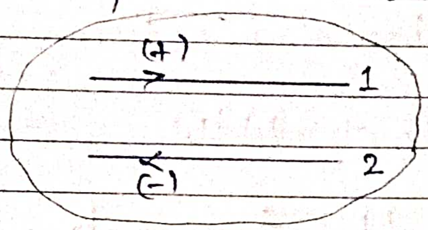
DATE 01/08/2023
PAGE NO.

Module - 3

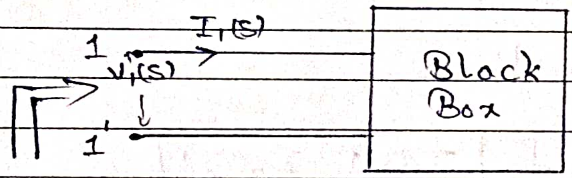
Two - Port Network

Port - A pair of available terminals



One - Port Network

* One pair of terminals are available.
Eg : TV, AC, etc.



Schematic diagram of One - Port N/W.

* Our prime focus is to find voltage & current. So, rest of the things are kept ~~at~~ altogether ~~inside a~~ terming it a Black box.

$$Z_{11'} = \frac{V_1(S)}{I_1(S)} \quad \Omega\ (\text{ohm})$$

Driving point Impedance function

DATE
PAGE NO.

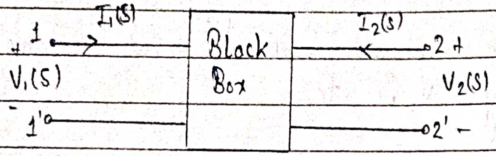
$$Y_{11'} = \frac{I_1(S)}{V_1(S)} \quad \mho \text{ (mho)}$$

Driving Point Admittance Function

Two-Port N/W

* 2 pairs of available terminals.
Eg: Transformer

Schematic diagram:



Parametric Equations

$$[V_1(S), V_2(S)] = f\,[I_1(S), I_2(S)]$$

$$\boxed{\begin{aligned} &V_1(S) = Z_{11} I_1(S) + Z_{12} I_2(S) \quad (1) \\ &V_2(S) = Z_{21} I_1(S) + Z_{22} I_2(S) \quad (2) \end{aligned}} \Rightarrow Z\text{-Parameter}$$

* Putting $I_2(S) = 0$
from eq^n (1):

$$Z_{11} = \left.\frac{V_1(S)}{I_1(S)}\right|_{I_2(S)=0}$$ (Open Circuiting)

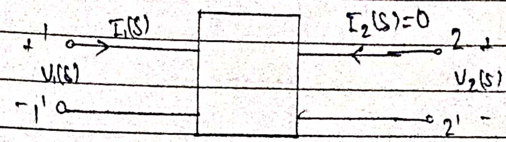
Driving Point Impedance / Open Circuit Impedance Parameter

DATE
PAGE NO.

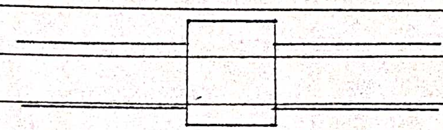
from eq^n (2):

$$Z_{21} = \left.\frac{V_2(S)}{I_1(S)}\right|_{I_2(S)=0}$$

Forward Transfer Impedance



* Putting $I_1(S) = 0$ i.e. Open Circuiting 11'



from eq^n (1):

$$Z_{12} = \left.\frac{V_1(S)}{I_2(S)}\right|_{I_1(S)=0}$$

Reverse Transfer Impedance

from eq^n (2):

$$Z_{22} = \left.\frac{V_2(S)}{I_2(S)}\right|_{I_1(S)=0}$$

Output Impedance

DATE
PAGE NO.

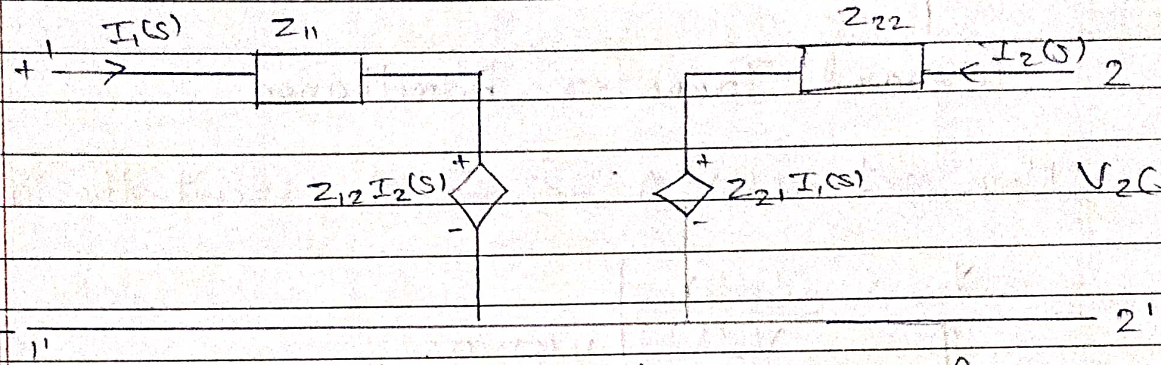
* $$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

(i) $Z_{12} = Z_{21}$ ⇒ Reciprocity requirement

(ii) $Z_{11} = Z_{22}$ ⇒ Symmetry requirement

DATE 3|8|23
PAGE NO.

Circuit Diagram corresponding to Eqⁿ 1 & 2 :-



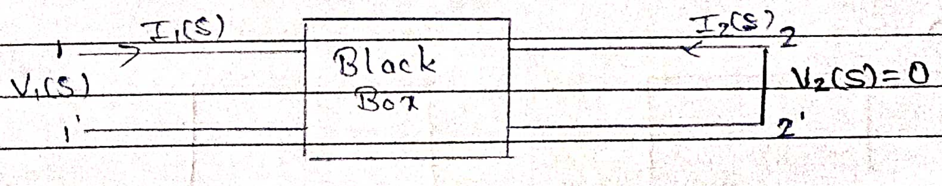
Two generator equivalent circuit diagram

Y-Parameter | Short Circuit Admittance Parameter

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1(S) = Y_{11} V_1(S) + Y_{12} V_2(S) \quad (1)$$

$$I_2(S) = Y_{21} V_1(S) + Y_{22} V_2(S) \quad (2)$$



Put $V_2(S)=0$ i.e. Short Circuit the port 22'.

$$\therefore Y_{11} = \left. \frac{I_1(S)}{V_1(S)} \right|_{V_2(S)=0}$$

Driving Point Admittance func.

$$Y_{21} = \left.\frac{I_2(S)}{V_1(S)}\right|_{V_1(S)=0}$$

Forward Transfer Admittance

Now put $V_1(S)=0$ i.e. Short Circuit the port $11'$

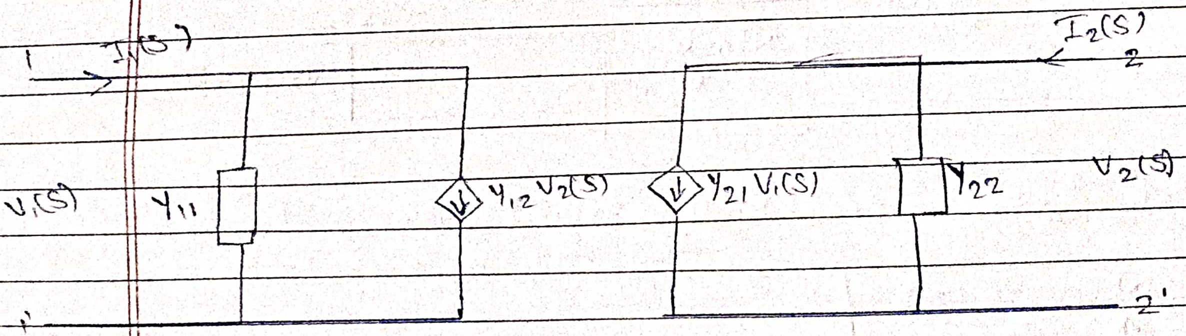
$$Y_{12} = \left.\frac{I_1(S)}{V_2(S)}\right|_{V_1(S)=0}$$

Reverse Transfer Admittance

$$Y_{22} = \left.\frac{I_2(S)}{V_2(S)}\right|_{V_1(S)=0}$$

O/p Admittance

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$



(i) $Y_{12} = Y_{21} \Rightarrow$ Condition for Reciprocity

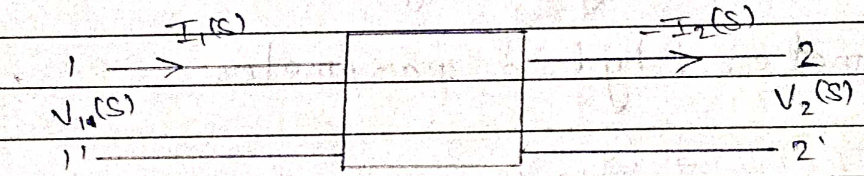
(ii) $Y_{11} = Y_{22} \Rightarrow$ Condition for Symmetry

T-Parameter / Chain / ABCD Parameter

$$(V_1, I_1) = f(V_2, -I_2)$$

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$



Put $-I_2 = 0$ i.e. Open Circuit port 22'

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0}$$ unitless

$$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0}$$ $(\mho)$

DATE
PAGE NO.

Put, $V_2 = 0$ i.e. Short circuit port 22'

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0} \quad (\Omega)$$

Reverse Transfer Impedance

$$D = \frac{I_1}{-I_2}\bigg|_{V_2=0}$$

(i) Condition for Recip rocity : $AD - BC = 1$

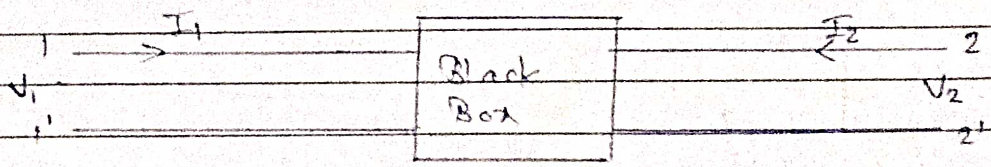
(ii) Condition for Symmetry : $A = B$

h-Parameter / hybrid parameter

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



DATE
PAGE NO.

