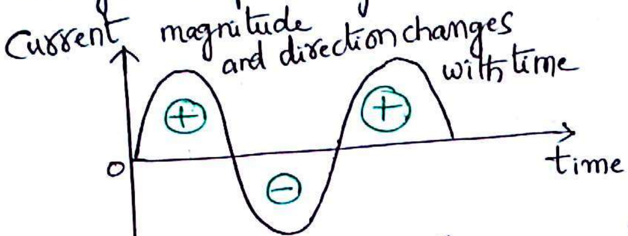
Alternating Quantities

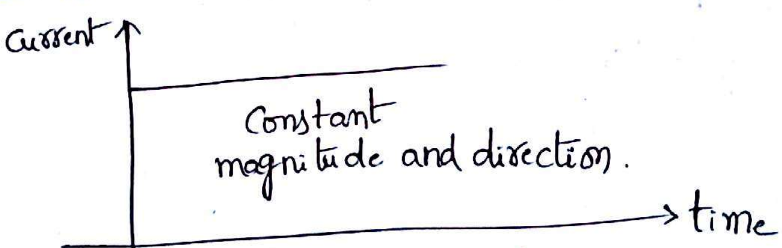
Electrical Supply used for commercial and domestic purpose is of alternating nature.

Alternating Current : (AC) An alternating current waveform is defined as the current that fluctuates with time periodically, with change in polarity and direction.



In alternating waveform there are two half cycles, one positive and other negative. These two half cycles make one cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly.

Direct Current : (DC) The d.c supply has constant magnitude with respect to time.



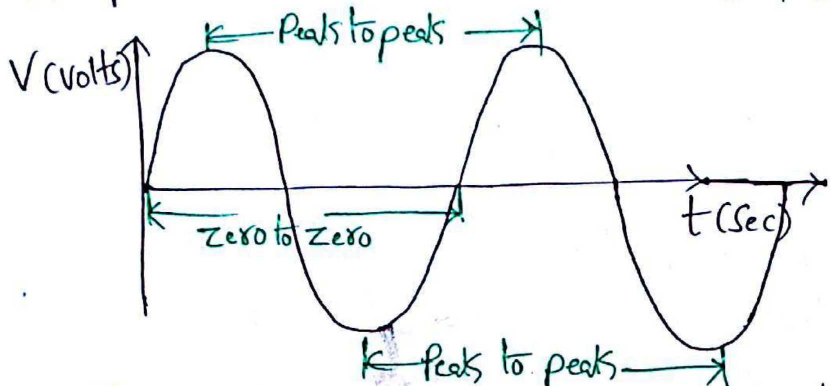
Standard Definitions Related to Alternating Quantity

1. Cycle :- The complete positive and negative portion of the wave is one cycle of the sine wave

2. Time period :- The time taken by an alternating quantity to complete its one cycle is known as time period denoted by T seconds.

After every 'T' seconds, the cycle of an alternating quantity repeats. So it is called Periodic wave.

The period can be measured in the fallowing different ways



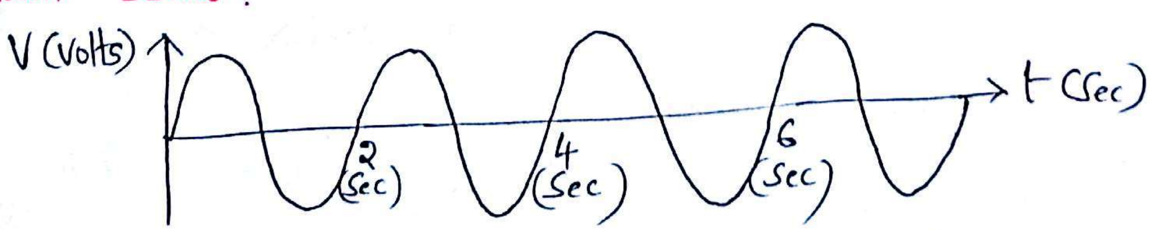
1. From Zero crossing of one cycle to zero crossing of the next cycle.
2. From positive peak of cycle to positive peaks of the next cycle
3. From negative peak of one cycle to negative peak of the next cycle.

3. Frequency :- (f) The number of cycles completed by an alternating quantity per second is known as its frequency. It is denoted by f and it is measured in cycles/second (or) Hertz (Hz).

As time period T is time for one cycle i.e Seconds/Cycle and frequency is cycles/seconds

So $$\boxed{f = \frac{1}{T} \ Hz}$$

1) Find the time period and frequency for the sine wave shown below?

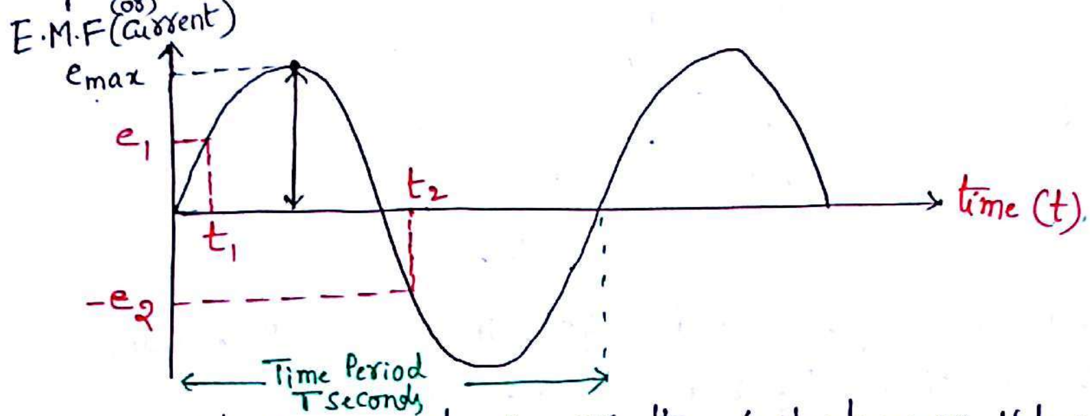


The sine wave takes two seconds to complete one period in each cycle

$$T = 2 \ Sec$$

$$f = \frac{1}{T} = \frac{1}{2} = 0.5 \ Hz.$$

4. Instantaneous Value : The Value of an alternating quantity at a particular instant is known as its instantaneous Value.



where e_1 and $-e_2$ are the instantaneous Values of an alternating e.m.f (or) Voltage at the instants t_1 and t_2 respectively.

5. Amplitude :- The maximum Value attained by an alternating quantity during positive (or) negative half cycle is called its amplitude. It is denoted as E_m (or) I_m.

Thus E_m is called peak value of the voltage while I_m is called peak Value of the current.

So the amplitude is also called peak value (or) maximum value of an alternating quantity.

6. Peak to Peak Value :- The Value of an alternating quantity from its positive peak to negative peak is called its peak to peak Value. It is denoted as I_{P-P} (or) V_{P-P}

$$\text{Amplitude} = \frac{\text{Peak to peak Value}}{2}.$$

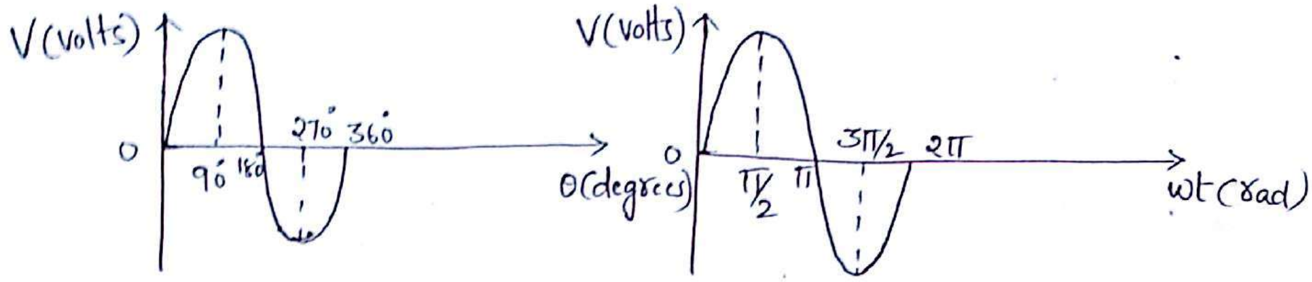
Angular relation of a Sine wave :-

A Sine wave can be measured along the X-axis on a time base which is frequency dependent.

A Sinewave can also be expressed in terms of an angular measurement. This angular measurement is expressed in degrees (or) radians.

1 Radian $= 57.3^\circ$, in a 360° revolution.

The angular measurement of a sine wave is based on 360° (or) 2π radians for a complete cycle shown below.



A sine wave completes a half cycle in 180° (or) π radians, a quarter cycle in 90° (or) $\pi/2$ radians, and so on.

where ω = Angular frequency

$\omega = 2\pi f$ units: radians/second

(or) $\omega = \dfrac{2\pi}{T}$

$\theta = \omega t$ (radians) $\theta = 2\pi f t$ radians.

The Sinewave equation :-

As the standard wave form of an alternating quantity is purely sinusoidal, the equation of an alternating voltage can be expressed as

$$V(t) = V_m \sin\omega t \quad \text{(or)} \quad e = E_m \sin\omega t$$

where V_m = Amplitude (or) maximum (or) peak value of the voltage

$V(t)$ = Instantaneous value of an alternating voltage

Similarly equation of an alternating current can be expressed as

$$i = I_m \sin\omega t$$

(or)

$$i(t) = I_m \sin 2\pi f t$$

I_m = Amplitude (or) maximum (or) peak value of the current

i = Instantaneous value of an alternating current.

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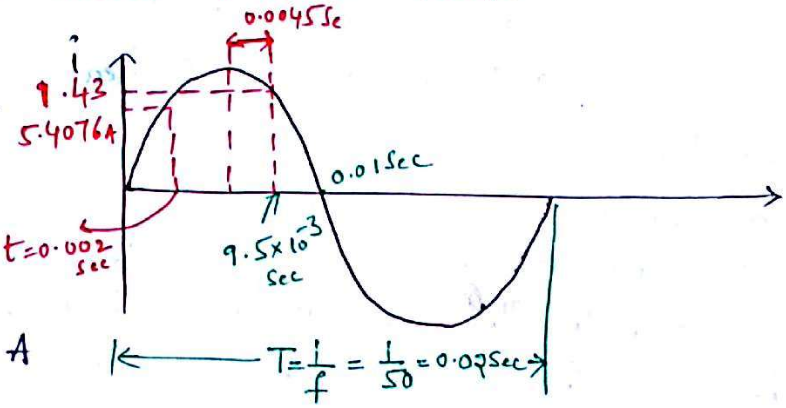
2) A Sinusoidal wave of frequency 50 Hz has its maximum value of 9.2 Amps. What will be its value at (a) 0.002 sec after the wave pass through zero in positive direction (b) 0.0045 sec after the wave passes through positive maximum. Show the values of current in a neat sketch of the waveform? (3)

<u>Sol.</u>



Given $I_m = 9.2$ Amps

$f = 50$ Hz

$i = I_m \sin 2\pi f t$

$i(t) = 9.2 \sin 100\pi t$ A

a) at $t = 0.002$ Sec

$$i(0.002) = 9.2 \sin(100\pi \times 0.002) = 5.4076 \text{ A}$$

use Sin in radians mode

(b) at $t = 0.0045$ sec after positive maximum

i.e $T = \frac{1}{f} = \frac{1}{50} = 0.02$ Sec

So positive maximum occurs at $T/4 = \frac{0.02}{4} = 0.005 = 5 \times 10^{-3}$ sec

after this 0.0045 Sec means value of i at

$t = 5 \times 10^{-3} + 0.0045 = 0.0095$ Se

$$i(0.0095) = 9.2 \sin(2\pi \times 50 \times 0.0095) = 1.4391 \text{ A}$$

Root mean Square Value (or) Effective Value :-

Root mean Square values are used to analyse the effects of an alternating quantity.

