UNIT IV Relations and digraphs.

Let A and B are two sets a sub set $A \times B$ is called a binary relation or a relation from A to B.

1. $R \subseteq A \times B$ then R is a relation from $A \to B$.
2. If A, B $(a,b) \in R$ is also can be written as aRb

Domain and Range :-

Let R be a relation from A to B.

1) Domain of R is denoted by 'dom R' and is defined as $\{a / a \in A;\ (a,b) \in R$ for some $b \in B\}$

2) Range :- Range of R is denoted by 'ran R' and is defined as $\{b / b \in B,\ (a,b) \in R$ for some $a \in A\}$.

Domain of $R \subseteq A$.

Range of $R \subseteq A$

Properties of a relation :-

A relation R on set 'a' is said to be

1) **Reflexive property** :- If x related to s (or) $(x,x) \in R\ \forall\ x \in A$

2) Irreflexive relation : If x is not related to x

$x \not R x$ (or) $(x,x) \notin R \ \forall x \in A$

3) symmetric relation :- If x related y, y related

xRy, yRx for all $x, y \in A$.

4) Transitive relation :- if $xRy, yRz, zRx \ \forall$

$x, y, z \in A$

5) a Asymmetric relation :-

if xRy, $y \not R x$ (or) $(y,x) \notin R \ \forall \ x, y \in A$

* Operations :-

1. let R and S be relations from A to B then

(i) $R \cup S$

(ii) $R \cap S$

(iii) $R - S$ (difference)

(iv) R'

(i) $R \cup S$: $\{(a,b) \in A \times B \mid (a,b) \in R$ (or) $(a,b) \in S\}$

is the union of a relation R and S

i.e, $(a,b) \in R \cup S \Leftrightarrow (a,b) \in R$ (or) $(a,b) \in S$

(ii) $R \cap S$: $\{(a,b) \in A \times B \mid (a,b) \in R$ (and) $(a,b) \in S\}$

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iv) $R-S$: $\{(a,b) \in A \times B \mid (a,b) \in R, (a,b) \notin S\}$

v) R' : $\{(a,b) \in A \times B \mid (a,b) \notin R\}$

Equivalence Relation :-

A relation R on set 'A' is said to be equivalence relation ~~of~~ on A if ~~R is a~~

i) R is reflexive on A

ii) R is symmetric on A

iii) R is transitive on A.

Inverse relation :-

Let R be a relation from A to B inverse of relation R from b to a is denoted by R^{-1} and is defined as

$$R^{-1} = \{ (b,a) / (a,b) \in R \}$$

Let $A = \{2,3,4\}$, $B = \{3,4,5,6,7\}$ define a relation R from A to B by ~~A, B~~ $(a,b) \in R$ if a/b

R = a/b difficible elem divisible elements.

$\{(2,4)(2,6)(3,3)(3,6)(4,4),(4,6)\}$

* **Representation of a Relation :-**

In this section we will discuss 2 alternative methods for representing relations, they are

1. Matrix method
2. directed graph method.

1. **Matrix method :-** A binary relation are from set A with n elements to a set B with m elements is represented by n x m matrix called relation matrix. it is denoted by $M_R[a_{ij}]$ where

$$[a_{ij}] = \begin{cases} 1 & \text{if } i^{th} \text{ element of A is related to } j^{th} \text{ element of B that is } (a_i, a_j) \in R \\ 0 & (a_i, a_j) \notin R \end{cases}$$

Example :- let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ the relation are from A to B is given by

$R = \{ ~~1, a~~ \}$ $\{ (1,a) (1,d) (2,b) (2,c) (3,a) (3,c) \}$

Sol:-

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

1) $A = \{1, 2, 3, 4, 5, 6\}$ define relation 'R' has less than on A. then find relation

sol:- $R = \{(1,2)(1,3)(1,4)(1,5)(1,6),(2,3),(2,4)(2,5)$
$(2,6)(3,4)(3,5)(3,6)(4,5)(4,6)(5,6)\}$

$$M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

2) $A = \{a_1, a_2\}$, $B = \{b_1, b_2, b_3, b_4, b_5\}$ and the relation matrix $M_R = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$

$$\begin{array}{c|c} & \begin{matrix} b_1 & b_2 & b_3 & b_4 & b_5 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$R = \{(a_1,b_2),(a_1,b_4),(a_2,b_1),(a_2,b_3),(a_2,b_4)\}$

* A relation matrix ~~replace~~ reflex some of the properties by of a relation for the matrix of a relation on a set which is square matrix can be used to determine whether the relation has certain properties.