Put, $V_2 = 0$ i.e. Short Circuit port 22'

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad (\Omega)$$

Driving Point Impedance

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad \text{(unitless)}$$

Current Gain

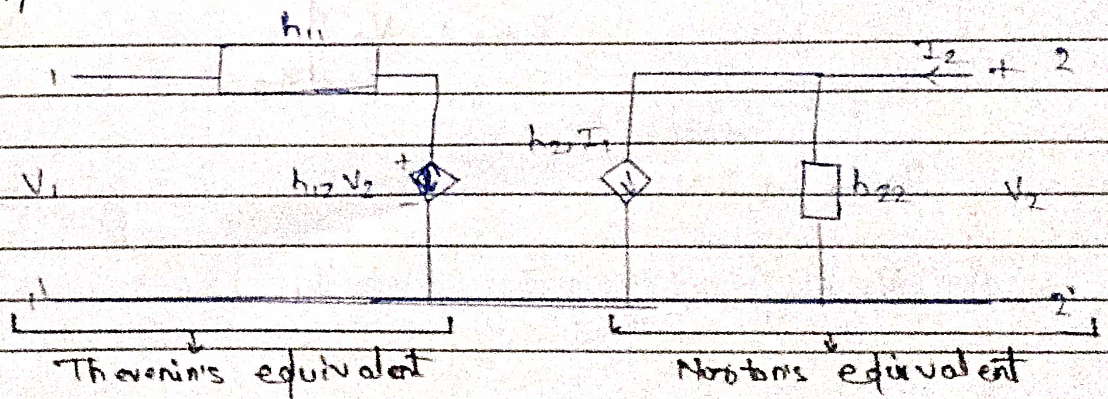
Put, $I_1 = 0$ i.e. Open Circuit port 11'

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{(unitless)}$$

Reverse Voltage Gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad (\mho)$$

o/p Admittance



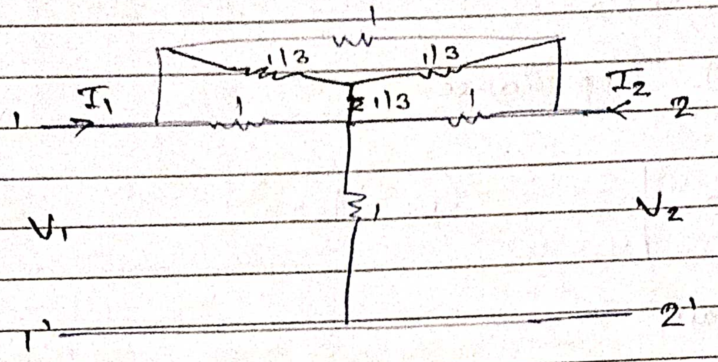
* Inverse T & h - parameters Assignment

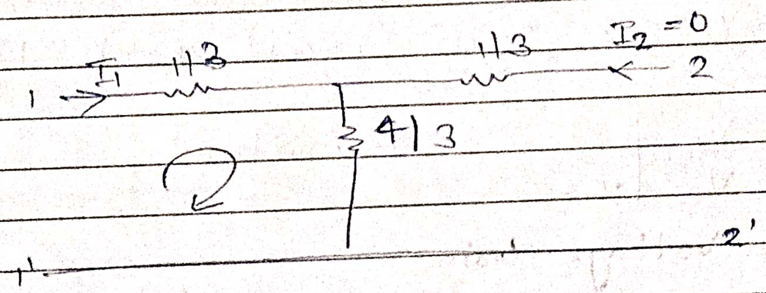
DATE
PAGE NO.

(i) $h_{12} = -h_{21} \Rightarrow$ Reciprocity

(ii) $h_{11}h_{12} - h_{21}h_{22} = 1 \Rightarrow$ Symmetry

Q.





Z-Parameters : $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Put, $I_2 = 0$;

$$V_1 = \left(\frac{1}{3} + \frac{4}{3}\right) I_1 = \frac{5 I_1}{3}$$

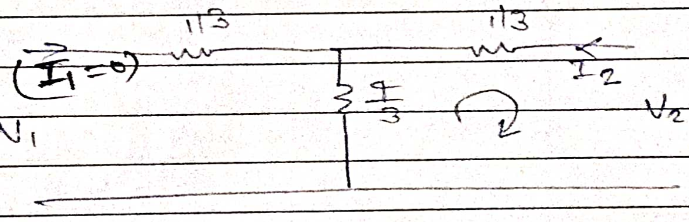
$$Z_{11} = \frac{V_1}{I_1} = \frac{\frac{5I_1}{3}}{I_1} = \frac{5}{3}\ (\Omega)$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$V_2 = \frac{4}{3} I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{4}{3}\ (\Omega)$$

Now, Put $I_1 = 0$



$$V_2 = \left(\frac{4}{3} + \frac{1}{3}\right) I_2$$

$$\therefore Z_{22} = \frac{V_2}{I_2} = \frac{5}{3}\ \Omega$$

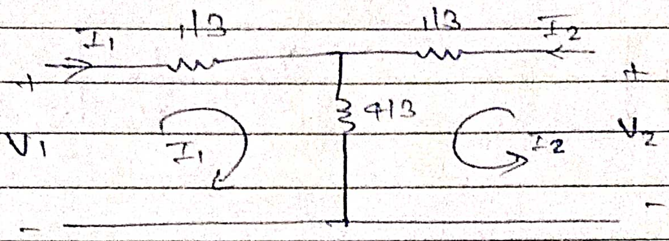
$$V_1 = \frac{4}{3} I_2$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{4}{3}\ \Omega$$

$$[Z] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ 4/3 & 5/3 \end{bmatrix}$$

DATE 4/8/23
PAGE NO.

Method II:



$$V_1 = \frac{1}{3} I_1 + \frac{4}{3}(I_1 + I_2)$$

$$= \frac{5}{3} I_1 + \frac{4}{3} I_2 \quad \text{— (1)}$$

$$V_2 = \frac{1}{3} I_2 + \frac{4}{3}(I_1 + I_2)$$

$$= \frac{4}{3} I_1 + \frac{5}{3} I_2 \quad \text{— (2)}$$

On Comparing :

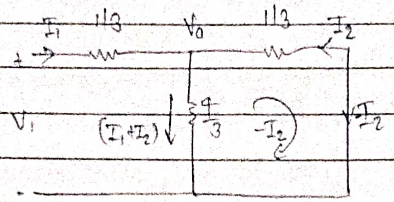
$Z_{11} = \frac{5}{3}$, $Z_{12} = \frac{4}{3}$

$Z_{21} = \frac{4}{3}$, $Z_{22} = \frac{5}{3}$

$[Y] = [Z]^{-1}$

For Y-parameter

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_1=0}$$



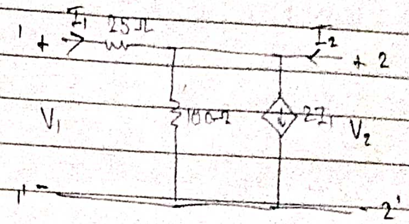
$$I_1 = \frac{V_0}{4/3} - I_2$$

$$= \frac{3V_0}{4} - I_2$$

$$V_1 = \frac{1}{3}I_1 + (I_1+I_2)\cdot\frac{4}{3}$$

$$V_0 = V_1 - \frac{1}{3}I_1$$

Q) Find Y-Parameters.

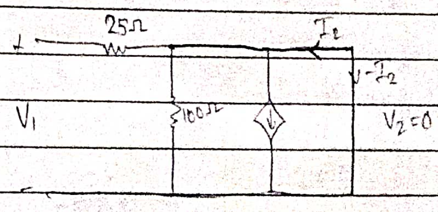


Sol^n: Put $V_2 = 0$:

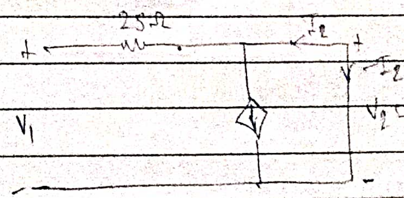
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$



100Ω is removed because of same potential



DATE
PAGE NO.

$-V_1 + 25I_1 + 0 = 0$
$\Rightarrow V_1 = 25I_1$ — (i)

$\cancel{\Rightarrow \frac{V_1}{I_1} = 25\Omega}$ $Y_{11} = \frac{I_1}{V_1} = \frac{1}{25}$ ℧

KCL at at

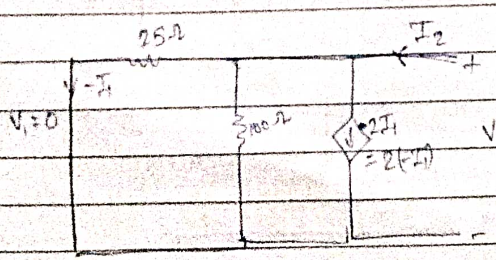
$I_1 = 2I_1 + (I_2)$

$\Rightarrow -I_1 = -I_2 \Rightarrow I_1 = I_2$ — (ii)

From (i) & (ii):

$V_1 = 25I_1$
$\cancel{\Rightarrow V_2 = 25I_2}$
$Y_{21} = \frac{I_2}{V_1} = \frac{1}{25}$

Now put $V_1 = 0$:-



DATE
PAGE NO.

$I_2 = -2I_1 + \frac{V_2}{100} + \frac{V_2}{25}$ — (iii)

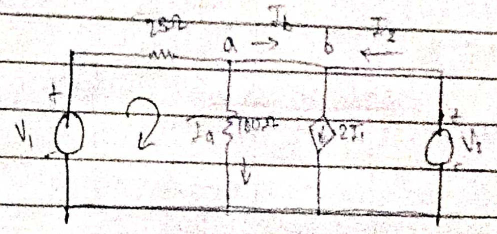
$-I_1 = \frac{V_2}{25}$ — (iv)

$Y_{12} = \frac{-1}{25}$ ℧ $\left(\frac{I_1}{V_2}\right)$

$I_2 = \frac{2V_2}{25} + \frac{V_2}{25} + \frac{V_2}{100}$

$\frac{I_2}{V_2} = \frac{13}{100} = Y_{22}$

2) $[Z] = [Y]^{-1}$



KCL at node b:-

$I_b + I_2 = 2I_1$

$\Rightarrow I_b = 2I_1 - I_2$ — (i)

KCL at node a:

$$I_1 = I_a + I_b$$

$$\Rightarrow I_a = I_1 - I_b$$
$$= I_1 - (2I_1 - I_2)$$
$$\Rightarrow I_a = -I_1 + I_2 \quad \text{--- (ii)}$$

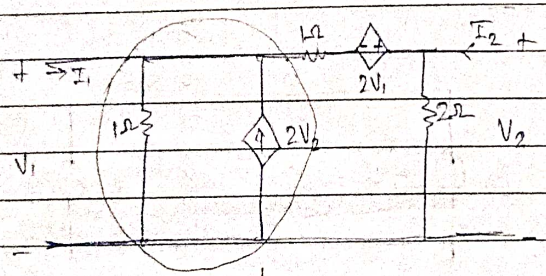
KVL in loop (I):

$$V_1 = 25I_1 + 100\overbrace{(-I_1 + I_2)}^{(I_a)}$$

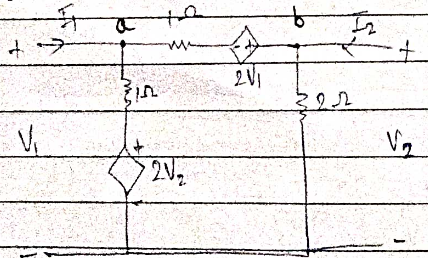
$$= -75I_1 + 100I_2 \quad \text{--- (1)}$$

loop (II):
$$V_2 = 100(-I_1 + I_2) \quad \text{--- (2)}$$

8.)



Sol^n: Method I:



KCL at node a:

$$I_1 = \frac{V_1 - 2V_2}{1} + \frac{V_1 + 2V_1 - V_2}{1}$$

$$\Rightarrow I_1 = V_1 - 2V_2 + V_1 + 2V_1 - V_2$$
$$\Rightarrow I_1 = 4V_1 - 3V_2 \quad \text{--- (1)}$$

KCL at node b:

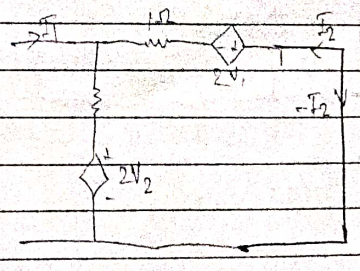
$$I_2 = \frac{V_2}{2} + \frac{V_2 - 2V_1 - V_1}{1}$$

$$= -3V_1 + 1.5V_2 \quad \text{--- (2)}$$

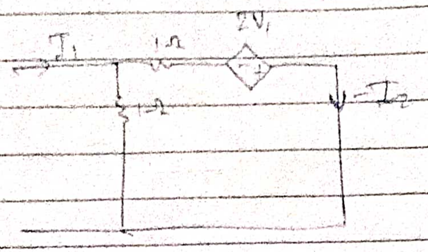
$$[Y] = \begin{bmatrix} 4 & -3 \\ -3 & 1.5 \end{bmatrix} \mho$$

Method II:

$V_2 = 0$



DATE
PAGE NO.



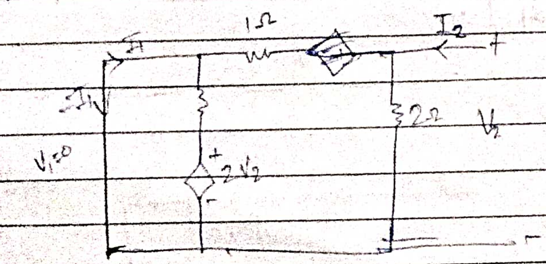
$-I_2 = \frac{V_1 + 2V_1}{1}$ $\quad$ $I_1 = \frac{V_1}{1} + \frac{V_1 + 2V_1}{1}$

$\Rightarrow \frac{I_2}{V_1} = -3\mho = Y_{21}$ $\quad$ $Y_{11} = \frac{I_1}{V_1} = 4\mho$

$-I_2 = \frac{V_1 + 2V_1}{1}$

$\Rightarrow Y_{21} = \frac{I_2}{V_1} = -3\mho$

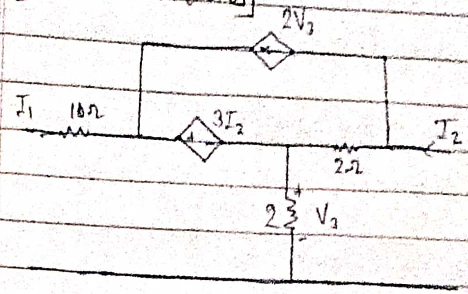
Now, $V_1 = 0$



DATE 7/8/23
PAGE NO.

[With Conditions]

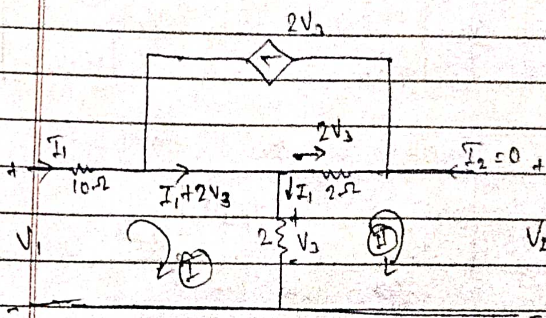
Q) Find Z-Parameters



Sol^n:

$$\boxed{\begin{array}{l} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \\ Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} \quad , \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} \end{array}}$$

Put, $I_2 = 0$



KVL in loop (I):

$-V_1 + 10I_1 + 2I_1 = 0$

$\Rightarrow V_1 = 12I_1$

$\therefore \underline{Z_{11}} = \frac{V_1}{I_1} = \underline{12\Omega}$

KVL in loop II :

$$-2I_1 + 2(2V_3) + V_2 = 0$$
$$\Rightarrow V_2 = 2I_1 - 4V_3$$

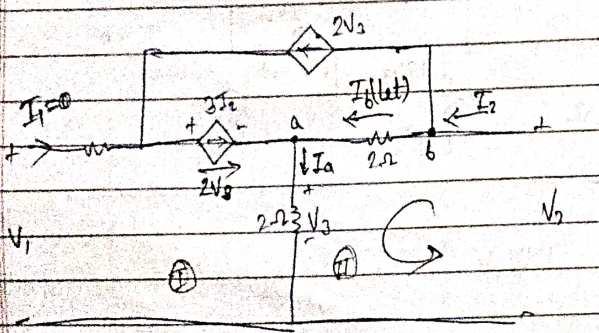
And also, $V_3 = 2I_1$

$$\text{So, } V_2 = 2I_1 - 4\times 2I_1$$
$$\Rightarrow V_2 = -6I_1$$

$$\therefore Z_{21} = \frac{V_2}{I_1} = -6\Omega$$

For Second half:

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} , \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



KCL at b : $I_2 = I_b + 2V_3$

$$\Rightarrow I_b = (I_2 - 2V_3)$$

Now, KCL at a: $2V_3 + I_b = I_a$

$$\Rightarrow 2V_3 + (I_2 - 2V_3) = I_a$$
$$\Rightarrow I_a = I_2$$

KVL in IInd loop:

$$-V_2 + 2I_b + 2\times I_a = 0$$
$$\Rightarrow V_2 = 2(I_2 - 2V_3) + 2(I_2)$$
$$\Rightarrow V_2 = 4I_2 - 4V_3$$

But, $V_3 = 2I_a = 2I_2$

$$\text{So, } V_2 = 4I_2 - 4\cdot 2I_2 = -4I_2$$

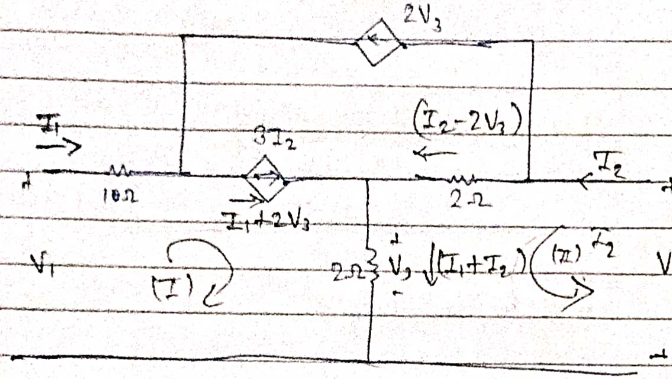
$$\therefore Z_{22} = \frac{V_2}{I_2} = -4\Omega$$

Now, KVL in Ist loop:

$$-V_1 + 3I_2 + 2I_2 = 0 \quad (\because I_a = I_2)$$
$$\Rightarrow Z_{12} = \frac{V_1}{I_2} = 5\Omega$$

$$\therefore Z = \begin{bmatrix} 12 & 5 \\ -6 & -4 \end{bmatrix} \quad \& \quad Y = [Z]^{-1}$$

Without Conditions



KVL in loop (I) :-

$-V_1 + 10I_1 + 3I_2 + 2(I_1+I_2) = 0$

$\Rightarrow V_1 = 12I_1 + 5I_2$

KVL in loop (II) :-

$-V_2 + 2(I_2 - 2V_3) + 2(I_1+I_2) = 0$

$\cancel{\Rightarrow V_2 = 2(I_1+I_2)}$

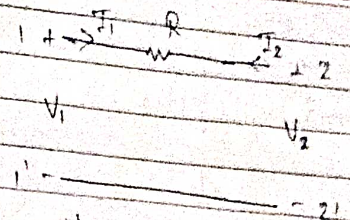
But, $V_3 = 2(I_1+I_2)$

$V_2 = 2I_2 - 8(I_1+I_2) + 2(I_1+I_2)$

$= -6I_1 - 4I_2$

$\therefore [Z] = \begin{bmatrix} 12 & 5 \\ -6 & -4 \end{bmatrix}$

Q)



Sol^n: Since it is Open Circuit, Single loop ⇒ So, same current flows through loop.

When, $I_1 = 0 \Rightarrow I_2 = 0$

$I_2 = 0 \Rightarrow I_1 = 0$

∴ Z parameter does not exist / not defined

Also, $I_1 = -I_2$

$V_1 = I_1 R$

$\frac{1}{R} = \frac{I_1}{V_1} = Y_{11}$

$I_1 = Y_{11}V_1 + Y_{12}V_2$

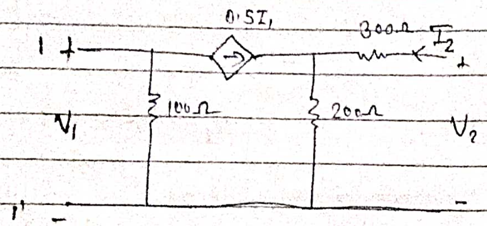
$I_2 = Y_{21}V_1 + Y_{22}V_2$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $\qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

$Y_{21} = \frac{-1}{R}$

DATE
PAGE NO.

Q.) Find Y-Parameter.

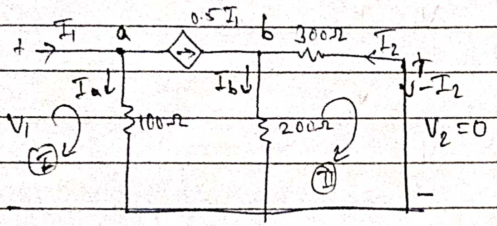


Soln: $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$, $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

Put $V_2 = 0$;



KCL at node a :-

$I_1 = I_a + 0.5I_1$

$\Rightarrow I_a = 0.5I_1$

KCL at node b :- $0.5I_1 = I_b - I_2$

$\Rightarrow I_b = 0.5I_1 + I_2$

DATE
PAGE NO.

KVL in loop (I) :-

$-V_1 + 100I_a = 0$

$\Rightarrow V_1 = 100 \times 0.5I_1 = 50I_1$

$\therefore Y_{11} = \frac{I_1}{V_1} = \frac{1}{50}\ ℧$

KVL in loop (II) :-

$-I_b \times 200 + 300(I_2) = 0$

$\Rightarrow -(0.5I_1 + I_2)200 + 300(I_2) = 0$

$\Rightarrow -100I_1 - 200I_2 - 300I_2 = 0$

$\Rightarrow -100I_1 = 500I_2$

But, $\frac{I_1}{V_1} = \frac{1}{50}$

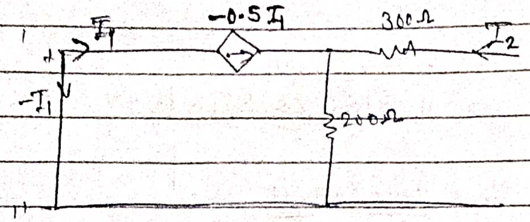
So, $-100 \times \frac{V_1}{50} = 500I_2$

$\therefore Y_{21} = \frac{I_2}{V_1} = \frac{-1}{250}\ ℧$

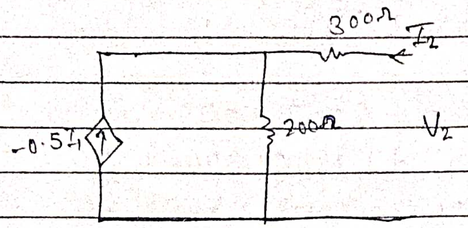
Now for second half :-

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$, $Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

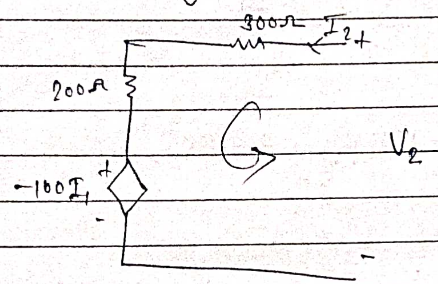
Put, $V_1 = 0$:



$\Downarrow$



$\Downarrow$



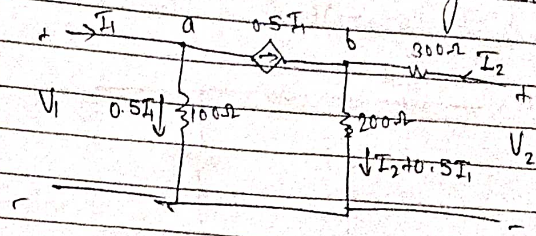
$$-V_2 + 500 I_2 - 100 I_1 = 0$$

$$\Rightarrow V_2 = 500 I_2 - \cancel{100 I_1}$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{1}{500} \mho$$

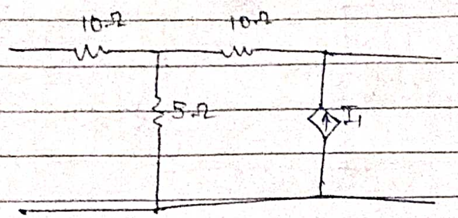
$$Y_{12} = 0$$

Alternate Method :- (without using condition)



A, B, C, D parameter -> always +ve)

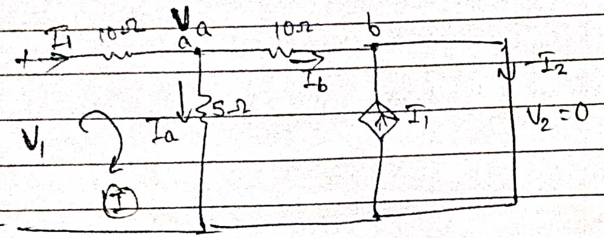
Q.> Find T-Parameter.



Sol^n: $\left[\begin{array}{l} V_1 = AV_2 + B(-I_2) \\ I_1 = CV_2 + D(-I_2) \end{array}\right]$

Put, $V_2 = 0$;

$\left[B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} , \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \right]$



KCL at a :-

$\frac{V_a - V_1}{10} + \frac{V_a - 0}{5} + \frac{V_a - 0}{10}$

KCL at b :-

$I_b + I_1 = -I_2$

$\Rightarrow I_b = -I_1 - I_2$ —(1)

KCL at a :-

$I_1 = I_a + I_b$

$\Rightarrow I_a = I_1 - I_b = I_1 + I_1 + I_2$

$\Rightarrow I_a = 2I_1 + I_2$

KVL in outer path:

$-V_1 + 10I_1 + 10I_b = 0$

$\Rightarrow -V_1 + 10I_1 + 10(-I_1 - I_2) = 0$

$\Rightarrow V_1 = -10I_2$

$B = \frac{V_1}{-I_2} = 10\Omega$

KVL in loop I: $-V_1 + 10I_1 + 5I_a = 0$

$\Rightarrow -10I_2 + 10I_1 + 5(2I_1 + I_2) = 0$

$\Rightarrow -5I_2 + 20I_1 = 0$

$\Rightarrow \frac{I_1}{-I_2} = \frac{15}{20} = \frac{3}{4}$

$\therefore D = \frac{3}{4}$

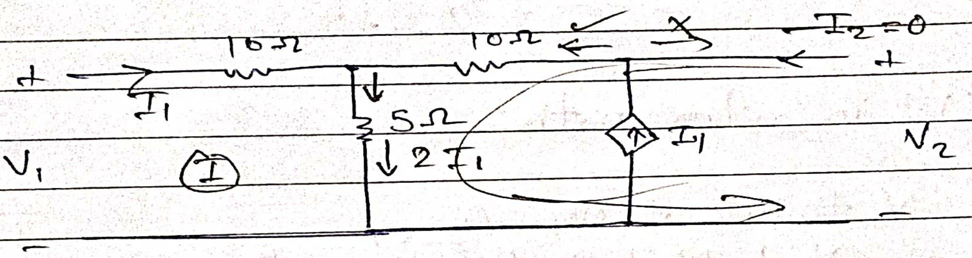
DATE
PAGE NO.

~~Put, $-I_2 = 0$~~

$V_1 = AV_2 + B(-I_2)$
$I_1 = CV_2 + D(-I_2)$

Put, $-I_2 = 0$:

$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0}$, $C = \left.\frac{I_1}{I_2}\right|_{-I_2=0}$



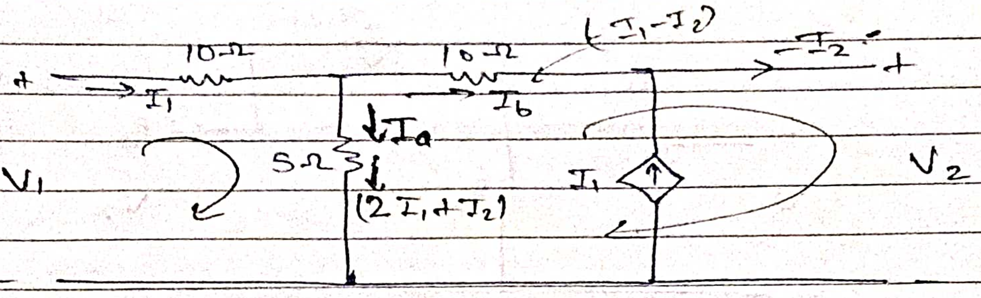
KVL in (I) Loop : $-V_1 + 10I_1 + 10I_1 = 0$
$\Rightarrow V_1 = 20I_1$
$\Rightarrow \frac{V_1}{I_1} = 20\Omega$ — (A)

KVL in loop (II) : $-V_2 + 10I_1 + 10I_1 = 0$
$\Rightarrow \frac{V_2}{I_1} = 20\,\Omega$
$\Rightarrow \frac{I_1}{V_2} = \frac{1}{20}\,\mho$ — (B)

from (A) & (B) :- $A = 1$

DATE 8/8/23
PAGE NO.

Method II: Without Condition



$$V_1 = 10I_1 + 5(2I_1+I_2)$$

$$\Rightarrow V_1 = 20I_1 + 5I_2 \quad \text{--- (1)}$$

$$\cancel{V_2} = V_2 - 5\times I_a + 10 I_b = 0$$

$$\Rightarrow V_2 = 5(2I_1+I_2) - 10(I_1-I_2)$$

$$\Rightarrow V_2 = 20I_1 + 15I_2 \quad \text{--- (2)}$$

$$\boxed{\begin{aligned} V_1 &= AV_2 + B(-I_2) \\ I_1 &= CV_2 + D(-I_2) \end{aligned}}$$

$$I_1 = \cancel{\frac{V_1-5I_2}{20}} \quad \frac{V_2-15I_2}{20}$$

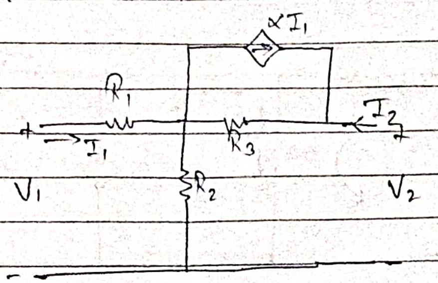
$$\cancel{V_2 = 20} \quad \cancel{V_1 =}$$

$$V_1 = 20\left(\frac{V_2-15I_2}{20}\right) + 5I_2$$

$$\Rightarrow V_1 = V_2 - 10I_2$$

$$20I_1 = V_2 - 15I_2$$

Q.) Calculate H-Parameters.



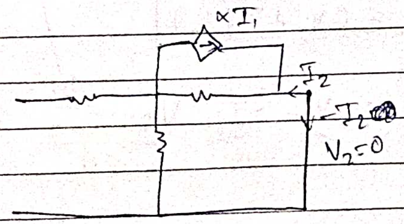
Solⁿ:

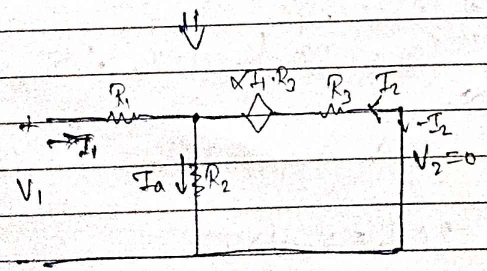
$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Put, $V_2 = 0$:

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$, $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$





$I_1 = I_a - I_2 \Rightarrow I_a = I_1 + I_2$

$-V_1 + I_1 R_1 + I_a R_2 = 0$

$\Rightarrow V_1 = (R_1+R_2) I_1 + R_2 I_2$ — (1)

KVL in loop II :-

$-I_a R_2 - \alpha R_3 I_1 - I_2 R_3 = 0$

$\Rightarrow -(I_1+I_2) R_2 - \alpha R_3 I_1 - R_3 I_2 = 0$

$\Rightarrow (-R_2 - \alpha R_3) I_1 + (-R_2 - R_3) I_2 = 0$

$\Rightarrow \frac{I_2}{I_1} = -\frac{(R_2 + \alpha R_3)}{R_2 + R_3}$ — (2)

$\Rightarrow I_2 = -\frac{I_1 (R_2 + \alpha R_3)}{R_2 + R_3}$

(1) $\Rightarrow V_1 = (R_1 + R_2) I_1 + R_2 \frac{(R_2 + \alpha R_3) \cdot (-I_1)}{R_2 + R_3}$

$= \frac{(R_1+R_2)(R_2+R_3) I_1 - R_2 (R_2 + \alpha R_3) I_1}{(R_2+R_3)}$

$= \frac{(R_1 R_2 + R_1 R_3 + \cancel{R_2^2} + R_2 R_3 - \cancel{R_2^2} - \alpha R_2 R_3) I_1}{R_2 + R_3}$

$\Rightarrow V_1 = \frac{R_1 R_2 + (1-\alpha) R_2 R_3 + R_1 R_3}{(R_2+R_3)} \cdot I_1$

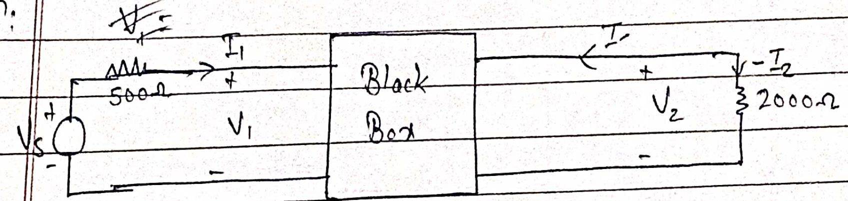
$\Rightarrow \frac{V_1}{I_1} = R_1 + \frac{(1-\alpha) R_2 R_3}{R_2 + R_3}$

DATE

PAGE NO.

Q.) The h-Parameter of a 2-Port network shown in figure are $h_{11} = 1000\,\Omega$, $h_{12} = 0.003$, $h_{21} = 100$, $h_{22} = 50 \times 10^{-6}\,\mho$. Find V_2 & Z-Parameter of the network if $V_s = 10^{-12}\angle 0°\,V$.

Sol^n:



$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$-V_s + 500 I_1 + V_1 = 0$

$\Rightarrow V_1 = V_s - 500 I_1$

$V_s - 500 I_1 =$

DATE 10/8/23
PAGE NO.