The root mean square value of a sine wave is a measure of the heating effect of the wave.

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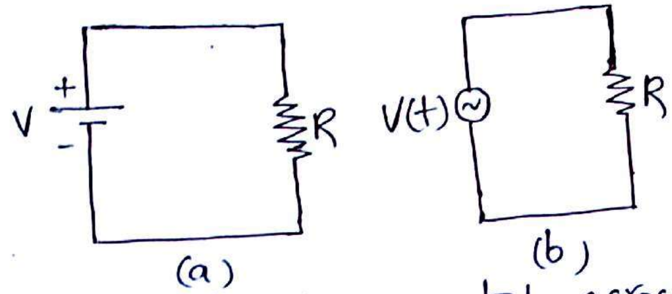


when a resistor is connected across a dc voltage source as shown in fig (a), a certain amount of heat is produced in the resistor in a given time.

A similar resistor is connected across an ac voltage source for the same time as shown in fig (b). The value of the ac voltage is adjusted such that the same amount of heat is produced in the resistor as in the case of the dc source. This value is called the rms value.

The rms value of a sine wave is equal to the dc voltage that produces the same heating effect.

The rms value of any function with period T has an effective value given by

$$\boxed{V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\, dt}}$$

Consider a function $v(t) = V_p \sin\omega t$

For sine wave $T = 2\pi$

The rms value, $V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_p \sin\omega t)^2\, d(\omega t)}$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \sin^2\omega t\, d(\omega t)}$$

$$= \sqrt{\frac{V_p^2}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

$$= \sqrt{\frac{V_p^2}{2\pi}\left[\frac{1}{2}(2\pi - 0) - \frac{1}{2}\left(\sin 2\omega t\right)_0^{2\pi}\right.}$$

$$= \sqrt{\frac{V_p^2}{4\pi}\left[2\pi - 0\right]} = \sqrt{\frac{V_p^2}{\cancel{4\pi}_2} \times \cancel{2\pi}} = \frac{V_p}{\sqrt{2}}$$

$$\therefore V_{rms} = \frac{V_p}{\sqrt{2}} = 0.707\, V_p$$

If the function consists of a number of sinusoidal terms, that is

$$v(t) = V_0 + (V_{c_1} \cos\omega t + V_{c_2} \cos 2\omega t + \cdots) + (V_{s_1} \sin\omega t + V_{s_2} \sin 2\omega t + \cdots)$$

The rms (or) effective value is given by

$$V_{rms} = \sqrt{V_0^2 + \frac{1}{2}(V_{c_1}^2 + V_{c_2}^2 + \cdots) + \frac{1}{2}(V_{s_1}^2 + V_{s_2}^2 + \cdots)}$$

3) A wire is carrying a direct current of 20A and a sinusoidal alternating current of peak value 20A. Find the rms value of the resultant current in the wire.?

The rms value of the combined wave

$$= \sqrt{20^2 + \frac{20^2}{2}}$$

$$= \sqrt{400 + 200} = \sqrt{600} = 24.5A$$

Note :- The rms value of the sinusoidal alternating voltage (or) current is 0.707 times the maximum (or) peak value of that alternating current (or) voltage.

Average Value :- The average value of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

The average value of any function with period T is given by

$$\boxed{V_{avg} = \frac{1}{T}\int_0^T v(t)\, dt}$$

Consider a function $V(t) = V_m \sin\omega t$.

$$V_{avg} = \frac{1}{T}\int_0^T V_m \sin\omega t \, d(\omega t)$$

$$= \frac{V_m}{\pi}(-\cos\omega t)_0^{\pi} = \frac{V_m}{\pi}[-\cos\pi + \cos 0]$$

$$= \frac{2V_m}{\pi} = 0.637\, V_m$$

Crest (or) Peak Factor (K_p)

The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s value.

$$K_p = \frac{\text{Maximum Value}}{\text{r.m.s value}}$$

The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

$$K_p = \frac{V_m}{0.707V_m} = 1.414$$

Form factor:-

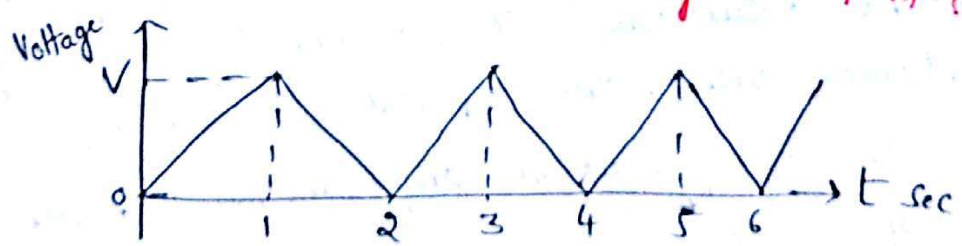
The form factor of an alternating quantity is defined as the ratio of r.m.s value to the average value

$$K_f = \frac{\text{rms Value}}{\text{Average Value}}$$

for sinusoidal alternating currents (or) voltages

$$K_f = \frac{0.707\,I_m}{0.637\,I_m} = 1.11$$

4) Find the form factor for the fallowing wave form.?



Sol

The time period of the wave-form T = 1 Sec.

From 0 to 1, Slope $= \frac{V-0}{1-0} = V$, i.e $v(t) = Vt$ $(y = mx + c)$

1 to 2, slope $= \frac{0-V}{2-1} = -V$

$v(t) = -Vt + c$

at $t = 2$, $v(t) = 0$

$0 = -2V + c \Rightarrow c = 2V$

$\therefore \; v(t) = -Vt + 2V = V[2-t]$

from 1 to 2

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Average Value $= \frac{\text{Area under the curve}}{\text{Base}} = \frac{\int_0^2 v(t)\,dt}{2}$

$= \frac{1}{2}\left[\int_0^1 v(t)\,dt + \int_1^2 v(t)\,dt\right]$

$= \frac{1}{2}\left[\int_0^1 Vt\,dt + \int_1^2 V(2-t)\,dt\right]$

$= \frac{V}{2}\left[\left(\frac{t^2}{2}\right)_0^1 + \left(2t - \frac{t^2}{2}\right)_1^2\right]$

$= \frac{V}{2}\left[\frac{1}{2} + \left[\left(4 - \frac{4}{2}\right) - \left(2 - \frac{1}{2}\right)\right]\right]$

$= \frac{V}{2}\left[\frac{1}{2} + 2 - \frac{3}{2}\right] = \frac{V}{2}\left[\frac{1}{2} + \frac{1}{2}\right] = \frac{V}{2}$

Rms Value $= \sqrt{\frac{1}{2}\int_0^2 v^2(t)\,dt}$

$= \sqrt{\frac{1}{2}\left[\int_0^1 (Vt)^2\,dt + \int_1^2 (V(2-t))^2\,dt\right.}$

$= \frac{1}{\sqrt{2}}\sqrt{V^2\left[\left(\frac{t^3}{3}\right)_0^1 + \int_1^2 (4 + t^2 - 4t)\,dt\right]}$

$= \frac{1}{\sqrt{2}}\sqrt{V^2\left[\frac{1}{3} + \left[4t + \frac{t^3}{3} - \frac{4t^2}{2}\right]_1^2\right.}$

$= \frac{1}{\sqrt{2}}\sqrt{V^2\left[\left(\frac{1}{3}\right) + \left(4\times 2 + \frac{8}{3} - 2\times 4\right) - \left(4 + \frac{1}{3} - 2\right)\right.}$

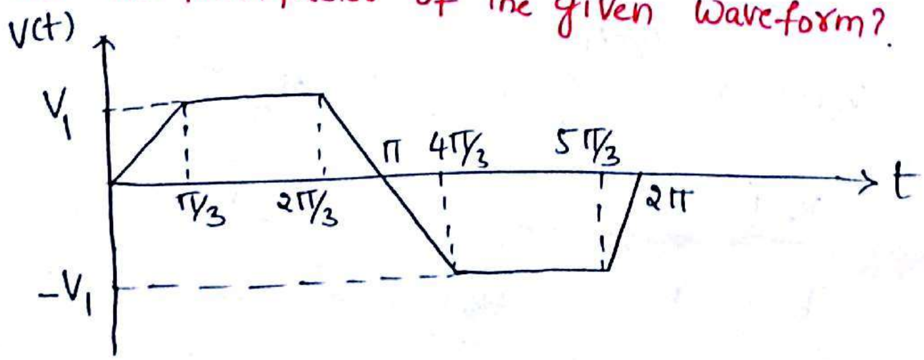
$= \frac{V}{\sqrt{2}}\sqrt{\left(\frac{1}{3} + \frac{8}{3} - \frac{7}{3}\right)}$

$= \frac{V}{\sqrt{2}}\sqrt{\frac{2}{3}} = \frac{V}{\sqrt{3}}$

$= 0.5773V$

Form factor $= \frac{\text{R.m.s Value}}{\text{Avg Value}} = \frac{0.5773V}{0.5V} = 1.1546$

5) Find the form factor of the given waveform?



Time period $T = 2\pi$

Hint: For symmetrical wave-form find R.m.s value by considering half wave that will be equals to R.m.s value for full wave.

