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A1-A

(1)

MATH ASSIGNMENT

1) i) $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$ —— (1)

Sol: Compare eq(1) with

$$M\,dx + N\,dy = 0$$

$M = (e^y+1)\cos x$. $\qquad N = e^y \sin x$

$M = e^y \cos x + \cos x$ $\qquad \frac{\partial N}{\partial x} = e^y \cos x$

$$\frac{\partial M}{\partial y} = e^y \cos x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The eq(1) is Exact D.E.

$$G.S \Rightarrow \int M\,dx + \int N\,dy = c$$

$$\int (e^y \cos x + \cos x)\,dx + \int 0\,dy = c$$

$$e^y \sin x + \sin x = c$$

$$\sin x(e^y+1) = c$$

ii) $2xy\,dy - (x^3 - y^2 + 1)\,dx = 0$

Sol: $(x^3 - y^2 + 1)\,dx - 2xy\,dy = 0$ —— (1)

~~$-y^2+1)\,dx + 2xy\,dy = 0$ —— (1)~~

Compare with $M\,dx + N\,dy = 0$

$M = x^3 - y^2 + 1$ $\qquad N = -2xy$.

$\frac{\partial M}{\partial y} = -2y$ $\qquad \frac{\partial N}{\partial x} = -2y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eq (1) is exact.

G.S $\Rightarrow \int M\,dx + \int N\,dy = 0$

$$\int (x^3 - y^2 + 1)\,dx + \int (0)\,dy = 0$$

$$\cancel{-y^2 \int x^3}$$

$$-y^2 \int 1\,dx + \int x^3\,dx + \int 1\,dx = 0$$

$$\therefore \frac{x^4}{4} - xy^2 + x = c.$$

iii) $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$

Sol: $\frac{dy}{dx} = -\frac{(y\cos x + \sin y + y)}{\sin x + x\cos y + x}$

$$(\sin x + x\cos y + x)\,dy = -(y\cos x + \sin y + y)\,dx$$

$(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0$ —— (1)

Compare (1) with $Mdx + Ndy = 0$

$M = y\cos x + \sin y + y$ $\qquad$ $N = \sin x + x\cos y + x$

$\frac{\partial M}{\partial y} = \cos x + \cos y + 1$ $\qquad$ $\frac{\partial N}{\partial x} = \cos x + \cos y + 1$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$\therefore$ Eq (1) is exact D.E.

G.S $\Rightarrow$ $\int M dx + \int N dy = c$

$\int (y\cos x + \sin y + y)\,dx + \int (0)\,dy = c$

$y\int \cos x\,dx + \sin y \int 1\,dx + y\int 1\,dx = c$

$y\sin x + \sin y(x) + y(x) = c$

$\boxed{y\sin x + x\sin y + xy = c}$

2) i) $(x^4 + y^4)dx - xy^3 dy = 0$ —— (1)

sol: compare with $Mdx + Ndy = 0$

$M = x^4 + y^4$ $\qquad$ $N = -xy^3$.

$$\frac{\partial M}{\partial y} = 4y^3. \qquad \frac{\partial N}{\partial x} = -y^3.$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\Rightarrow$ Eq (1) is not exact

M & N are homogeneous polynomials of x & y with degree 4.

$$Mx + Ny = (x^4+y^4)x + (-xy^3)y$$

$$= x^5 + \cancel{xy^4} - \cancel{xy^4} = x^5 \neq 0$$

$$I.F \Rightarrow \frac{1}{Mx+Ny} = \frac{1}{x^5}$$

Multiply eq (1) with I.F

$$\Rightarrow \frac{1}{x^5}\left[x^4+y^4\right]dx - \frac{1}{x^5}\left[xy^3\right]dy = 0$$

$$\left[\frac{1}{x} + \frac{y^4}{x^5}\right]dx - \left[\frac{y^3}{x^4}\right]dy = 0 \longrightarrow (2)$$

Compare eq (2) with $M_1 dx + N_1 dy = 0$

$$M_1 = \frac{1}{x} + \frac{y^4}{x^5} \qquad N_1 = \frac{-y^3}{x^4}$$

$$\frac{\partial M_1}{\partial y} = \frac{4y^3}{x^5} \qquad \frac{\partial N_1}{\partial x} = \frac{4y^3}{x^5}$$

$$\frac{\partial M_1}{\partial y} \quad \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ Eq (2) is exact

G.S $\Rightarrow$ $\int M_1\,dx + \int N_1\,dy = c$

$$\int \left[\frac{1}{x} + \frac{y^4}{x^5}\right] dx + \int 0\,dy = c$$

$$\log x + y^4 \frac{x^{-4}}{-4} = c$$

$$\log x - \frac{y^4}{4x^4} = c$$

ii) $y(2xy+1)\,dx + x(1+2xy-x^3y^3)\,dy = 0$ — (1)

