8.08.22

Circuit Theory

Network topology is the geometry or interconnection of different elements in the circuit.

Incidence Matrix: The relationship between different branches incidents at a particular vertex can be represented using a matrix. This is the incidence matrix.

Sign convention:

+1 : Entry in matrix when edge current is directed away from ~~matrix~~ vertex.

-1 : Edge current is directed towards vertex

0 : Edge current isn't incident on ~~matrix~~ vertex.

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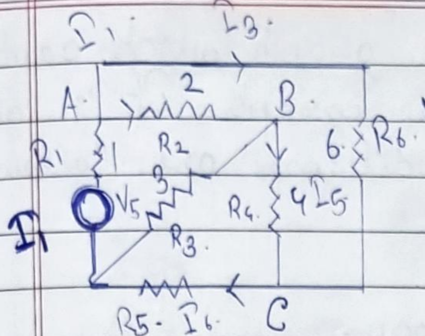
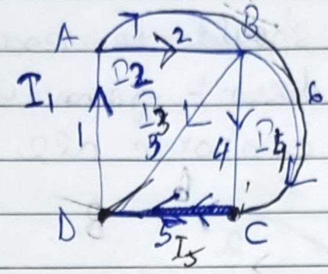


Degree of Vertex: The no of edges incident the vertex is called the degree of the vertex.



oriented graph.

$I_1 = I_2 + I_6$.

$I_1 - I_2 - I_6 = 0 \sim (1)$

$I_2 = I_4 + I_5$

$I_2 - I_4 - I_5 = 0 \sim (2)$ at node B.

KCL at C degree of vertex:

$I_4 + I_6 = I_5$.

$-I_5 + I_4 + I_6 = 0$

$\Rightarrow I_4 - I_5 + I_6 = 0 \sim (3)$

$I_1 = I_3 + I_5$

$I_1 - I_3 - I_5 = 0 \sim (4)$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = 0$$

At node A

$-I_1 + I_2 + I_6 = 0$ — (1)

At node B

$-I_2 + I_4 + I_3 = 0$ — (2)

At node C

$-I_4 + I_5 - I_6 = 0$ — (3)

At node D.

$I_1 - I_5 - I_3 = 0$ — (4)

Branches 1-6

$$\begin{array}{c} \text{Node} \\ A \\ B \\ C \\ D \end{array} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ \vdots \end{bmatrix}$$

$[A_i]$ =

node	1	2	3	4	5	6
a	-1	1	0	0	0	1
b	0	-1	1	1	0	0
c	0	0	0	-1	1	-1
d	1	0	-1	0	-1	0

(branches)

or

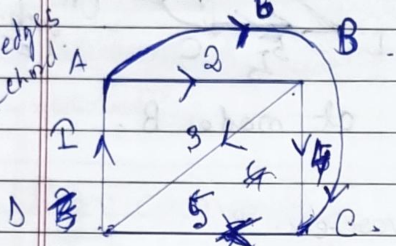
* Rank of matrix : n - 1
* sum of the columns is 0
* determinant of matrix is 0

Tree → **Spanning subgraph** : It is the graph which contains all the vertices of the graph. It is said to be a tree if these conditions are followed:

Imp.

(i) It should be connected.
(ii) It doesn't form a closed loop.
(iii) It contains all the vertices of parent graph.

Twigs - edges
Link - chord



Twigs / Links:
The branches of the tree are called as twigs. Branches not present in trees are called links.

Forest:
For non-connected subgraphs, each part of the subgraph has a tree and the collection of these trees is called forest.

Co-Tree
The complimentary subgraph of a tree of the subgraph is the co-tree and the edges of the co-tree are links / chords.

No. of trees of the graph :-

$= \det\{[A][A^T]\} = 3$

↳ reduced incidence matrix

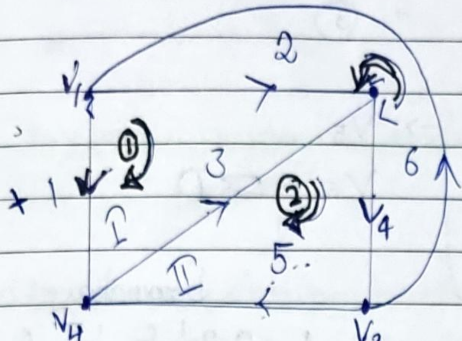
$$[A_i] = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \qquad [A] = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

$$A^T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

* The no. of twigs = no. of nodes − 1 (v − 1

= 3 − 1 (e − v + 1 + v − 1)

= 2 (e)



$V_1 \ldots V_6$ are the branch voltage.

KVL in loop ①:

$-V_1 + V_2 + V_3 = 0$ — (1)

KVL in loop ②:

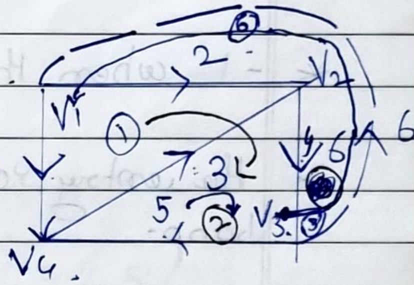
$V_3 + V_4 + V_5 = 0$ — (2)

KVL in outer group loop ③:

$-V_1 + V_2 + V_4 + V_5 = 0$ — (3)

$$\begin{bmatrix} -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{-1} \\ -1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \Rightarrow [B_f][V_b] = 0$$

B_f → fundamental loop matrix.



* no. of fundamental loop depends on no. of links.

* The direction of loop current is set according to the direction of link current.

* While forming a funda. loop, only one link is taken at a time.

① Select the properties.
② Mark the twigs.

$-V_1 + V_2 + V_4 + V_5 = 0 \quad \sim (1)$

$V_3 + V_4 + V_5 = 0 \quad \sim (2)$

$V_6 + V_1 - V_5 = 0 \quad \sim (3)$

$$\begin{bmatrix} -1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} * \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \rightarrow \begin{matrix} V_5 \\ V_6 \end{matrix} = 0$$

$[B_f]$ =

Branches.

$$\begin{array}{l|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \text{f-loop 2. } (1,2,3,4,5) & -1 & 1 & 0 & 1 & 1 & 0 \\ \text{f-loop 3. } (3,4,5) & 0 & 0 & 1 & 1 & 1 & 0 \\ \text{f loop-4. } (1,6,5) & 1 & 0 & 0 & 0 & -1 & 1 \end{array}$$

* If the direction of loop current and loop current coincide - matrix is +1.

* -1 when they don't co-incide.

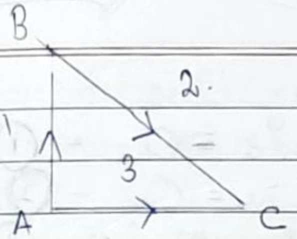
* the entry is 0 if branch is not a part of loop.

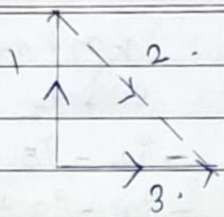
$$B_f = \begin{bmatrix} -1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$ (Sign Convention.)

Def

Cutset Branches : Cutset is defined as the min no. of branches removal of which seperates the graph into exactly 2 parts.

R.





No. of funda. cutset branches = no. of twigs. (edges)

* Only one twig is taken to form a fundamental cutset branch

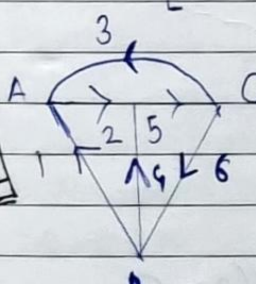
* direction / orientation of fundamental cutset branch depends on direction / orientation of twig.

Matrix is +1 ; if orientation of branch = twigs.
" " −1 ; if orientation ≠ twigs.

$[Q_f]$ =

	1	2	3
f-cutset-1 (1, 2)	1	−1	0
f-cutset-2 (3, 2)	0	1	1

$$(Q_f) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}_{2\times 3}$$



	1	2	3	4	5	6
f-cutset-2 (1, 2, 3)	−1	1	−1	0	0	0
f-cutset-4 (1, 4, 6)	−1	0	0	1	0	1
f-cutset-5 (3, 5, 6)	0	0	1	0	1	1

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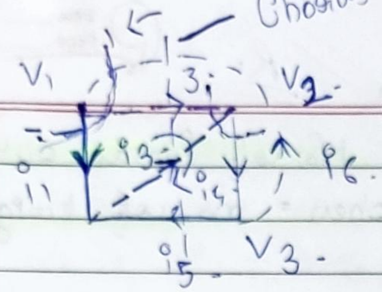
Links / Chords.

Funda f-cutset matrix



KCL eq.

$i_1 + i_2 - i_6 = 0$ — (1)

$-i_2 - i_5 + i_4 = 0$ — (2)

$-i_2 - i_3 + i_5 + i_6 = 0$ — (3)

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} i_1 & i_4 \\ i_2 & i_5 \\ i_3 & i_6 \end{bmatrix} = 0.$$

$\Rightarrow [Q_f][I_e] = 0$ or $[Q_f][I_e] = 0$

$$[Q_{11} \quad U]\begin{bmatrix} I_c \\ I_B \end{bmatrix} = 0.$$

$[Q_{11} I_c + U I_b] = 0.$

$Q_{11} I_c + I_b = 0.$ ✓ $I_b = -Q_{11} I_c$

The branch current can be expressed as a linear combi. of links / chords.

f-loop / circuit matrix using KVL.

$-V_1 + V_2 + V_4 + V_5 = 0$ — (1)

$V_3 + V_4 + V_5 = 0$ — (2)

$V_1 - V_5 + V_6 = 0$ — (3)

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} V_1 & V_4 \\ V_2 & V_5 \\ V_3 & V_6 \end{bmatrix}.$$

$[B_f][V_c$ or $V_b]. \Rightarrow B_f V_c = 0.$

$$\begin{bmatrix} U & B_{12} \end{bmatrix}\begin{bmatrix} V_c \\ V_b \end{bmatrix} = 0 \Rightarrow U V_c + B_{12} V_b = 0$$

$\Rightarrow V_c + B_{12} V_b = 0$

$\Rightarrow V_c = -B_{12} V_b$

The default voltage can be expressed as the linear quech ~~...~~ voltage.

