Vidya Jyothi Institute of Technology (Autonomous)

(Accredited by NBA & NAAC Approved By A.I.C.T.E., New Delhi, Permanently Affiliated to JNTUH)

Aziz Nagar, C.B.Post, Hyderabad -500075

Department of Humanities and Sciences

B.Tech First Year I Semester-2019-20

QUESTION BANK

Subject: Mathematics I (Matrices & Calculus)

Prepared by:

UNIT-I SHORT ANSWER QUESTIONS	
1	Define Real Matrix and Complex Matrix.
2	Define Symmetric Matrix and Skew-Symmetric Matrix.
3	Define Orthogonal Matrix.
4	Define Hermitian and Skew-Hermitian Matrices.
5	Define Unitary Matrix.
6	Show that every square matrix can be expressed as sum of symmetric and skew symmetric matrices.
7	Find the rank of $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 4\\ 7 & 10 & 12\end{bmatrix}$
8	Is the matrix $A=\begin{bmatrix}2 & -3 & 1\\ 4 & 3 & 1\\ -3 & 1 & 9\end{bmatrix}$ is orthogonal?
9	Show that $A=\frac{1}{2}\begin{bmatrix}-1 & 1 & 1 & 1\\ 1 & -1 & 1 & 1\\ 1 & 1 & -1 & 1\\ 1 & 1 & 1 & -1\end{bmatrix}$ is orthogonal.
10	Is the matrix $\frac{1}{2}\begin{bmatrix}1+i & -1+i\\ 1+i & 1-i\end{bmatrix}$ an unitary?
11	Show that $A=\begin{bmatrix}a+ic & -b+id\\ b+id & a-ic\end{bmatrix}$ is unitary if $a^2+b^2+c^2+d^2=1$.
12	Show that $A=\begin{bmatrix}2+3i & 1-i & 2+i\\ -2i & 4 & 2i\\ -4i & -4i & i\end{bmatrix}$ is skew-Hermitian matrix.
13	Define rank of a matrix.

14	Find the value of k such that the rank of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & k & 7\\ 3 & 6 & 10\end{bmatrix}$ is 2.
15	Find the value of k such that the rank of $A=\begin{bmatrix}1 & 1 & -1 & 1\\ 1 & -1 & k & -1\\ 3 & 1 & 0 & 1\end{bmatrix}$ is 2.
16	Define Homogeneous and Non-Homogeneous System of equations
17	Define trivial and non-trivial solution.
18	Define augmented matrix
19	Define Consistent and in-consistent System of equations.
20	Write the procedure to solve system of linear equations using consistency method.

UNIT-I LONG ANSWER QUESTIONS	
1	Determine the values of a, b, c when $A=\begin{bmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{bmatrix}$ is orthogonal.
2	Express the matrix A as sum of symmetric and skew symmetric matrices where $A=\begin{bmatrix}3 & -2 & 6\\ 2 & 7 & -1\\ 5 & 4 & 0\end{bmatrix}$
3	Find the rank of the following matrices by reducing them to their Echelon for $(a)\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ $(b)\begin{bmatrix}1 & 4 & 3 & -2 & 1\\ -2 & -3 & -1 & 4 & 3\\ -1 & 6 & 7 & 2 & 9\\ -3 & 3 & 6 & 6 & 12\end{bmatrix}$ $(c)\begin{bmatrix}1 & 2 & 3 & 4\\ -2 & -3 & 1 & 2\\ -3 & -4 & 5 & 8\\ 1 & 3 & 10 & 14\end{bmatrix}$
4	Find the rank of the following matrices, by reducing into their normal form $(a)\begin{bmatrix}2 & 1 & 3 & 5\\ 4 & 2 & 1 & 3\\ 8 & 4 & 7 & 13\\ 8 & 4 & -3 & -1\end{bmatrix}$ $(b)\begin{bmatrix}1 & 3 & 4 & 5\\ 1 & 2 & 6 & 7\\ 1 & 5 & 0 & 10\end{bmatrix}$ $(c)\begin{bmatrix}4 & 3 & 2 & 1\\ 5 & 1 & -1 & 2\\ 0 & 1 & 2 & 3\\ 1 & -1 & 3 & -2\end{bmatrix}$
5	Find whether the following equations are consistent or not, if so solve them $(a) x+y+z=6; 2x+3y-2z=2; 5x+y+2z=13.$ $(b) x+2y+2z=2; 3x-2y-z=5; 2x-5y+3z=-4; x+4y+6z=0.$

