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Introduction to Electrical Engineering

ohm's law, basic circuit components, Kirchoff's laws, Simple problems Basic definitions, types of elements, types of sources, resistive networks, inductive networks, capacitive networks, and series parallel circuits, star delta and delta star transformation. Network theorems - Superposition thevenins's, Norton's, Reciprocity, Maximum power transfer theorems and simple problems.

Basically all the materials in the universe are classified in to three types.

1. Solids
2. liquids
3. Gaseous

According to modern electron theory, every atom is composed of the three fundamental particles, which are invisible to eyes. These are the neutron the Proton and the electron, always the electrons revolving around the nucleus in a circular paths by Bhor's model (or) elliptical paths by Rutherford's model.

The innermost orbit electrons are having more attraction by the +ve neucleus [as they are nearer to the nucleus] than the far most orbit electrons. This outermost orbit electrons are called Valency electrons. By giving some external energy, called ionization energy, we can only remove the outermost orbit electrons, called the Valency electrons.

After ionization the Valency electrons will become free electrons. These free electrons, comes out from the atom after ionization and they are free to move throughout the material i.e conductor.

Generally in any conductor there are 10^{18} to 10^{23} atoms per unit volume [unit cube]. So there are 10^{18} to 10^{23} free electrons per unit volume in silver (Ag) conductor (i.e every conductor is a very rich of free electrons).

Based on conductivity all the materials are classified in to three types

1. conductors
2. Semi Conductors
3. Insulators.

1. Conductors :- which conducts the electricity. Because it has very rich amount of free electrons.

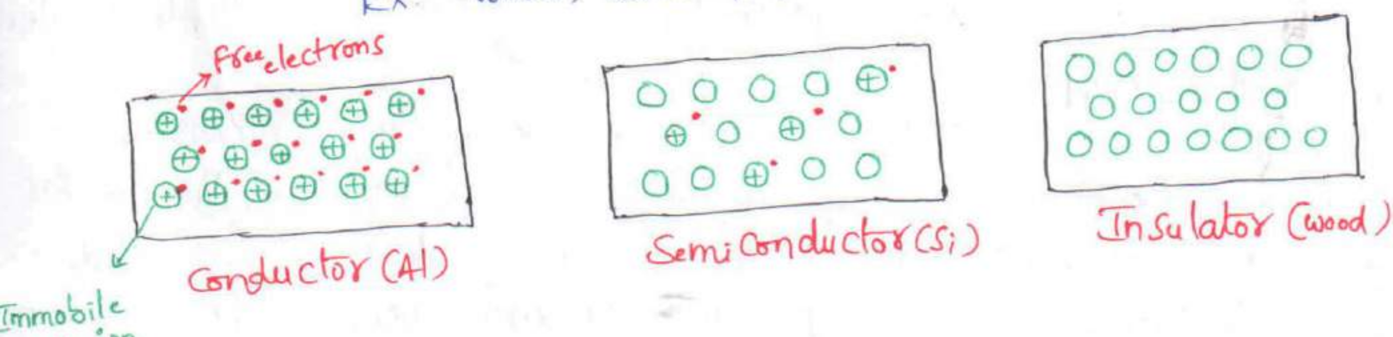
Ex: Alluminium (Al), Copper (Cu), Gold

2. Semi conductors :- which conducts the electricity partially. Because it has very less amount of free electrons.

Ex: Si, Ge

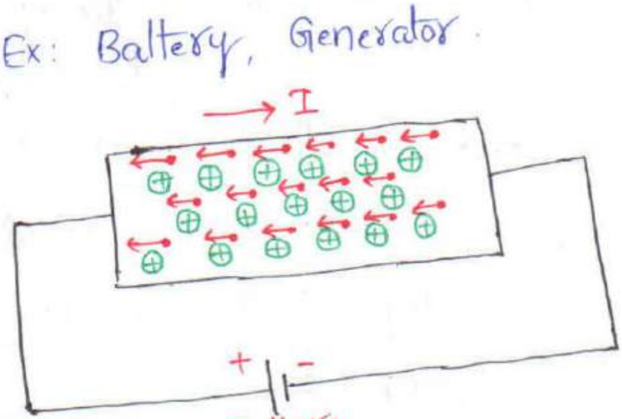
3. Insulators :- which doesn't conducts the electricity. Because it has zero free electrons. Ideally speaking the current flowing through the Insulator is zero.

Ex: wood, Rubber, plastic



Voltage :- A certain amount of energy (work) is required to overcome the force and move the charges through a specific distance (i.e the force of attraction between positive and negative charge particles) is called voltage.

Ex: Battery, Generator.



The electrons are attracted towords +ve plate of Battery and repelled from the -ve plate

Mathematically

$$V = \frac{W}{Q} \quad (\text{or}) \quad V = \frac{dw}{dq}$$

So it is expressed in terms of energy (w) per unit charge (Q)

where dw is the small change in energy and

dq is the small change in charge.

units: Volts (V) (or) J/C - J-Joules, C- coulomb

1) If 70 J of energy is available for every 30 C of charge, what is the voltage?

Given $W = 70\,J$

$Q = 30\,C$

$$V = \frac{W}{Q} = \frac{70}{30} = 2.33\,V$$

Current :- Current is defined as the rate of flow of electrons in a conductive (or) semiconductive material.

It is measured by the number of electrons that flow past a point in unit time.

Mathematically

$$I = \frac{Q}{t}$$

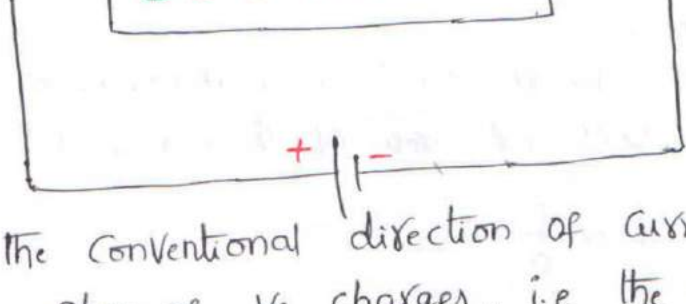
where I is the current, Q is the charge of electrons, and t is the time, (or)

$$i = \frac{dq}{dt}$$

where dq is the small change in charge

dt is the small change in time.

Unit: Ampere (A) (or) C/sec



* The conventional direction of current flow is opposite to the flow of -ve charges, i.e the electrons.

2). Five Coulombs of charge flow past a given point in a wire in 2s. How many amperes of current is flowing?

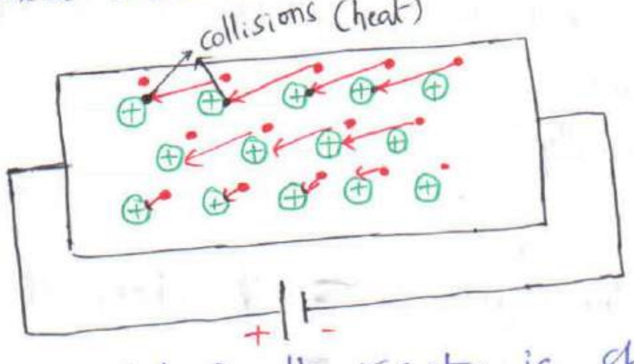
Given $Q = 5C$

$t = 2s$

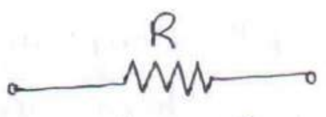
$$I = \frac{Q}{t} = \frac{5}{2} = 2.5\ A$$

Resistance:- The property of a material to restrict the flow of electrons is called resistance, denoted by R.

(when a current flows in a material, the free electrons move through the material and collide with other atoms. These collisions cause the electrons to loose some of their energy. This loss of energy per unit charge is the drop in potential across the material. The amount of energy lost by the electrons is related to physical property of the material. These collisions restrict the movement of electrons)



The Symbol for the resistor is shown below



unit: ohm (Ω)

The factors affecting the resistance of a material are,

1. Length of the material:-(L) The resistance of a material is directly proportional to the length. i.e as the length of the conductor increases resistance also increases.

$$R \propto L$$

2. Cross Sectional Area:- (a) The resistance of a material is inversely proportional to the cross sectional area of the material.

i.e $R \propto \frac{1}{a}$

3. The type and nature of the material :-

If the material is conductor, its resistance is less while if it is insulator, its resistance is very high

4. Temparature :- As temparature changes, the value of the resistance of the material also changes

→ So for a certain material at a certain constant temparature we can write a mathematical expression as,

$$R \propto \frac{l}{a}$$

→ If the effect of nature of material is considered through the constant of proportionality denoted by ρ (rho) called Resistivity (or) Specific resistance of the material.

$$R = \frac{\rho l}{a}$$

Resistivity :- The resistivity (or) Specific resistance of a material depends on the nature of material. It is denoted as 'ρ'

$$\rho = \frac{Ra}{l}$$

unit :- ohm-metre (Ω-m)

→ A material with higher value of resistivity is better insulator while lower value of resistivity is better conductor.

Conductance :- (G)

The reciprocal of resistance is called Conductance. It is denoted as G and is measured in mho (or) Siemens. ($\mho$).

$$G = \frac{1}{R} = \frac{1}{\rho}\left(\frac{a}{l}\right)$$

Conductivity :- (σ)

The reciprocal of resistivity is called conductivity and denoted as σ. It is measured in Siemens/m (or) $(\Omega\text{-m})^{-1}$

$$\sigma = \frac{1}{\rho} = \frac{l}{Ra}$$

3) If the piece of certain wire of 40 m length and 0.07 cm in radius has a resistance of 15 Ω, find the specific resistance of the material?

Given $l = 40m$, $r = 0.07\ cm$, $R = 15\ \Omega$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2 \times 0.07 \times 10^{-2})^2 = 1.5393 \times 10^{-6}\ m^2$$

$$R = \frac{\rho l}{a} \quad \text{i.e} \quad 15 = \frac{\rho \times 40}{1.5393 \times 10^{-6}}$$

$$\rho = 5.7726 \times 10^{-7}\ \Omega\text{-}m$$

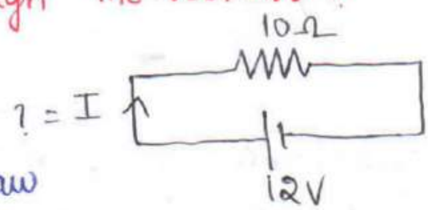
Ohm's law :- According to ohm's law there exist a linear relation between current and voltage i.e the current is directly proportional to the voltage and inversely proportional to the total resistance of the circuit under constant temparature.

$$I \propto V,\ I \propto \frac{1}{R} \Rightarrow I = \frac{V}{R}$$

Limitations:-

1. It is not applicable to the non linear devices such as diodes, zener diodes etc.
2. The temparature should be kept constant.

