Module-4

Inequalities, converges & limit theorem

1. Markov → upper
2. Chebyshev → lower
3. Chen of bounds
4. Modes of convergence
5. Centeral limit theorem
6. strong & weak laws of large no.

$$f_Z(z) = \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-(z-m_z)^2/2\sigma_z^2}$$

$m_z = 0$, $\sigma_z^2 = 1$

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$F_Z(z) = \int_{-\infty}^{z} f_Z(z)\,dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-z^2/2}\,dz$$

Q) The order at a restaurant i.i.d random variance with mean = \$8, S.D σ = \$2. Estimate the prob that 1st 100 customer spend a total of more than \$840.
Estimate the prob that 1st 100 customers spend a total of between \$780 and \$820.

mean = nm $n = 100$

$= 8 \times 100 = 800$

Variance $= n\sigma^2 = 100 \times 4 = 400$

$$Z_n = \frac{S_n - nm}{\sigma\sqrt{n}} \qquad Z_{100} = \frac{S_{100} - 800}{20}$$

$S = x_1 + x_2 \ldots$

$P[S_{100} > 840]$

$P[20 Z_{100} + 800 > 840]$

$$P\left[Z_{100} > \frac{840 - 800}{20}\right]$$

$P[Z_{100} > 2]$

$\approx \phi(2) = 2.28(10^{-2})$

$P[780 \le S_{100} \le 820]$

$= P[-1 \le Z_{100} \le 1] = 1 - 2\phi(1) = 0.684$

Convergence Modes

let us consider sequence of RV's $X_1, X_2 \ldots X_n$ then for specific (s) is a sequence of no. that might not converse

→ Convergence everywhere (e)

A sequence of no x_n tends to a limit x if given $\epsilon > 0$, we can find a no. n_0 such that $|x_n - x| < \epsilon$ for every $n > n_0$ in other words

limit of seq. X_n is a RV of X

$$X_n \to X \text{ as } n \to \infty$$

→ Convergence almost everywhere (a.e) – If the set of outcomes 'S' such that limit $X_n(s) = X(s)$ as $n \to \infty$ exist and its prob equals 1, then the sequence X_n converges everywhere (or with prob)

$$\boxed{P\{X_n \to X\} = 1 \text{ as } n \to \infty} \quad (3)$$

event $\{X_n \to X\}$ consist of all outcomes 's' such then $X_n \to X$.

X_n is a sequence of RV's, X is a RV

3) Convergence in the mean square sense (MS)
The sequence X_n tends to RV 'x' in MS

$$\boxed{E\{|X_n - X|^2\} \to 0 \text{ as } n \to \infty} \to (4)$$

if limit in the mean sense & written as
$\lim X_n = X$ as $n \to \infty$.

4) Convergence in probability (p).
The probability $P\{|X_n - X| > \epsilon\}$ of the event $\{|X_n - X|\}$ is a sequence of numbers depending on ϵ, if this sequence tends to 0

$$\boxed{P\{|X_n - X| > \epsilon\} \to 0 \text{ as } n \to \infty} \quad (5)$$

for any $\epsilon > 0$, then the sequence X_n tends to RV 'X' in probability. This is also called stochastic convergence.

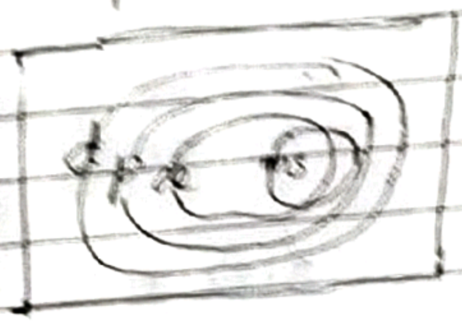
5) Convergence in distribution (d) – let $F_n(x)$ and $F(x)$ are the distribution of X_n & X are respectively if

$$\boxed{F_n(x) \to F(x) \text{ as } n \to \infty} - (6)$$

for every point x of continuity of $F(x)$, then the sequence X_n tends to RV 'X' in distribution

In this case, sequence $X_n(s)$ need not converge for any s.