1. R is reflexive if all the elements in the main diagonal M_R are equal 1

2. R is symmetric if $a_{ij} = a_{ji}$ $\forall i,j$

$$M_R = (M_R)^T$$

ex:- 1. suppose the relation R on a set is represented by the matrix $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is symmetric (or) and R is reflexive

sol:- diagonal are 1 then it is reflexive and

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \boxed{1} & \boxed{1} \\ 0 & \boxed{1} & 1 \end{bmatrix}$$

symmetric

R

2) digraph of a relation.

a relation can also be represented pictorially by drawing its graph. The procedure for drawing digraph of a relation is given as follows

let R be a relation on set $A = \{x_1, x_2, \ldots\}$

The elements of A are represented by pt an arrow is drawn from the vertex x_i to the vertex x_j, if x_i is related to x_j. this is called an directed edge.

→ an element of the form (x, x) in a relation corresponds to a directed edge from $(x$ to $x)$ such an edge is called "loop".

Q: Draw the directed graph of a relation

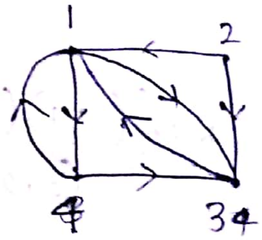
$R = \{(1,1)(1,3)(2,1)(2,3)(2,4)(3,1)(3,2)(4,1)\}$

set $A = \{1,2,3,4\}$

Sol:-

Identify the relation

set $A = \{1,2,3,4\}$

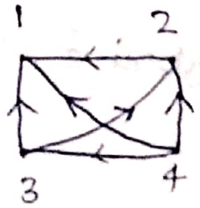


Sol: $R = \{(1,3)(1,4)(2,1)(2,3)(3,1)(4,1)(4,3)\}$

Let $A = \{1,2,3,4\}$ and $R = \{(x,y) / x > y\}$

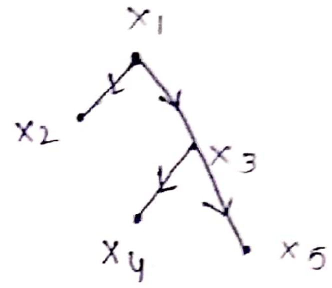
Draw the graph of R and also matrix.

$R = \{(2,1)(3,1)(3,2)(4,1)(4,2)(4,3)\}$



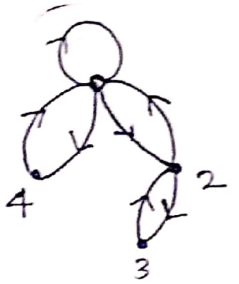
$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

4. determine the properties of relation given by the graph shown below and also write the corresponding relation matrix.



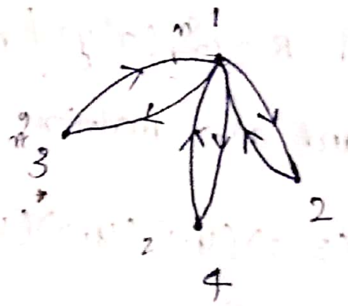
$R_1 = \{(x_1,x_3)(x_1,x_5)(x_1,x_2)(x_3,x_4)$
$(x_3,x_5)(\quad \}$

prop. ~~sym~~ irreflexive, Transitive



$R = \{(1,1)(1,2)(1,4)(2,1)(2,$
$(3,2)(4,1)\}$

reflexive, symmetric, ~~irreflexive~~, Asymmetric



$R = \{(1,2)(1,3)(1,4)(2,1)$
$(3,1)(4,1)\}$

Symmetric, irreflexive,

Let P be a partition of set A define a relation R on A as aRb then a and b are members of the same block.

