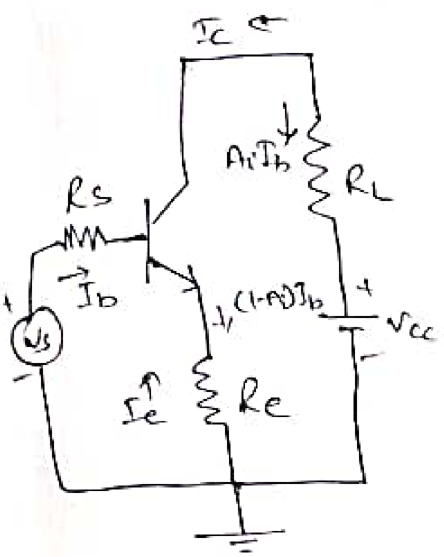
$$\boxed{I_b = \frac{-I_e}{(1+h_{fe})}}$$

Dual of Miller's Theorem:

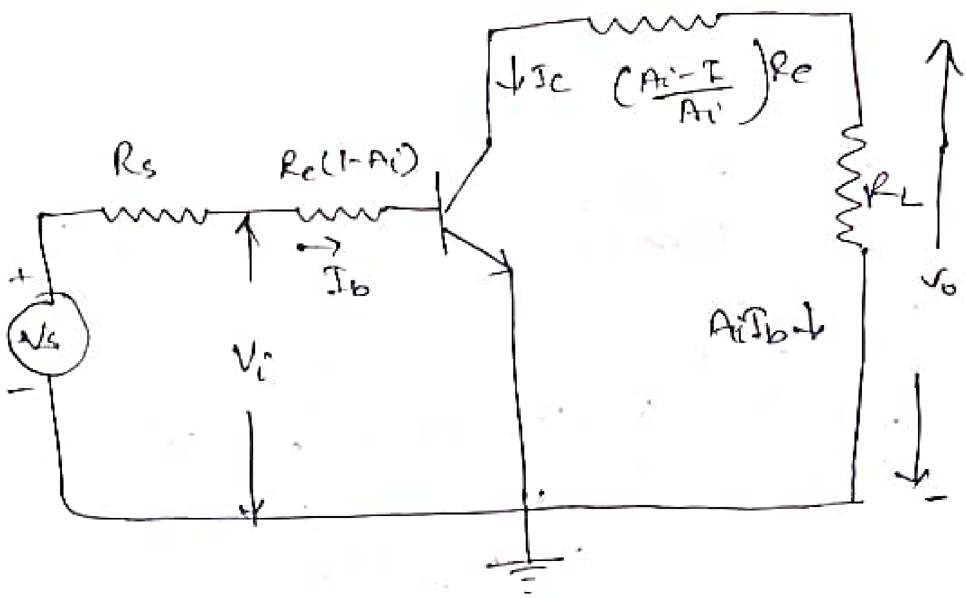




21/08/22 : CE with Re.



Transistor Amplifier stage with ~~unpolarised~~ unbipassed emitter resistor.



Small signal Equ. Circuit. (Miller's)

$A_i = -\frac{I_c}{I_b} = -h_{fe}$

Validity of Approximation:

Effective load impedance, $\boxed{R_L' = R_L + \left(\frac{A_i-1}{A_i}\right)R_e}$ — (VIII)

$h_{oe}R_L' = h_{oe}(R_L+R_e) \leq 0.1$

$A_i = -h_{fe}$

$h_{fe} >> 1$

$A_v = -\frac{R_L}{R_e}$

$A_v >>>$

$R_e << R_L$

i,f → approx
0,r → exact.

The exact solution:

$$A_i = \frac{-h_{fe}}{1+h_{oe}R_L'}$$

$$\boxed{A_i = \frac{-h_{fe}}{1+h_{oe}\left(R_e\left(\frac{A_i-1}{A_i}\right)+R_L\right)}} \quad \text{— (X)}$$

$h_{oe}R_e \ll h_{fe}$

$$A_i \approx -h_{fe}$$

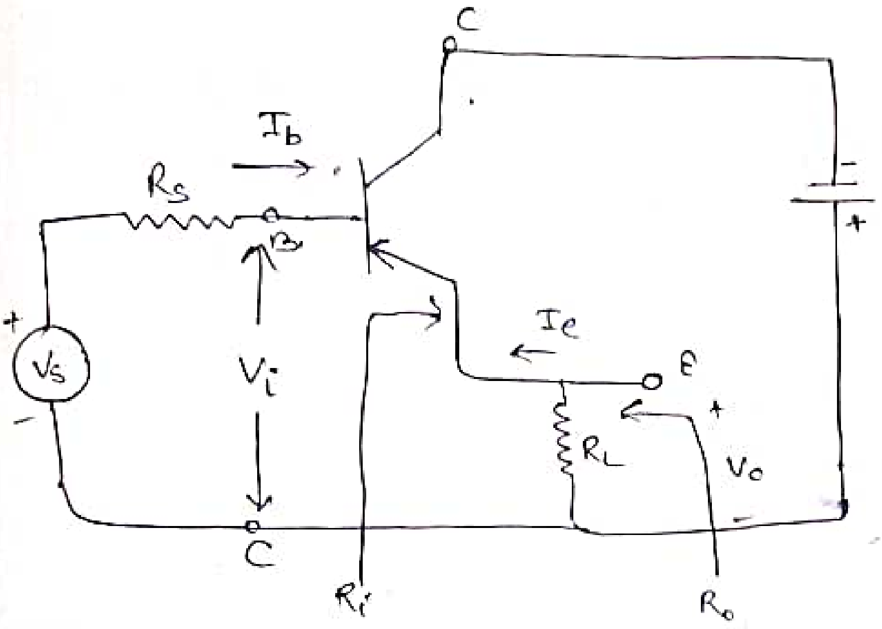
$$\boxed{A_i = \frac{h_{oe}R_e - h_{fe}}{1+h_{oe}(R_L+R_e)}} \quad \text{— (XI)}$$

$$R_i = \frac{V_i}{I_b} = (1-A_i)R_e + h_{ie} + h_{re}A_iR_L' \quad \text{— (XII)}$$

$$A_v = \frac{A_iR_L}{R_i} \quad \text{— (XIII)}$$

$$R_o = \frac{1}{h_{ob}} + (R_s+h_{ie})\left(1+\frac{1}{h_{oe}R_e}\right) \quad \text{— (XIV)}$$

Emitter follower:



CC

$A_v \approx 1$

Re = very high (kΩ)

R_o = Low (Ω)

Impedance matcher

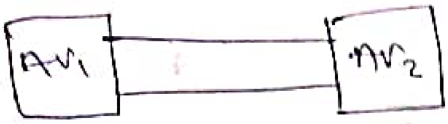
* Buffer circuit
* works upto 500kΩ (Re).
* emitter following change at the base of transistor.
* change to base will change the output.
* No phase shift b/w current and voltage.

$$A_i = -\frac{I_e}{I_b} = \frac{-h_{fc}}{1+h_{oc}R_L} = \frac{1+h_{fe}}{1+h_{oe}R_L} \quad \text{— (i)}$$

$$R_i = \frac{V_i}{I_b} = h_{ic} + h_{rc}A_iR_L = h_{ie} + A_iR_L \quad \text{— (ii)}$$

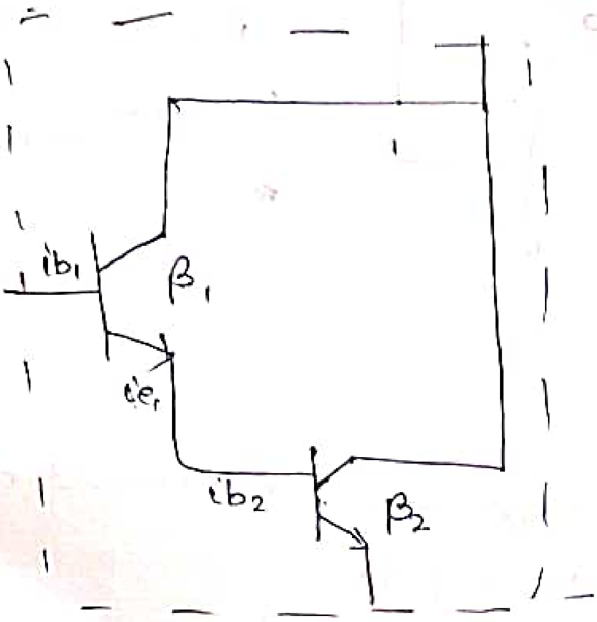
$$A_v = \frac{V_o}{V_i} = \frac{A_iR_L}{R_i} = \frac{R_i - h_{ie}}{R_i} = 1 - \frac{h_{ie}}{R_i} \quad \text{— (iii)}$$

Cascading :



Sydney Darlington
. Bell - lab
1953.