Interrelationship of Matrices :

(1) Relation between incidence Matrix [A] and Fundamental Cut Set Matrix [Q_f]

From incidence matrix $[A][I_e] = 0$ (1)
Fundamental cutset matrix $[Q_f][I_e] = 0$ (2) } Use KCL

Edge current / Branch current $= \begin{bmatrix} I_e \\ I_b \end{bmatrix}$ ~ (3)
↳ I_e

$[A] = [A_{11} \; A_{12}]$ - (4)

Use (3) & (4)

$[A_{11} \; A_{12}] \begin{bmatrix} I_e \\ I_b \end{bmatrix} = 0$

$A_{11} I_e + A_{12} I_b = 0$

$I_b = -A_{11} I_e / A_{12}$ $\Rightarrow$ $I_b = -A_{11} I_e A_{12}^{-1}$ - (5)

$[U \; Q_{11}] \begin{bmatrix} I_b \\ I_e \end{bmatrix} = 0 \Rightarrow UI_b + Q_{11} I_e = 0$
$\Rightarrow I_b + Q_{11} I_e = 0$
$\Rightarrow I_b = -Q_{11} I_e$ - (6)

Equate (5) & (6)

$\boxed{-Q_{11} = -A_{11} A_{12}^{-1}}$

This eq shows that rows of funda cutset ma[trix] are a linear combi of rows of incidence matrix.

Relationship based on node & mesh :-

(1) Node : $[Q_f][I_e] = 0$; $[B_f][V_e] = 0$

$[Q_{11} \; U] \begin{bmatrix} I_e \\ I_b \end{bmatrix} = 0 \Rightarrow Q_{11} I_e = -I_b$

$V_e = \begin{bmatrix} V_c \\ V_b \end{bmatrix}$, $I_e = \begin{bmatrix} I_c \\ I_b \end{bmatrix}$, $B_f = [U \; B_{11}]$

$[U \; B_{11}]\begin{bmatrix} V_c \\ V_b \end{bmatrix} = 0 \Rightarrow V_e = -B_{11} V_b$

$\Rightarrow V_b = B_f^T [V_c]$

Further, V_t can be written as
From branch trans. we have;
branch voltage $V_b = B_f^T [V_e]$
" current $I_b = Q_f^T [I_e]$.

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$[B_f][Q_f] = 0$ or $[Q_f][B_f^T] = 0$

$\hookrightarrow [B_f]\begin{bmatrix} U \\ Q_{11}^T \end{bmatrix} = 0$ $\hookrightarrow \cancel{[B_{11} \; U]}$

x_1, x_2, x_3

$B_{11} + Q_{11}^T = 0$

$[B_{ft}] = -Q_{ft}^T \leftarrow B_{11} = -Q_{11}^T$ — (1)

$[B_f][V_e] = 0$ or $[B_f][V_b] = 0 \leftarrow$ KVL - (A)

$[Q_f][I_e] = 0$ (B) or $[Q_f][I_b] = 0$

KCL $\longrightarrow$

$[A][I_e] = 0$ - (C)

$[B_f][V_e] = 0$

$[B_{11} \; U][V_e] = 0$

$[B_{11} \; U]\begin{bmatrix} V_{et} \\ V_{el} \end{bmatrix} = 0$

$\cancel{V_{et}} = B_{11} + V_{et} + V_{el} = 0$

$V_{et} = -\frac{V_{el}}{B_{11}} = -B_{11}^{-1} V_{el}$

$e = b$

$V_{eb} = Q_{11}^T V_{el}$

$[V_b] = [Q_{fe}^T][V_e]$

Eq. (2) gives relationship between link voltage & Q_f voltage.

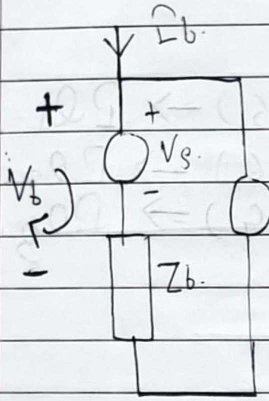
Relation between branch voltage & node voltage.

$V_e = [Q_f^T][V_t]$.

$V_b = [A^T][V_n]$ ← node voltage
↳ branch v. or $[V_e] = [A^T][V_n]$.

Relation between branch & link current

$[I_e] = [B_f^T][I_l]$. or $[I_b] = [B_f^T][I_l]$.



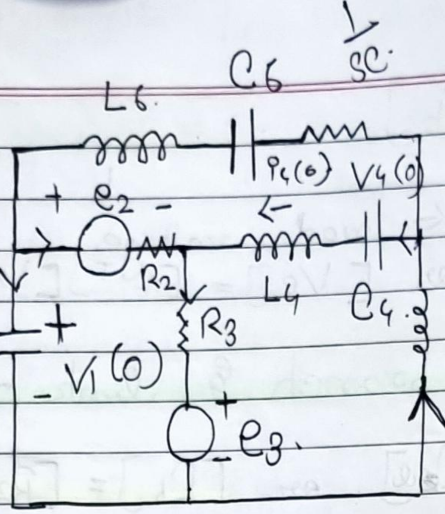
$-V_b + V_s + Z_b(I_b - I_s) = 0$.
$V_b = V_s + Z_b[I_b - I_s]$
$V_b = V_s + Z_b I_b - Z_b I_s$.

$[B_f][V_b] = [B_f][V_s] + [B_f][Z_b][I_b]$
$- [Z_b][I_s]$.

$\because [I_b] = [B_f^T][I_l]$.
$[B_f][V_b] = [B_f][V_s] + [B_f][Z_b][B_f][I_l] -$
$[B_f][Z_b][I_s]$.

from KVL $[B_f][V_b] = 0$
$0 = [B][V_s] + [B_f][Z_b][B_f^T][I_l] - [B_f][Z_b][I_s]$
$\underbrace{[B_f][Z_b][B_f^T][I_l]} = [B_f][Z_b][I_s] - [B_f][V_s]$.

$[Z_l][I_l] = 0$
↳ loop impedence matrix.
→ branch impedence matrix.

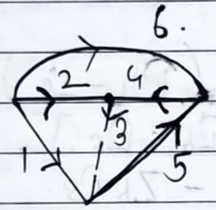


$X_L = j\omega L$

$X_C = \frac{1}{j\omega C}$

$X_L = SL$

$X_C = \frac{1}{SC}$



Loop 01 (1, 5, 6) → I_{l_1}

Loop 02 (2, 4, 6) → I_{l_2}

Loop 03 (3, 5, 4) ⇒ I_{l_3}

$$B_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

23.08.22

Transform the 't' domain into 's' domain

$L \to sL$ ← Impedence value. sC ← admittance

$1/sL$ ← Admittance value.

$j\omega L$ ← immitance

$$B_f = \begin{bmatrix} 1/SC_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/SC_4 + SL_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & SL_5 & 0 \end{bmatrix}$$

$V_L = L \frac{d\,I f(e)}{dt}$ $\frac{1}{T}$

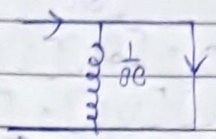
$\mathcal{L}\, V(t) = \mathcal{L}\, L \frac{dI(t)}{dt}$

Differentiation theorem in s domain.

$\mathcal{L}\, L \frac{di(t)}{dt} = L\,(s\,i(s) - i(0^-))$

$V(s) = SL\, i(S) - L\, i(0^+)$.

$i(s) = Vs / sL + i(0^+)/S$.

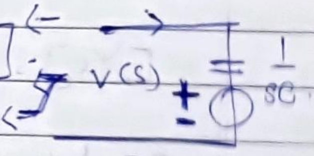


$i(t) = C\, \frac{dv(t)}{dt}$

$i(s)\ \cancel{\mathcal{L}\, i(t)} = C\,[S\,v(s) - v(0^+)]$

$i(s) = SC\,v(s) - Cv(0^+)$

$v(s) = \frac{i(s)}{sC} + \frac{v(0^+)}{S}$.



$[V_s] = \begin{bmatrix} V_1(0)/5 & V_4(+0)/S - S(0^+)_4 \\ e_2 & S\,i_5(0) - 1 \\ e_3 & 0 \end{bmatrix}$

26-08-22

Equilibrium eq. on node basis.

$I_b = I_s + (V_b - V_s)\,Y$.

Multiply both sides by reduced incidence matrix

$A I_b = A I_s + AY\,(V_b - V_s)$ ①

$V_b = A^T V_m$ ②

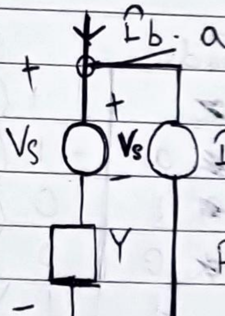


Fig. 1-

Use eq. ② in eq. ①.

$A I_b = A I_s + AY\,(A^T V_m - V_s)$

$[A][I_b] = [A][I_s] + [A][Y][A^T][V_m] - [A][Y][V_s]$.

By KCL → $[A][A_b] = 0$.

$0 = [A][I_s] + [Y_m][V_m] - [A][Y][V_s]$.

$[Y_m][V_m] = [A][Y][V_s] - [A][I_s]$.

$[I_n] = [A][Y][V_s] - [A][I_s]$.

Multiply both sides by fundamental cutset matrix

$Q_f I_b = Q_f I_s + Q_f Y\,(V_b - V_s)$ ①

$V_b = Q_f^T V_f$ (2).

Use eq (2) in (1).

$Q_f I_b = Q_f I_s + Q_f Y (Q_f^T \cdot V_t - V_s)$

$[Q_f][I_b] = [Q_f][I_s] + [Q_f][Y][Q_f^T][V_t] - [Q_f][Y][V_s]$

By KCL $[Q_f][I_b] = 0$.

