Vidya Jyothi Institute of Technology (Autonomous)

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Aziz Nagar, C.B.Post, Hyderabad -500075

Department of Humanities and Sciences

B.Tech First Year II Semester-2019-20

QUESTION BANK

Subject: Mathematics II (Differential Equations and vector Calculus)

UNIT I: First Order First Degree Ordinary Differential equations & Applications

Prepared by: Mathematics Faculty, Dept of H&S, VJIT

UNIT-I SHORT ANSWER QUESTIONS	
1	Define Differential Equation, Ordinary and Partial Differential Equations
2	Define Solution, General solution and Particular Solution of a Differential Equation.
3	Define Exact differential equation.
4	Define Linear differential equation in y and Bernoulli's Differential Equation in y.
5	Write the Necessary and Sufficient condition for exactness.
6	Define Integrating factor.
7	Define Orthogonal Trajectories of family of curves.
8	State Newton's Law of Cooling.
9	State law of natural growth or decay.
10	Form the differential equation for the following equations (i) $\log\left(\frac{y}{x}\right) = cx$ (ii) $Sin^{-1}x + Sin^{-1}y = c$ (iii) $y = Ae^{-2x} + Be^{5x}$
11	Solve the differential equation $\frac{dy}{dx} + \frac{4x}{1+x^2}y = \frac{1}{(x^2+1)^2}$
12	Solve the differential equation $\frac{dy}{dx} + 3\frac{y}{x} = \frac{\sin x}{x^3}$
13	Solve the following differential equations (i) $x\,dy - y\,dx + 2x^3$dx=0 (ii) $x\,dy - y\,dx = a(x^2+y^2)dy$ (iii) $(y - x^2)dx + (x^2 Coty - x)dy = 0$
14	Solve the differential equations (i)$(y^2 - 2xy)dx = (x^2 - 2xy)dy$ (ii) $(x^2 - y^2)dx = 2xydy$
15	Find the Integrating Factor of $x^2ydx - (x^3 + y^3)dy = 0$
16	Find the integrating factor of $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
17	Solve the following differential equations (i) $x\frac{dy}{dx} + y = logx$ (ii) $\frac{dy}{dx} - \frac{2y}{x+1} = e^x(x+1)^2$
18	Write the procedure to solve Bernoulli's differential equation.
19	Find Orthogonal Trajectories of the family of circles $x^2 + y^2 = a^2$, where a is a parameter.
20	Find Orthogonal Trajectories of the family of circles (i) $r = ae^{\theta}$ (ii) $r = a\cos\theta$

UNIT-I LONG ANSWER QUESTIONS	
1	Solve the following differential equations (i) $(e^y + 1)Cosxdx + e^y sinxdy = 0$ (ii) $2xydy - (x^3 - y^2 + 1)dx = 0$ (iii) $\frac{dy}{dx} + \frac{yCosx+Siny+y}{SInx+xCosy+x} = 0$
2	Solve the following differential equations (i) $(x^4 + y^4)dx - xy^3dy = 0$. (ii) $y(2xy+1)dx + x(1 + 2xy - x^3y^3)dy = 0$ (iii) $(xy\ \sin xy + \cos xy)y\,dx + (xy\ \sin xy - \cos xy)x\,dy = 0$

	(iv) $2xydy-(x^2+y^2+1)dx=0$. (v) $2xydx+(y^2-x^2)dy=0$. (vi) $y(xy+e^x)dx-e^x dy=0$
3	Solve the following differential equations (i) $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$ (ii) $\frac{dy}{dx}+\frac{y}{xlogx}=\frac{Sin2x}{logx}$ (iii) $(1+y^2)dx=(\tan^{-1}y-x)\,dy$ (iv) $\frac{dy}{dx}+2y=e^x+x, y(0)=1$ (v) $(1+x^2)dy+2xydx=Cotx\ dx$
4	Solve the following differential equations (i) $x\frac{dy}{dx}+y=x^3y^6$ (ii) $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ (iii) $(1-x^2)\frac{dy}{dx}+xy=y^3\ \sin^{-1}x$ (iv) $e^x\frac{dy}{dx}=2xy^2+ye^x$
5	Find the orthogonal trajectories of the family of curves (i) $ay^2=x^3$ (Family of Semi-cubical parabolas) (ii) $x^2+y^2+2gx+c=0$ (Family of circles) (iii) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ (iv) $r=a(1-\cos\theta)$ (v) $r^n\sin n\theta=a^n$ (vi) $r^n=a^n\cos n\theta$
6	(i) Show that the family of parabolas $y^2=4a(x+a)$ are self orthogonal (ii) Show that the family of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal.
7	(i) If the temperature of the body is changing from 100°c to 70°c in 15 minutes, find when the temperature will be 40°c, if the temperature of air is 30°c. (ii)A murder victim is discovered and a lieutenant from the Forensic science laboratory is summoned to estimate the time of death. The body is located in a room that is kept at a constant temperature of 68°F. The lieutenant arrived at 9.40PM and measured the body temperature as 94.4°F at that time. Another measurement of the body temperature at 11PM is 89.2°F. Find the estimated time of death. (iii)A body kept in air with temperature 25°c cools from 140°c to 80°c in 20 minutes. Find when the body cools down to 35°c.
8	(i) An object cools from 120°F to 95°F in half an hour when surrounded by air whose temperature is 70°F. Find its temperature at the end of another half an hour. (ii) If air is maintained at 20°C and the temperature of the body cools from 100°C *to* 80°C in 10 minutes, find the temperature of the body after 20 minutes and when it will be40°C ?
9	(i)The number N of bacteria in a culture grows at a proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hours? (ii) The rate at which bacteria multiply is proportional to the instantaneous N number present. If the original number double in 2hours, when it will be tripled?
10	(i)A radioactive substance disintegrates at a rate proportional to its mass. When its mass is 10 mgm, the rate of disintegration is 0.0051 mgm per day. How long it will take for the mass to be reduced from 10 mgm to 5 mgm? (ii)If 30% of a radioactive substance disappears in 10 days, how long it will take for 90% of it to disappear?

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Department of Humanities and Sciences

B. Tech I Year II Semester-2019-20

QUESTION BANK

Subject: Mathematics II (Differential Equations and vector Calculus)

UNIT II: Second and Higher Order Linear Differential Equations

Prepared by: Mathematics Faculty, Dept of H&S, VJIT

UNIT-II SHORT ANSWER QUESTIONS	
1	Define differential and inverse differential operators.
2	Explain complementary function and particular integral.
3	Write the general form of n^{th} order Linear differential equation with constant coefficients.
4	Solve the differential equation $(D^3-3D+4)y=0$
5	Solve $y''+y'-2y=0, y'(0)=1, y(0)=4$
6	Solve $\frac{d^3x}{dt^3}-x=0$
7	Solve the differential equation $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=0$
8	Solve the differential equation $(D^2+4)y=0$
9	Solve the differential equation $(D^2-2D+4)y=0$
10	Find $\frac{1}{D+1}(x)$
11	Find $\frac{1}{(D-1)(D-2)}(e^{2x})$
12	Find the Particular integral of $(D^3+2D^2+D)y=Sin^2x$
13	Find the Particular integral of $\left(\frac{d^3y}{dx^3}+4\frac{dy}{dx}\right)=\sin 2x$
14	Find the Particular integral of $(D^3+2D^2+D)y=e^{2x}$
15	Write the procedure for method of variation of parameters

UNIT-II LONG ANSWER QUESTIONS	
1	Explain the procedure to solve higher order linear differential equation when $Q(x)=e^{ax}$ and solve the following differential equations (i) $(4D^2-4D+1)y=100$ (ii) $(D^3-6D^2+11D-6)y=e^{-2x}+e^{-3x}$
2	Explain the procedure to solve higher order linear differential equation when $Q(x)=e^{ax}$ and solve the following differential equations (i) $y''-4y'+3y=4e^{3x}, y(0)=-1, y'(0)=3$ (ii) $(D^3-5D^2+7D-3)y=e^{2x}\cosh x$
3	Explain the procedure to solve higher order linear differential equation when $Q(x)=Cos\ bx\ or\ Sin\ bx$ and solve the following (i) $(D^2-4)y=2\cos^2 x$ (ii) $y''+4y'+4y=4\cos x+3\sin x, y(0)=0, y'(0)=0$
4	Explain the procedure to solve higher order linear differential equation when $Q(x)=$

	$Cos\ bx\ or\ Sin\ bx$ and solve the following (i) $(D^2+1)y=\sin x \sin 2x$ (ii) $(D^3+2D^2+D)y=e^{-x}+\sin 2x$
5	Explain the procedure to solve higher order linear differential equation when $Q(x)=x^n$ and solve the following (i) $(D^2+D+1)y=x^3$ (ii) $y'''+2y''-y'-2y=1-4x^3$ (b) $(D^2-6D+25)y=e^{2x}+\sin x+x$
6	Explain the procedure to solve higher order linear differential equation when $Q(x)=v(x)e^{ax}$ and solve the following (i) $(D^2+2)y=e^x\cos x$ (ii) $(D^3-7D^2+14D-8)y=e^x\cos 2x$ (iii) $(D^2-1)y=x\sin x+(1+x^2)e^x$
7	Explain the procedure to solve higher order linear differential equation when $Q(x)=x^n v(x)$ and solve the following $(D^2-2D+1)y=x^2e^{3x}-\sin 2x+3$ (ii) $(D^2+4)y=x\sin x$ (iii) $(D^2-2D+1)y=xe^x\sin x$
8	Explain the method of variation of parameters to solve higher order linear differential equation and solve the differential equation $\frac{d^2y}{dx^2}+4y=\tan 2x$
9	By using the method of variation of parameters, solve the differential equation $(D^2+1)y=Coses\ x$
10	Solve the differential equations $(D^2+a^2)y=\sec ax$ by using method of variation of parameters

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Subject: Mathematics II (Differential Equations and vector Calculus)

UNIT III: Laplace Transforms

Prepared by: Mathematics Faculty, Dept of H&S, VJIT

UNIT-III SHORT ANSWER QUESTIONS	
1	Define Laplace Transform and inverse Laplace transform.
2	Write the sufficient conditions for exiting the Laplace transformations
3	Define function of exponential order.
4	Define Unit step function and find Laplace transform of Unit step function
5	State first and Second shifting theorems.
6	Write the change of scale property and find $(i)\ L\left[e^{-3t}\sinh 3t\right]$ $(ii)\ If\ L\left[f(t)\right]=\dfrac{9s^2-12s+15}{(s-1)^3},\ find\ L\left[f(3t)\right]$ using $change\ of\ scale\ property.$
7	State theorem on Laplace transform of derivatives and integrals.
8	Define Convolution Product and State Convolution theorem
9	Write the procedure to solve differential equation using Laplace transform
10	Find the Laplace Transform of the following functions (i) $4t^2+\sin 3t+e^{2t}$ (ii) $3\cos 3t\cos 4t$ (iii) $(\sin 2t-\cos 2t)^2$
11	Find $(i)\ L\left[e^{2t}+4t^3-\sin 2t\cos 3t\right]$ (ii) $L\left[e^{3t}-2e^{-2t}+\sin 2t+\cos 3t+\sinh 3t-2\cosh 4t+9\right]$
12	Find $(i)\ L\left[e^{at}\sin bt\right]$ $(ii) L\left[e^{at}\cos bt\right]$ $(iii)\ L\left[e^{at}\cosh bt\right]$ $(iv)\ L\left[e^{at}\sin bt\right]$
13	Laplace Transform of the following functions. (i) $e^{-t}(3\sinh 2t-5\cosh 2t)$ (ii) $e^{-3t}(2\cos 5t-3\sin 5t)$ (iii)$e^{-t}(\sin 2t\cos t)$ (iv) $\cosh 2t\sin 3t$
14	Define Dirac Delta function and find Laplace transform of Dirac Delta function
15	Find $(i)\ L^{-1}\left[\dfrac{1}{s^3}\right]$ $(ii)\ L^{-1}\left[\dfrac{3(s^2-2)^2}{2s^5}\right]$ $(iii)\ L^{-1}\left[\dfrac{s}{s^2-a^2}\right]$ $(iii)\ L^{-1}\left[\dfrac{4}{(s+1)(s+2)}\right]$
16	Find the inverse Laplace transform of the following functions $(i)\ \dfrac{s+3}{s^2-10s+29}$ $(ii)\ \dfrac{1}{s(s^2-1)(s^2+1)}$ $(iii)\ \dfrac{s}{s^2+4s+3}$
17	Find $(i)\ L^{-1}\left[\dfrac{1+e^{-\pi s}}{s^2+1}\right]$ $(ii)\ L^{-1}\left[\dfrac{e^{-2s}}{s^2+4s+5}\right]$ $(iii)\ L^{-1}\left[\log\left(\dfrac{s+1}{s-1}\right)\right]$
18	Find $(i)\ L^{-1}\left[\dfrac{s+1}{\left(s^2+2s+2\right)^2}\right]$ $(ii)\ L^{-1}\left[\dfrac{1}{s^2(s^2+a^2)}\right]$
19	$If\ L\left[f(t)\right]=\dfrac{1}{(s-1)^2},\ find\ L\left[\dfrac{1}{s(s-1)^2}\right].$

UNIT-III LONG ANSWER QUESTIONS	
1	Find the Laplace Transform of the following functions. $(i)\, e^{-3t}\cosh 4t\sin 3t \quad (ii)\sin 2t\cos t\cosh 2t \quad (iii)\, e^{2t}\sin t\cos 2t$
2	State Laplace transform of a function which is multiplied by 't' and find the Laplace transform of the following functions (i) $t^2e^{-2t}\cos t$ (ii) $t^2\sin t\cos 2t$ $(iii)\, te^{-t}\sin 2t$ $(iv)\, t\sin 3t\cos 2t$ $(v)\, te^{-t}\cosh t$ $(vi)\, t^2e^{-2t}$ $(vii)\, te^{2t}\sin 3t$ $(viii)$ $t\sin^2 3t$
3	State and Laplace transform of function which is divided by 't' and solve the following $(i)\, L\left[\frac{\sin 3t\cos t}{t}\right]$ $(ii)\, L^{-1}\left\{\frac{1-\cos t}{t^2}\right\}$ $(iii)\; L^{-1}\left\{\frac{\cos 4t\sin t}{t}\right\}$ $(iv)\frac{e^{-at}-e^{-bt}}{t}$ $(v)\; \frac{\text{sint}}{\text{t}}$ $(iii)\frac{e^{-t}\sin t}{t}$
4	Using Laplace transforms (i) Show that $\int_0^\infty t^2e^{-4t}\sin 2t\,dt = \frac{11}{50}$ (ii) Evaluate $\int_0^\infty te^{-t}\sin t\,dt$ (iii) Evaluate $\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}dt$ (iv) Evaluate $\int_0^\infty \frac{\cos at-\cos bt}{t}dt$ (v)Evaluate $\int_0^\infty te^{-t}\sin t\,dt$ (vi) Evaluate $\int_0^\infty \frac{e^{-at}\sin^2 t}{t}dt$
5	Find $L[f(t)]$ where $f(t)$ is given by $f(t)=t, 0<t<b \, and \; f(t)=2b-t, b<t<2b, 2b\, being\ the\ period\ of\; f(t)$
6	Find the inverse Laplace transform of the following functions $(i)\frac{5s-2}{s^2(s+2)(s-1)}$ $(ii)\frac{s}{(s^2+1)(s^2+9)(s^2+25)}$ $(iii)\frac{s}{s^4+4a^4}$ $(iv)\frac{1}{s(s^2-1)(s^2+1)}$
7	State inverse Laplace transform of derivatives and Find $(i)\, L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$ $(ii)\, L^{-1}\left[\cot^{-1}s\right]$ $(iii)\, L^{-1}\left[\log\left(\frac{s^2+4}{s^2+9}\right)\right]$ $(iv)\, L^{-1}\left[\log\left(1+\frac{16}{s^2}\right)\right]$
8	State convolution theorem. Evaluate the following using convolution theorem $(i)\, L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$ $(ii)\; L^{-1}\left\{\frac{s^2}{\left(s^2+4\right)\left(S^2+9\right)}\right\}$
9	Solve the differential equation by using Laplace transform $(i)\,(D^2+4D+5)y=5, y(0)=0, y'(0)=0.$ $(ii)\frac{d^2x}{dt^2}-4\frac{dx}{dt}-12x=e^{3t}, given\, that\; x(0)=1 and\; x'(0)=-2$ $(iii)\; \frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t, given\, that\; y(0)=10 and\; y'(0)=1$ $(iv)\frac{d^2x}{dt^2}+9x=\sin t, given\, that\; x(0)=1 and\; x(\frac{\pi}{2})=1$ $(v)\; y''-3y'+2y=4t+e^{3t}, given\, that\; y(0)=1 and\; y'(0)=1$

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Subject: Mathematics II (Differential Equations and vector Calculus)