4) A 10 Ω resistor is connected across a 12V battery. How much current flows through the resistor?

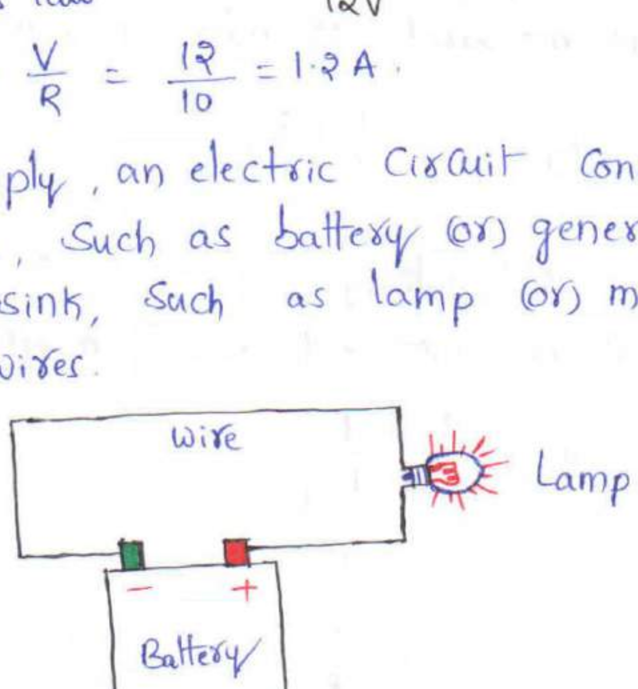


From ohm's law

$$I = \frac{V}{R} = \frac{12}{10} = 1.2\ A.$$

Circuit :- Simply, an electric circuit consists of three parts

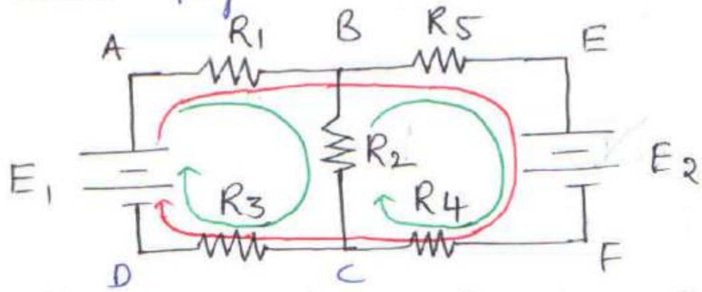
1. energy source, such as battery (or) generator
2. The load (or) sink, such as lamp (or) motor
3. Connecting wires.



Networks Analysis

The network analysis (or) circuit analysis means to find a current through (or) voltage across any branch of networks (or) circuit.

Networks :- Interconnection of two (or) more simple circuit elements is called an electric networks. Such a networks is shown in the below fig.



Network element (or) Circuit element :- By definition, a simple circuit element is the mathematical model of two terminal electrical devices and it can be completely characterised by its voltage and current.

(In fig $E_1, E_2, R_1, R_2, R_3, R_4$ and R_5 are called network elements).

Branch :- A part of the network which connects the various points of the network with one another is called a branch

(In fig AB, BC, CD, BE, EF, CF and AD are called various branches).

Junction point :- A point where three (or) more branches meet is called ~~node~~ a junction point.

(C and B are the junction points)

Node :- A point at which two (or) more elements are joined together is called node. The junction points are also the nodes of the networks.

Mesh (or) loop :- A loop can be defined as a closed path which originates from a particular node, terminating at the same node, without travelling through any node twice.

In the fig paths A-B-C-D-A, A-B-E-F-C-D, and BE-F-C-B.

Electrical Power :-

The rate at which an electrical work is done in an electric circuit is called an Electrical power

$$\text{Electrical Power } P = \frac{\text{Electrical work}}{\text{Time}} = \frac{W}{t} \ J/sec$$

$$(or) \quad P = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt}$$

$$\Rightarrow P = V \times I \ J/sec \ (or) \ Watts$$

unit : J/Sec (or) Watts (W)

Acc to ohm's law

$$V = IR \quad (or) \quad I = \frac{V}{R}$$

$$\Rightarrow P = VI = IR \times I = I^2R. \ W$$

$$\Rightarrow P = VI = V \times \frac{V}{R} = \frac{V^2}{R} \ W.$$

5. What is the power in watts if energy equal to 50J is used in 2.5 s?

Given energy (or) work = 50 J

$t = 2.5$ s

$$\therefore \text{Power } P = \frac{\text{energy}}{\text{time}} = \frac{50}{2.5} = 20\,W.$$

Electrical Energy :-

An Electrical energy is the total amount of electrical work done in an electric circuit.

$$\text{Electrical energy } E = \text{Power} \times \text{time} = VI \times t \ \text{Joules}$$

unit : joules (or) watt-Sec

$$1\,wh = 1 \text{ watt} \times \text{hour} = 1 \text{ watt} \times 3600 \text{ Sec} = 3600 \text{ Watt-Sec}$$

i.e joules

$$1\,kwh = 1000 \text{ watt} \times \text{hour} = 10^3 \times 3600\,J = 3.6 \times 10^6\,J$$

Note :- The electricity bills we are getting are charged based on kwh (or) unit.

The difference between a mesh and a loop is that a mesh doesnot contain any other loop within it. Thus a mesh is a smallest loop. A mesh is always a loop but a loop may (or) may not be a mesh. In the fig path A-B-C-D-A is a mesh while path A-B-E-F-C-D-A. is a loop.

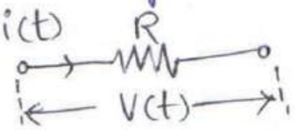
Basic Circuit Elements :-

The three basic passive circuit components are,

i) Resistance (R) (ii) Inductance (L) (iii) Capacitance (C)

(i) Resistance (R) :-

It is the property of the material by which it opposes the flow of current through it.



where $V(t) \rightarrow$ Time Varying Voltage across 'R'

$i(t) \rightarrow$ Time Varying current flowing through 'R'

$$R = \frac{V(t)}{i(t)} \ \Omega$$

The Power is given by,

$$P(t) = V(t)\, i(t) \ W$$

The energy converted to heat energy in time 't' is given by

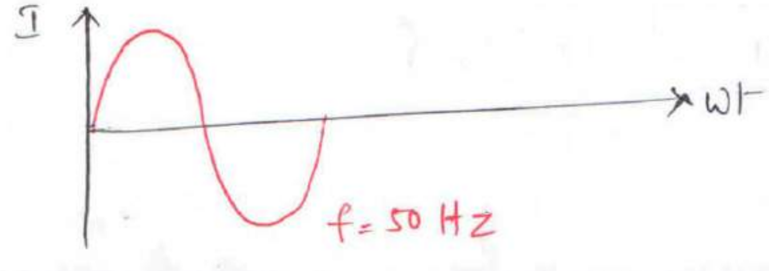
$$W = \int_{-\alpha}^{t} P(t)\, dt = \int_{-\alpha}^{t} V(t)\, i(t)\, dt \ J$$

If the Voltage and current are d.c in nature

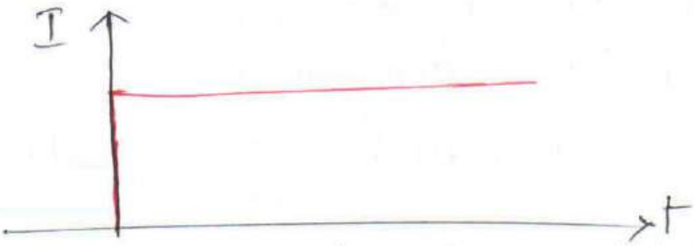
$$R = \frac{V}{I} \ \Omega, \quad P = VI \ W, \quad E = VIt \ J.$$

(ii) Inductance (L) :-

A.C :- A.C means alternating current. The frequency of AC in India is 50Hz



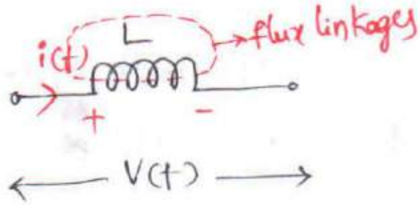
D.C :- D.C means Direct Current. The frequency of DC is Zero



Coil :- when a Conductor is wounded like a Spring, Then it is said to be a coil.

→ when a time varying current is flowing through the coil, a time varying magnetic flux will be produced by Faradays law. The total flux produced is

$$\psi = N\phi \quad (wb)$$



Where, N = the number of turns

ϕ = flux per turn

i.e $\psi(t) = N.\phi(t)$ (wb)

This total flux produced is praportional to the current flowing through the coil.

i.e $\psi \propto i$

$$\psi = Li$$

Where L = Inductance parameter of the coil. units: Henry (H)

The Voltage drop across the coil is

$$V = \frac{d\psi}{dt} \text{ / By Faradays law.}$$

$$V = \frac{d(Li)}{dt}$$

i.e The voltage drop across Inductor $\boxed{V = L\frac{di}{dt} \cdot V}$

$$\rightarrow \quad di = \frac{1}{L} v\,dt$$

Take Integration on both side

i.e The current flowing through the Inductor $\boxed{i = \frac{1}{L}\int v\,dt \;\; A}$

The power absorbed by Inductor is

$$P(t) = Vi = L\,i(t)\,\frac{di(t)}{dt} \quad (W)$$

The energy stored in the Inductor in the form of an electromagnetic field is,

$$W = \int P(t)\,dt = \int L\,i(t)\,\frac{di(t)}{dt}\,dt \quad (J)$$

$$P = Li\frac{di}{dt} = \frac{d}{dt}\left[\tfrac{1}{2}Li^2\right]$$

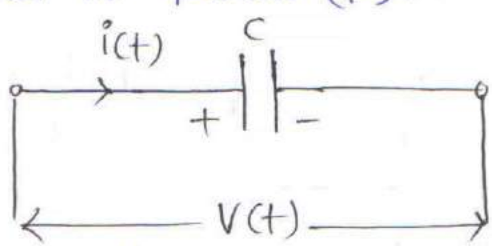
Since $W = \int P\,dt \ (J)$

$$= \int \frac{d}{dt}\left[\tfrac{1}{2}Li^2\right]dt$$

$$W = \tfrac{1}{2}Li^2 \ (J) = \text{Total energy stored}$$

(iii) Capacitance (C):-

An element in which energy is stored in the form of an electrostatic field is known as capacitance. It is made up of two conducting plates seperated by a dielectric material. It is denoted as 'c' and is measured in farads (F).



