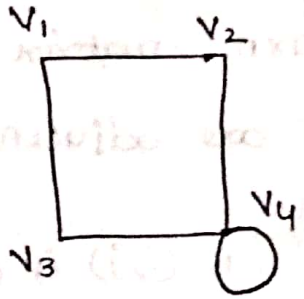
UNIT V

GRAPHS (Graph theory)

degree of a vertex :-

It is a no. of edges including self loops incident at vertex 'v'

Ex :-



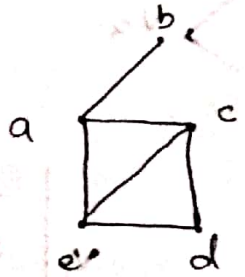
$V_1 = 2$
$V_2 = 2$
$V_3 = 2$
$V_4 = 4$

- if the degree of vertex is 'zero' than that vertex is called 'isolated vertex'.
- if the vertex of degree is '1' than that vertex is called as 'pendant'

adjacency list :-

It is a way to represents a graph without multiple edges it's specifies all the vertices that are adjacent to each vertex of a graph.

Ex :-



a = b, c, e
b = a
c = a, e, d
d = e, c
e = a, c, d.

matrix

	a	b	c	d	e
a	0	1	1	0	1
b	1	0	0	0	0
c	1	0	0	1	1
d	0	0	1	0	1
e	1	0	1	1	0

* ~~Adjustency~~ Adjacency matrix :-

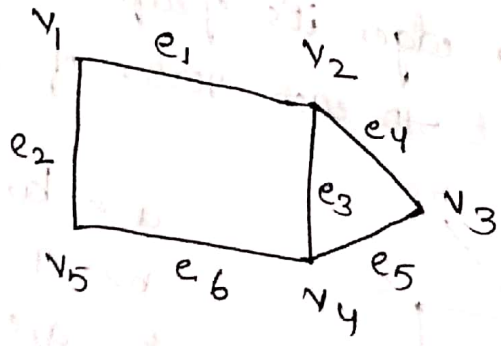
A graph is a nxm matrix with '1' at (i,j) entry when i and j are adjacent to each other & otherwise it is '0'

$$A = [a_{ij}] = \begin{cases} 1 & \text{if } (i,j) \text{ is edge} \\ 0 & \text{otherwise} \end{cases}$$

* Incidence matrix :-

Let $G = (V, E)$ be a undirected graph where $V = V_1, V_2, \ldots V_n$, $e = e_1, e_2, \ldots e_n$. the nxm matrix where $I = m_{ij}$, where $m_{ij} = \begin{cases} 1 & \text{if } V_i \text{ is incident} \\ 0 & \text{otherwise} \end{cases}$ is called incidence matrix

Ex :-



	e_1	e_2	e_3	e_4	e_5
V_1	1	1	0	0	0
V_2	1	0	1	1	0
V_3	0	0	0	1	1

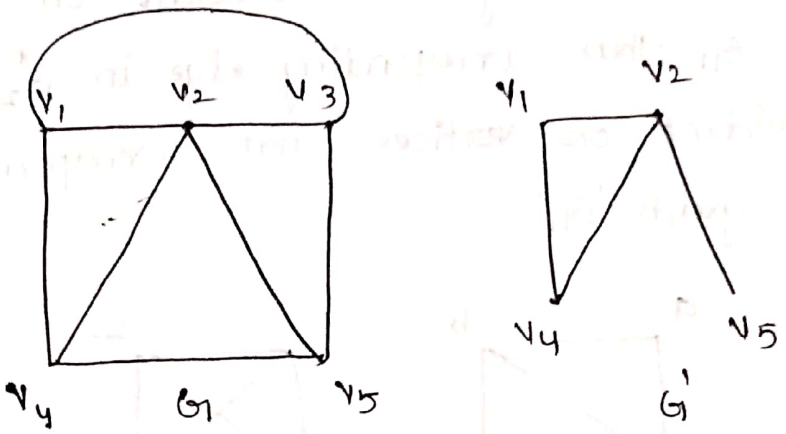
* sub graph :-

A sub graph of graph 'G' is a graph 'G'' such that vertices of G'

$$V(G') \subseteq V(G)$$

$$E(G') \subseteq E(G)$$

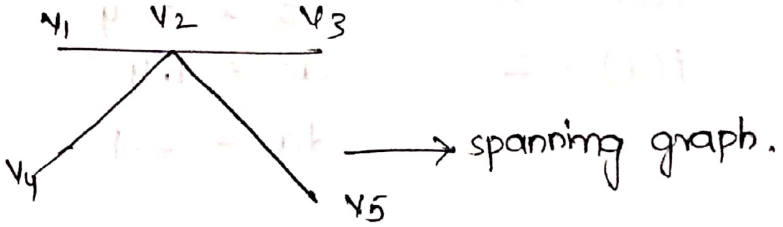
Ex:-



A sub graph 'H' of 'G' is called spanning sub graph of 'G' if and only if ~~vertices of sub graph~~.

