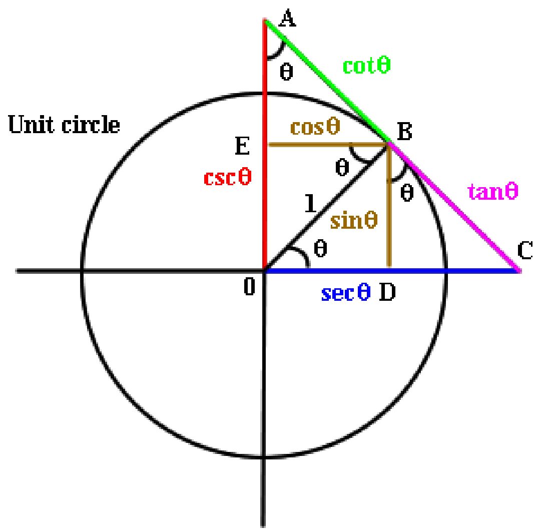
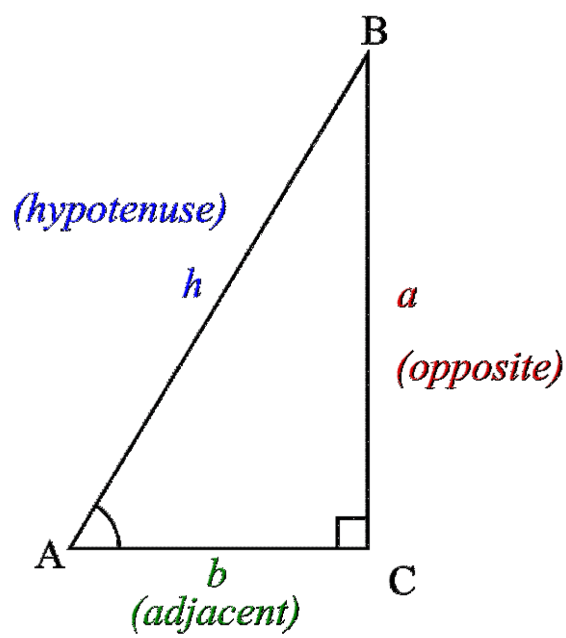


Trigonometry

An Overview of Important Topics

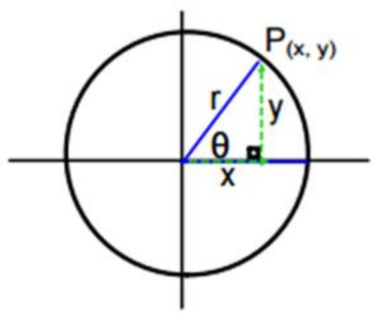


TRIGONOMETRIC FUNCTIONS

Definitions of trig ratios and functions

In Trigonometry there are six trigonometric ratios that relate the angle measures of a right triangle to the length of its sides. (Remember a right triangle contains a 90° angle)

A right triangle can be formed from an initial side *x* and a terminal side *r*, where *r* is the radius and hypotenuse of the right triangle. (see figure below) The Pythagorean Theorem tells us that $x^2 + y^2 = r^2$, therefore r = $\sqrt{x^2 + y^2}$. θ (theta) is used to label a non-right angle. The six trigonometric functions can be used to find the ratio of the side lengths. The six functions are sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot). Below you will see the ratios formed by these functions.



sin θ = $\frac{y}{r}$, also referred to as $\frac{opposite\ side}{hypotenuse}$

cos θ = $\frac{x}{r}$, also referred to as $\frac{adjacent\ side}{hypotenuse}$

tan θ = $\frac{y}{x}$, also referred to as $\frac{opposite\ side}{adjacent\ side}$