The charge stored in a capacitor is directly proportional to the voltage applied across capacitor

i.e $q \propto V$

$q = CV$

C = Capacitance parameter of the capacitor

units: Farads (F)

$$\Rightarrow i = \frac{dq}{dt}$$

$$i = \frac{d(CV)}{dt}$$

$$\boxed{i = C\frac{dv}{dt} \ A}$$

$$dV = \frac{1}{C} i\, dt \Rightarrow \boxed{V = \frac{1}{C}\int i\, dt} \; V$$

The power absorbed by the capacitor is

$$p(t) = Vi = C\, V(t) \frac{dV(t)}{dt}$$

The energy stored in the capacitor is given by

$$W = \int p(t)\, dt = \int C V(t) \frac{dV(t)}{dt} dt$$

$$= \int C V(t)\, dV(t) = C \frac{V^2(t)}{2}$$

$$\boxed{W = \frac{1}{2} C V^2(t) \text{ joules}}$$

Voltage - Current Relationships for passive Elements

Element	Basic relation	Voltage across, if current known	Current through, if voltage known	Energy
R (Resistor)	$R = \frac{V}{i}$	$V_R(t) = R\, i_R(t)$	$i_R(t) = \frac{1}{R} V_R(t)$	$W = \int_{-\alpha}^{t} i_R(t) V_R(t) dt$
L (Inductor)	$L = \frac{N\phi}{i}$	$V_L(t) = L \frac{di_L(t)}{dt}$	$i_L(t) = \frac{1}{L}\int_{-\alpha}^{t} V_L(t) dt$	$W = \frac{1}{2} L i^2(t)$
C (Capacitor)	$C = \frac{q}{V}$	$V_C(t) = \frac{1}{C}\int_{-\alpha}^{t} i_C(t) dt$	$i_C(t) = C \frac{dV_C(t)}{dt}$	$W = \frac{1}{2} C V^2(t)$

Kirchoff's Laws :-

There are two Kirchhoff's laws.

1. KCL 2. KVL

1. Kirchhoff's Current law :- (KCL)

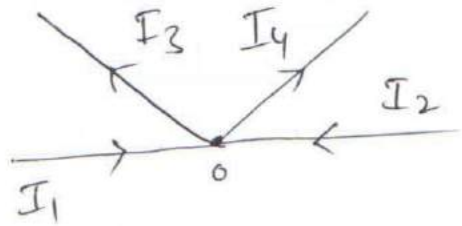
The law can be stated as,

"The algebraic Sum of all the Currents meeting at anode is always zero".

$\sum I$ at node $= 0$

Sign Convention:- Currents flowing towards a node are assumed to be positive while currents flowing away from a junction point (or) node assumed to be negative

KCL at node 'o'



$\sum I$ at node 'o' $= 0$

$I_1 + I_2 - I_3 - I_4 = 0$

$$\boxed{I_1 + I_2 = I_3 + I_4}$$

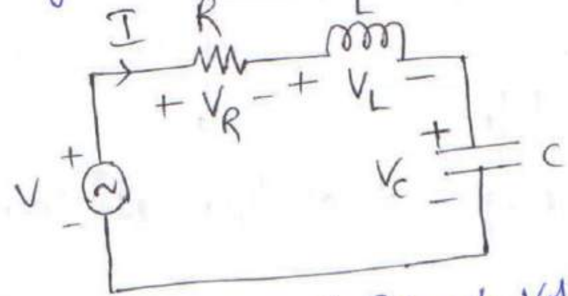
i.e. The total entering current is equals to leaving currents

2. Kirchhoff's Voltage Law (KVL)

It is always defined in a loop (or) mesh i.e. closed path.

Definition:-

" The algebraic sum of branch voltages around the loop is zero."



By KVL $\Rightarrow$ $\sum$ Branch Voltages $= 0$

$$V - V_R - V_L - V_C = 0$$

$$\boxed{V = V_R + V_L + V_C}$$

So it can be also stated as

" In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f s (voltage) in the path"

(or)

Sum of all the potential rises must be equal to sum of all the potential drops.

Sign Convention:-

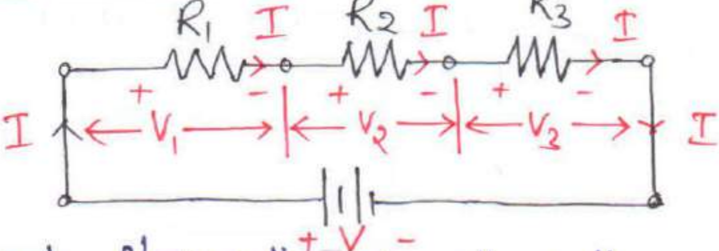
→ while moving in a closed path for KVL, if we go from -ve marked terminal to +ve marked terminal, that voltage must be taken as positive. This is called potential rise.

→ While moving in a closed path for KVL, if we go from +ve marked terminal to -ve marked terminal, that voltage must be taken as ~~posi~~ negative. This is called Potential drop.

Series and Parallel Circuits

A series circuit is one in which several resistances are connected one after the other. Such connection is also called end to end connection (or) Cascade connection. There is only one path for the flow of current.

Resistors in Series



The current flowing through all of them is same indicated as I amperes.

→ Let V_1, V_2, and V_3 be the voltage drops across resistances R_1, R_2 and R_3 respectively.

Then according to ohm's law

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

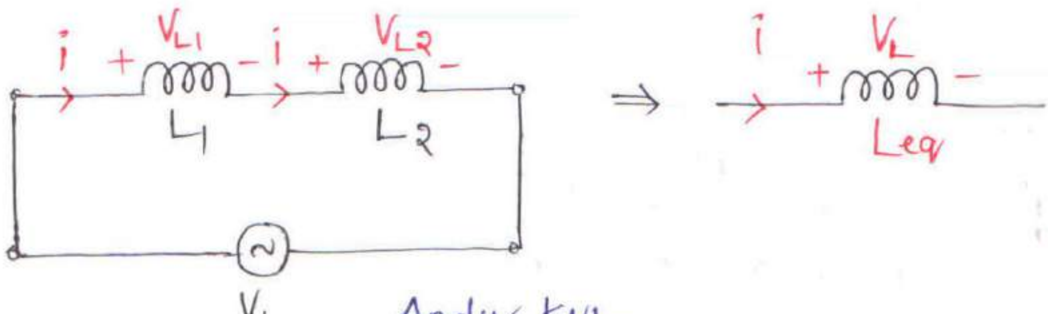
Apply KVL to the loop

$$V = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$V = I(R_1 + R_2 + R_3)$$
$$\Rightarrow V = I\,R_{eq}$$
$$\therefore R_{eq} = R_1 + R_2 + R_3$$

i.e total (or) equivalent resistance of the series circuit is arithmetic sum of the resistances conned in series. ⑧

For n resistances in series $R = R_1 + R_2 + R_3 + \cdots + R_n$

Inductors in Series :



Apply KVL

$$V_L = V_{L_1} + V_{L_2}$$

From ohm's law

$$L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

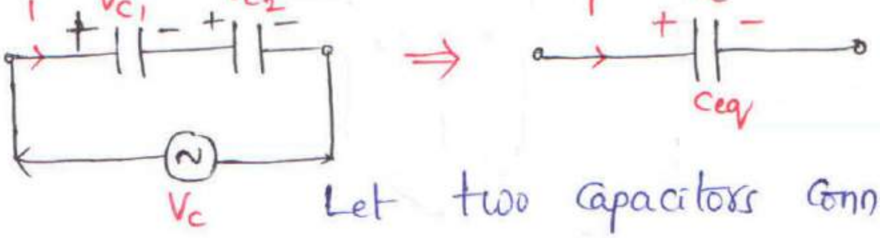
$$\boxed{\therefore \ L_{eq} = L_1 + L_2}$$

∴ The total equivalent inductance of the series circuit is sum of the inductances connected in series.

For n inductances in series

$$\boxed{L_{eq} = L_1 + L_2 + \cdots + L_n}$$

Capacitors in Series



Let two capacitors connected in series

Apply KVL

$$V_c = V_{C_1} + V_{C_2}$$

From Ohm's law

$$V_{C_1} = \frac{1}{C_1}\int i\,dt \qquad V_{C_2} = \frac{1}{C_2}\int i\,dt$$

$$\frac{1}{C_{eq}}\int i\,dt = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$$\therefore \quad \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

For n Capacitors in series

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\cdots + \frac{1}{C_n}}$$

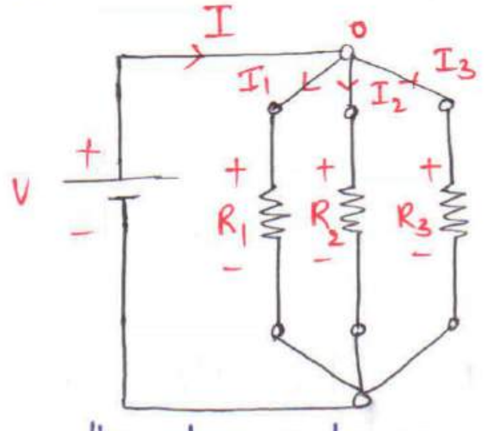
Parallel Circuits :-

The parallel circuit is one in which several resistances are connected across one another.

→ The Voltage across all the elements connected in parallel is same.

Resistors in parallel

Let three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of Voltage 'V'.



The Voltage across the two ends of each resistances R_1, R_2, and R_3 is the same and equals to the supply Voltage V.

i.e $V_{R_1} = V_{R_2} = V_{R_3} = V$

Apply KCL at node 'o'

$$I - I_1 - I_2 - I_3 = 0$$

$$I = I_1 + I_2 + I_3$$

From ohm's law

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

R_{eq} = Total (or) equivalent resistance of the circuit.