$AV_1 + AV_2$



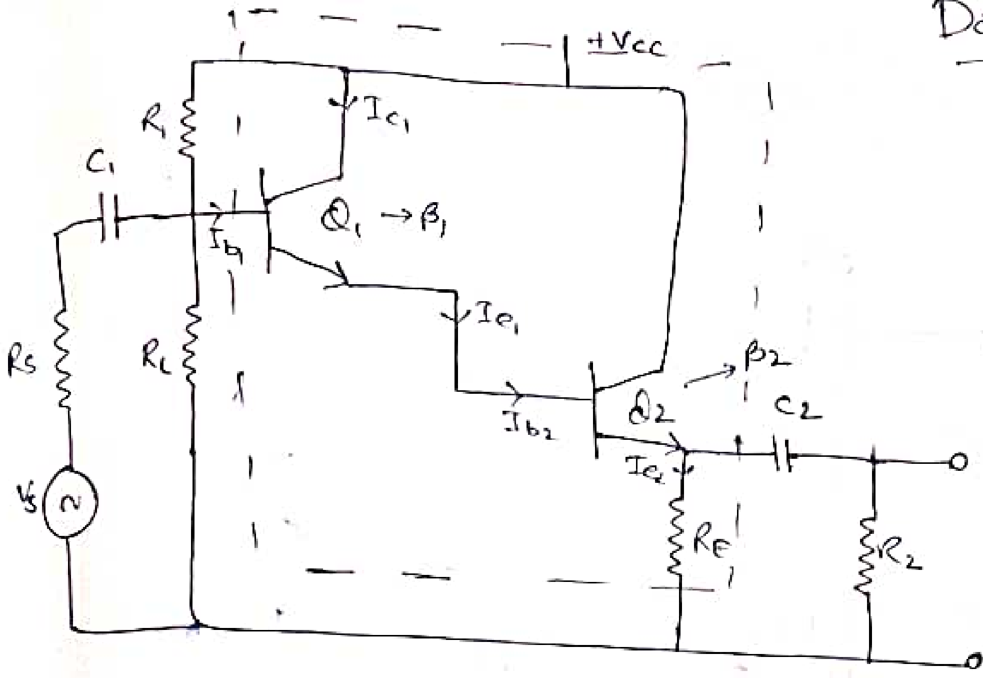
$\beta = \beta_1 \cdot \beta_2$

$= \beta^2$ (for same transistor)

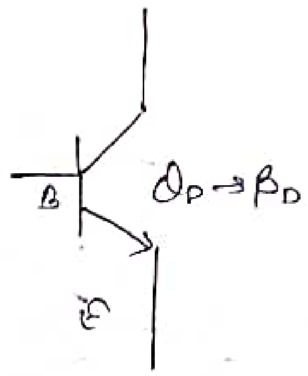
~~Sydney~~ Darlington Pair
or
Super-β. Transistor.

* >>> 500 KΩ for Buffering.
* $R_i \rightarrow$ high
* $R_o \rightarrow$ low
* Dissipates a lot of heat
* $V_{BE} = V_{BE_1} + V_{BE_2}$
* Slow and noisy.

Darlington Pair:



$\Rightarrow$



Current gain $= \dfrac{I_{e_2}}{I_{b_1}}$

$$I_e \approx I_c = \beta I_b$$

current gain $= \dfrac{\beta_2 I_{b_2}}{I_{b_1}}$

$I_{e_1} = I_{b_2}$ (same branch)

$$= \beta_2 \left(\frac{I_{e_1}}{I_{b_1}} \right)$$

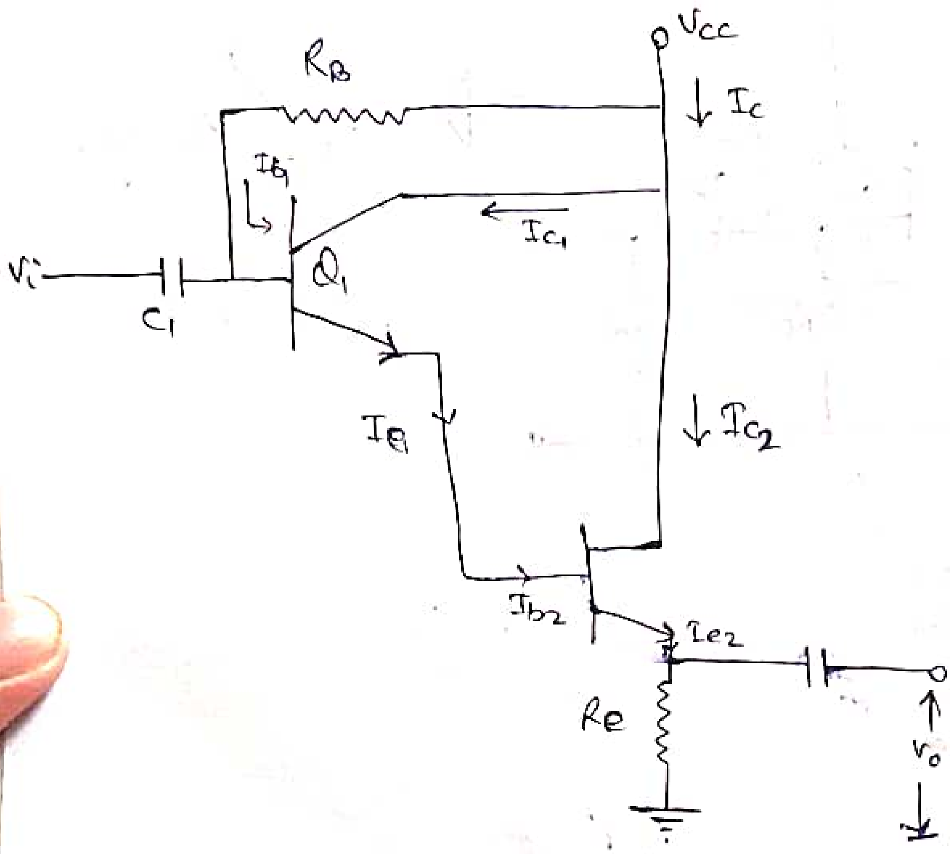
$$= \beta_2 \beta_1 \left(\frac{I_{b_1}}{I_{b_1}} \right)$$

$$\beta_{total} = \beta_1 \beta_2$$

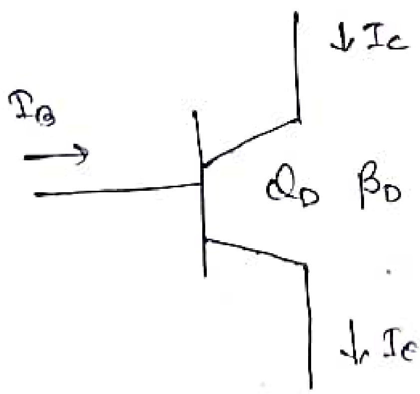
$\beta_1 = \beta_2 = \beta$ (if same transistor)

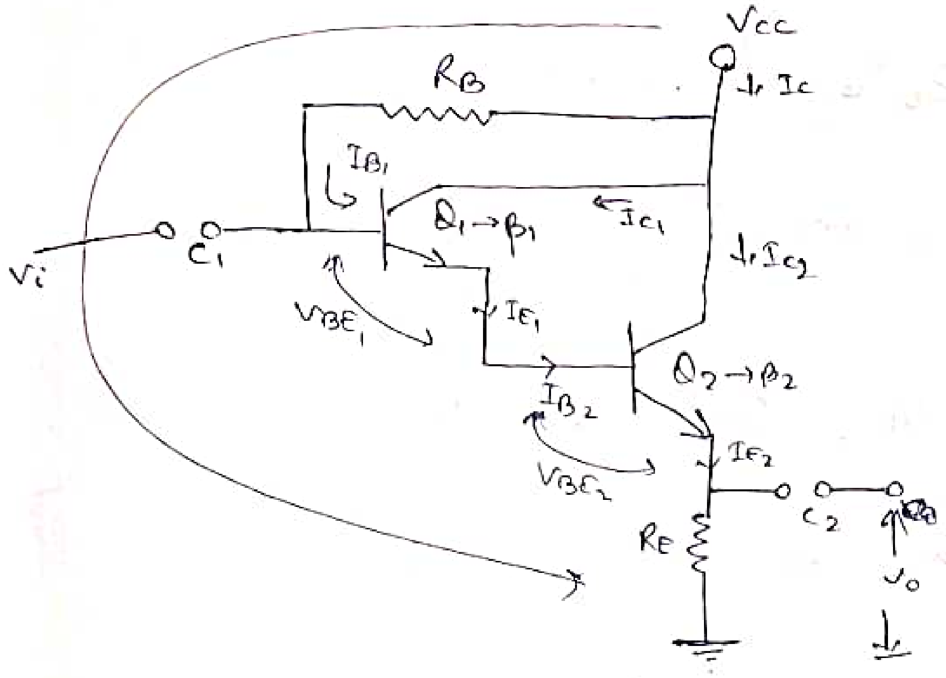
$\rightarrow$ $\boxed{\beta_{total} = \beta^2}$ super-β-Transistor.

DC Analysis of Darlington Pair:



C_1 and C_2 are coupling capacitors.
for DC analysis reactance would be infinity.
Hence capacitor will be open circuited.