These three functions have 3 reciprocal functions

csc θ = $\frac{r}{y}$, which is the reciprocal of sin θ

sec θ = $\frac{r}{x}$,which is the reciprocal of $\cos\theta$

cot θ = $\frac{x}{y}$, which is the reciprocal of $\tan\theta$

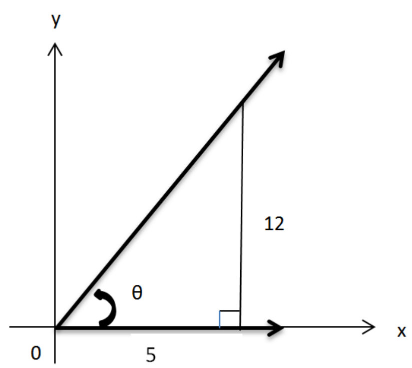
You may recall a little something called SOH-CAH-TOA to help your remember the functions!

SOH… Sine = opposite/hypotenuse

…CAH… Cosine = adjacent/hypotenuse

…TOA Tangent = opposite/adjacent

Example: Find the values of the trigonometric ratios of angle θ



Before we can find the values of the six trig ratios, we need to find the length of the missing side. Any ideas? Good call, we can use r = $\sqrt{x^2+y^2}$ (from the Pythagorean Theorem)

r = $\sqrt{5^2+12^2}$ = $\sqrt{25+144}$ = $\sqrt{169}$ = 13

Now we can find the values of the six trig functions

sin θ = $\frac{opposite}{hyptoenuse}$ = $\frac{12}{13}$

cos θ = $\frac{adjacent}{hypotenuse}$ = $\frac{5}{13}$

tan θ = $\frac{opposite}{adjacent}$ = $\frac{12}{5}$

csc θ = $\frac{hyptoenuse}{opposite}$ = $\frac{13}{12}$

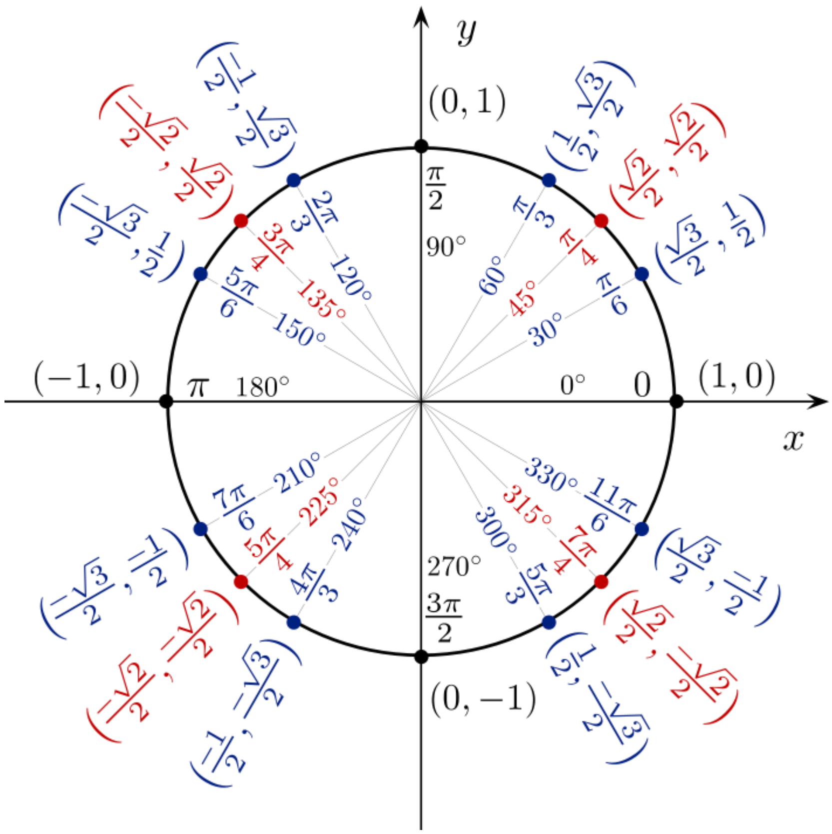
sec θ = $\frac{hypotenuse}{adjacent}$ = $\frac{13}{5}$

cot θ = $\frac{adjacent}{opposite}$ = $\frac{5}{12}$

Find the value of trig functions given an angle measure

Suppose you know the value of θ is 45°, how can this help you find the values of the six trigonometric functions?

