3. Recurrence Relations

Generating function : Let $A = \{a_n\}_{n=0}^{\infty}$

be the sequence of real numbers

then $a_0 + a_1 x + a_2 x^2 + \ldots + a_r x^r + \ldots = \sum_{n=0}^{\infty} a_n x^n$

is called power series and we can write as

$$g(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_r x^r + \ldots$$

where x is an indeterminent.

* Let $A = \{a_n\}_{n=0}^{\infty} = (a_0, a_1, a_2, \ldots, a_n)$ be the sequence then its generating function is defined as $g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

* we can easily obtain generating fun^n from sequence. for example the generating function

$$\{a_n\}_{n=0}^{\infty} = \{\alpha^n\}_{n=0}^{\infty} = \alpha^0 + \alpha^1 + \alpha^2 + \ldots$$

* The infinite series in the above equation can be written in closed form as $\frac{1}{1-\alpha x}$ which is rather compact way to represent the sequence $\{a_n\} = \{1, \alpha, \alpha^2, \ldots\}$

problem

1. Find the generating function for the sequence

$$\{3^n\}_{n=0}^{\infty}$$

Sol:- $1+3x+3^2x^2+\dots+\{a_n\}\dots + = \sum_{n=0}^{\infty} 3^n x^n$

2. find the generating function for the sequence

$1, 1, 1, 1, \dots$

Sol:- $= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$= 1 + 1x + 1x^2 + 1x^3 + \dots$

$= 1 + x + x^2 + x^3 + \dots = \frac{1}{(1-x)}$

3. find the generating fun^n for the sequence

$1, -1, 1, -1, \dots$

Sol:- $a_0 + a_1 x + a_2 x^2 + \dots$

$1 - x + 1x^2 - 1x^3 + \dots$

$1 - x + x^2 - x^3 + \dots = \frac{1}{(1+x)}$

4) find the generating fun^n for the sequence

$1, 2, 3, 4, \dots$

Sol:- $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$1 + 2x + 3x^2 + 4x^3 + \dots = \frac{1}{(1-x)^2}$

$= \frac{1}{} \quad \frac{1}{}$

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5) $1, -2, 3, -4, \ldots$

Sol:- $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

$1 - 2x + 3x^2 - 4x^3 + \ldots = \frac{1}{(1+x)^2}$

6) $0, 1, 2, 3, 4, \ldots$

Sol:- $0 + x + 2x^2 + 3x^3 + 4x^4 + \ldots = \frac{1}{x(1+x)^2}$

(or) $\Rightarrow \frac{x}{(1-x)^2}$

7) $0, 1, -2, 3, -4, \ldots$

Sol:- $0 + x - 2x^2 + 3x^3 - 4x^4 + \ldots = \frac{1}{x(1+x)^2}$

(or) $= \frac{x}{(1+x)^2}$

8. Find the generating funn for the sequence

$1^2, 2^2, 3^2, \ldots$

Sol:- $1 + 2^2 x + 3^2 x^2 + 4^2 x^3 + \ldots = \frac{1}{(1-x^2)^2}$

9. In

→ In general since $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$

$$\boxed{= \sum_{r=0}^{\infty} \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} x^r}$$

for any real numbers 'n' we know that

$f(x) = (1+x)^n$

is a generating function for the sequence is 'a

$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \binom{n}{3}, \dots \binom{n}{n}, 0, 0, 0 \dots$

where n is real number

Then expression written as

$$\boxed{(1+x)^n = \sum_{r=0}^{\infty} \binom{n}{r} x^r}$$ for any real numbers 'n'

1) find the sequence generate that by the following function $(3+x)^3$

$$(1+x)^n = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

$n = 3$

$x = x/3$

$$3^3\left(1+\frac{x}{3}\right)^3 = 3^3\left[1 + 3\left(\frac{x}{3}\right) + \frac{3(3-1)}{2!}x^2 + \frac{3(3-1)(3-2)}{3!}x^3 + \dots\right.$$

$$= 3^3\left(1 + x + \frac{6}{2!}x^2 + \frac{6}{3!}x^3 + \dots\right.$$

$$= 3^3\left(1 + x + 3x^2 + 1x^3 + \dots\right.$$

$$= 3^3 + 3^3 x + 3^2 x^2 + 3^3 x^3 + \dots$$

$$= 27 + 27x + 9x^2 + 27x^3 + \dots$$

$$= 27, 27, 9, 1, 0, 0, 0, \dots$$

(2) $(2+x)^3$

$$2^3\left(1+\frac{x}{2}\right)^3$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$

$n=3$, $x=\frac{x}{2}$

$$= 2^3\left(1+3\left(\frac{x}{2}\right)+\frac{3(3-1)}{2!}\left(\frac{x}{2}\right)^2+\frac{3(3-1)(3-2)}{3!}\left(\frac{x}{2}\right)^3+\dots\right)$$

$$= 2^3\left(1+\frac{3x}{2}+\frac{6}{2!}\left(\frac{x^2}{4}\right)+\frac{6}{3!}\left(\frac{x^3}{27}\right)+\dots\right)$$

$$= 2^3+2^{\cancel{3}\,2}\cdot\frac{3x}{\cancel{2}}+2^3\cdot 3\cdot\frac{x^2}{4}+2^3\cdot\frac{x^3}{27}+\dots$$

$$= 8+12x+6x^2+\frac{8x^3}{27}+\dots$$

$$= 8, 12, 6, \frac{8}{27}, 0, 0, \dots$$

(3) $2x^2(1-x)^{-1}$

Sol:- $= 2x^2(1+x+x^2+x^3+\dots)$

$= 2x^2+2x^3+2x^4+2x^5+\dots$

$x=1$ $= 2, 2, 2, 2, 2, \dots$

(4) $\frac{1}{(1-x)}+2x^3$

Sol:- $(1-x)^{-1}+2x^3$

$= 1+x+x^2+x^3+\dots+2x^3$

$= 1+x+x^2+3x^3+x^4+\dots$

$= 1, 1, 1, 3, 1, 1, \dots$

(5) $3x^3+e^{2x}$

(6) find the generating function for the sequence

$1, 1, 0, 1, 1, \dots$

Sol:- $(1-x)^{-1}-x^2$

7) find the generating function for the sequence
$1^2, 2^2, 3^2, \ldots$

sol:- Given

$1^2, 2^2, 3^2, 4^2, \ldots$

$= a_0 x^0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

$= 1 + 4x + 9x^2 + 16x^3 + \ldots$

$0, 1, 2, 3, \ldots$ apply derivation

$$\frac{d}{dx}\left(0 + x + 2x^2 + 3x^3 + \ldots\right) = \frac{d}{dx}\left(\frac{x}{(1-x)^2}\right)$$

$$1 + 4x + 9x^2 + 16x^3 + \ldots = \frac{d}{dx}\left(\frac{x}{(1-x)^2}\right)$$

$$1 + 4x + 9x^2 + \ldots = \cancel{(1-x)^2 \; x^3 \; 2}$$

$$= \frac{(1-x)^2 \cdot 1 - x \, 2(1-x)(0-1)}{((1-x)^2)^2}$$

$\frac{u}{v} = \frac{vu' - uv'}{v^2}$

$$= \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = \frac{(1-x)^2 + \cancel{2x + 2x}(1-x)}{(1-x)^4}$$

$$= \frac{1 + x^2 - 2x + 2x - 2x^2}{(1-x)^4} = \frac{(1-x)^2 \cancel{+ 2x} - 2x + 2x^2}{(1-x)^4}$$