Comparision convergence modes —



Each point in rectangle represent a random sequence

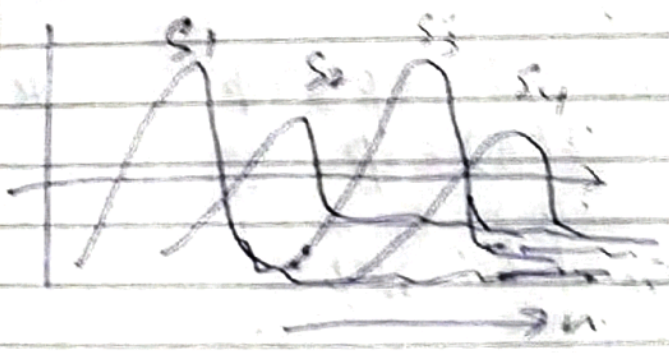
All the sequence converges in the stated mode written on each circle

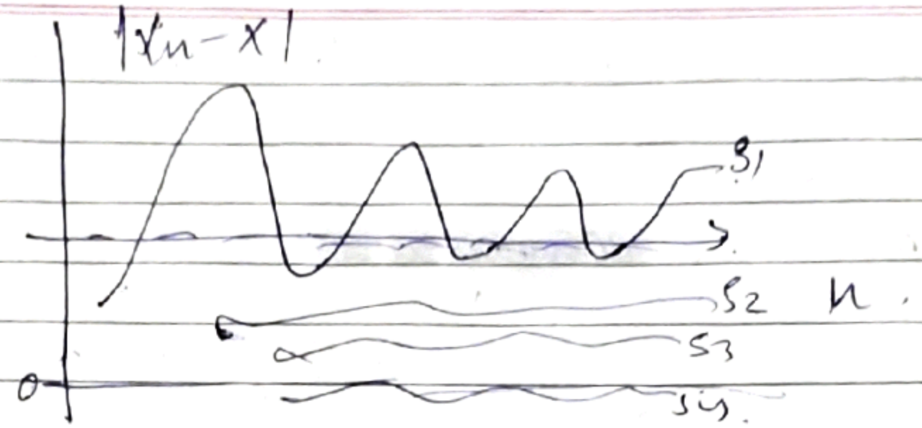
→ The curve d shows that if a sequence converges at all then it also converges in distribution

→ If a sequence converges in ms then it also converges in prob. that means

$$\boxed{P\{|X_n - X| > \epsilon\} \le \frac{E\{|X_n - X|^2\}}{\epsilon^2}}$$

→ ae convergence in eqn (3) ⇒ also converges in p. the converse is not necessarily true.





In this fig sequences drawn as curve and each curve represent a particular sequence $|X_n(s) - X(s)|$

→ Convergence in P means for specific $n > n_0$ only small percent of this curve will have ordinates that exceeds ϵ (fig 1-4)

→ It is also possible that not of these less than ϵ for every.

Convergence (a.e) → demands that most curve will be below ϵ for every $n > n_0$ fig (1-4)

The Law of Large Numbers

Event A occur with prob p in k times in n trials $P(A) = p$

then $\boxed{k \simeq np}$

The approximation $k \simeq np$, means the ratio $\frac{k}{n}$ close to p

In the sense that, for any $\epsilon > 0$, prob that

$$\left|\left(\frac{k}{n}\right) - p\right| < \epsilon \to 1 \text{ as } n \to \infty$$

i.e $\boxed{P\left\{\left|\frac{k}{n} - p\right| < \epsilon\right\} \to 1 \text{ as } n \to \infty}$

To reestablish this result as a limit of sequence of random variables

R.V's $X_i = \begin{cases} 1, & \text{if } A \text{ occurs at } i\text{th trial} \\ 0, & \text{Otherwise.} \end{cases}$

Sample mean is

$$\bar{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$ of the RV's

p in prob as $n \to \infty$.

proof $E\{X_i\} = E\{\bar{X}_n\} = p$

$\sigma_{x_1}^2 = pq \quad \sigma_{x_n}^2 = pq/n.$

further more

$$pq = p(1-p) \leq 1/4$$

Hence

$$P\{|\bar{X}_n - p| < \epsilon\} \geq 1 - \frac{pq}{n\epsilon^2} > 1 - \frac{1}{4n\epsilon^2} \text{ as } \to 1 \text{ } n \to \infty$$

This establishes eq^n (2) because $\bar{X}_n$ as $\frac{K}{n}$. if a occurs K time

Probabili

Strong law of large no.

