UNIT – II

~~Long~~ Short Answer Questions

1. **Differential operator:**

The part $\frac{d}{dx}$ of the symbol $\frac{dy}{dx}$ may be regarded as an operator such that when it operates on y, the result is the derivative of y.

Similarly, $\frac{d^2}{dx^2}, \frac{d^3}{dx^3}, \ldots, \frac{d^n}{dx^n}$ may be regarded as operators.

Let $D = \frac{d}{dx}, D^2 = \frac{d^2}{dx^2}, \ldots, D^n = \frac{d^n}{dx^n}$

Thus, D is a differential operator.

Inverse operator:

$$\cancel{D^{-1} F(x)} \quad D^{-1}[F(x)] = \frac{1}{D} F(x) = \int F(x)\, dx.$$

If $D\, u(x) = v(x)$ then $u(x) = D^{-1} v(x)$

Here $D^{-1} = \frac{1}{D}$ is known as Inverse operator of D such that $D D^{-1} F(x) = F(x)$ i.e., $D D^{-1} = 1$.

When D is a differential operator, then D^{-1} or $\frac{1}{D}$ represents integral operator.

2. **Complementary Function:**

Consider, Non Homogeneous linear D.E

$$f(D)y = Q(x) \quad \text{—(1)}, \text{ provided } Q(x) \neq 0.$$

$f(D)y = 0$ —(2) is known as corresponding Homogeneous linear d.e

Let $y_1, y_2, y_3, \ldots, y_n$ be n linearly independent solutions of D.E (2), then

$y = c_1 y_1 + c_2 y_2 + c_3 y_3 + \cdots + c_n y_n$

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where $c_1, c_2, c_3, \ldots, c_n$ are arbitrary constants, is also solution of D.E ②. i.e., general sol. of D.E ②.

This solution, $y = c_1 y_1 + c_2 y_2 + c_3 y_3 + \cdots + c_n y_n$ is called complementary function (C.F) of D.E ①.

Particular Integral.

Consider. $f(D)y = Q(x)$ — ①.

For $y = \frac{1}{f(D)} Q(x)$, Equation ① becomes identity.

$\therefore y = \frac{1}{f(D)} Q(x)$ is ~~called~~ a solution of D.E ①.

This solution, $y = \frac{1}{f(D)} Q(x)$ is called particular integral of D.E ② and it does not contain any arbitrary constant.

3. General Linear D.E with constant coefficients:

A general linear differential equation of n^{th} order with constant coefficients is

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_n y = Q(x),$$

where $a_0, a_1, a_2, \ldots, a_n$ are all constants and $Q(x)$ is a function of x only.

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4. Solve the differential equation $(D^2-3D+4)y=0$

$(D^2-3D+4)y=0$

auxilary term

$m^2-3m+4=0$

$m = \frac{3 \pm i\sqrt{7}}{2}$

The roots are complex

$C.F = e^{3/2x}\left[C_1\cos\frac{\sqrt{7}}{2}x + C_2 \sin\frac{\sqrt{7}}{2}x\right]$

General solution

$y = e^{3/2x}\left[C_1\cos\frac{\sqrt{7}}{2}x + C_2\sin\frac{\sqrt{7}}{2}x\right]$ //

5. Solve $y'' + y' - 2y = 0$, $y'(0)=1$, $y(0)=4$

symbolic equation

$(D^2+D-2)y=0$

auxilary term

$m^2+m-2=0$

$m = 1, -2$

The roots are real and distinct

$C.F = C_1e^{x} + C_2e^{-2x}$

The general solution

$y = C_1e^{x} + C_2e^{-2x}$ —①

Give $y(0)=4$

$y=4 \quad x=0$

sub in eqn ①

$C_1+C_2=4$ —②

$y = C_1e^{x} + C_2e^{-2x}$

$y' = C_1e^{x} - 2C_2e^{-2x}$

$y'(0)=1$

$y'=1 \quad x=0$

$1 = +C_1 - 2C_2$ —③

Solve ③ & ②

$C_1 + C_2 = 4$

$+C_1 - 2C_2 = -1$

$3C_2 = 3$

$\boxed{C_2 = 3/3} = 1$

$C_1 + C_2 = 4$

$C_1 = 4 - 1$

$\boxed{C_1 = 3}$ //

$\therefore \boxed{y = 3e^{x} + 1e^{-2x}}$

6. $\frac{d^3x}{dt^3} - x = 0$

symbolic equation

$(D^3-1)x = 0$

auxilary form

$m^3 - 1 = 0$

$m = 1, \frac{-1 \pm \sqrt{3}i}{2}$

The roots are real and complex

$C.F = C_1e^t + e^{-1/2t}[C_2\cos\frac{\sqrt{3}}{2}t + C_3\sin\frac{\sqrt{3}}{2}t]$

The General solution

$x = C_1e^t + e^{-1/2t}[C_2\cos\frac{\sqrt{3}}{2}t + C_3\sin\frac{\sqrt{3}}{2}t]$. 4

7. solve the D.E $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

symbolic equation

$(D^3 - 6D^2 + 11D - 6)y = 0$

auxilary form

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 1, 2, 3$

The roots are real and distinct

$C.F = C_1e^x + C_2e^{2x} + C_3e^{3x}$

The General solution

$y = C_1e^x + C_2e^{2x} + C_3e^{3x}$ 4

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8. solve the D.E $(D^2+4)y=0$

~~symbolic~~ equation auxilary

$$m^2+4=0$$

$$m=\pm 2i$$

The roots are complex

$$C.F = e^{0.x}[c_1\cos 2x + c_2\sin 2x]$$

The General solution

$$y = c_1\cos 2x + c_2\sin 2x$$

9. solve the D.E $(D^2-2D+4)y=0$

auxilary form

$$m^2-2m+4=0$$

$$m = 1\pm i\sqrt{3}$$

The roots are complex

$$C.F = e^{1.x}[c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x]$$

The General solution

$$y = e^{x}[c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x]$$

10. Find $\frac{1}{D+1}(x)$.

$$= \frac{1}{1+D}\cdot x$$

$$P.I = [1+D]^{-1}\, x$$

$$= [1-D]\,x$$

$$= x - Dx$$

$$P.I = x-1.$$

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11. $\frac{1}{(D-1)(D-2)} \cdot e^{2x}$