$$= \frac{1 - x^2}{(1-x)^4} \qquad = 1 + x^2 \cancel{- 2x + } 2x^2$$

$\because (1^2 - x^2) = (1+x)(1-x)$

$$= \frac{(1+x)(1-x)}{(1-x)^4} \Rightarrow \frac{(1+x)}{(1-x)^3}$$

1) $0^2, 1^2, 2^2, 3^2, \ldots$

$1^2x^2 + 2^2x^3 + 3^2x^4 + \ldots$

$= x^2 + 4x^3 + 9x^4 + \ldots$

$1, 2, 3 \ldots$ Apply derivation

$\frac{d}{dx}\left(\cancel{x^2 + 2x^2 + 3}\right.$

$\frac{d}{dx}\left(\cancel{x + 2x^2 + 3x^3 +} \ldots\right)$

$(1 + 2x + 3x^2 + 4x^3 + \ldots) =$

$0 + 2 +$

$\frac{x(1+\cancel{x})}{(\cancel{1}-x)^3}$

2) $1^3 + 2^3 + 3^3 + \ldots$

$a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$

$= 1^3 + 2^3x + 3^3x^2 + 4^3x^3 + \ldots$

$= \frac{d}{dx}(0^2 + x + 2^2x^2 + 3^2x^3 + \ldots) = \frac{d}{dx}\frac{x(1+x)}{(1-x)^3}$

$= 1^3 + 2^3x + 3^3x^2 + \ldots = \frac{d}{dx}\frac{x(1+x)}{(1-x)^3}$

$x(1+x)$
$= x + x^2$

$\frac{vu' - uv'}{v^2} = \frac{(1-x)^3(1+2x) - (x+x^2)(3(1-x)^2(0-1))}{((1-x)^3)^2}$ $\frac{u}{v}$ rule

$= \frac{(1-x)^3(1+2x) - (x+x^2)(3-3x)(-1)}{(1-x)^5}$

$= \frac{(1-x)^3(1+2x) + (3x - 3x^2 + 3x^2 - 3x^3)}{(1-x)^5}$

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$$\frac{(1-x)^2}{(1-x)^6}\left[(1-x)(1+2x)+3x+3x^2\right]$$

$$\frac{1}{(1-x)^4}\left[x^2+4x+1\right]$$

10) find the generating fun^n for $(1,1,1,1,1,1)$ for the sequence

sol:- $\frac{1-x^6}{1-x}$ $\qquad\qquad$ $\frac{1}{1-x}$

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* calculate the coefficient of generating fun^n

$$\frac{1}{(1-x)^n}=\sum_{r=0}^{\infty}c(n-1+r,r)\,x^r$$

$$\frac{1}{(1+x)^n}=\sum_{r=0}^{\infty}c(n-1+r,r)(-1)^r x^r$$

$$\frac{1}{(1-ax)^n}=\sum_{r=0}^{\infty}c(n-1+r,r)\,a^r x^r$$

$$\frac{1}{(1+ax)^n}=\sum_{r=0}^{\infty}c(n-1+r,r)(-a)^r x^r$$

① find the co-efficient of

x^{20} in $(x^3+x^4+x^5+\cdots)^5$

sol:- $x^{20} = (x^3+x^4+x^5+\cdots)^5$

$x^{20} = (x^3)^5(1+x+x^2+\cdots)^5$

$x^{20} = x^{15}(1-x)^{-1}$

$x^{20} = x^{15}c(n-1+r, r)x^r$

$n = 5$

$x^{20} = x^{15} + x^r$

$x^{20} - x^{15} = x^r$

$x^5 = x^r$

$\boxed{r = 5}$

$c(5-1+5, 5)$

$c(9,5) = \dfrac{9!}{4!\,5!}$

2) find coefficient x^{27} in $(x^4+x^5+x^6+\dots)^5$

Sol:- $x^{27} = (x^4)^5(1+x+x^2+\dots)^5$

$= c(n-1+r, r)\, x^r$ $\quad r = 7$

$c(5-1+7, 7)$ $\quad n = 5$

$c(11, 7) = \dfrac{11!}{7!\,4!}$

3) find the coefficient x^{15} in $(x^2+x^3+x^4+x^5)$.

$(x+x^2+x^3+x^4+x^5+x^6+x^7)$

$(1+x+x^2+\dots x^{15})$

Sol:- $x^{15} = x^2(1+x+x^2+x^3)\, x(1+x+x^2+x^3+x^4+x^5+x^6)$

$(1+x+x^2+\dots+x^{15})$

$x^{15} = x^2 \cdot \dfrac{1-x^4}{1-x} \cdot x \cdot \dfrac{1-x^7}{1-x} \cdot \dfrac{1-x^{16}}{1-x}$

$x^{15} = \dfrac{x^3(1-x^4)(1-x^7)(1-x^{16})}{(1-x)^3}$

$x^{15} = x^3(1-x^7-x^4+x^4-x^{16}+x^{23}+x^{20}-x^{27})$

$(1-x)$

$x^{15} = x^3(1-x^7-x^4+x^4)\, c(2+r, r)x^r \; (c(3-1+r, r)$

$x^{15} = x^3(c(2+r, r)\, x^r$ —— (1)

$= x^{3+r}\; c(2+12, 12) \Rightarrow c(14, 12)$

$\boxed{r = 12}$

$= \dfrac{14!}{12!\,12!}$

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$x^{15} = -x^{10} c(2+r, r)\, x^r$ — ②.

$x^{15} = -x^{10+r}$

$c(2+25, 25)$

$x^{15} = x^{14} c(2+r, r) x^r$

$c(27, 25)$

$= x^{14+r}$

$= \frac{27!}{25!\,2!}$

$\boxed{r=1}$ $(2+1, 1)$

$(3, 1)$

$\frac{3!}{1!\,2!}$

$x^{15} = -x^7 c(2+r, r)\, x^r$ — ③

$x^{15} = -x^{7+r}$ $(10, 8)$

$\frac{10!}{8!\,2!}$

$x^{15} = c(14, 12) - c(7, 5) - c(10, 8) - c(3, 1)$

4) find the coefficient of

x^5 in $(1-2x)^{-7}$

$n = 7$

Sol:- $x^5 = \frac{1}{(1+2x)^7}$

$c(n-1+r, r), a^r x^r$.

$x^5 = c(7-1+r, r)(2)^r . x^r$

$x^5 = c(6+r, r)(2)^r, x^r$

$\boxed{r=5}$

$= c(6+5, 5)(2)^5$

$x^5 = c(11, 5).\,32$

(5) find coefficient of x^{10} in $\frac{x^3 - 5x}{(1-x)^3}$

Sol:- $x^{10} = x^3 - 5x$ $(1-x)^{-3}$

$n = 3$

$c(n-1+r, r)$

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$c(3-1+r, r)$

$x^{10} = x^3 - 5x \cdot c(2+r, r)\, x^r$

$x^{10} = x^{3+r} c(2+r, r) - 5x^{1+r} c(2+r, r)$

$x^{10} = x^{3+r} c(9, 7)$ $\quad$ $x^{10} = -5 \cdot x^{1+r}$

$\boxed{r = 7}$ $\quad$ $c(9, 7)$ $\quad$ 9

$c(11, 9)$

$x^{10} = c(9,7) - c(11 \cdot 9)\, 5$

(6) Find the co-efficient x^{15} in $\dfrac{(1+x)^4}{(1-x)^4}$

Sol :- $x^{15} = (1+x)^4 (1-x)^{-4}$

$= (1+x)^4 (n-1+r, r)$ $\quad$ $n = 4$

$= (1+x)^4 c(4-1+r, r)\, x^r$

$= (1+x)^4 c(3+r, r)\, x^r$

$x^{15} = \left(1 + 4x + \dfrac{4(3)}{2!} x^2 + \dfrac{4(3)(2)}{3!} x^3 + \cdots \right) c(3+r, r) x^r$

$\boxed{r = 15}$

$x^{15} = \boxed{c(18, 15) \quad — (1)}$

$x^{15} = 4 \cdot x^{r+1}$ $\quad$ $\boxed{4 \cdot c(17, 14)}$ — (2)

$\boxed{r = 14}$

$x^{15} = x^{2+r} \cdot 6$ $\quad$ $\boxed{6 \cdot c(16, 13)}$ — (3)

$\boxed{r = 13}$

$x^{15} = 4 \cdot x^{3+r}$ $\quad$ $c(3+r, r)$

$(r = 12)$

$4c(15, 12)$

$1 + 4C(18,15) + x^2 C(17,14) + 6x^3 C(16,13) + 4C(15,12)$

(7) find the generating funⁿ of x^{12} in $(1+2x+3x^2)(x - x^2 + x^3 - x^4 + \cdots)^3$.