Inter-relationship between different Parameters

Express Z-Parameter in terms of Y-Parameter

Z Parameters are:

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

Y-Parameters are:

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{(3)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (4)}$$

from eqn 4 :- $V_2 = \dfrac{I_2 - Y_{21} V_1}{Y_{22}}$ — (5)

Using eqn 5 in 3 :-

$$I_1 = Y_{11} V_1 + Y_{12} \left(\frac{I_2 - Y_{21} V_1}{Y_{22}} \right)$$

$$\Rightarrow Y_{22} I_1 = Y_{22} Y_{11} V_1 + Y_{12} I_2 - Y_{21} Y_{21} V_1$$

$$\Rightarrow (Y_{11} Y_{22} - Y_{12} Y_{21}) V_1 = Y_{22} I_1 - Y_{12} I_2$$

$$V_1 = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2 \quad \text{—(6)}$$

where, $(\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21})$

Comparing (1) & (6) :-

$$\therefore \quad Z_{11} = \frac{Y_{22}}{\Delta Y}, \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

from eqn 3 :-

$$V_1 = \frac{I_1 - Y_{12} V_2}{Y_{11}} \quad \text{—(7)}$$

Using eqn (7) in (4) :-

$$I_2 = Y_{21} \left(\frac{I_1 - Y_{12} V_2}{Y_{11}} \right) + Y_{22} V_2$$

$$\Rightarrow Y_{11} I_2 - Y_{21} I_1 = (Y_{11} Y_{22} - Y_{12} Y_{21}) V_2$$

$$\Rightarrow \Delta Y V_2 = -Y_{21} I_1 + Y_{11} I_2$$

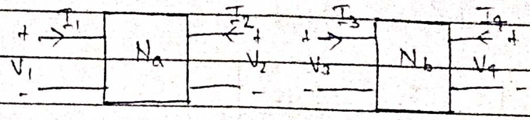
$$V_2 = \frac{-Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2 \quad \text{—(8)}$$

$$\therefore \quad Z_{21} = \frac{-Y_{21}}{\Delta Y}, \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

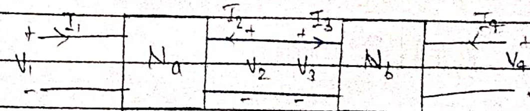
Q) Express : (i) T-parameter in terms of H-para.
(ii) Y- " " " "

Connections of 2-Port Network System

1. Cascade Connection
2. Series "
3. Parallel "
4. Series-Parallel "
5. Parallel-Series "



1.) Cascade Connection (only T-parameters is used)



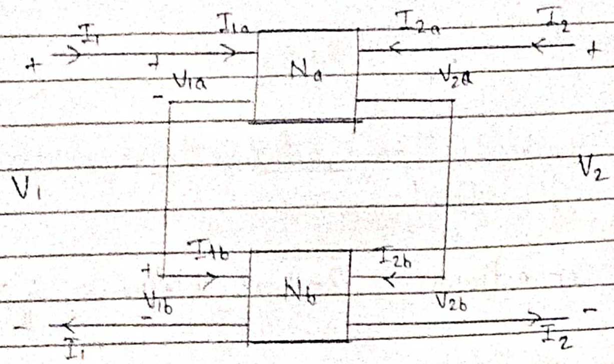
[T_a] [T_b]

$V_2 = V_3$
$I_2 = -I_3$

$$\boxed{T = [T_a][T_b]}$$

DATE
PAGE NO.

2.) Series Connection



$$V_1 = V_{1a} + V_{1b} \qquad V_2 = V_{2a} + V_{2b}$$

$$I_1 = I_{1a} = I_{1b} \qquad I_2 = I_{2a} = I_{2b}$$

$$[Z_a] = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22b} \end{bmatrix}$$

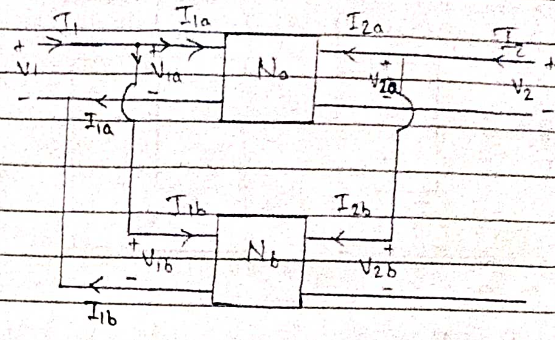
$$[Z_b] = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix}$$

$$[Z] = [Z_a] + [Z_b]$$

$$= \begin{bmatrix} \underbrace{Z_{11a}+Z_{11b}}_{Z_{11}} & \underbrace{Z_{12a}+Z_{12b}}_{Z_{12}} \\ \underbrace{Z_{21a}+Z_{21b}}_{Z_{21}} & \underbrace{Z_{22a}+Z_{22b}}_{Z_{22}} \end{bmatrix}$$

DATE
PAGE NO.

3.) Parallel Connection



$$V_1 = V_{1a} = V_{1b} \qquad V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \qquad I_2 = I_{2a} + I_{2b}$$

$$Y_{11} = Y_{11a} + Y_{11b}$$

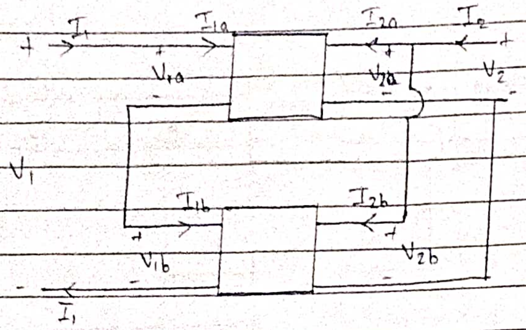
$$Y_{12} = Y_{12a} + Y_{12b}$$

$$Y_{21} = Y_{21a} + Y_{21b}$$

$$Y_{22} = Y_{22a} + Y_{22b}$$

DATE
PAGE NO.

4) Series-Parallel Connection



$V_1 = V_{1a} + V_{1b}$ $\quad$ $V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} = I_{1b}$ $\quad$ $I_2 = I_{2a} + I_{2b}$

$h_{11} = h_{11a} + h_{11b}$

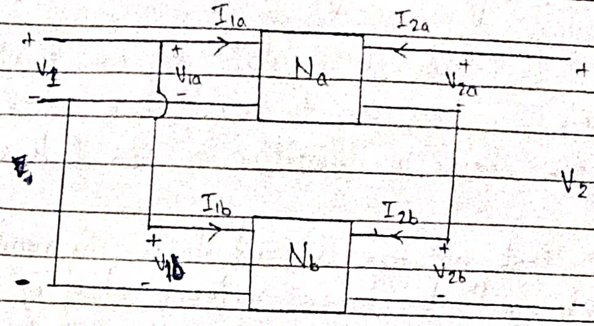
$h_{12} = h_{12a} + h_{12b}$

$h_{21} = h_{21a} + h_{21b}$

$h_{22} = h_{22a} + h_{22b}$

DATE
PAGE NO.

5) Parallel-Series Connection



$V_1 = V_{1a} = V_{1b}$ $\quad$ $V_2 = V_{2a} + V_{2b}$

$I = I_{1a} + I_{2b}$ $\quad$ $I_2 = I_{2a} = I_{2b}$

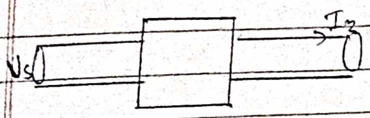
DATE 11/8/23
PAGE NO.

Reciprocity and Symmetry of 2 Port Network

* A 2-Port network is said to be Reciprocal if ratio of response to excitation remains invariant to the change in position of the ~~~~ two.

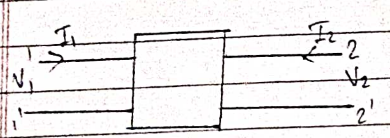
* A 2-Port network is said to be Symmetrical if ports can be interchanged without changing the current and voltage.

R:

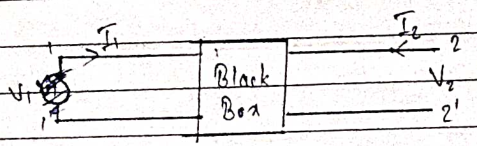


$$\left[\frac{I_2}{V_s} = \frac{I_1}{V_s}\right]$$

S:



$$\left[\frac{V_1}{I_1} = \frac{V_2}{I_2}\right]$$

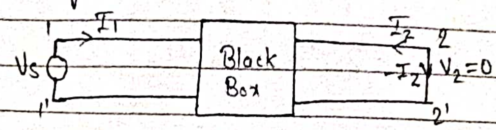


$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

DATE
PAGE NO.

Let, V_s is the excitation voltage applied at 1,1' and I_2 is the response obtained after Short Circuiting the 22' port.



Adopting the changes in eq^n 1 & 2:-

$V_s = Z_{11} I_1 - Z_{12} I_2$ — (3)

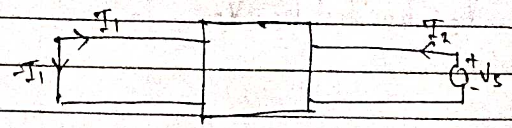
$0 = Z_{21} I_1 - Z_{22} I_2$ — (4)

From eq^n 4; $I_1 = \frac{Z_{22} I_2}{Z_{21}}$ (5)

$V_s = Z_{11}\left(\frac{Z_{22} I_2}{Z_{21}}\right) - Z_{12} I_2$

$\Rightarrow V_s = \frac{Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$ (6)

Now, interchange the position of response & excitation.



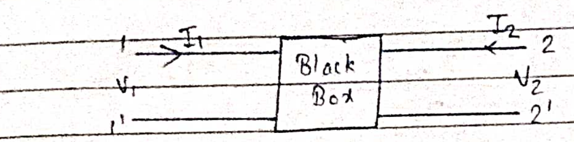
$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

So, $0 = -Z_{11} I_1 + Z_{12} I_2$

$V_S = -Z_{21} I_1 + Z_{22} I_2$

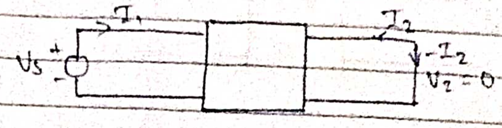
h-Parameter



$V_1 = h_{11} I_1 + h_{12} V_2$ — (1)

$V_2 = h_{21} I_1 + h_{22} V_2$ — (2)

Connect V_s at 11' and Short Circuit 22' i.e. $V_2 = 0$.

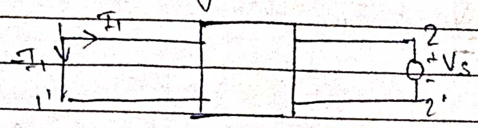


$V_s = h_{11} I_1 + 0 \Rightarrow I_1 = \frac{V_s}{h_{11}}$

$-I_2 = h_{21} I_1 \Rightarrow -I_2 = \frac{h_{21} V_s}{h_{11}}$

$\Rightarrow \frac{I_2}{V_s} = -\frac{h_{21}}{h_{11}}$

Now, Interchange Response & Excitation



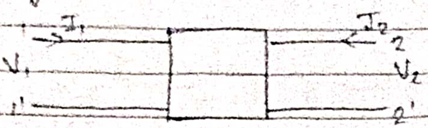
Connect V_s at 22' and short circuit 11' i.e $V_1 = 0$

$\cancel{V_s = h_{11} I_1 + 0 \Rightarrow I_1 = \frac{V_s}{h_{11}}}$

$0 = -h_{11} I_1 + h_{12} V_2$

Assignment: T, Y Parameter

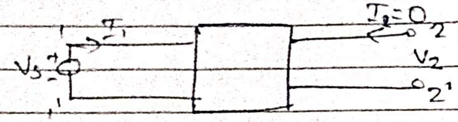
Symmetry



$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

Connect a voltage source 'V_s' at 11' and make 22' open.

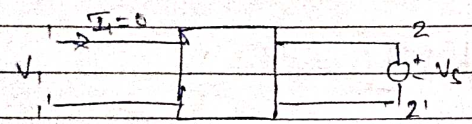


Hence, eqn (1) becomes:-

$$V_s = Z_{11} I_1$$

$$\Rightarrow \frac{V_s}{I_1} = Z_{11} \quad \text{--- (3)}$$

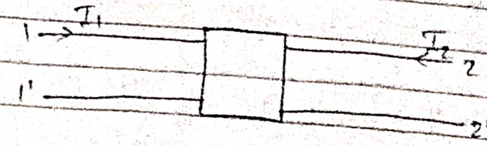
Now, connect V_s at 22' & Open Circuit 11'.



from eqn 2: $V_s = Z_{22} I_2$

Hence, $\frac{V_s}{I_2} = Z_{22}$ --- (4)

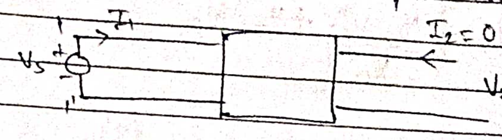
Y-Parameter



$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (2)}$$

Connect V_s at 11' & Open circuit 22'



$$I_1 = Y_{11} V_s + Y_{12} V_2 \quad \text{--- (3)}$$

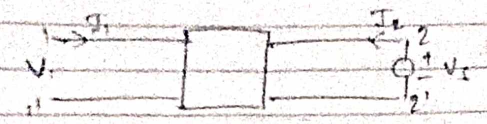
$$0 = Y_{21} V_s + Y_{22} V_2 \quad \text{--- (4)}$$

from 4: $V_2 = -\frac{Y_{21} V_s}{Y_{22}}$

$$I_1 = Y_{11} V_s - \frac{Y_{12} Y_{21} V_s}{Y_{22}}$$

$$= \frac{(Y_{11} Y_{22} - Y_{12} Y_{21}) V_s}{Y_{22}}$$

$$\Rightarrow \frac{V_s}{I_1} = \frac{Y_{22}}{Y_{11} Y_{22} - Y_{12} Y_{21}}$$



$$0 = Y_{11} V_1 + Y_{12} V_s \quad \text{— (6)}$$

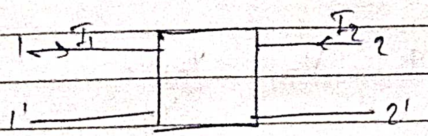
$$I_2 = Y_{21} V_1 + Y_{22} V_s \quad \text{— (7)}$$

from eqⁿ 6 : $V_1 = -\dfrac{Y_{12} V_s}{Y_{11}}$

$$I_2 = Y_{21} \left(-\frac{Y_{12} V_s}{Y_{11}} \right) + Y_{22} V_s$$

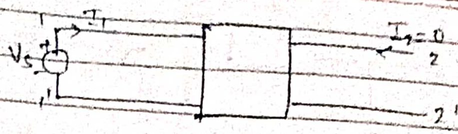
$$\Rightarrow \frac{V_s}{I_2} = \frac{Y_{11}}{Y_{11} Y_{22} - Y_{12} Y_{21}} \quad \text{— (8)}$$

H-Parameter



$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{— (1)}$$

$$V_2 = h_{21} I_1 + h_{22} V_2 \quad \text{— (2)}$$



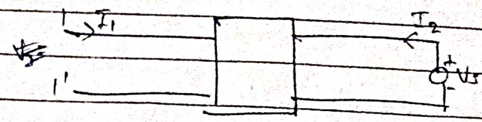
$$V_s = h_{11} I_1 + h_{12} V_2$$

$$0 = h_{21} I_1 + h_{22} V_2$$

$$\Rightarrow V_2 = \frac{-h_{21} I_1}{h_{22}}$$

So, $V_s = h_{11} I_1 + h_{12} \cdot \left(\dfrac{-h_{21} I_1}{h_{22}} \right)$

$$\Rightarrow \frac{V_s}{I_1} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}$$



$$V_1 = h_{12} V_s$$

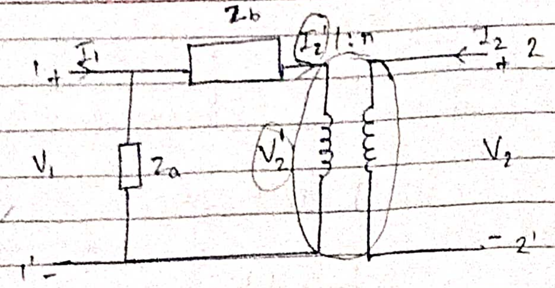
$$I_2 = h_{21} V_s$$

$$\Rightarrow \frac{V_s}{I_2} = \frac{1}{h_{22}}$$

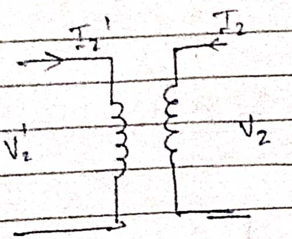
DATE 14/8/23

PAGE NO.

Q.)



Soln: $a = \frac{1}{n}$ (given)

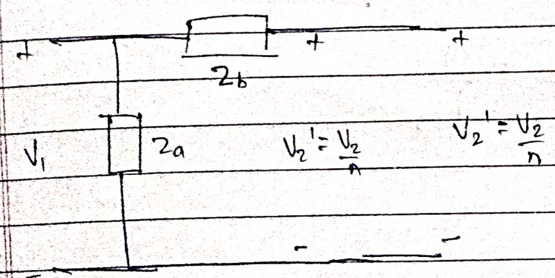


$\frac{V_1}{V_2} = \frac{1}{n}$, $\frac{I_2'}{I_2} = -n$

$\frac{I_2'}{I_2} = -n$

$\Rightarrow I_2' = -n I_2$ — (1)

$\frac{V_2'}{V_2} = \frac{1}{n} \Rightarrow V_2' = \frac{V_2}{n}$ — (2)



DATE

PAGE NO.

$V_1 = Z_a (I_1 - I_2')$

$= Z_a I_1 + n I_2 Z_a$

$= Z_a I_1 + n Z_a I_2$ — (3)

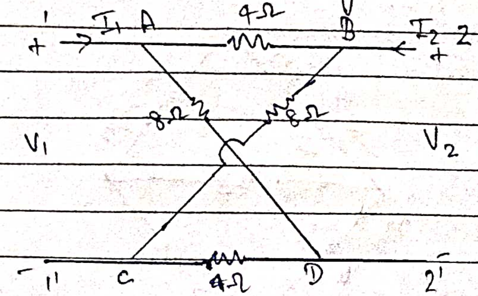
$V_2' + (I_2' - I_1) Z_a + I_2' Z_b = 0$

$\Rightarrow V_2' = Z_a I_1 - (Z_a + Z_b) I_2'$

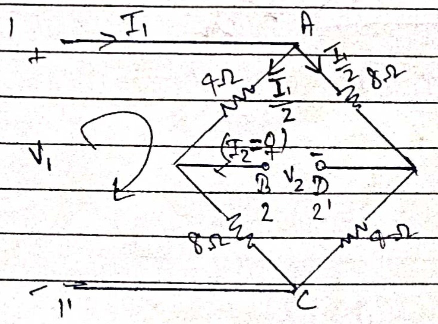
$\Rightarrow \frac{V_2}{n} = Z_a I_1 + n (Z_a + Z_b) I_2$

$\Rightarrow V_2 = n Z_a I_1 + n^2 (Z_a + Z_b) I_2$ — (4)

Q.) Find Z-Parameter (Symmetrical Lattice Network)



Soln:



Put, $I_2 = 0$:-

$$-V_1 + 4\frac{I_1}{2} + 8\frac{I_1}{2} = 0$$

$$\Rightarrow -V_1 + 2I_1 + 4I_1 = 0$$

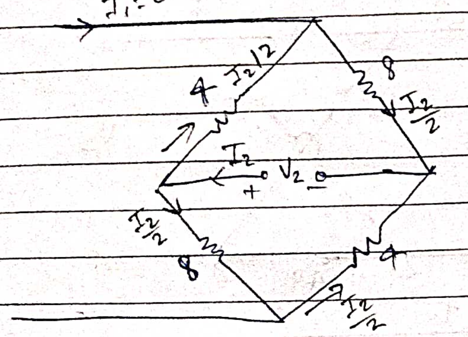
$$\Rightarrow \frac{V_1}{I_1} = 6\Omega = Z_{11}$$

Now, $-V_2 - \frac{I_1}{2} \times 4 + \frac{I_1}{2} \times 8 = 0$

$$\Rightarrow V_2 = -I_1 + 4I_1 = 2I_1$$

$$\therefore Z_{21} = \frac{V_2}{I_1} = 2\Omega$$

Put, $I_1 = 0$:-



$$-V_2 + 4\frac{I_2}{2} + 8\frac{I_2}{2} = 0$$

$$\Rightarrow Z_{22} = \frac{V_2}{I_2} = 6\Omega$$

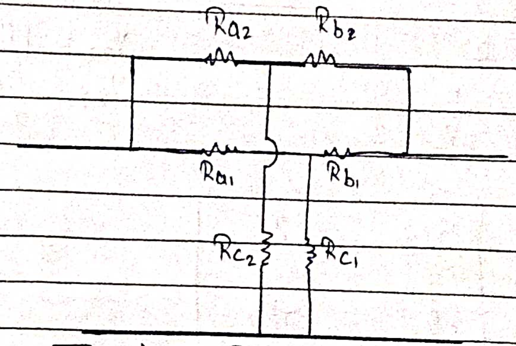
Assignment : Image Impedance

$$-V_1 - 4 \times \frac{I_2}{2} + 8\frac{I_2}{2} = 0$$

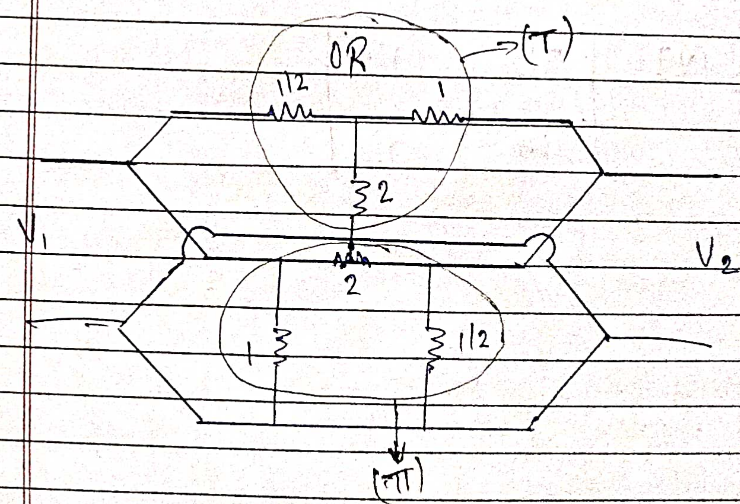
$$\Rightarrow \frac{V_1}{I_2} = 2\Omega = Z_{12}$$

$$\therefore [Z] = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

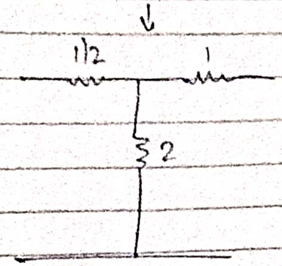
Q.)