$\frac{1}{D^2-2D-D+2} \cdot e^{2x}$

$P.I = \frac{1}{D^2-3D+2} \cdot e^{2x}$ put $D = a = 2$

~~P.I =~~ $\frac{1}{2^2-3.2+2} \cdot e^{2x}$

$4-6+2$

$P.I = x.\frac{1}{2D-3} \cdot e^{2x}$

put $D=2$

$= x\frac{1}{1} \cdot e^{2x}$

$P.I = x.e^{2x}$ //

The General ~~solution~~

12. Find P.I of $(D^3+2D^2+D)y = \sin^2 x$

$P.I = \frac{1}{f(D)} \cdot Q(x)$

$\cos 2x = 1-2\sin^2 x$

$2\sin^2 x = 1-\cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$= \frac{1}{D^3+2D^2+D} \cdot \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{2}\left[\frac{1}{D^3+2D^2+D} \cdot 1.e^{0.x} - \frac{1}{D^3+2D^2+D} \cdot \cos 2x\right]$

put $D=0$ put $D^2=-2^2$

$= \frac{1}{2}\left[\frac{x}{3D^2+4D+1} \cdot 1.e^{0.x} - \frac{1}{-4D-8+D}\cos 2x\right]$

$= \frac{1}{2}\left[x + \frac{1}{8+3D} \cdot \cos 2x\right]$

$= \frac{1}{2}\left[x + \frac{8-3D}{(8+3D)(8-3D)} \cdot \cos 2x\right]$

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$$= \frac{1}{2}\left[x + \frac{8-3D}{(64-9D^2)} \cdot \cos 2x\right]$$

put $D^2 = -2^2 = -4$

$$= \frac{1}{2}\left[x + \frac{(8-3D)\cos 2x}{100}\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{100}\left[8\cos 2x - 3D\cos 2x\right]\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{100}\left[8\cos 2x - 3(-2)\sin 2x\right]\right]$$

$$P.I = \frac{1}{2}\left[x + \frac{1}{100}(8\cos 2x + 6\sin 2x)\right]$$ //

13. Find P.I of $\left(\frac{d^3y}{dx^3} + 4\frac{dy}{dx}\right) = \sin 2x$

~~symbotic~~ symbolic form

$$(D^3 + 4D)y = \sin 2x$$

$$P.I = \frac{1}{f(D)} \cdot Q(x)$$

$$= \frac{1}{D^3+4D} \cdot \sin 2x$$

$\cancel{D^2} = -2^2 =$

$$= x\frac{1}{3D^2+4} \cdot \sin 2x$$

put $D^2 = -2^2$

$$= \frac{x\,1}{3(-4)+4} \cdot \sin 2x$$

$$= \frac{x\,1}{-12+4} \sin 2x$$

$$P.I = -\frac{x}{8}\sin 2x$$ //

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14. Find the P.I of $(D^3+2D^2+D)y = e^{2x}$

$$(D^3+2D^2+D)y = e^{2x}$$

$$P.I = \frac{1}{f(D)} \cdot Q(x)$$

$$P.I = \frac{1}{D^3+2D^2+D} \cdot e^{2x}$$

put $D = 2$

$$P.I = \frac{1}{8+8+2} \cdot e^{2x}$$

$$P.I = \frac{1}{18} \cdot e^{2x}$$

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LONG - ANSWER QUESTIONS

1. Explain the procedure to solve higher order linear D.E when $Q(x) = e^{ax}$

Sol procedure to find the P.I when $Q(x) = e^{ax}$ where a is a constant

let $f(D)y = Q(x)$

$$P.I = \frac{1}{f(D)} \cdot Q(x)$$

$$P.I = \frac{1}{f(D)} \cdot e^{ax}$$

$$= \frac{1}{f(D)} \cdot e^{ax}$$

put $D = a$ in $f(D)$

$$P.I = \frac{1}{f(a)} \cdot e^{ax}$$

when $f(a) = 0$

$$P.I = x \cdot \frac{1}{f'(D)} \cdot e^{ax}$$

put $D = a$

$$P.I = x \cdot \frac{1}{f'(a)} \cdot e^{ax}$$

when $f'(a) = 0$

$$P.I = x^2 \cdot \frac{1}{f''(D)} \cdot e^{ax}$$

put $D = a$ $f'(D) \neq 0$

$$P.I = x^2 \cdot \frac{1}{f''(D)} \cdot e^{ax}$$

$$P.I = x^2 \frac{1}{f''(a)} \cdot e^{ax}$$

In General

$$\frac{1}{(D-a)^k} \cdot e^{ax} = \frac{x^k}{k!} \cdot e^{ax}$$

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solve the following differential equations

i) $(4D^2-4D+1)y=100$ — ①

Auxilary form

$4m^2-4m+1=0$

$m=\frac{1}{2},\frac{1}{2}$

the roots are real and repeated

$C.F = (C_1+C_2x)e^{1/2x}$

$P.I = \frac{1}{f(D)} \cdot Q(x)$

$= \frac{1}{4D^2-4D+1} \cdot 100 \cdot e^{0.x}$

$= 100 \cdot \frac{1}{4D^2-4D+1} \cdot e^{0x}$

put $D=0$

$= 100 \cdot \frac{1}{1} \cdot e^{0x}$

$P.I = 100$

General solution

$y = C.F + P.I$

$y = (C_1+C_2x)e^{1/2x} + 100$ //

ii) $(D^3-6D^2+11D-6)y = e^{-2x}+e^{-3x}$

Auxilary form

$m^3-6m^2+11m-6$

$m = 1, 2, 3$

The roots are real and distincts.

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$C.F = c_1e^{x} + c_2e^{2x} + c_3e^{3x}$

$P.I = \frac{1}{f(D)} \cdot Q(x)$

$P.I = \frac{1}{(D^3-6D^2+11D-6)} \cdot [e^{-2x} + e^{-3x}]$

$= \left[\frac{1}{D^3-6D^2+11D-6} \cdot e^{-2x} + \frac{1}{D^3-6D^2+11D-6} \cdot e^{-3x}\right]$

put $D=-2$ put $D=-3$

$= \left[-\frac{1}{60} \cdot e^{-2x} + \frac{1}{(-120)} \cdot e^{-3x}\right]$

$P.I = -\frac{1}{120}\left[2e^{-2x} + e^{-3x}\right]$

The General solution

$y = C.F + P.I$

$y = c_1e^{x} + c_2e^{2x} + c_3e^{3x} - \frac{1}{120}\left[2e^{-2x} + e^{-3x}\right]$