$$V(H) = V(G)$$

Ex:-



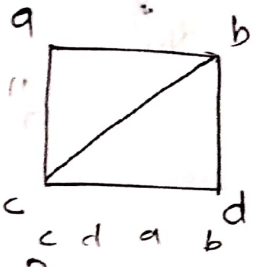
* Isomorphism :

The graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are called isomorphic. if there is 1,2,1 and onto function from V_1 to V_2 with the property that a and b are adjacent in G_1. if and only if f(a), f(b) are adjacent in G_2, for all a and b in V_1.

properties of Isomorphism :-

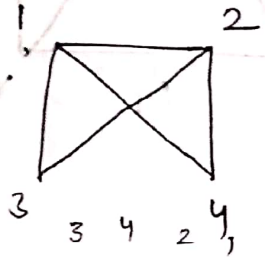
1. same no. of vertices
2. " " edges
3. equal no. of vertices with a given degree
4. suppose the edge 'e' is incident on v_1 and v_2 in G_1 then corresponding edge in G_2 must be incident on vertices that corresponds to vertices in graph 'G_1'

Ex :-



c d a b

3, 2, 2, 3



3 4 2 1

3, 3, 2, 2

$f(a) = 1$

$f(b) = 3$

$f(c) = 4$

$f(d) = 2$

ab = 1, 3 ← checking edges from

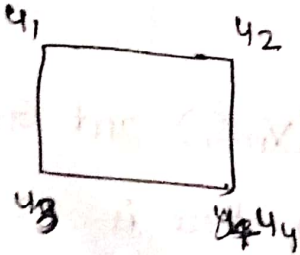
bd = 3, 2

dc = 2, 4

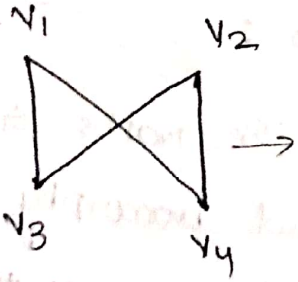
ac = 1, 4

da = 2, 1

(a) Show that 2 graphs are isomorphism



2, 2, 2, 2



2, 2, 2, 2

$f(u_1) = v_1$

$f(u_2) = v_4$

$f(u_3) = v_2$

$f(u_4) = v_3$

ab = v_1, v_4

bc = 4, 2

ad = 1, 3

bd = 4, 3

$u_1u_2 = v_1v_4$

$u_2u_4 = v_4v_2$

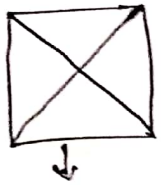
$u_3u_4 = v_3v_2$

$u_1u_3 = v_1v_3$

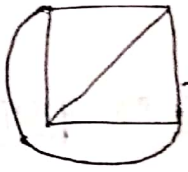
Planar graphs :-

A graph 'G' is said to be planar, if it can be drawn in the plane without its edge crossing otherwise graph is a non-planar graph.

Ex :-



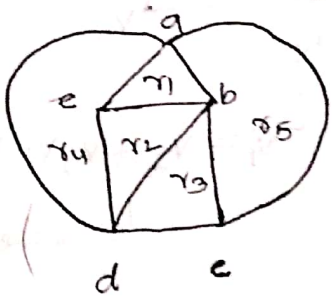
Non planar graph



→ planar graph.

A planar graph 'G' dividing the plane into regions or phases. A region is characterised by the cycle that form its boundary. This regions are connected adjacents of the regions

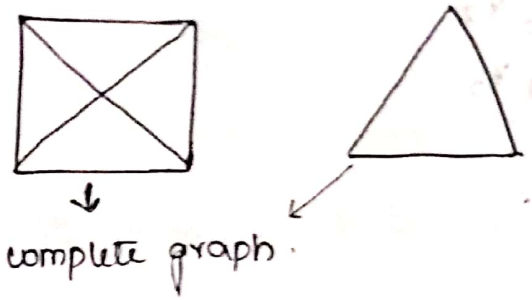
Ex :-



In planar graph 'G' determines a region of infinity area called the exterior region of 'G'. r_6 is the Exterior region.

