Sa2's

6 i) $L\{e^{-3t}\sinh 3t\}$

$L[e^{-at} f(t)] = \{\bar{F}(s)\}_{s \to s+a}$

$a=3 \quad f(t) = \sinh(3t)$

$L[e^{-at} \sinh 3(t)] = \frac{3}{s^2-9}$

$L[e^{-3t} \sinh 3t] = \left[\frac{3}{s^2-9}\right]_{s \to s+3}$

$= \frac{3}{(s+3)^2-9}$

ii) $L(f(t)) = \frac{9s^2-12s+15}{(s-1)^3}$

$L(f(at)) = \frac{1}{a}\bar{F}\left(\frac{s}{a}\right)$

$\bar{F}(s) = \left(\frac{9s^2-12s+15}{(s-1)^3}\right)$

$f(3t) = \frac{1}{a}\bar{F}\left(\frac{s}{a}\right)$

$= \frac{1}{3}\left(\frac{9s^2-12s+15}{(s-1)^3}\right)_{s \to s/3}$

$= \frac{1}{\cancel{3}}\left(\frac{\left(9\cdot\frac{s^2}{9} - 12\cdot\frac{s}{3} + 15\right)\cdot 3^{\cancel{3}}}{(s-3)^3}\right)$

$= \frac{9(s^2-4s+15)}{(s-3)^3}$

SHOT ON POCO X3

(a) $f(t) = 4t^2 + \sin 3t + e^{2t}$

$L(f(t)) = 4L(t^2) + L(\sin 3t) + L(e^{2t})$

$$= 4 \cdot \frac{2!}{s^{2+1}} + \frac{3}{s^2+9} + \frac{1}{s-2}$$

$$= \frac{8}{s^2+1} + \frac{3}{s^2+9} + \frac{1}{s-2}$$

ii) $3\cos 3t \cos 4t = f(t)$

$f(t) = \frac{3}{2}\, 2\cos 3t \cos 4t$ $\qquad \cos(A+B) + \cos(A-B) = 2\cos A \cos B$

$= \frac{3}{2}(\cos 7t + \cos t)$

$$L(f(t)) = \frac{3}{2}\left[L(\cos 7t) + L(\cos t)\right]$$

$$= \frac{3}{2}\left[\frac{s}{s^2+49} + \frac{s}{s^2+1}\right]$$

$$= \frac{3s}{2}\left\{\frac{2s^2+50}{(s^2+49)(s^2+1)}\right.$$

$$= \frac{3s(s^2+25)}{(s^2+49)(s^2+1)}$$

iii) $(\sin 2t - \cos 2t)^2$

$= 1 - 2\sin 2t \cos 2t$

$f(t) = 1 - \sin 4t$

$L(f(t)) = L(1) - L(\sin 4t)$

$$\boxed{L(f(t)) = \frac{1}{s} - \frac{4}{s^2+16}}$$

SHOT ON POCO X3

11) i) $L[e^{2t}+4t^3-\sin 2t\cos 3t$

$L(e^{2t}) = \frac{1}{s-2}$

$L(4t^3) = 4\cdot\frac{3!}{s^{3+1}} = \frac{24}{s^4}$

$L(\sin 2t\cos 3t) = \frac{1}{2}(2\sin 2t\cos 3t)$

$= \frac{1}{2}(\sin 5t - \sin t)$

$= \frac{1}{2}[L(\sin 5t) - L(\sin t)]$

$= \frac{1}{2}\left[\frac{5}{s^2+25} - \frac{1}{s^2+1}\right]$

$= \frac{1}{s-2} + \frac{24}{s^4} - \frac{1}{2}\left[\frac{5}{s^2+25} - \frac{1}{s^2+1}\right].$

ii) $L[e^{3t} - 2e^{-2t} + \sin 2t + \cos 3t + \sinh 3t - 2\cosh 4t + 9$

$L[e^{3t}] = \frac{1}{s-3}$

$L[2e^{-2t}] = 2\frac{1}{s+2} = \frac{2}{s+2}$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[\cos 3t] = \frac{s}{s^2+9}$

$L\{\sinh 3t\} = \frac{3}{s^2-9}$

$L(2\cosh 4t) = 2L(\cosh 4t) = \frac{2\cdot s}{s^2-16}$

$L(9) = \frac{9}{s}$

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SHOT ON POCO X3

$= \frac{1}{s-3} - \frac{2}{s+2} + \frac{2}{s^2+4} + \frac{s}{s^2+9} + \frac{3}{s^2-9} - \frac{2s}{s^2-16} + \frac{4}{s}$.