6	Discuss for what values of λ & μ the simultaneous equations $x+y+z=6; x+2y+3z=10; x+2y+\lambda z=\mu$ have (i) no solution, (ii) an unique solution, (iii) infinite number of solutions.
7	Solve the following system of equations $(a) x+3y-2z=0; 2x-y+4z=0; x-11y+14z=0.$ $(b) 4x+2y+z+3w=0; 6x+3y+4z+7w=0; 2x+y+w=0.$ $(c) x+y+w=0; y+z=0; x+y+z+w=0; x+y+2z=0$
8	Show that the only real number λ for which the system $x+2y+3z=\lambda x; 3x+y+2z=\lambda y; 2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them, when $\lambda=6$
9	Solve the following system of equation by using LU decomposition method $2x+y+z=2; x+3y+2z=2; 3x+y+2z=2.$
10	Solve the following system of equation by using Gauss elimination method $2x+y+z=10;\ 3x+2y+3z=18;\ x+4y+9z=16.$

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Department of Humanities and Sciences

B. Tech I Year I Semester-2019-20

QUESTION BANK

Subject: Mathematics I (Matrices & Calculus)

Prepared by:

UNIT-II SHORT ANSWER QUESTIONS	
1	Find the characteristic roots of the matrix $A=\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$.
2	IF $A=\begin{bmatrix}1&0\\0&3\end{bmatrix}$, find A^{50}.
3	Find the eigen values of the matrix A and A^{-1} where $A=\begin{bmatrix}1&3&4\\0&2&5\\0&0&3\end{bmatrix}$
4	Prove that if λ is an Eigen value of a non-singular matrix A, then $\frac{\lvert A\rvert}{\lambda}$ is an Eigen value of the matrix adj A.
5	Prove that if λ is an Eigen value of an orthogonal matrix then $\frac{1}{\lambda}$ is also its Eigen value

6	Prove that the Eigen values of A^{-1} are the Eigen values of A when A is orthogonal
7	Find the eigen values of the matrix $A=\begin{bmatrix}3 & 2 & 2\\ 1 & 2 & 2\\ -1 & -1 & 0\end{bmatrix}$
8	Find the sum and product of the eigen values of the matrix $(i) A=\begin{bmatrix}1 & 2 & -1\\ 0 & 2 & 2\\ 0 & 0 & -2\end{bmatrix}$ (ii) $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$
9	State Cayley-Hamilton theorem.
10	Define model matrix and spectral matrix.
11	If $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ find value of the matrix $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$
12	Write the matrices of the following Quadratic forms **i.** $\mathbf{2x^2+3y^2+6xy}$ **ii.** $\mathbf{2x^2+5y^2-6z^2-2xy-yx+8zx}$
13	Write the Quadratic form corresponding to the following matrix $\begin{bmatrix}2 & 4 & 5\\ 4 & 3 & 1\\ 5 & 1 & 1\end{bmatrix}$
14	Define Rank , Index and Signature of the Quadratic forms.
15	Define orthogonal set of vectors.
16	Define orthogonal transformation.
17	Define Rank , Index and Signature of the Quadratic forms.
18	Discus the nature of the following Quadratic form $x^2+4xy+6xz-y^2+2yz+4z^2$
19	Find the nature, index and signature of a quadratic form $2x^2+2y^2+2z^2+2yz$
20	Find the nature, index and signature of a quadratic form $3x^2+5y^2+3z^2-2xy-2yz+2zx$

UNIT-II LONG ANSWER QUESTIONS	
1	Find the Eigen values and Eigen vectors of the following matrices. $(i)\ A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ $(ii)\ A=\begin{bmatrix}2 & -2 & 2\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$ $(iii)\ A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$
2	Prove that the sum of Eigen values of a matrix is equal to its trace and product of the Eigen values is equal to its determinant.
3	Prove that the Eigen values of a real symmetric matrix are always real.
4	Prove that the two eigenvectors corresponding to the two different Eigen values are linearly independent
5	Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 2 & -1\\ 3 & 1 & 0\\ -2 & 1 & 4\end{bmatrix}$ and hence find A^{-1} and A^4
6	Find A^{-1} and A^4 for $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$ by Cayley-Hamilton theorem
7	Determine the diagonal matrix which is orthogonal similar to the matrix

	$A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$
8	Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ and hence find A^4
9	Reduce the Quadratic for form $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$ to the canonical form by orthogonal transformation
10	Reduce the Quadratic for form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to the canonical form by orthogonal transformation.