2 Solve the following D.E

i) $y'' - 4y' + 3y = 4e^{3x}, \quad y(0) = -1, \; y'(0) = 3$

symbolic equation

$(D^2-4D+3)y = 4e^{3x}$

auxilary form

$m^2-4m+3 = 0$

$m = 1, 3$

The roots are real and distinct

$C.F = c_1e^{x} + c_2e^{3x}$

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$$P.I = \frac{1}{f(D)} \cdot Q(x)$$

$$P.I = \frac{1}{(D^2-4D+3)} \cdot 4e^{3x}$$

$$= 4 \cdot \frac{1}{(D^2-4D+3)} \cdot e^{3x}$$

~~put~~ D=

$$= 4 \cdot x \frac{1}{2D-4} \cdot e^{3x}$$

= put D=3

$$= \cancel{4}^2 \cdot \frac{x}{\cancel{2}} \cdot e^{3x}$$

$$P.I = 2xe^{3x}$$

General solution

$$y = c_1 e^x + c_2 e^{3x} + 2x \cdot e^{3x} \quad -(1)$$

$$\therefore c_1 = -3, \; c_2 = 2$$

$$y = c_1 e^x + \cancel{y} c_2 e^{3x} + 2x e^{3x} \; -(1)$$

given condition $y(0) = -1$

$y = -1, \quad x = 0.$

Substitute in eqn (1)

$$-1 = c_1 + c_2$$

$$c_1 + c_2 = -1 \quad -(2)$$

$y'(0) = 3 \Rightarrow y' = 3 \quad x = 0.$

$$y' = c_1 e^x + 3c_2 e^{3x} + 6x \cdot e^{3x}$$

$$3 = c_1 + 3c_2 \quad -(3)$$

solve (2) & (3)

$$c_1 + 3c_2 = 3$$

$$c_1 + c_2 = -1$$

$$- \quad - \quad +$$

$$2c_2 = 4$$

$$c_2 = 2$$

$$c_1 + c_2 = -1$$

$$c_1 = -3$$

ii) $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$

Auxilary form

$$m^3 - 5m^2 + 7m - 3 = 0$$

$$m = 3, 1, 1$$

The roots are real and repeated

$$C.F = (c_1 + c_2 x)e^x + c_3 e^{3x}$$

$$P.I = \frac{1}{f(D)} \cdot Q(x).$$

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$$P.I = \frac{1}{D^3-5D^2+7D-3} . e^{2x} \cdot \cosh x$$

$$= \frac{1}{D^3-5D^2+7D-3} . e^{2x}\left[\frac{e^x+e^{-x}}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^3-5D^2+7D-3} . [e^{3x}+e^x]\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^3-5D^2+7D-3} \cdot e^{3x} + \frac{1}{D^3-5D^2+7D-3} . e^x\right]$$

$$= \frac{1}{2}\left[\frac{x}{3D^2-10D+7} . e^{3x} + \frac{x}{3D^2-10D+7} . e^x\right]$$

27 -

3 - 10 +

put $D = 3$

$$= \frac{1}{2}\left[\frac{x}{4} \cdot e^{3x} + \frac{x^2}{6D-10} . e^x\right]$$

$D = 1$

$$= \frac{1}{2}\left[\frac{x}{4} e^{3x} - \frac{x^2}{4} e^x\right]$$

$$P.I = \frac{1}{8}\left[xe^{3x} - x^2e^x\right]$$

The General solution

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x)e^x + c_3 e^{3x} + \frac{1}{8}\left[xe^{3x} - x^2 e^x\right]$$

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3. Explain the procedure to solve higher order linear D.E when $Q(x) = \cos ax$ or $\sin ax$

sol procedure to find the P.I

$f(D)y = Q(x)$ where $Q(x) = \cos ax$ or $\sin ax$

$P.I = \frac{1}{f(D)} \cdot Q(x)$ let $Q(x) = \sin ax$

$P.I = \frac{1}{f(D)} \cdot \sin ax$

we try to get a function of D^2. say that is $\phi(D^2)$ replace the D^2 by $-a^2$. If $(-a^2) \neq 0$ then we use this formula

$\therefore P.I = \frac{1}{f(D)} \sin ax$

$= \frac{1}{\phi(D^2)} \sin ax$ put $D^2 = -a^2$

$= \frac{1}{\phi(-a^2)} \cdot \sin ax$

$P.I = \frac{1}{\phi(-a^2)} \cdot \sin ax$, If $\phi(-a^2) \neq 0$

solve the following D.E

1) $(D^2 - 4) = 2\cos^2 x$

Auxilary form

$m^2 - 4 = 0$

$m = \pm 2$

The roots are real and distinct

$C.F = c_1 e^{2x} + c_2 e^{-2x}$

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$$P.I = \frac{1}{D^2-4} \cdot 2\cos^2 x$$

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = 1 + \cos 2x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$P.I = \frac{1}{D^2-4} \cdot \cancel{2}\left(\frac{1+\cos 2x}{\cancel{2}}\right)$$

$$= \left[\frac{1}{D^2-4} \cdot 1 \cdot e^{0x} + \frac{1}{D^2-4} \cdot \cos 2x\right]$$

put $D = 0$. put $D^2 = -2^2 = -4$

$$= \left[-\frac{1}{4} - \frac{1}{8} \cdot \cos 2x\right]$$

$$P.I = -\frac{1}{8}\left[2 + \cos 2x\right]$$

General solution

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{8}\left[2 + \cos 2x\right]$$

ii) $y'' + 4y' + 4y = 4\cos x + 3\sin x$, $y(0) = 0$, $y'(0) = 0$

symbolic equation

$$(D^2 + 4D + 4)y = 4\cos x + 3\sin x$$

auxilary form

$$m^2 + 4m + 4 = 0$$

$$m = -2, -2$$

The roots are real and repeated

$$C.F = (c_1 + c_2 x)e^{-2x}$$

$$P.I = \frac{1}{D^2+4D+4} \cdot (4\cos x + 3\sin x)$$

$$= \left[\frac{1}{D^2+4D+4} \cdot 4\cos x + \frac{1}{D^2+4D+4} \cdot 3\sin x\right]$$