Then R is an equivalence relation on 'a'.

(i) if $a \in A$ then a & a are in the same block so aRa

(ii) if aRb then a and b are in the same block then bRa

(iii) If aRb and bRa. bRc. (a, b, c) are in the same block. then aRc.

Let $A = \{1, 2, 3, 4\}$ and partition $P = \{\{1,2,3\}\{4\}\}$ be a partition of A, find the equivalence relation determined by P.

Sol:-

$$A = \{1, 2, 3, 4\}$$

$$P = \{\underbrace{\{1, 2, 3\}}_{P_1} \underbrace{\{4\}}_{P_2}\}.$$

$$R = \{(1,1) (2,2) (3,3)$$

Reflexive

$$(1,2)(2,1) (1,3) (3,1) (2,3) (3,2)$$

Symmetric

$$(4,4)\}$$

Transitive -

(a) let $A = \{1,2,3,4\}$, $R = \{(1,1)(1,2)(2,1)(2,2)$
$\cancel{(2,3)}(3,4)(4,3)(3,3)(4,4)\}$

be an relation R. determine A/R.

Sol:-

related

$1 = \{1,2\}$

$3 = \{3,4\}$

$P = \{\{1,2\}\{3,4\}\}$

$A/R = [1][3]$

General procedure for construction of A/R :-

procedure :-

1. choose $a \in A$ and find $[a]$
2. If ~~a∉~~ $a \neq A$ the choose $b \in A$ and $b \notin [a]$ and find b ?
3. If $[a] \cup [b]$ is not equal to A choose $c \in a$ and $c \in [a] \cup [b]$ and find c
4. Repeat steps until all elements ~~are~~. of a R included in the equivalence class.

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① $A = \{1,2,3,4,5,6\}$, $R = \{(1,1)(1,5)(2,2)(2,3)(2,6)(3,2)(3,3)(3,6)(4,4)(5,1)(5,5)(6,2)(6,3)(6,6)\}$

determine partition and A/R

Sol:- ~~r = {1,2,3,4~~

$1 = \{1,5\}$

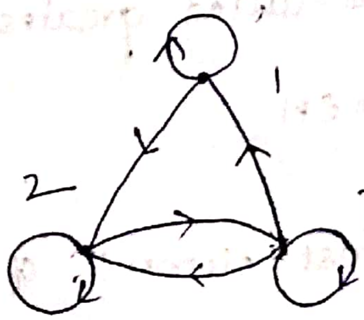
$2 = \{2,3,6\}$ $\qquad P = \{\{1,5\}\{2,3,6\}\{4\}\}$

$4 = \{4\}$

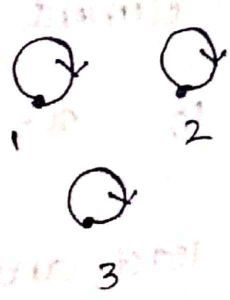
$A/R = [1][2][4]$.

② find whether the relations for the directed graph shown in the following figure reflexive, symmetric transitive and also write corresponding relation matrix.

①

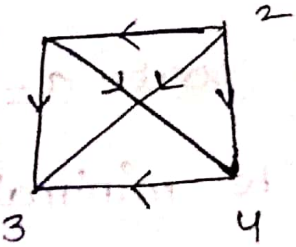


②



(1,1)(1,2)(2,2)(2,3)(3,2)(3,3)(3,1) of

③



(ii) (1,1)(2,2)(3,3)

(iii) (1,3)(1,4)(4,3)(2,3)(2,1)(2,4)

1. reflexive, ~~symmetric~~, transitive.

2. reflexive.

3

* compatability Relation :-

The relation R on set n is compalability relation. if and only if its Satisfies two properties Reflexive and symmetric

a compatabilitt represented by ≈

Ex:- let a = {1,2,3} let the relation R = {(1,1)(2,2)(3,3)(2,3)(1,2)(2,1)(3,2)(1,3)(3,1)} define on A. check this relation compalability relation or not.