- A graph in which every vertex is connected to every other vertex is called "complete graph".

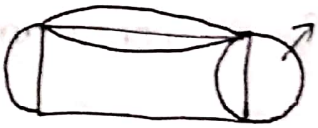
Ex:-



complete graph.

— A graph containing more edges is called "multi graph"

Ex:-



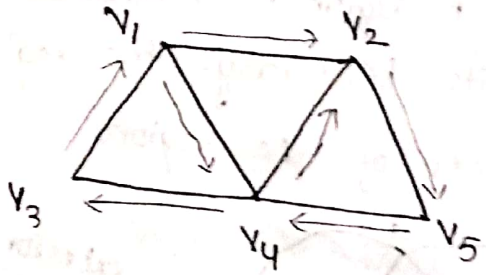
multi graph.

***

* Eulerian (or) Euler graphs :-

A Euler path in a graph is a path that includes each edge of the graph exactly one's and intersects each vertex of the graph atleast one's..

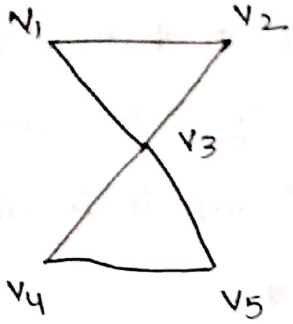
Ex:-



$V_1 - V_2 - V_5 - V_4 - V_3 - V_1 - V_4 - V_5$

↳ Euler graph

- A Euler circuit is an euler path whose end point are identical. A graph is said to be Eulerian graph if it has a euler circuit.



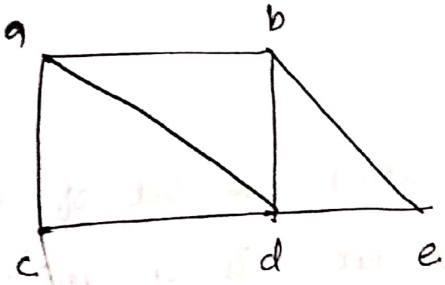
$v_1 - v_2 - v_3 - v_5 - v_4 - v_3 - v_1$

↳ Euler circuit

- Euler graph is always connected because Euler path contains all the edges of the graph.
- A connected graph is Euler graph if its has atmost two (or) odd degree vertices.
- A connected graph is Euler circuit if each vertex has even degree for a vertices.

ex :-

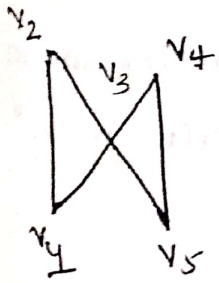
a-c-d-b-e-d-a



a = 3
b = 3
c = 2
d = 4
e = 2

↑

Euler graph.



$v_1 = 2$

$v_2 = 2$

$v_3 = 4$ ✓

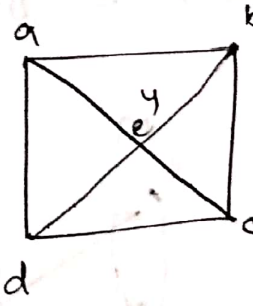
$v_4 = 2$

$v_5 = 2$

Even

Euler circuit-

$v_1 - v_2 - v_3 - v_5 - v_4 - v_3 - v_1$



a = 3
b = 3
c = 3
d = 3
e = 4

Euler graph path

a - b - d - c -

+ A graph with n vertices so that each of n vertices are adjacent to each of other n-1 vertices is called "complete graph" and it is denoted by 'K_n' where n = 1, 2, 3, 4, 5 ...