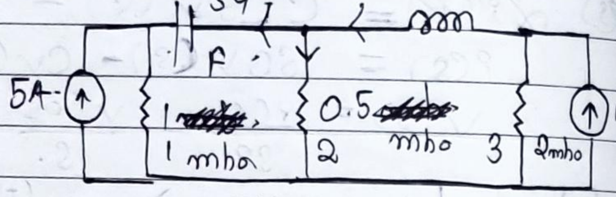
$0 = [Q_f][I_s] + [Y_e][V_t] - [Q_f][Y][V_s]$.

$[Y_L][V_t] = [Q_f][Y][V_s] - [Q_f][I_s]$.

$Y_e = [Q_f][Y][Q_f^T]$.

↳ branch admitance.

$[Y_b]$.



$1F = \frac{1}{S}$ ← impedence

$1H = S$ ←

← elastance. Fig 2(a)

Oriented Graph.

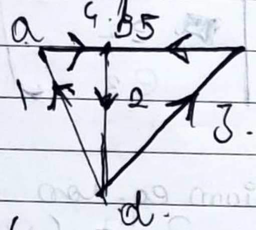


Fig 2(b).

Node	1	2	3	4	5
a	-1	0	0	1	0
b	0	1	0	-1	-1
c	0	0	-1	0	1
d	1	-1	1	0	0

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1/5 \end{bmatrix} \qquad V_n = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$[Y_n] = [A][Y][A^T]$$

3x5 5x5 5x3.

$$V_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad I_s = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix} \quad 5 \times 1$$

Same direction : +ve.

Different direction : -ve.

SIGN CONVENTION

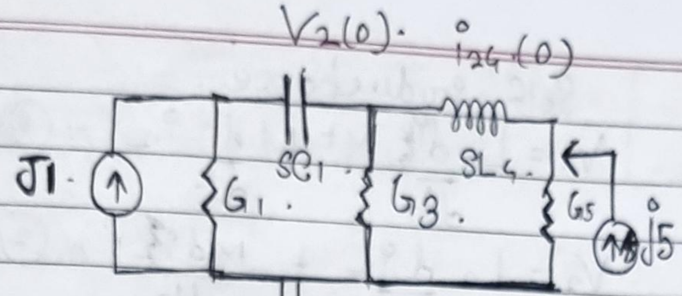


Fig 3.

⇓

Transformed N/W.

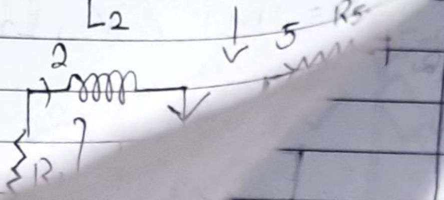
$$[Y] = \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 \\ 0 & G_3 & 0 & 0 & 0 \\ 0 & 0 & G_5 & 0 & 0 \\ 0 & 0 & 0 & SC_1 & 0 \\ 0 & 0 & 0 & 0 & 1/SC_4 \end{bmatrix}$$

$i = C\,dv/dt$ $\quad$ $i(s) = sCV(s) - cv(0)$

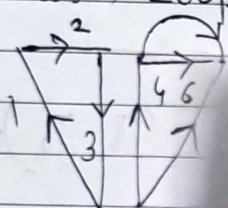
$i = 1/L \int v\,dt$ $\quad$ $i(s) = 1/sL\ v(s) + V(0^+)$

$$\begin{bmatrix} J_1 \\ 0 \\ J5 \end{bmatrix} \quad \begin{bmatrix} \bar{C}V_{C_2}(0) \\ i_{L4}(0)/5 \end{bmatrix}$$

$M_{12} = M_{21} = M$



Mesh / Loop Analysis →



$$\begin{bmatrix} \dots & 2 & 0 & 0 \\ \dots & 2 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/4 & 1/3 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 1/4 & 1/5 \end{bmatrix}$$

$$\begin{bmatrix} \dots & \\ \dots 4 & 0 \\ \dots 0 & 1/5 \end{bmatrix}$$

$[Z_b] = \begin{bmatrix} R \dots \\ \dots 5 \end{bmatrix}$ (Ans.)

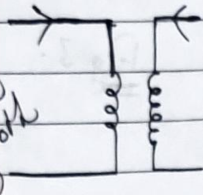
Multiply Loop Admittance

$[V_L] = [Q_f][V_B][Y_b]$. Loop Admittance.

or

$\times [V_{ba} \rightarrow V_{tb}]$.

Self inductance.



$V_1 = L_1 \frac{d i_1}{dt} M + M \frac{d}{dt} i_2 \sim (1)$

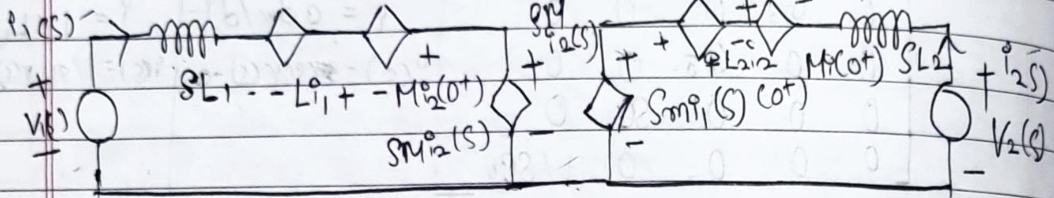
$V_2 = L_2 \frac{d i_2}{dt} + \frac{M d i_1}{dt} \sim (2)$

Take laplace trans. on both sides of (1) & (2)

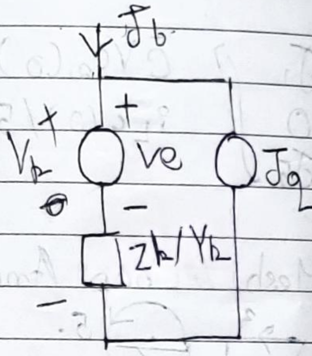
$V_1(S) = SL_1 i_1(S) - L_1 i_2(0^+) + SMi_2(0) - Mi_2(0^+)$

$= SL_1 i_1(S) + SMi_2(S) - L_1 i_2(0^+) - Mi_2(0^+)$ - (3)

$V_2(S) = SL_2 i_2(S) - L_2 i_2(0^+) + SMi_1(S) - Mi_1(0^+)$ (4)



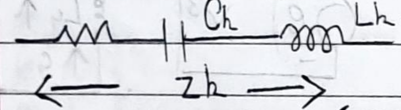
$K = |M| / \sqrt{L_1 L_2}$



$Z_h = -R + 1/SC + SC.$

$Y_h = G + SC + 1/SC.$

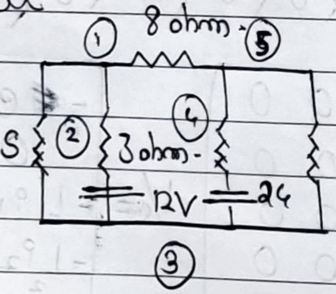
$V_h = V_e + [J_b - J_q] Z_h.$

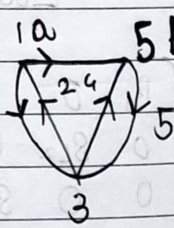


$V_h = V_e + (J_b - J_q) R_h + \frac{1}{C_h} \int (J_b - J_q) dt + L_h + \frac{d}{dt}(J_b - J_q).$

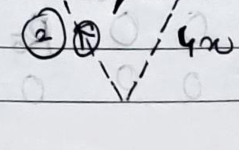
$\mathcal{L} \frac{df(t)}{dt} = SF(S) \ominus \quad \mathcal{L} \int_0^t f(t)\, dt = \frac{f(s)}{s} + \frac{f(0^+)}{s}$

29.08.22





Oriental Graph.



$(Q_f) = \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}_{2\times 5}$; $V_S = \begin{bmatrix} 0 \\ -12 \\ 0 \\ -24 \end{bmatrix}_{5\times 1}$

admittance

Branch Voltage $\Rightarrow$ $Y_B = \begin{bmatrix} 1/4 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/8 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 1/5 \end{bmatrix}$

Loop Admittance $[Y_L] = [Q_f][Y_B][Q_f]$.

$= \cancel{\begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}} \cancel{\begin{bmatrix} 1/4 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/8 & 0 & 0 \end{bmatrix}}$

$Y = $ Rough. $1/4 = 4)10(\boxed{\cdot 22}$ $\frac{8}{2}$

$= \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 1+0 & 1+1 & 0 & 0 \\ 0 & 0 & 1+1 & 0+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix}$.

$\begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/8 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/3 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 1/4 & 1/5 \end{bmatrix}$

$= \begin{bmatrix} 0.708 & 0.125 \\ 0.125 & 0.575 \end{bmatrix}$ (Ans).

Multiply Loop Admittance

$[Y_L][V_L] = [Q_f][V_B][Y_b]$. Loop Admittance.

or

$\times [V_{ba} \rightarrow V_{tb}]$.

$$\begin{bmatrix} V_{ba} \\ V_{bb} \end{bmatrix} = \begin{bmatrix} 0.708 & 0.125 \\ 0.125 & 0.575 \end{bmatrix} \cancel{\begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}} =$$

$$= \begin{bmatrix} -4 \\ -6 \end{bmatrix} \Rightarrow \begin{bmatrix} V_{ba} \\ V_{bb} \end{bmatrix} = \begin{bmatrix} -3.96 \\ -9.57 \end{bmatrix}$$

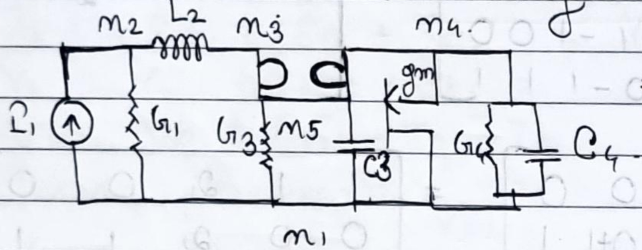
$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_{b1} \end{bmatrix} = \begin{bmatrix} -1\ 1\ -1\ 0\ 0 \\ 0\ 0\ -1\ 1\ 1 \end{bmatrix} \begin{bmatrix} -3.96 \\ -9.57 \end{bmatrix} \rightarrow \begin{bmatrix} Z_{12} = Z_{21} \\ Z_{13} = Z_{31} \\ Z_{23} = Z_{32} \end{bmatrix}$$

Incase of (Independent Sources) :-

$$I_b = [Y_b][V_b - V_s]$$

2.9.22

Network Theory Control Process.