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$$= \left[4 \cdot \frac{1}{D^2+4D+4} \cdot \cos x + 3 \cdot \frac{1}{D^2+4D+4} \cdot \sin x \right]$$

put $D^2 = -1^2 = -1$

$$=$$

$$= \left[\frac{4}{-1+4D+4} \cdot \cos x + \frac{3}{-1+4D+4} \cdot \sin x \right]$$

$$= \left[\frac{4}{3+4D} \cos x + \frac{3}{3+4D} \sin x \right]$$

$$= \left[\frac{4(3-4D)}{(3+4D)(3-4D)} \cdot \cos x + \frac{3(3-4D)}{(3+4D)(3-4D)} \cdot \sin x \right]$$

$$= \left[\frac{4(3-4D)}{(9-16D^2)} \cos x + \frac{3(3-4D)}{(9-16D^2)} \cdot \sin x \right]$$

put $D^2 = -1^2 = -1$

$$= \left[\frac{4(3-4D)}{25} \cdot \cos x + \frac{3(3-4D)}{25} \cdot \sin x \right]$$

$$= \frac{\cancel{3}4}{25} \left[3\cos x - 4D\cos x \right] + \frac{3}{25} \left[3\sin x - 4D\sin x \right]$$

$$P.I = \frac{4}{25} \left[3\cos x + 4\sin x \right] + \frac{3}{25} \left[3\sin x - 4\cos x \right]$$

General solution

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x)e^{-2x} + \frac{4}{25} \left[3\cos x + 4\sin x \right] + \frac{3}{25} \left[3\sin x - 4\cos x \right] //$$

$$y = (c_1 + c_2 x)e^{-2x} + \frac{1}{25} \left[12\cos x + 16\sin x + 9\sin x - 12\cos x \right]$$

$$y = (c_1 + c_2 x)e^{-2x} + \sin x \quad + \frac{1}{\cancel{25}} \left[\cancel{25} \sin x \right]$$

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3) (ii) $y = (c_1 + c_2 x)e^{-x} + \sin x$

$y(0) = 0, \quad y'(0) = 0$

$y = 0, \quad x = 0$

$0 = (c_1 + c_2 \cdot 0)e^{-0} + 0.$

$c_1 = 0$

$\therefore \boxed{y = xe^{-2x} + \sin x}$

$y = c_1 e^{-x} + c_2 x e^{-x} + \sin x$

$y' = -c_1 e^{-x} + c_2 \{-x \cdot e^{-x} - e^{-x}\} + \cos x$

$y'(0) = 0$

$y = 0 \quad x = 0$

$0 = -c_1 - c_2 + 1$

$c_1 + c_2 = 1$

$c_2 = 1$

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4. Solve the following D.E

i) $(D^2+1) = \sin x \sin 2x$

Auxilary form

$m^2+1 = 0$

$m = \pm i$

The roots are complex and distinct

$C.F = e^{0 \cdot x}[c_1 \cos x + c_2 \sin x]$

$C.F = c_1 \cos x + c_2 \sin x$

$P.I = \frac{1}{D^2+1} \frac{2 \sin x \sin 2x}{2}$

$= \frac{1}{2}\left[\frac{1}{D^2+1} \cos(x-2x) - \cos(x+2x)\right]$

$= \frac{1}{2}\left[\frac{1}{D^2+1} \cos(-x) - \cos(3x)\right]$

$= \frac{1}{2}\left[\frac{1}{D^2+1} \cdot \cos x - \frac{1}{D^2+1} \cdot \cos 3x\right]$

$\cancel{D^2 = -1^2 = -1}$ $\quad D^2 = -3^2 = -9.$

$= \frac{1}{2}\left[\frac{x}{2D} \cdot \cos x + \frac{1}{8} \cos 3x\right]$

$= \frac{1}{2}\left[\frac{x}{2} \overset{\sin x}{\cancel{\cos x}} + \frac{1}{8} \cos 3x\right]$

$P.I = \frac{x}{4} \overset{\sin x}{\cancel{\cos x}} + \frac{1}{16} \cos 3x$

The General solution

$y = c_1 \cos x + c_2 \sin x + \frac{1}{16}\left[4 \overset{\sin x}{\cancel{\cos x}} + \cos 3x\right]$

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4) iii) $(D^3+2D^2+D)y = e^{-x} + \sin 2x$

Auxilary form

$m^3+2m^2+m=0$

$m = -1, -1, -1/3$

The roots are real and repeated

$C.F = (C_1+C_2x)e^{-x} + C_3 e^{-1/3x}$

$P.I = \left[\frac{1}{D^3+2D^2+D} e^{-x} + \frac{1}{D^3+2D^2+D} . \sin 2x\right]$

$D^2 = -2^2 = -4.$

$= \left[\frac{x}{3D^2+4D+1} . e^{-x} + \frac{1}{-4D-8+D} . \sin 2x\right]$

$= \left[\frac{x^2}{6D+4} . e^{-x} - \frac{1\,(8-3D)}{(8+3D)(8-3D)} . \sin 2x\right]$

$= \left[\frac{x^2}{-2} - \frac{(8-3D)}{(64-9D^2)} . \sin 2x\right]$

$D^2 = -2^2 = -4$

$= \left[-\frac{x^2}{2} - \frac{(8-3D)}{100} . \sin 2x\right]$

$= \left[-\frac{x^2}{2} - \frac{1}{100}(8\sin 2x - 3D\sin 2x)\right]$

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$= \left[-\frac{x^2}{2} - \frac{1}{100}(8\sin 2x - \frac{8}{2}6\cos 2x)\right]$

$P.I = -\left[\frac{x^2}{2} + \frac{1}{50}(4\sin 2x - 3\cos 2x)\right]$

General solution

$C.F = (C_1 + C_2 x)e^{-x} + C_3 e^{-1/3 x} - \left[\frac{x^2}{2} + \frac{1}{50}(4\sin 2x - 3\cos 2x)\right]$

5. Explain the procedure to solve higher order D.E when $Q(x) = x^n$

Sol let $f(D) \cdot y = Q(x)$

where $Q(x) = x^n$

let $P.I = \frac{1}{f(D)} \cdot Q(x)$

$P.I = \frac{1}{f(D)} \cdot x^m$

Take the lowest degree of term common from $f(D)$ so as it reduse to this term 1 $[1 \pm Q(D)]^n$ and now take it into numerator. we get $[1 \pm Q(D)]^{-n}$ then which can be expanded with the help of binomial expansion