First way: You can familiarize yourself with the unit circle we talked about.



An ordered pair along the unit circle (x, y) can also be known as (cos θ, sin θ), since the r value on the unit circle is always 1. So to find the trig function values for 45° you can look on the unit circle and easily see that sin 45° = $\frac{\sqrt{2}}{2}$, cos 45° = $\frac{\sqrt{2}}{2}$

With that information we can easily find the values of the reciprocal functions

csc 45° = $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$, sec 45° = $\sqrt{2}$

We can also find the tangent and cotangent function values using the quotient identities

USING DEFINITIONS AND FUNDAMENTAL IDENTITIES OF TRIG FUNCTIONS

Fundamental Identities

Reciprocal Identities

$\sin\theta = 1/(\csc\theta)$

$\csc\theta = 1/(\sin\theta)$

$\cos\theta = 1/(\sec\theta)$

$\sec\theta = 1/(\cos\theta)$

$\tan\theta = 1/(\cot\theta)$

$\cot\theta = 1/(\tan\theta)$

Quotient Identities

$\tan\theta = (\sin\theta)/(\cos\theta)$

$\cot\theta = (\cos\theta)/(\sin\theta)$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Complementary Angle Theorem

If two acute angles add up to be 90°, they are considered complimentary.

The following are considered cofunctions:

sine and cosine $\quad$ tangent and cotangent $\quad$ secant and cosecant

The complementary angle theorem says that cofunctions of complimentary angles are equal.

Sum and Difference Formulas

In this section we will use formulas that involve the sum or difference of two angles, call the sum and difference formulas.

Sum and difference formulas for sines and cosines

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

How do we use these formulas?

Example Find the exact value of $\cos 105°$

Well we can break 105° into 60° and 45° since those values are relatively easy to find the cosine of.

Therefore $\cos 105° = \cos(60° + 45°)$= $\cos 60° \cos 45° - \sin 60° \sin 45°$

Using the unit circle we obtain,

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{1}{4}(\sqrt{2} - \sqrt{6})$

Example Find the exact value of $\sin 15° =$

$\sin(45° - 30°)$

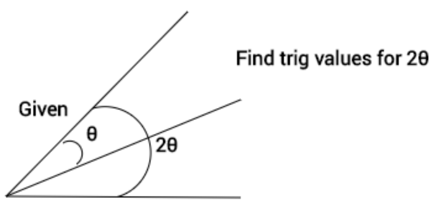
$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

Double and Half Angle Formulas

Below you will learn formulas that allow you to use the relationship between the six trig functions for a particular angle and find the trig values of an angle that is either half or double the original angle.



Double Angle Formulas

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half Angle Formulas

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ $\quad\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$ $\quad\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

Lets see these formulas in action!

Example Use the double angle formula to find the exact value of each expression

$\sin 120°$

$$\sin 120° = \sin 2(60°) = 2\sin 60° \cdot \cos 60° = \frac{\sqrt{3}}{2}$$

Product to Sum Formulas

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

Example Use the product-to-sum formula to change $\sin 75° \sin 15°$ to a sum

$$\sin 75° \sin 15° = \frac{1}{2}[\cos(75° - 15°) - \cos(75° + 15°)] = \frac{1}{2}[\cos 60° - \cos 90°]$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0\right] = \frac{1}{4}$$

Sum to Product Formulas

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Example Use the sum-to-product formula to change $\sin 70° - \sin 30°$ into a product

$$\sin 70° - \sin 30° = 2\cos\left(\frac{70° + 30°}{2}\right)\sin\left(\frac{70° - 30°}{2}\right) = 2\cos 50° \cdot \sin 20°$$