Sol:- $x^{12} = (1+2x+3x^2)(x - x^2 + x^3 - x^4 + \cdots)^3$

$= (1+2x+3x^2)\, x(1 - x + x^2 - x^3 + \cdots)^3$

$= (1-x)^{-2} x \left(\frac{1}{1+x}\right)^3$

$n=3$

$x^{12} = x C(n-1+r, r)(-1)^r x^r$

$= x C(2+r, r)(-1)^r x^r$

$= x^{1+r}$

Sol:- $x^{12} = (1+2x+3x^2)(x - x^2 + x^3 - x^4 + \cdots)^3$

$= x^3(1+2x+3x^2)\left(\frac{1}{(1+x)^3}\right)$

$\boxed{n=3}$

$= x^3(1+2x+3x^2((1+x)^{-1})^3$

$= (x^3 + 2x^4 + 3x^5) \cdot C(n-1+r, r)(-1)^r x^r$

$x^3 \cdot x^r (-1)^r C(3-1+r, r) + 2x^r \cdot x^4 (-1)^r C(2+r, r)$

$3+r=12$ $\quad 3x^5 \cdot (-1)^r \cdot x^r C(2+r, r)$

$r=9$

$r=8$ $\quad C(2+9, 9)(-1)^9 + 2(-1)^8 C(2+8, 8)$

$r+4=12$

$r=8$ $\quad + 3 \cdot (2+7, 7)(-1)^7$

$r+5=12$

$r=7$ $\quad = -C(11, 9) + 2C(10, 8) - 3C(9, 7)$

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* **Rule of counting :-**

Suppose we wish to determine the no. of integral solutions of equation,

$e_1 + e_2 + e_3 + \dots e_n = r$. where $(0 \le r \le n)$

under the contrain that e_1 can take the integer values $a_{11}, a_{12}, a_{13}, \dots$ simi

similarly

e_2 can take value $a_{21}, a_{22}, a_{23} \dots$

e_3 " " " $a_{31}, a_{32}, a_{33} \dots$

⋮

e_n can take $a_{n1}, a_{n2}, \dots$

to solve this problem first define the function $f_1(x), f_2(x), \dots f_n(x)$ as follows.

$$f_1(x) = x^{a_{11}} + x^{a_{12}} + x^{a_{13}} + \dots$$

similarly

$$f_2(x) = x^{a_{21}} + x^{a_{22}} + x^{a_{23}} + \dots$$

⋮

$$f_n(x) = x^{a_{n1}} + x^{a_{n2}} + \dots$$

and consider $f(x) = f_1(x), f_2(x), \dots f_n(x)$

and determine coeff of x^r in this fun'

This coeff happens to the equal to the no. of solution that are desired to find. the function $f(x)$ is called generating function

Q) find the no. of non-negative integrals of the eqn $e_1+e_2+e_3+e_4+e_5+e_6 = 20$ where $1 \le e_1 \le 5$, $2 \le e_2, \dots e_6$

sol:- $r = 20$, $n = 5$

$$e_1 + e_2 + \dots e_n = r$$

$$f_1(x) = x + x^2 + x^3 + x^4 + x^5$$

$$f_2(x) = (x^2 + x^3 + x^4 + x^5 + \dots)$$

$$f_3(x) = x^2 \quad ''$$

$$f_4(x) = \quad ''$$

$$f_5(x) = \quad ''$$

$$f_6(x) = \quad ''$$

(or)

$$f_2(x) = (x^2 + x^3 + x^4 + x^5 + \dots)^5$$

because all f function remains same, so be conv eqn as $p^5{}_4$

$$f(x) = f_1(x) + f_2 x + \dots f_n(x)$$

$$x^{20} = (x + x^2 + x^3 + x^4 + x^5)\underbrace{(1 + x + x^2 + \dots)^5}_{(x^2)^5}$$

$$x^{20} = x(1 + x + x^2 + x^3 + x^4)x^{10}(1 + x + x^2 \dots)^5$$

$$x^{20} = x^{11}(1 + x + x^2 + x^3 + x^4)(1 + x + x^2 + \dots)$$

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$$x^{20} = (x^{11}+x^{12}+x^{13}+x^{14}+x^{15})(n-1+r, r)\,x^r$$

$$= (x^{11}+x^{12}+x^{13}+x^{14}+x^{15})(4+r, r)\,x^r$$

$r = 9, 8, 7, 6, 5$

$x^{20} = x^{10} \cdot x^r$

$x^2 \quad 20 = 10 + r$

$r = 10$

$C(13, 9) + C(12, 8) + C(11, 7) + C(10, 6) + C(9, 5)$

(2) find the generating function that determines no. of non-negative integral solution of the eqn

$e_1 + e_2 + e_3 + e_4 + e_5 = 20$ where $0 \le e_1 \le 3$

$r = 20$. $\quad 0 \le e_2 \le 4$

$2 \le e_3 \le 6$

$2 \le e_4 \le 5$

$e_5 \le 9$ with odd number

sol:- $f_1(x) = x^0 + x^1 + x^2 + x^3$

$f_2(x) = x^0 + x^1 + x^2 + x^3 + x^4$

$f_3(x) = x^2 + x^3 + x^4 + x^5 + x^6$

$f_4(x) = x^2 + x^3 + x^4 + x^5$

$f_5(x) = \cancel{x^5 + x^7 + x^9}\ x^1 + x^3 + x^5 + x^7 + x^9$

$f(x) = f_1(x) \cdot f_2(x) \cdot f_3(x) \cdot f_4(x) \cdot f_5(x)$

$$x^{20} = (1+x+x^2+x^3)(1+x+x^2+x^3+x^4)(x^2+x^3+x^4+x^5+x^6)(x^2+x^3+x^4+x^5)(x^1+x^3+x^5+x^7+x^9)$$

$$= x \cdot x^2 \cdot x^2 (1+x+x^2+x^3)(1+x+x^2+x^3+x^4)(1+x+x^2+x^3+x^4)(1+x^2+x^4+x^6+x^8)$$

$$= x^5(1-x^4)^2(1-x^5)^2(1-x)^{-4}(1+x^2+x^4+x^6+x^8)$$

=

(3) using the generating funⁿ find the no. of non-negative integral solution of eqn.

$$x_1 + x_2 + x_3 + x_4 = 25$$

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* Recurrance Relations :-

A recurrance relation uses a previous values in a sequence in to compute the current value. A recurrance relation is also called as 'referance equation'

→ A recurrance relation is a formula that relates to many integers, where $n \geq 1$ then the sequence of n terms are $a_0, a_1, a_2, \dots a_{n-1}$.