O/P voltage $= V_E$

Apply KVL:

$$V_{CC} - I_{B_1}R_B - V_{BE_1} - V_{BE_2} - I_{E_2}R_E = 0 \quad \text{--- (1)}$$

$I_{B_1} = I_B$, $I_{E_2} = I_E$

$I_E = I_C + I_B$

$I_E = I_{C_2} + I_{B_2}$ (with reference to Darlington Pair)

$= \beta_2 I_{B_2} + I_{B_2}$

$\Rightarrow I_E = I_{B_2}(\beta_2 + 1)$

$\approx I_{B_2}\beta_2$

$I_{B_2} = I_{E_1}$ (same branch)

$\Rightarrow I_E = \beta_2 I_{E_1}$

$I_E = \beta_2 \beta_1 I_{B_1}$

$$\Rightarrow \boxed{I_E = \beta_D I_{B_1}} \quad \text{—— (11)}$$

from (1) and (11)

$$V_{CC} - I_{B_1}R_B - V_{BE_1} - V_{BE_2} - \beta_D I_{B_1} R_E = 0$$

$$V_{BE_1} + V_{BE_2} = V_{BE}$$

$$\Rightarrow V_{CC} - I_B R_B - V_{BE} - \beta_D I_B R_E = 0$$

$$\Rightarrow V_{CC} - V_{BE} - I_B (R_B + \beta_D R_E) = 0$$

$$\Rightarrow \boxed{I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E}}$$

Slly:

$$\boxed{I_E = \frac{\beta_D (V_{CC} - V_{BE})}{R_D + \beta_D R_E}}$$

$$\boxed{V_E = V_{BE} = I_E R_E}$$

$$\boxed{V_E = \frac{\beta_D R_E (V_{CC} - V_{BE})}{R_D + \beta_D R_E}}$$

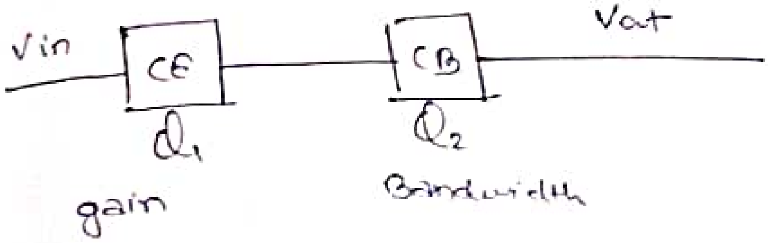
Cascode Amplifier :

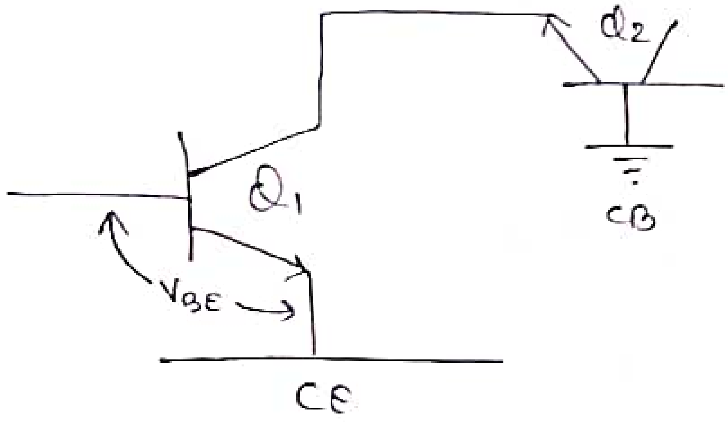
Cascoding provides high gain and high b/w.

Stability high

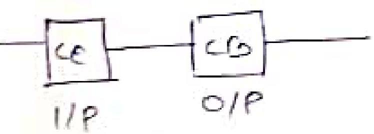
~~Re and Ro high~~

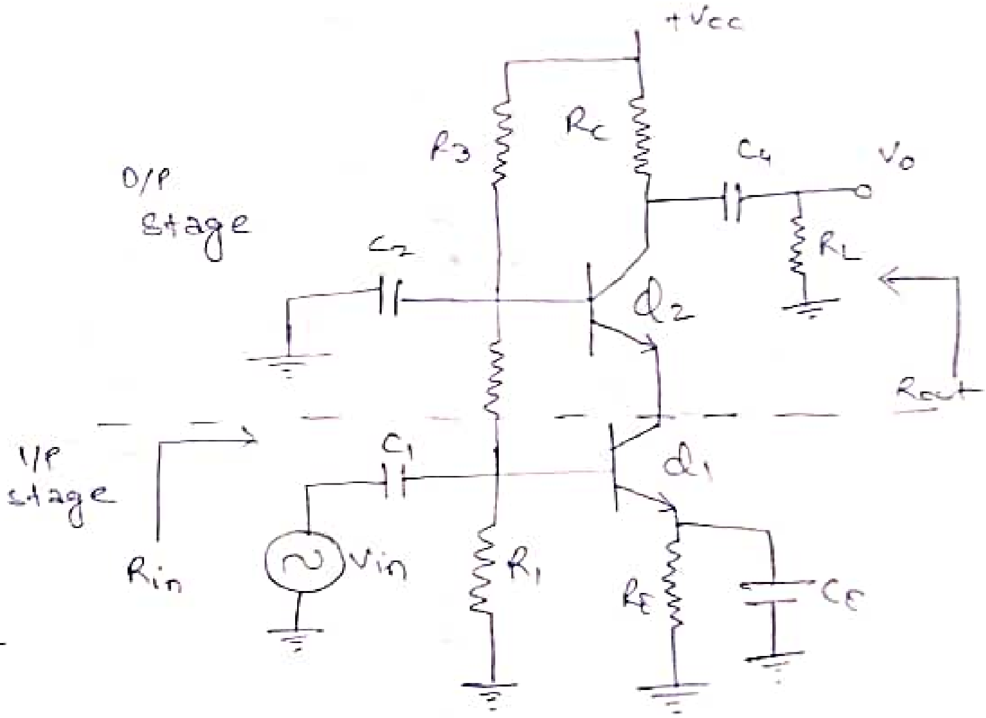
High input and O/P impedance.

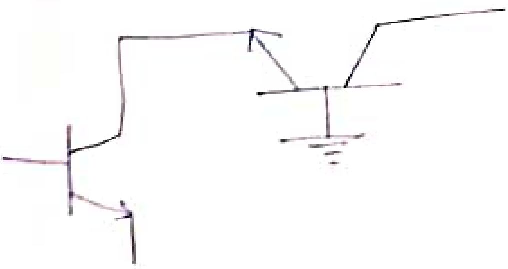




Cascode Amplifiers:







AC Analysis:

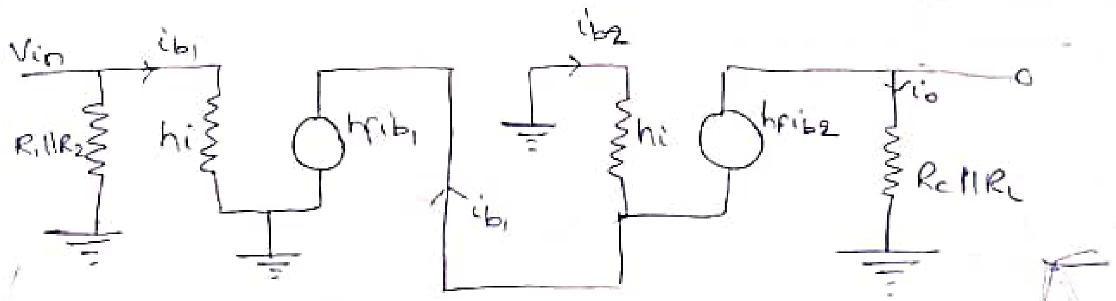
→ Ground each DC component

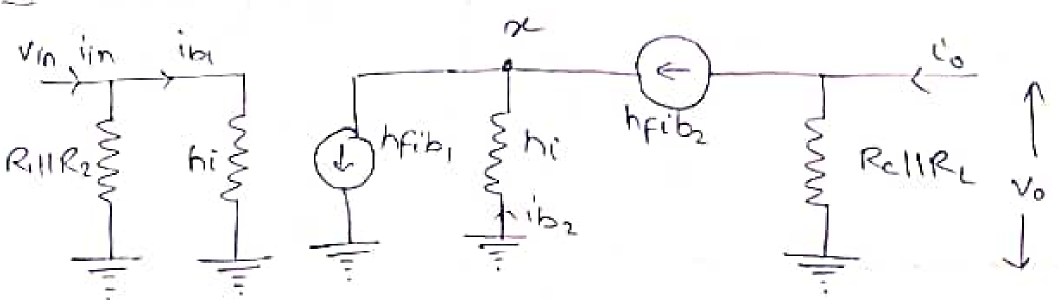
→ Use only AC I/P, or O/P

→ Short ckt all capacitors

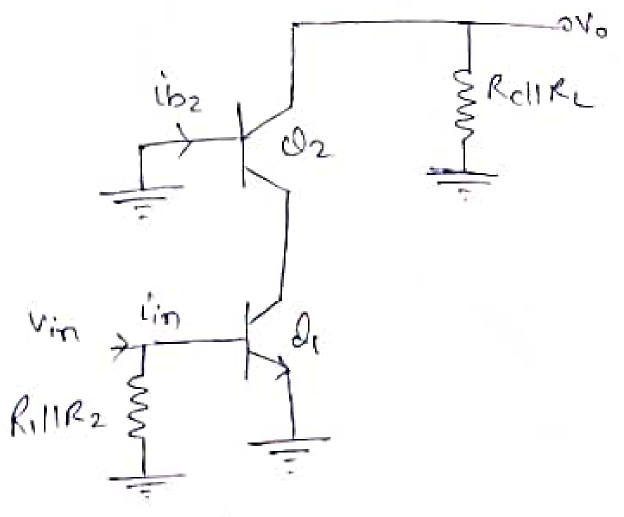
→ Replace transistor with its simplified value/diagram.