Sol:-

→ **warshall's algorithm :-**

warshall's algorithm is an efficient Method for computing transitive closer of a relation

① A = {a,b,c,d} and relation R = {(a,b)(b,a),(b,c)(c,d)} find the transitive closer

Sol:-

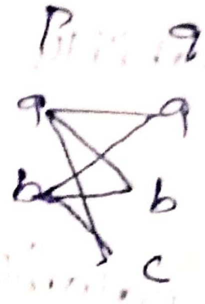
$$
W_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}
$$

p q

b b

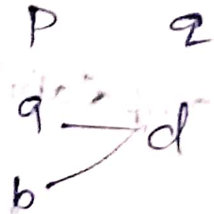
(b, b)

$$W_1 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$



(a,a)(a,b)(a,c)(b,a)
(b,c)

$$W_2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 0 \\ b & 1 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$



(a,d)(b,d)

$$W_3 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

p	q
a	ϕ
b	
c	

$$W_4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

R = {(a,b) (b,a) (b,c) (c,d) (a,d) (b,b) (b,d) (a,a) (a,c) }

A = {a, b, c, d} and R = {(a,d)(b,a)(b,c)(c,a)(c,d)(d,c)} (2)

find transitive closure

$$W_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 0 & 1 & 0 \end{array}$$

p q

b d

c

(b,d) (c,d)

$$W_1 = \begin{array}{c|cccc} & & & & \\ \hline a & 0 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 0 & 1 & 0 \end{array}$$

p q

ϕ a

c

d

$$W_2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 0 & 1 & 0 \end{array}$$

p q

b a

d d

(b,a)(b,d),(d,a)

(d,d)

$$W_3 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 1 & 0 & 1 & 1 \end{array}$$

p q

a a

b c

c d

d

$$W_4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 1 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 0 & 1 & 1 \\ d & 1 & 0 & 1 & 1 \end{array}$$

R = (a,a)(a,c)(a,d)

(b,a)(b,c)(b,d)

(c,a)(c,c)(c,d)

(d,a)(d,c)(d,d)

(3) $A = \{1,2,3,4\}$ and $R = \{(1,1)(2,1)(2,2)$ and

$(2,4)(3,1)(3$

$(3,3)(3,4)\}$ is

$$W_0 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

p	q
1	1
2	
3	

$$W_1 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$(1,1)(2,1)(3,$

p	q
2	1
3	2
	3
	4

$$W_2 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

p	q
2	1
3	2
	3
	4

$$W_3 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$(2,1)(2,2)(2,3)$
$(2,4)(3,1)(3,2)$
$(3,3)(3,4)$

p	q
2	$\emptyset$
3	

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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Order relations:- ③

A binary relation are in a set P is called partial order relation (or) partial ordering in P if R is reflexive, anti-symmetric and transitive. we denote partial ordering by the symbol $\leq$. if $\leq$ is a partial ordering on P then the order pair $(P, \leq)$ is called partially ordered set (or) poset.

→ Let $(P, \leq)$ be a poset if for every a $(x, y) \in P$ we have either $x \leq y$ (or) $y \leq x$ then $\leq$ is called simple ordering (or) linear ordering on P and $(P, \leq)$ is called totally order (or) simply order set (or) a chain.

* Hasse diagram:-

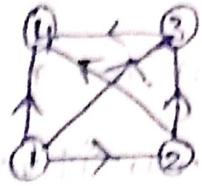
A partial ordering $\leq$ on a set P can be represented by means of diagram known as Hasse diagram. In such a diagram each element is represented by a small circle.

The circle for $x \in P$ is drawn below the circle if $x \leq y$ and a line is drawn between x and y if y covers x.

(1) let $P = \{1, 2, 3, 4\}$, and the relation ... are Refl ...