$X_1, X_2 \ldots X_n$ sequence of RV's fi^{te} finite mean $E[X] =$ mean & $Var[X] = \sigma^2$

then $P[\lim_{n \to \infty} M = m] = 1$

$\underbrace{}$ Sample mean $M_n = \frac{1}{n} \sum_{i=1}^{n} X_i$

Weak law of large no.

$X_1, X_2 \ldots X_n$ sequence of RV's. with $E\{X\} =$

for $\epsilon > 0$ $\lim_{n \to \infty} P[|M_n - m| < \epsilon] = 1$

$P[\lim_{n \to \infty} M_n = m] = 1$ } for la

mean fine
value of sample mean M
close to true mean.

with Prob 1 every sequence, Sample mean approaches to $E[X] = m$

e.g. (1) Tossing a fair dice $S = \{1, 2, 3, 4, 5, 6\}$

(2) X_n represent the temp^r at the end of the nth hour $T = \{1, 2, 3, 4, \ldots\}$ T discrete.

$S = \{$any temp$^r\}$, $T = \{1, 2 \ldots 24\}$ T contin^u

(3) $X(t)$ represent no. of calls received $S \to \{0, 1, \ldots$

T - anything

(4) $X(t)$ represent max^m temp^r $(0, t)$ $S \to$ anyth

Mod-5 Random Process.

Random variable → a funcⁿ that assigns a real no. to every outcome of a random experiment.

Random process → It is a collection or ensemble of random variables $\{X(s,t)\}$ that are functions of real variable namely time 't' and s, where $t \in T$, $s \in S$

Based on $X(s,t)$ random process can be defined as

① $X(s,t)$ is a RV if t is constant or fixed & s is variable.

② $X(s,t)$ is a Random process if both s and t are variable.

③ $X(s,t)$ is time fⁿ if s is fixed.

④ $X(s,t)$ is a number if S & t both are fixed

Classification of Random process.

Based on continous or discrete nature of S and T as RP is classified into 4 types.

① If both T & S are discrete then the RP is as a discrete Random sequence
② If T is discrete & S is continous → continous random sequence
③ If T is continous & S is discrete → discrete RP.
④ If T & S are continous → continous RP

First order cdf $F(x;t) = P[X(t) \le x]$

pdf $f(x,t) = \frac{d}{dx} F(x,t)$

Description of Random process

cdf of a RP $\rightarrow X(t)$ or $X(s,t)$ is defined as $F_{X,T}(x,t) = P\{X(t) \le x\}$

$F(x,t) = P\{X(t) \le x\} \Rightarrow$ 1st order distribution

$F(x_1, x_2, t_1, t_2) = P\{X(t_1) \le x_1; X(t_2) \le x_2\}$

$f(x_1, x_2, x_n; t_1, t_2, t_3 \ldots t_n) \longrightarrow$ 2nd order

$= \frac{\partial^n F(x_1, x_2, \ldots x_n; t_1, t_2 \ldots t_n)}{\partial x_1 \partial x_2 \ldots \partial x_n}$

the pdf of RP $X(t)$ is

$$f(x,t) = \frac{\partial F(x,t)}{\partial x}$$

$$f(x_1, x_2; t_1, t_2) = \frac{\partial^2 F(x_1, x_2; t_1, t_2)}{\partial x_1 \partial x_2}$$

Stationary process $\Rightarrow$ If certain probability distributions or averages do not depend on t then RP $\{X(t)\}$ is called stationary process

1. Strict sense stationary process (SSS)
2. Wide sense stationary process (WSS).

A random process is called strongly stationary process or strict sense process if all the finite dimensional distribution are invariant under the translation of time.

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(1) i.e. $f(x,t) = f(x, t+t_0)$ where $t_0 > 0$ and it is possible if $f(x,t)$ is independent of t.

(2) it is possible if $f(x,t)$ is independent of t_0.