UNIT IV: Multiple Integrals and Vector Calculus

Prepared by: Mathematics Faculty, Dept of H&S, VJIT

UNIT-IV SHORT ANSWER QUESTIONS	
1	Evaluate $\int_{y=0}^{2}\int_{x=0}^{3} xy\,dx\,dy$
2	Evaluate $\int_{0}^{\frac{\pi}{2}}\int_{-1}^{1} x^2y^2\,dx\,dy$
3	Evaluate(i) $\int_1^e \int_0^{\log y} \frac{1}{\log y}\,dxdy$ (ii) $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dxdydz$
4	Evaluate $\int_0^3\int_0^1 (x^2+3y^2)\,dy\,dx$
5	Evaluate(i) $\int_0^5\int_0^{x^2} x(x^2+y^2)\,dx\,dy$(ii) $\int_0^1\int_0^x e^x\,dxdy$
6	(i)Evaluate $\int_0^1\int_1^2\int_2^3 xyz\,dx\,dy\,dz$ (ii) Evaluate $\int_0^1\int_0^1\int_0^1 x^2y^3z^4\,dxdydz$
7	Write the Physical representation of Gradient and Divergent.
8	Define Gradient, Divergence and Curl of a vector function.
9	Define Solenoidal and Irrotational Vectors.
10	Define directional derivative of Scalar point function
11	(i)Find a unit normal vector to the surface $x^3+y^3+3xyz=3$ at the point (1,2,-1) (ii)Find a unit normal vector to the surface $z=x^2+y^2$ at (-1,-2,5)
12	Find the Directional derivative of $\frac{1}{r}$ in the direction of $\bar{r}=xi+yj+zk$ at (1,1,2)
13	Find the greatest value of the directional derivative of the function $f=x^2yz^3$ at (2,1,-1).
14	(i) Prove that $\nabla \log\lvert\bar{r}\rvert=\frac{\bar{r}}{r^2}$ (ii)If $\bar{f}=xy^2i+2x^2yzj-3yz^2k$ then find div$\bar{f}$ at (1,-1,1)
15	Show that the vector $3y^4z^2i+z^3x^2j-3x^2y^2k$ is solenoidal.
16	If $\bar{f}=xy^2\,i+2x^2yz\,j-3yz^2\,k$ find curl$\bar{f}$ at the point (1,-1,1)
17	If $\bar{F}=grad(x^3+y^3+z^3-3xyz)$ then find curl$\bar{F}$
18	If $\bar{f}=(x+3y)i+(y-2z)j+(x+pz)k$ is solenoidal then find ‘p’
19	Prove that $\bar{F}=yz\bar{i}+zx\bar{j}+xy\bar{k}$ is irrotational
20	Show that curl$(\bar{r}r^n)=\bar{0}$ (OR) Prove that $r^n\bar{r}$ is irrotational

UNIT-IV LONG ANSWER QUESTIONS	
1	(i)Evaluate $\iint_R y\,dxdy$ where R is the region bounded by the parabola $y^2=4x$ and $x^2=4y$.

	(ii)Evaluate $\iint (x^2 + y^2)\, dx\, dy$ in the positive quadrant for which x+y ≤ 1
2	(i)Evaluate $\iint_R xydxdy$ where R is the region bounded by X-axis and x=2a and the curve x^2=4ay. (ii)Change of order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$
3	(i)Evaluate the following integral by changing of order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dx\, dy$ (ii)By changing the order of the integration, evaluate $\int_0^1 \int_1^{2-x} xy\, dx\, dy$
4	(i)Evaluate $\int_0^{\pi} \int_0^{a\sin\theta} r\, dr\, d\theta$ (ii) Evaluate $\iint r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.
5	(i) Evaluate by transforming into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2 + y^2}\, dx\, dy$ (ii) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\; dz\, dx\, dy$
6	(i)Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dx\, dy\, dz$ (ii)Find the Directional derivative of $\emptyset = x^2yz + 4xz^2$ at (1,-2,-1) in the direction 2i-j-k
7	(i)Find the Directional derivative of a scalar point function $\phi(x, y, z) = 4xy^2 + 2x^2yz$ at the point A(1,2,3) in the direction of the line AB, where B=(5,0,4) (ii)Find the Directional derivative of $\emptyset = x^2yz + 4xz^2$ at (1,-2,-1) in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at (-1,2,1)
8	(i)Find the Directional derivative of $f(x, y, z) = xy^2 + yz^3$ at (2,-1,1) in the direction of the vector i+2j+2k (ii)Evaluate the angle between the normal to the surface $xy = z^2$ at (4,1,2) and (3,3,-3)
9	(i)Find div$\bar{f}$, where$\bar{f} = r^n\bar{r}$. Find n if it is solenoidal (ii)Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential
10	(i)If $\bar{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$ show that $\bar{F}$ is irrotational and find scalar potential. (ii)Prove that $div(grad\, r^m) = m(m + 1)r^{m-2}$ (or) $\nabla^2(r^n) = n(n + 1)r^{n-2}$