12) i) $L[e^{at} \sin bt]$

$L[e^{at} f(t)] = [\bar{f}(s)]_{s \to s-a}$

$f(t) = \sin bt$

$L(f(t)) = \frac{b}{s^2+b^2}$

$L[e^{at} \cdot f(t)] = \left[\frac{b}{s^2+b^2}\right]_{s \to s-a}$

$= \left\{\frac{b}{(s-a)^2+b^2}\right\}$.

ii) $L[e^{at} \cos bt]$

$f(t) = \cos bt$

$L(f(t)) = \frac{s}{s^2+b^2}$

$L[e^{at} \cos bt] = \left\{\frac{s}{s^2+b^2}\right]_{s \to s-a}$

$= \left[\frac{s-a}{(s-a)^2+b^2}\right]$

iii) $L[e^{at} \cosh bt]$

$f(t) = \cosh bt$

$L(f(t)) = \frac{s}{s^2-b^2}$

$L[e^{at} \cosh bt] = \left\{\frac{s}{s^2-b^2}\right]_{s \to s-a}$

$= \left\{\frac{s-a}{(s-a)^2-b^2}\right]$

SHOT ON POCO X3

10) $L\{e^{at} \sinh bt\}$

$f(t) = \sinh bt$

$L\{f(t)\} = \frac{b}{s^2+b^2} \cdot \frac{b}{s^2-b^2}$

$L\{e^{at} \sinh bt\} = \left\{\frac{b}{s^2-b^2}\right\}_{s \to s-a}$

$= \left\{\frac{b}{(s-a)^2-b^2}\right\}$.

13) 1) $e^{-t}(3\sinh 2t - 5\cosh 2t)$

$L(3 \cdot e^{-t} \sinh 2t) - L(5e^{-t} \cosh 2t)$

$L(e^{-at} f(t)) = [\bar{f}(s)]_{s \to s+a}$

$3 L[e^{-t} \sinh 2t] - 5 L[e^{-t} \cosh 2t]$

$3\left[\frac{2}{s^2+4}\right]_{s \to s+1} - 5\left[\frac{s}{s^2-4}\right]_{s \to s+1}$

$= \frac{6}{(s+1)^2+4} - \frac{5s}{(s+1)^2-4}$

ii) $e^{-3t}(2\cos 5t - 3\sin 5t)$

$L(2e^{-3t} \cos 5t) - L(3 \cdot e^{-3t} \sin 5t)$

$2\left[\frac{s}{s^2+25}\right]_{s \to s+3} - 3\left[\frac{5}{s^2+25}\right]_{s \to s+3}$

$\frac{2s}{(s+3)^2+25} - \frac{15}{(s+3)^2+25} = \frac{2s-15}{(s+3)^2+25}$.

SHOT ON POCO X3

iii) $e^{-t}(\sin 2t \cos t)$

$e^{-t}(2\sin t \cos^2 t)$

$e^{-t}(2\sin t(1-\sin^2 t))$

$e^{-t}(2\sin t - 2\sin^3 t)$

$e^{-t}\left(2\sin t - \frac{3\sin t + \sin 3t}{2}\right)$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\frac{\sin 3\theta - 3\sin\theta}{-4} = \sin^3\theta$

$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$

$L(2 \cdot e^{-t}\sin t) - L\left(\frac{3}{2}e^{-t}\sin t\right) + L\left(\frac{3}{2}e^{-t}\sin 3t\right)$

$2\left(\frac{1}{s^2+1}\right)_{s \to s+1} - \frac{3}{2}\left(\frac{1}{s^2+1}\right)_{s \to s+1} + \frac{3}{2}\left(\frac{3}{s^2+9}\right)_{s \to s+1}$

$\frac{2}{(s+1)^2+1} - \frac{3}{2(s+1)^2+1} + \frac{9}{2(s+1)^2+1}$

$\frac{2}{(s+1)^2+1} + \frac{\cancel{6}\,3}{\cancel{2}(s+1)^2+1} = \frac{5}{(s+1)^2+1}$

iv) $\cosh 2t \sin 3t$

sol) $\left(\frac{e^{2t}+e^{-2t}}{2}\right)\sin 3t$

$\frac{1}{2}e^{2t}\sin 3t + \frac{1}{2}e^{-2t}\sin 3t$

$\frac{1}{2}\left[\frac{3}{s^2+9}\right]_{s \to s-2} + \frac{1}{2}\left[\frac{3}{s^2+9}\right]_{s \to s+2}$

$\frac{3}{2(s-2)^2+9} + \frac{3}{2(s+2)^2+9}$

$= \frac{3}{2}\left[\frac{1}{(s-2)^2+9} + \frac{1}{(s+2)^2+9}\right]$

$= \frac{3}{2}\left[\frac{2s^2+8}{((s-2)^2+9)((s+2)^2+9)}\right] = \frac{3(s^2+4)}{((s-2)^2+9)((s+2)^2+9)}$.

SHOT ON POCO X3

LAD'S

1) i) $e^{-3t}\cosh 4t \sin 3t$

Sol) $e^{-3t}\left[\frac{e^{4t}+e^{-4t}}{2}\right]\sin 3t$

$\left(\frac{e^{-t}+e^{-7t}}{2}\right)\sin 3t$

$\frac{1}{2}\left[e^{-t}\sin 3t + e^{-3t}\sin 3t\right]$

$\frac{1}{2}\left[\left[\frac{3}{s^2+9}\right]_{s\to s+1} + \left[\frac{3}{s^2+9}\right]_{s\to s+3}\right]$

$\frac{1}{2}\left[\frac{3}{(s+1)^2+9} + \frac{3}{(s+3)^2+9}\right]$

$\frac{3}{2}\left[\frac{1}{(s+1)^2+9} + \frac{1}{(s+3)^2+9}\right]$

$\frac{3}{2}\left[\frac{s^2+9+6s+s^2+2s+1+9+9+9}{((s+1)^2+9)((s+3)^2+9)}\right]$

$\frac{3}{2}\left[\frac{2s^2+8s+28}{((s+1)^2+9)((s+3)^2+9)}\right]$

$= \frac{3(s^2+4s+14)}{((s+1)^2+9)((s+3)^2+9)}$

SHOT ON POCO X3

ii) $\sin 2t \cos t \cosh 2t$

Sol) $\left(\frac{e^{2t}+e^{-2t}}{2}\right)\cdot\frac{1}{2}\cdot 2\sin 2t\cos t$

$\frac{1}{4}\left(e^{2t}+e^{-2t}\right)\left(\sin 3t+\sin t\right)$

$\frac{1}{4}\left[e^{2t}\sin 3t+e^{2t}\sin t+e^{-2t}\sin 3t+e^{-2t}\sin t\right]$

$=\frac{1}{4}\left[\left\{\frac{3}{s^2+9}+\frac{1}{s^2+1}\right\}_{s\to s-2}+\left\{\frac{3}{s^2+9}+\frac{1}{s^2+1}\right\}_{s\to s+2}\right]$

$=\frac{1}{4}\left[\frac{3}{(s-2)^2+9}+\frac{1}{(s-2)^2+1}+\frac{3}{(s+2)^2+9}+\frac{1}{(s+2)^2+1}\right]$

$=\frac{3}{4}\left[\frac{(s-2)^2+9+(s+2)^2+9}{((s-2)^2+9)((s+2)^2+9)}\right]+\frac{1}{4}\left[\frac{(s-2)^2+1+(s+2)^2+1}{((s-2)^2+1)((s+2)^2+1)}\right]$

$=\frac{3}{4}\left[\frac{2s^2+26}{((s-2)^2+9)((s+2)^2+9)}\right]+\frac{1}{4}\left[\frac{2s^2+10}{((s-2)^2+1)((s+2)^2+1)}\right]$

$=\frac{3}{2}\left[\frac{s^2+13}{((s-2)^2+9)((s+2)^2+9)}\right]+\frac{1}{2}\left[\frac{s^2+5}{((s-2)^2+1)((s+2)^2+1)}\right]$

iii) $e^{2t}\sin t\cos 2t$

$e^{2t}\sin t\,(1-2\sin^2 t)$

$L\{e^{2t}\sin t\}-L\{2e^{2t}\sin^3 t\}$

$L[e^{2t}\sin t]-2L\left[e^{2t}\,\frac{3\sin t-\sin 3t}{4}\right]$

$\left(\frac{1}{s^2+1}\right)_{s\to s-2}-\frac{1}{2}\left[\frac{3}{s^2+1}-\frac{3}{s^2+9}\right]_{s\to s-2}$

$\frac{1}{(s-2)^2+1}-\frac{3}{2}\left[\frac{1}{(s-2)^2+1}-\frac{1}{(s-2)^2+9}\right]$

SHOT ON POCO X3

$10+2(s^2+4-4s)$

$10+8+2s^2-8s$

$\frac{3}{2}\left[\frac{1}{(s-2)^2+9}\right]-\frac{1}{2}\left[\frac{1}{(s-2)^2+1}\right]$

$= \frac{3(s^2+4-4s)+3-((s^2+4-4s)+9)}{2((s-2)^2+9)((s-2)^2+1)}$

$= \frac{2s^2+2-16s}{2((s-2)^2+9)((s-2)^2+1)}$

$= \frac{s^2+1-8s}{((s-2)^2+9)((s-2)^2+1)}$

2) i) $t^2 e^{-2t}\cos t$

$f(t) = e^{-2t}\cos t$

$\bar{f}(t) = \left(\frac{s}{s^2+1}\right]_{s \to s+2}$

$= \left(\frac{s+2}{(s+2)^2+1}\right)$

$L(t^2 \cdot e^{-2t}\cos t) = (-1)^n \frac{d^n}{ds^n}(\bar{f}(s))$

$= (-1)^2 \frac{d^2}{ds^2}\left(\frac{s+2}{(s+2)^2+1}\right)$

$= \frac{d}{ds}\left(\frac{(s+2)^2+1-(s+2)\cdot 2(s+2)}{((s+2)^2+1)^2}\right.$

$= \frac{d}{ds}\left(\frac{-(s+2)^2+1}{((s+2)^2+1)^2}\right.$

$= \frac{((s+2)^2+1)^2 \cdot -2(s+2)-((s+2)^2-1)\cdot 2((s+2)^2+1)\cdot 2(s+2)}{((s+2)^2+1)^4}$

$$\left(\left((s+2)^2+1\right)^2 - 2(s+2) - 4(s+2)\left((s+2)^4-1\right)\right.$$

$$= \frac{-2(s+2)\left(\left((s+2)^2+1\right)^2 + (s+2)^4 - 1\right)}{\left((s+2)^2+1\right)^4}$$

$$= \frac{-2(s+2)\left(\left((s+2)^2+1\right)^2 + \left((s+2)^2+1\right)\left((s+2)^2-1\right)\right)}{\left((s+2)^2+1\right)^4}$$

$$= \frac{-2(s+2)\left(\cancel{(s+2)^2+1}\right)\left((s+2)^2+1+(s+2)^2-1\right)}{\left((s+2)^2+1\right)^4}$$

$$= \frac{-2(s+2)\cdot 2(s+2)^2}{\left((s+2)^2+1\right)^3}$$

$$= \frac{4(s+2)^2}{\left((s+2)^2+1\right)^3}.$$

SHOT ON POCO X3

ii) $t^2 \sin t \cos 2t$

Sol) $\cancel{t^2 \sin t (1-2\sin^2 t)}$

$\frac{t^2}{2}(2\sin t \cos 2t)$

$\frac{t^2}{2}(\sin 3t - \sin t)$

$\frac{t^2}{2} L\{\sin 3t - \sin t\}$

$L\left\{\frac{t^2}{2}\left(\frac{3}{s^2+9} - \frac{1}{s^2+1}\right)\right\}$

$= (-1)^2 \frac{d^2}{ds^2} \cdot \left[\frac{3}{2} \cdot \frac{1}{s^2+9} - \frac{1}{2} \cdot \frac{1}{s^2+1}\right]$

$= \frac{1}{2}\frac{d}{ds}\left[\frac{3}{s^2+9} - \frac{1}{s^2+1}\right]$

$= \frac{1}{2}\frac{d}{ds}\left[\frac{3-(2s)}{(s^2+9)^2} + \frac{2s}{(s^2+1)^2}\right]$

$= \frac{1}{2}\frac{d}{ds}\left[\frac{2s}{(s^2+1)^2} - \frac{6s}{(s^2+9)^2}\right]$

$= \frac{d}{ds}\left[\frac{s}{(s^2+1)^2} - \frac{3s}{(s^2+9)^2}\right]$

$= \frac{(s^2+1)^2 1 - s \cdot 2(s^2+1)2s}{(s^2+1)^4} - 3\frac{((s^2+9)^2 \cdot 1 - s \cdot 2(s^2+9) \cdot 2s)}{(s^2+9)^4}$

$= \frac{(s^2+1)^2 - 4s^2(s^2+1)}{(s^2+1)^4} - 3\frac{((s^2+9)^2 - 4s^2(s^2+9))}{(s^2+9)^4}$

$= \frac{(s^2+1)(s^2+1-4s^2)}{(s^2+1)^4} - \frac{3((s^2+9)(s^2+9-4s^2))}{(s^2+9)^4}$

$= \dfrac{1-3s^2}{(s^2+1)^3} + \dfrac{3(-3s^2+9)}{(s^2+9)^3}$

$= \dfrac{1-3s^2}{(s^2+1)^3} + \dfrac{(9s^2-27)}{(s^2+9)^3}.$

iii) $t \cdot e^{-t} \sin 2t$

sol) $\bar{f}(t) = e^{-t} \sin 2t$

$L(\bar{f}(t)) = \left[\dfrac{2}{s^2+4}\right]_{s \to s+1}$

$= \dfrac{2}{(s+1)^2+4}$

$L[t \cdot e^{-t} \sin 2t] = -1 \cdot \dfrac{d}{ds}\left[\dfrac{2}{(s+1)^2+4}\right]$

$= (-1) \cdot 2 \cdot \dfrac{(-1) \cdot 2(s+1)}{((s+1)^2+4)^2}$

$= \dfrac{4(s+1)}{((s+1)^2+4)^2}.$

SHOT ON POCO X3

iv) $t \sin 3t \cos 2t$

Sol) $\frac{t}{2}(\sin 5t + \sin t)$

$$\frac{t}{2}\left(\frac{5}{s^2+25} + \frac{1}{s^2+1}\right)$$

$$= \frac{1}{2} \cdot \frac{d}{ds}\left(\frac{5}{s^2+25} + \frac{1}{s^2+1}\right)$$

$$= \frac{1}{2}\left[\frac{-10s}{(s^2+25)^2} - \frac{2s}{(s^2+1)^2}\right]$$

$$= -\left[\frac{5s+1}{(s^2+25)^2(s^2+1)^2}\right]$$

$$= -\left[\frac{5s}{(s^2+25)^2} + \frac{1}{(s^2+1)^2}\right].$$

v) $t \cdot e^{-t} \cosh t$

Sol) $t \cdot \left[\frac{s}{s^2+1}\right] s \to s+1$

$$t\left[\frac{s+1}{(s+1)^2+1}\right]$$

$$= \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right]$$

$$(-1)\frac{((s+1)^2+1)1 - (s+1)2(s+1)}{((s+1)^2+1)^2}$$

$$(-1)\frac{(s+1)^2+1-2(s+1)^2}{((s+1)^2+1)^2} = \boxed{\frac{+(s+1)^2+1}{((s+1)^2-1)^2}}$$

SHOT ON POCO X3

vi) $t^2 \cdot e^{-2t}$

$t^2 \left[\frac{1}{s+2}\right]$

$(-1)^n \frac{d}{ds}\left[\frac{d}{ds}\ \frac{1}{s+2}\right]$

$\frac{d}{ds}\left(\frac{-1}{(s+2)^2}\right.$

$\frac{-2}{(s+2)^3}$.

vii) $t \cdot e^{2t}\ \sin 3t$

Sol) $\bar{f}(s) = \left[\frac{3}{s^2+9}\right]_{s \to s-2}$

$\therefore \frac{3}{(s-2)^2+9}$

$L(\bar{f}(s)) = (-1)^n \frac{d}{ds}\left(\frac{3}{(s-2)^2+9}\right]$

$= -1\left[3 \cdot (-1)\left((s-2)^2+9\right)^{-2} \cdot 2(s-2)\right]$

$= +1\left[\frac{6(s-2)}{((s-2)^2+9)^2}\right]$.

viii) $t \sin^2 3t$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta$

Sol) $\cancel{t}\ (\cancel{t}-\cos^2 \cancel{3t}$

$t\left(\frac{1-\cos 6t}{2}\right)$

$L\left(\frac{1-\cos 6t}{2}\right) = \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+36}\right)$.

$L(t\sin^2 3t) = -1 \cdot \frac{1}{2}\ \frac{d}{ds}\left(\frac{1}{s} - \frac{s}{s^2+36}\right)$

SHOT ON POCO X3

$= \frac{-1}{2} \frac{d}{ds}\left[\frac{1}{s} - \frac{s}{s^2+36}\right]$

$= -\frac{1}{2} \left\{ \frac{-1}{s^2} - \left[\frac{s^2+36 - s \cdot 2s}{(s^2+36)^2} \right]\right.$

$= \frac{-1}{2}\left[\frac{-1}{s^2} - \left[\frac{-s^2+36}{(s^2+36)^2}\right]\right.$

$= \frac{-1}{2}\left[\frac{-1}{s^2} + \left[\frac{(s^2-36)}{(s^2+36)^2}\right]\right.$

$= \frac{1}{2}\left[\frac{1}{s^2} - \frac{s^2-36}{(s^2+36)^2}\right].$

3) i) $L\left[\frac{\sin 3t \cos t}{t}\right]$

$L\left(\frac{f(t)}{t}\right) = \int_s^\infty \bar{f}(s) \cdot ds$

$f(t) = \sin 3t \cos t$

$f(t) = \frac{1}{2}\left(\sin 4t + \sin 2t\right)$

$= \frac{1}{2}\left[\frac{4}{s^2+16} + \frac{2}{s^2+4}\right]$

$= \frac{2}{s^2+16} + \frac{1}{s^2+4}$

$L\left[\frac{\sin 3t \cos t}{t}\right] = \int_s^\infty \frac{2}{s^2+16} + \frac{1}{s^2+4}\right]$