(3) Ass $E\{X(t)\} = m =$ a constant

(4). Auto correlation f^n $R(t_1,t_2)$ or R_c

f^n of time difference $\varepsilon = t_1 - t_2$. → $R(t_1,t_2) = E[X(t_1)\,X(t_2)]$ is a also a f^n of $e = t_1 - t_2$

(2) Wide sense stationary process (WSS)

(1) if $E\{X(t)\} = m =$ a constant.

(2) $Var\{X(t)\} = \sigma^2 =$ a constant.

(3) $R(t_1,t_2) = E[X(t_1)\,X(t_2)]$

$=$ fun^n of $(t_1 - t_2)\ \varepsilon_1$

Mean. ⇒ t = continuous $X(t)$

$$\boxed{\mu(t) = E\{X(t)\}}$$

Autocorrelation

$R_{XX}(t_1,t_2) = R_x(t_1,t_2) = R(t_1,t_2)$

$= E\{X(t_1) \times X(t_2)\}$

#

$C_{XX}(t_1,t_2)$ or $C_x(t_1,t_2)$ or $C(t_1,t_2) = R(t_1,t_2) -$

$\mu(t_1, t_2)$

Stationary process

Q) Show that random process $X(t) = A\cos(\omega_0 t + \theta)$ is WSS process. if A and ω_0 are constant and θ is uniformly distributed RV over $(0, 2\pi)$

$$f_\theta(\theta) = \frac{1}{2\pi}, \text{ for } (0, 2\pi).$$

① Mean $E[X(t)] = \int_0^{2\pi} x(t)\, f_\theta(\theta)\, d\theta.$

$$= \int_0^{2\pi} A\cos(\omega_0 t + \theta) \cdot \frac{1}{2\pi}\, d\theta$$

$$= \frac{A}{2\pi} \int (\cos\omega_0 t \cos\theta - \sin\omega_0 t \sin\theta)\, d\theta.$$

$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$$= 0$$

$$= \text{a const.}$$

② Autocorrelation $R(t_1, t_2) = E[X(t_1) X(t_2)]$

$$= E[A^2 \cos(\omega_0 t_1 + \theta) \cdot \cos(\omega_0 t_2 + \theta)]$$

$$= A^2 \cdot E\left[\cos(\omega_0 t_1 + \theta)\cos(\omega_0 t_2 + \theta)\right]$$

$\frac{\cos(\omega(t_1+t_2) + 2\theta) + \cos(\omega(t_1-t_2))}{2}$

$$= \frac{A^2}{2} E\left[2\cos(\omega_0 t_1 + \theta)\cos(\omega_0 t_2 + \theta)\right]$$

$$= \frac{A^2}{2} E\left(\cos((t_1+t_2)\omega_0 + 2\theta) + \cos[\omega_0(t_1 - t_2)]\right)$$

$$= \frac{A^2}{2} \int_0^{2\pi} \frac{1}{2\pi} x'(t)\, d\theta.$$

$$\Rightarrow \frac{A^2}{4\pi} \int_0^{2\pi} \{ \cos (t_1 + t_2) w_0 + 2\theta] + \cos [w_0 (t_1 - t_2) d\theta \}$$

$$= \frac{A^2}{2} \cos [w_0 (t_1 - t_2)]$$

$= f^n$ of $t_1 - t_2$ i.e time dependent.

Q) Verify that the process $X(t) = Y(\cos wt)$ is S S S where Y is uniformly distributed b/w 0 to 1.

$\rightarrow$ $F(x,t) = F(x, t + t_0)$

$f(x,t) = f(x, t + t_0)$

$x(t) = Y \cos wt.$

$$F(x,t) = P[X(t) \le x] = P[Y \cos wt \le x]$$

$$= \begin{cases} P \{ Y \le \frac{x}{\cos wt} \}, & \text{if } \cos wt > 0 \\ P \{ Y \ge \frac{x}{\cos wt} \}, & \text{if } \cos wt < 0 \end{cases}$$

i.e $F_{X(t)}(x,t) = \begin{cases} F_Y\left(\frac{x}{\cos\omega t}\right), & \text{if } \cos\omega t \\ 1 - F_Y\left(\frac{x}{\cos\omega t}\right), & \text{if } \cos\omega t \end{cases}$

$\therefore f_{X(t)}(x,t) = \frac{1}{|\cos\omega t|} \times f_Y\left[\frac{x}{\cos\omega t}\right]$

$=$ funⁿ of 't'.