5. i) solve the following D.E

$(D^2 + D + 1)y = x^3$

Auxilary form

$m^2 + m + 1 = 0$

$m = \frac{-1 \pm \sqrt{3}i}{2}$

The roots are complex

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$$C.F = e^{-\frac{1}{2}x}\left[c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right]$$

$$P.I = \frac{1}{f(D)} \cdot Q(x)$$

$$= \frac{1}{D^2+D+1} \cdot x^3$$

$$= \frac{1}{[1+(D+D^2)]} \cdot x^3$$

$$= [1+\underbrace{(D+D^2)}_{x}]^{-1} \cdot x^3$$

$$\therefore [1+x]^{-1} = 1 - x + x^2 - x^3 + x^4 - \cdots$$

$$= \left[1 - (D+D^2) + (D+D^2)^2 - (D+D^2)^3\right] \cdot x^3$$

$$= [1 - D - \cancel{D^2} + \cancel{D^2} + D^4 + 2D^3 - D^3 - D^6 - 3D^4 - 3D^5] \cdot x^3$$

$$= [1 - D + D^3] \cdot x^3$$ neglecting higher powers

$$= [x^3 - Dx^3 + D^3x^3]$$

$Dx^3 = 3x^2$
$3D^2x^2 = 6x$
6

$$= [x^3 - 3x^2 + 6]$$

$$P.I = [x^3 - 3x^2 + 6]$$

The General solution

$$y = P.I + C.F$$

$$y = e^{-1/2x}\left[c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right] + [x^3 - 3x^2 + 6]$$

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5(ii) $y''' + 2y'' - y' - 2y = 1 - 4x^3$

symbolic form

$(D^3 + 2D^2 - D - 2)y = 1 - 4x^3$

Auxiliary form

$m^3 + 2m^2 - m - 2 = 0$

$m = 1, -1, -2$

The roots are real and distinct

$C.F = c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x}$

$$P.I = \frac{1}{(D^3 + 2D^2 - D - 2)} \cdot (1 - 4x^3)$$

$$= \left[\frac{1}{D^3 + 2D^2 - D - 2} \cdot 1 \cdot e^{0x} - \frac{4}{D^3 + 2D^2 - D - 2} \cdot x^3 \right]$$

$$= -\left[\frac{1}{2} + 4 \cdot \frac{1}{(-2)\left[1 - \frac{(D^3 + 2D^2 - D)}{2}\right]} \cdot x^3\right]$$

$$= -\left[\frac{1}{2} - 2 \cdot \left[1 - \frac{(D^3 + 2D^2 - D)}{2}\right]^{-1} \cdot x^3\right] \qquad [1 - x]^{-1} = 1 + x + x^2 + x^3 + x^4 \ldots$$

$$= -\frac{1}{2} + 2\left[1 + \left(\frac{D^3 + 2D^2 - D}{2}\right) + \frac{(D^3 + 2D - D)^2}{4} + \frac{(D^3 + 2D - D)^3}{8}\right] x^3$$

$$= -\frac{1}{2} + 2\left[1 + \frac{D^3 + 2D^2 - D}{2} + \frac{(D^6 + 4D^4 + D^2 + 4D^4 - 4D^2 - 2D^4)}{4} + \frac{D^9 + 2D^3 - D^3 + 3(D^3 + 2D)(2D - D)(D + D)}{8}\right] x^3$$

$$= -\frac{1}{2} + 2\left[1 + \frac{D^3 + 2D^2 - D}{2} + \frac{D^2}{4} + D^3\right] \cdot x^3$$

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$= -\frac{1}{2} + \frac{2}{4}\left[4 + 2D^3 + 4D^2 - 2D + D^2 + 4D^3\right]x^3$

$= -\frac{1}{2} + \frac{1}{2}\left[4x^3 + 6D^3 + 3D^2 - 2D\right]\cdot x^3$

$3x^2$

$6x$

$= -\frac{1}{2} + \frac{1}{2}\left[4x^3 - 2Dx^3 + 3D^2x^3 + 6D^3\cdot x^3\right]$

$= -\frac{1}{2} + \frac{1}{2}\left[4x^3 - 6x^2 + 18x + 36\right]$

$= -\frac{1}{2} + \frac{2}{2}\left[2x^3 - 3x^2 + 9x + 18\right]$

The General solution

$y = C_1e^{x} + C_2e^{-x} + C_3e^{-2x} - \frac{1}{2} + 2x^3 - 3x^2 + 9x + 18$

(3)(iii) $(D^2 - 6D + 25)y = e^{2x} + \sin x + x$

Auxilary ~~symbolic~~ form

$m^2 - 6m + 25 = 0.$

$m = 3 \pm 4i$

The roots are complex

$C.F = e^{3x}\left[C_1\cos 4x + C_2\sin 4x\right]$

$P.I = \frac{1}{D^2 - 6D + 25}\left[e^{2x} + \sin x + x\right]$

$P.I = \left[\frac{1}{D^2 - 6D + 25}e^{2x} + \frac{1}{D^2 - 6D + 25}\cdot\sin x + \frac{1}{D^2 - 6D + 25}\cdot x\right]$

$y_{p_1} = \frac{1}{D^2 - 6D + 25}\cdot e^{2x}$

$4 - 12 + 25$

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$\frac{-12}{17}$

put $D = 2$

$y_{p_1} = \frac{1}{17}\cdot e^{2x}$ — (1)

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$$y_{P_2} = \frac{1}{D^2-6D+25} \cdot \sin x$$

put $D^2 = -1^2 = -1$

$$= \frac{1}{-1+6D+25} \sin x = \frac{1}{24+6D} \cdot \sin x$$

$$= \frac{(24-6D)}{(24+6D)(24-6D)} \cdot \sin x$$

$$= \frac{(24-6D)}{(24^2-36D^2)} \cdot \sin x$$

put $D^2 = -1^2 = -1$

$$= \frac{(24-6D)}{612} \cancel{*} \sin x$$

$$= \frac{1}{612}\left[24\sin x - 6D\sin x\right]$$

$$y_{P_2} = \frac{1}{612}\left[24\sin x - 6\cos x\right] \quad \text{--- (2)}$$

$$y_{P_3} = \frac{1}{D^2-6D+25} \cdot x$$

$$= \frac{1}{25\left[1+\frac{D^2-6D}{25}\right]} \cdot x$$

$$= \frac{1}{25}\left[1+\frac{D^2-6D}{25}\right]^{-1} \cdot x$$

$$= \frac{1}{25}\left[1 - \frac{D^2-6D}{25}\right] \cdot x$$

$$= \frac{1}{25}\left[x - \frac{1}{25}\left(D^2x - 6Dx\right)\right]$$

$$= \frac{1}{25}\left[x - \frac{1}{25}[-6]\right]$$

$$= \frac{1}{25}\left[x + \frac{6}{25}\right] \quad \text{--- (3)}$$

$$P.I = \frac{1}{17}e^{2x} + \frac{1}{612}\left[24\sin x - 6\cos x\right] + \frac{1}{25}\left[x + \frac{6}{25}\right]$$