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B. Tech I Year I Semester-2019-20

QUESTION BANK

Subject: Mathematics I (Matrices & Calculus)

Prepared by:

UNIT-III SHORT ANSWER QUESTIONS	
1	Define Sequence, Convergent and divergent sequence
2	Define Series, Convergent and Divergent Series
3	State Auxiliary- test
4	State Comparison Test
5	State D'Alembert's Ratio Test
6	State Raabe's Test
7	State Cauchy's n^{th} Root test
8	State Integral Test
9	Sate Leibnitz's test
10	Define Alternating Series, Absolute and Conditional Convergence.
11	Test the convergence of the series $(a)\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n} (b)\sum_{n=1}^{\infty} \frac{n+1}{n^p}$
12	Test the convergence of the series $(a)\sum_{n=1}^{\infty} \frac{1}{(4n^2 - 1)} (b)\sum_{n=1}^{\infty} \sqrt{n^4+1} - \sqrt{n^4-1}$
13	Test the convergence of the series $(a)\sum_{n=1}^{\infty} \frac{n^2}{2^n} (b)\sum_{n=1}^{\infty} \frac{n^p}{\angle n}$
14	Test the convergence of the series $(a)\sum_{n=1}^{\infty} \frac{1\cdot 2\cdot 3\cdots(2n+1)}{2\cdot 5\cdot 8\cdots(3n+2)} (b)\sum_{n=1}^{\infty} \frac{3^{n+1}}{(n+1)2^n}$
15	Test the convergence of the series $(a)\sum_{n=1}^{\infty} \frac{1}{(\log\log n)^n} (b)\sum_{n=1}^{\infty} \frac{1}{n\log n}$

16	Test for the convergence of $\sum_{n=1}^{\infty}\frac{x^n}{n^{n-1}}$
17	Test the convergence of the series $(a)\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}(b)1+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}-\frac{1}{5^2}+....$
18	Show that the following Series $s=1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+.......$ converges
19	Examine the convergence of $\frac{1}{1\cdot3\cdot5}-\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}-\frac{1}{7\cdot9\cdot11}+....$
20	Examine the convergence of $1+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}-\frac{1}{5^2}+....$

UNIT-III LONG ANSWER QUESTIONS	
1	Test the convergence of the series $(a)\sum_{n=1}^{\infty}\frac{1}{(\sqrt{n}+\sqrt{n+1})}(b)\sum_{n=1}^{\infty}\sqrt[3]{n^3+1}-n(c)\sum_{n=1}^{\infty}\left(\frac{2^n+3}{3^n+1}\right)^{\frac{1}{2}}$
2	Test the convergence of the series $\frac{1}{4\cdot7\cdot10}+\frac{1}{7\cdot10\cdot13}+\frac{1}{10\cdot13\cdot16}+....$
3	Test for the convergence of the series $(a)x+\frac{1}{2}\frac{x^2}{3}+\frac{1\cdot3}{2\cdot4}\frac{x^5}{5}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\frac{x^7}{7}+.....(b)\frac{x}{1\cdot2}+\frac{x^2}{2\cdot3}+\frac{x^3}{3\cdot4}.......(c)\sum_{n=1}^{\infty}\frac{x^{2n}}{(n+2)\sqrt{n+1}},(x>0)$
4	Test for the convergence of the series $(a)\frac{2}{1}+\frac{2\cdot5\cdot8}{1\cdot5\cdot9}+\frac{2\cdot5\cdot8\cdot11}{1\cdot5\cdot9\cdot13}+.....(b)\sum_{n=1}^{\infty}\frac{1\cdot3\cdot5\cdots(2n+1)}{2\cdot5\cdot8\cdots(3n+2)}$
5	Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1}+\frac{\sqrt{3}-1}{4^2-1}+\frac{\sqrt{4}-1}{5^2-1}+\frac{\sqrt{5}-1}{6^2-1}+....$
6	Examine the following series for absolute and conditional convergence of the series $\frac{1}{5\sqrt{2}}-\frac{1}{5\sqrt{3}}+\frac{1}{5\sqrt{4}}-\frac{1}{5\sqrt{5}}+......$
7	Test for the convergence of $(a)\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}(b)\frac{2}{1^2}x+\frac{3^2}{2^3}x^2+.....\frac{(n+1)^n}{n^{n+1}}x^n+....,(x>0)$
8	Show that the series $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+.......$ is convergent
9	Test the series for absolute/conditional convergence $(a)\sum_{n=2}^{\infty}\frac{(-1)^n}{n(\log n)^2}\quad(b)\sum\frac{(-1)^n(x+2)}{2^n+5}\quad(c)\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}\quad(d)\sum\frac{(-1)^n\,\mathrm{Sin}(\frac{1}{\sqrt{n}})}{n-1}$