For 'n' resistances are connected in parallel,

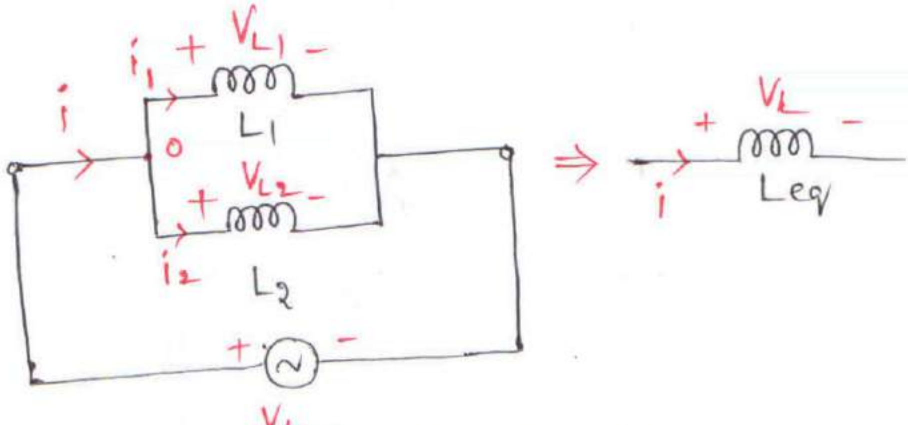
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\cdots + \frac{1}{R_n}$$

For two resistances R_1 and R_2 in parallel

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Inductors in Parallel :-

Let two inductors L_1 and L_2 are connected in Parallel.



For parallel combination $V_L = V_{L_1} = V_{L_2}$

Apply KCL at node 'o'

$$i = i_1 + i_2$$

From ohm's law.

$$i = \frac{1}{L_{eq}} \int V_L \, dt$$

$$\frac{1}{L_{eq}}\int V_L\,dt = \frac{1}{L_1}\int V_L\,dt + \frac{1}{L_2}\int V_L\,dt$$

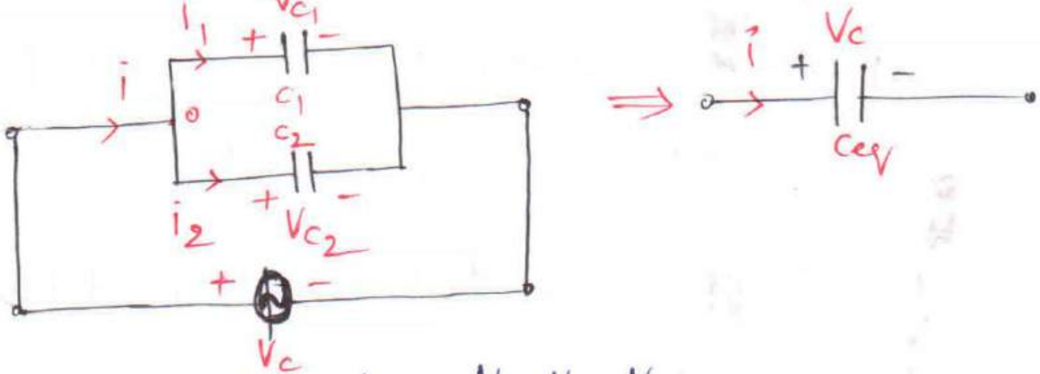
$$\therefore \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

For 'n' inductors in parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$$

Capacitors in parallel

let two capacitors C_1 and C_2 are connected in parallel



For parallel combination $V_c = V_{c_1} = V_{c_2}$

Apply KCL at node 'o'

$$i = i_1 + i_2$$

$$C_{eq}\frac{dV_c}{dt} = C_1\frac{dV_c}{dt} + C_2\frac{dV_c}{dt}$$

$$\therefore \boxed{C_{eq} = C_1 + C_2}$$

For n capacitors connected in parallel

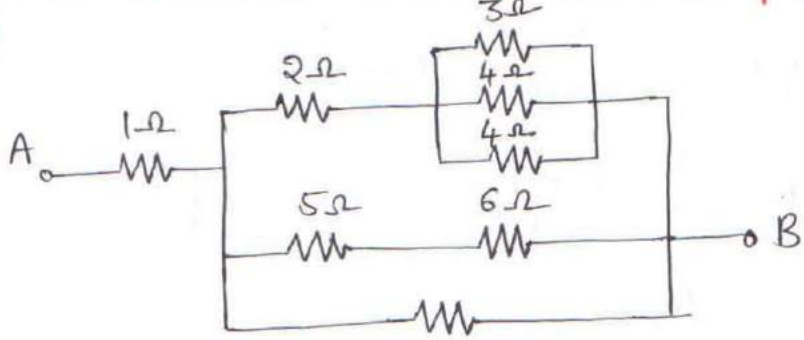
$$C_{eq} = C_1 + C_2 + \cdots + C_n.$$

Note :-

i) In Series circuit the current flowing through each element is Same

ii) In parallel circuit the voltage across each element is Same

6) Find the equivalent resistance between the two points A and B shown in fig? (10)



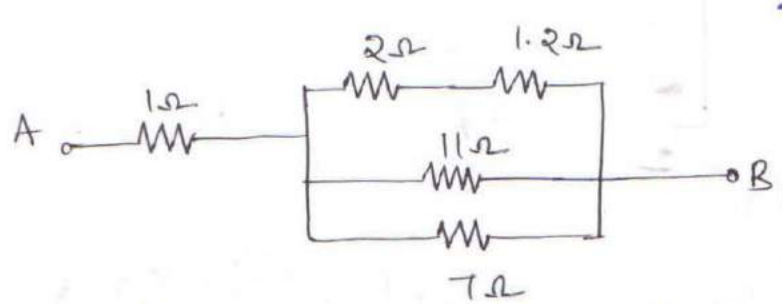
A) In fig 5Ω and 6Ω are in series

So equivalent resistance is $5+6=11\Omega$

and 3Ω, 4Ω and 4Ω are connected in parallel

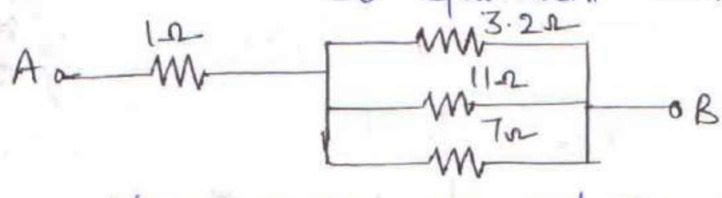
So equivalent resistance is $\frac{1}{R_{eq}} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4}$

$\therefore R_{eq} = \frac{3\times4\times4}{3\times4+4\times4+4\times3} = \frac{12}{10} = 1.2\Omega$



Again 2Ω and 1.2Ω are in series

So equivalent resistance $= 2+1.2 = 3.2\Omega$



Now, 3.2Ω, 11Ω and 7Ω are connected in parallel

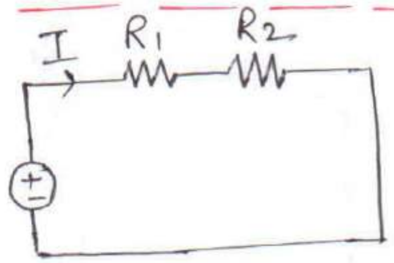
So equivalent resistance $\frac{1}{R_{eq}} = \frac{1}{3.2}+\frac{1}{11}+\frac{1}{7}$

$\therefore R_{eq} = \frac{11\times7\times3.2}{11\times7+7\times3.2+3.2\times11} = 1.8304\Omega$

Now, 1Ω and 1.8304Ω are in series

$\therefore R_{AB} = 1+1.8304 = 2.8304\Omega$

Voltage Division in Series Circuit of Resistors



From ohm's law

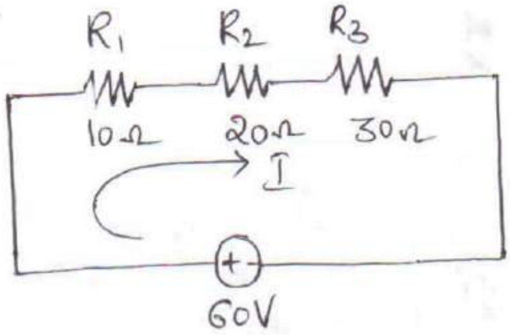
i.e $I = \frac{V}{R_1+R_2}$

$V_{R_1} = IR_1 \quad V_{R_2} = IR_2$

$\Rightarrow V_{R_1} = \left(\frac{V}{R_1+R_2}\right) \times R_1 \quad V_{R_2} = \left(\frac{V}{R_1+R_2}\right) \times R_2$

So this circuit is a Voltage divider circuit.

7) Find the voltage across the three resistances shown in fig



Sol:

$I = \frac{V}{R_1+R_2+R_3}$

$= \frac{60}{10+20+30} = 1A$

$\therefore V_{R_1} = 1 \times 10 = 10V$

$V_{R_2} = 1 \times 20 = 20V$ (or)

$V_{R_3} = 1 \times 30 = 30V$

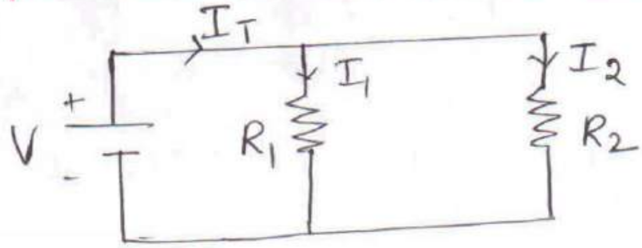
By Voltage division rule

$V_{R_1} = \frac{V \times R_1}{R_1+R_2+R_3} = \frac{60 \times 10}{10+20+30} = 10V$

$V_{R_2} = \frac{V \times R_2}{R_1+R_2+R_3} = \frac{60 \times 20}{10+20+30} = 20V$

$V_{R_3} = \frac{V \times R_3}{R_1+R_2+R_3} = 30V$

Current division in parallel Circuit of Resistors



$I_T = I_1 + I_2 \rightarrow (1)$

$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}$

$\therefore V = I_1 R_1$ (or) $V = I_2 R_2$

$I_1 R_1 = I_2 R_2$

$\Rightarrow I_1 = I_2\left(\frac{R_2}{R_1}\right) \rightarrow (2)$

Sub ② in ①

$$I_T = I_1 + I_2 = I_2\left(\frac{R_2}{R_1}\right) + I_2$$

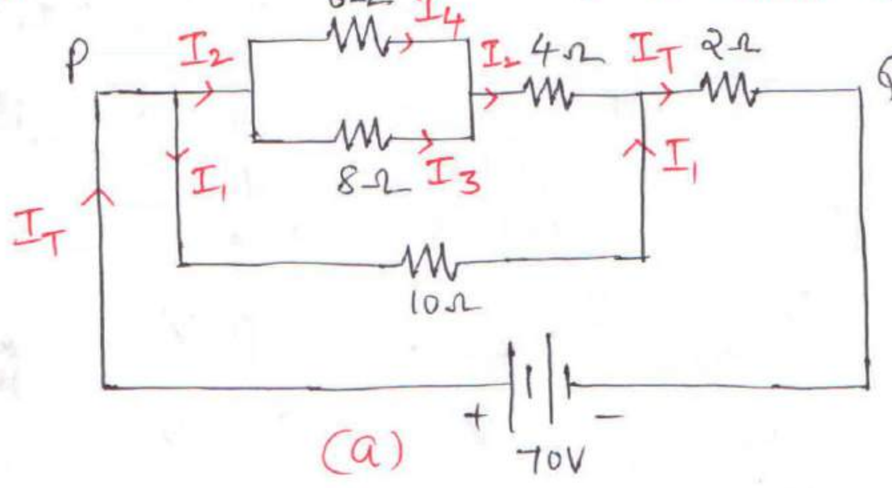
$$= I_2\left(\frac{R_2}{R_1} + 1\right)$$

$$\therefore \boxed{I_2 = I_T \times \left(\frac{R_1}{R_1 + R_2}\right)}$$

$$I_1 = I_T - I_2 = I_T - I_T\left(\frac{R_1}{R_1+R_2}\right) = I_T\left(1 - \frac{R_1}{R_1+R_2}\right)$$