K_n

K_1 =

K_2 =

K_3 =

K_4 =

K_5 =

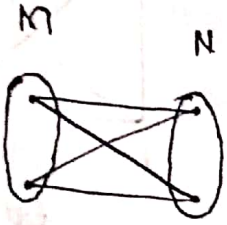
K_7 =

K_6 =

⋮

- A graph in which the set of vertices can be partition into two sets of vertices m, n in such a way that each edge joins one vertex in 'm' to a vertex in n is called "A bipartiate graph"

Ex:-



- A graph with m+n vertices, so that each of the 1st m vertices are adjacent to each of the 2nd n vertices and there are no edges b/w 1st m vertices and there are no edges b/w 2nd n vertices is called "complete bipartiate graph" and is denote

by $k_{m,n}$

$K_{1,1}$ 

$K_{1,2}$ 

$K_{1,3}$ 

$K_{2,2}$ 

$K_{2,3}$ 

$K_{2,4}$ 

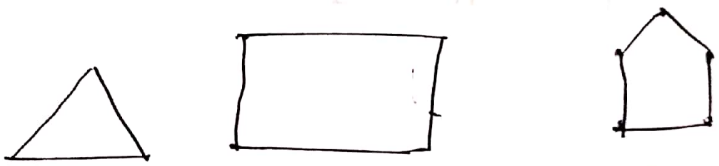
$K_{3,3}$ 

- The number of edges in K_n is $\left[\frac{n(n-1)}{2}\right]$

- $K_{m,n}$ has $m+n$ vertices and $m*n$ edges.

- A graph in which all the edges form a cycle is called "A cycle graph".

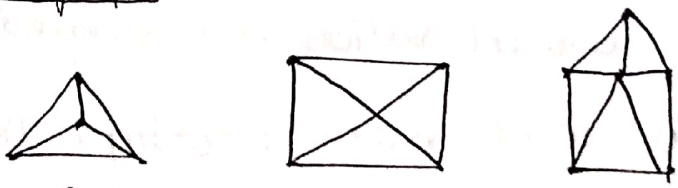
Ex :-



C_3 C_4 C_5

A graph obtained from a cycle graph by joining a single new vertex to a vertex of the cycle is called "wheel graph".

Ex :-



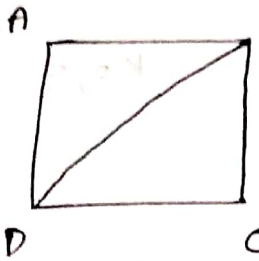
C_3 C_4 C_5

Hamilton cycle graph (or) Hamilton path :-

A path in a graph 'G' is called a hamilton path, if it contains every vertex of 'G'

- A cycle in a graph 'G' is called Hamilton cycle if it contains every vertex of 'G'
- A graph 'G' is said to be hamilton graph if it contains a Hamilton cycle

Ex:-

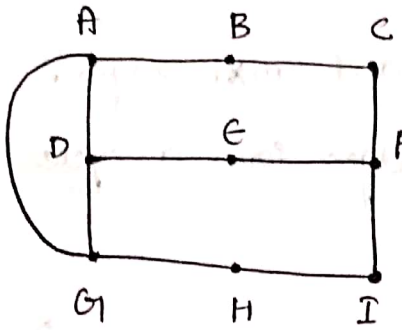


Hamilton graph.

A - B - C - D - A

↓

Hamilton cycle



A - B - C - F - E - D - G - H - I

Hamilton path.

* chromatic number :-

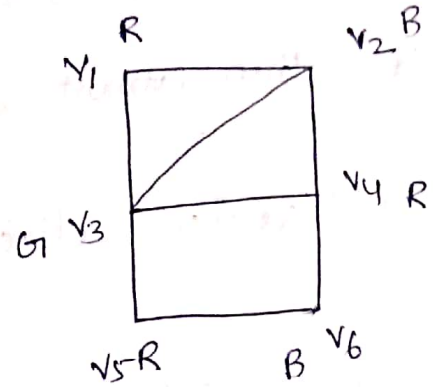
Vertex colouring :-

The assignment of colours to the vertices of graph 'G' one colour to each vertex so that adjucent vertices are assigned different colours is called vertex colouring: An 'n' colouring of 'G' is a colouring of graph using 'n' colours

- if 'G' has an 'n' colouring then 'G' is said to be 'n' colouring.

The chromatic number:- of graph 'G' is the minimum number of colours to the vertices of the graph 'G' we denote the chromatic number of graph 'G' by $\chi(G)$. if $\chi(G) = K$ then graph G is K chromatic.