Twin-T Parallel Circuit
[(T+π) network combination]



T



$$Z_{11} = 2 + \frac{1}{2} = \frac{5}{2}\ \Omega$$

$$Z_{12} = 2\ \Omega$$

$$Z_{21} = 2\ \Omega$$

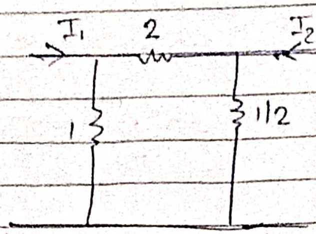
$$Z_{22} = 2 + \frac{1}{2} = \frac{5}{2}\ \Omega$$

$$[Z] = \begin{bmatrix} 5/2 & 2 \\ 2 & 5/2 \end{bmatrix}$$

$$Y = [Z]^{-1}$$

$$[Y] = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix}$$

Π



$$Y_{11} = 2 + \frac{1}{2} = \frac{5}{2}$$

$$Y_{12} = 1/2$$

$$Y_{21} = 1/2$$

$$Y_{22} = \frac{1}{2} + 2 = \frac{5}{2}$$

DATE 14/8/23

Network Function

$$H(S) \triangleq \frac{\propto \text{Zero State Response}}{\propto \text{Excitation}}$$

Response = Zero Input Response + Zero State Response

- Response ↓ o/p ↓ Complete Solution
- Zero Input Response ↓ No Excitation or No input
- Zero State Response ↓ All the initial conditions are zero

1. Driving Point Immitance Function (Impedance + Admittance)

2. Forced T.F Impedance $= \frac{V_2(S)}{I_1(S)}$ (T.F ↓ Transfer)

3. Reverse Transfer Impedance $= \frac{V_1(S)}{I_2(S)}$

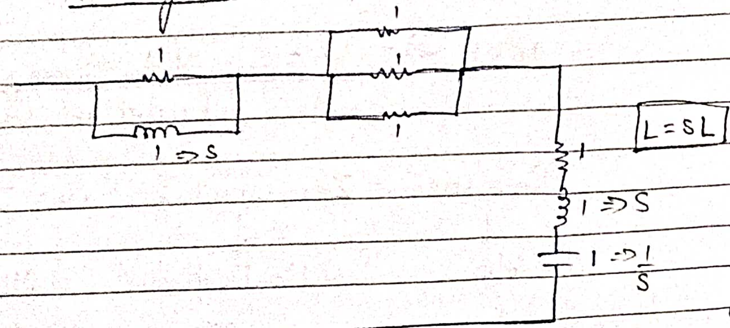
4. Voltage T.F function $= \frac{V_2(S)}{V_1(S)}$

5. Current T.F function $= \frac{I_2(S)}{I_1(S)}$

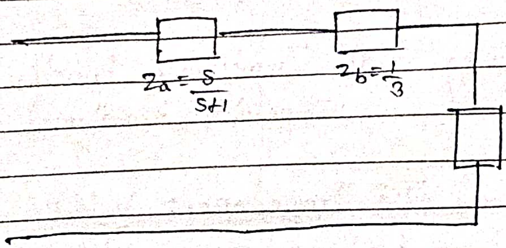
DATE
PAGE NO.

Input Impedance
(OR

Q.) Find out Driving Point Impedance Function.



Solⁿ:



$Z_c = 1 + S + \frac{1}{S}$

$= \frac{S^2+S+1}{S}$

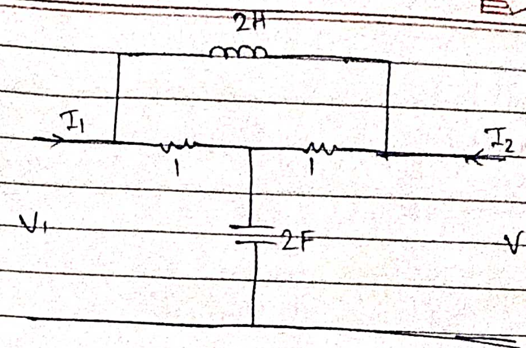
$Z_{11} = \frac{V_1}{I_1} = Z_a + Z_b + Z_c$

$= \frac{S}{S+1} + \frac{1}{3} + \frac{S^2+S+1}{S}$

$= \frac{3S^2 + S(S+1) + (S+1)(S^2+S+1).3}{3S(S+1)}$

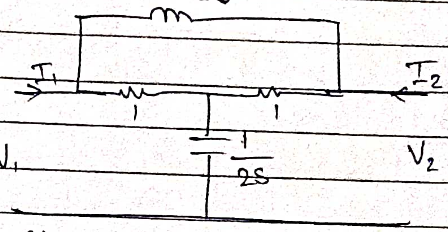
DATE
PAGE NO.

Q.)

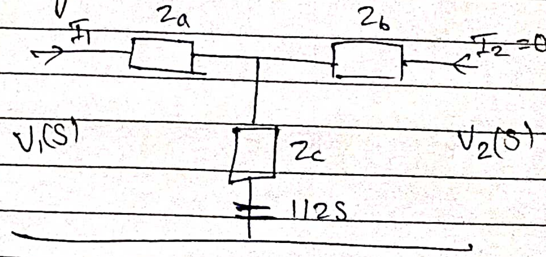


⇓ Transformed network

Solⁿ:



Using Star :-



$Z_a = \frac{2S \times 1}{1+1+2S} = \frac{S}{S+1}$

$Z_b = \frac{1}{2+2S}$

$Z_c = \frac{1 \times 2S}{2+2S} = \frac{S}{S+1}$

DATE
PAGE NO.

Find $\frac{V_2(S)}{V_1(S)}$.

DATE 17/8/23
PAGE NO.

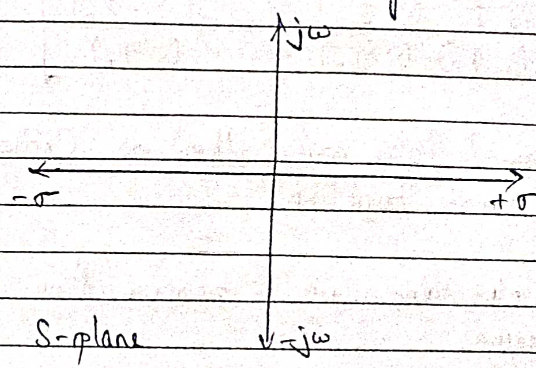
Poles and Zeroes of the Network Function

$$H(S) = \frac{a_n S^n + a_{n-1} S^{n-1} + a_{n-2} S^{n-2} + \dots + a_0}{b_m S^m + b_{m-1} S^{m-1} + b_{m-2} S^{m-2} + \dots + b_0}$$

$$= K \frac{\prod_{i=1}^{n} (S - Z_i)}{\prod_{j=1}^{m} (S - P_j)}$$

K is a Scaling Factor : $K = \frac{a_n}{b_m}$

S is a Complex Frequency



S-plane

$\boxed{S = \sigma \pm j\omega}$ → angular frequency (rad/sec)

σ → Attenuation Factor having unit "Neper".

DATE
PAGE NO.

* When each factor of Numerator is equated to zero, then frequency obtained is called as Zeroes of Network Function.

$S-Z_1=0$, $S-Z_2=0$,
$\Rightarrow S=Z_1$ $\Rightarrow S=Z_2$

* If factor of denominator is equated to zero, then freq. obtained is called Poles of N/w function.

$S-P_1=0$, $S-P_2=0$
$\Rightarrow S=P_1$ $\Rightarrow S=P_2$

$$\boxed{K=\frac{(S-Z_1)(S-Z_2)\ldots\ldots(S-Z_n)}{(S-P_1)(S-P_2)\ldots\ldots(S-P_n)}}$$

* Zeroes & Poles are called as Critical Frequency of Network Function.

* In a N/w func., no. of zeroes & no. of poles are same.

* Multiplicity → repetition

DATE
PAGE NO.

Note:

1.

Eg: $H(S)=\dfrac{S(S+1)}{(S^2+2S+2)\cdot(S^3)}$

$=\dfrac{S(S+1)}{(S+1-j)(S+1+j)\cdot S^3}$

Numerator: $S(S+1)=0$

$\Rightarrow S=0$, $S=-1$
2 Zeroes

Denominator: $(S+1-j1)(S+1+j1)\cdot S^3=0$

$S^3=0$
$\Rightarrow S=0,0,0$

... given, then $\boxed{S=0}$

$S+1-j1=0$ $\quad\quad$ $S+1+j1=0$

$\Rightarrow S=j1-1$ $\quad\quad$ $\Rightarrow S=-1-j1$

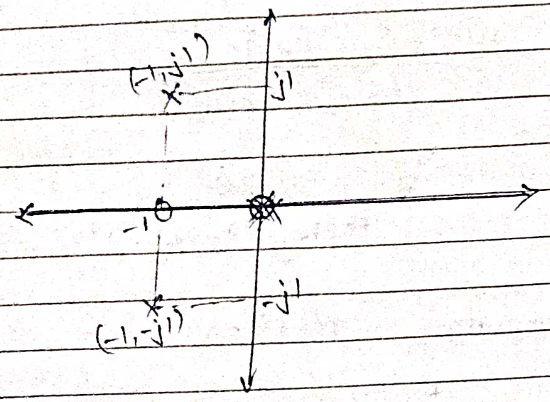
$\therefore$ 5 Poles i.e. $0, 0, 0, -1\pm j1$

Poles − Zeroes = $5-2=3$

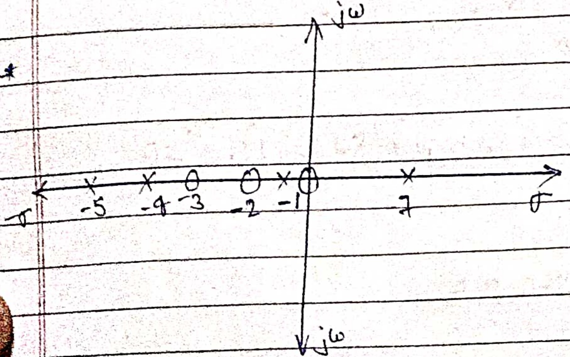
$\therefore$ 3 zeroes are present at ∞.

Representation of Poles and Zeroes in S-Plane

Zeroes − O $\quad\quad$ Poles − X



*



$$H(S)=\frac{S(S+2)(S+3)}{(S+1)(S+4)(S+5)(S-7)}$$

Let, $S=7$, then find k.

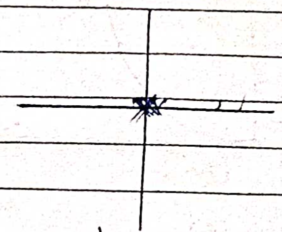
$$H(7)=\frac{K\,7\,(7+2)(7+3)}{(7+1)(7+4)\,7+}$$

* The location of zeroes doesn't decide Stability

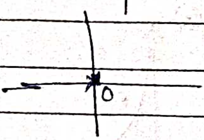
If all Poles are present in left half of S Plane $\Rightarrow$ Stable

If any one pole is present in right half of S Plane $\Rightarrow$ Unstable

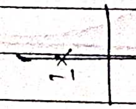
*



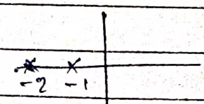
$H(S)=\frac{1}{S}$ → Unit step func. in S-domain



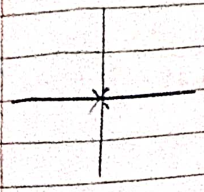
$H(S)=\frac{1}{S^2}$ → Unit ramp. func. in S-domain



$$H(S)=\frac{1}{S+1}$$



$$H(S)=\frac{1}{(S+1)(S+2)}$$

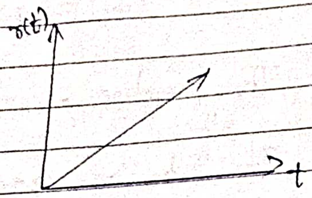
1. 