Hence not SSS.

(1) Mean of the process $\{X(t)\}$

$m(t) = E\{X(t)\}$

(2) Autocorrelation

$R(t_1, t_2) = E\{X(t_1) \cdot X(t_2)\}$

(3) Autocovariance

Variance $C(t_1,t_1)$

$C(t_1,t_1) - m(t)^2$

$C(t_1, t_2) = E\{[X(t_1) - m(t_1)] \cdot [X(t_2) - m(t_2)]\}$

$= R(t_1, t_2) - m(t_1)\, m(t_2)$

(4) Correlation coefficient

$\rho(t_1, t_2) = \frac{C(t_1, t_2)}{\sqrt{C(t_1,t_1) \cdot C(t_2,t_2)}}$

$R_{XY}(t_1, t_2) = E\{X(t_1) \cdot Y(t_2)\}$

Autocorrelation.

$$R(t_1, t_2) = E[X(t_1) X(t_2)]$$

$$= E[X(t) X(t - \tau)]$$

$$\tau = t_1 - t_2$$

where $X(t)$ is a random process.

Properties of Autocorrelation f^n

→ $R(\tau)$ is even f^n of τ

i.e. $R(\tau) = R(\tau)$

proof → $R(\tau)$ is maximum at $\tau = 0$.

$$|R\tau| \leq R(0)$$

3) If ACF of real stationary process $\{X(t)\}$ is continuous at $\tau = 0$, then it is continuous at every point.

$R(\tau)$ is continuous at $\tau = 0$ $\lim_{\tau \to 0} R(\tau) = R(0)$

$R(\tau)$ is continuous for all τ.

$$\lim_{h \to 0} \{R(\tau + h)\} = R(\tau)$$

mean $m_X = \sqrt{\lim_{\tau \to \infty} R(\tau)}$

$R_{XY}(\tau) = E\{X(t) Y(t - \tau)\}$ cross correlat^n.

6) Cross covariance of two process $\{X(t)\}$ & $\{Y(t)\}$ is.

$$C_{XY}(t_1, t_2) = R_{XY}(t_1, t_2) - M_x(t_1) \cdot M_y(t_2)$$

7) Cross correlation coeff. of two processes $\{X(t)\}$ & $\{Y(t)\}$ is

$$\rho_{XY}(t_1, t_2) = \frac{C_{XY}(t_1, t_2)}{\sqrt{C_{XX}(t_1, t_1) \times C_{YY}(t_2, t_2)}}$$

$X(t) = A\cos(\omega_0 t + \theta)$

$Y(t) = A\sin(\omega t)$

Autocorrelation f^n.

Ergodicity

A process is said to be ergodic if its assembled mean is equal to time avg.

for exp → Assemble mean of a discrete random process $\{X(t)\}$ is

$$m_x = \sum x_i p_i$$

and time avg of a single sample fun^n.

$$\overline{X_T} = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt.$$

cos ωt
sin ωt

$$\boxed{m_x = \overline{X_T}}$$

Correlation Ergodic.

The stationary process $\{X(t)\}$ is correlation ergodic if process $\{Y(t)\}$ is mean ergodic, where $Y(t) = X(t)\,X(t+\tau)$

i.e $$\overline{Y_T} = \frac{1}{2T}\int_{-T}^{T} X(t)\,X(t+\tau)\,dt$$

tends to $E\{X(t+\tau)\,X(t)\} = R(\tau)$ as $T \to \infty$

Q) If WSS process is $X(t) = 10\cos(100t+\theta)$ where θ is uniformly distributed over $(-\pi,\pi)$. Prove that $X(t)$ is correlation ergodic.

$$\gamma_T = \frac{1}{2T}\int_{-T}^{T} X(t)\,X(t+\tau)\,d\tau$$

$$R(\tau) = E\{X(t+\tau)\,X(\tau)\}$$

at $T \to \infty$

Power spectral density (PSD)

if $X(t)$ is a stationary process

$$\rightarrow \quad S(\omega) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= F.T\,[R(\tau)]$$

$$\omega = 2\pi f$$

Then $S(f) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j2\pi f\tau}\, d\tau$

Q2) Given that PSD, the ACF $R(\tau)$ is -

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\, e^{j\omega\tau}\, d\omega$$

ie

$$R(\tau) = \int_{-\infty}^{\infty} S(f)\, e^{j2\pi f\tau}\, df$$

$$= FT\{S(\omega)\}$$

Properties

① $S(0) = \int_{-\infty}^{\infty} R(\tau)\, d\tau$
at $f = 0$
= Area under $R(\tau)$

② $E\{x^2(t)\} = R(0) = \int_{-\infty}^{\infty} S(f)\, df$

Mean square value of $x(t)$

Total Area under curve PSD.

③ PSD of real random process is an even f^n $S(\omega) = S(-\omega)$.

④ The power spectral density of $x(t)$ is real complex. is real f^n of ω and non-ve

$$\boxed{S^*(\omega) = S(\omega)}$$

⑤ The PSD & ACF of a real WSS process from a fourier cosine Transform pair

⑥ $R(\tau) = \text{FICT of } \left[\frac{1}{2} S(\omega)\right]$

proof // Wiener → Khinchin Theorem.

#

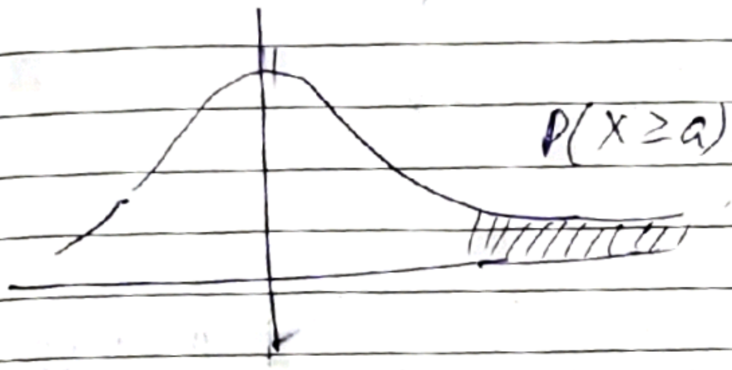
Markov's → upper bound
Mean → markov's

Let X be a positive Random Variable and $a > 0$ real no. then

$$P(X \geq a) \leq \frac{E(X)}{a}$$

$E(X)$ = mean
X = R.V
a = upper bound

$x \geq a$ → upper bound
$x \leq a$ lower bound



Let X be a discrete random variable with pmf (summation) $p(x)$.

$1 + 2 = 3$
$1 \geq 3$

$$\Rightarrow E(X) = \sum x p(x)$$

$$= \sum_{x<a} x p(x) + \sum_{x \geq a} x p(x)$$

$$\geq \sum_{x \geq a} x p(x) \quad \because x \geq a$$

$x = a$

$$\geq \sum_{x \geq a} a\, p(x)$$

$$\geq a \sum_{x \geq a} p(x)$$

$$\geq a\, P(X \geq a)$$

$$\boxed{P(X \geq a) \leq \frac{E(X)}{a}}$$

X positive continuous R.V with pdf $f(x)$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx \qquad x > 0$$

$$= \int_{0}^{\infty} x f(x)\,dx.$$

$$= \int_{0}^{a} x f(x)\,dx + \int_{a}^{\infty} x f(x)\,dx$$

$$= \geq \int_{a}^{\infty} x f(x)\,dx$$

$$\geq a \int f(x)\,dx$$

$$\geq a\, P(X \geq a)$$

$$P(X \geq a) \leq \frac{E(X)}{a}$$

Only have to know expected value (not say, varian

→ It does not depend on any property of probabili distribution of the R.V.

Q) $G(X) = 70\%$

$a = 90\%$

$$P(X \geq 0.9) \leq \frac{E(X)}{0.9}, \quad \leq \frac{7}{9}$$

$= 77.8\%$

Binomial.

Q.) $n = 20 \rightarrow$ times $\rightarrow$ Binomial

$E(X) = np$ $\quad p = \frac{20}{100} =$

Varia $= npq.$

$E(X) = np = \cancel{20}^{4} \times \frac{\cancel{20}}{\cancel{100}_{5}} = 4.$

$P(X \geq 16) \leq \frac{E(X)}{16} \leq \frac{4}{16} = \frac{1}{4}$

Q.)

x	0	1
$p(x)$	$\frac{24}{25}$	$\frac{1}{25}$

$E(X) = 0 \times \frac{24}{25} + 1 \times \frac{1}{25}$ $\qquad P(X \geq 5)$

$= \frac{1}{25}$

$P(X \geq 5) \leq \frac{E(X)}{5} = \frac{\frac{1}{25}}{5} \leq \frac{1}{125}$

Chebyshev's

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Chebyshev's

$P(|X-\mu|) \geq k\sigma \leq \frac{1}{k^2}$.

μ = mean

σ = variance.

By Markov's inequality Z is RV also

$$P(|Z| \geq a) \leq \frac{E|Z|}{a}$$

$$Z = (x-\mu)^2. \qquad a = k^2\sigma^2$$

$$P(|x-\mu| \geq k\sigma) \leq \frac{E(|x-\mu|^2)}{k^2\sigma^2}$$

$$\leq \frac{\sigma^2}{k^2\sigma^2}$$

$$\leq \frac{1}{k^2}.$$

→ four continous variable

$\sigma^2 = E(|x-\mu|^2)$ $\qquad (x-\mu) = k^2\sigma^2$.

$$= \int_{-\infty}^{\infty} |x-\mu|^2 f(x)\,dx.$$

$$= \int_{|x-\mu|<k\sigma} |x-\mu|^2 f(x)\,dx + \int_{|x-\mu|\geq k\sigma} |x-\mu|^2 f(x)\,dx$$

$$\geq \int_{|x-\mu|>k\sigma} |x-\mu|^2 f(x)$$

$$\# \geq \int_{|x-\mu| \geq k\sigma} k^2\sigma^2 f(x)\,dx$$

$$\cancel{P\left|(x-\mu)\right| \geq k\sigma} \quad k^2$$

$$\sigma^2 \geq \int k^2\sigma^2 f(x)\,dx$$

$$\frac{1}{k^2} \geq \int f(x)\,dx$$

$$P\left|(x-\mu)\right| \geq k\sigma \leq \frac{1}{k^2}$$

$\#$ $\sigma^2 = E\left[(x-\mu)^2\right]$ for discrete X.

$$= \sum (x-\mu)^2 f(x)$$

$$= \sum_{|x-\mu| < k\sigma} (x-\mu)^2 f(x) + \sum_{|x-\mu| \geq \sigma} (x-\mu)^2 f(x)$$

$$\geq \sum_{|x-\mu| \geq k\sigma} (x-\mu)^2 f(x)$$

$$\geq \sum_{|x-\mu| \geq k\sigma} k^2\sigma^2 f(x)$$

$$\frac{1}{k^2} \geq \sum_{|x-\mu| \geq k\sigma} f(x)$$

$$\left(P\,|x-\mu| \geq k\sigma\right) \leq \frac{1}{k^2}$$

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Q) Suppose that $X(t)$ $\mu(t) = 3$. → fixed mean

$R(t_1, t_2) = 4 + 4e^{-0.2|t_1 - t_2|}$.

find V, Cov. $Z = X(5)$ & $W = X(8)$

$E(Z) = 3 \quad E(W) = 3$

$Var(X(5)) = R(5,5) - (\mu(5))^2$

$= 13 - 9 = 4$

$Var(X(8)) = R(8,8) - (\mu(8))^2$.

$= 13 - 9$

Covariance $C(5,8) = R(5,8) - \mu(5)\mu(8)$

$= 9 + 4e^{-0.6} - 9 = 2.19$.

Q) $X(t) = A\cos(wt + \theta)$ RV in $(0, 2\pi)$

$E(X) = E(A\cos(wt+\theta)) =$ uniform distri

mean $= \frac{1}{b-a}$.

$= \frac{1}{2\pi - 0} = \frac{1}{2\pi}$

$\int x f(x)$

$= \int_0^{2\pi} A\cos(wt+\theta) \frac{1}{2\pi} d\theta$

$= \frac{A}{2\pi}\left[\sin(wt+\theta)\right]_0^{2\pi}$

$= \frac{A}{2\pi}\left[\sin(2\pi + wt) - \sin wt\right]$

$= 0$