So consider $T = \pi$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/3} V_1^2(t)\,dt + \int_{\pi/3}^{2\pi/3} V_2^2(t)\,dt + \int_{2\pi/3}^{\pi} V_3^2(t)\,dt\right.}$$

$\Rightarrow$ $V_1(t)$ from $(0,0)$ to $(\pi/3, V_1)$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$y = V_1(t)$ $\quad (V_1(t) - 0) = \left(\frac{V_1 - 0}{\pi/3 - 0}\right)(t - 0)$

$x = t$

$$\boxed{V_1(t) = \frac{3V_1 t}{\pi}}$$

$$\Rightarrow \int_0^{\pi/3}\left(\frac{3V_1 t}{\pi}\right)^2 dt = \frac{9V_1^2}{\pi^2}\left(\frac{t^3}{3}\right)_0^{\pi/3}$$

$$= \frac{9V_1^2}{\pi^2}\left(\frac{\pi^3}{27\times 3}\right) = \boxed{\frac{V_1^2\pi}{9}} \rightarrow (1)$$

$\Rightarrow$ $V_2(t)$ it is parallel to x-axis

So $V_2(t) = V_1$

$$\Rightarrow \int_{\pi/3}^{2\pi/3} V_1^2\,dt = V_1^2\left(\frac{2\pi}{3} - \pi/3\right) = \boxed{\frac{V_1^2\pi}{3}} \rightarrow (2)$$

$\rightarrow$ $V_3(t)$ from $(2\pi/3, V_1)$ to $(\pi, 0)$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$V_3(t) - V_1 = \left(\frac{0 - V_1}{\pi - 2\pi/3}\right)(t - 2\pi/3)$$

$$V_3(t) - V_1 = -\frac{3V_1}{\pi}(t - 2\pi/3)$$

$$V_3(t) - V_1 = -\frac{3V_1 t}{\pi} + \frac{\cancel{3}V_1}{\cancel{\pi}} \times \frac{2\cancel{\pi}}{\cancel{3}}$$

$$V_3(t) = -\frac{3V_1 t}{\pi} + 2V_1 + V_1 = -\frac{3V_1 t}{\pi} + 3V_1$$

$$V_3(t) = 3V_1(1 - t/\pi)$$

$$\Rightarrow \int_{2\pi/3}^{\pi} \left(3V_1 - \frac{3V_1 t}{\pi}\right)^2 dt$$

$$\Rightarrow \int_{2\pi/3}^{\pi} \left[\frac{9V_1^2 t^2}{\pi^2} - \frac{18V_1^2 t}{\pi} + 9V_1^2\right] dt$$

$$\Rightarrow 9V_1^2 \left[\frac{1}{\pi^2}\left(\frac{t^3}{3}\right) - \frac{2}{\pi}\left(\frac{t^2}{2}\right) + t\right]_{2\pi/3}^{\pi}$$

$$\Rightarrow 9V_1^2 \left[\left(\frac{1}{\cancel{\pi^2}}\left(\frac{\pi^{\cancel{3}}}{3}\right) - \frac{\cancel{2}}{\cancel{\pi}} \times \frac{\pi^{\cancel{2}}}{\cancel{2}} + \pi\right) - \left(\frac{1}{\pi^2} \frac{8\pi^3}{27\times 3} - \frac{2}{\pi}\left(\frac{4\pi^2}{9\times 2}\right) + \frac{2\pi}{3}\right)\right]$$

$$\Rightarrow 9V_1^2 \left(\left(\frac{\pi}{3} \cancel{-\pi} \cancel{+\pi}\right) - \left(\frac{8\pi}{27\times 3} - \frac{4\pi}{9} + \frac{2\pi}{3}\right)\right)$$

$$9V_1^2 \left[\frac{\pi}{3} - \frac{8\pi}{81} + \frac{4\pi}{9} - \frac{2\pi}{3}\right]$$

$$\Rightarrow \cancel{9}V_1^2 \left[\frac{27\pi - 8\pi + 36\pi - 54\pi}{81\cancel{9}}\right)$$

$$= \frac{V_1^2 (63\pi - 62\pi)}{9} = \boxed{\frac{V_1^2 \pi}{9}} \rightarrow 3$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\left[1 + 2 + 3\right]}$$

$$= \sqrt{\frac{1}{\pi}\left(\frac{V_1^2\pi}{9} + \frac{V_1^2\pi}{3} + \frac{V_1^2\pi}{9}\right)} = \sqrt{\frac{1}{\cancel{\pi}}\left(\frac{5V_1^2\cancel{\pi}}{9}\right)}$$

$$= \frac{\sqrt{5}\,V_1}{3} = 0.7453V_1$$

The average value is also to be obtained over half cycle

$$V_{avg} = \frac{\int_0^{\pi} v(t)\,dt}{\pi} = \frac{1}{\pi}\left[\int_0^{\pi/3} v_1(t)\,dt + \int_{\pi/3}^{2\pi/3} v_2(t)\,dt + \int_{2\pi/3}^{\pi} v_3(t)\,dt\right]$$

$$\Rightarrow \int_0^{\pi/3} v_1(t)\,dt = \int_0^{\pi/3} \frac{3V_1 t}{\pi}\,dt = \frac{3V_1}{\pi}\left[\frac{t^2}{2}\right]_0^{\pi/3} = \boxed{\frac{V_1\pi}{6}} \rightarrow 1$$

$$\int_{\pi/3}^{2\pi/3} v_2(t)\,dt = \int_{\pi/3}^{2\pi/3} V_1\,dt = V_1 (t)\Big|_{\pi/3}^{2\pi/3} = V_1\left(2\pi/3 - \pi/3\right)$$

$$= \boxed{\frac{V_1\pi}{3}} \rightarrow (2)$$

$$\int_{2\pi/3}^{\pi} v_3(t)\,dt = \int_{2\pi/3}^{\pi}\left[-\frac{3V_1 t}{\pi} + 3V_1\right]dt$$

$$= 3V_1\left[\frac{-t^2}{2\pi} + t\right]_{2\pi/3}^{\pi}$$

$$= 3V_1\left[\left(-\frac{\pi^{\cancel{2}}}{2\cancel{\pi}} + \pi\right) - \left(-\frac{\cancel{4}\pi^2}{9\times2\times\pi} + 2\pi/3\right)\right]$$

$$= 3V_1\left[\pi/2 + \frac{2\pi}{9} - \frac{2\pi}{3}\right] = 3V_1\left[\frac{9\pi + 4\pi - 12\pi}{18}\right]$$

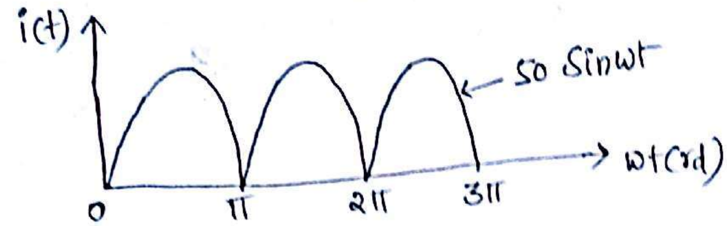
$$= \cancel{3}V_1\left[\frac{\pi}{\cancel{18}\,6}\right] = \boxed{\frac{V_1\pi}{6}} \rightarrow (3)$$

$$\therefore V_{avg} = \frac{1}{\pi}\left[(1) + (2) + (3)\right]$$

$$= \frac{1}{\pi}\left[\frac{V_1\pi}{6} + \frac{V_1\pi}{3} + \frac{V_1\pi}{6}\right] = \frac{2V_1}{3} = 0.666V_1$$

$$K_f = \text{form(form)-factor} = \frac{\text{R.m.s}}{\text{Avg Value}} = \frac{0.7453V_1}{0.666V_1} = 1.1179.$$

6) The current of the fallowing wave-form is passed through 5 ohms resistor Find the power consumed?



Sol Let us find the r.m.s Value of the current,

$T = \pi$ (Time period)

$i(t) = 50\sin\omega t$

$$I_{rms} = \sqrt{\frac{\int_0^{\pi} i^2(t)\,d(\omega t)}{\pi}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} 2500\sin^2\omega t\,d(\omega t)}$$

$$= \frac{50}{\sqrt{\pi}}\sqrt{\int_0^{\pi}\sin^2\omega t\,d(\omega t)} = \frac{50}{\sqrt{\pi}}\sqrt{\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d(\omega t)}$$

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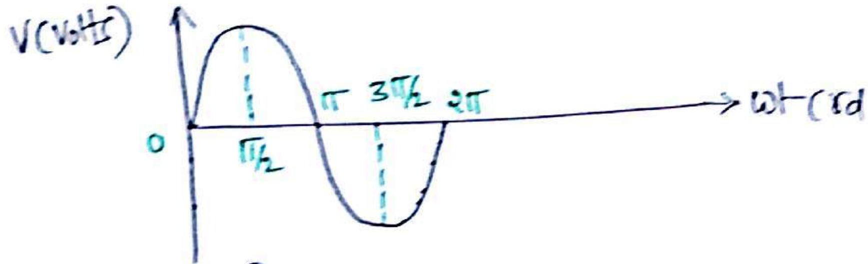
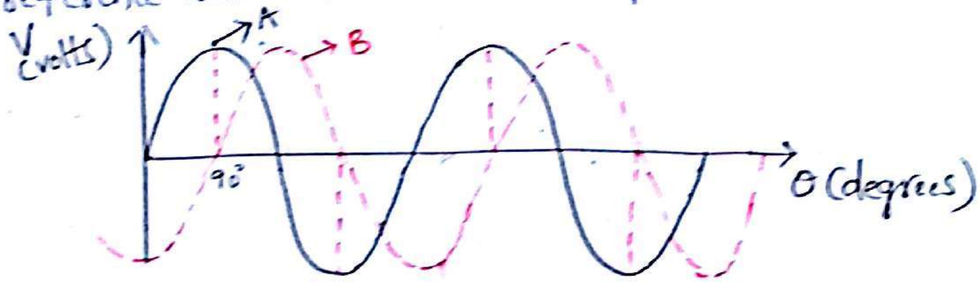
Phase of a Sinewave

The phase of a Sine wave is an angular measurement that Specifies the position of the sinewave relative to a reference.



Reference wave

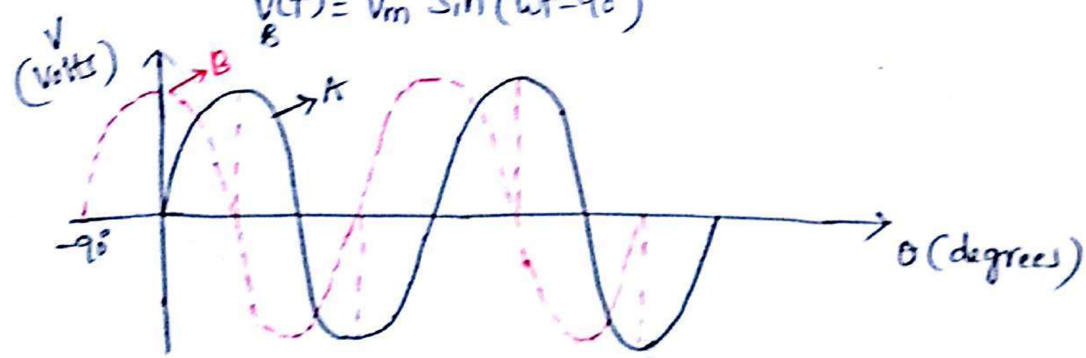
When the Sine wave is shifted left (or) right with respect to the reference wave there occurs a phase shift.



The Sine wave is shifted to the right by 90° (π/2 rd) shown by the dotted lines. There is a phase angle of 90° between A and B.