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QUESTION BANK

Subject: Mathematics II (Differential Equations and vector Calculus)

UNIT V: Vector Integration

Prepared by: Mathematics Faculty, Dept of H&S, VJIT

UNIT-V SHORT ANSWER QUESTIONS	
1	Define Circulation.
2	Define work done by a force.
3	Evaluate $\int \overline{F}\cdot d\overline{r}$ where $\overline{F} = x^2 i + y^2 j$ and c is the curve $y = x^2$ in the $xy-plane$ from (0,0) to (1,1).
4	Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ and C is the parabola y = 2x^2 from (0,0) to (1,2).
5	Prove that the scalar field $\overline{F} = \left(x^2 + xy^2\right)i + \left(y^2 + x^2 y\right)j$ is conservative.
6	State Gauss Divergence theorem
7	State Green's theorem
8	State Stoke's theorem
9	Using Green's theorem evaluate $\int_c (2xy - x^2)dx + (x^2 + y^2)dy$ where c is the closed curve of the region bounded by y=x^2 and y^2 =x.
10	Evaluate by Stoke's theorem $\int_c (e^x dx + 2y dy - dz)$ where c is the curve x^2+y^2=9 and z=2.

UNIT-V LONG ANSWER QUESTIONS	
1	If $\bar{f} = xy\, i - z\, j + x^2 k$ and C is the curve x=t^2,y=2t,z=t^3 from t=0 to t=1 then evaluate $\int_c \bar{f}\cdot d\bar{r}$
2	Find the work done by force $\bar{f} = (x^2 - y^2 + x)i - (2xy + y)j$ which moves a particle on XY-plane from (0,0) to (1,1) along the parabola y^2=x.
3	Evaluate $\int_c (x^2 + xy)dx + (x^2 + y^2)dy$ where C is the square formed by the lines y=±1 and x=±1.
4	Find the work done by force $\bar{f} = (2y + 3)i + xzj + (yz - x)k$ when it moves a particle from the point (0,0,0) to (2,1,1) along the curve x=2t^2, y=t,z=t^3.
5	Prove that force field given by $\bar{f} = 2xyz^3\, i + x^2 z^3\, j + 3x^2 yz^2\, k$ is conservative. Find the work done by moving a particle from (1,-1,2) to (3,2,-1) in this force field.
6	(i)Evaluate $\int_s \bar{f}\cdot\bar{n}ds$ where $\bar{f} = zi + xj - 3y^2 zk$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between z=0 and z=5.

	(ii)Evaluate $\int_s \bar{f} \cdot \bar{n} ds$ where $\bar{f} = (x + y^2)i - 2xj + 2yz\,k$ and S is the surface of the plane 2x+y+2z=6 in the first octant.
7	(i)Evaluate by Green's theorem $\oint (y - \sin x)\, dx + \cos x\ dy$ where C is the triangle enclosed by the lines y=0, x= $\frac{\pi}{2}$, $\pi y = 2x$. (ii)Verify Green's theorem for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by x=0,y=0 and x+y=1.
8	(i)Verify Green's theorem in the plane for $\oint (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where 'C' is the square with vertices (0,0),(2,0),(2,2) and (0,2). (ii)Verify Green's Theorem for $\int_c (xy + x^2)dx + x^2 dy$ where C is bounded by y=x and y=x^2
9	(i)Use divergence theorem to evaluate $\iint \bar{F}.ds$ where $\bar{F} = 4x\,i - 2y^2\,j + z^2 k$ and S is the surface bounded by the region $x^2 + y^2 = 4$, z=0 and z=3. (ii)Verify divergence theorem for F = $4xi - 2y^2j + z^2k$ taken over the surface bounded by the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.
10	(i)Apply Stoke's theorem to evaluate $\int_c (x + y)dx + (2x - z)dy + (y + z)dz$ where C is the boundary of the triangle with vertices (0,0,0),(1,0,0) and (1,1,0). (ii)Verify Stoke's theorem for $\bar{f} = (x^2 + y^2)i - 2xy\,j$ taken round the rectangle bounded by the lines x=±a, y=0 and y=b.