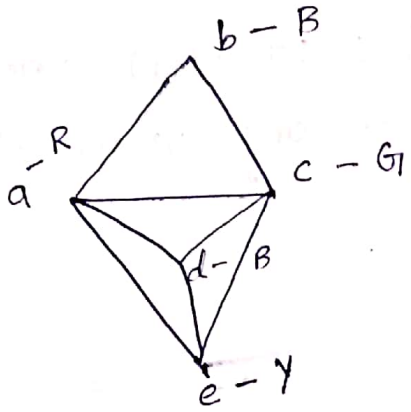
Eg:-



chromatic number = 3

$\chi(G_1) = 3$

(b)



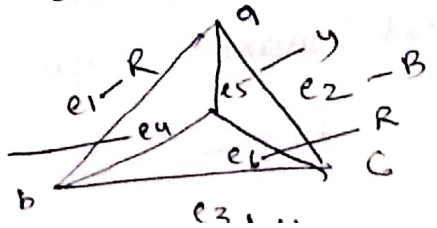
$\chi(G) = 4$

colour chromatic number:-

Assigning colours to the edges so that no to adjacent edges recieve the colour is called "colouring of G".

edge chromatic number:- The edge chromatic number is the min no. of colour to the all edges of graph G, so that edges with end pts are colour are common are coloured with different colours.

Eg:-



* Rules for finding the chromatic number of graph G

1. If $\chi(G) \leq |V|$ where V is the no. of vertices of graph G.

2. If H is a sub-graph of G then $\chi(H) \leq \chi(G)$

3. If degree of vertices = d then atmost d colours $\deg(v)$ are required to colour the vertices adjacent to 'v'.

4. $\chi(G) = \max \{ \chi(C) / C$ is a connected component of G$\}$

5. for any graph 'G' $\chi(G) \leq 1 + \Delta(G)$ where $\Delta(G)$ is the largest degree of any vertex of graph G

* Eulers formula :-

If G is a connected planar graph then any drawing of G to the plane as a planar graph will always form

$$|R| = |E| - |V| + 2$$

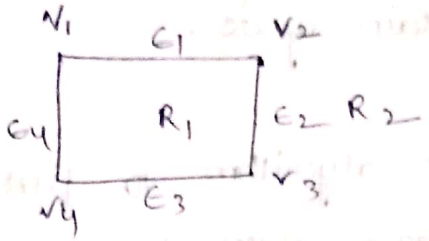
EVLER's formula

including the exterior region where R, E, V denote respectively the no. of regions edges & vertices of graph 'G'.

— If G is a connected plane graph then

$$|V| - |E| + |R| = 2$$

Ex:-



$4 - 4 + 2 = 2$

$2 = 2$

* Trees and these properties :-

A tree is a connected undirected simple acyclic graph. In other coord a tree is a simple graph G such that there is a unique simple undirected graph (or) path between each pair of vertices of graph G.

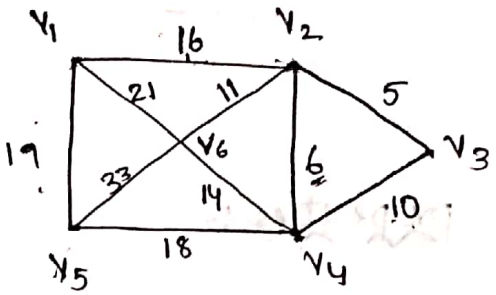
- A rooted tree is a tree in which a particular vertex is designated as be root.
- A rooted tree is a directed tree if there is a root from which there is a directed path to be each of the tree.
- The level of vertex 'v' in a rooted tree is the length of the simple path from the root.
- The height of the rooted tree is a max level number that occurs in the tree.

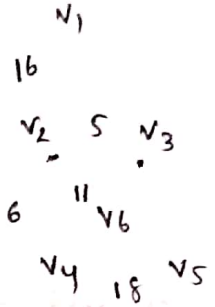
$\Rightarrow (h+1)$

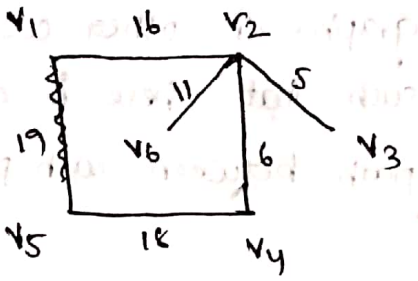
* Spanning tree :- A tree is a spanning tree of graph 'G' is a sub graph of graph 'G' that contains all the vertices of graph.

— it shouldn't contain cyclic.