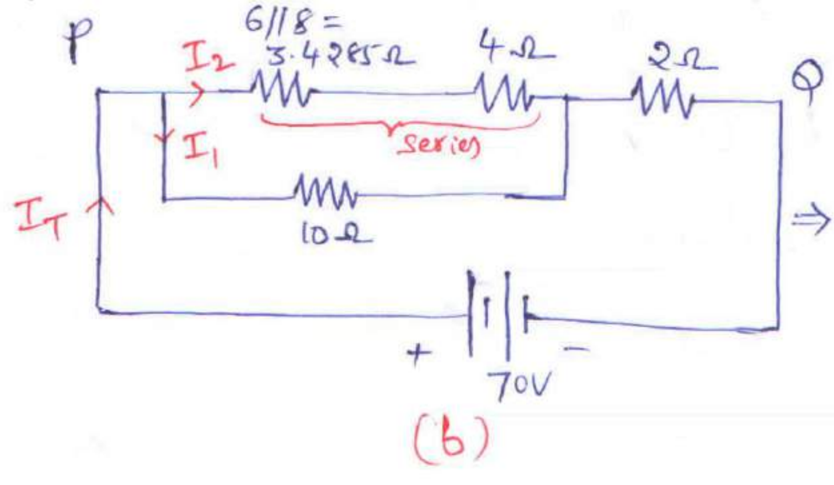
$$\boxed{I_1 = I_T \times \frac{R_2}{R_1 + R_2}}$$

8) Find the effective resistance of the fallowing combination of resistances and the Voltage drop across each resistance when the applied Voltage is 70V across P and Q as shown in fig. ?



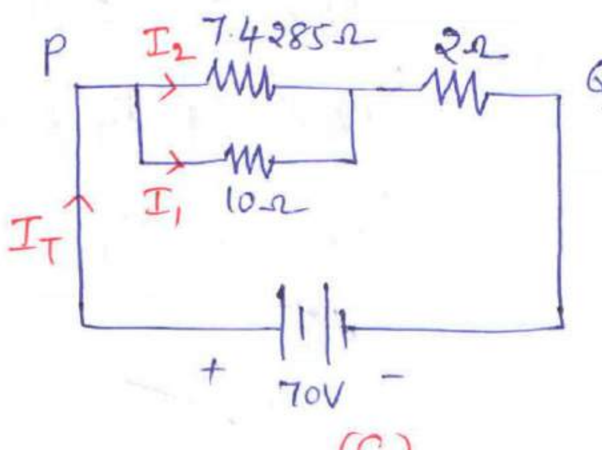
(a)

⇓

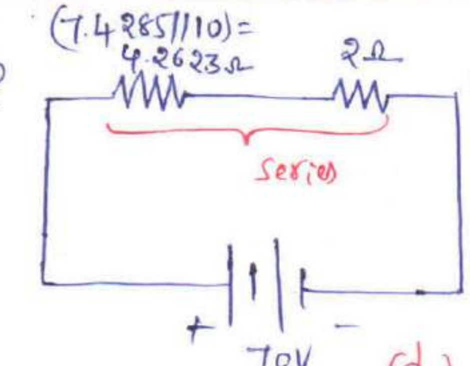


(b)

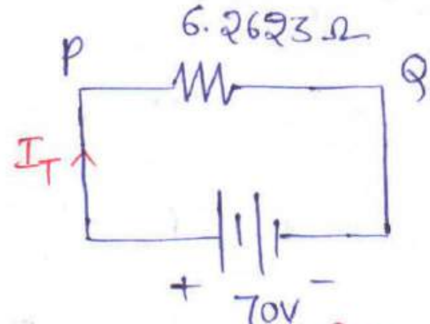
⇒



(C)



(d)



(e)

So the effective resistance $R_{PQ} = 6.2623\,\Omega$

$$\therefore\ I_T = \frac{70}{6.2623} = 11.178\,A$$

Using current division rule for the fig (c)

$$I_1 = I_T \times \frac{7.4285}{7.4285+10} = 4.7643\,A$$

$$I_2 = I_T \times \frac{10}{10+7.4285} = 6.4136\,A$$

Using current division rule for fig (a)

$$I_3 = I_2 \times \frac{6}{6+8} = \frac{6.4136 \times 6}{14} = 2.7487\,A$$

$$I_4 = I_2 \times \frac{8}{6+8} = \frac{6.4136 \times 8}{14} = 3.6649\,A$$

Voltage drops across resistances

$$V_{R_{6\Omega}} = I_4 \times 6 = 21.9894\,V$$

$$V_{R_{8\Omega}} = I_3 \times 8 = 21.9894\,V$$

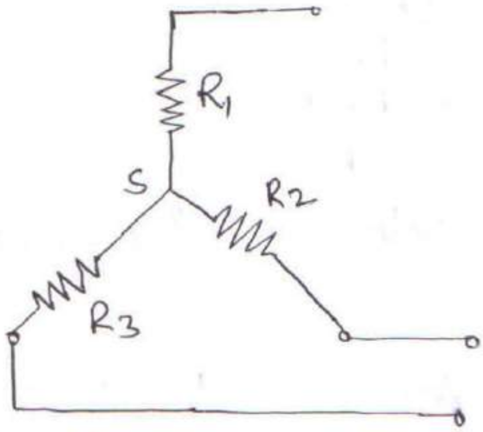
$$V_{R_{4\Omega}} = I_2 \times 4 = 25.6544\,V$$

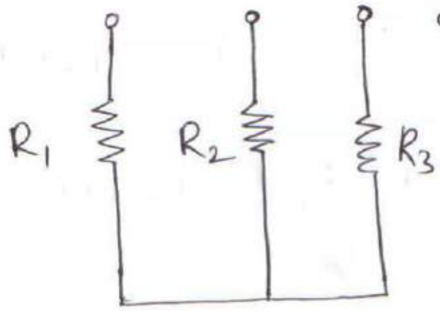
$$V_{R_{2\Omega}} = I_T \times 2 = 22.356\,V$$

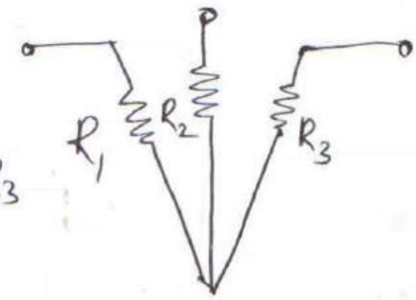
$$V_{R_{10\Omega}} = I_1 \times 10 = 47.643\,V$$

Star and Delta Connection of Resistances

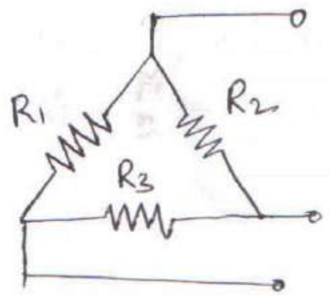
Star connection:- If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called star point, then that resistances are said to be connected in Star.

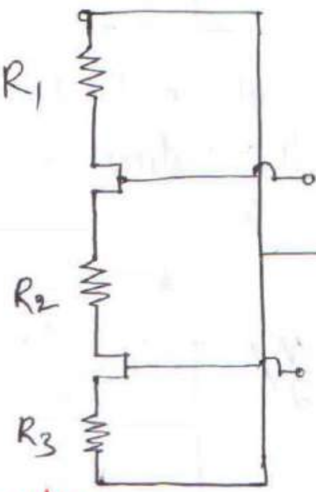


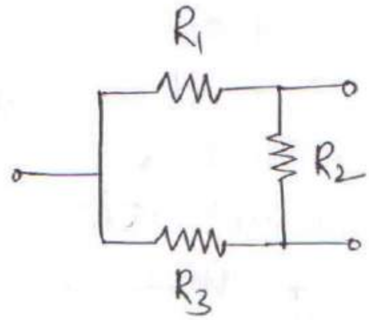




Delta connection:- If the three resistances are connected in such a manner that one end of the first connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in Delta.

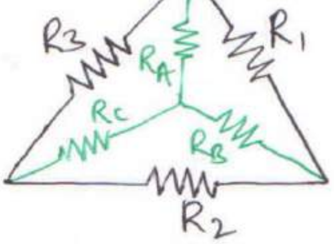






Delta to star transformation:-

we have to represent Delta equivalent star values



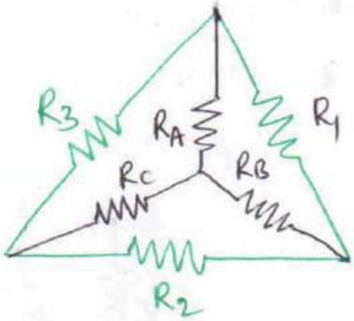
$$R_A = \frac{R_1 \times R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 \times R_2}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 \times R_3}{R_1 + R_2 + R_3}$$

Star-to-Delta transformation:-

We have to represent star equivalent Delta values.



$$R_1 = R_A + R_B + \frac{R_A \times R_B}{R_C}$$

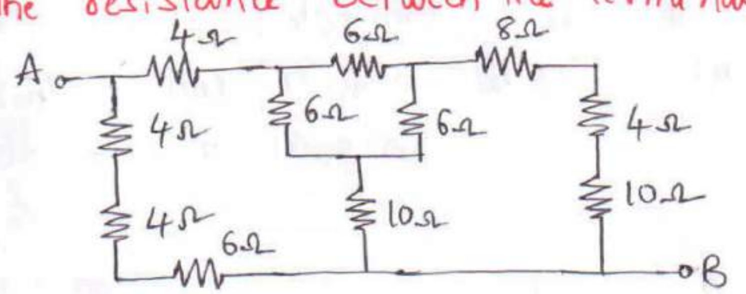
$$R_2 = R_B + R_C + \frac{R_B \times R_C}{R_A}$$

$$R_3 = R_C + R_A + \frac{R_A \times R_C}{R_B}$$

Note:- (i) If $R_1 = R_2 = R_3 = R$, then the values of Star connection are $R_A = R_B = R_C = R/3$

(ii) If $R_A = R_B = R_C = R$, then the values of Delta connection are $R_1 = R_2 = R_3 = 3R$

Q) Find the resistance between the terminals A and B.?