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The General solution

$$y = e^{3x}[c_1\cos 4x + c_2\sin 4x] + \frac{1}{17}e^{2x} + \frac{1}{612}[24\sin x - 6\cos 3x]$$
$$+ \frac{1}{25}\left[x + 6/25\right]$$

6. Explain the procedure to solve higher order linear D.E $Q(x) = e^{x}.V(x)$.

Let $f(D)y = Q(x)$
where $Q(x) = e^{x}.V(x)$.

$$P.I = \frac{1}{f(D)}.Q(x)$$

$$P.I = \frac{1}{f(D)}.e^{x}.V(x)$$

put $D = D+a$ in $f(D)$

$$P.I = e^{ax}.\frac{1}{f(D+a)}.V(x)$$

where $\frac{1}{f(D+a)}.V(x)$ can be evaluated using previous methods

i) solve the following D.E

$(D^2+2)y = e^{x}\cos x$

sol auxilary form

$$m^2+2=0$$
$$m = \pm\sqrt{2}i$$

The roots are complex

$$C.F = e^{0.x}[c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x]$$

$$C.F = c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x.$$

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$$P.I = \frac{1}{D^2+2} \cdot e^x \cdot \cos x$$

put $D = D+1$

$$= e^x \frac{1}{(D+1)^2+2} \cdot \cos x$$

$$= e^x \frac{1}{D^2+2D+3} \cdot \cos x$$

put $D^2 = -1^2 = -1$

$$= e^x \frac{1}{-1+2D+3} \cos x = \frac{e^x}{2+2D} \cdot \cos x$$

$$= e^x \frac{(2-2D)}{(2+2D)(2-2D)} \cdot \cos x$$

$$= e^x \frac{(2-2D)\cos x}{(4-4D^2)}$$

put $D^2 = -1^2 = -1$

$$= \frac{e^x}{8}\left[2\cos x - 2D\cos x\right]$$

$$P.I = \frac{e^x}{4}\left[\cos x + \sin x\right]$$

The General solution

$$y = C.F + P.I$$

$$y = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x + \frac{e^x}{4}\left[\cos x + \sin x\right]$$

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11) $(D^3-7D^2+14D-8)y = e^x \cos 2x$

Auxiliary form

$m^3-7m^2+14m-8=0$.

$m = 4, 2, 1$

The roots are real and distinct

$C.F = C_1e^x + C_2e^{2x} + C_3e^{4x}$

$$P.I = \frac{1}{D^3-7D^2+14D-8} e^x . \cos 2x$$

put $D = D+1$

$$= e^x . \frac{1}{(D+1)^3-7(D+1)^2+14(D+1)-8} . \cos 2x$$

$$= e^x \left[\frac{1}{D^3+\cancel{1}+3D^2+3D-7D^2-\cancel{7}-\cancel{14D}+\cancel{14D}+\cancel{14}-\cancel{8}} . \cos 2x \right]$$

$$= e^x \left[\frac{1}{D^3-4D^2+3D\ \cancel{+8}} . \cos 2x \right]$$

put $D^2 = -2^2 = -4$

$$= e^x \left[\frac{1}{-4D+16+3D} . \cos 2x \right] = e^x \left[\frac{1}{16-D} . \cos 2x \right]$$

$$= e^x \left[\frac{16+D}{(16-D)(16+D)} . \cos 2x \right]$$

$$= e^x \left[\frac{(16+D)\cos 2x}{(16^2-D^2)} \right] = e^x \left[\frac{(16+D)\cos 2x}{260} \right]$$

$D^2 = -2^2 = -4$

$$= \frac{e^x}{260} \left[16\cos 2x + D\cos 2x \right]$$

$$P.I = \frac{e^x}{260} \left[16\cos 2x + \frac{1}{2}\sin 2x \right]$$

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General solution

$$y = C_1 e^{x} + C_2 e^{2x} + C_3 e^{4x} + \frac{e^{x}}{260}\left[16\cos 2x + \frac{1}{2}\sin 2x\right]$$

iii) $(D^2-1)y = x\sin x + e^{x} + e^{x}.x^2$

Auxilary form

$m^2-1=0$

$m = \pm 1$

The roots are real and distinct

$C.F = e^{0.x}\left[C_1\cos x + C_2\sin x\right]$

$$P.I = \frac{1}{D^2-1}.\left[x\sin x + e^{x} + e^{x}.x^2\right]$$

$$= \left[\frac{1}{D^2-1}.x\sin x + \frac{1}{D^2-1}e^{x} + \frac{1}{D^2-1}.e^{x}.x^2\right]$$

$$y_{P_1} = \frac{1}{D^2-1}.x\sin x$$

$$y_{P_1} = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)}.~~\sin x~~ \, Q(x)$$

$$= \left[x - \frac{2D}{D^2-1}\right]\frac{1}{D^2-1}.\sin x$$

put $D^2 = -1^2 = -1$

$$= \left[x - \frac{2D}{D^2-1}\right]\frac{1}{(-2)}\sin x$$

$$= \left[-\frac{x\sin x}{2} + \frac{D}{D^2-1}.\sin x\right]$$

$$= \left[-\frac{x}{2}\sin x + D\left[-\frac{1}{2}\sin x\right]\right]$$

$$y_{P_1} = -\frac{1}{2}\left[x\sin x + \cos x\right] \quad - (1)$$

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$$y_{P_2} = \frac{1}{D^2-1} \cdot e^x$$

$$= \frac{x}{2D} \cdot e^x$$

put $D=1$

$$y_{P_2} = \frac{x}{2} \cdot e^x \quad \text{— (2)}$$

$$y_{P_3} = \frac{1}{D^2-1} \cdot e^x \cdot x^2$$

put $D = D+1$

$$= e^x \frac{1}{(D+1)^2-1} \cdot x^2$$

$$= e^x \left[\frac{1}{D^2+\cancel{1}+2D\cancel{-1}} \cdot x^2 \right]$$

$$= e^x \left[\frac{1}{2D\left[1+\frac{D}{2}\right]} \cdot x^2 \right]$$

$$= e^x \frac{1}{2D} \left[1+\frac{D}{2}\right]^{-1} x^2$$

$$= e^x \frac{1}{2D} \left[1 - \frac{D}{2} + \frac{D^2}{4} - \frac{D^3}{8}\right] \cdot x^2$$

$$= e^x \left[\frac{1}{2D} - \frac{1}{4} + \frac{D}{8} - \frac{D^2}{16} \right] \cdot x^2$$

$$= e^x \left[\frac{1}{2D} \cdot x^2 - \frac{1}{4}x^2 + \frac{D}{8} \cdot x^2 - \frac{D^2}{16} \cdot x^2 \right] \qquad x^2 = 2x$$

$$= e^x \left[\frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{4}x^2 + \frac{2x}{8} - \frac{2}{16} \right]$$

$$= e^x \left[\cancel{\frac{x^2}{4}} - \cancel{\frac{1}{4}x^2} + \frac{x}{4} - \frac{1}{8} \right] = e^x \left[\frac{2x-1}{8} \right] \text{— (3)}$$

$$P.I = -\frac{1}{2}\left[x\sin x + \cos x\right] + \frac{x}{2}e^x + e^x\left(\frac{2x-1}{8}\right)$$