→ We will get R_i, R_o, A_i, A_v





(internal impedance very high)



I/P Impedance, $R_i = R_1 \| R_2 \| h_i$

O/P Impedance, $R_o = R_C \| R_L$

$$i_o = -h_f i_{b_2} \quad \text{--- (III)}$$

Nodal analysis at x

$$i_{b_2} + h_f i_{b_2} = h_f i_{b_1}$$

$$i_{b_2}(h_f + 1) = h_f i_{b_1}$$

$$i_{b_2} = \frac{h_f i_{b_1}}{h_f + 1} \quad \text{--- (IV)}$$

$$i_{b_1} = \frac{V_{in}}{h_i} \quad \text{--- (V)}$$

Putting (V) into (IV)

$$i_{b_2} = \frac{h_f}{h_f + 1}\left(\frac{V_{in}}{h_i}\right)$$

Put i_{b_2} in (III)

Hence $$i_o = \frac{-h_f^2 V_{in}}{h_i(1 + h_f)} \quad \text{--- (VI)}$$

$$V_o = i_o(R_C \| R_L)$$

$$= \frac{-h_f^2 (R_C \| R_L)}{h_i(1 + h_f)}$$

$$\boxed{A_v = \frac{V_o}{V_{in}} = \frac{-h_f^2 (R_C \| R_L)}{h_i(1 + h_f)}}$$

for current gain:

$$i_{b1} = \left(\frac{R_1 || R_2}{h_i + R_1 || R_2} \right) i_{in} \quad \text{--- (VII)}$$

from (VII) and (IV)

$$i_{b2} = \frac{h_f [R_1 || R_2] \, i_{in}}{(1+h_f)(h_i + R_1 || R_2)} \quad \text{--- (VIII)}$$

from (III)

$$i_{b2} = \frac{-i_o}{h_f}$$

~~from~~

$$\Rightarrow \frac{-i_o}{h_f} = \frac{h_f \, i_{in} \, [R_1 || R_2]}{(1+h_f)(h_i + R_1 || R_2)}$$

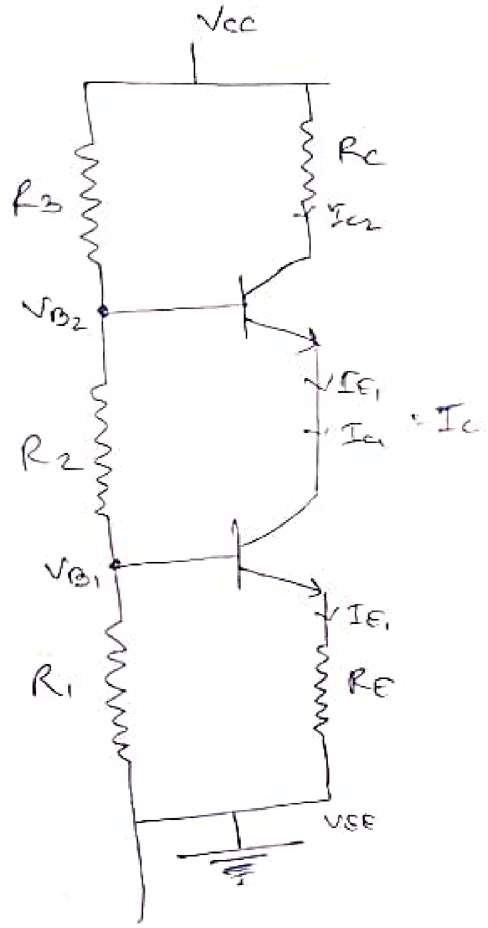
$$\Rightarrow \boxed{\frac{-i_o}{h_f} = \frac{-h_f^2 \, (R_1 || R_2)}{(1+h_f)(R_1 || R_2 + h_i)} = A_i}$$

DC Analysis:

→ DC component (+Vcc)

→ AC component nullify

→ Capacitor O.C.



$$V_{B_1} = \left[\frac{R_1}{R_1+R_2+R_3}\right](V_{cc}+V_{EE}) \quad \text{——— (i)}$$

$$V_{B_2} = \left[\frac{R_1+R_2}{R_1+R_2+R_3}\right](V_{cc}+V_{EE}) \quad \text{——— (ii)}$$

$$I_{B_1} = I_{B_2} = 0$$

$$I_{C_1} = I_{C_2} = I_{E_1} = I_{E_2} = I_C$$

Applying KVL in lower loop

$V_{B_1} = V_{BE} + I_E R_E - V_{EE}$

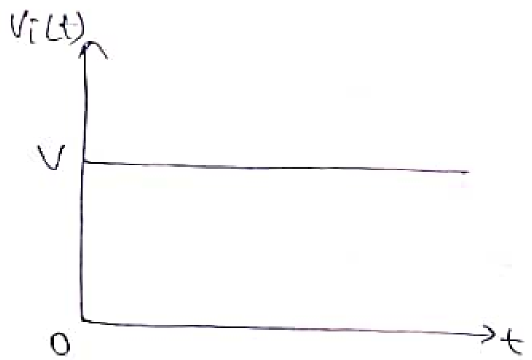
$$\boxed{I_E = \frac{V_B - V_{BE} + V_{EE}}{R_E}}$$

Linear Wave shaping:

* Sinosidal
* Non-sinosidal

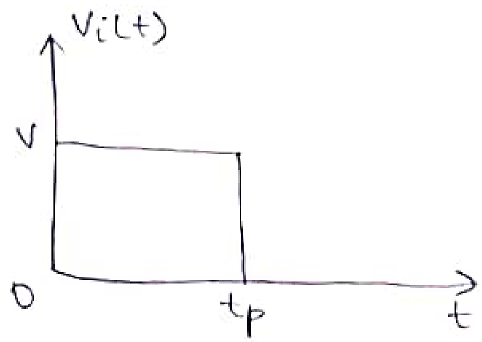
Non-sinosidal:

(i) Step Signal



$V_i(t) = V$
$t > 0$
$V_i(t) = 0$, otherwise

(ii) Pulse I/P

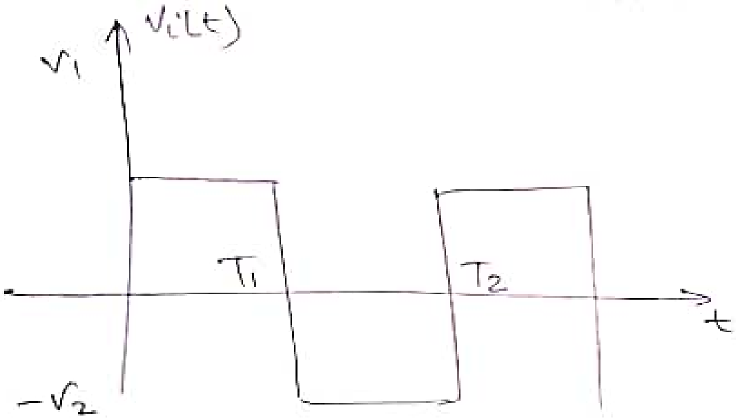


$V_i(t) = V \quad 0 \le t \le t_p$
$= 0$, otherwise

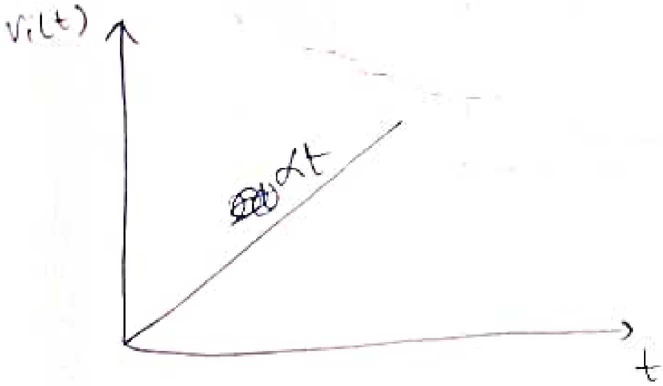
(iii) Square wave i/p

$V_i(t) = V_1 \quad t \le T_1$

$= -V_2 \quad t < T_2$ and $t > T_1$



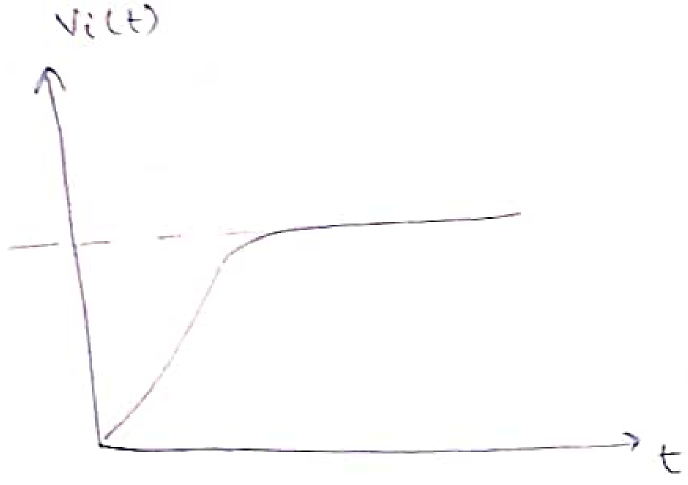
(iv) Ramp i/p



$V_i(t) = \alpha t \quad , t \ge 0$

$= 0 \quad ,$ otherwise

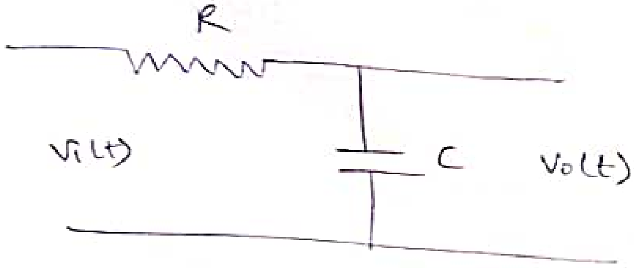
1) Experimental I/P



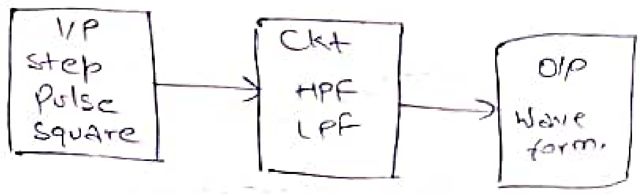
$$V_i(t) = V(1-e^{-t/\tau}) \; ; \; t \geq 0$$
$$= 0 \; ; \; t<0$$

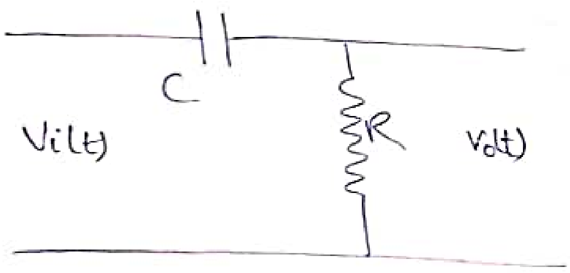
$\tau \rightarrow$ time constant.