Here the waveform B is lagging behind waveform A by 90°. In otherwords the sine wave A is leading the waveform B by 90°.

$$V_B(t) = V_m \sin(\omega t - 90°)$$



The Sinewave A is lagging behind the waveform B by 90°

$$V_B(t) = V_m \sin(\omega t + 90°)$$

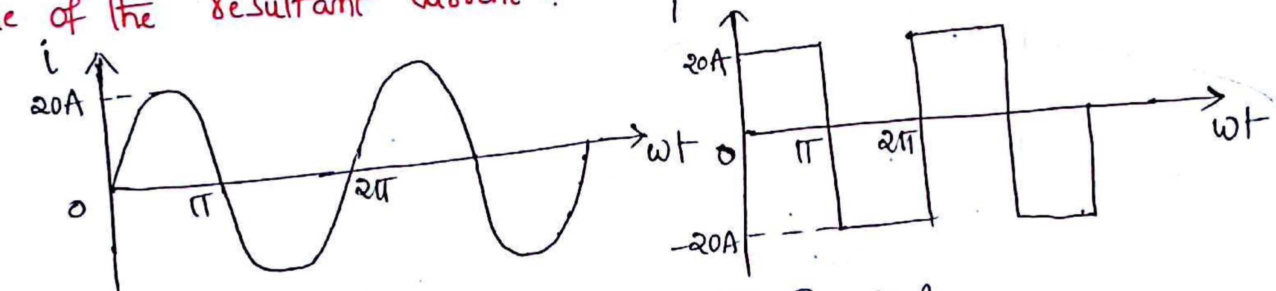
$$= \frac{50}{\sqrt{2\pi}}\sqrt{\int_0^{\pi}(1-\cos 2\omega t)\,d(\omega t)} = \frac{50}{\sqrt{2\pi}}\sqrt{\left(\omega t - \frac{\sin 2\omega t}{2}\right)_0^{\pi}}$$

$$I_{rms} = \frac{50}{\sqrt{2\pi}}\sqrt{(\pi-0)-(0)} = \frac{50}{\sqrt{2}\cancel{\sqrt{\pi}}} \times \cancel{\sqrt{\pi}} = \frac{50}{\sqrt{2}} = 35.3553\,A$$

Power Consumed by resistor $= I_{rms}^2 \times R = (35.3553)^2 \times 5$

$= 6250\,W.$

7) A resistor carries two alternative currents having same frequency and phase and having the same peak value of 20A. One is sinusoidal and the other is rectangular in waveform. Find the R.m.s value of the resultant current?



Consider sinusoidal current with $I_m = 20\,A$.

For sinusoidal current

$$I_{rms-1} = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.1421\,A$$

Consider rectangular current with $I_m = 20\,A$

$$I_{rms-2} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d(\omega t)}$$

$i = 20$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi}(20)^2\, d(\omega t)}$$

$$= \sqrt{\frac{400}{\pi}\left(\omega t\right)_0^{\pi}} = \frac{20}{\sqrt{\pi}}\sqrt{(\pi-0)} = \frac{20}{\cancel{\sqrt{\pi}}} \times \cancel{\sqrt{\pi}}$$

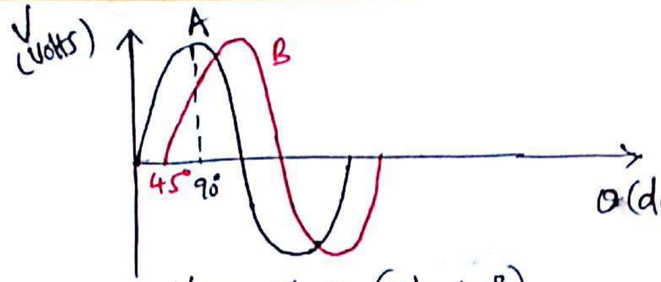
$= 20$

∴ Total R.m.s current

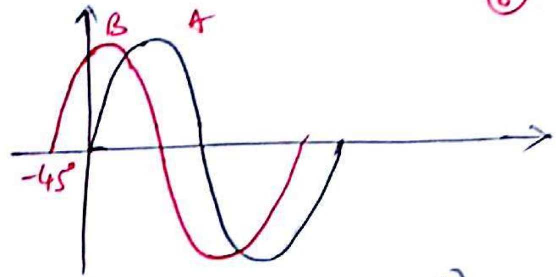
$$I_{rms} = \sqrt{I_{rms-1}^2 + I_{rms-2}^2}$$

$$= \sqrt{(14.1421)^2 + (20)^2}$$

$= 24.4948\,A.$



$V_B(t) = V_m \sin(\omega t - 45^\circ)$



$V_{(t)B} = V_m \sin(\omega t + 45^\circ)$

Phasor representation of an alternating quantity

The Sinusoidally Varying alternating quantity can be represented graphically by a straight line with an arrow in the phasor representation method. This is Similar to vector such a line is called a phasor.

The phasors are assumed to be rotate in anticlock wise direction with a constant speed ω rd/sec.

→ In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant.

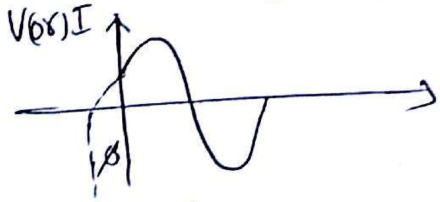
→ In general phase ϕ of an alternating quantity Varies from $\phi = 0$ to 2π (rd) (or) $\phi = 0^\circ$ to 360°

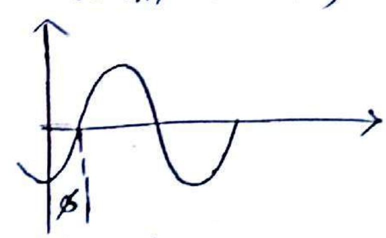
let $v = V_m \sin(\omega t \pm \phi)$ equation in terms of phasor

$\phi = 0$, $V = V_m \sin\omega t$

$\phi = +Ve$
$V = V_m \sin(\omega t + \phi)$

$\phi = -Ve$
$V = V_m \sin(\omega t - \phi)$



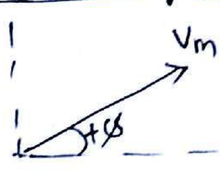


Phasor diagram

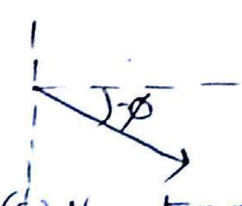
phasor diagram

phasor diagram

(a) Zero phase



(b) Positive phase



(c) Negative phase

Note :-

1. The phase is measure with respect to reference direction i.e Positive x-axis direction.
2. The phase measured in anticlockwise direction is positive while the phase measured in clock wise direction is negative.

→ The difference between the phases of the two alternating quantities is called the phase difference. which is nothing but the angle difference b/w the two phasors representing the two alternating quantities.

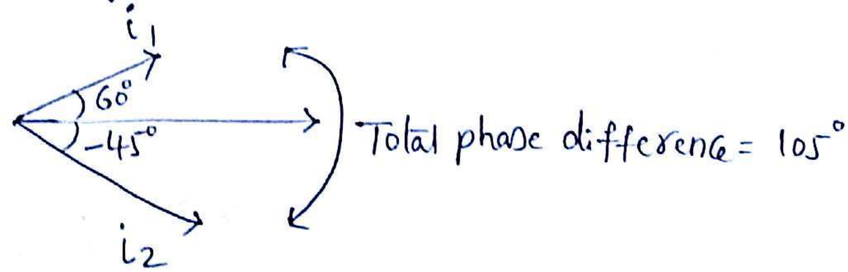
Phasor diagram :- The diagram in which different alternating quantities of the same frequency, Sinusoidal in nature are represented by individual phasors indicating exact phase inter relationship is known as phasor diagram.

8) Two sinusoidal Currents are given by $i_1 = 10 \sin(\omega t + \pi/3)$ and $i_2 = 15 \sin(\omega t - \pi/4)$. Calculate the phase difference between them in degrees and draw the phasor diagram ?

Given $i_1 = 10 \sin(\omega t + \pi/3)$

$i_2 = 15 \sin(\omega t - \pi/4)$

phasor diagram



1) Zero phase difference :-

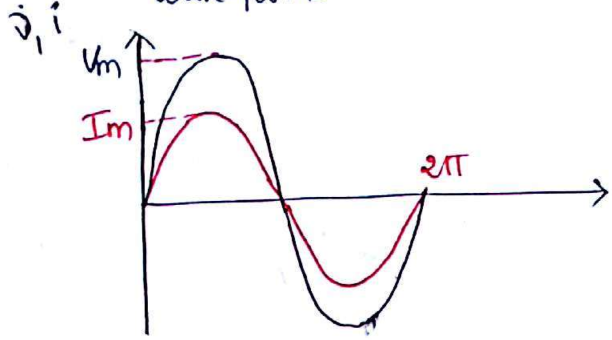
Consider two alternating quantities having same frequency f Hz and different maximum values

$v = V_m \sin\omega t$

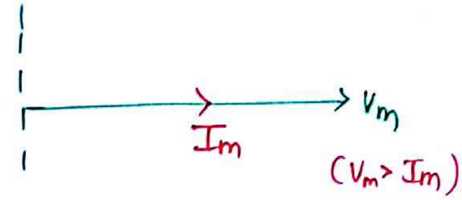
$i = I_m \sin\omega t$

let $V_m > I_m$

Wave-forms



phasor representation



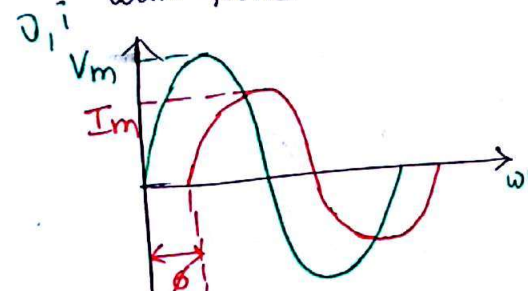
Note: The length of phasor equals to maximum value.

When such phase difference between two alternating quantities is zero, the quantities are said to be in phase.

2) Lagging phase difference :-

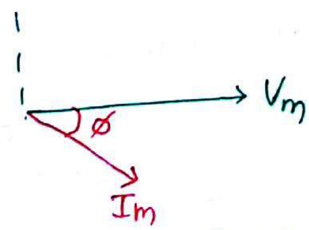
let $v = V_m \sin\omega t$ $\quad i = I_m \sin(\omega t - \phi)$ $\quad (V_m > I_m)$

wave forms



There exist a phase difference of ϕ between the two phasors

phasor diagram

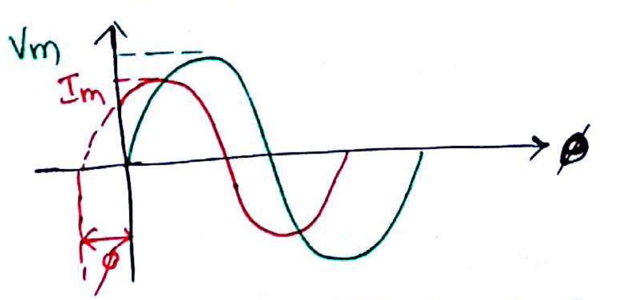


The current i is said to be lagging with the voltage v by an angle ϕ.

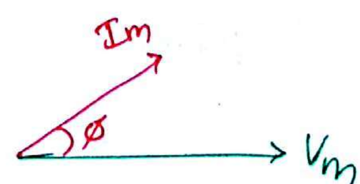
3) leading phase difference :-

let $v = V_m \sin\omega t$ $\quad i = I_m \sin(\omega t + \phi)$ $\quad (V_m > I_m)$

wave forms



phasor diagram



The current i is said to be leading with the voltage v by an angle ϕ.