→ Ex :-

1) $s_n = n + s_{n-1}$

2) $a_n = a_{n-1} + d$

3) $P_n = \gamma P_{n+1}$

4) $a_n - 3a_{n-1} + 2a_{n-2} = 0$.

5) $a_n - 3a_{n-1} + 2a_{n-2} = n^2 + 1$

6) $a_n - (n-1)a_{n-1} * - (n-1)a_{n-2} = 0$

7) $a_n - 3(a_n - 1)^2 + 2a_{n-2} = n$

8) $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 5n$

9) $a_n = a_0 a_{n-1} + a_1 a_{n-2} + \dots + a_{n-1} a_0$

$a_n^2 + (a_n - 1)^2 = -1$

→ suppose n and k are non-negative integers a recurrance relation of the form

$$c_0(n)a_n + c_1(n)(a_{n-1}) + \dots + c_k(n)a_{n-k} = f(n).$$

for $n \geq k$.

where $c_0(n)$, $c_1(n)$, ... $c_k(n)$ and $f(n)$ are functions of n is said to be linear recurrance relation.

→ If $c_0(n)$ and $c_k(n)$ are not identically '0' then it is said to be linear recurrance relation of degree 'k'.

→ If $c_0, c_1, c_2, \dots c_n$ are constants then it is said to be constant pos. coefficient.

→ If $f(n)$ is identically 'zero' then it is said to be Homogeneous. otherwise it is said to be inhomogeneous.

problem

(1) The recurrance relation $a_n = a_{n-1} + 5$

where $n \geq 2$, $a_1 = 2$

$n = 2$ $\qquad a_{n-1} \dots 2, 5, 7, 12, 1$

$a_2 = a_1 + 5$

$= 2 + 5 = 7$

$n = 3$ $\quad a_3 = a_2 + 5$

$= 7 + 5 = 12$

The recurrance relation $a_n = a_{n-1} + a_{n-2}$

$a_1 = a_2 = 1$

$a_3 = a_2 + a_1$

$= 1 + 1 = 2$

$1, 1, 2, 3, 5, \ldots$

Fibonacci recurrance relation :-

The recurrance relation

$F_n = F_{n-1} + F_{n-2}$ where $n \geq 2$ with the initial condition $F_0 = F_1 = 1$ is known as fibonacci recurrance relation.

→ The numbers F_n generated by the fibonacci relation with the initial condition $F_0 = F_1 = 1$ are called "Fibonacci numbers".

→ The sequence of fibonacci numbers $\{F_n\}_{n=0}^{\infty}$ is a fibonacci sequence

Properties :-

i) $F_0 + F_1 + F_2 + \ldots + F_n = F_{n-2} - 1$

ii) $F_n^2 = F_{n-1}F_{n+1} + (-1)^n \quad n \geq 2.$

iii) $F_1 + F_3 + \ldots + F_{2n-1} = F_{2n} - 1$

* Solving Recurrance Relation :-

we consider 3 methods to solve the recurrance relation they are

1. substitution method.
2. generating function
3. characteristic group roots

1. substitution method :- In this method the recurrance relation for a_n is used repeatedly to solve for general expression for a_n in terms of 'n'..

Q. solve the recurrance relation $a_{n+1} = 5a_n$ for $n \geq 0$ given that $a_0 = 2$

Sol:-

$a_{0+1} = 5a_0$

$a_1 = 5 \cdot 2 \Rightarrow a_1 = 10$

$a_2 = 5a_1$

$= 5 \cdot 10 \Rightarrow 50$ (or) $5 \cdot 5 \cdot 2$

$a_3 = 5a_2$

$= 5 \cdot 5 \cdot 5 \cdot 2$

$a_4 = 5a_3$

$= 5 \cdot 5 \cdot 5 \cdot 5 \cdot 2$

$\boxed{a_n = 5^n \cdot 2}$

2Q) solve the recurrance relation by using substitution method

$$a_n = a_{n-1} + n \quad \text{where } n \geq 1,\ a_0 = 2$$

sol:- $a_1 = a_0 + 1$

$a_1 = 2+1 \Rightarrow 3$

$a_2 = a_{2-1} + 2$

$= a_1 + 2 \Rightarrow 3+2 = 5$

(or)

$= 2+1+2$

$a_3 = a_2 + 3$

$= 2+1+2+3$

$a_4 = a_3 + 4$

$= 2+1+2+3+4$

$\vdots$

$a_n = 2+1+2+3+4+ \cdots + n$

$= a_0 + (1+2+3+4+\cdots+n)$

$= a_0 + \frac{n(n+1)}{2} \Rightarrow 2 + \frac{n(n+1)}{2}$

3Q) solve the recurrance relation by using substitution method.. $a_n = a_{n-1} + n^2$, where $a_0 = 7$, $n \geq 1$

sol:-

$a_1 = a_0 + 1^2$

$a_1 = 7+1$

$a_2 = a_1 + 2^2$

$= 7 + 1^2 + 2^2$

$a_3 = a_2 + 3^2$

$= 1^2 + 2^2 + 7 + 3^2$

$\vdots$

$a_n = 7 + 1^2 + 2^2 + 3^2 + \cdots + n^2$

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$$= 7 + \frac{n(n+1)(2n+1)}{6} \;//$$

4Q) solve recurrance relation by using substitution method

$$a_n = a_{n-1} + \frac{1}{n(n+1)}, \quad n \geq 1, \quad a_0 = 1$$

Sol:-

$$a_1 = a_0 + \frac{1}{2}$$

$$a_1 = 1 + \frac{1}{2}$$

$$a_2 = a_1 + \frac{1}{6}$$

$$a_2 = 1 + \frac{1}{2} + \frac{1}{6}$$

$$a_3 = a_2 + \frac{1}{12}$$

$$a_3 = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12}$$

$$\vdots$$

$$a_n = 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots \frac{1}{n(n+1)}$$

(or)

$$= 1 + \frac{1}{2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)}$$

$$= 1 + \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cdots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 + \left(1 - \frac{1}{n+1}\right)$$

$$\boxed{a_n = \frac{2n+1}{n+1}} \;//$$

5Q) solve the recurrance relation by using substitution method

$$4a_r - 5a_{r-1} = 0, \quad r \geq 1, \quad a_0 = 1$$

Sol:- $4a_r = 5a_{r-1}$

$a_r = \frac{5}{4} a_{r-1}$

$a_1 = \frac{5}{4} a_0 \Rightarrow \frac{5}{4} \cdot 1$

$a_2 = \frac{5}{4} \cdot a_1 \Rightarrow \frac{5}{4} \cdot \frac{5}{4} \cdot 1$

$a_3 = \frac{5}{4} \cdot a_2 \Rightarrow \frac{5}{4} \cdot \frac{5}{4} \cdot \frac{5}{4} \cdot 1$

$\vdots$

$a_r = \left(\frac{5}{4}\right)^r \cdot 1$

Q:- solve the recurrence relation by using substitution method

$$a_n = a_{n-1} + 3^n, \quad n \geq 1;\ a_0 = 1$$

Sol:- $a_1 = a_0 + 3^1$

$= 1 + 3$

$a_2 = a_1 + 3^2$

$= 1 + 3 + 3^2$

$a_3 = a_2 + 3^3$

$= 1 + 3 + 3^2 + 3^3$

$\vdots$

$a_n = 1 + 3 + 3^2 + 3^3 + \cdots 3^n$

$= 1 + \sum_{n \geq 1} 3^n$

Q:- If a_n is a solution of $a_{n+1} = k a_n$ for $n \geq 0$, $a_3 = 153/49$, $a_5 = \frac{1377}{2401}$

Sol:-

8Q) solve the recurrance relation by using substitution method.

$$a_r = 2a_{r-1} + 1 , \quad a_1 = 7, \quad r > 1$$

sol:-

$$a_2 = 2a_1 + 1$$
$$= 2 \cdot 7 + 1$$

$$a_3 = 2a_2 + 1$$
$$= 2 \cdot 2 \cdot 7 + 1$$

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9. $a_n = a_{n-1} + n^3 , \quad a_0 = 5 , \quad n \geq 1$

10. $a_n = a_{n-1} + 2n + 1 , \quad a_0 = 1, \quad n \geq 1$

11. $a_n = a_{n-1} + 3n^2 + 3n + 1 , \quad a_0 = 1 , \quad n \geq 1$

12. $a_n = a_{n-1} + n \cdot 3^n , \quad a_0 = 1 , \quad n \geq 1$

2. Generating function

characteristic grouping

This method is somewhat general metho to solve homogeneous, linear, recurrance relati of degree ~~a~~ 'k' for these we requie the defination of characteristics of equation of a homogeneous, linear recurrance relation

let $a_n + c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_n a_{n-k} = 0$

$n \geq k$, $c_k \neq 0$ be a linear recurrance relation of degree k. Then the equation

$$r^k + c_1 r^{k-1} + c_2 r^{k-2} + \dots c_k = 0$$

is said to be the characteristic equation of the given recurrance relation

→ The characteristic eqn of the recurrance relation

Two types of characteristic rules.

Ex:- $a_n - 3a_{n-2} + 2a_{n-2} = 0$

$$r^2 - 3r + 2 = 0$$

$$r = 1, 2$$

1) If the characteristic eqn of a linear homogeneous recurrance relation of degree 'k' has k different ~~states~~ roots say $\alpha_1, \alpha_2, \dots, \alpha_k$

then $\boxed{a_n = c_1 \alpha^n + c_2 \alpha_2^{n} + c_3 \alpha_3^n + \dots c_k \alpha_k^n}$

2) If the characteristic eqn of a linear, homogeneous recurrance relation of degree k' has K same roots.

α repeatdly k times then

$$\boxed{a_n = (D_1 + D_2 n + D_3 n^2 + \cdots\cdots)\alpha^n}$$

where $D_1, D_2, \ldots D_k$ are constants, is the general solution of the given recurrance relation

problems.

1) $a_n - 3a_{n-1} - 4a_{n-2} = 0$, $n \geq 2$ using characteristic roots.

sol:-

$$r^2 - 3r - 4 = 0$$

$$r^2 - 4r + r - 4 = 0$$

$$r(r-4) + (r-4) = 0$$

$$r = 4, -1$$

$$a_n = c_1(-1)^n + c_2(4)^n$$

2) $a_n - 7a_{n-1} + 12a_{n-2} = 0$, $n \geq 2$

sol:-

$$r^2 - 7r + 12 = 0$$

$$r^2 - 4r - 3r + 12 = 0$$

$$r = 4, 3$$

$$a_n = c_1(4)^n + c_2(3)^n$$

3) $a_n - 6a_{n-1} + 9a_{n-2} = 0$

$$r^2 - 6r + 9 = 0$$

$$r^2 - 3r - 3r + 9 = 0$$

$$r = 3, 3$$

$$a_n = (D_1 + D_2 n)3^n$$

$a_n - 7a_{n-1} + 10a_{n-2} = 0$ where $n \geq 2$, $a_0 = 10$, $a_1 = 41$

$r^2 - 7r + 10 = 0$

$r = 5, 2$

$a_n = c_1(2)^n + c_2(5)^n$

$n=0$ $a_0 = c_1 + c_2$

$10 = c_1 + c_2 \Rightarrow c_1 = 10 - c_2$

$n=1$ $a_1 = c_1(2) + c_2(5)$

$41 = c_1(2) + c_2(5)$

$41 = (10 - c_2)2 + 5c_2$

$41 = 20 - 2c_2 + 5c_2$

$c_2 = 7$, $c_1 = 3$.

(or)

$20 = 2c_1 + 2c_2 \rightarrow \times 2$

$41 = 2c_1 + c_2(5)$

$21 = 3c_2$

$c_2 = 7$

$c_1 = 3$

$a_n + a_{n-1} - 6a_{n-2} = 0$, $a_0 = -1$, $a_1 = 8$

$r^2 + r - 6 = 0$

$r^2 + 3r - 2r - 6 = 0$

$r = -3, 2$

$a_n = (-3)^n c_1 + c_2(2)^n$

$-1 = c_1 + c_2$

$8 = (-3)^n c_1 + (2)^n c_2$

$-1 = c_1 + c_2$

$8 = c_1(-3)^n + c_2(2)^n \Rightarrow$

$-1 = 2c_1 + 2c_2$

$8 = -3c_1 + 2c_2$

$6 = 5c_1$

$c_1 = -1/6$

$c_1 = 6/5$

$-1 = \frac{6}{5} + c_2$

$a_n = (-2)(-3)^n + 2^n$ $\quad -\frac{9}{6} = 5c_2 \Rightarrow -1 = \frac{6}{5} + c_2 \Rightarrow -1 - \frac{6}{5} = c_2$

$-11/5 = c_2$ //

6) $a_n - 3a_{n-1} + 3a_{n-2} - a_{n-3} = 0$, $r = 1,1,1$

7) $a_n - 9a_{n-1} + 27a_{n-2} - 27a_{n-3} = 0$, $r = 3,3,3$

6 sol:- $r^3 - 3r^2 + 3r - 1 = 0$

$r^3 - 3r^2 + 3r = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & 0 & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$r^2 - r - r + 1$

$(r-1)(r^2 - 2r + 1) = 0$

$\cancel{(r-1)(r(r-2)-1(r-1))} = 0$

$(r-1)(r^2 - r - r + 1) = 0$

$(r-1)(r(r-1) - 1(r-1)) = 0$

$(r-1)(r-1)(r-1) = 0$

$(r-1)^3 = 0$

$r = 1$,,

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(*) solving recurrence relation by using generating characteristi method.

The equivalent expressions for generating function in recurrance relation is given as

$A(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$$A(x) = \sum_{n=0}^{\infty} a_n x^n$$

* 3) Generating function method :-

solve the reccurrance relation $a_n - 2a_{n-1} - 3a_{n-2} = 0$
where $n \geq 2$, $a_0 = 3$, $a_1 = 1$

$a_n - 2a_{n-1} - 3a_{n-2} = 0$

multiply both sides x^n.

$a_n x^n - 2x^n a_{n-1} - 3a_{n-2} x^n = 0$ — homogeneous RR.

$\sum_{n=2}^{\infty} a_n x^n - 2\sum_{n=2}^{\infty} a_{n-1} x^n - 3\sum_{n=2}^{\infty} a_{n-2} x^n = 0$

$\rightarrow a_1 x^2 + a_2 x^3 + \dots$

$\rightarrow a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$

$A(x) - a_0 - a_1 x) - 2x(A(x) - a_0) - 3x^2(A(x)) = 0$

$\because A(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$A(x) - a_0 - a_1 x = a_2 x^2 + \dots$

$A(x) - a_0 - a_1 x - 2x A(x) + 2x a_0 - 3x^2(A(x)) = 0$

$(A(x) - 2x A(x) - 3x^2 A(x)) = a_0 + a_1 x + 2x a_0$

$A(x)(1 - 2x - 3x^2) = 3 + x + 6x$

$$A(x) = \frac{3 - 5x}{1 - 2x - 3x^2}$$

$$A(x) = \frac{3 - 5x}{(1+3x)(1+x)}$$

By using partial fraction.

$$\frac{A}{1+3x} + \frac{B}{1+x}$$

$A(1+x) + B(1+3x)$ ~~=~~ $3 - 5x$

~~$A + Ax + B + 3Bx = a - 3 - 5x$~~

~~$(A + B + x(A+3B) = a\ 3 - 5x$~~

~~$A + B = 3$~~ ~~$A + 3B$~~ $= -5$

~~$B = 3 - A$~~ $A + 6 - 3A = -5$

~~$B = 3 - A$~~ $\quad -2A + 6 = -5$

~~$B = 2$~~ $\quad A = 1$

$$\Rightarrow \frac{1}{1+3x} + \frac{2}{1+x}$$

$1(\cancel{1+x}) + 2(\cancel{1+3x})$

$1(1-3x)^{-1} + 2(1+x)^{-1}$

$$\sum_{n=0}^{\infty} a^n x^n = \sum_{n=0}^{\infty} 3^n x^n + 2\sum_{n=0}^{\infty} (-1)^n x^n$$

Remove x^n on b.s

$$\boxed{a_n = 3^n + 2(-1)^n}$$

2) ~~find the generating~~ solve the recurrance relation

$a_n - 4a_{n-1} + 3a_{n-2} = 0$ where $n \geq 2$, $a_0 = 2$, $a_1 = 4$

Sol:- multiply x^n on b.s

$$a_n x^n + 4a_{n-1} x^n + 3a_{n-2} x^n = 0$$

$$\sum_{n=2}^{\infty} a_n x^n - 4\sum_{n=2}^{\infty} a_{n-1} x^n + 3\sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$\to a_1 x^2 + a_2 x^3 + \dots$

$\to a_2 x^2 + a_3 x^3 + \dots$

$$(A(x) - a_0 - a_1 x) - 4x(A(x) - a_0) + 3x^2 A(x) = 0$$

$$A(x) - a_0 - a_1 x - 4x A(x) + 4x a_0 + 3x^2 A(x) = 0$$

$$(A(x) - 4x A(x) + 3x^2 A(x)) = a_0 + a_1 x + 4x a_0$$

$$A(x)(1 - 4x + 3x^2) = 2 + 4x - 8x$$

$$A(x) = \frac{2-4x}{1-4x+3x^2}$$ $\frac{1-x-3x+3x^2}{1-4x+3x^2}$

$$A(x) = \frac{2-4x}{(1-3x)(1-x)}$$

by using partial fraction

$$\frac{A}{1-3x} + \frac{B}{1-x} =$$

$A+B=2$ $\quad -A-3B=-4$

$A = 4+3B$

$4B=6$

$B = 3/2$

$A = 4+3(\frac{3}{2})$

$A = \frac{8+9}{2}$

$A = 17/2$

$A(1-x) + B(1-3x) = 2-4x$

$A - Ax + B - 3xB = 2-4x$

$\cancel{(A+B) - (Ax+3Bx) = 2-4x}$

$\cancel{A+B=2}$ $\quad \cancel{-(A+3B) = -4}$

$\cancel{A=2-B}$ $\quad \cancel{-3B = -4+A}$

$\cancel{B = \frac{-4+2-B}{-3}}$

put $x=1$

$\cancel{B = \frac{-2-B}{-3}} \Rightarrow \frac{2-B}{3}$

$B=1$, $A=1$

$$\frac{1}{1-3x} + \frac{1}{1-x}$$

$$1(1-3x)^{-1} + 1(1-x)^{-1}$$

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} 3^n x^n + \sum_{n=0}^{\infty} (1)^n x^n$$

remove x^n b.s

$a_n = 3^n + (1)^n$

$$\boxed{a_n = 1 + (3)^n}$$

by

(3) $a_n - 7a_{n-1} - 10a_{n-2} = 0, \ n \geq 2, \ a_0 = 10, a_1 = 41$

Sol:- $a_n - 7a_{n-1} - 10a_{n-2} = 0.$

$$a_n x^n - 7a_{n-1} x^n - 10a_{n-2} x^n = 0$$

$$\sum_{n=2}^{\infty} a_n x^n - 7\sum_{n=2}^{\infty} a_{n-2} x^n - 10\sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$(A(x) - a_0 - a_1 x) - 7x(A(x) - a_0) - 10x^2 A(x) = 0$$

$$A(x)(1 - 7x - 10x^2) = a_0 + a_1 x - 7a_0 \cdot$$

$$A(x) = \frac{10 + 41x - 70x}{1 - 7x - 10x^2}$$

$$A(x) = \frac{10 - 29x}{(1-5x)(1-2x)}$$

$1 - 2x - 5x - 10x^2$ 2

$1 - 7x - 10x^2$ 4

$$\frac{A}{(1-5x)} + \frac{B}{(1-2x)}$$

$$A(1-2x) + B(1-5x) = 10 - 29x$$

put $x = +1/2$

$$A(1-1) + B\left(1 - \frac{5}{2}\right) = 10 - \frac{29}{2}$$

$$B\left(1 - \frac{5}{2}\right) = 10 - \frac{29}{2}$$

$$B\left(-\frac{3}{2}\right) = 10 - \frac{29}{2}$$

$$B = \frac{49}{2} \times -\frac{2}{3}$$

B

$a_1 = 41$

$A(1-2x) + B(1-5x) = 10 - 29x$

put $x = 1/2$

$B\left(-\frac{3}{2}\right) = -\frac{9}{2}$

$B = 3$

$A + B = 10$

$A = 7$

0

$\frac{7}{(1-5x)} + \frac{3}{(1-2x)}$

$7(1-5x)^{-1} + 3(1-2x)^{-1} = 0$

$x) = 0$

$\sum_{n=0}^{\infty} a_n x^n = 7 \sum_{n=0}^{\infty} 5^n x^n + 3 \sum_{n=0}^{\infty} 2^n x^n$

$\boxed{a_n = 7(5^n) + 3(2^n)}$

$x - 10x^2$

$1 - 10x^2$

solution

Q solve the recurrence relation $a_n - 6a_{n-1} = 0$ where $n \geq 1$, $a_0 = 1$

Sol:-

$a_n - 6a_{n-1} = 0$

$x^2 - 6x = 0$ multipey x^n b. s.

$x - 6 = 0$ $\quad a_n x^n - 6a_{n-1} x^n = 0$.

$x = 6$. $\quad \sum_{n=1}^{\infty} a_n x^n - 6 \sum_{n=1}^{\infty} a_{n-1} x^n$

$\hookrightarrow a_1 x^1 +$

$a_0 x + a_1 x^2$

$= a_0 + a_1 x + \cdots$

$(A(x) - a_0 - \cancel{a_1}) - 6x(A(x) - a_0)$

$A(x) - \cancel{a_0} - \cancel{a_1 x} - 6x(A(x) - \cancel{a_0}) = 0$

a_0

$A(x)(1-6x) = \cancel{a_0} + \cancel{a_1 x} - 6x\cancel{a_0}$ $6x$

by

$A(x) = \frac{1 - 6x}{1 + 6x} \Rightarrow A(x) = \frac{1}{1-6x}$

$$a_n \quad \frac{1}{1-6x}$$

$$a_n = \frac{1\,(1-6x)^{-1}}{}$$

$$\sum a_n x^n = \sum 6^n x^n$$

$$\boxed{a_n = 6^n}$$

(6) solve recurrance relation $a_n - 7a_{n-1} + 12a_{n-2} = 0,\ n \geq 2$

Sol:-

$$a_n - 7a_{n-1} + 12a_{n-2}$$

$$\sum_{n\geq 2}^{\infty} a_n x^n + 7\sum_{n\geq 2}^{\infty} a_{n-1} x^n + 12 \sum_{n\geq 2}^{\infty} a_{n-2} x^n$$