$R = \{(1,1)(1,2)(1,3)(1,4)(2,2)(2,3)(2,4)(3, ... (1,1)$

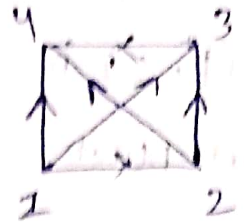
$(4,4)\}$



Remove

Remove Reflexive:

$(1,1)(2,2)(3,3)(4,4)$



Remove transitive aRb

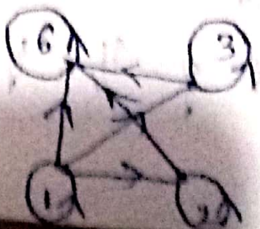
$(1,3)(1,4)(2,4)$. bRc

~~aRc~~

(2) $A = \{1, 2, 3, 6\}$

$R = \{(1,1)(1,2)(1,3)(1,6)(2,2)(2,6)(3,3)$

$(3,6)(6,6)\}$



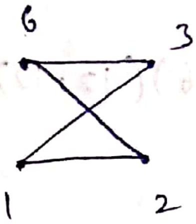
(4)

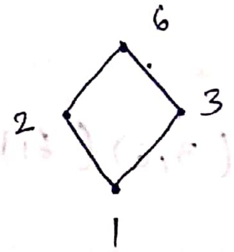
C_3 are Reflexive

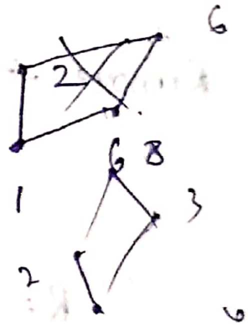
(1,1) (2,2) (3,3) (6,6)

(3) Remove transitive

(1,6)







6/2 = 3

2/6

6/2

(3) $x = \{2, 3, 6, 12, 24, 36\}$ and the relation $R \preceq$ be such that $x \leq y$ if x divides y

Sol:- y is divisible x 2,4 4/2 = 2

R = { (2,2) ~~(2,3)~~ (2,6) (2,12) (2,24) (2,36) ~~(3,2)~~
(3,3) (3,6) (3,12) (3,24) (3,36) ~~(6,2)~~ ~~(6,3)~~
(6,6) (6,12) (6,24) (6,36) ~~(12,2)~~ ~~(12,3)~~
~~(12,6)~~ (12,12) (12,24) (12,36) ~~(24,2)~~ ~~(24,3)~~
~~(24,6)~~ ~~(24,12)~~ (24,24) (24,36) ~~(36,2)~~ ~~(36,3)~~
~~(36,6)~~ ~~(36,12)~~ ~~(36,24)~~ (36,36). }

there exist no 'x' in 'a'

R = { ~~(2,2)~~ (2,6) ~~(2,12)~~ ~~(2,24)~~ ~~(2,36)~~ (
~~(3,12)~~ (3,24) (~~3,36~~) ~~(6,6)~~ (6,
~~(6,36)~~ (~~12,12~~) (12, 24) (~~12,36~~)

36

~~(24,36)~~ (~~36,36~~)

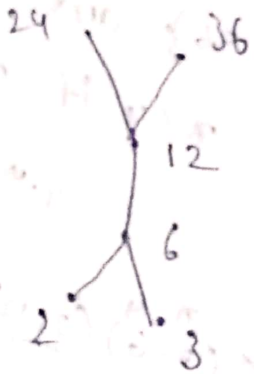
36
12
3

→ Remove Reflexive

(2,2)(3,3)(6,6)(12,12)(24,24)(36,

→ Remove transitive

R = { (2,6)(3,6)(6,12)(12,24)(12,36)



X = { 2, 3, 4, 9, 12, 18 } , { x, y }
consider pair as (x, y)
slash
y is divisible by x
Hasse diagram graph

R = { (2,2)(2,4) ~~(2,12)~~ (2,18) ~~(3,3)~~ (3,9)
(3,12) ~~(3,18)~~ ~~(4,4)~~ (4,12) ~~(9,9)~~ (9,18)
~~(12,12)~~ ~~(18,18)~~ }