Mathematical Representation of phasor

The algebraic operations such as addition, subtraction etc with waveforms are more complicated and time consuming. Hence it is necessary to represent the phasors mathematically.

Any phasor can be represented mathematically in two ways:

1) polar co-ordinate system
2) Rectangular co-ordinate system.

Polar co-ordinate system

Consider an alternating current $i = I_m \sin(\omega t + \phi)$

Thus its maximum value is I_m and phase is ϕ. The phase ϕ is always measured with respect to X-axis direction.

→ while representing this phasor by polar system, it is represented as $r \angle \phi$

where $r = \frac{I_m}{\sqrt{2}}$ (Rms value)

ϕ = phase angle with respect to +ve X-axis

Polar form

→ So draw a line at an angle ϕ measured w.r.t +ve x-axis from the origin and measure a distance equal to $r = \frac{I_m}{\sqrt{2}}$

Polar representation of phasor mathematically

$= r \angle \pm \phi$

Q) Represent $V = 40 \sin(\omega t + \pi/3)$ in polar coordinate system?

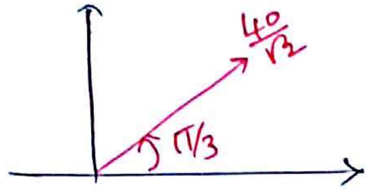
Given $V = 40 \sin(\omega t + \pi/3)$

Polar form $r \angle \pm \phi$

$r = \frac{40}{\sqrt{2}}$ $\phi = +\pi/3$ $\therefore$ Polar form $= \frac{40}{\sqrt{2}} \angle +\pi/3$

Scanned by CamScanner

i.e



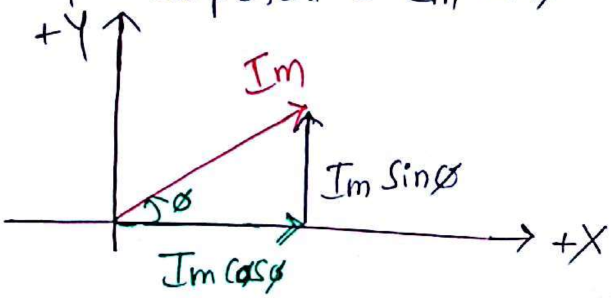
2). Rectangular Co-ordinate System:-

Mathematically an alternating quantity can be divided in to two components, X-component and Y-component.

If $i = I_m \sin(\omega t + \phi)$

$X\text{-component} = I_m \cos\phi$

$Y\text{-component} = I_m \sin\phi$



→ X and Y-components can be +ve (or) −ve

→ To indicate that X and Y components are perpendicular to each other, the operator 'j' is used in mathematical representation of phasor in rectangular co-ordinate system.

Mathematical representation of rectangular co-ordinate system

$$= \pm X \pm jY$$

Mathematically value of $j = \sqrt{-1}$

but in the phasor representation $j = 1\angle 90^\circ$

i.e rotation through 90°

10) Represent $i = 20 \sin(\omega t + \pi/6)$ in rectangular co-ordinate system?

Given $i = 20 \sin(\omega t + \pi/6)$

$I_m = 20$ and $\phi = \pi/6$

$X\text{-component} = I_m \cos\phi = 20 \times \cos \pi/6 = 10\sqrt{3}$

$Y\text{-component} = I_m \sin\phi = 20 \times \sin(\pi/6) = 20 \times 1/2 = 10$

Then $i = 20\sin(\omega t + \pi/6)$ in rectangular co-ordinate system

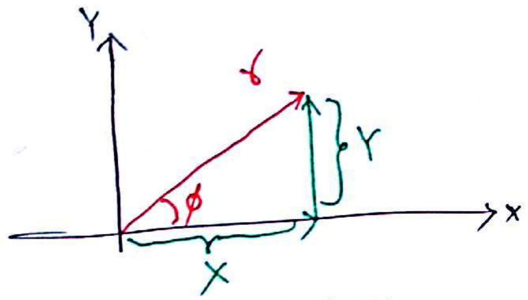
is $X + jY$

i.e $10\sqrt{3} + j\,10$

Polar to Rectangular Conversion

Polar form for phasor is $r\angle\phi$

Polar to Rectangular means find X and Y interms of r and ϕ



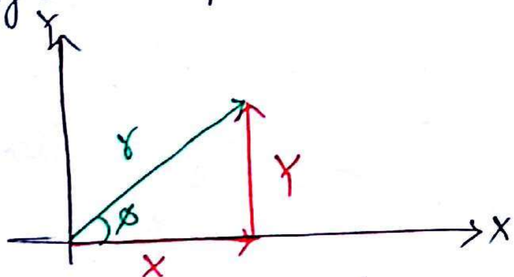
$X = r\cos\phi$

$Y = r\sin\phi$

Rectangular to polar Conversion :-

Rectangular form of phasor is $X + jY$

Rectangular to polar means find r and ϕ interms of X and Y



Find r and ϕ interms of X and Y

From diagram $\quad r = \sqrt{X^2 + Y^2} \qquad \phi = \tan^{-1}\left(\frac{Y}{X}\right)$

$\therefore$ Polar representation $= r\angle\phi = \sqrt{X^2+Y^2}\ \angle \tan^{-1}\left(\frac{Y}{X}\right)$

Note: polar form always gives r.m.s Value of an alternating quantity.

11) Write polar form of the Voltage given by $v = 100\sin(100\pi t + \pi/6)$ and also obtain its rectangular form?

$V_m = 100$ and $\phi = \pi/6 = 30^\circ$

$\therefore \quad V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7106\ V.$

$\therefore$ In polar form $= 70.7106 \angle 30$

Rectangular form $= X + jY$

$= r\cos\phi + j\, r\sin\phi$

$= 61.2371 + j\, 35.3553 \ V$

12) Find rms value and phase of the current $I = 25 + j40\ A$?

Sol Converts Rectangular to polar form

$= r\angle\phi = I_{rms}\angle\phi$

$r = I_{rms} = \sqrt{25^2 + 40^2} = 47.699$

$\phi = \tan^{-1}\left(\frac{Y}{X}\right) = \tan^{-1}\left(\frac{40}{25}\right) = 57.99$

$I = 47.1699 \angle 57.99^\circ$

13) A Voltage is defined as $-V_m \cos\omega t$. Express it in polar-form?

$v = -V_m \cos\omega t$

$= -V_m \sin(\omega t + \pi/2)$

$= V_m \sin(\omega t + \pi + \pi/2)$

$= V_m \sin(\omega t + 3\pi/2)$

$\left(\begin{array}{l} \sin(90+\theta) = \cos\theta \\ \sin(180+\theta) = -\sin\theta \end{array} \right)$

$\therefore$ In polar form $v = \frac{V_m}{\sqrt{2}} \angle +3\pi/2$

(or)

$v = \frac{V_m}{\sqrt{2}} \angle -90^\circ$

Note :- 1. For addition and subtraction of alternating quantities use rectangular form of representation of phasors

2. For multiplication and division use polar form of representation of phasors.

14) Three Voltages represented by $v_1 = 20 \sin\omega t$, $v_2 = 30 \sin(\omega t - \pi/4)$ and $v_3 = 40 \cos(\omega t + \pi/6)$ act together in a circuit. Find expression for the resultant voltage?

In polar form

$V_1 = \frac{20}{\sqrt{2}} \angle 0^\circ = 14.1421 \angle 0^\circ$

$V_2 = \frac{30}{\sqrt{2}} \angle -45^\circ = 21.2132 \angle -45^\circ$

$V_3 = 40 \cos(\omega t + \pi/6) = 40 \sin(90 + \omega t + 30^\circ)$

$= 40 \sin(\omega t + 120^\circ)$

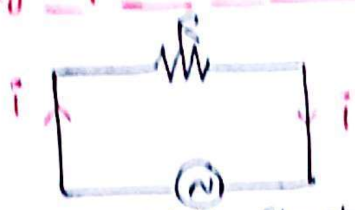
$$V_3 = \frac{40}{\sqrt{2}} \angle 120^\circ = 28.2848 \angle 120^\circ$$

$$\therefore V_R = V_1 + V_2 + V_3 = 15 + j9.4948$$

$$= 17.7524 \angle 32.33^\circ \; V$$

A.C through pure Resistance, Inductance and Capacitance

1) A.C through pure Resistance :-



$V = V_m \sin\omega t$

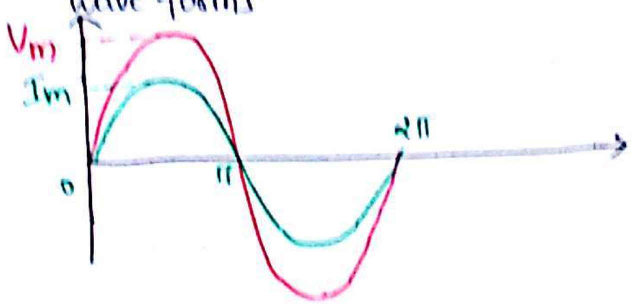
According to ohm's law

$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = \left(\frac{V_m}{R}\right)\sin\omega t$$

$$\therefore I_m = \frac{V_m}{R}$$

$$\phi = 0$$

Wave forms

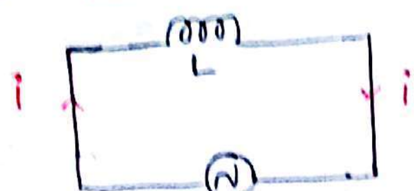


phasor diagram

$\boxed{\text{Inphase}}$

In purely resistive circuit, the current and voltage applied are inphase with each other.

2) AC through pure Inductance :-



$V = V_m \sin\omega t$

The current flowing through the Inductor

$$i = \frac{1}{L}\int v\,dt = \frac{1}{L}\int V_m \sin\omega t \; d(t)$$

$$= \frac{V_m}{L}\left(-\frac{\cos\omega t}{\omega}\right) = \frac{V_m}{L\omega} - \sin(\pi/2 - \omega t)$$

$i = \frac{-V_m}{L\omega} \cos\omega t = \frac{-V_m}{L\omega} \sin(\pi/2 - \omega t)$ $\therefore \sin(90-\theta) = \sin\theta$

$i = \frac{V_m}{L\omega} \sin(\omega t - \pi/2)$

$i = \frac{V_m}{X_L} \sin(\omega t - \pi/2)$

$X_L = L\omega$ = Inductive Reactance.

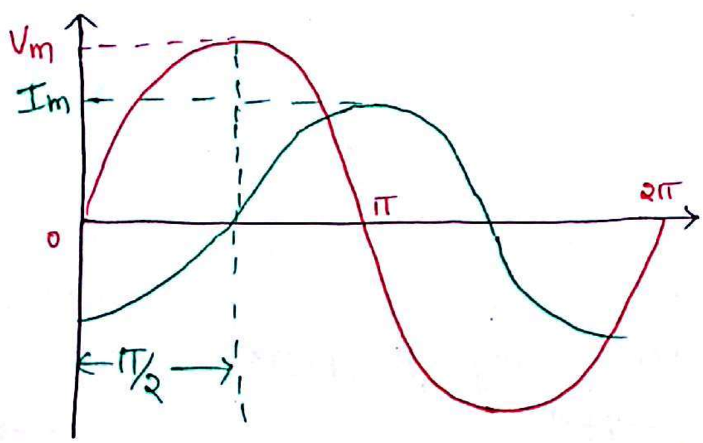
$$\boxed{i = I_m \sin(\omega t - \pi/2)}$$

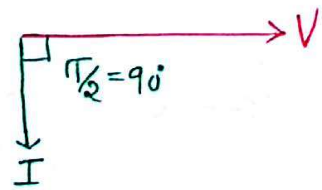
The above equation purely shows that the current is sinusoidal and having phase angle of $-\pi/2$ radians. i.e 90°. This means that the current lags voltage applied by 90°.

→ The negative sign indicates lagging nature of the current.

waveforms

Phasor diagram.





In purely inductive circuit, current lags voltage by 90°.

Inductive Reactance :- ($X_L = L\omega$)

The inductive reactance is defined as the opposition offered by the inductance of a circuit to the flow of an alternating current.

$$X_L = L\omega = L 2\pi f \ \Omega.$$

It is measured in ohms.

Power :-

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$P = V \times i = V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$$

$$= V_m \sin\omega t \times I_m - \sin(\pi/2 - \omega t)$$

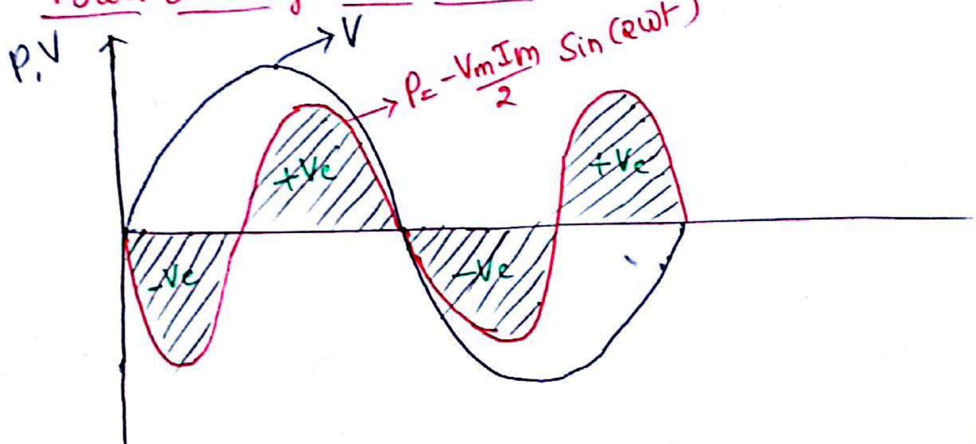
$$= -V_m I_m \sin\omega t \cos(\omega t)$$

$$= -\frac{V_m I_m}{2} \sin(2\omega t)$$ (Double the frequency of Voltage)

avg power taken by Inductor

$$P_{avg} = 0$$

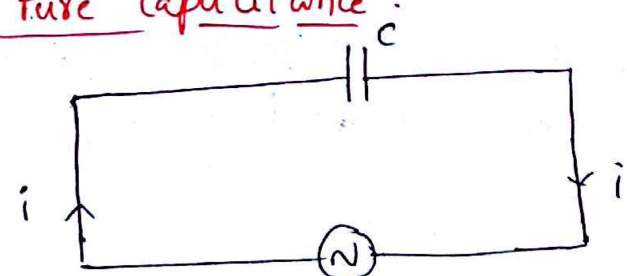
Power & Voltage wave form



It can be observed from the wave-form that when * power curve is positive, energy gets stored in the magnetic fied established due to the increasing current while during negative power curve, this power is returned back to the supply.

* Pure inductance never consumes power.

A.C through Pure Capacitance :



The current flowing through the capacitance is

$$i = C\frac{dv}{dt} = C\frac{d}{dt}[V_m \sin\omega t] = CV_m \cos\omega t \cdot \omega$$

$$i = V_m C\omega \cos\omega t = \frac{V_m}{(1/C\omega)} \cos\omega t.$$

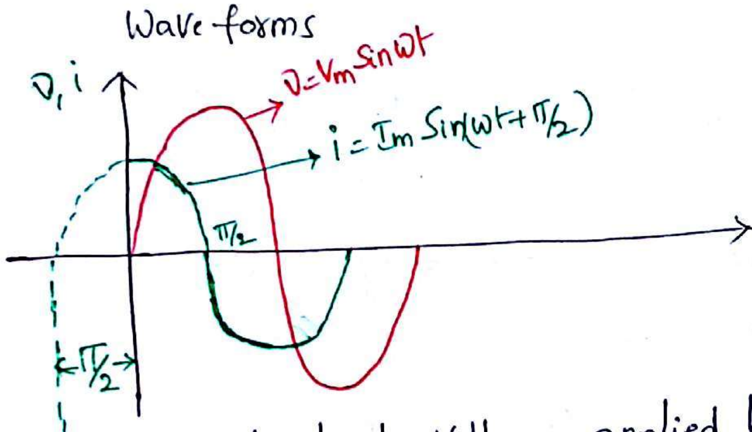
$$i = \frac{V_m}{X_c} \cos\omega t = \frac{V_m}{X_c} \sin(\pi/2 + \omega t)$$ 13 $\sin(90+\theta) = \cos\theta$

$$X_c = \text{Capacitive reactance} = 1/C\omega = \frac{1}{2\pi f C}$$

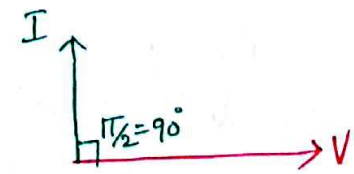
$$\therefore \quad i = \frac{V_m}{X_c} \sin(\omega t + \pi/2)$$

$$\boxed{i = I_m \sin(\omega t + \pi/2)}$$

Wave forms



phasor diagram



Current leads voltage applied by 90°. The positive sign indicates leading nature of the current.

Power :-

The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$P = V \times i = V_m \sin\omega t \times I_m \sin(\omega t + \pi/2)$$

$$= \frac{V_m I_m}{2} \times 2 \sin\omega t \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

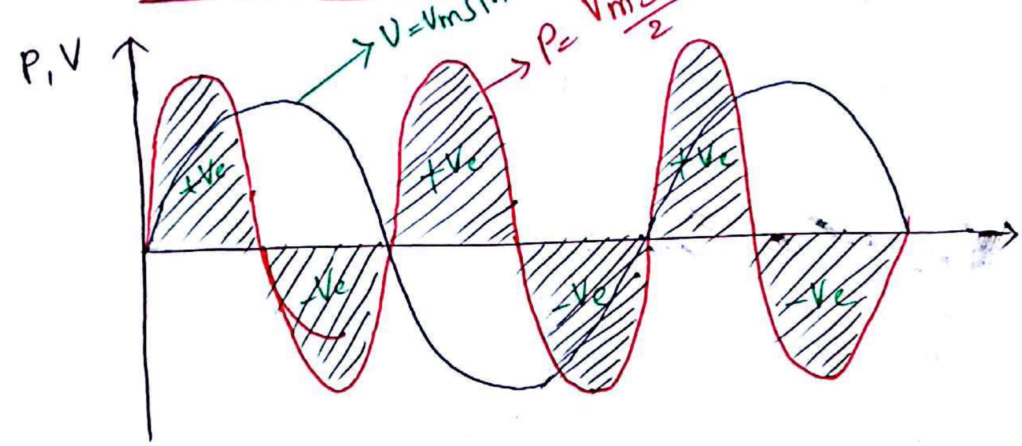
Power curve is a sine wave of frequency double that of applied voltage.

→ The average value of sine curve over a complete cycle is always zero.

$$P_{av} = 0$$

Hence average power consumption is zero.

waveforms (P & V) :-



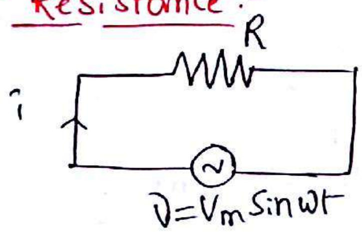
Impedance :-

The opposition offered by an electric circuit to the flow of an alternating current is called an impedance. It is denoted by Z.

It is defined as the ratio of an alternating voltage to an alternating current through the circuit.

Impedance is complex and is expressed in polar (or) rectangular form.

For Resistance :-



$V = V_m \sin \omega t$

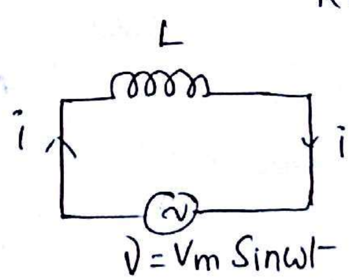
$i = I_m \sin \omega t = \frac{V_m}{R} \sin \omega t$

$$\therefore \quad Z = \frac{v}{i} = \frac{V_m \sin \omega t}{\frac{V_m}{R} \sin \omega t} = R$$

in Rectangular form $\quad Z = R + j0$

polar form $\quad Z = R \angle 0$

For pure Inductance :-



$v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \pi/2)$

$= \frac{V_m}{L\omega} \sin(\omega t - \pi/2)$

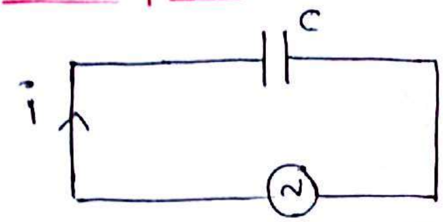
In polar form $\quad v = \frac{V_m}{\sqrt{2}} \angle 0^\circ$

$i = \frac{V_m}{\sqrt{2}\, L\omega} \angle -\pi/2$

$$Z = \frac{v}{i} = \frac{\frac{V_m}{\sqrt{2}} \angle 0^\circ}{\frac{V_m}{\sqrt{2}\, L\omega} \angle -\pi/2} = L\omega \angle \pi/2 = jL\omega \qquad \therefore j = 1\angle 90^\circ$$

$$\therefore Z = jL\omega = jX_L$$

For pure Capacitance :-



$v = V_m \sin\omega t$

$v = V_m \sin\omega t$

$$i = \frac{V_m}{(1/C\omega)} \sin(\omega t + \pi/2)$$

In polar form $\quad v = \frac{V_m}{\sqrt{2}} \angle 0^\circ$

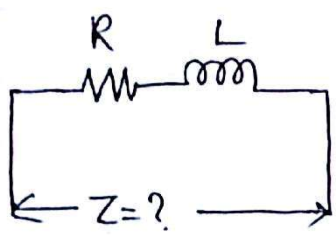
$$i = \frac{V_m}{\sqrt{2}\, X_C} \angle \pi/2$$

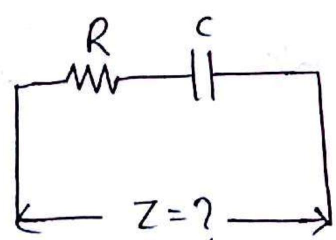
$$\therefore Z = \frac{v}{i} = \frac{\frac{\cancel{V_m}}{\cancel{\sqrt{2}}} \angle 0^\circ}{\frac{\cancel{V_m}}{\cancel{\sqrt{2}}\, X_C} \angle \pi/2} = X_C \angle -\pi/2 = -jX_C$$