$$(A(x) - a_0 - a_1 x) - 7x(A(x) - a_0) + 12(A(x))x^2$$

$$A(x) - 7xA(x) + 12x^2 A(x) = a_0 + a_1 x - 7x a_0$$

$$A(x)(1 - 7x + 12x^2) = a_0 + a_1 x - 7x a_0$$

$$A(x) = \frac{a_0 - 7x a_0 + a_1 x}{1 - 7x + 12x^2}$$

$1 - 4x - 3x + 12x^2$

$1 - 7x + 12x^2$

$$A(x) = \frac{a_0(1-7x) + a_1 x}{(1-3x)(1-4x)}$$

$$A(x) = \frac{a_0(1-7x) + a_1 x}{(1-3x)\ (1-4x)}$$

$$a_0(\cancel{1-3x})(1-\cancel{4x}) + \cancel{a_1x(1+23x)} =$$

$$\frac{A}{1-3x} + \frac{B}{1-4x} = \cancel{x}$$

$$a_n = A(1-3x)^{-1} + B(1-4x)^{-1} = \cancel{x}$$

$$\Sigma a_n x^n = A \Sigma 3^n x^n + B \Sigma 4^n x^n$$

$$\boxed{a_n = A3^n + B4^n}$$

$a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0, \ n \ge 3, \ a_0 = 1, a_1 = 1,$

$a_2 = 10$

$$\sum_{n\ge 3}^{\infty} a_n x^n + 9 \sum_{n \ge 3}^{\infty} a_{n-1} x^n + 26 \sum_{n\ge 3}^{\infty} a_{n-2} x^n - 24 \sum_{n \ge 3}^{\infty} a_{n-3} x^n = 0$$

$$(A(x) - a_0 - a_1 x - a_2 x^2) - 9x(A(x) - a_0 - a_1 x) +$$

2

$$26x^2(A(x) - a_0) - 24(A(x))\, x^3 = 0$$

$$A(x) - 9x(A(x)) + 26x^2 A(x) - 24x^3(A(x)) = a_0 + a_1 x +$$

$$a_2 x^2 - 9x a_0 - 9x a_1 x$$

$$+ 26x^2 a_0 \cancel{=0}$$

$$A(x)(1 - 9x + 26x^2 - 24x^3) = 1 + x + 10x^2 - 9x - 9x^2$$

$$+ 26x^2$$

$$A(x) = \cancel{\frac{27x^2 - 8x + 1}{1 - 9x + 26x^2 - 24x^3}}$$

$$= \cancel{\frac{27x^2 - 8x + 1}{(x-2)(x-3)(x-4)}}$$

$$= \cancel{\frac{A}{(x-2)}} + \cancel{\frac{B}{(x-3)}} + \cancel{\frac{C}{(x-4)}}$$

$$= Ax + B$$

$$= \frac{1 - 8x + 27x^2}{1 - 9x + 26x^2 - 24x^3}$$

$$\begin{array}{c|cccc} 2 & 1 & -9 & 26 & -24 \\ & 0 & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$$(x-2)(x^2 - 7x + 12)$$

$$\begin{array}{cccc} a^3 & a^2 & a^1 & a^0 \\ \hline -24 & 26 & -9 & 1 \end{array}$$

by

7) $a_n - 3a_{n-1} - 2 = 0$, $n \geq 1$, $a_0 = 1$

8) $a_n - 9a_{n-1} + 20a_{n-2} = 0$, $n \geq 2$, $a_0 = -3$, $a_1 = -10$

9) $a_n + 3a_{n-1} - 10a_{n-2} = 0$, $a_0 = 1$, $a_1 = 4$.

* Inhomogeneous recurrance relation:

The inhomogeneous recurrance where

$a_0 + c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_n a_{n-k} = f(n)$ for $n \geq k$

$c \neq 0$ and $f(n)$ is some specified function of n

→ The solve of linear inhomogeneous recurrance relation with constant coefficient is the sum of 2 parts. They are homogeneous & particular solution.

A solution which satisfies the recurrance relation with $f(n)$ on the right hand side is called particular solution.

→ The General solution is $\boxed{a_n = a_n^{(h)} + a_n^{(p)}}$

→ The homogeneous solution is denoted by $a_n^{(h)}$ and the particular soln is denoted by $a_n^{(p)}$.

→ we follow the same procedure as in solving homogeneous recurrance relation for determinatin the homogeneous solution

→ There homogeneous no general procedure for determining the particular solution. it depends on the nature

If $f(n)$, to determine the particular solution we use the following rules.

① If $f(n)$ is of the form of a polynomial degree m in n i.e, $b_0 + b_1 n + b_2 n^2 + \dots b_m n^m$ then the particular solution will be the form $\boxed{a_0 + a_1 n} + \dots + a_m n^m$ provides that 1 is not the characteristics roots.

② If $f(n)$ is of the form $(b_0 + b_1 n + b_2 n^2) + \dots b_m n^m) a^n$ then the particular soln is of the form.

$$(a_0 + a_1 n + a_2 n^2 + \dots + a_m n^m) a^n$$

where a is not the characteristics root of recurrance relation.

③ If a is the characteristics root of the multiplicity are where $f(n)$ is of the form $(b_0 + b_1 n + b_2 n^2 + \dots + b_m n^m) a^n$ then

particular soln $n(a_0 + a_1 n + a_2 n^2 + \dots + a_m n^m) a^n$ by

then general solution = homogeneous + particular soln

Q: Solve the inhomogeneous recurrance relation

$$a_n - 9a_{n-1} + 20a_{n-2} = 1$$

Sol: $a_n = a_n^{(h)} + a_n^{(p)}$

$r^2 - 9r + 20 = 0$

$r = 4, 5$

$a_n^{(h)} = c_1(4)^n + c_2(5)^n$

$a_0 - 9a_0 + 20a_0 = 1$

$a_0 = \frac{1}{12}$

$a_n^{(p)} = 1/12$

$a_n = c_1(4^n) + c_2(5^n) + 1/2$

Q) $a_n - a_{n-1} - 6a_{n-2} = -30$ where $a_0 = 20,\ a_9 = -5$

Q) $a_n - 7a_{n-1} + 10a_{n-2} = 6 + 8n$

* $a_n - 7a_{n-1} + 10a_{n-2} = 1 \cdot 4^n$:

Sol: homogeneous

$r^2 - 7r + 10 = 0$

$r = 5, 2$ $a_n = c_1(5)^n + c_2(2)^n$

particular

$a_n^{(p)} = a_0 \cdot 4^n$ — ① Sub above a^n

$(a_0 \cdot 4^n) - 7(a_0 4^{n-1}) + 10(a_0 4^{n-2}) = 4^n$

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$$q_0 4^n - 7q_0 4^{n-1} + 10 q_0 4^{n-2} = 4^n$$

$$q_0 4^n \left(1 - \frac{7 4^{n-1}}{4^n} + \frac{10 4^{n-2}}{4^n}\right) = 4^n$$