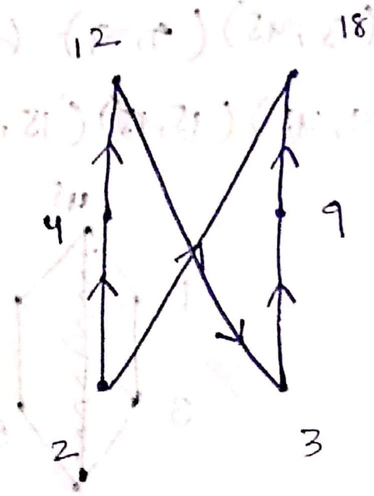
5

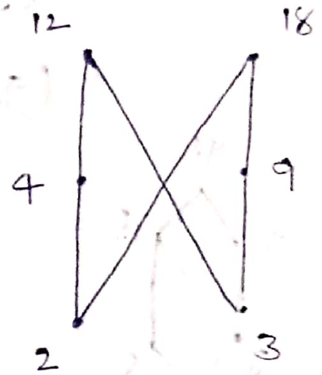
Reflexive (2,2) (3,3) (4,4) (9,9) (12,12) (18,18)

Antisymmetric (~~2,1~~

transitive (2,12) (3,18)

Remove





5. {D_{12}, / }, draw the Hasse diagram

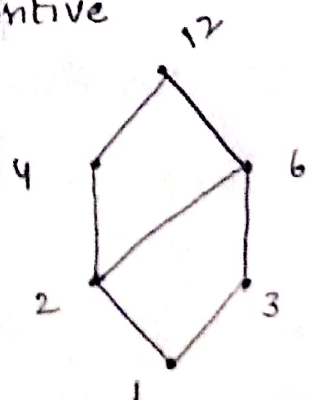
D - division ~ divisible by 12

X = { 1, 2, 3, 4, 6, 12 }

R = { (~~1,1~~) (1,2) (1,3) (~~1,4~~) (~~1,6~~) (~~1,12~~) (~~2,2~~)
(2,4) (2,6) (~~2,12~~) (~~3,3~~) (3,6) (~~3,12~~)
(~~4,4~~) (4,12) (~~6,6~~) (6,12) (~~12,12~~) }

= Remove reflexive

- Remove transitive

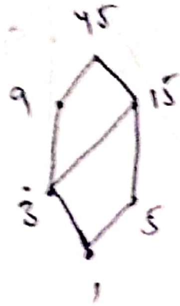


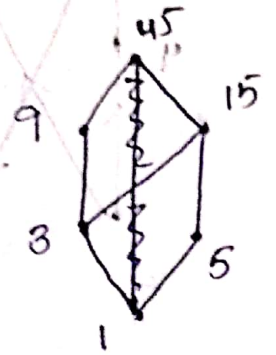
... to be maximal element of A, if

6) $\{D_{45}, |\}$ draw Hasse diagram

$A =\{1, 3, 5, 9, 15, 45\}$

$R = \{ \cancel{(1,1)} (1,3) \cancel{(1,9)} \cancel{(1,15)} \cancel{(1,45)} \cancel{(3,3)} (3,9)$
$(3,15) \cancel{(3,45)} \cancel{(9,9)} \cancel{(5,5)} (5,15) (1,5)$
$\cancel{(9,9)} (9,45) \cancel{(15,15)} (15,45) \cancel{(45,45)} \}$





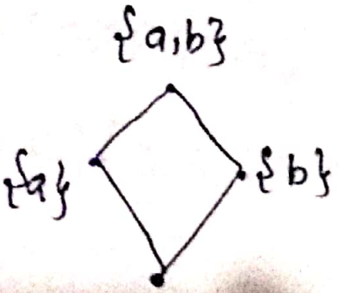
7) let a, A, B any finite set and P(A) be the power set of A, then $\subseteq$ be the inclusion relation on elements of P(A). draw the Hasse diagram of $(P(A), \subseteq)$ for

(i) $A = \{a\}$

(ii) $A = \{a, b\}$

(i) $R = \{\phi, \{a\}\}$

(ii) $R = \{\phi, \{a\}, \{b\}, \{a,b\}\}$



$(5,15)$ $(3,$ ion. if and only

$(45,45)$ $(3,$ reflexive

✓ symmetric

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Special elements in poset :-

Let $(P, \leq)$ be a poset and $A \leq P$

I) an element $a \in A$ is called least element of 'A'

if $a \leq x \quad \forall x \in A$

(ii) an element $a \in A$ is called greatest element of 'A'

if $x \leq a \quad \forall x \in A$

NOTE : least and greatest element of 'A' exists then

it is unique.