$\cancel{2}\left(\frac{1}{\cancel{4}_{2}} \cdot \tan^{-1}\left(\frac{s}{4}\right) + \frac{1}{2}\tan^{-1}\left(\frac{s}{2}\right)\right]_s^\infty$

$= \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{s}{4}\right) + \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{s}{2}\right)\Big)$

$= \frac{1}{2}\left(\cot^{-1}\left(\frac{s}{4}\right) + \cot^{-1}\left(\frac{s}{2}\right)\right)$

SHOT ON POCO X3

ii) $L\left[\frac{1-\cos t}{t^2}\right]$

$\bar{f}(s) = L[1-\cos t]$

$= \frac{1}{s} - \frac{s}{s^2+1}$

$L\left[\frac{1-\cos t}{t^2}\right] = \int_s^\infty \frac{1}{s} - \frac{s}{s^2+1}$

$\left[\log s - \frac{1}{2}\log(s^2+1)\right]_s^\infty$

$= \frac{1}{2}\log(s^2+1) - \log s$

$= \log\left[\frac{\sqrt{s^2+1}}{s}\right].$

iii) $L\left[\frac{\cos 4t \sin t}{t}\right]$

$f(s) = \frac{1}{2}$

$\bar{f}(s) = \frac{1}{2}\left[\sin 5t + \sin(-3t)\right]$

$= \frac{1}{2}\left[\sin 5t - \sin 3t\right]$

$= \frac{1}{2}\left[\frac{5}{s^2+25} - \frac{3}{s^2+9}\right]$

$L\left[\frac{\cos 4t \sin t}{t}\right] = \frac{1}{2}\int_s^\infty \frac{5}{s^2+25} - \frac{3}{s^2+9}$

$= \frac{1}{2}\left[\tan^{-1}\left(\frac{s}{5}\right) - \tan^{-1}\left(\frac{s}{3}\right)\right]_s^\infty$

$= \frac{1}{2}\left[\frac{\pi}{2} - \tan^{-1}\frac{s}{5} - \frac{\pi}{2} + \tan^{-1}\frac{s}{3}\right]$

$= \frac{1}{2}\left[\tan^{-1}\left(\frac{s}{3}\right) - \tan^{-1}\left(\frac{s}{5}\right)\right].$

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SHOT ON POCO X3

iv) $\frac{e^{-at}-e^{-bt}}{t}$

$f(t) = e^{-at} - e^{-bt}$

$\bar{f}(s) = \frac{1}{s+a} - \frac{1}{s+b}$

$L\left[\frac{e^{-at}-e^{-bt}}{t}\right] = \int_s^\infty \left[\log(s+a) - \log(s+b)\right]$

$= \left\{\log\left(\frac{s+a}{s+ab}\right]_s^\infty$

$= \log\left[\frac{1+\frac{a}{s}}{1+\frac{b}{s}}\right]_s^\infty$

$= 0 - \cancel{\log}\left[\log\left[\frac{s+a}{s+b}\right]\right.$

$= \log\left[\frac{s+b}{s+a}\right].$

v) $\frac{\sin t}{t}$

Sol) $f(t) = \sin t$

$\bar{f}(s) = \frac{1}{s^2+1}$

$L\left(\frac{\sin t}{t}\right) = \int_s^\infty \frac{1}{s^2+1}$

$= (\tan^{-1}(s))_s^\infty$

$= \frac{\pi}{2} - \tan^{-1}(s)$

$= \cot^{-1}(s).$

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vi) $\frac{e^{-t}\sin t}{t}$

sol) $f(t) = e^{-t} \sin t$

$$L(f(t)) = \left[\frac{1}{s^2+1}\right]_{s \to s+1}$$

$$= \frac{1}{(s+1)^2+1}$$

$$L\left[\frac{e^{-t}\sin t}{t}\right] = \int_s^\infty \frac{1}{(s+1)^2+1}$$

$$= \int_s^\infty \tan^{-1}(s+1)$$

$$= \frac{\pi}{2} - \tan^{-1}(s+1)$$

$$= \cot^{-1}(s+1)$$

4) $\int_0^\infty t^2 \cdot e^{-4t} \sin 2t$

$f(t) = t^2 \sin 2t$

$\bar{f}(s) = \frac{2}{s^2+4}$

$$L(t^2 \cdot \sin 2t) = (-1)^2 \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right)$$

$$= \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{2}{s^2+4}\right)\right)$$

$$= \frac{d}{ds}\left(\frac{-2 \cdot 2s}{(s^2+4)^2}\right)$$

$$= \frac{d}{ds}\left(\frac{-4s}{(s^2+4)^2}\right) = \frac{-4s}{(s^2+4)^2} = 1$$

$$= \left[\frac{\cancel{4s} \cdot 2s}{(\cancel{s^2}+4)^3}\right] = \frac{\cancel{8s}^3}{(s^2+4)^3} \cdot \frac{12s^2-16}{(s^2+4)^3}$$