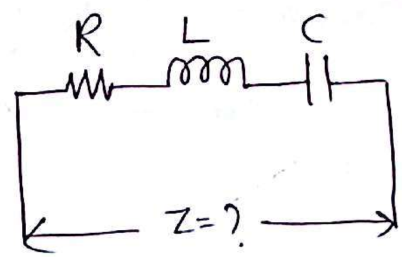
$$-j = 1 \angle -\pi/2$$

$$\therefore Z = -jX_C$$

Series Circuits







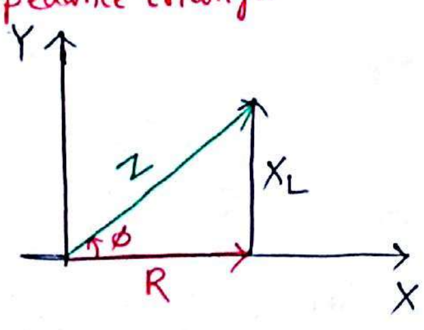
$Z = R + jX_L$

$Z = R - jX_C$

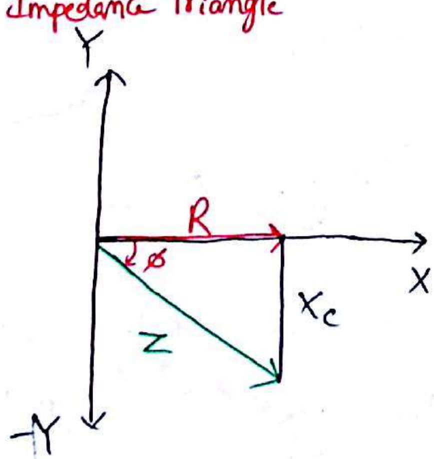
$Z = R + jX_L - jX_C$

$= R + j(X_L - X_C)$

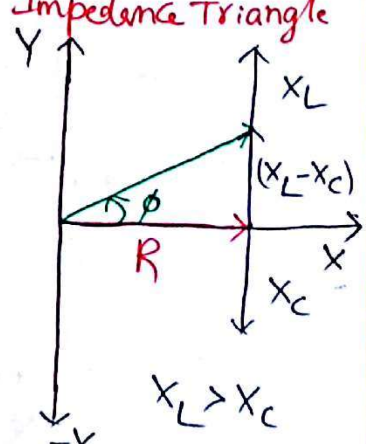
Impedance triangle



Impedance Triangle



Impedance Triangle



polar-form

$Z = R + jX_L$

$Z = \sqrt{R^2 + X_L^2}\,\angle \tan^{-1}\left(\frac{X_L}{R}\right)$

$= Z \angle \phi$

Polar-form

$Z = R - jX_C$

$Z = \sqrt{R^2 + X_C^2}\,\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$

$= Z \angle -\phi$

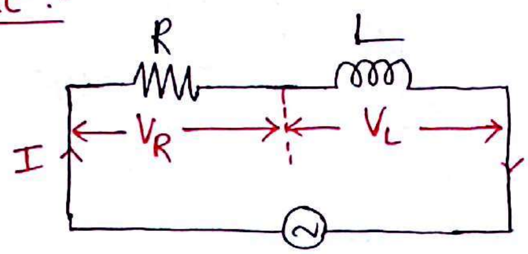
polar-form

$Z = R + j(X_L - X_C)$

$Z = \sqrt{R^2 + (X_L - X_C)^2}\,\angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$= Z \angle \phi$

1. Series RL Circuit :-



$V = V_m \sin \omega t$

where

V_R = Drop across pure resistance = $I \times R$

V_L = Drop across pure Inductance = $I \times X_L$

Impedance $Z = R + jX_L$

$$Z = \frac{V}{I}$$

$$V = IZ = I(R + jX_L)$$

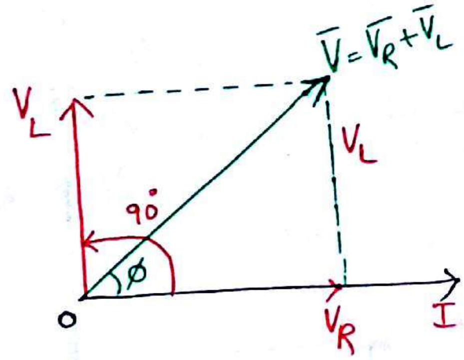
$$V = IR + jIX_L$$

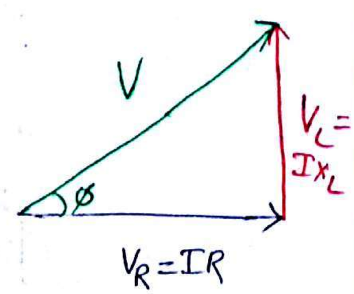
$$\boxed{\bar{V} = \bar{V}_R + \bar{V}_L}$$ (From K.V.L)

Note :- In Series circuit take current as reference in phasor diagram.

phasor diagram :-

Voltage Triangle





It canbe observed that current lags voltage by angle ϕ

$v(t) = V_m \sin \omega t \qquad i(t) = I_m \sin(\omega t - \phi)$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{I X_L}{I R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Power and Power triangle:-

The expression for the current in the series R-L circuit is,

$i = I_m \sin(\omega t - \phi)$ as current las voltage.

the instantaneous power is given by

$$P = V \times i = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m \left[\sin(\omega t) \cdot \sin(\omega t - \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right] = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

$$\left(\because \; 2\sin A \sin B = \cos(A-B) - \cos(A+B)\right)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence average power consumed is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P_{av} = V I \cos\phi}$$ where V and I are r.m.s values

In series RL circuit

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

Multiply voltage equation by current I, Then we get The expression for total power input i.e

$$\overline{VI} = \overline{V_R I} + \overline{V_L I}$$

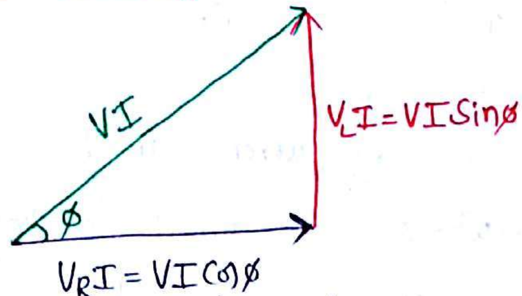
From voltage triangle

$$V_R = V\cos\phi$$

$$V_L = V\sin\phi$$

$$\overline{VI} = \overline{V\cos\phi I} + \overline{V\sin\phi I}$$

Power triangle



So three sides of this triangle are

1) VI 2) $VI\cos\phi$ 3) $VI\sin\phi$

1. Apparent power (S) :-

It is defined as the product of r.m.s value of Voltage (V) and current (I). It is denoted by S.

$$S = VI \quad VA$$

It is measured in unit Volt-amperes (VA) (or) KVA

2. Real (or) True power (P) :-

It means the useful power from the source to the load, which is also called true power.

It is defined as the product of the effective values of both voltage and current multiplied by cosine of the phase angle between voltage and the current.

It is measured in watts (w), (or) KW

$$P = VI\cos\phi \quad \text{watts.}$$

3. Reactive power (Q) :-

It is defined as the product of the r.m.s values of both voltage and current multiplied by sine of the phase angle between voltage and the current

→ It is also defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in Volt-amp reactive (VAR) (or) KVAR

$$Q = VI\sin\phi \quad VAR$$

Power factor ($\cos\phi$):-

It is defined as the ratio of true power to apparent power

(or)

It is defined as the cosine of phase angle between Voltage and Current.

$$\text{Power factor } (\cos\phi) = \frac{\text{True Power}}{\text{Apparent power}} = \frac{VI\cos\phi}{VI} = \cos\phi$$

→ As the phase angle between Voltage and total current increases, the power factor decreases. The smaller the power factor, the smaller the power dissipation. The power factor varies from 0 to 1.

→ It is also defined as the ratio of resistance to the impedance (from impedance triangle)

$$\cos\phi = \frac{R}{Z}$$

→ Note:- The nature of power factor is always determined by position of current with respect to the voltage.

→ If current lags voltage, power factor is said to be lagging power factor

→ If current leads voltage, power factor is said to be leading power factor

→ If current is inphase with voltage, power factor is said to be unity power factor.

For pure Resistive :-

$\phi = 0$, $\cos\phi = \cos(0) = 1$ unity power factor

For pure Inductive :-

$\phi = \pi/2$ lag. $\cos(\pi/2) = 0$ Zero lagging power factor

For pure Capacitive :-

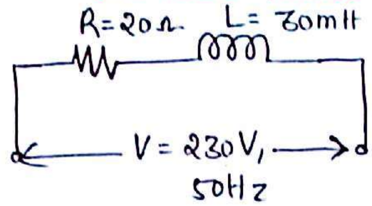
$\phi = \pi/2$ lead $\cos(\pi/2) = 0$ Zero leading power factor.

A 20 Ω resistance and 30 mH inductance are connected in series and the circuit is fed from a 230V, 50Hz AC supply. Find

i) Reactance across the inductance, impedance, admittance, current

ii) Voltage across Resistance (iii) Voltage across the inductance

iv) Real, reactive and active powers (V) Power factor.



(i) $X_L = L\omega = L \times 2\pi f$

$= 30 \times 10^{-3} \times 2\pi \times 50 = 9.4248 \ \Omega$

$Z = R + jX_L = 20 + j\,9.4248 \ \Omega$

$= 22.1094 \angle 25.2317^\circ \ \Omega$

admittance $Y = \frac{1}{Z} = \frac{1}{22.1094 \angle 25.2317} = 0.04522 \angle -25.2317^\circ \ ℧$

$I = \frac{V}{Z} = \frac{230 \angle 0^\circ}{22.1094 \angle 25.2317} = 10.4028 \angle -25.2317^\circ \ A$

(ii) $V_R = IR = (10.4028 \angle -25.2317^\circ) \times 20 \angle 0^\circ = 208.056 \angle -25.2317^\circ \ V.$

(iii) $V_L = I \times jX_L = 10.4028 \angle -25.2317 \times j\,9.4248$

$= 98.0443 \angle 64.7683^\circ \ V$

(iv) $P = VI\cos\phi = 230 \times 10.4028 \times \cos(-25.2317^\circ) = 2164.365 \ W$

$Q = VI\sin\phi = 230 \times 10.4028 \times \sin(-25.2317^\circ) = -1019.9359 \ VAR$

The negative sign indicates lagging nature of reactive Volt-amperes.

$S = VI = 230 \times 10.4028 = 2392.644 \ VA$

(V) Power factor $= \cos\phi = \cos(-25.2317^\circ) = 0.90459$ lagging

A series circuit with a resistance $R = 10\Omega$ and inductance 20 mH has a current of $i = 2\sin 500t$. obtain the total voltage across the series circuit and angle by which the current lags the voltage?

