Count min sketch with max heap (25pts)

In the class, we studied count-min sketch, an elegant algorithm to estimate the frequency of elements. Here we want to find the top-k frequent elements with one pass over an insert-only stream, using this algorithm. Note that to get top-k elements with only one pass, we must maintain an additional data structure, e.g., a heap, to store the current top-k frequent elements in the stream.

Let w be the width (num of columns) and d be the depth (num of rows) of the count min sketch. The high-level idea of the algorithm is the following:

1. Initialize a heap and a matrix of size $w \times d$.

2. For each element e coming from the stream, you should update the count min sketch matrix using d different hash functions (as taught in the class). Then, check if the heap size is already k. If no, insert a tuple (the count of e, e) into the heap; Else, replace the tuple of the least frequent element in the heap, say h_0, with a tuple of (estimated count of e, e), **if e has an estimated frequency larger than h_0.**

Note the heap will have size at most k, and it stores the (estimated) top-k frequent elements. We need only $O(wd + k)$ words of memory to implement the whole idea. Please implement this algorithm as a class in the cell below.

What is heap and how to use it in Python:

A heap is a specialized tree-based data structure which is essentially an almost complete tree that satisfies the heap property: in a max heap, for any given node C, if P is a parent node of C, then the key (the value) of P is greater than or equal to the key of C. A brief introduction can be found here).

In python, the most common implementation of heap can be found here. The heapq module implement min heap by storing the data in a list, and exposing all heap-operation APIs needed. You can also find examples of heapq here. Note when the data stored in the list are tuples, the first element of each tuple will be considered as the "key".

In [ ]:

```
import heapq
import time
import matplotlib.pyplot as plt

class CountMinSketchHeap:
    """
    The CountMinSketchHeap class implements the Count-Min Sketch algorithm for approxi
        of item frequencies. It also maintain a heap of size k to store the top-k freq
    """

    def __init__(self, width, depth, k, hashfunc):
        """
        Following variables are initialized here.
        width: an integer representing the number of columns in the hash table.
        depth: an integer representing the number of rows in the hash table.
        k: the number of top frequent elements to store.
        hashfunc: a callable hash function.
```

```
        counts: a two-dimensional list representing the hash table.
        keys: a list that stores the current top k frequent elements.
        heap: a max heap of size at most k, it stores (count, elment) pairs of the top
        hash_execution_time: an integer used to compute the efficiency of the hash fur
        """
        self.width = width
        self.depth = depth
        self.k = k
        self.hashfunc = hashfunc
        self.counts = [[0] * width for _ in range(depth)]
        self.keys = []
        self.heap = []
        self.hash_execution_time = 0

    def hash(self, key):
        """
        DONT CHANGE THIS!
        This function takes a key and generates in total self.depth hash values using
            pairwise independent hash functions.
        """
        hashes = []
        st = time.time()
        for i in range(self.depth):
            h = self.hashfunc((i, key)) % self.width
            hashes.append(h)
        self.hash_execution_time += (time.time()-st)
        return hashes

    def update(self, key, count=1):
        """ This function updates both heap and count min sketch when we get a new ele
        Please implement below. """

        hashes = self.hash(key)
        for i in range(self.depth):
            self.counts[i][hashes[i]] += count
        # Update heap and keys
        if key not in self.keys:
            # Add (count,key) to heap
            heapq.heappush(self.heap, (self.query(key)+count, key))
            self.keys.append(key)
        else:
            # Update count
            for i, (count_i, key_i) in enumerate(self.heap):
                if key_i == key:
                    self.heap[i] = (count_i + count, key)
                    break
            # reorder to satisfy heap property
            #heapq.heapify(self.heap)
        if len(self.heap) > self.k:
            smallest_count, smallest_key = heapq.heappop(self.heap)
            self.keys.remove(smallest_key)


    def query(self, key):
        """ This function returns the estimated frequency of an element(key) via count
        Please implement below. """
        hashes = self.hash(key)
        count = [0]*self.depth
        for i in range(self.depth):
            count[i] = (self.counts[i][hashes[i]])
        min_count = min(count)
        return min_count

    def topk(self):
```

```
        return sorted(self.heap, reverse=True)
```

Below is a set of hash functions implemented in hashlib package.