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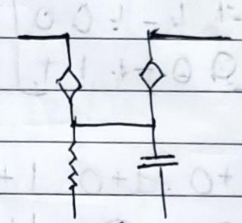

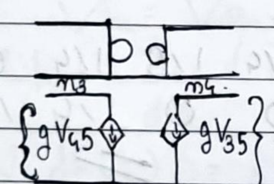

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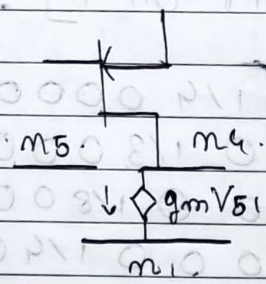
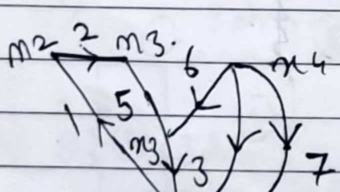




[A]

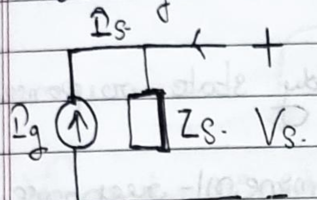
	1	2	3	4	5	6	7
n2	-1	1	0	0	0	0	0
n3	0	-1	0	0	1	0	0
n4	0	0	0	1	0	1	1
n5	0	0	1	0	-1	-1	0
n1	1	0	-1	-1	0	0	-1

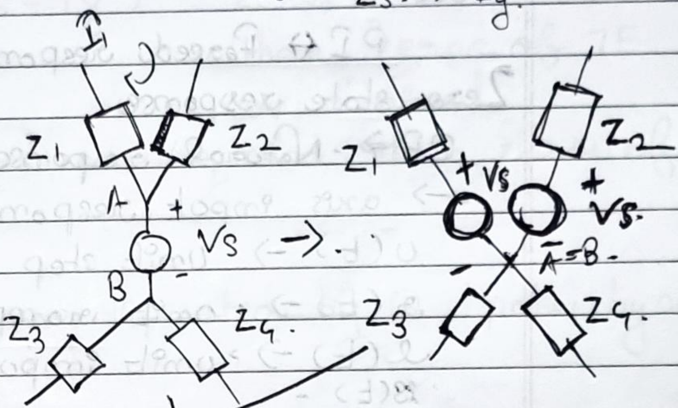
$$Y_b = \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/SL_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3+SC_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_4+SC_4 \end{bmatrix}$$

$$[Y_L] = [A][Y_b][A].$$

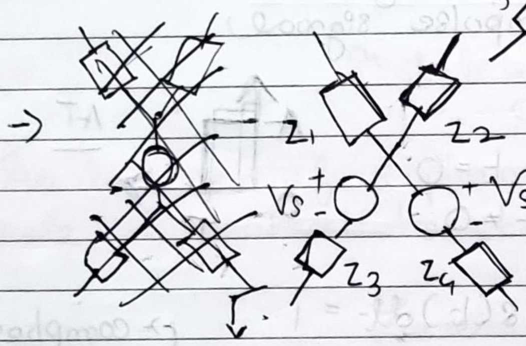
$$I_s = \begin{bmatrix} I_1 \\ 0 \\ 0 \\ g_m V_5 \end{bmatrix}_{7\times 1} \rightarrow \begin{bmatrix} gV_{45} \\ -gV_{35} \\ 0 \end{bmatrix}$$

$V_s = V_g + Z_s I_s$. $\quad V_g = V_s - Z_s I_s$. $\quad \frac{V_s}{Z_s} = \frac{V_g}{Z_s} + I_s \rightarrow I_g$





$\rightarrow$

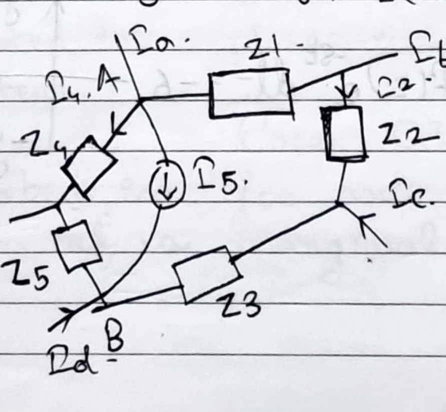


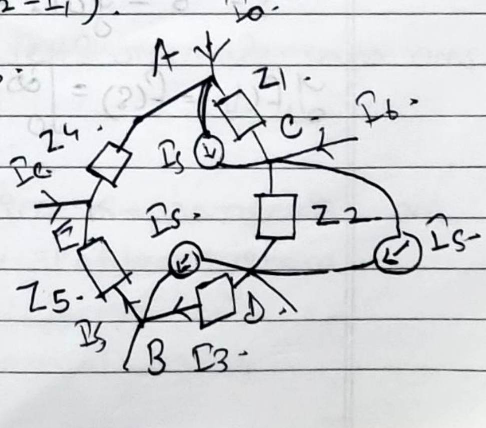
$$I_1 Z_1 + V_{Z_1 Z_2} - Z_2 I_1 = 0$$
$$V_{Z_1 Z_2} = Z_2 I_1 - Z_1 I_1$$

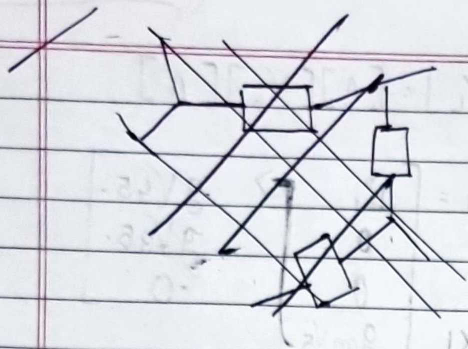
$$-V_5 + Z_1 I_1 + V_{Z_1 Z_2} + Z_2 I_1 + I_3 = 0.$$
$$V_{Z_1 Z_2} = -(I_1 Z_1 + I_2 Z_2)$$

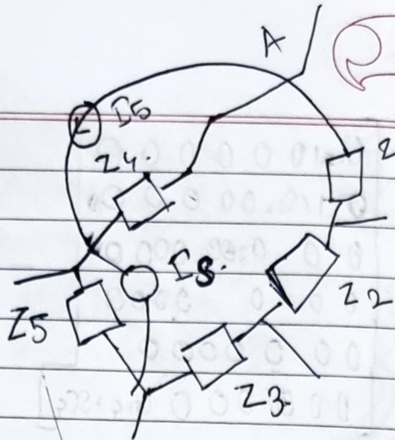
$$-V_{Z_2 Z_3} + (I_2 - I_1) Z_4 - V_5 + Z_2 (I_2 - I_1) = 0.$$
$$V_{Z_2 Z_3} = (I_2 - I_3) Z_4 - V_5 + Z_2 (I_2 - I_1).$$

$I_a = I_1 + I_5 + I_4$.









$$\frac{dY(t)}{dt} + aY(t) = x(t).$$

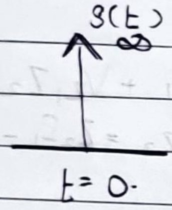
PI ↔ Forced response ↔ Steady state response ↔ Zero state response

CF → Natural response → pramimemt response ↔ ↔ axis impot response.

$u(t)$ → unit step signal

$r(t)$ → unit ramp signal

$\delta(t)$ → unit impulse signal



$\delta(t) = 1$ at $t = 0$

$= 0$ at $t \neq 0$

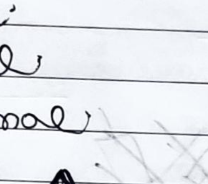
$0 \times \infty = 1$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Lapls Transformation : $(\mathcal{L})\, f(t) = f(s)$ → complex variable

$s = \sigma \pm j\omega$ → rad s^{-1}

$$\mathcal{L}f(t) = f(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt$$



07.9.22

Representation of poles & zeros.

$T(s) = (b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_0) / (a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_n)$.

General exp for TF ↑

Scaling factor : $\dfrac{\sum_{i=1}^{m} (s - z_i)}{\sum_{j=1}^{m} (s - P_j)}$ ← when numerator factors are equated to zeros, ← then, zeros of TF are obtained.

When dominator factors are equated, then zeros of TF are obtained.

In S-planes zeros are seperated by O, poles by x

Stability

The location of s-planes decides an important role in content to stability of system.

Case - 01

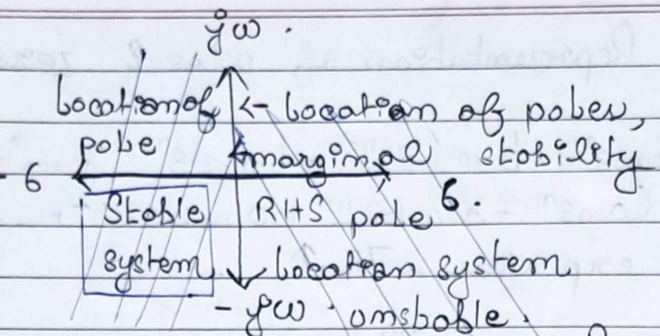
Poles located in 6-plane in LHS give absolute stability of system.

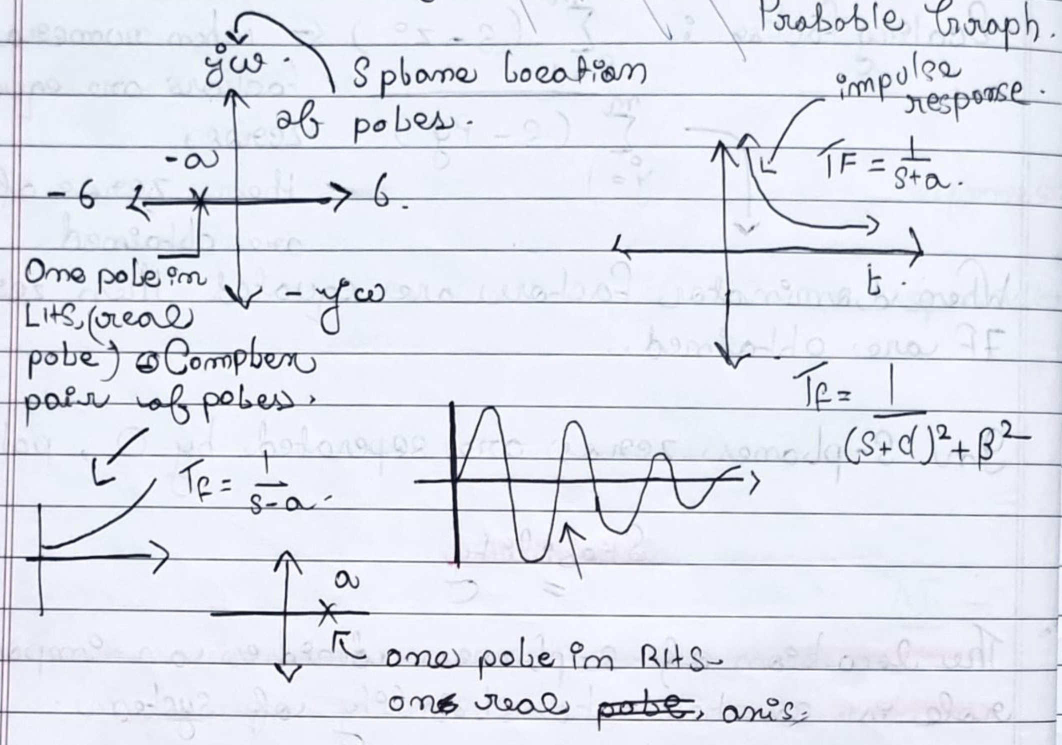
Case - 02

Poles located in S-plane in RHS reveals an unstable system.

Case - 03.

Poles located in 'jω' axis in s-plane is an indication of a marginal stable system.





12.09.22

Two port N/W

A pair of available terminals.

Devices.

↓	↓	↓
One port n/w (one pair of terminals)	Two port n/w (two pairs of terminals)	Multiport n/w (multiple pair of terminals)

Ex : fan, microwave, tubelight etc.

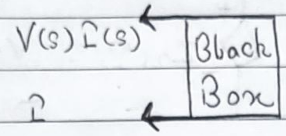
Ex : Transformer, BJT, UJT op-amp.

classmate

Date
Page

Schematic Diagram.

One port n/w

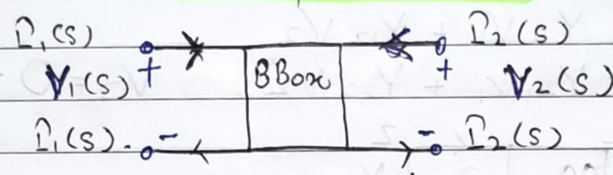


Relationship between voltage & current (V_s & I_s).

$Z(s) = V(s) / I(s)$ ← point impedance.

$Y(s) = I(s) / V(s)$ ← point admittance

Two port n/w.

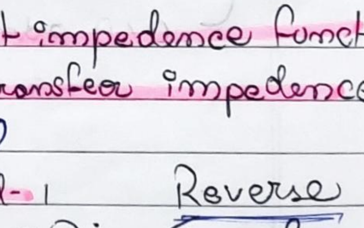


There are 4 variables & the relationship between them is established by different equations also known as characteristic equations.

$(V_1 V_2) = f(I_1, I_2)$ ← Z - parameter, open c/ct impedence.

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$



Put $I_2 = 0$ i.e. output port-2;

$V_1 = Z_{11} I_1$.

✓ $\frac{V_1}{I_1} = Z_{11} \quad | \; I_2 = 0.$ Z_{11} & Z_{22} are Driving point impedence function

✓ $Z_{21} = \frac{V_2}{I_1} \quad | \; I_2 = 0.$ Forward transfer impedence.

(Z_{21} & Z_{12} are)

✓ Put $I_1 = 0$. i.e. open ckt port-1

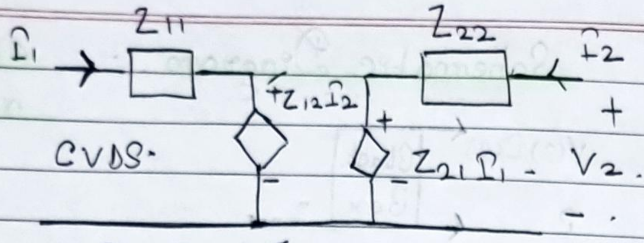
$Z_{12} = V_1 / I_2 \quad | \; I_1 = 0.$ Reverse Transfer Impedence.

$Z_{22} = V_2 / I_2 \quad | \; I_1 = 0.$

↳ O/P impedence.

If the two port under consideration is reciprocal, then

$Z_{12} = Z_{21}$



CDVS (This generates eq.)
(current def. voltage source)

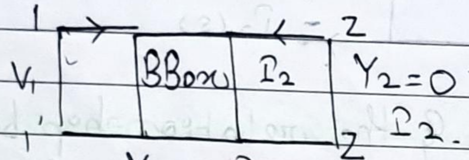
Y-parameter short ckt admittance parameter

$(I_1 I_2) = f(V_1, V_2)$.

↳ Taylor series.
Halkias.

$I_1 = Y_{11}V_1 + Y_{12}V_2$

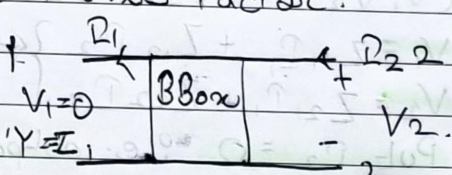
$I_2 = Y_{21}V_1 + Y_{22}V_2$. $V_2 = 0$.



$Y_{21} = I_2 / V_1 \mid V_2 = 0$.

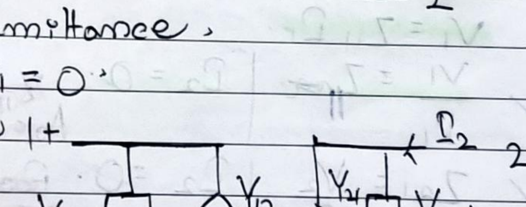
$Y_{11} = I_1 / V_1 \mid V_2 = 0$.
Driving point admittance factor.

$Y_{12} = \dfrac{I_1}{V_2} \Big| V_1 = 0$.



Reverse transfer admittance.

$Y_{22} = I_2 / V_2 \mid V_1 = 0$.
O/P admittance



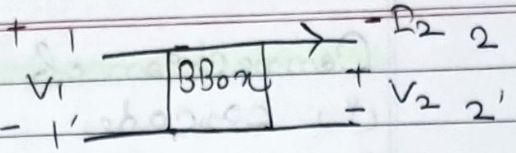
VDCS.

T-Parameter, ABCD Parameter, chain para.
Exclusively for overhead transmission

$V_1 I_1 = f(V_2, -I_2)$

$V_1 = AV_2 + B(-I_2)$ $I_1 = CV_2 + D(-I_2)$

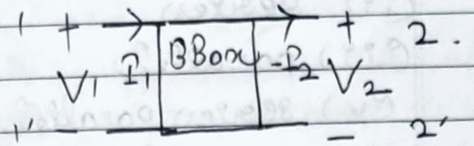
$\cancel{A = \text{Pot } I_2 = 0}$ i.e. open ckt port 2



Pot $I_2 = 0$.

$A = V_1 / V_2 \mid -I_2 = 0$.

Reverse voltage gain.



$C = I_1 / V_2 \mid -I_2 = 0$.

Reverse transfer admittance ($\mho$) -

Pot $V_2 = 0$ i.e. ckt port 2, 2' (Short ckt).

$B = V_1 / -I_2 \mid V_2 = 0$ (Ω) Reverse trans. imp

$D = I_1 / -I_2 \mid V_2 = 0$ Reverse current gain

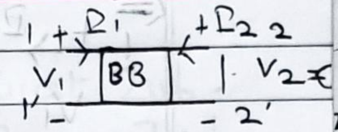
Hybrid / h-parameter.

Popularly used for small signal analysis of transmitter.

$(V_1, I_2) = f(I_1, V_2)$. $V_1 = h_{11} I_1 + h_{12} V_2$.

$I_2 = h_{21} I_1 + h_{22} V_2$

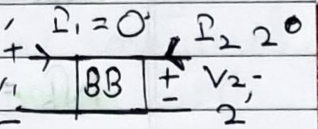
Pot $V_2 = 0$. i.e. short ckt port 2, 2'



Driving point imp -> $h_{11} = V_1 / I_1 \mid V_2 = 0$.

current gain -> $h_{21} = I_2 / I_1 \mid V_2 = 0$.

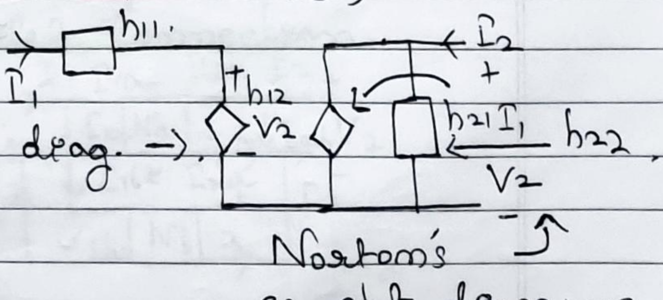
Pot $I_1 = 0$ i.e. short ckt port 1, 1'



$h_{12} = V_1 / V_2 \mid I_1 = 0$: voltage gain

O/P admittance $h_{22} = I_2 / V_2 \mid I_1 = 0$ ($\mho$)

Thevenin's eq. diag. ->

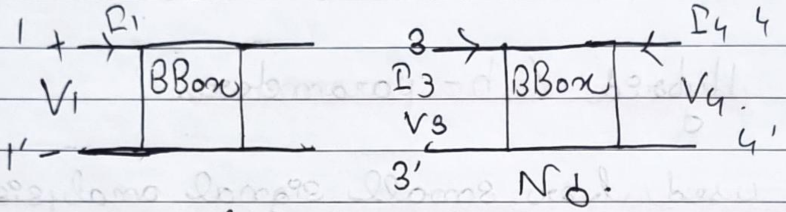


Norton's eq ckt diagram.

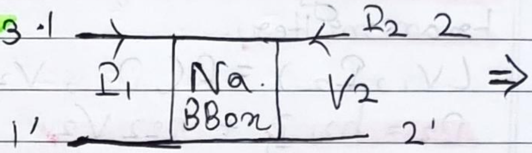
Connection of two port n/ws

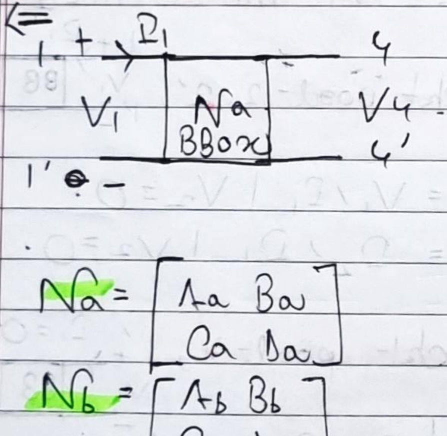
(i) cascade connection
(ii) series " "
(iii) parallel " "
(iv) series parallel " "
(v) parallel series " "

Cascade connection: When output of one device becomes input of another device it is known as cas. con.



$V_2 = V_3$. $I_2 = -I_3$





Find T-parameter of Na.

$$N_a = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}$$

Find T-parameter of N_b.

$$N_b = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

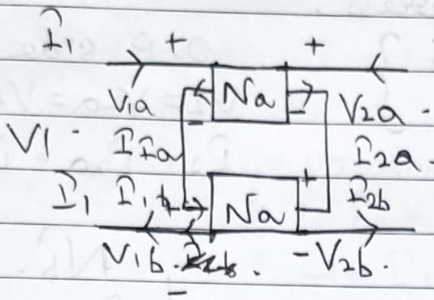
The overall T-parameter will be

$$[T] = [T_a][T_b]$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

13.9.22

Series Connection.



Two nos. of 2 port n/w. are connected in series.

Write Z parameters of each two port n/ws.

Z-parameters n/w Na

$V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a}$

$V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}$

Z-parameters for n/w N_{12}

$V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b}$.

$V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$.

Using series props.

$I_1 = I_{1a} = I_{1b}$.

$I_2 = I_{2a} = I_{2b}$,

$V_1 = V_{1a} + V_{1b}$ - (A)

$V_2 = V_{2a} + V_{2b}$. - (B)

$Z_{11} = Z_{11a} + Z_{11b}$.

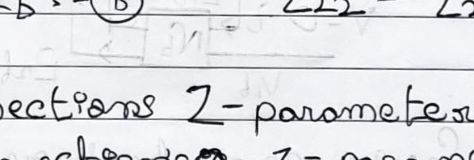
$Z_{12} = Z_{12a} + Z_{12b}$.

$Z_{21} = Z_{21a} + Z_{21b}$

$Z_{22} = Z_{22a} + Z_{22b}$.

* For series connections Z-parameters are used & the overall connections Z-parameter will be of additive nature.

Parallel Connection.



Parallel features.

I/P side $\rightarrow \left\{ \begin{array}{l} V_1 = V_{1a} = V_{1b} \\ I_1 = I_{1a} + I_{1b} \end{array} \right\}$

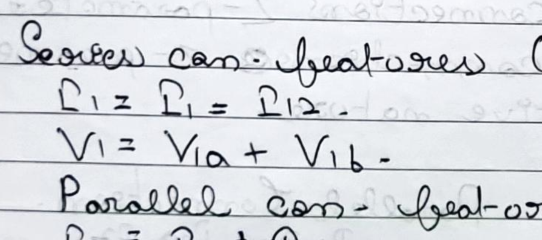
O/P side
$V_2 = V_{2a} = V_{2b}$
$I_2 = I_{2a} = I_{2b}$.

Na.	Nb.
$I_{1a} = Y_{11a} V_{1a} + Y_{12a} V_{2a}$	$I_{1b} = Y_{11b} V_{1b} + Y_{12b} V_{2b}$
$I_{2a} = Y_{21a} V_{1a} + Y_{22a} V_{2a}$	$I_{2b} = Y_{21b} V_{1b} + Y_{22b} V_{2b}$

$I_1 = I_{1a} + I_{1b}$.
$I_1 = (Y_{11a} + Y_{11b}) V_1 + (Y_{12a} + Y_{12b}) V_2$
$I_2 = (Y_{21a} + Y_{21b}) V_1 + (Y_{22a} + Y_{22b}) V_2$

$Y_{11} = Y_{11a} + Y_{11b}$. $\quad Y_{21} = Y_{21a} + Y_{21b}$.
$Y_{12} = Y_{12a} + Y_{12b}$. $\quad Y_{22} = Y_{22a} + Y_{22b}$.

Series Parallel Connection.



Parallel
Series
g-parameters

Series con- features (I/P)
$I_1 = I_1 = I_{12}$.
$V_1 = V_{1a} + V_{1b}$.
Parallel con- features (O/P)
$I_2 = I_{2a} + I_{2b}$.
$V_2 = V_{2a} = V_{2b}$.

$V_1 = (h_{11a} + h_{11b}) I_1 + (h_{12a} + h_{12b}) V_2$
$I_2 = (h_{21a} + h_{21b}) I_1 + (h_{22a} + h_{22b}) V_2$

Imp Inter-relationship between different parameters.

Q. Express z-parameters in terms of T-para.

Z-T

Step ① Write z-parametric equation

$V_1 = Z_{11} I_1 + Z_{12} I_2$ - ①.

$V_2 = Z_{21} I_1 + Z_{22} I_2$ - ②.

Step ② Write t-parametric equations-

$V_1 = A V_2 + B(-I_2)$ - ③.

$I_1 = C V_2 + D(-I_2)$ - ④

Step ③ Compare the variables in eq ① & ③,

Replace V_2 in ③ using ④

Step ④ From eq. ④

$V_2 = (I_1 + D I_2)/C$ - ⑤

Use V_2 from ⑤ in ③

$$V_1 = A\left[\frac{I_1 + D I_2}{C}\right] + B(-I_2)$$

$$= \frac{A}{C} I_1 + \frac{AD}{C} I_2 + B(-I_2)$$

$$V_1 = \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2 \quad - ⑥.$$

Compare eq ① & ⑥

$Z_{11} = A/C$

$Z_{12} = ((AD - BC)/C)$

~~Step~~ Compare ② & ⑤

All variables in eq ⑤ are present in eq ② but their positions are different.

$$V_2 = \frac{I_1}{C} + \frac{D I_2}{C} \quad - (5)$$

Compare (2) & (5)

$Z_{21} = 1/C$.

$Z_{22} = D/C$.

Re. Step (1). Write h parameters

h-T

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad (2)$$

Step (2) T-parameter

$$V_1 = A V_2 + B(-I_2) \quad - (3)$$

$$I_1 = C V_2 + D(-I_2) \quad - (4)$$

Re 2 Step (1)

Z-Y

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad - (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad - (2)$$

Step (2)

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad - (3)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad - (4)$$

Reciprocity Conditions.

Def — A 2 port n/w is said to be reciprocal if the reciprocal of the ratio of the response to the excitation remains unchanged even if their positions are interchanged.

Parameter.	Reprocity Condition
Z	$Z_{12} = Z_{21}$
Y	$Y_{12} = Y_{21}$.
T	$AD - BC = 1$.
h.	$h_{12} = -h_{21}$.

9.09.22

Substitution Theorem.

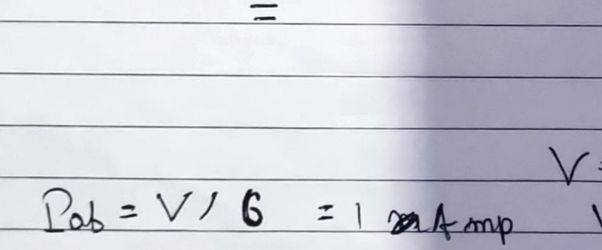
↳ Non Linear & time varying

It states that a particular edge / branch N/W are circuit can be replaced by a suitable voltage source, current source combination of voltage source along with impedence provided that the voltage & c in ckt remain untouched.

Limitations :

① Not useful for circuit / N/W having cou elements.

② The voltage & current of the branch to b placed must be known.



$$\frac{V-10}{2} + \frac{V}{6} + \frac{V}{6} =$$

$$V\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{6}\right] =$$

$$\frac{3+2}{6} V = 5.$$

$$V = 6V.$$

$I_{ab} = V/6 = 1$ Amp $\quad V_{ab} = I_{ab} \times 1.$

$= 1$ Volt.

Replace 1Ω resistance by 1 volt voltage sourc

$$\frac{V-10}{2} + \frac{V}{6} + \frac{V-1}{5} = 0.$$

$$\frac{26V}{30} = \frac{26}{5}. \Rightarrow V = 6 \text{ Volt}.$$

$I_{ab} = (V-1)/5 = 6-1/5 = 1\ Amp.$

~~Using KCL~~ Using KVL : $\frac{V-10}{2} + \frac{V}{6} + 1 = 0.$

$3V - 30 + V + 6 = 0.$

$4V = 24. \Rightarrow V = 6V.$

Using KVL

$-6 + 5\times 1 + V_{ab} = 0.$

$V_{ab} = 1V.$

$V_{ab} = V + IR.$

$1 = V + R. \quad - \text{(~)}$

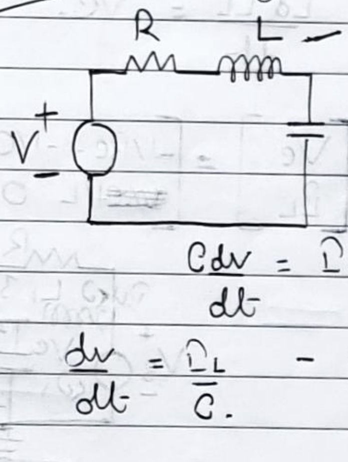
Space State Analysis.

system matrix, input matrix, input coupling matrix

$X = AX + BU \leftarrow$ state eq

Output matrix $- Y = CX + DU \leftarrow$ O/P eq.

Output coupling matrix, transmission matrix.



L - Inductor Current.

C - Capacitor Voltage.

$C\frac{dv}{dt} = I_L.$

Using KVL.

$\Rightarrow \frac{dv}{dt} = \frac{I_L}{C}. \quad - (1)$

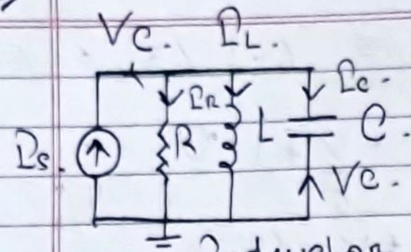
$-V + RI_L + L\frac{dI_L}{dt} + V_C = 0.$

$L\frac{dI_L}{dt} = V - RI_L - V_C. = -\frac{R}{L}I_L - \frac{V_C}{L} + \frac{V}{L} - (2)$

State eq.

$$\begin{bmatrix} \dot{V}_C \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}_{2\times 2} \begin{bmatrix} V_C \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix}_{2\times 1} V$$

20.9.22



← State Variables

Inductor current I_L } State
Capacitor voltage V_C } Variables.

$I_R + I_L + I_C = I_S$.

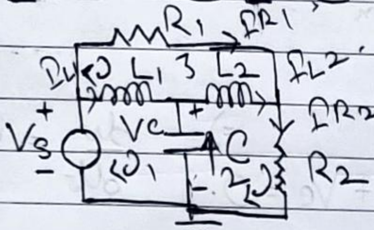
$\frac{V_C}{R} + \cancel{\frac{1}{L}\int V dt}\ I_L + C\frac{dV_C}{dt} = I_S$.

$C\frac{dV_C}{dt} = I_S - I_L - \frac{V_C}{R}$ ⇒. $\frac{dV_C}{dt} = \frac{I_S}{C} - \frac{I_L}{C} - \frac{V_C}{RC}$.

$X = AX + BU$.

$L\frac{dI_L}{dt} = V_C$. $\frac{dI_L}{dt} = \frac{V_C}{L}$ - (2)

$$\begin{bmatrix} V_C \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & -1/C \\ 1/L & 0 \end{bmatrix}\begin{bmatrix} V_C \\ I_L \end{bmatrix} + \begin{bmatrix} 1/C \\ 0 \end{bmatrix} I_S.$$



KCL at b

$C\frac{dV_C}{dt} - I_{L1} - I_{L2} = 0$

$C\frac{dV_C}{dt} = I_{L1} - I_{L2}$ ⇒.

$\frac{dV_C}{dt} = \frac{I_{L1}}{C} - \frac{I_{L2}}{C}$ - (1)

Three state variables.
Three state equation
$L_1 \to I_{L1} \to I_{L1}$
$L_2 \to I_{L2} \to I_{L2}$
$C \to V_C \to V_C$ }

Applying KVL in loop 1.

$$L_1 \frac{dI_{L1}}{dt} + V_C - V_S = 0$$

$$\frac{dI_{L1}}{dt} = \frac{V_S}{L_1} - \frac{V_C}{L_1} \sim (2)$$

~~F~~KVL in loop III

$R I_{R1} - L_2\, dI_{L2}/dt - L_1\, dI_{L1}/dt = 0$.

KVL in loop II

$-V_C + L_2\, dI_{L2}/dt + I_{R2} R_2 = 0$

$L_2 \frac{dI_{L2}}{dt}$ = unknown value.

$I_{R1} = [L_2\, dI_{L2}/dt + L_1\, dI_{L1}/dL]\, 1/R_1$.

$$L_2 \frac{dI_{L2}}{dt} = L_1 \frac{dI_{L1}}{dt} - R I_{R1}.$$

$$L_2 \frac{dI_{L2}}{dt} = L_1 \frac{dI_{L1}}{dt} - \frac{R}{R_1}\left[L_2 \frac{dI_{L2}}{dt} + L_1 \frac{dI_{L1}}{dL}\right]$$

Representation of state space equation

1. Phase variable representation.

$$7\frac{d^3Y}{dt^3} + 5\frac{d^2Y}{dt^2} + 2\frac{dY}{dt} + 3Y = 6U.$$

$7\dddot{Y} + 5\ddot{Y} + 2\dot{Y} + 3Y = 6U$

State variables are ⇒

x_1, x_2 & x_3.

Let $Y = X_1$ (1) $\ddot{Y} = X_3$ (3)

$\dot{Y} = X_2$ (2) $\dot{Y} = \dot{X}_1$,

Use eq (2) ⇒ $\dot{X}_1 = X_2$ — (A)

Differentiate eq. (1)

Differentiate eq. (2). (B)

$\dot{Y} = \dot{x}_2$

Use eq. (3),

$\dot{x}_2 = x_3$.

Differentiate eq. (3).

$\ddot{Y} = \dot{x}_3$

$7\dot{x}_3 + 5x_3 + 2x_2 + 3x_1 = 60$.

$\dot{x}_3 = (60 - 5x_3 - 2x_2 - 3x_1)/7$. (c)

$= \cancel{20} - \cancel{\frac{5x_3}{3}} - \cancel{\frac{2x_2}{3}} - \cancel{x_1}$

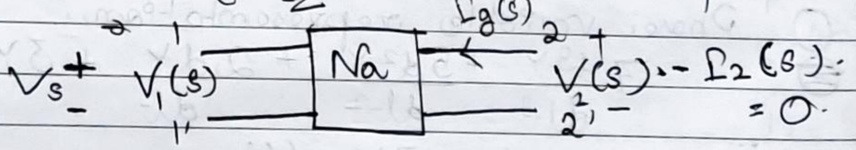
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3/7 & -2/7 & -5/7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{6}{7} \end{bmatrix} U.$$

$$[Y] = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

$Y + 6Y + 7\ddot{Y} + 6\dot{Y} + 5Y = 60$.

10.10.22

~~Rep~~ Reciprocity condition for Z-parameter



Condition (1)

Apply a voltage source V_s at 1, 1' &

short circuit the port 2, 2'.

Before making any changes at the terminals,

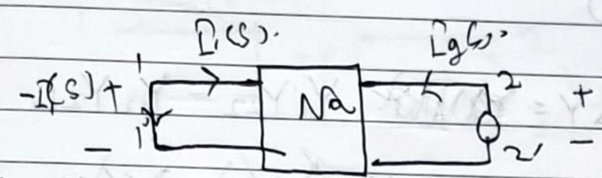
Z parametric eq. are :

$V_1(s) = Z_{11} I_1(s) + Z_{12} I_2(s)$ — ①.
$V_2(s) = Z_{21} I_1(s) + Z_{22} I_2(s)$ — ②

Apply changes using con. ①.
From eq. ① & ②.

$V_S = Z_{11} I_1(s) + Z_{12}(-I_2(s))$ — ③
$0 = Z_{21} I_1(s) + Z_{22}(-I_2(s))$ — ④

$$\frac{I_2(s)}{V_S} = \frac{-Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \quad \text{— ⑤}$$



Apply a voltage source ~~at~~ VS at 2, 2'
& SS the port 1 1'.

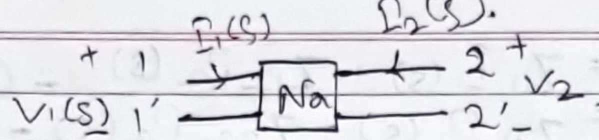
Before making any changes at the terminals,
Z parametric eqn. are :.

$V_1(s) = Z_{11} I_1(s) + Z_{12} I_2(s)$ — ①.
$V_2(s) = Z_{21} I_1(s) + Z_{22} I_2(s)$ — ②
Applying changes using con. ②

$0 = Z_1(-I_1(s)) + Z_{12}(I_2(s))$ — ⑥
$V_S = Z_{21}(-I_1(s)) + Z_{22}(I_2(s))$ — ⑦.

$$\frac{I_1(s)}{V_S} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

For requirement of reciprocity from ⑤ & ⑧
$Z_{12} = Z_{21}$.
This is applicable for all ntwks except ones having dependent sources.



$$I_1(s) = Y_{11}V_1(s) + Y_{12}V_2(s).$$
$$I_2(s) = Y_{21}V_1(s) + Y_{22}V_2(s)$$

Apply current source $I(s)$ at 1, 1' & open circuit the port 2, 2'

$$\frac{V_2(s)}{I(s)} = \cancel{V_2(s) = Y_{21}V_1(s)}\ \frac{-Y_{12}}{\Delta Y}.$$

where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$.

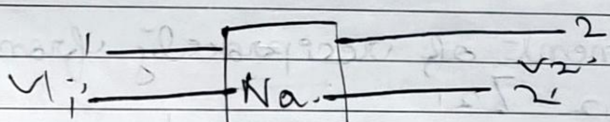
$$\cancel{\frac{V_2(s)}{I_1(s)} = (I_1(s) - Y_{11}V_1(s))/Y_{12}}$$

15.5.22

Symetry of two-port ntwk.

Definition : A two-port network is said to be symmetrical if ports can be interchanged without altering the port voltages & currents.

Currents parameters.	Symmetry condition.
Z	$Z_{11} = Z_{22}$
Y	$Y_{11} = Y_{22}$.
T	$A = D$.
h	$h_{11}h_{22} - h_{12}h_{21} = 1$.