(iii) an let $(P, \leq)$ be a poset $A \leq P$ and element

$a \in A$ is said to be minimal element of 'A'

if there exist no 'x' in 'A' such that $x < a$

(iv) let $(P, \leq)$ be a poset $A \leq P$ and element $a \in A$

is said to be maximal element of 'A' if

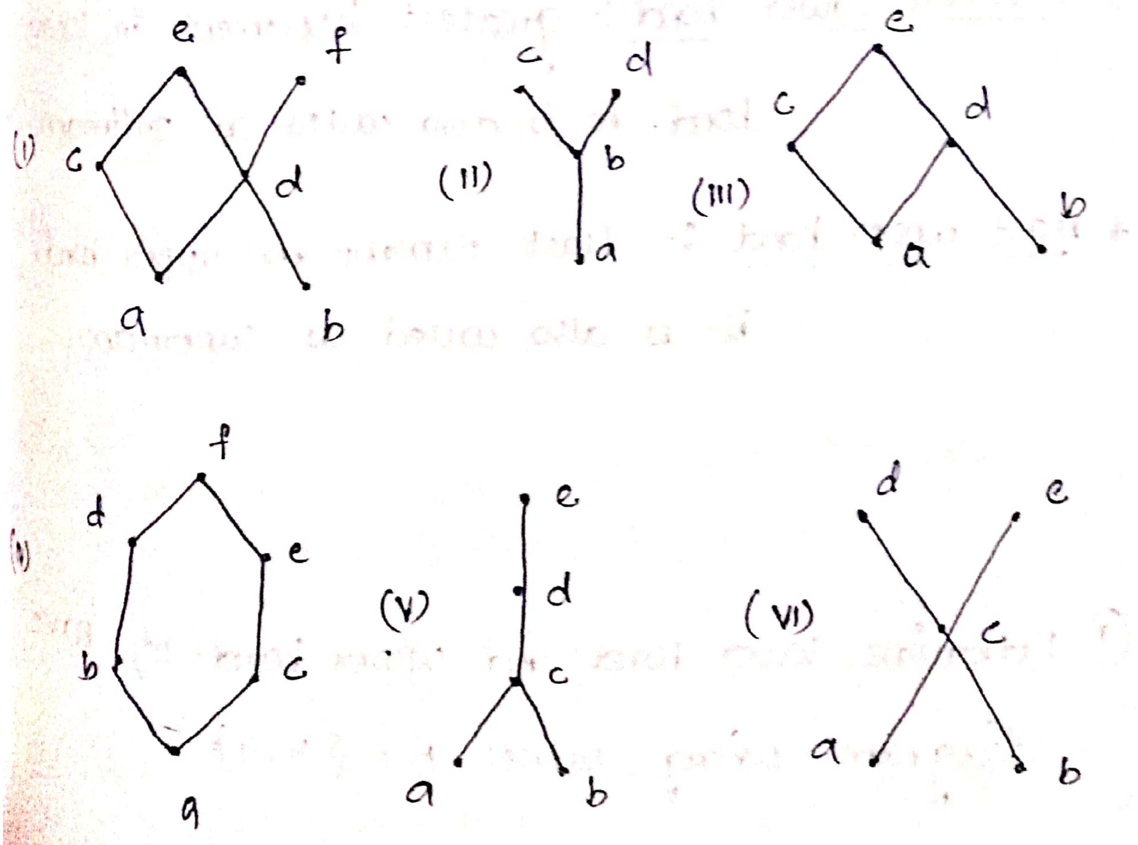
there exist no 'x' in 'A' such that $a < x$

NOTE :- minimal (or) maximal elements is not unique

- least element of 'a' is minimal but reverse is not possiable

- greatest element of 'a' is maximal but reverse is not possiable.