10	Examine the convergence of $(a)\frac{1}{1\cdot2\cdot3}-\frac{5}{3\cdot4\cdot5}+\frac{9}{5\cdot6\cdot7}-\frac{13}{7\cdot8\cdot9}+\cdots\cdots(b)\frac{1}{5\cdot9\cdot13}-\frac{1}{9\cdot13\cdot17}+\frac{1}{13\cdot17\cdot21}-\cdots\cdots$

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Subject: Mathematics I (Matrices & Calculus)

Prepared by:

UNIT-IV SHORT ANSWER QUESTIONS	
1	State Rolle's theorem
2	State Lagrange's theorem
3	State Cauchy's mean value theorem.
4	Explain geometrical interpretation of Rolle's theorem
5	Explain geometrical interpretation of Lagrange's theorem.
6	Verify Rolle's theorem for $f(x)=\lvert x\rvert \; in \; [-1,1]$
7	Verify Lagrange's theorem for $f(x)=\log_e x \; on \; [1,e]$
8	Find the value of c of Cauchy's mean value theorem for $f(x)=\sqrt{x} \; and \; g(x)=\frac{1}{\sqrt{x}} \; in \; [a,b], 0<a<b$
9	Show that, for any x>0, $1+x<e^x<1+xe^x$
10	Apply CMVT to the function f(x) = e^x and g(x)= e^{-x} in the interval [a,b]
11	Define Beta function
12	Define Gamma function
13	Show that $\mathrm{B}(m,n)=\mathrm{B}(n,m)$
14	Show that $\mathrm{B}(m,n)=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$
15	Show that $\Gamma(n+1)=n\Gamma(n)$
16	Express $\int_0^1\frac{x}{\sqrt{1-x^5}}dx$ as beta function
17	Show that $\Gamma(-1/2)=-2\sqrt{\pi}$
18	$Compute \quad \Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$

19	Evaluate $\int_0^\infty e^{-y} y^{-1/2}\, dy$
20	If m, n positive integers then $B(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$

UNIT-IV LONG ANSWER QUESTIONS	
1	Verify Rolle's mean value theorem for the following i. $f(x) = (x-a)^m (x-b)^n$ where m,n are positive integers in [a,b]. ii. $f(x) = x(x+3)e^{-\frac{x}{2}}\ in\ -3 \le x \le 0$ iii. $f(x) = e^{-x} \sin x$ in $[0,\pi]$
2	i. Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem ii. For *0 < a < b*, prove that $1 - \frac{a}{b} < \log\left(\frac{b}{a}\right) < \frac{b}{a} - 1$. Hence prove that $\frac{1}{6} < \log\left(\frac{6}{5}\right) < \frac{1}{5}$
3	Prove that if $0 < a < 1,\ 0 < b < 1\ and\ \ a < b, then\ \ \frac{b-a}{1-a^2} < Sin^{-1}b - Sin^{-1}a < \frac{b-a}{1-b^2}$ $and\ hence\ deduce\ that\ \frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1}\frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$
4	If a<b prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using Lagrange's mean value theorem and hence deduce the following (i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$ (ii) $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$
5	Using Cauchy's mean value theorem i. prove that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta, 0 < \alpha < \theta < \beta < \frac{\pi}{2}$ ii. prove that the mean value 'C' of the functions $f(x) = \sqrt{x}\ and\ g(x) = \frac{1}{\sqrt{x}}$ is geometric mean of a and b, a>0, b>0 . iii. find 'C' for Sin x and Cos x in $\left[0, \frac{\pi}{2}\right]$

6	Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, where $m>0,\ n>0$
7	Show that $\int_0^1 x^m(\log x)^n\,dx=\dfrac{(-1)^n n!}{(m+1)^{n+1}}$ where n is positive integer and $m>-1$, evaluate $\int_0^1 x(\log x)^3\,dx$
8	Prove that i. $\int_0^{\pi/2}\sin^2\theta\cos^4\theta\,d\theta=\dfrac{\pi}{32}$ ii. $\int_0^{\pi/2}\dfrac{d\theta}{\sqrt{\sin\theta}}\times\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta=\pi$
9	Prove that *i.* $\int_0^{\infty}x^{-3/2}(1-e^{-x})dx=2\sqrt{\pi}$ *ii.* $\int_0^1\dfrac{x^2}{\sqrt{1-x^4}}dx\times\int_0^1\dfrac{1}{\sqrt{1+x^4}}dx=\dfrac{\pi}{4\sqrt{2}}$
10	Show that i. $\int_0^1 y^{q-1}\left(\log\dfrac{1}{y}\right)^{p-1}dy=\dfrac{\Gamma(p)}{q^p}(p,q>0)$ ii. $\Gamma\left(\dfrac{1}{2}\right)=\sqrt{\pi}$

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QUESTION BANK

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Prepared by:

UNIT-V SHORT ANSWER QUESTIONS	
1	Define Total derivative.
2	Define exact derivative.
3	State chain rule of partial differentiation.
4	Define Jacobian of a function.
5	Show that JJ'=1 for $x=u(1-v),y=uv$
6	Write the conditions for functional dependency and independency.