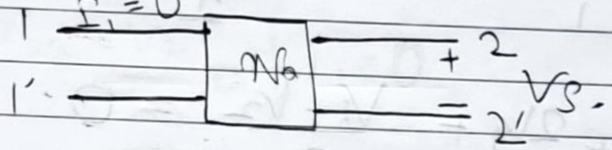
Z parameter -

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1).$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2).$$

Apply a voltage source V_S at $1,1'$ & open circuit the port $2 2'$.

Then from eq (1). $V_S = Z_{11} I_1$ $\quad Z_{11} = V_S / I_1$ $-(3)$

Apply V_S at $2 2'$ & open circuit at port $1,1'$

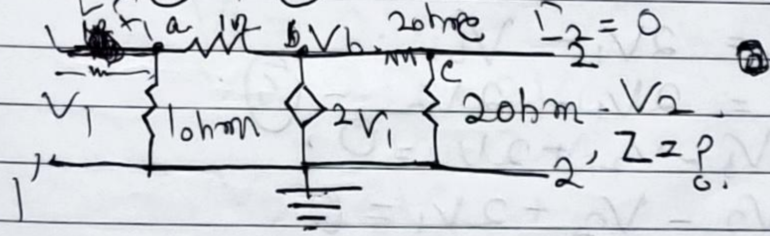


from eq. (1).

$$V_2 = 0 + Z_2 I_2$$

$$\frac{V_3}{I_2} = Z_{22}$$

In order for the z-parameter. to be symmetrical from eq (2) & (3).



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2.$$

Put $I_2 = 0$ i.e. open circuit.

$$Z_{11} = \frac{V_1}{I_1} \Big| I_2 = 0$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

✓ KCL at a.

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_a}{1}$$

$$I_1 = 2V_1 - V_a \quad -① ✓$$

✓ KCL at b.

$$\frac{V_b - V_1}{1} + 2V_1 + \frac{V_b - V_c}{2} \quad \because V_c = V_2$$

$$\frac{V_b - V_1}{1} + 2V_1 + \frac{V_b - V_2}{2} = 0$$

$$2V_b - 2V_1 + 4V_1 + V_b - V_2 = 0$$

$$3V_b - V_2 + 2V_1 = 0 \quad - ②$$

✓ KCL at C.

$$0 = \frac{V_2}{2} + V_2 - V_b,$$

Also $\underline{V_a = V_1}$

$$0 = 2V_2 - V_b \quad - ③$$

$\boxed{3V_2 - 2V_b}$

$$V_b = 2V_2$$

$$I_1 = 2V_1 - V_b$$

$$I_1 = 2V_1 - 2V_2 \quad - ④$$

$$3V_b - V_2 + 2V_1 = 0$$

$$6V_2 - V_2 + 2V_1 = 0$$

$$5V_2 + 2V_1 = 0$$

$$V_2 = -\frac{2V_1}{5}$$

$$I_1 - 2V_1 - 2\left(-\frac{2V_1}{5}\right)$$

Put

Z_1

Z

$$\cancel{2V} + \frac{4V_1}{5}$$

$$= \frac{14V_1}{5}$$

Put $I_1 = 0$ & reopen circuit $1, 1'$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} \quad ; \; V_h = V_1$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

$$I_a = \frac{V_1}{1} = V_1 \quad - (1)$$

$$I_b = V_1 + 2V_1$$

$$= 3V_1 \quad - (2)$$

$$I_c = \frac{V_2}{2} \quad - (3)$$

$$I_2 = I_b + I_c$$

$$I_2 = 3V_1 + V_2/2 \quad - (4)$$

$$V_b = 2V_1$$

$$V_2 = 2I_b + 2V_1$$

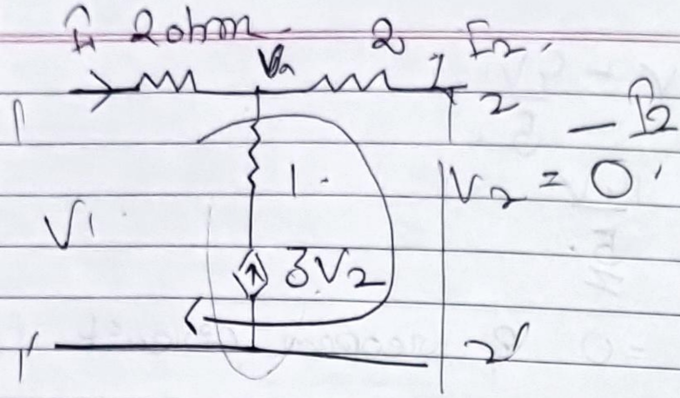
$$= 6V_1 + 2V_1$$

$$V_2 = 8V_1$$

$$I_2 = 3V_1 + \frac{8V_1}{2} = 3V_1 + 4V_1$$

$$= 7V_1$$

$$I_2 = 7V_1$$



$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

$Y_{11} = \frac{I_1}{V_1} \Big| V_2 = 0$

$Y_{21} = \frac{I_2}{V_1} \Big| V_2 = 0$

$\frac{V_a - V_1}{2} - 3V_2 - I_2 = 0$

$V_a - V_1 - 6V_1 - 2I_2 = 0$ - ①

$V_a - 7V_1 - 2I_2 = 0$ - ②

KVL in outer loop.

$V_1 = 2I_1 - 2I_2$.

$V_1 = 2I_1 + V_a$ - ③

$7V_1 = V_a - 2I_2$ - ④

Substituting value of ④ in eq ③

$\frac{V_a}{7} - \frac{2I_2}{7} = 2I_1 - 2I_2$

$V_a - 2I_2 = 7(2I_1 - 2I_2)$

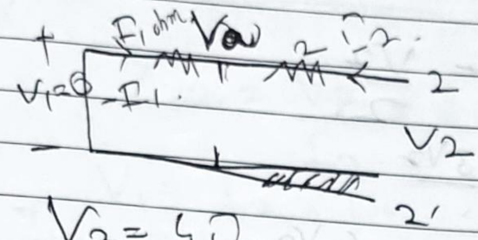
$V_a - 2I_2 = 14I_1 - 14I_2$.

$12I_2 = 14I_1 - V_a$.

$I_2 = \frac{7I_1}{6} - \frac{V_a}{12}$

$I_2 = -\frac{7I_1}{4}$.

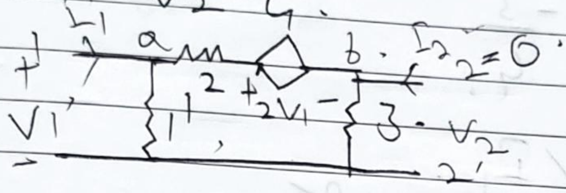
$Y_{22} = I_2 / V_2 \mid V_1 = 0$.



$V_2 = 4I_2$

$\frac{I_2}{V_2} = \frac{1}{4}$ mho. — (2)

V Imp.



PoL $I_2 = 0$

$Z_{11} = V_1 / I_1 \mid I_2 = 0$

$Z_{21} = V_2 / I_1 \mid I_2 = 0$

KCL at node 'a'

$I_1 = V_1 + V$.

$2I_1 = V_1 - V_2$.

$I_1 = \frac{V_1}{2} - \frac{V_2}{2}$ — (1).

KCL @ node b.

$0 = \frac{V_2}{3} + \frac{V_2 + 2V_1 - V_1}{2}$

$2V_2 + 3V_2 + 6V_1 - 3V_1 = 0$.

$5V_2 + 3V_1 = 0$

$\cancel{3}V_1 = -5V_2/3$

$I_1 = -\frac{8V_2}{6}$.

$I_1 = -4V_2/3$.

$\frac{V_2}{I_1} = \frac{3}{4}$ V

$3V_1 = -\frac{5V_2}{3}$

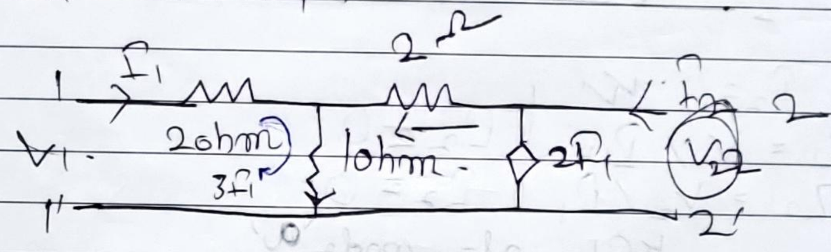
$I_1 = -\frac{5V_2}{6} - \frac{V_2}{2}$

$= \frac{-5V_2 - 3V_2}{6}$

$= \frac{-8V_2}{6} = -\frac{4V_2}{3}$

$\frac{V_2}{I_1} = -\frac{3}{4}$

$Z_{11} = V_1 / I_1$



$Z_{11} = V_1 / I_1 \mid I_2 = 0$

$Z_{21} = V_2 / I_1 \mid I_2 = 0$

$V_1 = 2I_1 + 3I_1 \times 1$

$= 5I_1$

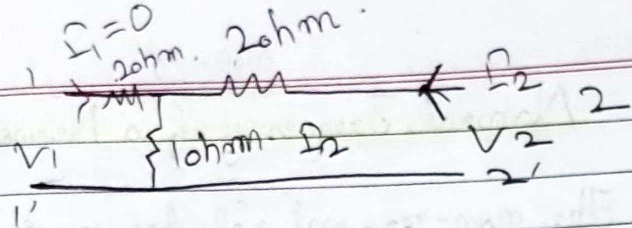
$\frac{V_1}{I_1} = 5$ ohm

KVL in middle loop -

$V_2 = 2 \times 2 I_1 + 3I_1 \times 1$

$V_2 = (4+3) I_1$

$Z_{21} = \frac{V_2}{I_1} = 7$ ohm



$$Z_{22} = \frac{V_2}{I_2} = 3\,ohm.$$

$$V_1 = I_2 \times 1.$$

$$Z_{12} = \frac{V_1}{I_2} = 1\,ohm$$