) determine least, greatest, minimal and maximal for the following Hasse diagram



(I) minimal - (a, b)
maximal - (e, f)

(II) minimal - (a)
maximal - (d)
least - (d)

(III) minimal - (a, b)
maximal - (e)
greatest (e)

(IV) minimal - (a)
maximal (f)
least - (a)

(V) minimal - (a, b)
maximal - (e)
greatest - (e)

(VI) minimal (a, b)
maximal (d, e)

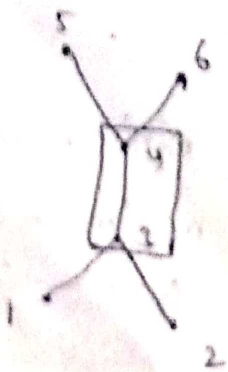
* Lower Bond :- let ~~(a, b)~~ (A, R) be a poset $a \in A$ lower bond of B. B is the subset of A if ~~A R~~ $a R x$ $\forall x \in B$

* Upper Bond :- let (A, R) be a poset $a \in A$ is upper bond of B. B is the subset of 'A' if $x R a$ $\forall x \in B$.

* Greatest lower bond :- greatest element in lower bond. it is also called as Infimam

* least upper bond :- least element in upper bond. it is also called as supermam.

① Determine lower bond and upper bond of Hasse diagram using subset $B = \{3, 4\}$



Lower bond = 1, 2, 3

upper bond = 4, 5, 6

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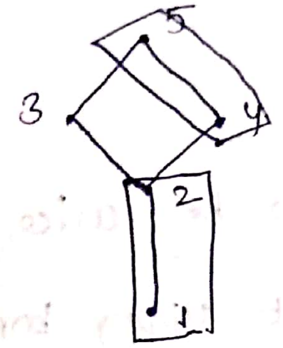


$B = \{4,5\}$, $B = \{1,2\}$.

Lower bond = 2,1,4 LB = ~~5~~ 1

upper bond = 5 UB = 5,2,4,3

A = {1, 2, 3, 4, ~~5~~ 6, 8, 12, 24} divides the relation.

draw the Hasse diagram to

lower bond - (8, 12)

upper bond - (8, 12)

Greatest Lower bond - (8, 12)

Least ~~to~~upper bond - (8, 12)

greatest & least

{ ~~(1,1)~~ (1,2) (1,3), ~~(1,4)~~ ~~(1,6)~~ ~~(1,8)~~ ~~(1,12)~~ ~~(1,24)~~

(2,4) (2,6) ~~(2,8)~~ ~~(2,12)~~ (2,24)

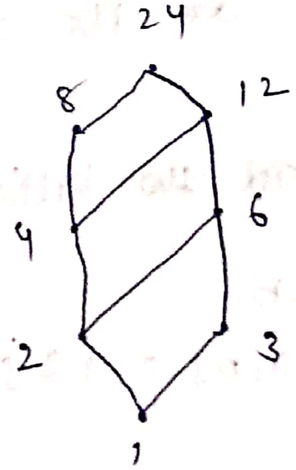
(3,6) ~~(3,12)~~ ~~(3,24)~~

(4,8) (4,12) ~~(4,24)~~

(6,12) ~~(6,24)~~

(8,24)

(12,24)



Lower bond = {8,12}

Upper bond = {8,2}, 24

GLB {8,12} = 4

* Latice :-

A poset $(L, \leq)$ is said to be latice, if every 2 elements in latice have least upper bond and greatest lower bond. the greatest lower bond of a subset $\{a, b\} \leq L$ will be denoted by

$a \wedge b$ (or) $a \times b$
(meet) (product)

Least upper bond of a subset $\{(a, b)\} \leq L$ will be denoted by

$a \vee b$ (or) $a \oplus b$
(join) sum.

NOTE :- Every lattice is a poset but all posets are not a lattice.

Q Find the lattice or not ?

LB
$\{a, b\} \text{ , } \{\phi\}$

UB
$\{a, b\} \text{ , } \{d, e\}$