7	Find $\frac{\partial(x,y)}{\partial(u,v)}$ if x=u(1+v),y=v(1+u)
8	If $x = r\cos\theta, y = r\sin\theta\, and\;\; z = z,\;\; find\; \frac{\partial(x,y,z)}{\partial(r,\theta,z)}$
9	If $x = \frac{1}{2}(u^2 - v^2), y = uv, z = w\;\; find\; \frac{\partial(u,v,w)}{\partial(x,y,z)}$
10	If $u = x^2 - y^2, v = 2xy,\; find\; \frac{\partial(u,v)}{\partial(x,y)}$
11	Define stationary point.
12	Define saddle point.
13	Define extreme points.
14	Define maxima and minima of a function.
15	Write the sufficient conditions for extreme of f(x,y) at (a,b)
16	Find the stationary points of $u = x^3y^2(12-3x-4y)$
17	Find the stationary points of $u = x^3 + 3x^2 + y^2 + 4xy$
18	Find the maximum value of $u = -x^2 - y^2$
19	Find the stationary points for the function $f(x,y) = \sin x \sin y \sin(x+y)$
20	Write about Method of Lagrange's multipliers

UNIT-V LONG ANSWER QUESTIONS	
1	Solve the following i. If $x + y + z = u, y + z = uv, z = uvw, then\;\; find\; \frac{\partial(x,y,z)}{\partial(u,v,w)}$ ii. If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, $then\;\; find\; \frac{\partial(u,v,w)}{\partial(x,y,z)}$ iii. If $u = \frac{2yz}{x}, v = \frac{3zx}{y}, w = \frac{4xy}{z},\;\; then\;\; find\; \frac{\partial(x,y,z)}{\partial(u,v,w)}$
2	If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$ then show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$ and hence find $\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$
3	If $u = x^2 - y^2, v = 2xy\;\; where\; x = r\cos\theta, y = r\sin\theta\;\; show\; that\; J\left(\frac{u,v}{r,\theta}\right) = 4r^3$
4	$If\;\; x = r\cos\theta, y = r\sin\theta, Find\;\; \frac{\partial(x,y)}{\partial(r,\theta)}\; and\; \frac{\partial(r,\theta)}{\partial(x,y)}. Also\; show\; that\; \frac{\partial(x,y)}{\partial(r,\theta)}.\frac{\partial(r,\theta)}{\partial(x,y)} = 1$
5	Determine whether the following functions are functionally dependent or not. If functionally dependent then find functional relation (i) $u = \frac{x^2 - y^2}{x^2 + y^2}, v = \frac{2xy}{x^2 + y^2}$ (ii) $u = x + y - z, v = x - y + z, w = x^2 + y^2 + z^2 - 2yz$ (iii) $u = xy + yz + zx, v = x^2 + y^2 + z^2$
6	Find the maximum and minimum values of the following functions

	i. $f(x,y)=2(x^2-y^2)-x^4+y^4$ ii. $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$ iii. $f(x,y)=x^3y^2(1-x-y)$ iv. $f(x,y)=x^4+y^4-2x^2+4xy-2y^2,\ x>0,\ y>0.$ v. $f(x,y)=x^3+y^3-3axy$
7	i. Find maximum value of $x+y+z$ *subject to* $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ ii. Find the minimum value of $x^2+y^2+z^2$ given that $xyz=a^3$ iii. Find the minimum value of $u=x^2y^3z^4$ if $2x+3y+4z=a$.
8	A rectangular box open at the top is to have volume of 32 cubic ft. find the dimension of box requiring least material for its construction.
9	Solve the following i. Divide 24 into three points such that continued product of the first, square of second and cube of third is maximum ii. Find the Three positive numbers whose sum is 100 and whose product is maximum. iii. Show that rectangular parallelepiped of maximum volume that can be inscribed in the given sphere is a cube.
10	Use the method of the Lagrange's multipliers to find volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.