$4^n \cdot 4^{-1}$

$$q_0\left(1 - \frac{7}{4} + \frac{10}{16}\right) = 1$$

$$q_0\left(\frac{16-28+10}{16}\right) = 1$$

$$q_0 = \frac{\cancel{16}\ 8}{-\cancel{2}} \Rightarrow \boxed{q_0 = -8}$$

$$a_n = c_1(5)^n + c_2(2)^n - 8 \cdot 4^n$$

$$a_n + 5a_{n-1} + 6a_{n-2} = 42 \cdot 4^n$$

$$r^2 + 5r + 6 = 0$$

$$r = -3, -2$$

$$a_n = c_1(-3)^n + c_2(-2)^n$$

$a_n^{(p)}$ $\quad q_0 = 42 \cdot 4^n$ $\quad$ 42

$(42 \cdot 4^n) + 5 \cdot 42 \cdot 4^n$

$$q_0 4^n + 5q_0 4^{n-1} + 6q_0 4^{n-2} = 42 \cdot 4^n$$

$$q_0 \cdot 4^n \left(1 + \frac{5}{4} + \frac{6}{16}\right) = 42 \cdot 4^n$$

$$q_0\left(\frac{16+20+6}{16}\right) = 42$$

$$q_0 = \frac{16}{\cancel{42}} \times \cancel{42}$$

$$q_0 = 16$$

$$a_n = c_1(-3)^n + c_2(-2)^n + 16 \cdot 4^n$$

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* $a_n + a_{n-1} = 3n \cdot 2^n$

$0 + h_n \cdot a^n$

$(q_0 + q_1 0)\, 2^n.$ $\qquad a_n^{(p)} = (q_0 + q_1 n) 2^n +$

$r^2 + r = 0 \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (q_0 + q_1 n) 2^{n-1} = 3n$

$r = -1, 0$

$= q_0 2^n + q_1 n 2^n + q_0 2^{n-1} +$

$a_n' = c_1(-1)^n + c_2(0)^n \qquad q_1 n 2^{n-1} = 3n \cdot 2^n$

$(q_0 + q_1 n) 2^n + (q_0 + q_1 (n-1)) 2^{n-1} = q_0 2^n ($

$= q_0 2^n (1 + \frac{q_1 n}{2}) + 0$

$a_n^{(p)} = (q_0 + 3n) 2^n + (q_0 + 3n) \frac{2^n}{2} = 3n \cdot 2^n$

$= q_0^n + 3n 2^n + q_0 2^{n-1} + 3n 2^{n-1} = 3n \cdot 2^n$

$= \cancel{q_0 2^n + 3n 2^n}$

$= (q_0 + 3n) 2^n (1 + \frac{1}{2}) = 3n \cdot 2^n$

$= q_0^n + 3n \cdot 2^n (\frac{3}{2}) = 3n \cdot 2^n$

$= q_0 + 3n (\frac{3}{2}) = 3n$

$= q_0 = 3n - 3n(\frac{3}{2})$

$q_0 = 3n(1 - \frac{3}{2})$

$$\boxed{a_n = c_1(-1)^n + (\frac{2}{3} + 2n) 2^n}$$

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$a_n - 3a_{n-1} - 4a_{n-2} = 4^n.$ Rule - 3

$n(q_0 + q_1 n \dots q_n n^n)q^n$

$r^2 - 3r - 4 = 0$

$r^2 - 4r + r - 4 = 0$

$r = 4, -1$

$a_n = c_1(4)^n + c_2(-1)^n.$

$a_n^{(p)} = a_n - 3a_{n-1} - 4a_{n-2} = 1 \cdot 4^n$

$n(q_0 \cdot 4^n)$

$= n(q_0 \cdot 4^n) - 3nq_0 4^{n-1} - 4nq_0 4^{n-2} = 4^n.$

$= 4^n\left((n \cdot q_0) - \frac{3nq_0}{4} - \frac{4nq_0}{16}\right) = \frac{1}{4^n}$

$= n \cdot q_0\left(1 - \frac{3}{4} - \frac{4}{16}\right) = 1$

$= n \cdot q_0\left(\frac{16 - 12 - 4}{16}\right) = 1$

$= nq_0 4^n - 3(n-1)q_0 4^{n-1} - 4(n-2)q_0 4^{n-2} = 4^n$

$= 4^n\left(nq_0 - \frac{3(n-1)q_0}{4} - \frac{4(n-2)q_0}{4^2}\right) = 4^n$

$= 4nq_0 - 3nq_0 + 3q_0 - nq_0 + 2q_0 = 4$

$= 5q_0 = 4 \Rightarrow q_0 = \frac{4}{5}$

$a_n = nq_0 \cdot 4^n \Rightarrow n\left(\frac{4}{5}\right)4^n.$

$c_1(4)^n + c_2(-1)^n + n\left(\frac{4}{5}\right)4^n$ //

Binomial - Theorem :-

any sum of two unlike symbols such as $x+y$ is called a binomial (or) a binomial expression.

The binomial theorem is a ~~pro~~ fromula for a pow of a binomial. If n is a +ve integer then

$$(x+y)^n = \sum_{r=0}^{n} nC_r \, x^{n-r} y^r$$

$$(x+y)^n = nC_0 x^n + nC_1 x^{n-1} y + \cdots + nC_n y^n$$

the expansion of $(x+y)^n$ contains $(n+1)$ terms

→ The sum of powers of x and y in each term is equal to n

→ The $(r+1)^{th}$ term of expansion $(x+y)^n$ is

$nC_r x^{n-r} y^r$ which is called a general term. it is denoted by T_{r+1}

→ The integers $nC_1, nC_2, \ldots nC_n$ are called binomi coefficients of the expansion $(x+y)^n$

→ The expression $(x+y)^n = (y+x)^n$

① Expand $(x+y)^7$

$n = 7 \quad n+1 = 8$

$$7C_0 x^7 + 7C_1 x^6 y + 7C_2 x^5 y^2 + 7C_3 x^4 y^3 + 7C_4 x^3 y^4 + 7C_5 x^2 y^5 + 7C_6 x^1 y^6 + 7C_7 x \, y^7$$

Find the midterm of $\left(2x - \frac{1}{3x}\right)^{10}$ $\quad$ ||||||0|||||

$(x+y)^n = {}^nC_r x^{n-r} y^r$ $\qquad$ $T_{r+1} = 6$

$r = 5$

${}^{10}C_5 (2)^5 \left(-\frac{1}{3}\right)^5$

$\left(x - \frac{3}{y}\right)^9$ $\qquad$ $\rightarrow n+1 = 10$

↳ ||||00||||

5,6

${}^nC_r x^{n-r} y^r$

$T_{r+1} = T_5$, $T_{r+1} = T_6$

$r = 4$ $\qquad$ $r+1 = 5$ $\qquad$ $T_{5+1} = T_6$

$r = 5-1$ $\qquad$ $r = 5$

$r = 4$

$T_5 = {}^9C_4 x^5 \left(\frac{-3}{y}\right)^4$

$T_6 = {}^9C_5 x^4 \left(\frac{-3}{y}\right)^5$

Find the coefficient of $x^9 y^3$ in the expansion of $(2x - 3y)^{12}$

${}^nC_r x^{n-r} y^r$ $\qquad$ $(n+1) =$ ||||||0|||||| (13)

x^{n-r} $\quad$ y^3

$\cancel{{}^{12}C_6}$ $\quad$ ${}^{12}C_3 x^9 y^3$ $\qquad$ x^3 $\quad$ $T_{r+1} = T_7$

$= {}^{12}C_3 (2)^9 (-3)^3$ $\qquad$ y^3 $\quad$ $r = 6$

$x^{n-r} = x^{12-9}$

$n - r = 12 - 9 = x^3$

$r = 3$

Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$

${}^{12}C_r (x^2)^{12-r} \left(\frac{1}{x}\right)^r$

$$^{12}C_r \, x^{24-2r} \cdot x^{-r}$$

$$x^{24-2r-r} = x^0$$

$$24 - 3r = x^0$$

$$r = 8$$

$$\Rightarrow \, ^{12}C_8$$