1) using kruskkal's algorithm to find minimum spanning tree for the graph given below.

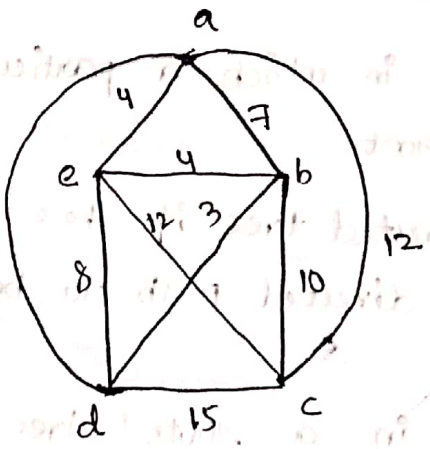


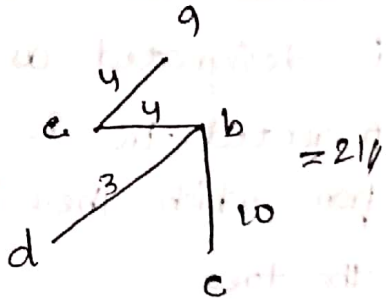




~~57~~ ~~56~~

2)





$= 21$

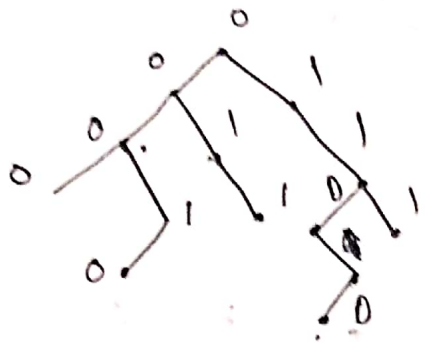
(**) binary trees :-

A binary tree is a directed tree where $T = (V, E)$ together with an edge labelly $E \rightarrow \{0,1\}$ such that every vertex has atmost 1 edge incident from it labelled with 0 and atmost 1 edge incident from it labelled with 1. each edge u, v

labelled with '0' is called left edge

- each edge u, v labelled with '1' is called right edge

ex:-



- Binary search tree :-

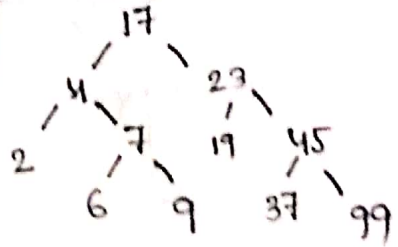
A binary search tree is a binary tree with vertex labelling $l : V \longrightarrow A$ where

$A = A_1, A_2, \ldots A_n$ is a totally ordered set with $A_1 < A_2 < A_3 < \ldots A_n$ where the labelling 'l' satisfies the properties.

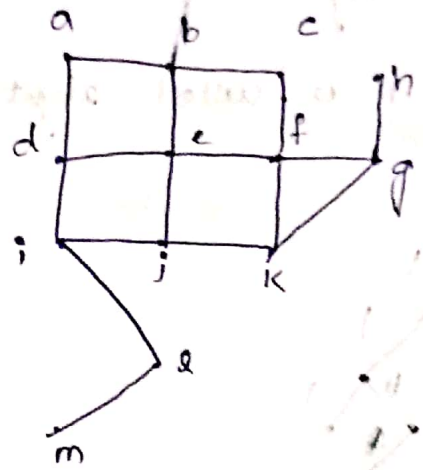
1. for each vertex 'u' in the left sub tree of a vertex 'v' $l(u) \leq l(v)$
2. for each vertex 'u' in the right sub tree of a vertex v $l(u) \geq l(v)$

ex:- Built a binary search tree for the sequence of numbers.

17, 23, 4, 7, 9, 19, 45, 6, 2, 37, 99

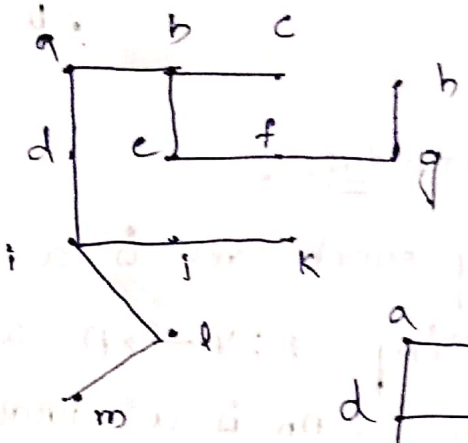


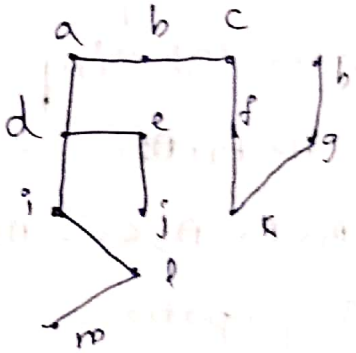
*



— BFS

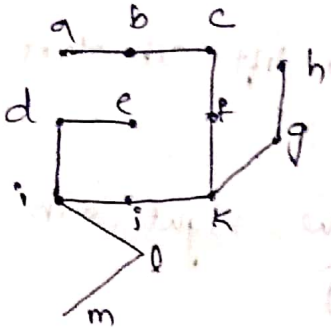
adjucents .



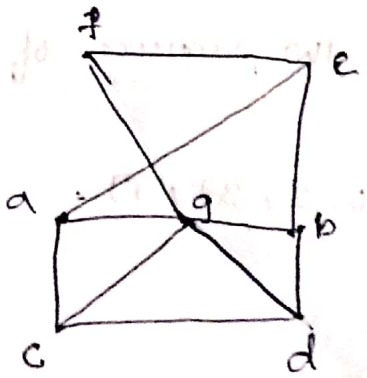


— DFS

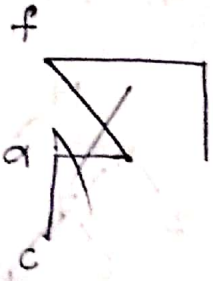
backtracking



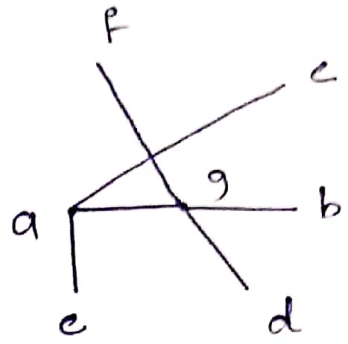
*



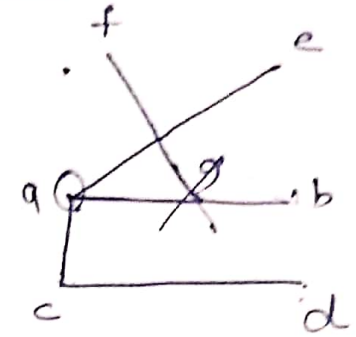
Construct BFS and DFS

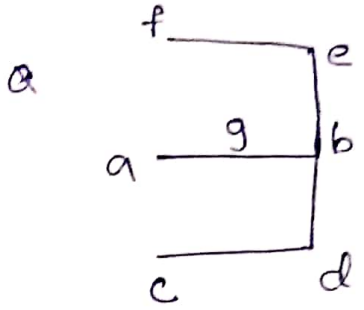


BFS

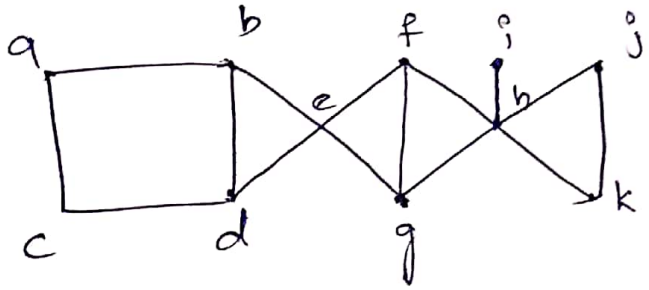


DFS backtracking

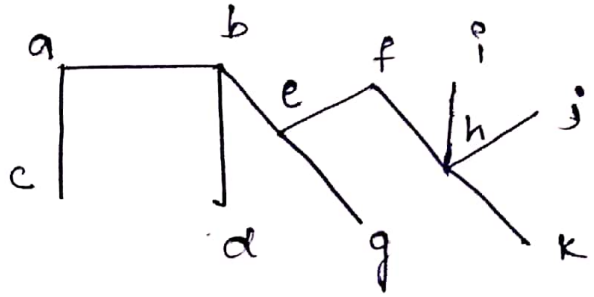




* construct BFS and DFS



BFS



DFS