31/08/2022



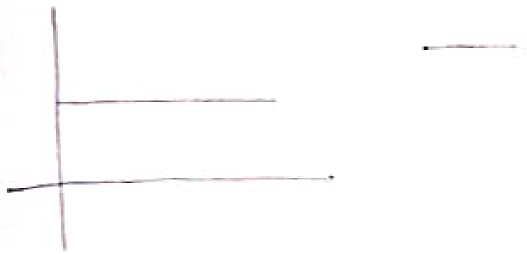
Low pass filter



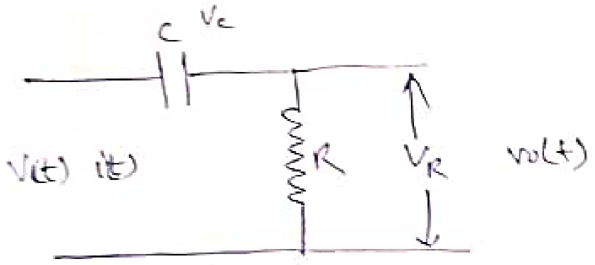


High Pass filter

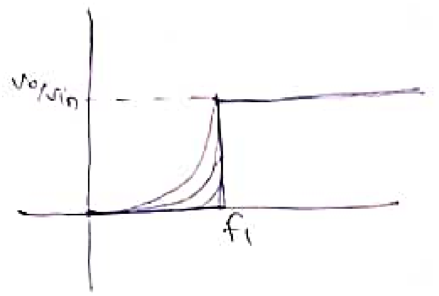
(i) Step Signal



High Pass Circuit



$V_i(t) = V_c + V_R$



Capacitance Reactance

$X_C = \frac{1}{\omega C}$

$X_C = \frac{1}{2\pi f C}$

$f_1 \rightarrow$ cut off frequency

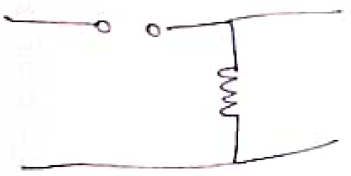
$f_1 = \frac{1}{2\pi RC}$

$f_1 >$ pass (gain increases)

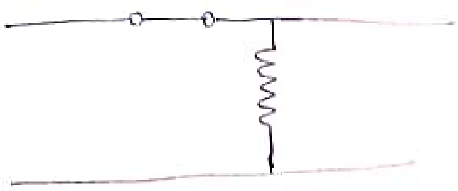
$f_1 <$ rejected (gain 0)

$f < f_1 \Rightarrow X_C \uparrow$

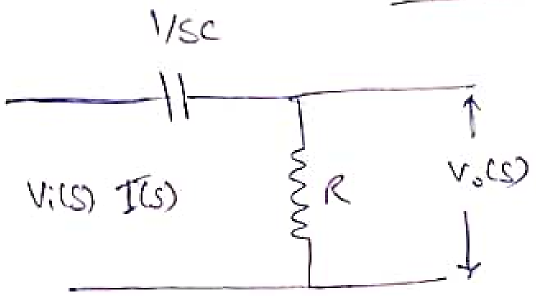
$X_C \propto \frac{1}{f}$



$f > f_1 \Rightarrow X_C \downarrow$



Laplace Transformation:



$$V_i(s) = \left(R + \frac{1}{SC}\right) I(s) \quad \text{—— (1)}$$

$$V_o(s) = I(s) R$$

A = Transfer function

$$A = \frac{V_o(s)}{V_{in}(s)} = \frac{I(s) R}{(R + 1/SC)\, \cancel{I(s)}}$$

$$= \frac{R}{R + 1/SC}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + 1/RSC}}$$

Putting, $S = J\omega$

$$\boxed{A = \frac{1}{1 + \frac{1}{RCJ\omega}}}$$

$$\omega = 2\pi f$$

$$A = \frac{1}{1 + \frac{1}{RC2\pi f J}}$$

$$= \frac{1}{1 + \frac{J}{RC2\pi f J^2}} = \frac{1}{1 - \frac{J}{2\pi RCf}}$$

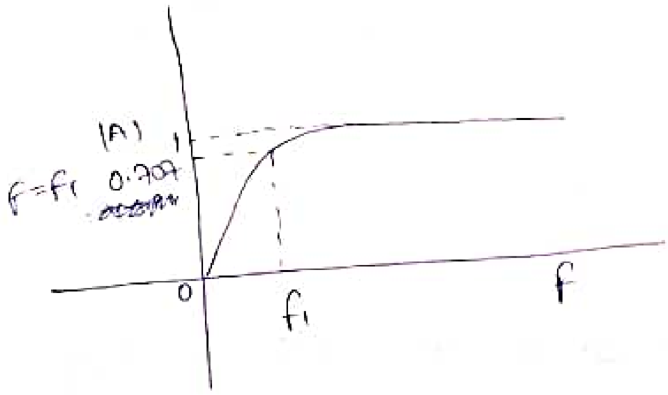
$$A = \frac{1}{1 - \frac{jf_1}{f}} \qquad \left(\because f_1 = \frac{1}{2\pi RC} \right)$$

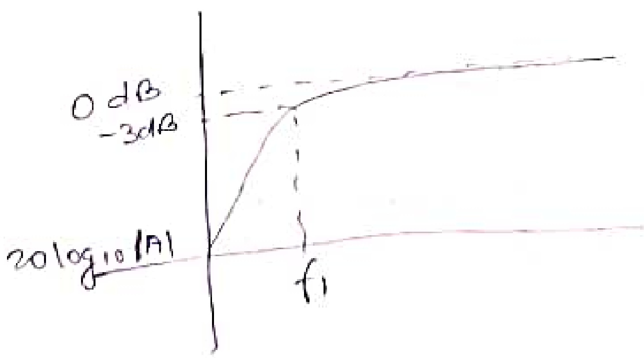
$$\Rightarrow \boxed{A = \frac{1}{1 - j\left(\frac{f_1}{f}\right)}}$$

$$\boxed{|A| = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}}$$

$f = 0 \Rightarrow A = 0$

$f = \infty \Rightarrow A = 1$

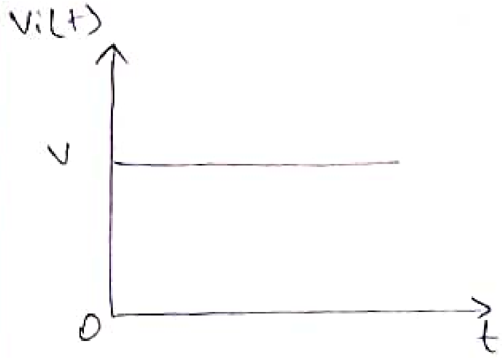




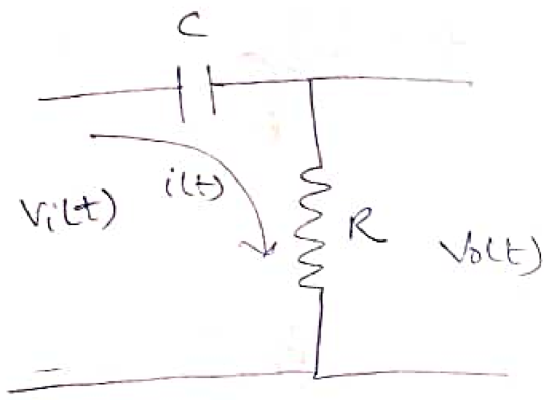
$$\frac{V_o(s)}{V_i(s)} = A\angle\theta$$

$V_i = V_m \sin\omega t \qquad V_o = V_m \sin(\omega t + \theta)$

High Pass RC circuit with step I/P



$\Rightarrow$



Analysis of O/P

→ Diff. eqn. method

→ Laplace method.

Case	Condition	V_i	V_c	V_R
1.	$t=0$	V	0	V
2.	$t>0$	V	expon. increases	exponentially decreases
3	$t=\infty$ (steady state conditions)	V	V	0

Method 1 : Expression for o/p voltage (Differenti method).

Applying KVL:

$$V_i(t) = \frac{1}{C}\int i(t)\,dt + i(t)R \quad \text{—— (1)}$$

Diff. wrt dt

$$\frac{dV}{dt} = \frac{1}{C} i(t) + \frac{di(t)R}{dt}$$

$$\Rightarrow R\frac{di}{dt} + \frac{1}{C} i = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{i}{RC} = 0$$

$$D = d/dt$$

$$\Rightarrow Di + \frac{i}{RC} = 0$$

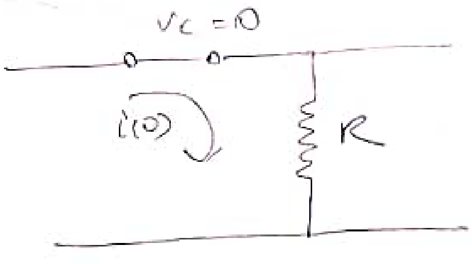
$$= \underbrace{\left(D + \frac{1}{RC}\right)}_{D = -1/RC} i = 0$$

$$i(t) = C_1 e^{-t/RC} \quad \text{——} \quad (\text{ii})$$

C_1 can be found through initial conditions

$t = 0,$

$i(0) = \frac{V}{R}$



$$\Rightarrow \boxed{\frac{V}{R} = C_1}$$

$$\Rightarrow \boxed{i(t) = \frac{V}{R} e^{-t/RC}} \quad \text{——} \quad (\text{iii})$$

$V_0 = i(t) R$

$= \frac{V}{\cancel{R}} e^{-t/RC} \times \cancel{R}$

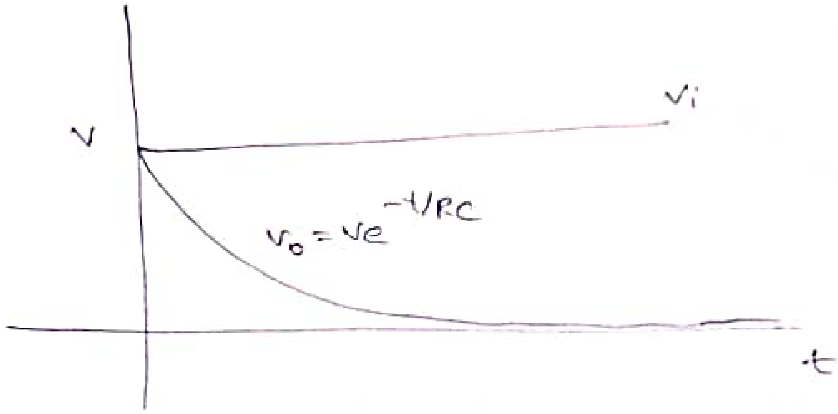
$\Rightarrow \boxed{V_0 = V e^{-t/RC}}$ —— (IV)

At $t = 0$

$V_0 = V$

At $t = \infty$

$V_0 = 0$



Method 2: Using Laplace Transformation Method:

for HPF,

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+1/sRC} = \frac{s}{s+\frac{1}{RC}} \quad \text{—— (i)}$$

$$V_i(t) = V$$

Laplace Transform of I/P signal (step)

$$V_i(s) = \frac{V}{s}$$

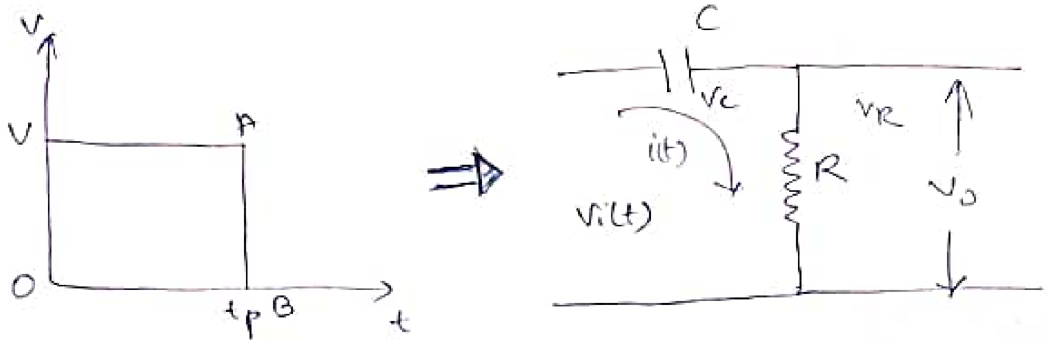
$$V_o(s) = \left(\frac{V}{\cancel{s}}\right)\left(\frac{\cancel{s}}{s+\frac{1}{RC}}\right)$$

$$\boxed{V_o(s) = \frac{V}{s+1/RC}}$$

Now, taking Inverse Laplace Transform:

$$\boxed{V_o(t) = Ve^{-t/RC}} \quad \text{—— (vi)}$$

High Pass RC Ckt with Pulse I/P



Diff. Method:

Case I $t=0$, $V_i = V$, $V_C = 0$, $V_R = V$

Case II $0 < t < t_p$, $V_i = V$, ~~VR~~ V_C = starts charging exponentially

V_R = starts decreasing exponentially with eqn.

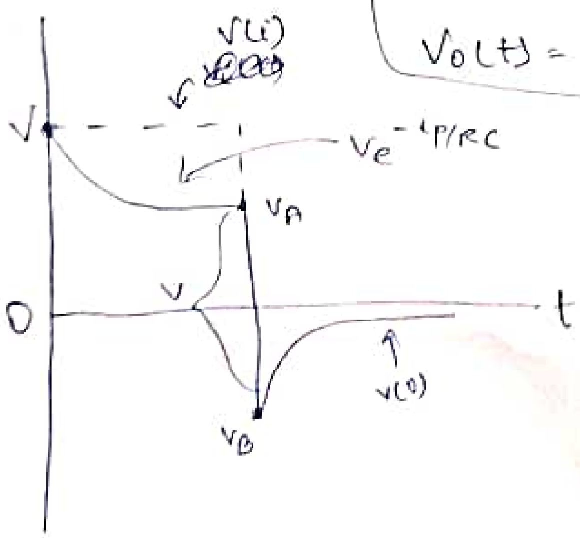
$$Ve^{-t/\tau} = Ve^{-t/RC}$$

Case III $t = t_p$, $V_i = V$, ~~V_C ... V_R~~
(upper corner) $V_A = Ve^{-t_p/RC}$

Case IV $t = t_p$ $V_i = V$ $V_B = V_A - V$
(lower corner) $= V(e^{-t_p/RC} - 1)$

Case V $t > t_p$ $V_i = 0$ ~~$V_o = V_B e^{(t-t_p)/RC}$~~
(steady state) $V_o(t) = V_B e^{-(t-t_p)/RC}$

$$\boxed{V_o(t) = V(e^{(-t_p/RC)} - 1)\, e^{-(t-t_p)/RC}}$$



Method 2: Using Laplace Transform Method:

for HPF,

$$\frac{V_o(s)}{V_i(s)} = \frac{s}{s+\frac{1}{RC}}$$

$$\Rightarrow V_o(s) = \left(\frac{s}{s+\frac{1}{RC}}\right) V_i(s) \quad \text{— (ii)}$$

$$V_i(s) = \int_0^{\infty} v_i(t) e^{-st} dt$$

$$= \int_0^{t_p} V e^{-st} dt$$

$$= V \int_0^{t_p} e^{-st} dt$$

$$= \frac{-V}{s} \left[e^{-st}\right]_0^{t_p}$$

$$= \frac{-V}{s} \left[e^{-st_p} - e^{0}\right]$$

$$= \frac{-V}{s} \left[e^{-st_p} - 1\right]$$

$$\Rightarrow V_i(s) = \frac{-V}{s} (e^{-st_p} - 1)$$

$$= \frac{V}{s} (1 - e^{-st_p}) \quad \text{— (iii)}$$

from (ii) and (iii)

$$V_o(s) = \left(\frac{\cancel{s}}{s+1/RC}\right) \frac{V}{\cancel{s}} (1 - e^{-st_p})$$

$$= \left(\frac{V}{s+1/RC}\right) (1 - e^{-st_p})$$

$$V_o(s) = V\left[\frac{1}{s+1/RC} - \frac{e^{-st_p}}{s+1/RC}\right] \quad \text{— (iv)}$$

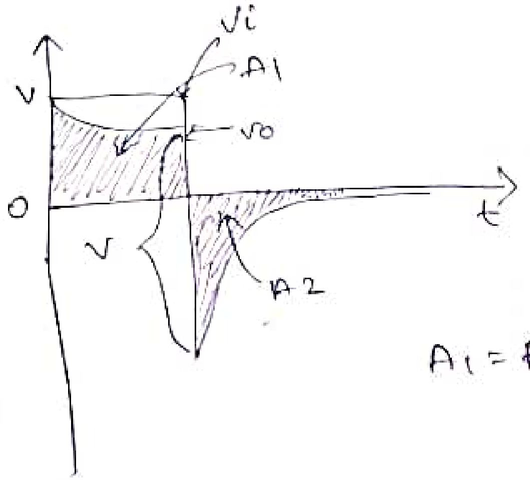
Taking reverse laplace Transform:

$$\boxed{V_o(t) = V\left[e^{-t/RC} - e^{-(t-t_p)/RC}\right]}$$

$$\boxed{V_o(t) = V\left(e^{-t_p/RC} - 1\right) e^{-(t-t_p)/RC}} \quad \text{— (V)}$$

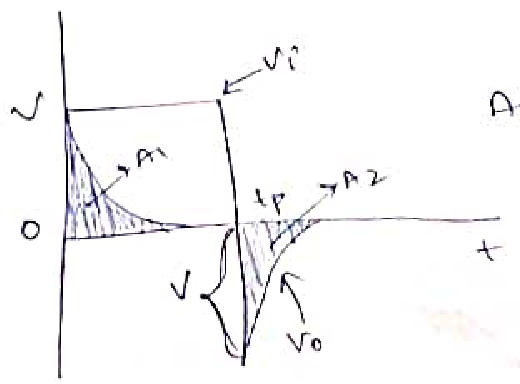
(i) $\tau = RC$

$RC \gg t_p$



$A_1 = A_2$

(ii) $RC \lll t_p$



$A_1 = A_2$

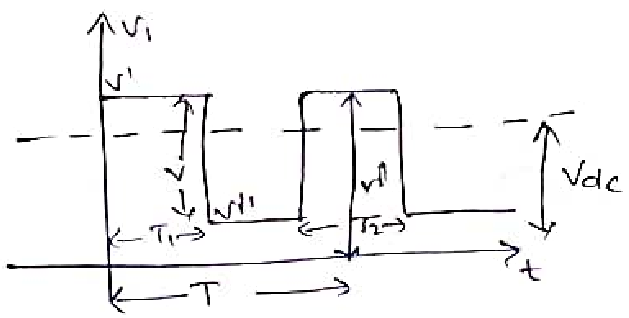
for larger time constant RC >>> tp,

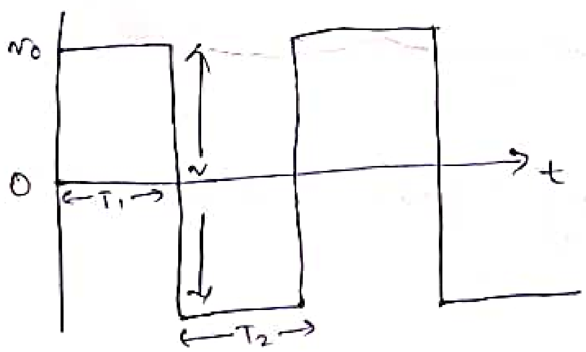
there is only a slight tilt to the o/p Pulse and under is very small.

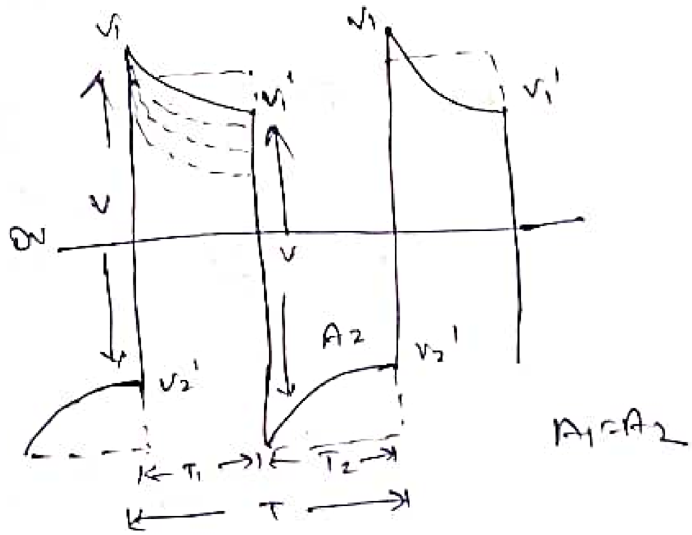
However, the -ve portion decreases very slowly.

for small time constant RC <<< tp, the o/p consists of a +ve spike or pip of amplitude V at the beginning of pulse and a -ve spike of same size at the end of pulse.

High pass RC circuit with square wave I/P:







RC >> T

$$V_1' = V_1 e^{-T_1/RC} \quad \text{---} \; (1)$$

$$V_1' - V_2 = V \quad \text{---} \; (2)$$

$$V_2' = V_2 e^{-T_2/R_2} \quad \text{---} \; (3)$$

$$V_1 - V_2' = V \quad \text{---} \; (4)$$

$$T_1 = T_2 = \frac{T}{2}$$

$$V_1' = -V_2'$$

$$V_1 = -V_2$$

from (2)

$$V_1' + V_1 = V$$

$$V_1 e^{-T_1/RC} + V_1 = V$$

$$V_1 \left[1 + e^{-T_1/RC}\right] = V$$

$$V_1 = \frac{V}{1 + e^{-T_1/RC}} = \frac{V}{1 + e^{-T/2RC}} \quad \text{---} \; (5)$$

$$e^{x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$$

As $RC >> T/2 = \frac{T}{2RC} << 1$

$$V_1 = \frac{V}{1 + 1 - \frac{T}{RC}} \quad \text{(neglecting higher order)}$$

$$= \frac{V}{2 - \frac{T}{RC}} = \frac{V}{2\left(1 - \frac{T}{4RC}\right)}$$

$$= \frac{V}{2}\left(1-\frac{T}{4RC}\right)^{-1}$$

$$V_1 = \frac{V}{2}\left(1+\frac{T}{4RC}\right) \quad \text{--- (6)}$$

$$V_1' = V_1 e^{-T/RC} = V_1 e^{-T/2RC}$$

$$V_1' = \frac{Ve^{-T/2RC}}{1+e^{-T/2RC}}$$

dividing num. and denominator by $e^{-T/2RC}$

$$= \frac{V}{1+e^{T/2RC}}$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+ \cdots$$

$$= \frac{V}{1+1+T/2RC} \quad \text{(Neglecting higher order)}$$

$$V_1' = \frac{V}{2\left(1+\frac{T}{4RC}\right)}$$

$$V_1' = \frac{V}{2}\left(1+\frac{T}{4RC}\right)^{-1} \quad \text{--- (7)}$$

Now % tilt is given by

$$\%\ \text{tilt} = \frac{V_1-V_1'}{V/2} \times 100\%$$

$$= \frac{\frac{V}{2}\left(1+\frac{T}{4RC}\right) - \frac{V}{2}\left(1-\frac{T}{4RC}\right)}{V/2} \times 100\%$$

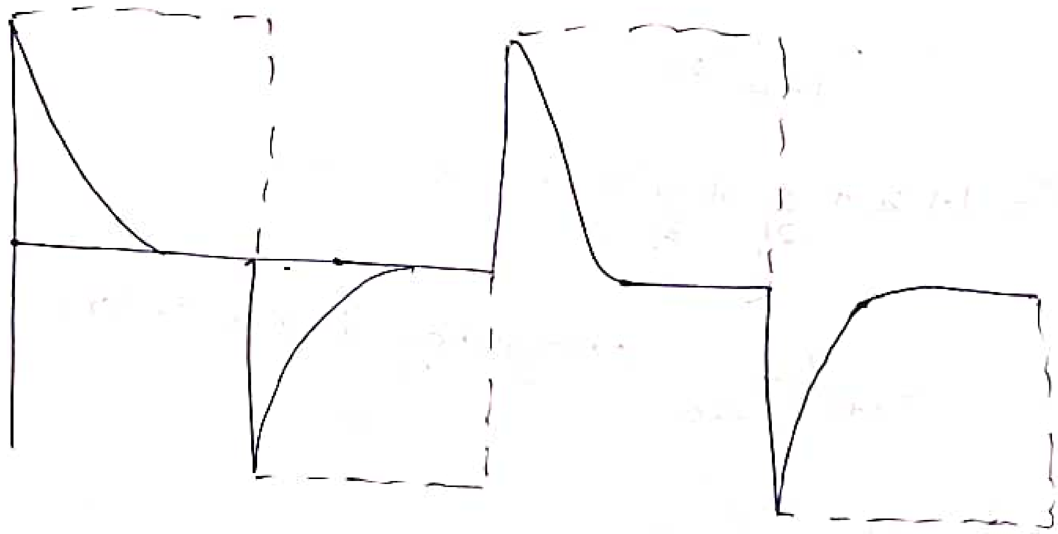
$$\boxed{\%\ \text{tilt} = \frac{T}{2RC} \times 100\%} \quad \text{--- (10)}$$

$$= \frac{1}{2fRC} \times 100\% \qquad (T = 1/f)$$

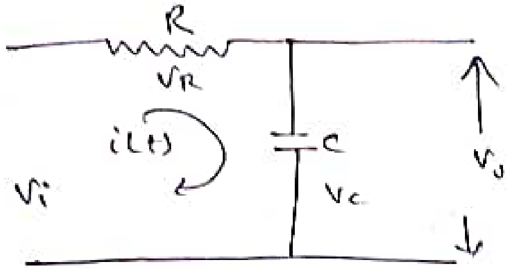
$$= \frac{T}{2fRC} \times 100\%$$

$$\boxed{\% \text{ tilt} = \frac{\pi f_1}{f} \times 100\%}$$

$f_1 \rightarrow$ cut off frequency.
(-3dB)



Low Pass RC Circuit



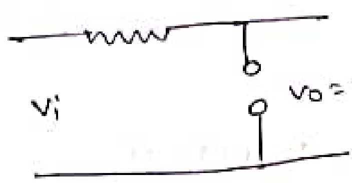
$$f_1 = \frac{1}{2\pi RC}$$

Capacitive Reactance

$$X_L = \frac{1}{\omega C}$$
$$= \frac{1}{2\pi f C}$$

$f\uparrow$ $X_c\downarrow$ short ckt

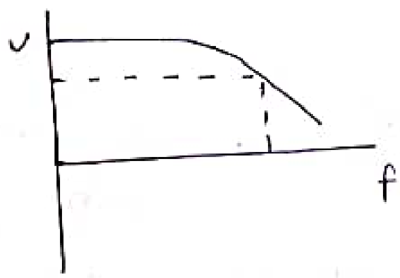
$f\downarrow$ $X_c\uparrow$ open ckt

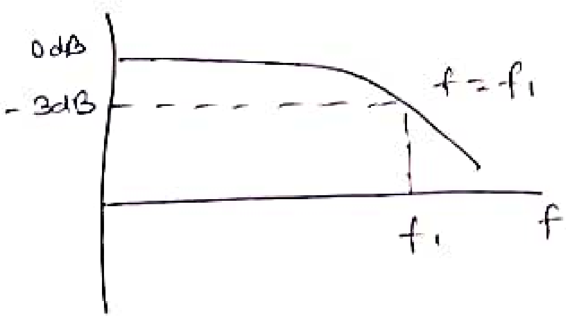


(i) $f_1 << f$

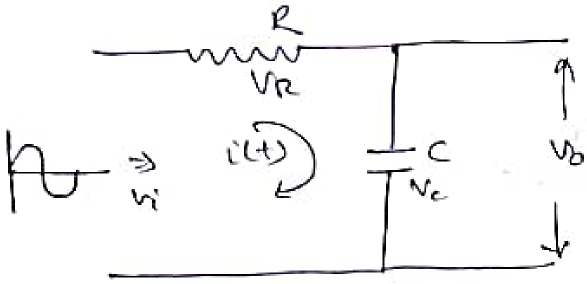
(ii) $f_1 >> f$

(iii) $f_1 = f$





Low pass RC circuit with sinusoidal input:



$v_i(t) = V_m \sin \omega t$

$V_i(s) = \left(R + \frac{1}{sC}\right) I(s)$

$I(s) = \frac{V_i(s)}{R + \frac{1}{sC}}$

$$= \frac{V_i(s)}{R\left(1 + \frac{1}{RSC}\right)} \quad \text{— (I)}$$

$$V_o(s) = \frac{1}{SC} I(s) \quad \text{— (II)}$$

Transfer function:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + SRC} = \frac{1}{1 + j2\pi fRC} \quad \text{— (III)}$$

Polar form $\frac{V_o(s)}{V_i(s)} = A \angle \theta$ — (IV)

A = gain

A is the mag. of steady state gain.

$\angle\theta$ = o/p leading i/p

$$A = \frac{1}{\sqrt{1 + (f/f_2)^2}}$$

$$f_2 = \frac{1}{2\pi RC}$$

$$\theta = -\tan^{-1}\left(\frac{f}{f_2}\right) \quad \text{— (VI)}$$

Applying (v) and (vi) o/p signal is given by,

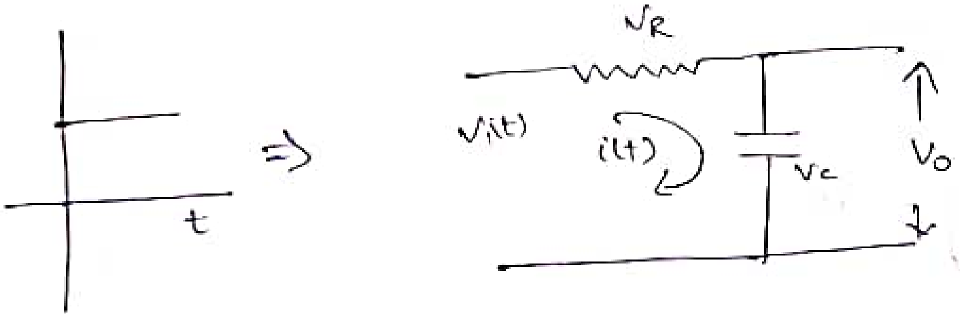
$$V_o(t) = A V_m \sin(\omega t + \theta) \quad \text{— (vii)}$$

As the phase angle θ is negative

Hence the o/p voltage $v_o(t)$ lags behind i/p signal $v_i(t)$ by a phase angle θ.

The gain A falls to 0.707 of its low frequency value at freq. f_2. Hence f_2 is called upper 3dB freq.

Low Pass RC circuit with step I/P:



$$V_i(t) = V_R + V_c$$

$$V_i(t) = i(t)R + \frac{1}{C}\int i(t)\,dt \quad \text{— (ii)}$$

$$V = i(t)R + \frac{1}{C}\int i(t)\,dt \quad \text{— (iii)}$$

diff. eqn (iii) wrt t, we have

$$R\frac{di}{dt} + \frac{1}{C}i = 0 \quad \text{— (iv)}$$

$$\boxed{\frac{di}{dt} + \frac{1}{RC}i = 0} \quad \text{— (v)}$$

first order diff. eqn.

$$\frac{d}{dt} = D$$

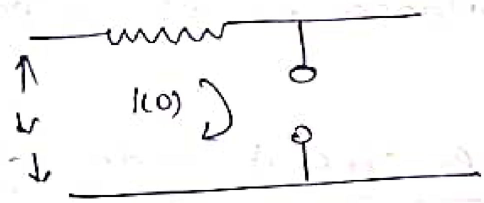
$$\left(D + \frac{1}{RC}\right) i = 0 \quad \text{— (vi)}$$

$$D = -\frac{1}{RC} \quad \text{(possible root)} \quad \text{— (vii)}$$

$$i(t) = C_1 e^{-t/RC} \quad \text{— (viii)}$$

At $t = 0$

$$i(0) = \frac{V}{R}$$



$$\frac{V}{R} = C_1 e^0$$

$$\boxed{C_1 = \frac{V}{R}} \quad \text{— (ix)}$$

$$\boxed{i(t) = \frac{V}{R} e^{-t/RC}} \quad \text{— (x)}$$

$$V_R = i(t) R$$

$$= V e^{-t/RC}$$

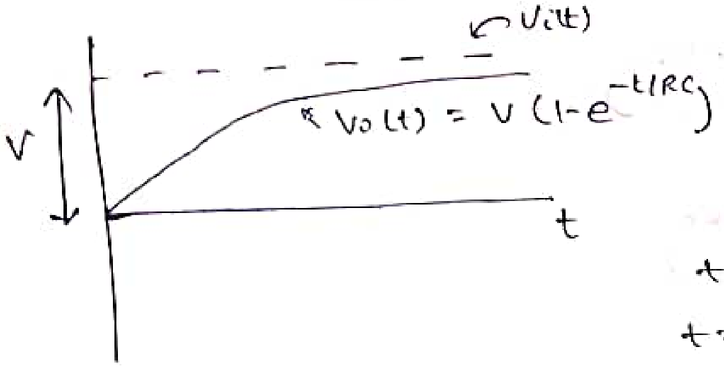
As we know,

$$V_i(t) = V_R + V_c$$

$$V = V e^{-t/RC} + V_c$$

$$V_c = V_o = V - V e^{-t/RC}$$

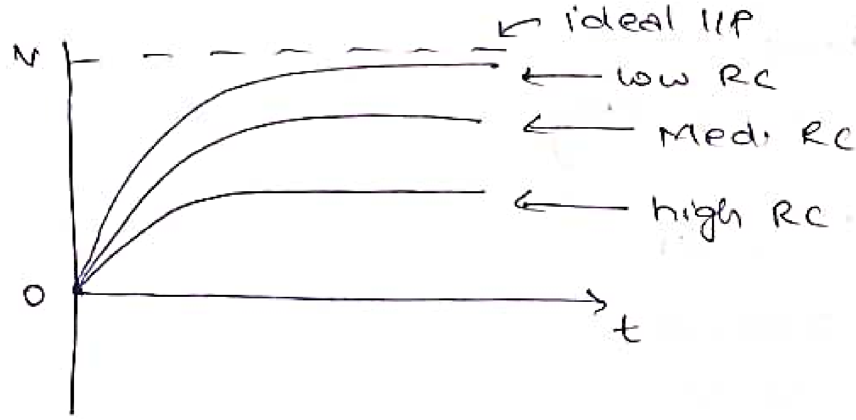
$$\boxed{V_c = V_o = V\left(1 - e^{-t/RC}\right)}$$



$t = 0 \qquad V_o(t) = 0$

$t = \infty \qquad V_o(t) = V$

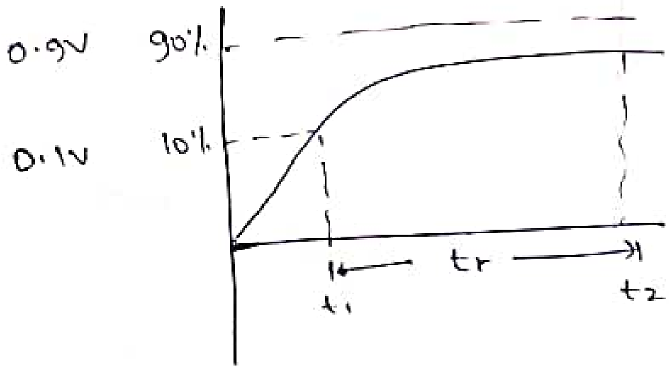
For different time constant (RC)



Rise Time:

$V_c \rightarrow (10\% - 90\%) V_o$

Time taken by the capacitor voltage to increase from 10% to 90% of its final volt V_o is called rise time.



$t_r = t_2 - t_1$

$$V_0(t) = V(1-e^{-t/RC})$$

At $t = t_1$

$$0.1V = V(1-e^{-t_1/RC})$$

$$e^{-t_1/RC} = 0.9$$

$$\boxed{t_1 = 0.105\,RC}$$

$t = t_2$, we have

$$0.9V = V(1-e^{-t_2/RC})$$

$$e^{-t_2/RC} = 0.1$$

$$t_2 = 2.303\,RC$$

$$t_r = t_2 - t_1$$

$$\boxed{t_r = 2.2\,RC}$$

let f_2 be upper 3dB freq.

$$f_2 = \frac{1}{2\pi RC}$$

$$t_r = 2.2 \times \frac{1}{2\pi f_2}$$

$$\boxed{t_r = \frac{0.35}{f_2}}$$