$H(S) = \frac{1}{S}$

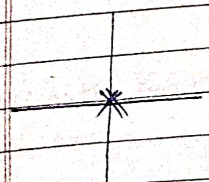
$u(t) = 1 \; ; \; \text{for } t \geq 0$
$\quad = 0 \; ; \; \text{for } t < 0$

2. 

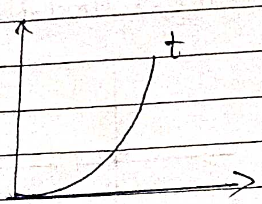
$H(S) = \frac{1}{S^2}$

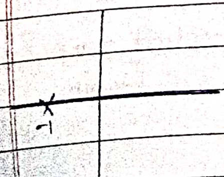


$r(t) = t \; ; \; t > 0$
$\quad = 0 \; ; \; t \leq 0$

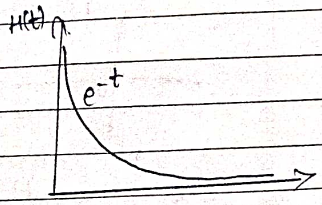
3. 

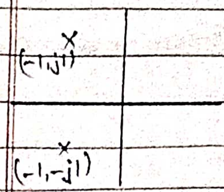
$\frac{1}{S^3}$



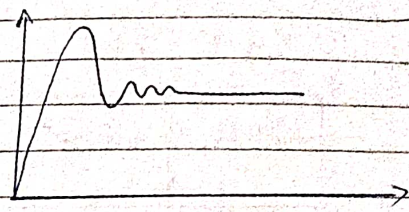
4. 

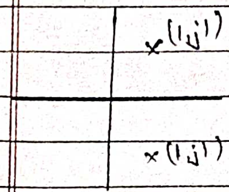
$H(S) = \frac{1}{S+1}$



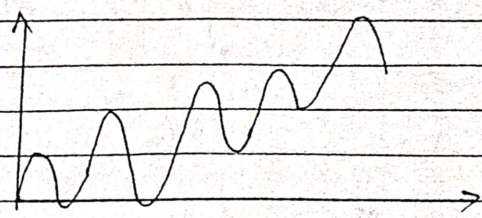
5. 

$H(S) = \frac{1}{(S+1+j1)(S+1-j1)}$



6. 

$H(S) = \frac{1}{(S-1-j1)(S-1+j1)}$



DATE 18/8/23
PAGE NO.

Some Important Laplace Transform and its Inverse

$\mathcal{L}\, f(t) = F(S)$

$$\boxed{F(S) = \int_0^\infty f(t) \cdot e^{-st}\, dt}$$

(i) $e^{-at} \longleftrightarrow \frac{1}{S+a}$

$$= \int_0^\infty e^{-at} \cdot e^{-st}\, dt$$

$$= \frac{1}{S+a}$$

(ii) $e^{at} \longleftrightarrow \frac{1}{S-a}$

$$\mathcal{L}\, e^{5t} = \frac{1}{S-5}$$

(iii) $\sin \omega t \longleftrightarrow \frac{\omega}{S^2+\omega^2}$

$$\sin 3t = \frac{3}{S^2+9}$$

DATE
PAGE NO.

(iv) $\cos \omega t \longleftrightarrow \frac{S}{S^2+\omega^2}$

$$\cos 2t = \frac{S}{S^2+4}$$

(v) $e^{-at} \sin \omega t \longleftrightarrow \frac{\omega}{(S+a)^2+\omega^2}$

(Damped Sinusoid func.)

$$e^{-2t} \sin \sqrt{3}t = \frac{\sqrt{3}}{(S+2)^2+(\sqrt{3})^2}$$

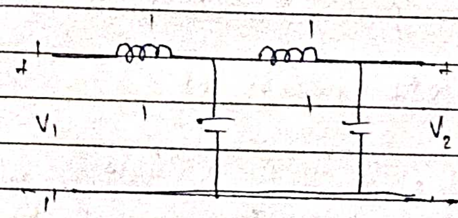
(vi) $e^{-at} \cos \omega t \longleftrightarrow \frac{S+a}{(S+a)^2+\omega^2}$

(Damped Cosine func.)

$$e^{-t} \cos 4t = \frac{S+1}{(S+1)^2+16}$$

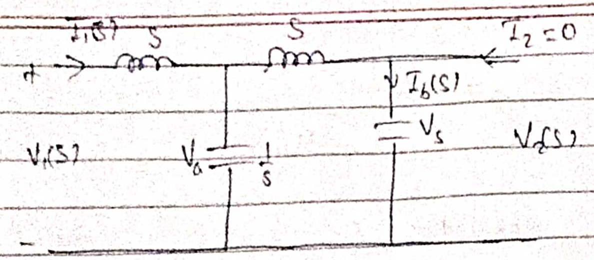
(vii) Impulse : $\mathcal{L}\, \delta(t) = 1$

Q.)



Ladder network

$\Downarrow$



$I_b(s) = \frac{V_2(s)}{1/s} = sV_2(s)$ — ①

KVL: $-V_a(s) + sI_b + V_2(s) = 0$

$\Rightarrow V_a(s) = (s^2+1)\cdot V_2(s)$ — ②

$I_a(s) = \frac{V_a}{1/s} = s\cdot V_a = s(s^2+1)\cdot V_2(s)$ — ③

But, $I_1(s) = I_a(s) + I_b(s)$

$= (s^3+s)\cdot V_2(s) + s\cdot V_2(s)$ — ④

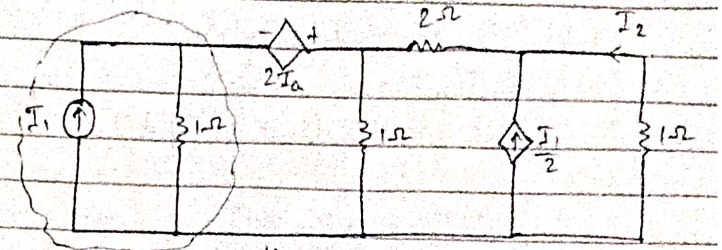
$-V_1(s) + I_1(s)\times s + V_a = 0$

$\Rightarrow V_1(s) = (s^3+2s)\cdot s\, V_2(s) + (s^2+1)\cdot V_2(s)$

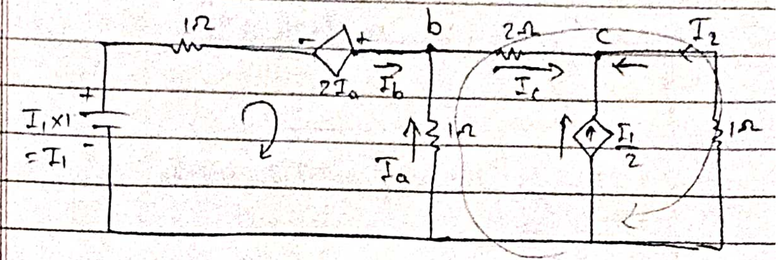
$V_1(s) = (s^4 + 3s^2 + 1)\cdot V_2(s)$

$\therefore \frac{V_2(s)}{V_1(s)} = \frac{1}{s^4+3s^2+1}$ (Voltage gain)

8.)



Sol^n:



KCL at C: $I_c + I_2 + \frac{I_1}{2} = 0$

$\Rightarrow I_c = -\left(I_2 + \frac{I_1}{2}\right)$

KCL at B: $I_b + I_a = I_c$

$\Rightarrow I_b = I_c - I_a = -\left(\frac{I_1}{2} + I_2 + I_a\right)$

KVL: $-I_1 + I_b \times 1 - 2I_a - I_a = 0$

$\Rightarrow I_1 = -\frac{I_1}{2} - I_2 - I_a - 3I_a$

$\Rightarrow I_1 + \frac{I_1}{2} = -I_2 - 4I_a$

$\Rightarrow \frac{3}{2}I_1 = -I_2 - 4I_a$

KVL: $I_a \times 1 + 2I_c - I_2 = 0$

$\Rightarrow I_a = -2I_c + I_2$

$= +2\left(I_2 + \frac{I_1}{2}\right) + I_2$

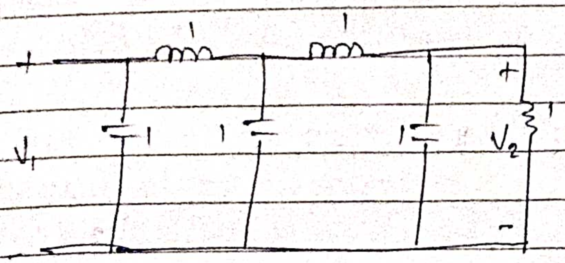
$\therefore I_a = 3I_2 + I_1$

So, $\frac{3}{2} I_1 = -I_2 - 4(3I_2 + I_1)$

$\Rightarrow \frac{3}{2} I_1 = -I_2 - 12I_2 - 4I_1$

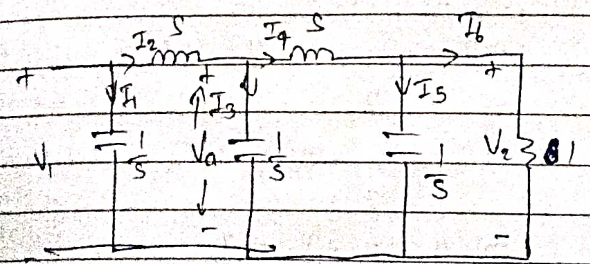
$\therefore \Rightarrow \boxed{\frac{I_2}{I_1} = \frac{-11}{26}}$

8)



$\Downarrow$

Sol^n:



$I_6 = \frac{V_2}{1} = V_2$

$I_5 = \frac{V_2}{1/s} = sV_2$

$I_4 = I_5 + I_6 = (1+s) \cdot V_2$

$-V_a + I_4 \times s + V_2 = 0$

$\Rightarrow V_a = \{(1+s)s + 1\} V_2$

$\therefore V_a = (s^2 + s + 1) \cdot V_2$

$I_3 = \frac{V_a}{1/s} = sV_a = s(s^2+s+1) \cdot V_2$

$= (s^3 + s^2 + s) \cdot V_2$

$I_2 = I_3 + I_4 = (s^3 + s^2 + s + 1 + s) V_2$

$= (s^3 + s^2 + 2s + 1) \cdot V_2$

$-V_1 + I_2 \times s + V_a = 0$

$\Rightarrow V_1 = s \times (s^3 + s^2 + 2s + 1) V_2(s) + (s^2 + s + 1) V_2$

$= (s^4 + s^3 + 2s^2 + s + s^2 + s + 1) V_2$

$\therefore \frac{V_2}{V_1} = \frac{1}{s^4 + s^3 + 3s^2 + 2s + 1}$