$i(t) = 2\sin 500t$

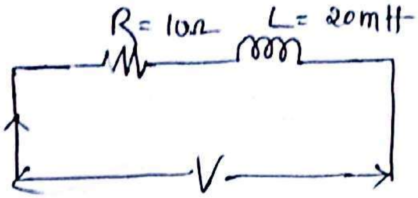
$\omega = 500$ rd/sec

$2\pi f = 500$, $f = \frac{500}{2\pi} =$

$X_L = L\omega = 500 \times 20 \times 10^{-3} = 10\Omega$

$Z = R + jX_L = 10 + j10\ \Omega$

$= 14.1421\angle 45^\circ\ \Omega$

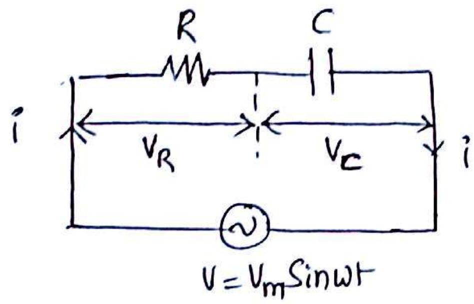


The phase of the current is 0°, i.e $I = 1.4142\angle 0^\circ$ A

$V = I \times Z = 1.4142\angle 0^\circ \times 14.1421\angle 45^\circ = 20\angle 45^\circ$ A

Current lags Voltage by 45°

Series R-C Circuit :-



$Z = R - jX_C$

$V_R = I \times R$

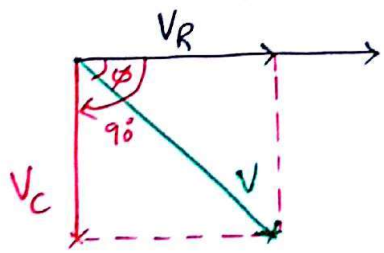
$V_C = I \times (-jX_C)$

From K.V.L

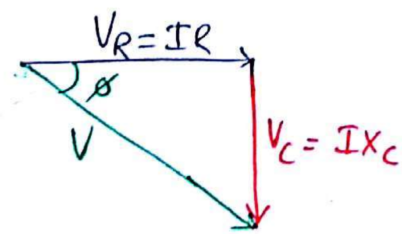
$\bar{V} = \bar{V}_R + \bar{V}_C = I \times R + I(-jX_C)$

$V = IZ = I(R - jX_C)$

Phasor diagram :-



Voltage Triangle



ϕ is the phase angle between Voltage (V) and Current (I)

It can be seen that Current leads Voltage by angle ϕ hence

$v(t) = V_m \sin\omega t \qquad i(t) = I_m \sin(\omega t + \phi)$.

Power and Power angle

The Current leads Voltage by angle ϕ, hence its expression is,

$$i = I_m \sin(\omega t + \phi)$$

$$P = V \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m \left[\sin\omega t \cdot \sin(\omega t + \phi)\right] = V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi).$$

Now, second term is cosine term whose average value over a cycle is zero. Hence average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

$$\boxed{P = VI\cos\phi} \text{ Watts.}$$

In Series RC circuit

$$\bar{V} = \bar{V_R} + \bar{V_c}$$

Multiply Voltage equation by Current I

$$\overline{VI} = \overline{V_R I} + \overline{V_c I}$$

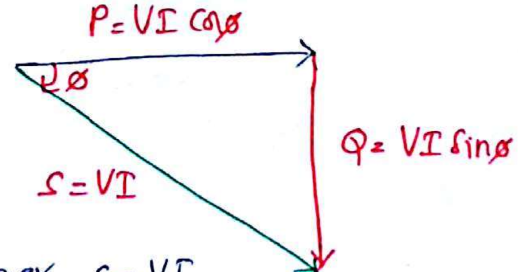
From Voltage triangle

$$V_R = V\cos\phi$$

$$V_c = V\sin\phi$$

$$VI = \overline{V\cos\phi I} + \overline{V\sin\phi I}$$

Power triangle



Apparent power $S = VI$

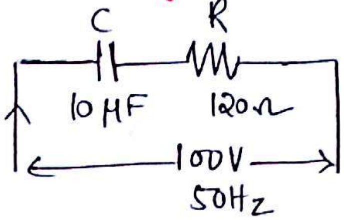
True (or) average power $= P = VI\cos\phi$ W

Reactive power $= Q = VI\sin\phi$ VAR

Power factor $= \cos\phi$ leading

A Capacitor having a capacitance of 10 μF is connected in series with a resistance of 120 Ω across 100 V, 50 Hz. Calculate the power, current and the phase difference between current and voltage? 15

Sol



$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318.3098\ \Omega$$

$$Z = R - jX_C = (120 - j\,318.3098)\,\Omega = 340.178\angle{-69.344^\circ}\ \Omega$$

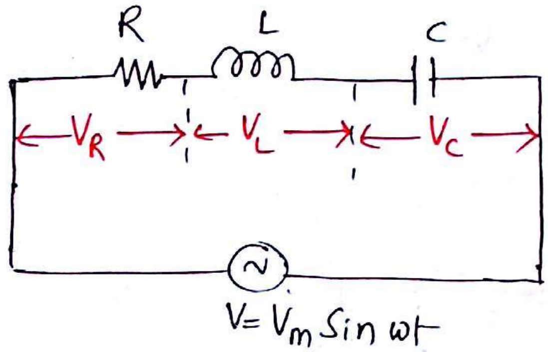
$$I = \frac{V}{Z} = \frac{100\angle 0^\circ}{340.178\angle{-69.344^\circ}} = 0.2939\angle 69.344^\circ\ A$$

phase difference between Voltage and Current $= \phi = 69.344^\circ$ leading

$$P = VI\cos\phi = 100 \times 0.2939 \times \cos(69.344^\circ) = 10.3697\ W.$$

(or) $P = I^2R = (0.2939)^2 \times 120 = 10.369\ W.$

Series R-L-C Circuit



$$Z = R + j(X_L - X_C)$$

$$V_R = I \times R$$

$$V_L = I \times (jX_L)$$

$$V_C = I \times (-jX_L)$$

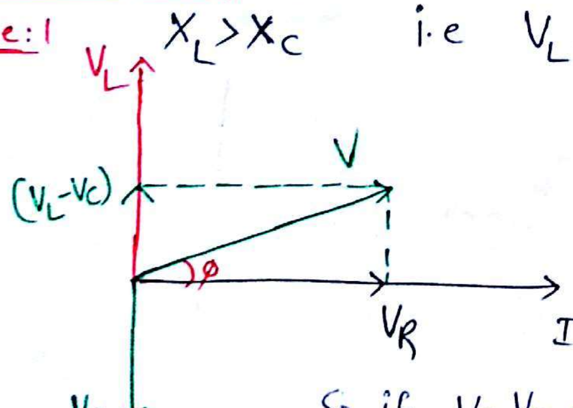
From K.V.L

$$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_C}$$

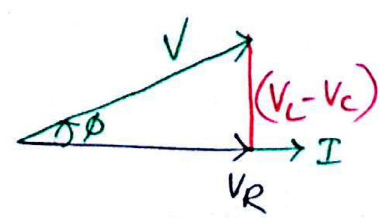
$$\bar{V} = I(R + j(X_L - X_C))$$

Phasor diagram

Case: 1 $X_L > X_C$ i.e $V_L > V_C$



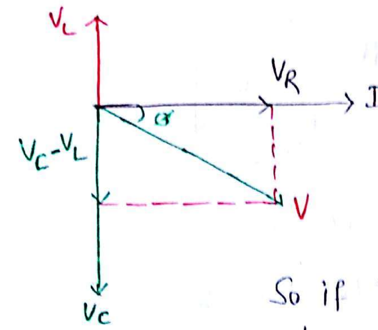
Voltage triangle

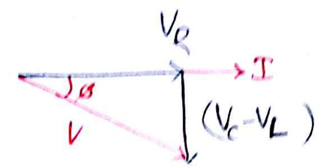


So if $V = V_m \sin\omega t$ then $i = I_m \sin(\omega t - \phi)$

as Current lags Voltage by angle ϕ for $X_L > X_C$.

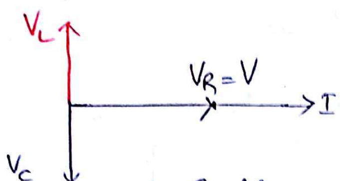
Case 2: $X_C > X_L$ $\quad V_C > V_L$





So if $V = V_m \sin \omega t$, then $i = I_m \sin(\omega t + \phi)$ as current leads voltage by angle ϕ for $X_C > X_L$.

Case 3: $X_L = X_C$, $V_L = -V_C$



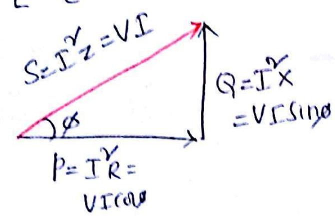
So if $V = V_m \sin \omega t$, then $i = I_m \sin(\omega t)$

Power and Power triangle :-

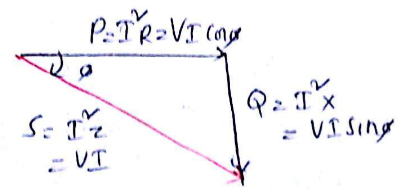
$P = VI \cos\phi \ W$ (for any case.

Power triangles

$X_L > X_C$



$X_L < X_C$

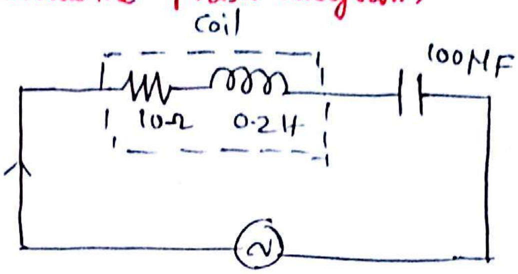


A coil having a resistance of 10 ohms and an inductance of 0.2H is connected in series with a 100×10^{-6} F Capacitor across a 230V, 50Hz Supply Calculate.

(19)

(i) The active and reactive Components of the Current

(ii) The voltage across the coil, draw the phasor diagram



$X_L = 2\pi f L$

$= 2\pi \times 50 \times 0.2$

$= 62.8318\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.8309\,\Omega$

$Z_{coil} = R + jX_L = 10 + j62.8318\,\Omega$

$= 63.6226 \angle 80.957^\circ\,\Omega$

$Z_T = Z_{coil} - jX_C = 10 + j62.8316 - j31.8309$

$= 10 + j31\,\Omega = 32.573 \angle 72.121^\circ\,\Omega$

$I = \frac{V}{Z_T} = \frac{230 \angle 0^\circ}{32.573 \angle 72.121^\circ} = 7.061 \angle -72.121^\circ$ A

$\phi_T = 72.121^\circ$ lagging

Active Component of Current $= I\cos\phi_T = 7.061 \times 0.307$

$= 2.1677$A

Reactive Component of current $= I\sin\phi_T = 7.061 \times 0.9517$

$= 6.72$A

$V_{coil} = I \times Z_{coil}$

$= 7.061 \angle -72.121^\circ \times 63.6226 \angle 80.957^\circ$.

$= 449.2391 \angle 8.836^\circ$ V

Phasor diagram