In [ ]:
```
# CODE FOR HASHING, DONT CHANGE

import hashlib
import mmh3

def md5_toint(key):
    a = hashlib.md5()
    a.update(bytes(str(key), 'utf-8'))
    return int.from_bytes(a.digest(),"big")

def sha256_toint(key):
    a = hashlib.sha256()
    a.update(bytes(str(key), 'utf-8'))
    return int.from_bytes(a.digest(),"big")

def blake2b_toint(key):
    a = hashlib.blake2b()
    a.update(bytes(str(key), 'utf-8'))
    return int.from_bytes(a.digest(),"big")

def murmur_toint(key):
    return mmh3.hash(str(key))
```

Test and plot (5pts)

In the folder, we include a text file of the book "blue castle." The next cell use the CountMinSketchHeap implemented above to estimate the top 10 frequent words in this book. Set width=200, range depth in `[10,15,20,25]`, and try different hash functions listed above. The test code collects for each hash function two performance metrics:

- The execution time of the hash function, which equals to hash_execution_time (a variable defined in CountMinSketchHeap).
- The sum of the difference between estimated and real frequencies of top-10 words.

In the last cell, please generate one plot that contains a curve for each hash function, with x-axis representing the execution time and y-axis representing the sum of the difference. Please add markers and legends properly.

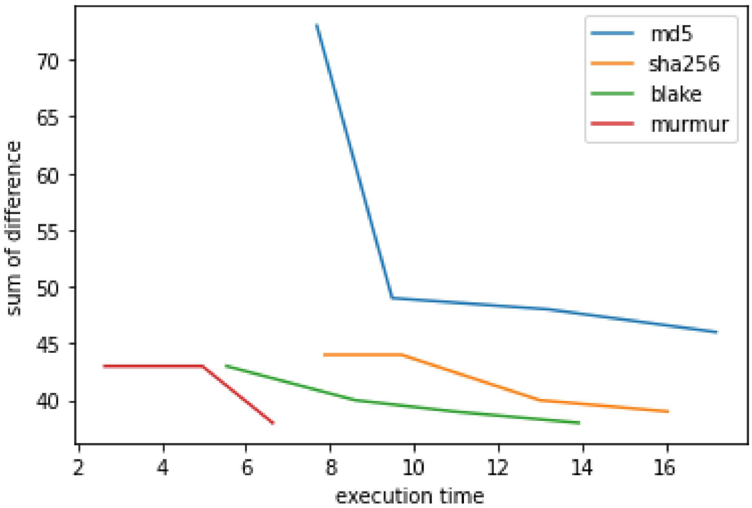
In [ ]:
```
# TEST CODE, DONT CHANGE
def count_min_test(depth, hashf):
    c = CountMinSketchHeap(200, depth, 10, hashf)
    d = {}
    f = open("bluecastle.txt","r",encoding="UTF-8")
    for line in f:
        for word in line.split():
            if word not in d:
                d[word]=0
            d[word]+=1
            c.update(word)
    return sum([i[0] for i in c.topk()])-sum(sorted([d[i] for i in d])[-10:]), c.ha
```

```
md5_time, md5_diff = [], []
sha256_time, sha256_diff = [], []
blake_time, blake_diff = [], []
murmur_time, murmur_diff = [], []
for depth in [10,15,20,25]:
    diff, timet = count_min_test(depth, md5_toint)
    md5_time.append(timet)
    md5_diff.append(diff)
    diff, timet = count_min_test(depth, sha256_toint)
    sha256_time.append(timet)
    sha256_diff.append(diff)
    diff, timet = count_min_test(depth, blake2b_toint)
    blake_time.append(timet)
    blake_diff.append(diff)
    diff, timet = count_min_test(depth, murmur_toint)
    murmur_time.append(timet)
    murmur_diff.append(diff)
```

In [ ]:

```
# Plot here
plt.plot(md5_time,md5_diff)
plt.plot(sha256_time,sha256_diff)
plt.plot(blake_time,blake_diff)
plt.plot(murmur_time,murmur_diff)
plt.legend(["md5", "sha256","blake","murmur"], loc ="upper right")
plt.xlabel("execution time")
plt.ylabel("sum of difference")

plt.show()
```



In [ ]: