UNIT-1 Fundamentals of logic

compound statements: A statement that is formed from atomic statement through the use of connective is called compound statements. Atomic statements are combined via. symbols of logic relation

(i) conjuction (∧): It is a compound statement formed by using 'AND' to combine 2 simple statements

P: Raman likes algebra

Q: " " physics.

$P \wedge Q$

Raman likes algebra and physics.

Truth table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

(II) Disjunction :-(V) :- It is compound statement formed by using the hard 'OR' to combine 2 simple statements.

Truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	~~F~~ T
F	F	F

(III) Negation :- let p be the preposition then we write negation of p as $\sim p$
($\sim$)

(or) $>p$ and define to be the preposition is table that p.

p	$>p$
T	F
F	T

Ex :- let 'p' be "it is cold' and q be 'q' be 'it is rains'

(i) $\sim p \longrightarrow$ It is not cold

(ii) $\sim p \wedge \sim q \longrightarrow$ It is not cold and not rains

Ex :- construct the truth table for $p \vee \sim q$

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Ex :- $p \wedge (q \vee r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
F	T	T	F	F
T	F	F	F	F
F	F	T	T	F
F	T	F	T	F
F	F	F.	F	F

(IV) conditional statements :
($\rightarrow$)

($p \rightarrow q$): p and q are any 2 statements then the statements $p \rightarrow q$ which is used as if p and q is called conditional statements. it is also called Implication.

P : object is in Telangana
Q : " " " India

P	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(V) Bi-conditional ; A biconditional is a compound ($\leftrightarrow$) statements formed by 2 conditions $p \rightarrow Q$ and $Q \rightarrow p$ under a ~~conjuga~~ condition

The bi-conditional $p \rightarrow q$ and $q \rightarrow p$ written symbolically '$p \leftrightarrow q$'. it is read as 'p' if and only if 'q'

Truth-table :

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

construct the truth table for $p \rightarrow (Q \rightarrow R)$

P	Q	R	$Q \rightarrow R$	$P \rightarrow (Q \rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
F	T	T	T	T
T	F	F	T	T
F	F	F	F	T
F	F	T	T	T
F	F	F	F	T

construct the truth table for

$(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$(p \wedge q)$

p	q	~p	~q	$(p \wedge q)$	$(\sim p \wedge q)$	$(p \wedge \sim q)$.
T	F	F	T	F	F	T
T	T	F	F	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge q)$	$(p \wedge \sim q) \vee (\sim p \wedge q)$
T	F	T
F	T	T
T	T	T
T	F	F

$(\neg p \vee q) \wedge (\neg q \vee p)$

$\vee$ - or

$\wedge$ - and.

P	Q	~P	~Q	ㄱP∪Q	ㄱQ∪P	∧
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	F	T	F	T	F	F

write the following statements in symbolic form with statement

p : pavan is rich

q : raghu is happy

(i) pavan is rich and raghu is not happy

$p \wedge \neg q$

(ii) pavan is not rich and raghu is happy

$\neg p \cap q$

R : Naveen is rich

H : Naveen is happy

(i) Naveen is poor but happy

(ii) Naveen is rich (or) unhappy

(iii) Naveen is neither rich (nor) happy

(iv) Naveen is poor (or) He is both rich and unhappy

① $\neg R \wedge H$

② $R \vee \neg H$

③ $\neg R \wedge \neg H$

④ $\neg R \cup (R \wedge \neg H)$

p : Naveen is smart

q : Amal is smart

① Naveen is smart and amal is not smart

$p \wedge \sim q$

② Naveen and amal are both smart

$p \wedge q$

③ Neither naveen nor amal are smart

$\sim (p \wedge q)$

④ it is not true that naveen and amal are both smart

$\sim (p \wedge q)$.

if neither sreenu takes calculus (or) swami takes graph theory then mahesh will take computer programer

$(p \vee Q) \rightarrow R$

Q) p : ΔABC is isoceles

q : ΔABC is equilateral

r : ΔABC is equi-angular.

a) $Q \rightarrow P$

b) $\neg P \rightarrow \neg Q$

c) $Q \leftrightarrow R$

d) $P \wedge \neg Q$

e) $R \rightarrow P$

a) In ΔABC is equilateral then ΔABC it is isoceles

b) If ΔABC is not isoceles that it is not equila

c) ~~I~~ ΔABC is equilateral if and only if it is equiangular

d) ΔABC is isoceles and not equilateral.

e) If ΔABC is equiangular then it is isoceles

Q) If P, Q, R are 3 statements with truth values T, T, F respectively. Find the truth values of the coloumns.

P	Q	R
T	T	F

(I) $P \vee Q$ — $T \vee T$ - T

(II) $P \wedge R$ — $T \wedge F$ - F

(III) $(P \vee Q) \wedge R$ — $T \vee T \wedge F$ - F

(IV) $P \wedge (\neg R \rightarrow T \wedge (T$ - F

(V) $(P \wedge \neg Q) \wedge \neg R$ — $T \wedge F \wedge F$ - F

(VI) $P \rightarrow R$ — $T \rightarrow F$ - F

(VII) $P \rightarrow Q$ - $T \rightarrow T$ - T

(VIII) $R \rightarrow P$ $F \rightarrow T$ - T

(IX) $(R \wedge P) \rightarrow Q$ - T

(X) $(P \wedge \neg Q) \rightarrow R$ - F

~~(XI)~~

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(XI) $(P \vee Q) \leftrightarrow (P \rightarrow \neg R)$ — T

(XII) $(P \leftrightarrow R) \rightarrow R$ — F

p	q	$p \rightarrow q$	$q \rightarrow p$ (converse).	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$ (Inverse)
T	T	T	T	F	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$\sim q \rightarrow \sim p$ (contrapositive)
T
F
T
T

converse

$q \rightarrow p$

Inverse

$\neg p \rightarrow \neg q$

Set :- The set is a collection of well defined objects called the elements of the set

operations on set :

(1) A and B be two sets.

The union of A and B is

$$A \cup B = \{ x / x \in A \text{ (or) } x \in B \text{ (or) both} \}$$

if $A_1, A_2, A_3, \ldots A_n$ are sets then there union is the set of objects and is denoted by

$$A_1 \cup A_2 \cup A_3 \ldots \cup A_n.$$

(II) Let 'U' be the universal set and A be the subset of 'U' the absolute complement of A is [A' (or) $\bar{A}$] is defined as $\{x / x \notin A$ (or) $x / x \in U$ and $x \notin A\}$

- If A and B are sets the relative complement of A with respect to B is defined as

$$B - A = \{x / x \in B \text{ and } x \notin A\}$$

(III) The intersection of two sets A and B is

$A \cap B = \{x / x \in A$ and $x \in B\}$. The intersection of n sets.

$A_1, A_2, A_3, \ldots A_n$ the set of all objects is denoted by $A_1 \cap A_2 \cap A_3 \ldots \cap A_n$

(IV) two sets A and B are called disjoints if

$A \cap B = \phi$ (i.e A and B have no common elements)

(V) Let A and B are two sets the symmetrical differance of two sets A and B is

$$A \Delta B = \{x / x \in A \text{ (or) } x \in B\}$$

but not both

The symmetrical differance of two sets is also called the bulliante sum of two sets (T or F)

$$(A + B) = (A - B) \cup (B - A)$$

Properties of set operations :-

1. Idempotent $A \cup A = A$ $A \cap A = A$
2. commutative $A \cup B = B \cup A$ $A \cap B = B \cap A$
3. Associative $A \cup (B \cup C) = (A \cup B) \cup C$ $(A \cap B) \cap C = A \cap (B \cap C)$

properties of complement :-

1. $A \cup A' = U$
2. $A \cap A' = \phi$
3. $U' = \phi$
4. $\phi = U$
5. $(A')' = A$
6. $(A \cup B) = A' \cap B'$
7. $(A \cap B)' = A' \cup B'$.

properties of symmetric difference

1. $A \Delta A = \phi$
2. $A \Delta \phi = A$
3. $A \Delta B = B \Delta A$
4. $A \Delta B = (A \cup B) - (B \cup A)$

properties of difference operations

1. $A' = U - A$

2. $A - B = A \cap B'$

3. $A - A = \phi$

4. $A - \phi = A$

5. $A - B = B - A \iff A = B$

6. $A - B = A \iff A \cap B = \phi$

7. $A - B = \phi \iff A \subseteq B$

Laws :-

1. Idempotent law :- (i) $A \cup A = A$

   (ii) $A \cap A = A$

2. Associative law :- (i) $(A \cup B) \cup C = A \cup (B \cup C)$

   (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

3. commutative law : (i) $A \cup B = B \cup A$

   (ii) $A \cap B = B \cap A$.

4. Distributive law : (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$

   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Absorption law : (i) $A \cup (A \cap B) = A$

   $A \cap (A \cap B) = A$

6. complement law : $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

7. Demorgan's law :

$$\phi' = U$$

$$U' = \phi$$

$$(A')' = A$$

$$A \cup \phi = A$$

$$A \cap U = A$$

$$A \cup A = U$$

$$A \cap \phi = \phi$$

$$A \cup A' = U$$

$$A \cap A' = \phi$$

* <u>power set</u> :- let A, R be the given set. The power set of A is denoted "p(A)". is a family of a sets such that 'x' is the proper subset x belongs to power set 'A'. symbolic of

$x \in p(A)$

$$\boxed{p(A) = \{x / x \subseteq A\}}$$

Ex :-

$A = \{a, b, c\}$

$p(A) = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{ab\}, \{b, c\}, \{a c\}, \{a, b, c\}\}$

* venn diagram :-

* well formed formulas :- A term in proposition of logic is sometimes called "A formula" and a term that is constructed correctly following the syntax rules is called as well coor formed formula. The set of well formed formula is define by saying how we can construct them.

Rules

1. A variable 'p' standing alone is a well formed formula.

2. If 'p' is a well formed formula then '~p' is well formed formula.

3. If p and q are well formed formula then $p \wedge q$, $p \vee q$, $p \Rightarrow q$, $p \Leftrightarrow q$ are well formed formula.

4. many formula that cannot be constructed using this rules are not well formed formulas

* Tautology :- A statement formula whose truth value is 'p' for all possiable assignments of truth values to the propositional variables is called tautology.

Ex :- $(p \wedge q) \rightarrow (p \vee q)$

T
T
T
T

* **contradiction** :- A statement formula whose truth value is false for all possible assignment of truth value values to the propositional variables is called contradiction.

Ex :- $p \wedge \sim p$
F
F

* **contigency** :- A statement formula which is neither a tautology nor a contradiction is called contigency.

(1) using the truth table to determine whether the given by tautology, contradiction or contigency

1. $[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\rightarrow \sim p$
T	T	F	F	T	F $\wedge$ T	F $\rightarrow$ F
T	F	F	T	F	T $\wedge$ F	F $\rightarrow$ F
F	T	T	F	T	F $\wedge$ T	F $\rightarrow$ T
F	F	T	T	T	T $\wedge$ T	T $\rightarrow$ T

2) $q \vee (\sim q \wedge p)$

3) $(q \wedge p) \vee (q \vee \sim p)$

1) T T T T } Tautology

(2)

q	$\vee$	$(\sim q$	$\wedge$	$p)$
T	V	F	$\wedge$	T
F	V	T	$\wedge$	T
T	V	F	$\wedge$	F
F	V	T	$\wedge$	T

$\Rightarrow$

q	$\vee (\sim q \wedge p)$
T	F
F	T
T	F
F	T

T F T T } contigency = F F F F } ~~contradiction.~~ ~~Tautology~~

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3) $(q \wedge p) \vee (q \vee \sim p)$

p	q	$\sim p$	$\sim q$	$q \wedge p$	$q \vee \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	F	T

T F T T } tautology ~~contigency~~

prove that the following questions are tautology

1) $P \wedge (P \rightleftarrows Q)$

2) ~~$(p \to Q)$~~ $\neg(\underset{A}{\underline{P \vee Q}}) \vee (\underset{B}{\underline{\neg P \wedge Q}}) \cup \underset{C}{\underline{P}}$

3) $((P \to R) \wedge (Q \to R)) \to ((P \vee Q) \to R)$

4) $[(P \to (Q \cup R)) \wedge (\neg Q)] \to (P \to R)$

5) $[((P \vee Q) \to R)) \wedge \sim P] \longrightarrow (Q \to R)$.

6) $(P \to Q) \rightleftarrows (\sim P \cup Q)$

7) $Q \cup (P \wedge \sim Q) \cup (\sim P \wedge \sim Q)$

8) $((P \to Q) \wedge (R \to S) \wedge (P \cup R)) \to (Q \cup S)$

$\to$

Sol:-

P	Q	$P \rightleftarrows Q$	$P \wedge (P \rightleftarrows Q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	F

contigency

(or)

Sol:-

P	Q	$P \veebar Q$	$P \underline{\wedge} Q$	$\sim P \wedge Q$	$\sim P \vee Q$
T	T	T	T	F	T
T	F	T	F	F	F
F	T	T	F	T	T
F	F	F	F	F	T

$\sim(P \cup Q) \vee (\sim P \wedge Q) \cup P$

$\Rightarrow$ (F) $\vee$ (F)	$\cup$ T	$\Rightarrow$	F	$\cup$	T	–	T
= (F) $\vee$ (F)	$\cup$ T		F	$\cup$	T	–	T
(F) $\vee$ (T)	$\cup$ F		T	$\vee$	F	–	T
(T) $\vee$ (F)	$\cup$ T		T	$\vee$	T	–	T

Tautology

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(3) $((P \Rightarrow R) \wedge (Q \Rightarrow R)) \rightarrow ((P \cup Q) \rightarrow R)$

P	Q	R	$P \Rightarrow R$	$Q \Rightarrow R$	$P \cup Q$
T	T	T	T	T	F
T	T	F	F	F	F
T	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	F	T
F	T	F	~~F~~ T	F	T
F	F	T	T	T	F
F	F	F	~~F~~ T	F	F

$((P \Rightarrow R) \wedge (Q \Rightarrow R))$		$(P \cup Q) \rightarrow R$	
(T) $\wedge$ (T)	– T	~~R~~ T $\rightarrow$ T	– T
(F) $\wedge$ (F)	– F	F $\rightarrow$ F	– T
(T) $\wedge$ (T)	– T	T $\rightarrow$ T	– T
(T) $\wedge$ (T)	– T	T $\rightarrow$ T	– T
(F) $\wedge$ (F)	– F	T $\rightarrow$ F	– F
(T) $\wedge$ (F)	– F	T $\rightarrow$ F	– F
(T) $\wedge$ (T)	– T	F $\rightarrow$ T	– T
(T) $\wedge$ (F)	– F	F $\rightarrow$ F	– T

$((P \to R) \land (Q \to R)) \to ((P \lor Q) \to R)$ *

T	$\to$	T	—	T	
F	$\to$	T	—	T	
T	$\to$	T	—	T	
T	$\to$	T	—	T	
F	$\to$	F	—	T	— tautology
F	$\to$	F	—	T	
T	$\to$	T	—	T	
F	$\to$	T	—	T	

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(4) $((P \to (Q \lor R)) \land (\sim Q)] \to (P \to R)$

P	Q	R	$Q \lor R$	$\sim Q$	$P \to R$	
T	T	T	~~F~~ T.	F	T	F
T	T	F	T.	F	F	F
T	F	T	T	T	T	T
F	T	T	T.	F	T	F
T	F	F	F.	T	F	F
F	T	F	T.	F	T	F
F	F	T	T.	T	T	T
F	F	F	F.T	T	T	T

$((P \to (Q \lor R) \land (\sim Q)] \to (P \to R)$

T	
T	
T	T
~~F~~	T
T	
T	

- contigency.

Equivalence of formula :-

Two formulas A and B are said to equivalent to each other if and only if $A \rightleftarrows B$ is a tautology. $A \rightleftarrows B$ if and only if truth tables of A and B are same.

① prove that $(P \cup Q) \Leftrightarrow \sim(\sim P \wedge \sim Q)$

P	Q	~P	~Q	~P ∧ ~Q	P ∪ Q	~(~P∧~Q)
T	T	F	F	F.	T	T
T	F	F	T	F	T	T
F	T	T	F	F.	T	T
F	F	T	T	T	F	F

$(P \cup Q) \Leftrightarrow \sim(\sim P \wedge \sim Q)$
T
T
T
T

2) $(p \vee q) \wedge \sim p \Leftrightarrow \sim p \wedge q$

P	Q	~P	P ∨ Q	~P ∧ q	~~p∧q~~ (p∨q)∧~p
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

equivalence of formula.

1. $P \wedge Q \Leftrightarrow Q \wedge P$
2. $\sim\sim P \Leftrightarrow P$ — commutative laws.
3. $P \vee Q \Leftrightarrow Q \vee P$

4. $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ — Associative laws
5. $(P \vee Q) \cup R \Leftrightarrow P \vee (Q \vee R)$

6. $P \wedge (Q \cup R) \Leftrightarrow (P \wedge Q) \cup (P \wedge R)$ — distributive laws
7. $P \vee (q \wedge R) \Leftrightarrow (P \vee q) \wedge (P \cup R)$

8. $\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$
9. $\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

10. $P \wedge P \Leftrightarrow P$ — Idempotent laws
11. $P \vee P \Leftrightarrow P$

12. $R \vee (P \wedge \sim P) \Leftrightarrow R$
13. $R \wedge (P \vee \sim P) \Leftrightarrow R$
14. $R \cup (P \wedge \sim P) \Leftrightarrow$ True
15. $R \wedge (P \vee \sim P) \Leftrightarrow$ False.
16. $P \rightarrow Q \Leftrightarrow \sim P \cup Q$
17. $\sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$
18. $P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

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19. $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge q) \rightarrow R$
20. $\neg(P \rightleftarrows q) \Leftrightarrow P \rightleftarrows \neg q$
21. $P \rightleftarrows q \Leftrightarrow (P \rightarrow q) \wedge (q \rightarrow P)$
22. $P \rightleftarrows q \Leftrightarrow (P \wedge q) \vee (\neg P \wedge \neg q)$

* **Identity law :-**

a) i) $p \vee F \Leftrightarrow p$ (ii) $p \vee T \Leftrightarrow T$

b) i) $p \wedge F \Leftrightarrow F$ (ii) $p \wedge T \Leftrightarrow p$

* **Compliment laws :-**

a. i) $p \wedge \sim p \Leftrightarrow F$ ✓ (ii) $p \vee \sim p \Leftrightarrow T$ ✓

b. i) $\sim\sim p \Leftrightarrow p$ (ii) $\sim T \Leftrightarrow F$, $\sim F \Leftrightarrow T$.

* **Demorgans law :-**

a) $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

b) $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

* **Absorption law :-**

a) $p \vee (p \wedge Q) \Leftrightarrow p$

b) $p \wedge (p \vee Q) \Leftrightarrow p$

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1) prove that $p \to (Q \to R)$ ~~is equivalence to $(p \wedge Q) \to R$~~

$p \to (Q \to R) \Leftrightarrow (p \wedge Q) \to R$ is equivalence

Sol :- $p \to (\underset{p}{Q} \to \underset{q}{R})$ (According to formula

ie. $p \to Q \Leftrightarrow \sim p \cup Q$)

$\Rightarrow \underset{p}{P} \to (\underset{q}{\underline{\sim Q \cup R}})$ $(p \to Q \Leftrightarrow \sim p \cup Q$.

$\Rightarrow \sim p \cup (\sim Q \cup R)$ (Associative law)

$\Rightarrow (\sim P \cup \sim Q) \cup R$ (demorgans law) $(\sim P \vee \sim Q)$ ...

is negation remove as ...

$\Rightarrow \sim(\underbrace{P \wedge Q}_{P}) \cup \underbrace{R}_{Q}$ ((or) changes to (and))

$\Rightarrow \sim(P \wedge Q) \longrightarrow R$ // $(\because \cancel{P \cup}\ \underline{P \longrightarrow Q} \Leftrightarrow \sim P \cup Q)$

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2. $(P \rightarrow Q) \wedge (R \rightarrow Q)$ ~~is~~ eq $\Leftrightarrow (\underline{P \cup R}) \longrightarrow Q$. prove that

Sol:- $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \cup R) \longrightarrow Q$

$\cancel{(\sim P \cup Q) \wedge (\sim R \cup Q)}$ $(\because P \rightarrow Q = \sim P \cup Q$

$\cancel{(\neg P}$ (~~distributive law~~)

[~~$P \vee Q) \wedge (P \cup R)$~~]

$= (\sim P \vee Q) \wedge (\sim R \vee Q)$

$= (\sim P \wedge \sim R) \vee Q$ — [demorgan's law].

$= \sim(\underbrace{P \vee R}_{P}) \vee \underbrace{Q}_{Q}$ — $[P \rightarrow Q \Leftrightarrow \sim P \cup Q]$

$= (P \vee R) \longrightarrow Q$ //.

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3) $\sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$

$\neg(\sim P \cup Q)$ $(P \rightarrow Q \Leftrightarrow \sim P \cup Q)$

$P \wedge \sim Q$

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4) $P \to (Q \cup R) \Leftrightarrow (P \to Q) \cup (P \to R)$

Sol:- $\underbrace{P}_{P} \to \underbrace{(Q \cup R)}_{Q} \Rightarrow$ $\quad (P \to Q \Leftrightarrow \sim P \cup Q)$

$= \sim P \cup (Q \cup R)$ $\quad$ (distributive law)

$= (\sim P \cup Q) \cup (\sim P \cup R)$ $\quad (P \to Q \Leftrightarrow \sim P \cup Q)$

$= (P \to Q) \cup (P \to R)$

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5) $\sim(p \vee (\sim p \wedge q)) \Leftrightarrow \sim p \wedge \sim q$

$\sim(\underbrace{p}_{p} \vee (\underbrace{\sim p \wedge q}_{q}))$ ($\because$ distributive law)

(~~demorgan's law~~)

$\cancel{\sim p \wedge \sim(\sim p \wedge q)}$

$\cancel{\sim(p \vee \sim p) \wedge \sim(p \vee q)}$

$\cancel{\sim(T) \wedge (\sim p \wedge \sim q)}$

$\cancel{F \wedge (\sim(p \vee q))}$

$\sim((p \vee \sim p) \wedge (p \vee q))$

$\sim(\underbrace{T}_{P} \wedge (\underbrace{p \vee q}_{Q}))$ ($\because p \wedge q \Leftrightarrow q \wedge p$

$\sim((\underbrace{p \vee q}_{P}) \wedge (\underbrace{T}_{T}))$

($\because \sim(p \wedge T) \Leftrightarrow p$)

$= \sim(p \vee q)$ (demorgan's law

$= \sim p \wedge \sim q$ //

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6) $\sim p \wedge (\sim Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

$(\sim p \wedge \sim Q) \wedge R \vee (Q \wedge R) \vee (P \wedge R)$

$\sim(\underbrace{P \cup Q}_{P}) \wedge \underbrace{R}_{Q} \vee (\underbrace{Q \vee P}_{P}) \wedge \underbrace{R}_{Q}$ (distributive law

$(\sim P \wedge \sim Q) \wedge R$

$(\sim P \wedge R) \wedge (\cancel{R \wedge \sim Q}) \vee (Q \vee P) \wedge R$

(6) $\sim p \wedge (\sim Q \wedge R) \vee (Q \wedge R) \vee (P \vee R) \Leftrightarrow R$.

$\sim(p \vee Q) \wedge R \vee (Q \vee P) \wedge R$ (∵

$(\sim\underbrace{(p \vee Q}_{\sim P} \vee \underbrace{(Q \vee P}_{P}))\wedge R$ (∵ $\sim P \vee P = T$

$\underbrace{T}_{T} \wedge \underbrace{R}_{P} = R$. (∵ $P \wedge T \Leftrightarrow P$)

---

(7) $(p \wedge q) \longrightarrow (p \vee q) \Leftrightarrow T$

$\sim(p \wedge q) \vee (p \vee q)$ ∵ $P \rightarrow Q = \sim P \vee Q$

$(\sim p \vee \sim q) \vee (p \vee q)$

$\cancel{(\sim p \vee \sim q \vee p)}\cancel{(\sim p \vee \sim q \vee q)}$

$\sim p \vee \sim q \vee p \vee q$

$T \vee T = T$ //.

---

(8) $A \rightarrow (P \vee C) \Leftrightarrow (A \wedge \sim P) \rightarrow C$

Sol:- $\underbrace{A}_{P} \rightarrow \underbrace{(P \vee C)}_{Q}$

$\sim A \vee (P \vee C)$

$(\sim A \vee P) \vee C$

$\sim (A \wedge \sim P) \vee C$.

$(A \wedge \sim P) \rightarrow C$.

q) $\sim(p \rightleftarrows q) \Leftrightarrow (p\vee q)\wedge\sim(p\wedge q)$

$\sim[(p\to q)\wedge(q\to p)]$

$\cancel{\sim[(\sim p\cup q)\wedge(\sim q\cup p)]}$

$\cancel{\sim[\sim(\sim p\cup q)\vee(\sim q\vee p)]}$

$\sim[(p\to q)\vee\sim(q\to p)]$ (∴ ~~demorgan's~~ law

$\cancel{\sim[(\sim p\cup q)\cup(\sim q\cup p)]}$; $\cancel{(\sim\sim p\wedge\sim q)\cup(\sim\sim q\wedge\sim p)}$

$\sim[\cancel{(p\wedge\sim q)\vee(q\wedge\sim p)}]$; $\cancel{(p\wedge\sim q)\cup(q\wedge\sim p)}$

; $\cancel{(p\wedge\sim q\vee q)\wedge(p\wedge\sim q\cup\sim p)}$

$\sim[(p\to q)\wedge(q\to p)]$

$\sim[(\sim p\cup q)\wedge(\sim q\cup p)]$

$\Rightarrow \sim(\sim p\vee q)\vee\sim(\sim q\vee p)$

$\Rightarrow (p\wedge\sim q)\vee(q\wedge\sim p)$

$\Rightarrow \cancel{p\wedge\sim q\vee q}\ [(\underbrace{p\wedge\sim q}_{p})\vee\underbrace{\sim p}_{q}]\wedge[(p\wedge\sim q)\vee q]$

$\Rightarrow (p\vee q)\wedge\sim(p\wedge q)$

<u>Law of Duality</u> :- Two formulas A and A^* are said to be duals of each other if either one can be obtained from the other by repeating and by "$\wedge$ by $\vee$ and $\vee$ by $\wedge$" the connectives and conjuction and
$(\wedge)\ (\vee)$ $(\wedge)$ & $(\vee)$

are duals of each other. If the formula 'A' contains special ~~forms~~ variables 'T' (or 'F'. n+ it's dual is obtained by replacing T by F and F by T in addition to the above mentioned
T/F
F/T
make interchanges.

Ex:- $A: \sim(P \vee Q) \wedge (P \vee \sim(Q \wedge \sim R))$

$A^*: \sim(P \wedge Q) \vee (P \wedge \sim(Q \vee \sim R))$

Normal forms :-

By constructing and comparing the tables we can determine wether 2 statements 'A' and 'B' are equipment. but this is very difficult method to follow on a computer because the no. of entries increases rapidly. a better method is to transform the statement formula 'A' and 'B' to same standard form $\bar{A}$ and $\bar{B}$ such that simple comparison of $\bar{A}$ and $\bar{B}$ shows $A \Leftrightarrow B$ the standard forms are called normal forms (or) commercial forms.

let $A(P_1, P_2, P_3, \ldots P_n)$ be a statement formulas where $P_1, P_2, P_3, \ldots P_n$ are the atomic varities by 'A' has truth values T for atleast one one combination of truth values assigned to $P_1, P_2, P_3 \ldots P_n$ then A is said to be satisfied.

The following problem of determining in a finite no. of steps whether a given statement formula is tautology / contradiction (or) atleast satisfy is known as "decision problem".

* A formula which is a product of conjuction of their variable and their negation is called an elementary product

* if p and q are atomic variables then

$p \wedge q$, $p \wedge \sim q$, $7p \wedge q$, $\sim p \wedge \sim q$

The sum of variable and their negation in a formula is called an elementary sum

→ if p and q are any 2 variables $p \vee q$, $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee \sim q$ are element are sums

<u>Sums</u>

1) <u>DNF</u> (<u>Disjunction</u> <u>Normal form</u>).

A statement is in DNF if it is Obtained by operation (or) among variables statement connected with <u>AND's</u> (or) <u>sum of elementary products</u>

ex: $(p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge Q)$.

product sums elementary product

A formula "A which is equipment to A" is called DNF of A if B is a sum of elementary products a d.N.F of a given formula is constant as follows

1) Replaces $(\rightarrow)$, $(\leftrightarrow)$ by using the logical connectiones

2) Use demorgans law to eliminate negation before sum (or) products.

3) Apply distributive law repeatirtly and elimin product of variables to obtain the require normal form.

** CNF (conjuction Normal form)

A compound statement is in CNF. by it is obtained by operating $(\wedge)$ among variables, statements conected with $(\vee)$ (or) sum of products of elemenating sums.

$$(p \vee q) \wedge (q \cup \sim p)$$

if formula 'B' which is equivalent to 'A' is called CNF if 'B' is a product of eliminativ sums

① Ex :- obtain DNF for the rules

$$p \wedge (p \rightarrow q)$$

$$\Rightarrow p \wedge (\sim p \wedge q)$$

$$\Rightarrow (p \wedge \sim q) \cup (p \wedge q)$$

Q. Show that $\sim(p \to (Q \wedge R))$ DNF

Sol:- $\sim(\underset{p}{\cancel{p \to}} (\underset{q}{Q \wedge R}))$

$\cancel{p \wedge \sim(Q \wedge R)}$

$\sim(\sim p \vee (Q \wedge R))$

$p \wedge \sim(Q \wedge R)$

$p \wedge (\sim Q \vee \sim R) \Rightarrow (p \wedge \sim q) \vee (p \wedge \sim R)$

$\sim(p \to Q) \Rightarrow p \wedge \sim Q$

$= \sim p \vee Q$

Q. $p \to [(p \to q) \wedge \sim(\sim \overset{q}{p} \vee \sim \overset{p}{q})]$. DNF

Sol:- $p \to [(\sim p \vee q) \wedge (\sim \overset{q}{p} \wedge \overset{p}{q})]$.

$\cancel{p \to [(\sim p \vee q \wedge p) \wedge (\sim p \vee q \wedge q)}$

$\sim p \vee \sim[\sim p \vee q] \wedge [\overset{q}{p} \wedge \overset{p}{q}]] \Rightarrow \sim p \vee [p \wedge \sim q] \wedge [\sim p \vee \sim q]$

$\sim p \vee [\sim p \wedge (q \wedge p) \vee (q \wedge (q \wedge p))]$

$q \wedge q = q$. distributive law

$\sim p \vee [F \vee (q \wedge p)]$

$\sim p \wedge p \Rightarrow F$

$F \wedge$ stow $= F$

$\sim p \vee [q \wedge p]$,,

Q. $p \wedge \sim(Q \wedge R) \vee \sim(p \to q)$

$\sim(\sim p \vee Q)$

$(p \wedge \sim Q)$

$p \wedge \sim(Q \wedge R) \vee (p \wedge \sim Q)$

$p \wedge (\sim Q \vee \sim R) \vee (p \wedge \sim Q)$.

$(p \wedge \sim Q) \vee (p \wedge \sim R) \vee (p \wedge \sim Q)$

$= (p \wedge \sim Q) \vee (p \wedge \sim R)$,,

$\rightarrow [p \wedge \sim(Q \vee \cancel{Q}\gamma)] \cup [((p \wedge q) \vee \sim \gamma) \wedge p]$

$[p \wedge \sim(Q \cup \gamma)] \cup [((p \wedge q) \vee \sim \gamma) \wedge p]$.

$(\cancel{p \wedge} \sim \cancel{Q}) \cup \cancel{(p \wedge \gamma)} \cancel{\cup} [\cancel{(}p$

$(p \wedge (\sim Q \wedge \sim R)) \cup [((p \wedge q) \vee \sim R) \wedge p]$.

$(p \wedge \sim Q \wedge \sim R) \cup [(p \wedge q \wedge p) \cup (\sim R \wedge p)$

$(p \wedge \sim Q \wedge \sim R) \vee (p \wedge q) \vee (\sim R \wedge p))$

$\rightarrow [Q \vee (\cancel{p} \wedge R) \wedge \sim((p \vee R) \wedge q)]$.

$[(Q \overset{\vee}{\cancel{\wedge}} p) \wedge (\cancel{Q} \overset{\vee}{\cancel{\wedge}} R) \wedge \sim (\cancel{p \wedge} q) \vee (\cancel{R \wedge} q)]$

$\cancel{(Q \vee p \wedge Q)} \vee (\cancel{Q \vee p} \wedge R)$

$(\cancel{Q \vee} p)$

$\cancel{Q \vee (p \wedge R) \wedge (\sim p \wedge \sim R) \overset{\vee \sim q}{\wedge q})]}$

$\cancel{Q \wedge \sim p \wedge \sim R \vee \sim q) \vee (p \wedge R \wedge \sim p \wedge \sim R) \vee \sim q}$

$(\cancel{T Q \wedge \sim p \wedge \sim R})$

$\rightarrow (Q \vee (p \wedge R) \wedge \sim((p \vee R) \wedge q)]$

$Q \vee (p \wedge R) \wedge (\sim p \wedge \sim R) \vee \sim q)]$

$Q \wedge (\sim p \wedge \sim R) \vee \sim q)] \vee [(p \wedge R) \wedge [\sim p \wedge \sim R) \vee \sim q]$

$(Q \wedge \sim p \wedge \sim R) \vee (Q \wedge \underset{F}{\sim q})] \vee [\underset{F}{p \wedge \sim p} \wedge \sim \gamma \wedge \underset{F}{\gamma}] \vee (p \wedge$

$\Rightarrow (q \wedge \sim p \wedge \sim R) \vee (p \wedge \gamma \wedge \sim q)$,

* $p \wedge (p \to q)$

$p \wedge (\sim p \vee q)$

* $\neg(p \vee q) \rightleftarrows (p \wedge q)$ CNF.

$\cancel{(p \wedge q) \to \sim(p \vee q)}) \wedge (\sim\cancel{(p \vee q) \to (p \wedge q)}))$

$\sim(\cancel{(p \wedge q) \vee (\sim(p \vee q))} \wedge ($

$P \to Q \wedge Q \to P$

$\neg(\underbrace{p \vee q}_{P}) \longrightarrow (\underbrace{p \wedge q}_{Q}) \wedge (p \wedge q) \to \sim(p \vee q)$

$(\sim p \wedge \sim q) \to (p \wedge q) \wedge (p \wedge q) \longrightarrow (\sim p \wedge \sim q)$

$\sim(\sim(p \vee q) \vee (p \wedge q)) \wedge ((p \wedge q) \vee (\sim(p \vee q))$

$\cancel{(p \vee q) \wedge \sim(p \wedge q)} \wedge \cancel{((p \wedge q) \vee (p \vee q)}))$

$\cancel{(p \wedge q)} \wedge \cancel{(\sim p \vee \sim q)} \wedge \cancel{(\sim p \vee \sim q)} \vee (p \wedge$

$(p \vee q) \vee (p \wedge q)) \wedge (\sim(p \wedge q) \vee \sim(p \vee q))$

$(p \vee q) \vee (p \wedge q) \wedge ((\sim p \vee \sim q) \vee (\sim p \wedge \sim q))$

$(p \vee q \vee p) \wedge (p \vee q \vee q) \wedge ((\sim p \vee \sim q \vee \sim p) \wedge$

$(\sim p \vee \sim q \vee \sim q)$

$\Rightarrow (p \vee q) \wedge (p \vee q) \wedge (\sim p \vee \sim Q) \wedge (\sim p \vee \sim Q)$

$(p \vee Q) \wedge (\sim p \vee \sim Q)$

CNF

* $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$(q \vee p) \wedge \underbrace{(q \vee \sim q)}_{T} \vee \sim (p \vee q)$

$\underline{(q \vee p)} \vee \sim \underbrace{(\sim p \hat{\vee} q)}_{\sim p} \wedge (T) \Rightarrow (q \vee p \vee \sim p) \wedge (p \vee q \vee$

$\wedge (q \vee$

$\boxed{p \vee \sim p = T}$

$T \wedge T = T$

---

* $[q \vee (p \wedge r)] \wedge \sim ((p \vee r) \wedge q))$ CNF

$[(q \vee p) \wedge (q \vee r)] \wedge \sim ((p \wedge q) \vee (q \wedge r)]$

demorgan

$\Rightarrow (q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim q \vee \sim r)$

---

* find the DNF of the form $(\neg p \rightarrow R) \wedge (p \rightleftarrows q)$

sol:- using truth table $\boxed{\text{DNF} - \text{True's}}$ $\boxed{\text{false} = \text{"}q\text{"}}$

P	q	R	$\sim p$	$\sim p \rightarrow R$	$p \leftrightarrow q$	$(\neg p \rightarrow R) \wedge$
T	T	T	F	T T	T	T
T	F	F	F	T T	F	F
F	T	F	T	T F	F	F
F	F	T	T	F T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	F	T	F	T	F

by truth table consider 'T' values, either or its is false '~' will apply to 'F' ones.

$$(p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge R) \vee (p \wedge q \wedge \sim R)$$

---

* truth table

$\boxed{\text{CNF} = \text{False}}$ "True = ~"

p	q	r	(A)
T	T	T	F ✓
T	T	F	F ✓
T	F	T	T
F	T	T	F ✓
T	F	F	T
F	T	F	T
F	F	T	F ✓
F	F	F	T

$(\sim p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee r) \wedge$

$(p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r$

NOTE :- The DNF (or) CNF of a statement formula is not unique. In order to arrive a unique normal form of a given formula we introduce to 2 more Normal forms. PDNF & PCNF.

* **PDNF** :- A minterms consist of conjuctions in which each statement variable (or) its negation but not both appears only one's(or) once.

For example, two variables p and q there are 2^2 minterms $(p \wedge q)$, $(p \wedge \sim q)$, $(\sim p \wedge q)$, $(\sim p \wedge \sim q)$.

Def :- An equivalent formula consisting of disjunction of minterms only is known as PDNF (sum of products canonical form)

Obtain PDNF given formula :-

(1) bi- constructing truth table :-

for every truth value 'T' of the given formula select the minterm which also has the value 'T' for the same combination of the truth value of the statement variable.

Ex :-

p	q	$p \to q$
T	T	T ✓
T	F	F
F	T	T ✓
F	F	T

only 'T' value

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

(ii) without constructing truth table:

Step 1: Replace the conditionals & bi-conditional by there equivalent formulas containing '$\wedge, \vee, \sim$'.

Step 2: The negations are applied to the variables by using Demorgan's law's followed by the distributive law's

Step 3: Any elementary product which is contradiction is dropped. minterms are obtain in the disjunction by introducing the missing factors identical minterms. appearing to the disjunctions are deleted

Ex:- $p \rightarrow q$

$\sim p \vee q$ — applying missing term.

$\neg p \wedge (q \vee \sim q) \vee q \wedge (p \vee \sim p)$

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p) \vee (q \wedge \sim p)$

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

(ii) $(p \wedge q) \vee (\neg p \wedge R) \vee (R \wedge q)$

$= (p \wedge q) \wedge (r \vee \sim r) \vee (\neg p \wedge R) \wedge (q \vee \sim q) \vee$

$(r \wedge q) \wedge (p \vee \sim p)$

$= (p \wedge q \wedge r) \vee (p \wedge \overset{\sim R}{\cancel{q}} \wedge \cancel{\sim}q) \vee (\sim p \wedge R \wedge q) \vee (\sim p \wedge r \wedge \sim q)$

$(p \wedge q \wedge R) \vee (\sim p \wedge q \wedge r)$.

$= (p \wedge q \wedge R) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge \sim R) \vee (\sim p \wedge \sim q \wedge R)$

(III) $P \vee (\neg p \wedge \sim q \wedge R)$

$P \wedge (Q \vee \sim Q) \cancel{\vee r \vee \sim r}) \vee (\sim p \wedge \sim q \wedge R)$

$(p \wedge Q \wedge R) \vee (\cancel{p \wedge \sim Q \wedge \sim r}) \vee (\cancel{\sim p \wedge \sim q \wedge R})$

$(p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge \sim q \wedge R)$.

$= (p \wedge q) \wedge (R \vee \sim R) \vee (p \wedge \neg Q) \wedge (R \vee \sim R) \vee (\sim p \wedge \sim q \wedge R)$

$= (P \wedge q \wedge R) \vee (p \wedge q \wedge \sim R) \vee (p \wedge \sim Q \wedge R) \vee (p \wedge \sim q \wedge \sim R) \vee (\sim p \wedge \sim q \wedge R)$

$= (p \wedge q \wedge R) \vee (p \wedge q \wedge \sim R) \vee (p \wedge Q \wedge R) \vee (p \wedge \sim q \wedge \sim R) \vee (\sim p \wedge \sim q \wedge R)$.

(IV) $P \rightleftarrows q$

(V) $(q \wedge \sim r \wedge \sim s) \vee (R \wedge s)$

a) $P \rightleftarrows q$

$= (p \to q) \wedge (q \to p)$

$= (\neg p \vee Q) \wedge (\neg q \vee p)$

$= (\neg p \wedge \neg Q) \vee (p \wedge Q)$

$\because P \rightleftarrows q = p \to q \wedge q \to p$

$\because P \to q = \neg p \vee Q$

b) $(q \wedge \neg r \wedge \neg s) \vee (r \wedge s)$ $\quad \because q \vee \sim q = T$

$= (q \wedge \neg r \wedge \neg s) \vee (r \wedge s \wedge (q \vee \sim q))$ $\quad \because p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

$= (q \wedge \neg r \wedge \neg s) \vee (r \wedge s \wedge q) \vee (r \wedge s \wedge \sim q)$ Distributive law

* $p \rightarrow ((p \rightarrow q) \wedge \neg(\neg q \vee \neg p))$

$\sim p \vee ((p \rightarrow q) \wedge \neg(\neg q \vee \neg p)))$

$\sim p \vee ((\sim p \vee q) \wedge (q \wedge p))$

$(\underbrace{\cancel{\sim p \vee \sim p}}_{T}) \vee (\underline{\cancel{\sim p \vee q}}) \wedge (\cancel{q \wedge p}))$

$(\cancel{\sim p \vee q}) \wedge (\cancel{q \wedge p}))$

$(\cancel{\sim p \wedge q \wedge p}) \vee (\cancel{q \wedge q \wedge p})$

$\cancel{\sim p \vee ((}\sim p \wedge q) \vee (\underbrace{\sim p \wedge p}_{F}) \vee (q \wedge q) \vee (q \wedge p))$

$\sim p \vee ((\cancel{\sim p \wedge q}) \vee \cancel{q \vee} (q \wedge \cancel{p}))$

$(\cancel{\sim p \vee \sim p} \wedge q) \vee (\cancel{\sim p \vee q}) \vee (\cancel{\sim p \vee q} \wedge p))$

$\sim p \vee ((\sim p \vee q) \wedge (q \wedge p))$

$\sim p \vee ((\sim p \wedge q) \wedge (\underbrace{\sim p \wedge p}_{T}) \vee (q \wedge p) \wedge (q \wedge p))$

$\sim p \vee ((\sim p \wedge q) \overset{\vee}{\cancel{\wedge}} (q \wedge (q \wedge p))$

$\sim p \vee (\sim p \wedge q) \overset{\vee}{\cancel{\wedge}} (q \wedge q \wedge p))$

$\sim p \vee ((\sim p \wedge q) \vee (q \wedge p))$

$(\sim p \wedge (q \vee \sim q)) \vee (\sim p \wedge q) \vee (q \wedge p))$

$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge q)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge q)$

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(2) $[\neg[(P \vee Q) \wedge R)] \wedge (P \overset{\vee}{\wedge} R)]$

$[(\sim P \wedge \sim Q) \vee \sim R] \wedge (P \overset{\vee}{\wedge} R)$

$(\neg P \wedge \neg Q \wedge P) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg R \wedge P) \vee (\neg R \wedge R)$

$\quad F \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad F$

$(\neg P \wedge \neg Q \wedge R) \vee (\neg R \wedge P) \wedge (Q \vee \neg Q)$

$(\neg P \wedge \neg Q \wedge R) \vee (\neg R \wedge P \wedge Q) \vee (\neg R \wedge P \wedge \neg Q)$

* S.T the following questions are equivalent given PDNF

(1) $P \vee (P \wedge Q) \Leftrightarrow P$

R.H.S. $P \vee (P \wedge Q)$

$P \wedge (Q \vee \neg Q) \vee (P \wedge Q)$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (P \wedge Q)$

$(P \wedge Q) \vee (P \wedge \neg Q)$

R.H.S. $P \wedge (Q \vee \neg Q)$

$(P \wedge Q) \vee (P \wedge \neg Q)$

L.H.S = R.H.S

(2) $(~~P \wedge Q) \vee (\neg P \wedge \neg Q~~) \Leftrightarrow ~~P \vee Q~~$

$P \vee (\neg P \wedge Q) \Leftrightarrow P \vee Q.$

L.H.S $P \wedge (Q \vee \sim Q) \vee (\neg P \wedge Q)$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

R.H.S $P \vee Q$

$P \wedge (Q \vee \neg Q) \vee (Q \wedge (P \vee \neg P))$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (Q \wedge \neg P)$

L.H.S = R.H.S

PCNF :- A MAX term consist of disjunction in which each variable (or) its negation but not both appears only once. For example, two variables P & Q are MAX terms given by

$\Rightarrow (P \vee Q), (\sim P \vee Q), (P \vee \neg Q), (\neg P \vee \neg Q)$

Definition :- An equivalent formula consisting of conjunction of MAX terms only is known as PCNF.

→ Every formula (which is not a tautology) has an equivalent PCNF which is unique ~~expect~~ except for the rearrangement of the factors in the MAX terms and conjunctions

→ by duality principle all the assertion made for the PDNF can also be made for the PCNF

Methods to obtain PCNF of a given formula :-

1) The methods for obtaining the PCNF for given

formula is similar to the PDNF

(2) If the PDNF & PCNF of a given formula 'A' containing in n' 'n' variables is known then the PDNF and PCNF of Negation 'A' will consist of disjunction (conjunction) of the remaining minterms (Max-terms). which do not appear in the PDNF and PCNF of 'A'

(3) From 'A' $\Leftrightarrow \neg\neg A$' can obtain the PCNF and PDNF of 'A' by repeated application of demorgan's law to the PCNF and PDNF of '$\neg A$'

(1) $P \wedge (P \to Q)$

sol:- $P \wedge (\neg P \vee Q)$

$(P \wedge \neg P) \vee (P \wedge Q)$ $\quad P \vee (Q \wedge \neg Q) \wedge (\neg P \vee Q)$

$\Rightarrow (P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee Q)$

(ii) $\neg(\underset{P}{\underline{P \vee Q}}) \Leftrightarrow (\underset{Q}{\underline{P \wedge Q}})$

$(P \to Q) \wedge (Q \to P)$

$(\neg(P \vee Q) \to (P \wedge Q)) \wedge ((P \wedge Q) \to \neg(P \vee Q))$

$\neg(\neg(P \vee Q) \vee (P \wedge Q)) \wedge (\neg(P \wedge Q) \vee \neg(P \vee Q))$

$(P \vee Q) \vee (P \wedge Q) \wedge (\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q)$

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$(p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\neg p \vee \neg q \vee \neg p) \wedge (\neg p \vee \neg q \vee \neg q)$

$(p \vee q) \wedge (p \vee q) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee \neg q)$

$(p \vee q) \wedge (\neg p \vee \neg q)$

(III) $(q \vee (p \wedge R)] \wedge \neg((p \vee R) \wedge q))$

(IV) $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

(III) III Sol:- $[q \vee (p \wedge R)] \wedge \neg[(p \vee R) \wedge q)]$

$(q \vee p) \wedge (q \vee R) \wedge (\neg p \wedge \neg R) \vee \neg q)$

$(q \vee p) \overset{\vee}{\wedge} (R \overset{\wedge}{\vee} \sim R) \wedge (q \vee R) \overset{\vee}{\wedge} (p \overset{\wedge}{\vee} \neg p) \wedge (\neg p \wedge \neg R)] \wedge$

$(\neg p \overset{\vee}{\wedge} \neg q) \wedge (\neg R \vee \neg q)$

$(q \vee p \vee R) \wedge (q \vee p \vee \neg R) \wedge (q \vee R \vee p) \wedge (q \vee R \vee \neg p) \wedge$

$(\neg p \vee \neg q \vee R) \wedge (\neg p \vee \neg q \vee \neg R) \wedge$

$(\neg R \vee \neg q \vee p) \wedge (\neg R \vee \neg q \vee \neg p)$.

$(p \vee q \vee R) \wedge (p \vee \neg R \vee q) \wedge (\neg p \vee q \vee R) \wedge (\neg p \vee \neg q \vee \neg R) \wedge$

$(\neg p \vee \neg q \vee R) \wedge (\neg p \vee \neg R \vee \neg q)$.

(V) $\neg(p \vee q) \rightleftarrows (p \wedge q)$

$[\neg(p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow \neg(p \vee q)$

$[(\neg p \wedge \neg q) \rightarrow (p \wedge q)] \wedge [p \wedge q \rightarrow \neg(p \vee q)]$

$(p \vee q) \vee (p \wedge q) \wedge (\neg p \vee \neg q \vee (\neg p \wedge \neg q)$

$(p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\neg p \vee \neg q \vee \neg p) \wedge (\neg p \vee \neg q \vee \neg q)$

$(p \vee q) \wedge (p \vee q) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee \neg q)$.

$(p \vee Q) \wedge (\neg p \vee \neg Q)$.

(vi) $(\neg p \to R) \wedge (q \rightleftarrows p)$

$(\neg(\neg p) \vee R) \wedge (q \to p) \wedge (p \to q)$

$(p \vee \overset{R}{q}) \wedge (\neg q \vee p) \wedge (\neg p \vee q)$

$(p \vee \overset{R}{Q}) \overset{v}{\wedge} (Q \wedge \neg Q) \wedge (\neg Q \vee p) \vee (R \wedge \neg R) \overset{v}{\wedge} (\neg p \vee q) \vee (R \wedge \neg R$

$\Rightarrow (p \vee R \vee Q) \wedge (p \vee R \vee \neg Q) \wedge (\neg Q \vee p \vee R) \wedge (\neg Q \vee p \vee \neg R) \wedge$
$(\neg p \vee Q \vee R) \wedge (\neg p \vee q \vee \neg R)$

$\to (p \vee Q \vee R) \wedge (p \vee R \vee \neg Q) \wedge (\neg Q \vee p \vee \neg R) \wedge (\neg p \vee Q \vee R) \wedge$
$(\neg p \vee q \vee \neg R)$.

(vii) $(p \wedge Q) \vee (\neg p \wedge q \wedge r)$

$(\underbrace{p \vee \neg p}_{T}) \wedge (p \vee q) \wedge (p \vee r) \wedge (Q \vee \neg p) \wedge (\underbrace{Q \vee q}_{Q}) \wedge$
$(q \vee r)$

$(p \vee q) \wedge (p \vee R) \wedge (Q \vee \neg p) \wedge Q \wedge (q \vee r)$

$(p \vee q) \wedge (p \vee R) \wedge (Q \vee \neg p) \wedge (Q \wedge Q \vee R)$

$(p \vee Q) \vee (R \wedge \neg R) \wedge (p \vee R) \vee (Q \wedge \neg Q) \wedge (Q \vee \neg p) \vee (R \wedge \neg R)$
$\wedge (Q \vee R) \vee (p \wedge \neg p)$

$= (\underline{p \vee Q \vee R}) \wedge (p \vee Q \vee \neg R) \wedge (\underline{p \vee R \vee Q}) \wedge (p \vee R \vee \neg Q) \wedge (\underline{Q \vee \neg p \vee R})$
$\wedge (Q \vee \neg p \vee \neg R) \wedge (\underline{Q \vee R \vee p}) \wedge (\underline{Q \vee R \vee \neg p})$

$(p \vee Q \vee R) \wedge (p \vee Q \vee \neg R) \wedge (p \vee R \vee \neg Q) \wedge (\underline{Q \vee \neg p \vee R})$
$\wedge (Q \vee \neg p \vee \neg R) \wedge (\underline{Q \vee R \vee \neg p})$

(VIII) $(p\wedge q)\vee(\neg p\wedge R)\vee(q\wedge R)$

$\Rightarrow (p\wedge q\wedge R)\vee(p\wedge q\wedge\neg R)\vee(\neg p\wedge R\wedge Q)\vee(\neg p\wedge R\wedge\neg Q)\vee$

$\underline{(p\wedge q\wedge R)}\vee(\underline{\neg p\wedge q\wedge R})$

PDNF

$A \Rightarrow (p\wedge q\wedge R)\vee(p\wedge q\wedge\neg R)\vee(\neg p\wedge R\wedge Q)\vee(\neg p\wedge R\wedge\neg Q).$

convert PCNF (or missing terms).

$\neg A = \neg[(\neg p\wedge\neg Q\wedge\neg R)\vee(\neg p\wedge Q\wedge\neg R)\vee(p\wedge\neg Q\wedge r)\vee(p\wedge\neg Q\wedge\neg R)$

$\Rightarrow \neg(\neg A) \Rightarrow$ PCNF $[(p\vee Q\vee R)\wedge(p\vee\neg Q\vee R)\wedge(\neg p\vee Q\vee\neg R)\wedge$

$(\neg p\vee Q\vee R)]$

(IX) $R\vee(\neg p\rightarrow(q\vee(\neg Q\rightarrow R))$

$p\vee(p\vee(q\vee(Q\vee R))$

$p\vee(p\vee(q\vee Q\vee R))$

$p\vee(p\vee q\vee R)$

$(p\vee p\vee q\vee R)$

$A \Rightarrow (p\vee q\vee R) \rightarrow$ PCNF

Missing terms

$\neg A = \neg[(\neg p\vee\neg Q\vee\neg R)\wedge(\neg p\vee\neg Q\vee R)\wedge(\neg p\vee Q\vee\neg R)\wedge$

$(p\vee\neg Q\vee\neg R)\wedge(\neg p\vee Q\vee R)\wedge$

$(p\vee Q\vee\neg R)\wedge(p\vee\neg Q\vee R)]$

$\neg(\neg A) \Rightarrow [(p\wedge Q\wedge R)\vee(p\wedge Q\wedge\neg R)\vee(p\wedge\neg Q\wedge R)\vee(\neg p\wedge Q\wedge R)$

$\vee(p\wedge\neg Q\wedge\neg R)\vee(\neg p\wedge\neg Q\wedge R)\vee$

$(\neg p\wedge Q\wedge\neg R)]$

* **RULES of inferance** :- logic is a study of inferen...

→ An inferance is a judgement derived from anothe judgement. The terms inferances and reasoning are used as synonyms.

→ logic is the branch of mathematics that tells whether and arguments is valid or invalid. In every argument there are two things.

(i) The premisis (set of elements)

(ii) The conclusion

→ The process of derivation by which one demor strats at a particular formula is a valid consequences of a given set of premises.

The two rules of inferances are called "Rule P ~~and~~ "Rule T"

Rule P :- A primises may be introduce at any point in the derivation

Rule T :- A formula 's' may be introduce in a deri vation. if 's' is a tautology implied by any one or more of the preceding form in the derivation.

Implication :-

I_1 $p \wedge q \Rightarrow p$

I_2 $p \wedge q \Rightarrow q$ (simplification).

I_3 $p \Rightarrow p \vee q$

I_4 $q \Rightarrow p \vee q$ addition.

I_5 $\neg p \Rightarrow p \rightarrow q$

I_6 $\neg q \Rightarrow p \rightarrow q$

I_7 $\neg(p \rightarrow q) \Rightarrow p$

I_8 $\neg(p \rightarrow q) \Rightarrow q$

I_9 $p, q \Rightarrow p \wedge q$

I_{10} $\neg p, p \vee q \Rightarrow q$ (disjunction syllogism).

I_{11} $p, p \rightarrow q \Rightarrow q$ (modus ponens)

I_{12} $\neg q, p \rightarrow q \Rightarrow \cancel{\neg q}, \neg p$ (modus tollens.

I_{13} $p \rightarrow q, q \rightarrow R \Rightarrow p \rightarrow R$ (hypothetical) syllogism)

I_{14} $p \vee q, p \rightarrow R, q \rightarrow R \Rightarrow R$ (dilemma)

E_{19} : $p \rightarrow (q \rightarrow R) \Leftrightarrow (p \wedge q) \rightarrow R$

E_{20} : $\neg(p \rightleftarrows q) \Leftrightarrow p \rightleftarrows \neg q$

E_{21} : $p \rightleftarrows q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

E_{22} : $p \rightleftarrows q \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$

1) S.T 'R' is a valid inferance from two primises $p \to q$, $q \to R$, P

sol:- 1) $p \to q$ Rule P

2) P Rule P

3) $P, P \to q$ I_{11}, ①, ②

4) q Rule T

5) $q \to R \Rightarrow R$ I_{11}, (4)(5) Rule

6) R. Rule T

(2) S.T 'MON' is a valid argument from two primises $7J \to (MON)$, $(HUG) \to 7J$, HVG

1) $7J \to (MVN)$, $(HVG) \to 7J$ Rule P

2) $(MVN) \to (HVG)$ (I_{13}) Rule T

3) HVG Rule P (I_{11}) $P, P \to q = q$

(4) MVN Rule T

(3) S.T $7P$ is a valid inferance from $p \to q$, $R \to 7q$, R.

sol:-

$$\boxed{\begin{array}{c} p \to q \\ R \to 7q \\ R \end{array}}$$

1) $R \to 7q, R$ Rule P

2) $7q$ Rule T

3) ~~p~~ $p \to q, \neg q$ Rule P

4) $\neg P$ Rule T (I_{12})

4) S.T 'R∨S' is valid from the primises C∨D, $(C \vee D) \to \neg H$, $(\neg H) \to (A \wedge \neg B)$, $(A \wedge \neg B) \to (R \vee S)$

1) $(C \vee D) \to \neg H$ P

2) $(\neg H \to (A \wedge \neg B)$ T I_{11} ① ②

(3) $(C \vee D) \to (A \wedge \neg B)$ P

(4) $(A \wedge \neg B \to (R \vee S)$ T I_{11} (3) (4)

(5) $(C \vee D) \to (R \vee S)$, $(C \vee D)$ P

(6) $R \vee S$ T I_{12} (5) (6)

(v) show that $R \wedge (P \vee q)$ is a valid conclution from the primises $p \vee q$, $q \to R$, $P \to M$, $\neg M$.

Sol:- (I) $\neg M$ Rule P

(II) $p \to m$ Rule P

(III) $\neg P$ Rule T I_{12} (I)(II)

(IV) $p \vee q$ Rule P

(V) $q \to R$ Rule P ~~I_{12} (IV)(V)~~

(VI) R Rule P I_{12} (IV) (V)

(VII) $P \vee q$ Rule p

(VIII) $R \wedge (p \vee q)$ Rule T I_9 (VII)(VIII).

6. Show that $S \vee R$ is a tautology implied by $p \vee q$, $p \Rightarrow R$, $q \Rightarrow S$

Sol:-

(I)	$p \vee q$	Rule P
(II)	$\neg p \Rightarrow q$	Rule T $(p \Rightarrow q \Leftrightarrow \sim p \vee q)$
(III)	$q \rightarrow S$	Rule P
(IV)	$\neg p \Rightarrow S$	Rule T $I_{13}(2, 3)$
(V)	$\neg S \Rightarrow p$	Rule T $(p \Rightarrow q \Leftrightarrow \sim q \Rightarrow \sim p)$
(VI)	$p \Rightarrow R$	Rule P
(VII)	$\neg S \Rightarrow R$	Rule T
(VIII)	$S \vee R$	Rule T

(7) Show that S is a valid inference from premises $p \Rightarrow \neg q$, $q \vee R$, $\neg S \rightarrow p$, $\neg R$

(I)	$q \vee R$	Rule P
(II)	$\neg q \rightarrow R$	Rule T
(III)	$p \Rightarrow \neg q$	Rule P
(IV)	$p \Rightarrow R$	Rule P
(V)	$\neg S \Rightarrow p$	
(VI)	$\neg S \Rightarrow R$	
(VII)	$\neg p \Rightarrow S$	
(VIII)	$\neg R$	

*

* show that $R \to s$ is a valid inferance from the premises $p \to (q \to s)$, $\neg R \vee p$, q

1) $p \to (q \to s)$ Rule P

2) $(p \wedge q) \to s$ Rule T E_{19}

3) $q \to s$ Rule T I_2

4 $\neg R \vee p$

5 $S, R \Rightarrow p, = S$

6. S

1)	$\neg R \vee p$	P
2)	R	P(additional)
3)	P	T
4)	$p \to (q \to s)$	P
5)	$q \to s$	T
6)	q	P
7)	S	T
8)	$R \to s$	CP

and

Rule CP :- If we can derived 's' from 'R' under the set of premises then we can derived $R \to s$ from the set of premises a lot.

→ Rule CP it also called reduction theorem and is generally used if the conculsion is of the form $R \to s$. In such case 'R' is taken as additional premises and s is derived from the given premises and using 'R'.

* show that $p \to q$ from the set of premises.

$R \to (s \to q)$.

$\neg p \vee R$

S

1) $(R \wedge S) \to q$

2) $\neg p \vee R$, p addition

3) S

4) R, $R \to (S \to q)$ (11)

5) S, $S \to q$ (4)

6) q

7) $p \to q$

* S.T $p \to S$.

1) $\neg p \vee q$ P

2) $\neg q \vee R$ P

3) $R \to S$. P

4) p addition.

5) q, $\neg q \vee R$ I_{10}

6) R

7) R, $R \to S$ I_{11}

8) S

9) $p \to S$ CP

*

* First order logic and other methods of proof rules :

certain declarative sentenance involved words that indicate quantity such as all, some, none etc... These words help to determine the answer to the question how many. since such words indicate quantity they are called "quantifiers"

1) The quantifier 'All' is called the universal quantifier and is denoted by "$\forall x$". It represents each of the following phrases, since they all essentially the same meaning

⇒ For all x, All x are such that, for every x, every x are such that, for each x, each x is such that.

2) The quantifier 'same' is a extantial quantifier and which is denoted by '$\exists x$'. it representes each of the following phrases.

⇒ There exist an x such that, for some x x is such that ; there is atleast 1 x such that, some x is such that

Ex

Example :- 1) All men are mortal   M(n) is a man
H(n) is a mortal

→ for all x, if 'x' is a man then x is a mortal].

$$\forall x[M(x) \rightarrow H(x)]$$

2) Every apple is Red.

→ every   M(x) : x is a apple
H(x) : x is a Red

every x is such that, x is a apple than x is a Red).

$$\forall x(M(x) \longrightarrow H(x)]$$

3) There exist a Man

M(x) : x is a man
H(x) : x is a exist

There exist an x, such that x is men

$$\exists x\ M(x).$$

4) some men are clever

M(x) : x is men
H(x) : x is clever

~~Some x is A~~

There exist an x, such that x is a men
x is clever

* All monkies have tails

(II) No monkies has a tails

(III) Some monkies have tails

(IV) Some monkies have no tails.

$M(x)$: x is a monkies

$H(x)$: x is tails

(I) ~~There exist an x,~~ For all x, ~~every~~ for 'x' is monkies, then x is tail.

$$\forall x\,[M(x) \rightarrow H(x)].$$

× (II) $\forall x\,[\neg M(x) \rightarrow \neg H(x)]$ ✓

(III) $\exists x\,[M(x) \wedge H(x)]$

(IV) $\exists x\,[\neg M(x) \wedge \neg H(x)]$.

Ex:

*(I) All men are good

(II) No men are good

(III) Some men are good

(IV) Some men are not good

(I) $\forall x\,(M(x) \rightarrow H(x)]$

(II) $\forall x\,[M(x) \rightarrow \neg H(x)]$.

(III) $\exists x\,[M(x) \wedge H(x)]$

(IV) $\exists x\,[\neg M(x) \wedge H(x)]$.

① show that $(\exists x) M(x))$ follows logically from the premises $(\forall x)(H(x) \rightarrow M(x))$ $(\exists x)H(x)$

Sol:-

Rules of inferance for quantified proposition :-

1. The elimination of quantifiers can we done by rules of specification. called "universal specification and extension specification".

2. To prefix the correct quantifier we need the rules of generalisation called UG and EG

(1) Rule US :- if a statement of the form ~~for all~~ $\forall x\, p(x)$ is assumed to be prove then the universal quantifier can be ~~brought~~ draw to obtain p of $p(t)$. is true for an orbitary object ~~x~~ t in the universe.

$$\therefore \frac{\forall x\, p(x)}{p(t)}$$

(II) Rule ES :- if a statement of the form $\exists x\, p$ is assumed to be prove then the there is an element 't' in the universe such that $p(t)$ is true

$$\therefore \quad \frac{\exists x\, p(x)}{p(t)}$$

(iii) Rule UG :- if a statement $p(t)$ is ~~prove~~ true for each element t of the universe then the universal quantifier may be prefix to obtain $\forall x(p(x))$

$$\therefore \quad \frac{p(t)}{\forall x\, p(x)}$$

(iv) Rule EG :- if a statement $p(t)$ is true for ~~so~~ some element 't' of the universe than the ~~e~~ $\exists x\, p(x)$ is true

$$\therefore \quad \frac{p(t)}{\exists x\, p(x)}$$

1. show that $\exists x\, M(x)$ follows logically from the premises $\forall x\,(H(x) \rightarrow M(x))$, $\exists x\, H(x)$

sol :-

1.	$\forall x(H(x) \rightarrow M(x))$	P
2.	$(H(t) \rightarrow M(t))$	US $\forall x\, p(x)$
3.	$\exists x\, H(x)$	P
4.	$H(t)$	ES
5.	$H(t), (H(t) \rightarrow M(t))$	T I_{11} (2)(4)
6.	$M(t)$	T I_{11} (2) (4)
7.	$\exists x(M(x))$	EG (6).

(2) consider the socrattise arguments given by

(i) all men are mortal

(ii) socrattise is a man

(iii) There fore, socrettis is a mortal.

sol:- $\forall x\,(M(x) \rightarrow H(x))$

$\exists x\,[M(x) \rightarrow \cancel{H}^{s}(x)]$

$\cancel{\forall x}\,(\cancel{S(x) \rightarrow H(x)})$

(1) for all x, if x is man than x is mortal

(ii) s:- socrattis

$M(s)$

(iii) $\therefore$, socrattise is a Mortal

$\therefore H(s)$

using (i),(ii) statements to prove (iii) statement

1. $\forall x\,(M(x) \rightarrow H(x))$ P

2. $M(s)$ P
I_{11} (1)+(2)

3 $\cancel{H(s)}$ $M(s) \rightarrow H(s)$ U.S

4. $H(s)$.

5) $\exists x\,(H(s))$

(3) Verify the Validity of the following arguments

(i) Tigers are dangerous animals

(ii) There are tigers

(iii) There are dangerous animals

(i) $M(x)$: 'x' is a tigers
$H(x)$: 'x' is a dangerous animals

for all x, if x is a tigers than 'x' is a dangeous animals

$$\forall x (M(x) \rightarrow H(x))$$

(ii) There are tigers

there exists x, such that x is a tigers.

$$\exists(x)\ M(s)$$

(iii) There are dangerous animals.

$$S(H).\ (\text{or})\ H(s)$$

using (i),(ii) statements to prove (iii) Statement

1)	$\forall x (M(x) \rightarrow H(s))$	P
2)	$M(s) \rightarrow H(s)$	U.S
3)	$M(s)$	P
4)	$H(s)$	I_{11} (3),(4)
5)	$\exists x\ H(s)$	EG

(4) Given argument which will established the validity of the following inferance.

5. (i) All integers are rational numbers

(ii) some integers are powers of 3

(iii) Therefore, some rational numbers are powers of '3'.

sol:- $M(x)$:- x is ...

$H(x)$:- x is an rational number

$R\cancel{S}(x)$:- 'x' is integer power '3'

$P\cancel{M}(x)$:- rational power '3'

~~$S(x)$~~ :-

(1) $\forall x\,(\overset{I}{\cancel{M}}(x) \rightarrow \overset{R}{\cancel{H}}(x))$

II) $\exists x\,(\overset{I}{S}(x) \wedge p(x))$

III) $\boxed{\exists x\,(R(x) \wedge p(x))}$ TO prove

1) $\forall x\,(\overset{I(x)}{\cancel{M}} \rightarrow \overset{R}{\cancel{H}}(x))$ P

2) $\overset{I}{\cancel{M}}(x) \rightarrow \overset{R}{\cancel{H}}(x)$ US

3) $\exists x\,(\cancel{I}(x) \wedge p(x))$ P

4) $I(x) \wedge p(x)$ U.S

5) $p(x)$ I_2 (4)

6) $p(x), \cancel{I}(x) \rightarrow R(x)$

7)

1) $\forall x\,(I(x) \rightarrow R(x))$ P

2). $I(\overset{t}{x}) \rightarrow R(\overset{t}{x})$ US

3) $\exists x\,(\overset{I}{\cancel{I}}(x) \wedge p(x))$ P

4 $\overset{I}{\cancel{R}}(\overset{t}{x}) \wedge p(\overset{t}{x})$

5. $p(x)$ $I(t)$ T I_1 (4)

6. $R(t)$ T I_{11} R(5)

7. $p(t)$ T I_2 (4)

T 8) $R(t) \wedge p(t)$

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(4) (i) Every living thing is a plant or an animal

(ii) John gold fish is alive and it is not a plant

(iii) all animals have hearts.

(iv) Therefore, John gold fish has a heart

(5) consider the arguments

1) All men are fallable

ii) All kings are men

(iii) Therefore, All kings are fallable

$\forall x [M(x) \Rightarrow H(x)]$

$[K(x) \rightarrow M(x)]$

$(K(x) \Rightarrow H(x))$

$M(x) \Rightarrow H(x)$

$K(x) \rightarrow M(x)$

$K(x) \Rightarrow H(x)$

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* Indireit method of proof :-

A set of formulas $H_1, H_2, \ldots$ is said to be consistant if there conju has the truth value 't' for some assignm orf truth values of atomic variable appearin in $H_1, H_2, \ldots H_n$.

→ A set of formulas is inconsistant if their conjection implies a contrdiction. That is

$H_1 \wedge H_2 \wedge \ldots \wedge H_m \Rightarrow R \wedge \neg R$

where R is any formula

> The techniques of indirect method of proof is as follows

1) Introduce the '$\neg$' of desired conclusion as a new premises. That is conclusion 'c' is false and consider $\neg c$ has additional premises.

2) From the additional premises (or) new premises together with given premises derived a contra diction. That is The new set of premises is inconsistant then they implie a contraction

3) ~~A set~~ Assert the derived conclusion is a logical inferance from the premises thus (follows logically from the premises

$H_1, H_2, \ldots \ldots H_n$

(1) By indirect proof show that $p \to q$, $q \to R$, $P \vee R$ conclude R.

sol:-

1) $P \to Q$	P	
2) $q \to R$	P	
3) $p \to R$	T	I_{13} (1)(2)
4) $P \vee R$	P	
5) $R \; P$	T	E_{14} ~~(4)(3)~~ I_3
6) $P, P \to R$	I_{11}	(5)(6)
7) R	↓ " " "	

wrong

1) $R \wedge \neg R$ - contradiction.

(or)

1) $q \to R$	P	
2) $\neg R$	(addition)	
3) $\neg q$	T	I_{12}
4) $p \to q$	P	
5) $\neg p$	T, I_{12}	
6) $P \vee R$	P	
7) R	T, I_{10}	

(2) prove by indirect method $\neg Q$, $P \to q$, $P \vee R$ to prove conclud R

$\neg R$	(addition)	
1) $\neg q$	P	
2) $p \to q$	P	
3) $\neg p$	T	I_{12} (1)(2)
4) $P \vee R$	P	
5) R	T	I_{10} (3)(4)
6) $R \wedge \neg R$ - contradiction		

1) $P \vee R$	P
2) $\neg R$	(addition)
3) P	T
4) $p \to q$	P
5) q	I_{11}
6) $\neg q$	P
7) $p \to q$	I_6
8) P	T (5)(6)
9) $P \wedge \neg P$	contradiction

1) $p \vee R$ Rule D

2) $\neg R$ additional Rule P

3) p I_{10}. (1) & (2) Rule T

4) $p \to Q$ Rule P

5) q I_{11}, Rule T

6) $\neg Q$ Rule P

7) $q \wedge \neg Q$ contradiction

***

(3) Use indirect method of proof

$p \to \neg s$ from $p \to (q \vee R)$

$q \to \neg p$

$s \to \neg R$

p

Sol:- $p \to \neg s$ - $\neg p \vee \neg s$ - $\neg(\neg p \vee \neg s)$

$\underline{p \wedge s}$ additional

1) $p \to (q \vee R)$ P

2) p P

3) $q \vee R$ T I_{10}. (1) (2)

4) $p \wedge s$ additional

5) s

6) $s \to \neg R$

7) $\neg R$

8) Q

9) $Q \to \neg p$

10) $\neg p$

11) $p \wedge \neg p$

P.S.T $\neg[p \wedge q]$ follows from $\neg p \wedge \neg q$

1) $\neg(p \vee q)$

2) $p \wedge q$ additional

3) q

4) $\neg q$

5) $\neg p$

6) p

7) $(p \wedge \neg p) \wedge (q \wedge \neg q)$

Atomic th

* Automatic theorem proving :-

In automatic theorem proving consists of 10 rules, axioms schema and rules of well-formed sequents and formulas.

1) Variables :- the captials letters A, B, C, ... are used as statement variables. They are also used as statement formulas.

2) connectives :- The connectives $\sim, \wedge, \vee, \rightarrow, \Leftrightarrow$ appear in the formula with the order of presecedence.

3) string of formulas :- A string of formulas is defined as follows.

1) Any formula is a string of formula

then 'β' and 'α' are also string of formulas

→ only those strings which are obtained by 1 and 2 are strings of formulas with exception of empty string which is also a string of formulas. (7)

→ **Sequents** :- if 'α' and 'β' are string of formulas then $\alpha \xrightarrow{S} \beta$ is called a sequent where α is anticident and 'β' is consequent.

→ in the same massive we shall use the symbol "$\xRightarrow{S}$" is applied to string of formulas.

(5) → **Axioms schemas** :- If 'α' and 'β' are strings of formula such that every formula in both 'α' and 'β' is a variable only then the sequents $\alpha \xrightarrow{S} \beta$ is an axiom if 'α' and 'β' have atleast one variable in common

(6) **Theorem** :- The following sequents are theorems of the our system

1) Every Axiom is a theorem
2) If a sequent 'α' is a theorem and a

sequent 'β' results from α through the use of one of the 10 rules of the system.

(3) Rules :-

The following 10 rules are used to combine formulas within strings by introducing connectives.

* Anticident Rules :-

Rule ¬ ⇒ if $\alpha, \beta \overset{s}{\Rightarrow} X, \gamma$ then $\alpha, \neg X, \beta \overset{s}{\Rightarrow} \gamma$

Rule ∧ ⇒ if $X, Y, \alpha, \beta \overset{s}{\Rightarrow} \gamma$ then $\alpha, X \wedge Y, \beta \overset{s}{\Rightarrow} \gamma$

Rule ∨ ⇒ if $X, \alpha, \beta \overset{s}{\Rightarrow} \gamma$ and $Y, \alpha, \beta \overset{s}{\Rightarrow} \gamma$ then

$\alpha, X \vee Y, \beta \overset{s}{\Rightarrow} \gamma$

Rule → ⇒ if $Y, \alpha, \beta \overset{s}{\Rightarrow} \gamma$ and $\alpha, \beta \overset{s}{\Rightarrow} X, \gamma$ then

$\alpha, X \rightarrow Y, \beta \overset{s}{\Rightarrow} \gamma$

Rule ⇄ ⇒ if $X, Y, \alpha, \beta \overset{s}{\Rightarrow} \gamma$ and $\alpha, \beta \overset{s}{\Rightarrow} X, Y, \gamma$

then $\alpha, X \rightleftarrows Y, \beta \overset{s}{\Rightarrow} \gamma$

* Consequent Rules :-

Rule ⇒ ¬ : if $X, \alpha \overset{s}{\Rightarrow} \beta, \gamma$ then $\alpha \overset{s}{\Rightarrow} \beta, \neg X, \gamma$.

Rule ⇒ ∧ : if $\alpha \Rightarrow \beta, X, \gamma$ and $\alpha \overset{s}{\Rightarrow} Y, \beta, \gamma$ then

$\alpha \overset{s}{\Rightarrow} \beta, X \wedge Y, \gamma$

$\underline{\text{Rule} \Rightarrow \vee}$:- if $\alpha \overset{s}{\Rightarrow} x, y, \beta, \gamma$ then $\alpha \overset{s}{\Rightarrow} \beta, x \vee y, \gamma$

$\underline{\text{Rule} \Rightarrow \rightarrow}$: if $x, \alpha \overset{s}{\Rightarrow} y, \beta, \gamma$ then $\alpha \overset{s}{\Rightarrow} \beta, x \rightarrow y, \gamma$

$\underline{\text{Rule} \Rightarrow \rightleftarrows}$: if $x, \alpha \overset{s}{\Rightarrow} y, \beta, \gamma$ and $y, \alpha \overset{s}{\Rightarrow} x, \beta, \gamma$

then $\alpha \overset{s}{\Rightarrow} \beta, x \rightleftarrows y, \gamma$

* show that $p \Rightarrow (\neg p \rightarrow q)$

$\neg p, p \overset{s}{\Rightarrow} q$

$$\frac{p \Rightarrow q, p}{\text{axiom}}$$

* $R \overset{s}{\Longrightarrow} p \vee \neg p \vee q$

$R \Rightarrow p, \neg p, q$ (Rule $\Rightarrow \vee$) then (1) (consequent)

if $R, p \overset{s}{\Rightarrow} p, q$ (Rule $\Rightarrow \neg$)

$\neg(p \wedge q) \overset{s}{\Rightarrow} \neg p \vee \neg q$ check whether its axiom (or) not

$\neg(p \wedge q) \Rightarrow \neg p, \neg q$

$\neg(p \wedge q), p, q \Rightarrow$

$\neg p, \neg q, p, q \Rightarrow$

$p, q \Rightarrow p, q$