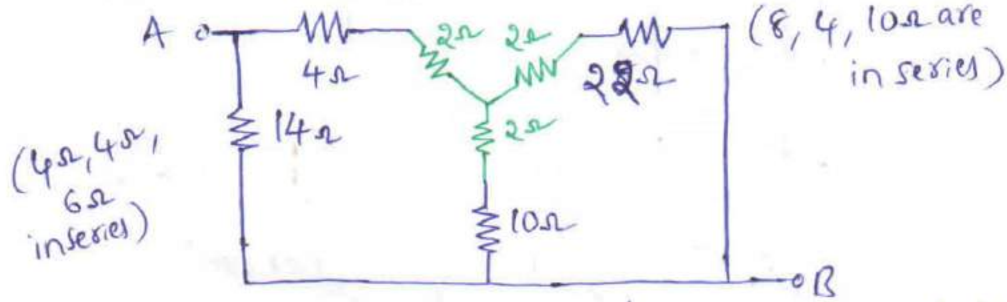


Sol

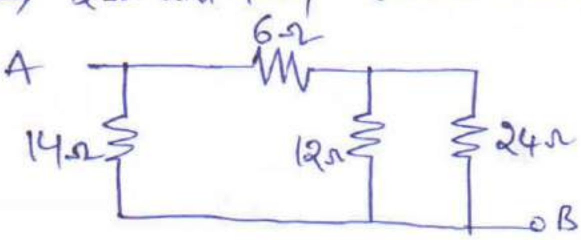
→ 6Ω, 6Ω, and 6Ω are connected in Delta connection, convert to star connection and the values are 2Ω, 2Ω, and 2Ω

→ 4Ω, 4Ω and 6Ω are in series.

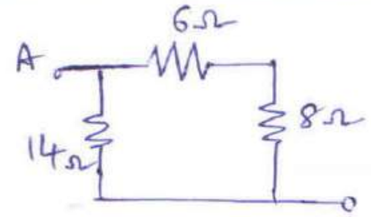
→ 8Ω, 4Ω and 10Ω are in series



→ 2Ω and 4Ω, 2Ω and 10Ω, 2Ω and 24Ω are in series



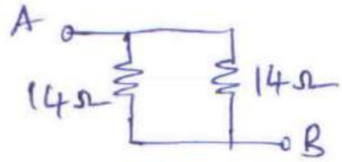
→ 12Ω and 24Ω are in parallel



$8\Omega \left(12 \| 24 = \frac{12 \times 24}{12+24} = 8\Omega \right)$

6Ω and 8Ω are in Series

⇒



⇒ 14Ω and 14Ω are in parallel

$7\Omega \left(14 \| 14 = \frac{14 \times 14}{14+14} = 7\Omega \right)$

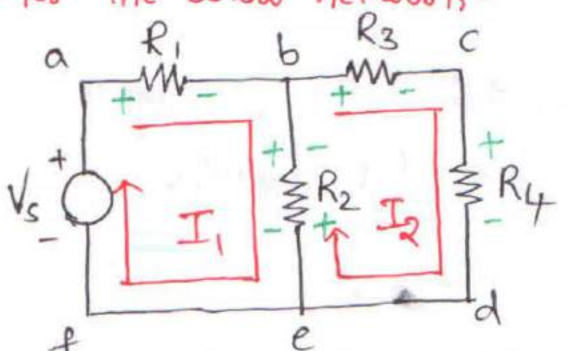
Mesh (or) Loop Analysis:-

Mesh (or) loop analysis is used to find the Values of Current in any branch.

Steps for the Mesh Analysis

1. choose the Various loops (or) Mesh
2. Assume the loop currents for Various loops
3. Assign the polarities for the Voltage drops due to assumed loop currents, across the Various elements.
4. Apply KVL to the Various loops and obtain the equations.
5. Solve the equations to find the Values of Various loop currents, with the help of loop currents we can find the branch current

Apply Mesh analysis for the below network.



1. There are two loops i.e abefa and bcdeb
2. Assume two loop currents I_1 and I_2
3. Assign the polarities for R_1, R_2, R_3 and R_4 based on assumed loop currents.

4. Apply KVL to loop-1 and loop-2

$$I_1 R_1 + R_2 (I_1 - I_2) = V_s \rightarrow (1)$$

$$R_3 I_2 + R_4 I_2 + R_2 (I_2 - I_1) = 0 \rightarrow (2)$$

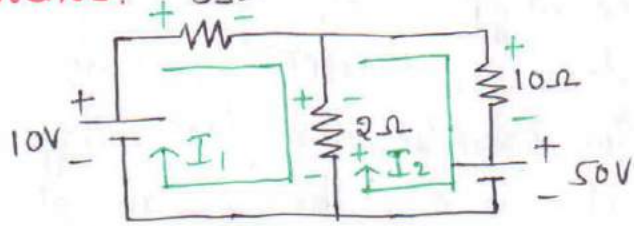
By rearranging (1) and (2) eq.

$$I_1 (R_1 + R_2) - I_2 R_2 = V_s \rightarrow (3)$$

$$-I_1 R_2 + I_2 (R_2 + R_3 + R_4) = 0 \rightarrow (4)$$

5. Solve equations (3) and (4) to find the values of I_1 and I_2. Then the current flowing through branch be is equals to $I_1 - I_2$.

10) Write the mesh current equations in the circuit shown, and determine the currents.



Assume two mesh currents I_1 and I_2 and assign the polarities for 5Ω, 2Ω, and 10Ω

Apply KVL to mesh-1

$$5I_1 + 2(I_1 - I_2) = 10$$

for Mesh-2

$$10I_2 + 50 + 2(I_2 - I_1) = 0$$

Rearrange the above equations

$$7I_1 - 2I_2 = 10 \rightarrow (1)$$

$$-2I_1 + 12I_2 = -50 \rightarrow (2)$$

Solve (1) and (2)

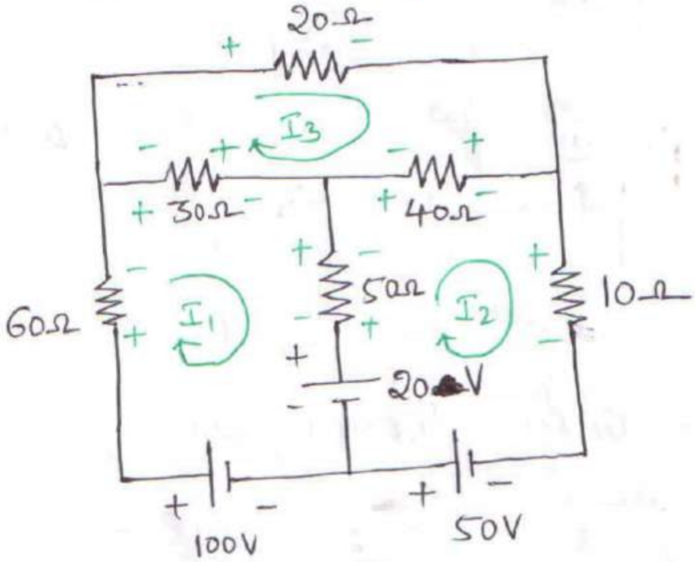
$$I_1 = 0.25\,A, \quad I_2 = -4.125\,A$$

Here the ~~second~~ current in the second mesh, I_2 is negative. that is the actual current I_2 flows opposite to the assumed direction of current in the circuit

The value of current flowing through 2Ω is

$$I_1 - I_2 = 0.25 - (-4.125) = 4.375\,A$$

11) Calculate the current in the 50 ohms resistor in the network shown in the fig. using mesh analysis.?



→ Assume mesh currents I_1, I_2, and I_3 and assign polarities.

Apply KVL to three loops.

loop-1 $\quad 30(I_1 - I_3) + 50(I_1 - I_2) + 20 - 100 + 60I_1 = 0$

$$140I_1 - 50I_2 - 30I_3 = 80 \rightarrow (1)$$

loop-2 $\quad 40(I_2 - I_3) + 10I_2 - 50 - 20 + 50(I_2 - I_1) = 0$

$$-50I_1 + 100I_2 - 40I_3 = 70 \rightarrow (2)$$

loop-3 $\quad 20I_3 + 40(I_3 - I_2) + 30(I_3 - I_1) = 0$

$$30I_1 + 40I_2 - 90I_3 = 0 \rightarrow (3)$$

Solve (1), (2) and (3)

By Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & 40 & -90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ 30 & 40 & -90 \end{vmatrix}} = 1.6489\,A.$$

$$I_2 = \frac{\begin{vmatrix} 140 & 80 & -30 \\ -50 & 70 & -40 \\ 30 & 0 & -90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ 30 & 40 & -90 \end{vmatrix}} = 2.1214A$$

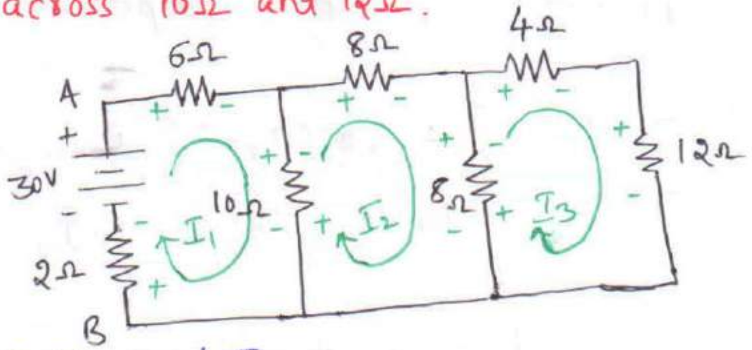
$$I_3 = \frac{\begin{vmatrix} 140 & -50 & 80 \\ -50 & 100 & 70 \\ 30 & 40 & 0 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ 30 & 40 & -90 \end{vmatrix}} = 1.4925A$$

Thus the current through 50Ω is

$$I_{50} = I_1 - I_2 = 1.6489 - 2.121 = -0.4721A$$

i.e $0.4721A \uparrow$

12) For the circuit shown in the fig find (i) current from battery (ii) potential difference across 10Ω and 12Ω.



→ Assume loop currents I_1 I_2 and I_3

→ Assign the polarity for all resistors.

→ Apply KVL to loop ① ② and ③

loop-1 $\quad -I_1 \times 6 - 10(I_1 - I_2) - 2I_1 + 30 = 0$

i.e $\quad 18I_1 - 10I_2 = 30 \rightarrow ①$

loop-2

$$-8 \times I_2 - 8(I_2 - I_3) - 10(I_2 - I_1) = 0$$

i.e $\quad 10I_1 - 26I_2 + 8I_3 = 0 \rightarrow ②$

loop-3

$$-4 \times I_3 - 12 \times I_3 - 8(I_3 - I_2) = 0$$

i.e $\quad 8I_2 - 24I_3 = 0 \rightarrow ③$

solving eq ①, ② and ③

$$I_1 = 2.1875\,A,\quad I_2 = 0.9375\,A,\quad I_3 = 0.3125\,A$$

(i) current from battery $= I_1 = 2.1875\,A$

(ii) $$I_{10\Omega} = I_1 - I_2 = 2.1875 - 0.9375 = 1.25\,A$$

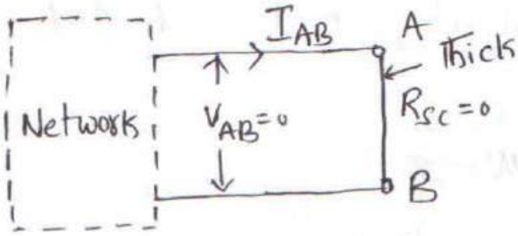
$$V_{10\Omega} = 10 \times I_{10} = 10 \times 1.25 = 12.5\,V$$

$$I_{12\Omega} = I_3 = 0.3125\,A$$

$$V_{12\Omega} = 12 \times I_{12} = 12 \times 0.3125 = 3.75\,V.$$

Short Circuit :-

When any two points in a network are joined directly to each other with a thick metallic conducting wire, the two points are said to be short circuited. The resistance of such short circuit is zero.

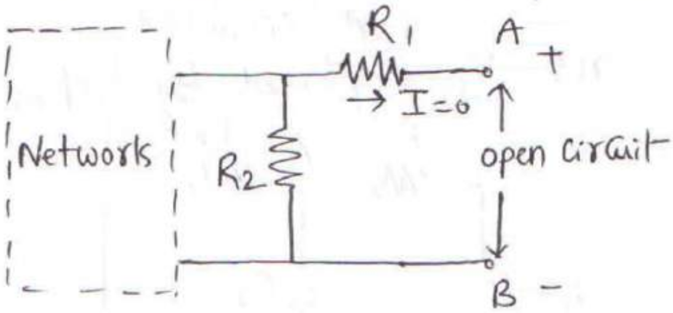


$$V_{AB} = I_{AB} \times R_{SC} = I_{AB} \times 0 = 0$$

Note: Voltage across short circuit is always zero, though current flows through the short circuited path.

open circuit :- when there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

The resistance of the open circuit is ∞ (Infinity).



$$I_{OC} = \frac{V_{AB}}{R_{OC}} = \frac{V_{AB}}{\infty} = 0$$

Note :- Current through open circuit is always zero though there exist a voltage across open circuited terminals.

THEOREMS

"Theorems are usefull to find the response in a particular element".

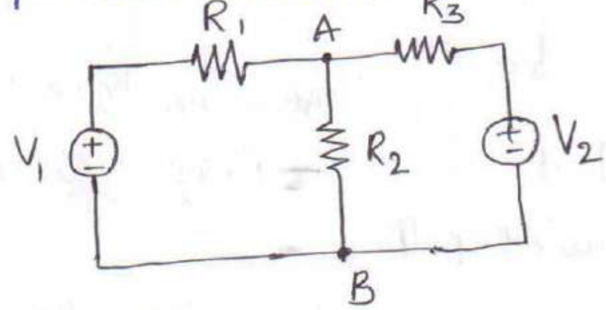
1. Super position theorem :-

statement :- In a linear network with several sources, The response (i.e current (or) voltage) in a particular element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative.

i.e other ideal voltage sources are replaced by short circuit, ideal current sources are replaced by open circuit.

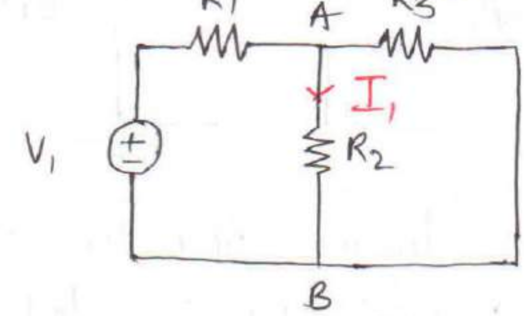
Explanation :-

→ Consider a network, shown in fig having two voltage sources V_1 and V_2. Calculate, the current in branch A-B of the network using super position theorem.



Total current flowing through branch A-B due to V_1 and V_2 = Current flowing through A-B due to V_1 alone and V_2 short circuited + Current flowing through A-B due to V_2 alone, V_1 short circuited

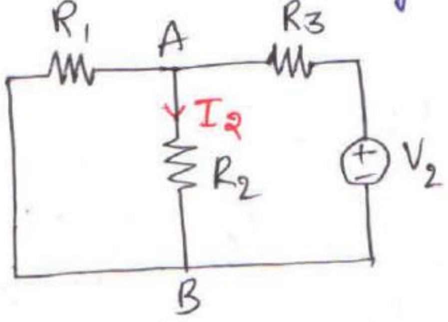
step : 1 V_1 acting alone and V_2 replaced by short circuited.



By using any technique find I_1

step 2 :- V_2 acting alone and V_1 replaced by short circuit.

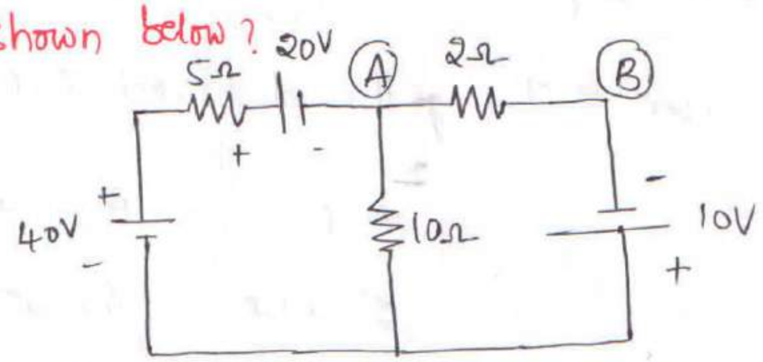
By using any reduction technique find I_2



∴ From Super position theorem

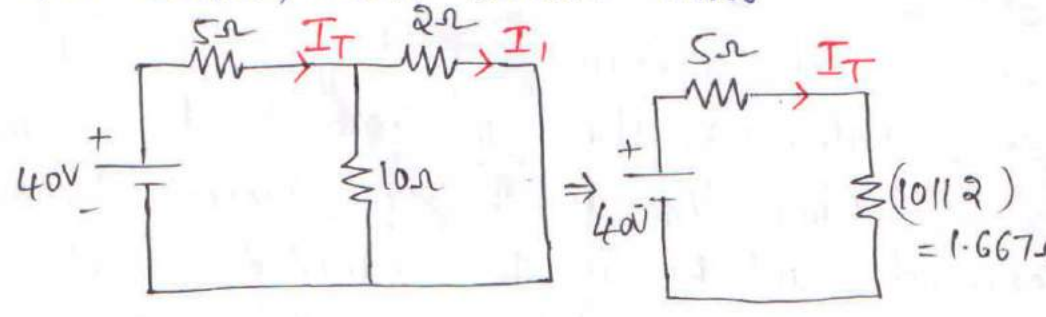
Total current flowing through branch A-B due to V_1 and V_2 $= I_1 + I_2$

13) Using Super position theorem Calculate current flowing in branch A-B for the circuit shown below?



Totally we have 3 sources (40V, 20V, 10V)

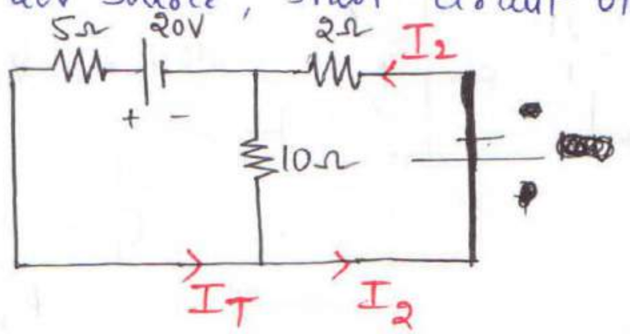
step 1 :- Consider 40V source, short circuit other

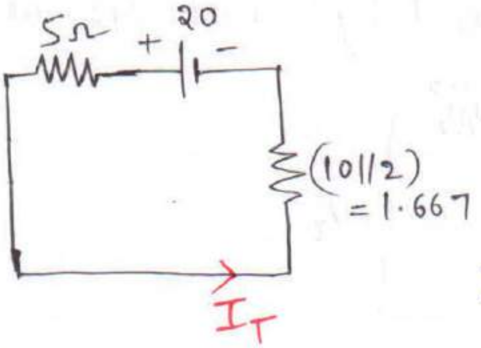


$$I_T = \frac{40}{5+1.667} = 6A.$$

$$I_1 = \frac{I_T \times 10}{10+2} = \frac{6 \times 10}{10+2} = \frac{\cancel{6} \times \cancel{10}}{\cancel{12}} = 5A$$

step 2 : Consider 20V source, short circuit other sources

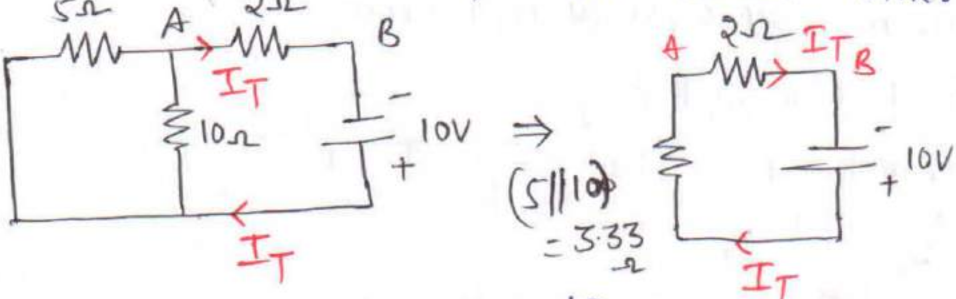




$$I_T = \frac{20}{5+1.667} = 3A$$

$$I_2 = \frac{I_T \times 10}{10+2} = \frac{3 \times 10}{12} = 2.5A \text{ (B to A)}$$

step 3:- consider 10V source, short circuit other.