$$\int_0^\infty e^{-4t} \sin 2t \cdot t^2 = \frac{12s^2-16}{(s^2+4)^3}$$

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$\int_0^\infty e^{-4t}(t^2 \sin t)$ is compose with $\int_0^\infty e^{-st} f(t) \cdot dt$

then $s = 4$

$$\int_0^\infty e^{-4t}(t^2 \cdot \sin t) = \frac{12s^2}{(s^2+4)^3} - \frac{16}{(s^2+4)^3}$$

$$= \frac{12 \times 16}{(16+4)^3} - \frac{16}{(20)^3}$$

$$= \frac{\cancel{176}\ \cancel{88}\ \cancel{44}\ \cancel{22}\ 11}{20 \times 20 \times 20}$$

$$= \frac{\cancel{176}\ \cancel{88}\ \cancel{44}\ \cancel{22}\ 11}{\cancel{800}\ \cancel{400}\ \cancel{200}\ \cancel{100}\ 50} = \frac{11}{50}$$

ii) $\int_0^\infty e^{-t} \cdot t \cdot \sin t \cdot dt$

$f(t) = t \sin t$

$\bar{f}(s) = \left(\frac{1}{s^2+1}\right)$

$$L(t \cdot \sin t) = (-1)^n \frac{d^n}{ds^n} (\bar{f}(s))$$

$$= (-1) \frac{d}{ds} \left(\frac{1}{s^2+1}\right)$$

$$= \frac{2s}{(s^2+1)^2}$$

$$\int_0^\infty e^{-t} \cdot t \cdot \sin t = \frac{2s}{(s^2+1)^2} = \frac{2}{(1+1)^2} = \frac{1}{2}$$

$s = 1$

iii) $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t} \, dt$

$\bar{f}(s) = \frac{1}{s+1} + \frac{1}{s+2}$

$\frac{e^{-t} - e^{-2t}}{t} = \int_s^\infty \frac{1}{s+1} + \frac{1}{s+2}$

$\left[\log\left[\frac{s+1}{s+2}\right]\right]_s^\infty$

$\log\left(\frac{1+\frac{1}{s}}{1+\frac{2}{s}}\right)_s^\infty$

$= -\log\left(\frac{s+1}{s+2}\right)$

$= \log\left(\frac{s+2}{s+1}\right)$

$\int_0^\infty e^{-0t} \cdot \left(\frac{e^{-t} - e^{-2t}}{t}\right) = \log\left[\frac{0+2}{0+1}\right] = \log(2).$

iv) $\int_0^\infty \frac{\cos at - \cos bt}{t} \cdot dt$

$\bar{f}(s) = \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$

$\left[\frac{\cos at - \cos bt}{t}\right] = \frac{1}{2}\int_s^\infty \left[\log\left[\frac{s^2+a^2}{s^2+b^2}\right]\right]$

$= \frac{1}{2}\left\{\log\left[\frac{1+\frac{a^2}{s^2}}{1+\frac{b^2}{s^2}}\right]\right]_s^\infty$

$= \frac{1}{2}\left[-\log\left[\frac{s^2+a^2}{s^2+b^2}\right]\right]$

$= \frac{1}{2}\log\left[\frac{s^2+b^2}{s^2+a^2}\right]$ $\quad s = 0$

$\int_0^\infty e^{-0t} \cdot \frac{\cos at - \cos bt}{t} \, dt = \frac{1}{2}\log\left[\frac{b^2}{a^2}\right]$

$= \log\left(\frac{b}{a}\right)^{\frac{1}{\cancel{2}} \cdot \cancel{2}} = \boxed{\log\left[\frac{b}{a}\right]}$

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v) $\int_0^\infty t \cdot e^{-t} \sin t \cdot dt$

$\int_0^\infty e^{-t} \cdot t \sin t$

$\bar{f}(s) = \frac{1}{s^2+1}$

$L(t \cdot \sin t) = (-1)^n \cdot \frac{d^n}{ds^n}\left(\frac{1}{s^2+1}\right)$

$= (-1) \cdot \frac{d}{ds}\left(\frac{1}{s^2+1}\right)$

$= \frac{2s}{(s^2+1)^2}$.