General solution

$$y = c_1 \cos x + c_2 \sin x - \frac{1}{2}(x\sin x + \cos x) + \frac{x}{2}e^x + e^x\left(\frac{2x-1}{8}\right)$$

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7. Explain the procedure to solve higher order L.D.E when $Q(x) = x^n V(x)$

Sol

let $f(D)y = Q(x)$

where $Q(x) = x \cdot V(x)$

$$P.I = \frac{1}{f(D)} \cdot x \cdot V(x)$$

$$P.I = \left[x - \frac{f'(D)}{f(D)}\right] \frac{1}{f(D)} \cdot V(x)$$

i) $(D^2-2D+1)y = x^2 e^{3x} - \sin 2x + 3$

Auxilary form

$m^2-2m+1 = 0$

$m = 1,1$

The roots real and repeated

$C.F = (C_1 + C_2 x)e^x$

$$P.I = \frac{1}{D^2-2D+1} [x^2 e^{3x} - \sin 2x + 3]$$

$$y_p = \left[\frac{1}{D^2-2D+1} \cdot x^2 e^{3x} - \frac{1}{D^2-2D+1} \sin 2x + \frac{1}{D^2-2D+1} \cdot 3 \cdot e^{0x}\right]$$

$$y_{p_1} = \frac{1}{D^2-2D+1} x^2 \cdot e^{3x}$$

put $D = D+3$

$$= e^{3x} \frac{1}{(D+3)^2 - 2(D+3)+1} x^2$$

$$= e^{3x} \frac{1}{D^2+9+6D-2D-6+1} \cdot x^2$$

$$= e^{3x} \frac{1}{D^2+4D+4} \cdot x^2$$

2m - -

$$= e^{3x} \frac{1}{(D+2)^2} \cdot x^2$$

$$= \frac{e^{3x}}{4\left(1+\frac{D}{2}\right)^2} \cdot x^2$$

$$= \frac{e^{3x}}{4} \left[1 + D/2\right]^{-2} \cdot x^2$$

$$= \frac{e^{3x}}{4} \left[1 - \cancel{2}\frac{D}{\cancel{2}} + 3\frac{D^2}{4}\right] \cdot x^2$$

$$= \frac{e^{3x}}{4} \left[1 - D + \frac{3}{4} \cdot D^2\right] x^2$$

$$= \frac{e^{3x}}{4} \left[x - 2x + \frac{3}{\cancel{4}2} \cdot x\right]$$

$$y_{P_1} = \frac{e^{3x}}{8} \left[\underbrace{2x - 4x}_{-2x} + 3\right] \quad \text{—} \; ①$$

$$y_{P_2} = -\frac{1}{D^2 - 2D + 1} \cdot \sin 2x$$

put $D^2 = -2^2 = -4$

$$= -\frac{1}{-4 - 2D + 1} \sin 2x$$

$$= \frac{1}{3 + 2D} \sin 2x$$

$$= \frac{(3-2D)}{(3+2D)(3-2D)} \cdot \sin 2x$$

$$= \frac{(3-2D)}{(9-4D^2)} \cdot \sin 2x \quad \text{put } D^2 = -2^2 = -4$$

$$= \frac{(3-2D)\sin 2x}{25}$$

$$= \frac{1}{25}\left[3\sin 2x - 2D\sin 2x\right]$$

$$y_{P_2} = \frac{1}{25}\left[3\sin 2x - 4\cos 2x\right] \quad \text{—} \; ②$$

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$$y_{P_3} = \frac{1}{D^2-2D+1} \cdot 3 \cdot e^{0x}$$

put $D=0$

$$y_{P_3} = 3 \quad - (3)$$

$$y_P = y_{P_1} + y_{P_2} + y_{P_3}$$

$$y_P = \frac{e^{3x}}{8}[3-2x] + \frac{1}{25}[3\sin 2x - 4\cos 2x] + 3$$

Generel solution

$$C.F = (c_1 + c_2 x)e^{x} + \frac{e^{3x}}{8}[3-2x] + \frac{1}{25}[3\sin 2x - 4\cos 2x] + 3.$$

ii) $(D^2+4)y = x\sin x$

Auxilary form

$$m^2 + 4 = 0$$

$$m = \pm 2i$$

The roots are complex

$$C.F = e^{0.x}[c_1 \cos 2x + c_2 \sin 2x]$$

$$P.I = \frac{1}{f(D)} \cdot Q(x)$$

$$= \frac{1}{D^2+4} \cdot x\sin x = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)} Q(x)$$

$$= \left[x - \frac{2D}{D^2+4}\right]\frac{1}{D^2+4} \cdot \sin x$$

put $D^2 = -1^2 = -1$

$$= \left[x - \frac{2D}{D^2+4}\right]\left(\frac{1}{3}\right)\sin x$$

$$= \frac{1}{3}\left[x\sin x - 2D\left[\frac{\sin x}{D^2+4}\right]\right]$$

$$= \frac{1}{3}\left[x\sin x - 2D\left(\frac{1}{3} \cdot \sin x\right)\right]$$

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$= \frac{1}{3}\left[x\sin x - \frac{2}{3}\cos x\right]$

General solution

$C.F = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{3}\left[x\sin x - \frac{2}{3}\cos x\right]$ //

(iii) $(D^2 - 2D + 1)y = xe^x \sin x$

Auxiliary form

$m^2 - 2m + 1 = 0$

$m = 1, 1$

The roots are real and repeated

$C.F = (c_1 + c_2 x)e^x$

$P.I = \frac{1}{D^2 - 2D + 1} \cdot e^x \cdot x\sin x$

$\cancel{D = D+1}$

$= e^x \frac{1}{(D-1)^2} \cdot x\sin x$

put $D = D+1$

$= e^x \frac{1}{(D+1-1)^2} \cdot x\sin x$

$= e^x \frac{1}{D^2} x\sin x$

$= e^x \left[x - \frac{f'(D)}{f(D)}\right] \frac{1}{f(D)} \cdot \sin x\right]$

$= e^x \left[x - \frac{2D}{D^2}\right] \frac{1}{D^2} \sin x\right]$

$= e^x \left[x - \frac{2}{D}\right] (-\sin x))$

$= e^x[-x\sin x - 2\cos x]$

General solution $y = (c_1 + c_2 x)e^x + e^x[-x\sin x - 2\cos x]$

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8. Explain the method of variation of parameters to solve higher order L.D.E and solve the D.E