$$I_T = I_3 = \frac{10}{2+3.33} = 1.875A$$

∴ current flowing through branch A-B is equals to

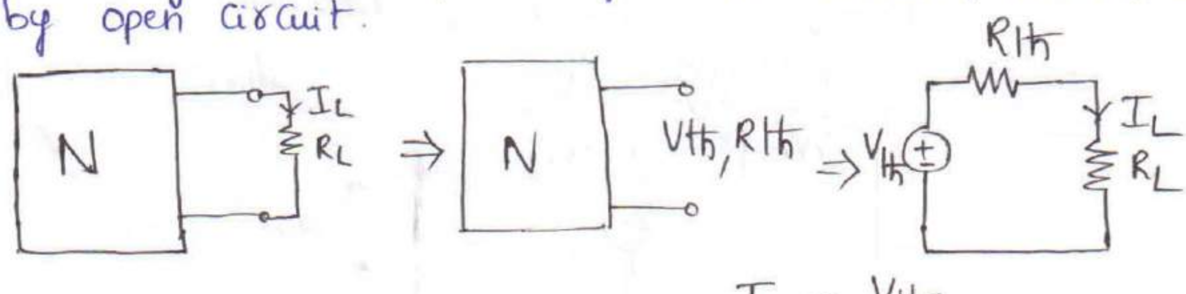
$$= I_1 (A-B) + I_2 (B-A) + I_3 (A-B)$$

$$= 5 - 2.5 + 1.875 = 4.375\,A \text{ (from A to B)}$$

Thevenin's Theorem

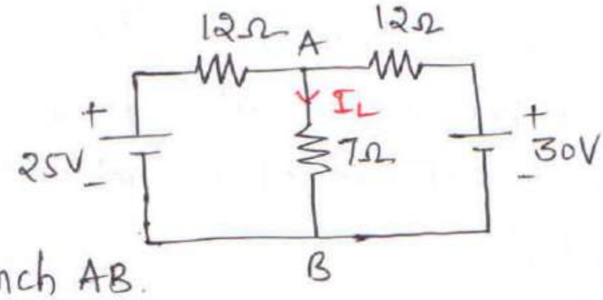
statement:- a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, across the two terminals of the load R_L.

where V_{th} is the open circuit voltage across the load terminals and R_{th} is the equivalent resistance of the networks as viewed through load terminals with R_L removed and all the active sources are replaced by their internal resistances i.e voltage source replaced by short circuit, current source replaced by open circuit.

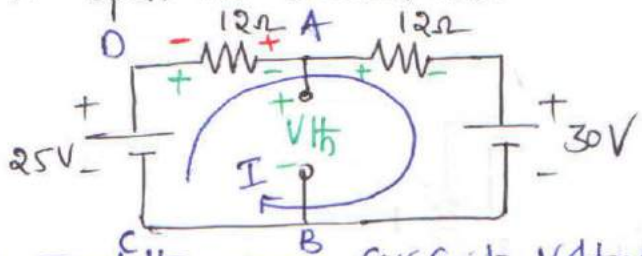


$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

14) Using thevenin's theorem, find the current through the 7Ω resistor.?



step 1: open the branch AB.



step 2: Find the open circuit voltage between A and B

$V_{th} = V_{AB}$

Apply KVL

$$-12I - 12I - 30 + 25 = 0$$

$$-24I - 5 = 0$$

$$I = -\frac{5}{24} = -0.20833A$$

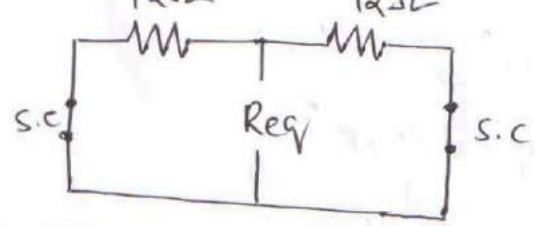
'−' (−Ve) represents the assumed current direction is wrong, so the current flowing in anty clocks wise direction

Apply KVL to loop ABCDA

$$-V_{th} + 25 + 12 \times 0.20833 = 0$$

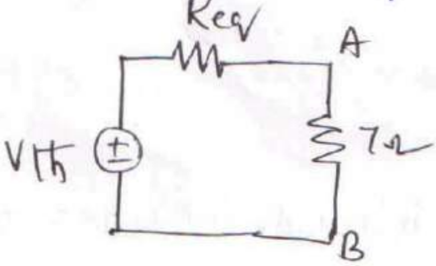
$$V_{th} = 25 + 2.5 = 27.5\ V.$$

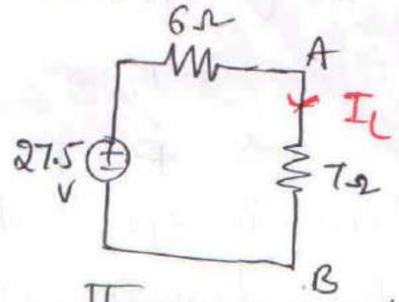
step 3: Calculate Req, and short circuit the voltage source.



So 12Ω and 12Ω are connected in parallel

$$\therefore R_{eq} = 12 // 12 = 6\Omega$$





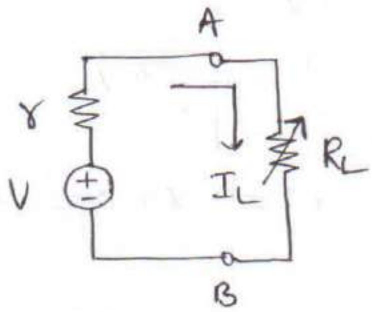
$$I_L = \frac{27.5}{6+7}$$
$$= 2.1153A \downarrow$$

Maximum power transfer theorem :

Statement :- In an active resistive network, maximum power transfer to the load resistance takes place when the load resistance equals the equivalent resistance of the network as viewed from the terminals of load.

Proof :-

Consider a network with voltage source V, and internal resistance 'r' connected to load resistance R_L.



$\rightarrow$ $$I_L \text{ (Load current)} = \frac{V}{R_L + r}$$

$\rightarrow$ The power consumed by the load resistance R_L is

$$P = I_L^2 R_L = \left[\frac{V}{r + R_L}\right]^2 R_L$$

$\rightarrow$ R_L is a variable quantity, we have to find the value of R_L for maximum power transfer with the help of Maximum theorem in mathematics i.e

$$\frac{dP}{dR_L} = 0 \Rightarrow \frac{d}{dR_L}\left[\frac{V}{(r+R_L)}\right]^2 R_L = 0$$

$$V^2 \frac{d}{dR_L}\left[\frac{R_L}{(r+R_L)^2}\right] = 0$$

$$(r+R_L)^2 \frac{dR_L}{dR_L} - R_L \frac{d(r+R_L)^2}{dR_L} = 0$$

$$(r+R_L)^2 - R_L\, 2(r+R_L) = 0$$

$$(r+R_L)\left[(r+R_L) - 2R_L\right] = 0$$

$$r - R_L = 0$$

$$\Rightarrow R_L = r$$

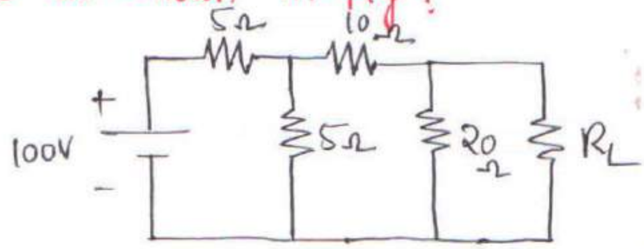
i.e when load resistance is equal to the internal resistance of source the maximum power transfer takes place.

The magnitude of maximum power transfer

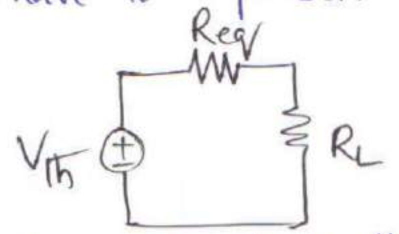
$$P_{max} = \left(\frac{V}{r+R_L}\right)^2 \times R_L \qquad (R_L = r)$$

$$= \frac{V^2}{(2R_L)^2} \times R_L = \frac{V^2}{4R_L^{\cancel{2}}} \times \cancel{R_L} = \frac{V^2}{4R_L} \text{ watts}$$

15) Find the value of R_L so that the maximum power is delivered to the load resistance as shown in fig?

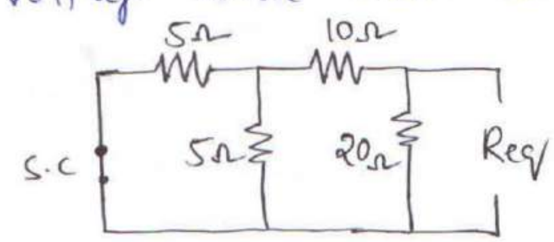


We have to represent the total network as like shown below

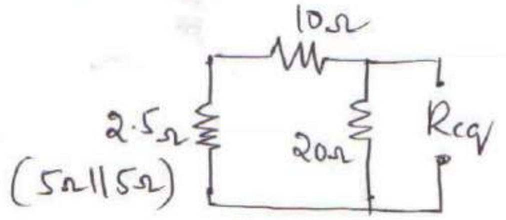


Then $R_L = R_{eq}$ for max power transfer

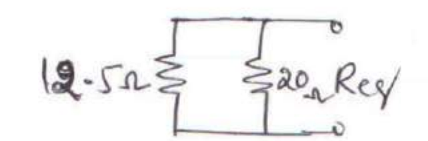
So we have to find the value of R_{eq} between load terminals. (Voltage source short circuited)



So 5Ω and 5Ω are in parallel



2.5Ω and 10Ω are in series



12.5Ω and 20Ω are in parallel

$$\text{So } R_{eq} = 12.5\Omega \,||\, 20\Omega = \frac{12.5 \times 20}{12.5+20} = 7.6923\Omega$$

$$\therefore R_L = R_{eq} = 7.6923\Omega$$