$\int_0^\infty e^{-t} \cdot t \cdot \sin t \cdot dt = \left[\frac{2s}{(s^2+1)^2}\right] s \to 1$

$= \frac{2}{4} = \frac{1}{2}$.

vi) $\int_0^\infty e^{-at} \cdot \frac{\sin^2 t}{t} \cdot dt$

$\int_0^\infty e^{-at} \frac{1 - \cos 2t}{2t} \cdot d$

$\bar{f}(s) = \left\{\frac{1}{s} - \frac{s}{s^2+4}\right\} \frac{1}{2}$

$L\left[\frac{1-\cos 2t}{2t}\right] = \frac{1}{2}\int_s^\infty \frac{1}{s} - \frac{s}{s^2+4}$

$= \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2+4)\right]_s^\infty$

$= \cancel{\left[\log\sqrt{s}\right]_s^\infty} - \frac{1}{4}\cancel{\log[s^2+4]}_s^\infty$

$= \frac{1}{2}\left[\log - \log\left(\sqrt{s^2+4}\right)\right]_s^\infty$

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$= \frac{1}{2}\left[\log\left[\frac{s}{\sqrt{s^2+4}}\right]_s^\infty\right.$

$= \frac{1}{2}\left[\log\left[\frac{1}{\sqrt{1+\frac{4}{s^2}}}\right]_s^\infty\right.$

$= \frac{1}{2}\left[-\log\frac{s}{\sqrt{s^2+4}}\right]$

$= \frac{1}{2}\left[\log\frac{\sqrt{s^2+4}}{s}\right]$

$= \frac{1}{2}\left[\log\sqrt{\frac{s^2+4}{s^2}}\right]$

$= \frac{1}{4}\left[\log\left[1+\frac{4}{s^2}\right]\right]$

$\int_0^\infty e^{-at}\frac{\sin^2 t}{t}.dt$ then $s = a$

$= \frac{1}{4}\left[\log\left[\frac{a^2+4}{a^2}\right]\right]$

$$\boxed{L\int_0^\infty e^{-at}\frac{\sin^2 t}{t}.dr = \frac{1}{4}\log\left\{\frac{a^2+4}{a^2}\right\}.}$$

period $L(f(t)) = \frac{\int_0^T e^{-st}.f(t).dt}{1-e^{-sT}}$ $T=2b$

So) $Lf(t) = \frac{\int_0^\infty e^{-st}\ f(t).dt}{1-e^{-2sb}}$

$$\frac{\int_0^b e^{-st}.\ t.dt - \int_b^{2b}(2b-t).e^{-st}.dt}{1-e^{-2sb}}$$

$\int_0^b e^{-st}.t.dt = t.\frac{e^{-st}}{-s} - \int\frac{e^{-st}}{-s}$

$= \left\{\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^b$

$= \frac{-be^{-sb}}{bs} - \frac{e^{-bs}}{s^2} + \frac{1}{s^2}$

$$\int_b^{2b} (2b-t)\, e^{-st} \cdot dt \qquad \left((2b-t)\frac{e^{-st}}{-s} - \left(-1\cdot\frac{e^{-st}}{-s}\cdot\frac{1}{-s}\right)\right.$$

$$\int_b^{2b} 2b\cdot e^{-st} - \int_b^{2b} t\cdot e^{-st} \qquad = \left[-(2b-t)\frac{e^{-st}}{s} + \frac{e^{-st}}{s^2}\right]$$

$$\left[2b\left(\frac{e^{-st}}{-s\cdot s}\right]_b^{2b} - \left[t\cdot\frac{e^{-st}}{-s} - \frac{e^{-st}}{s^2}\right)_b^{2b}$$

$$= \frac{-2b\,e^{-2bs}}{s^2} + \frac{2be^{-bs}}{s}, + \left[+\frac{2be^{-2bs}}{s} + \frac{e^{-2bs}}{s^2} - \frac{be^{-bs}}{s}, - \frac{e^{-bs}}{s^2}\right],$$

$$= \frac{be^{-bs}}{s} - \frac{e^{-bs}}{s^2} + \frac{e^{-2bs}}{s^2} - \frac{2be^{-2bs}}{s^2}$$

$$\int_0^b e^{-st}\cdot f(t)\cdot dt + \int_b^{2b} (2b-t)\, e^{-st}\cdot dt$$

$$= \cancel{\frac{-be^{-sb}}{s}} - \frac{e^{-bs}}{s^2} + \frac{1}{s^2} + \cancel{\frac{be^{-bs}}{s}} - \frac{e^{-bs}}{s^2} + \frac{e^{-2bs}}{s^2} - \frac{2b}{s}$$

$$= \frac{-2e^{-2bs} + 1 + e^{(-bs)^2}}{s^2}$$

$$\mathcal{L}\{f(t)\} = \frac{\int_0^T f(t)\cdot dt}{1-e^{-sT}}$$

$$= \frac{1 + e^{(-bs)^2} - 2e^{-2bs}}{(1-e^{-2bs})\, s^2}$$

$$= \frac{(1-e^{-bs})^2}{(1-e^{-2bs})\, s^2}$$