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

symbolic form

$$(D^2+4)y = \tan 2x$$

Auxilary form

$$m^2+4=0$$

$$m = \pm 2i$$

The roots are complex

$$C.F = e^{0x}[c_1 \cos 2x + c_2 \sin 2x]$$

$$= c_1 \cos 2x + c_2 \sin 2x$$

let $u(x) = \cos 2x$, $v(x) = \sin 2x$

let $y_p = Au(x) + Bv(x)$

$\therefore y_p = A\cos 2x + B\sin 2x$

where A, B are function of x

$$w = \begin{vmatrix} u(x) & v(x) \\ u'(x) & v'(x) \end{vmatrix}$$

$\cancel{\cos 2x = 1-\sin^2 2x}$

$\cancel{\sin^2 2x = \frac{1-\cos 2x}{2}}$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$\sin^2 2x = 1-\cos^2 2x$

$$= 2\cos^2 2x + 2\sin^2 2x = 2$$

$$A = -\int \frac{v(x) \cdot Q(x)}{w} dx = -\int \frac{\sin 2x \cdot \tan 2x}{2} dx$$

$$= -\frac{1}{2}\int \sin 2x \cdot \frac{\sin 2x}{\cos 2x} dx = -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x} dx$$

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$$= -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x}\,dx$$

$$= -\frac{1}{2}\int\left(\frac{1}{\cos 2x} - \frac{\cos^{2} 2x}{\cos 2x}\right)dx$$

$$= -\frac{1}{2}\int\left[(\sec 2x) - \cos 2x\right]dx$$

$$= -\frac{1}{2}\left[\frac{1}{2}\log(\sec 2x + \tan 2x) - \frac{1}{2}\sin 2x\right]$$

$$A = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right]$$

$$B = \int \frac{u(x)\cdot a(x)}{w}\,dx = \int \frac{\cos 2x\cdot\tan 2x}{2}\,dx$$

$$= \frac{1}{2}\int \cancel{\cos 2x}\cdot\frac{\sin 2x}{\cancel{\cos 2x}}\,dx = \frac{1}{2}\int \sin 2x\,dx$$

$$B = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cos 2x = -\frac{1}{4}\cos 2x$$

$$y_p = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right]\cos 2x - \frac{1}{4}\cos 2x\cdot\sin 2x$$

$$y_p = -\frac{1}{4}\log(\sec 2x + \tan 2x)\cos 2x + \cancel{\frac{1}{4}\sin 2x\cos 2x} - \cancel{\frac{1}{4}\cos 2x\sin 2x}$$

$$y_p = -\frac{1}{4}\log(\sec 2x + \tan 2x)\cos 2x$$

General solution

$$y = y_c + y_p$$

$$y = c_1\cos 2x + c_2\sin 2x - \frac{1}{4}\log(\sec 2x + \tan 2x)\cdot\cos 2x$$

$$y = c_1\cos 2x + c_2\sin 2x - \frac{1}{4}\log(\sec 2x + \tan 2x)\cdot\cos 2x$$

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9. $(D^2+1)y = \operatorname{cosec} x$

Auxilary form

$m^2+1=0$

$m = \pm i$

The roots are complex

$C.F = e^{0.x}[c_1\cos x + c_2\sin x]$

$C.F = c_1\cos x + c_2 \sin x$

$u(x) = \cos x, \quad v(x) = \sin x.$

$y_p = A\,u(x) + B\,v(x)$

$\therefore y_p = A\cos x + B\sin x$

where A, B are unknown functions of x

$$w = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$w = \cos^2 x + \sin^2 x = 1$

$$A = -\int \frac{v(x)\cdot Q(x)}{w}dx = -\int \frac{\sin x \cdot \operatorname{cosec} x}{1} dx$$

$$= -\int \sin x \cdot \frac{1}{\sin x}dx = -\int dx$$

$A = -x$

$$B = \int \frac{u(x)\cdot Q(x)}{w}dx = \int \frac{\cos x \cdot \operatorname{cosec} x}{1}dx$$

$$= \int \cos x \cdot \frac{1}{\sin x}dx = \int \cot x\, dx$$

$B = \log(\sin x)$

$y_p = -x\cos x + \log(\sin x)\sin x$

General solution

$y = y_c + y_p$

$y = c_1\cos x + c_2\sin x - x\cos x + \log(\sin x)\sin x$

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10. $(D^2+a^2)y = \sec ax$

Auxilary form

$$m^2+a^2=0$$

$$m = \pm ia$$

The roots are complex

$$y_c = e^{0.x}[c_1\cos ax + c_2\sin ax]$$

$$y_c = c_1\cos ax + c_2\sin ax$$

$$u(x) = \cos ax \ , \ v(x) = \sin ax$$

$$y_p = A\,u(x) + B\,v(x)$$

$$\therefore y_p = A\cos ax + B\sin ax$$

where A, B, are unknown functions of x

$$w = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix} = a\cos^2 ax + a\sin^2 ax$$

$$w = a$$

$$A = -\int \frac{v(x)\cdot Q(x)}{w}dx = -\int \frac{\sin ax \cdot \sec ax\,dx}{a} = -\frac{1}{a}\int \sin ax \cdot \frac{1}{\cos ax}dx$$

$$A = -\frac{1}{a}\int \tan ax\,dx = -\frac{1}{a^2}\log(\sec ax)$$

$$B = \int \frac{u(x)\cdot Q(x)}{w}dx = \int \frac{\cos ax\cdot \sec ax\,dx}{a} = \frac{1}{a}\int \cos ax\cdot\frac{1}{\cos ax}dx$$

$$B = \frac{1}{a}\int dx = \frac{1}{a}x.$$

$$y_p = \left[-\frac{1}{a^2}\log(\sec ax)\right]\cos ax + \frac{1}{a}x\cdot\sin ax$$

General solution

$$y = y_c + y_p$$

$$y = c_1\cos ax + c_2\sin ax + \frac{x}{a}\sin ax - \frac{1}{a^2}\log(\sec ax)\cos ax$$

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