Sol: Compare with $M dx + N dy = 0$

$M = y(2xy+1)$ $\qquad$ $N = x(1+2xy-x^3y^3)$

$= 2xy^2 + y$ $\qquad$ $= x + 2x^2y - x^4y^3$

$$\frac{\partial M}{\partial y} = 4xy+1 \qquad \frac{\partial N}{\partial x} = 1+4xy-4x^3y^3$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\Rightarrow$ Eq (1) is not exact

Consider $Mx - Ny = [2xy^2 + y]x - [x + 2x^2y - x^4y^3]y$

$= 2x^2y^2 + xy - xy - 2x^2y^2 + x^4y^4$

$= x^4y^4 \neq 0$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{x^4y^4}.$$

Multiply eq (1) with I.F

$$\frac{1}{x^4y^4}\left[y(2xy+1)\,dx + x(1+2xy-x^3y^3)\,dy\right] = 0$$

$$\frac{1}{x^4y^4}\left[2xy^2 + y\right]dx + \frac{1}{x^4y^4}\left[x + 2x^2y - x^4y^3\right]dy = 0$$

$$\left[\frac{2}{x^3y^2} + \frac{1}{x^4y^3}\right]dx + \left[\frac{1}{x^3y^4} + \frac{2}{x^2y^3} - \frac{1}{y}\right]dy = 0 \quad \text{--- (2)}$$

Compare (2) with $M_1\,dx + N_1\,dy = 0$

$$M_1 = \frac{2}{x^3y^2} + \frac{1}{x^4y^3} \qquad N_1 = \frac{1}{x^3y^4} + \frac{2}{x^2y^3} - \frac{1}{y}$$

$$\frac{\partial M_1}{\partial y} = \frac{-4}{x^3y^3} - \frac{3}{x^4y^4} \qquad \frac{\partial N_1}{\partial x} = \frac{-3}{x^4y^4} - \frac{4}{x^3y^3}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ Eq (2) is exact.

G.S $\Rightarrow \int M_1 dx + \int N_1 dy = c$

$$\int\left[\frac{2}{x^3y^2}+\frac{1}{x^4y^3}\right]dx+\int\left[-\frac{1}{y}\right]dy=c$$

$$\frac{2}{y^2}\int\frac{1}{x^3}dx+\frac{1}{y^3}\int\frac{1}{x^4}dx-\int\frac{1}{y}dy=c$$

$$\frac{\cancel{2}}{y^2}\left[\frac{x^{-2}}{-\cancel{2}}\right]+\frac{1}{y^3}\cdot\frac{x^{-3}}{-3}-\log y=c$$

$$-\frac{x^{-2}}{y^2}-\frac{x^{-3}}{3y^3}-\log y=c$$

$$-\frac{1}{x^2y^2}-\frac{1}{3x^3y^3}-\log y=c$$

iii) $(xy\sin xy+\cos xy)y\,dx+(xy\sin xy-\cos xy)x\,dy=0$ — (1)

Sol: Compare eq (1) with $M dx + N dy = 0$

$M = (xy\sin xy + \cos xy)y$ $\quad$ $N = (xy\sin xy - \cos xy)x$

$= xy^2\sin xy + y\cos xy$ $\quad$ $N = x^2y\sin xy - x\cos xy$

~~$\frac{\partial M}{\partial y} = 2xy\sin xy + xy^2\cos xy \cdot x + \cos xy + y[-\sin xy \cdot x]$~~

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}\left[xy^2\sin xy+y\cos xy\right]$$

$$=\frac{\partial}{\partial y}\left[xy^2\sin xy\right]+\frac{\partial}{\partial y}\left[y\cos xy\right]$$

$$=x\left[y^2\frac{d}{dx}(\sin xy)+\sin xy\frac{d}{dx}(y^2)\right]+y\frac{d}{dy}(\cos xy)+\cos xy\frac{d}{dx}(y)$$

$$= x\left[y^2 \cos xy(x) + \sin xy(2y)\right] + y(-\sin xy)(x) + \cos xy(1)$$

$$= x^2y^2 \cos xy + 2xy \sin xy - xy \sin xy + \cos xy$$

$$= \cos xy(x^2y^2+1) + xy \sin xy.$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[x^2y \sin xy - x\cos xy\right]$$

$$= y\left[x^2\frac{\partial}{\partial x}\sin xy + \sin xy \frac{\partial}{\partial x}x^2\right] - \left[x\frac{\partial}{\partial x}\cos xy + \cos xy \frac{\partial}{\partial x}(x)\right]$$

$$= y\left[x^2 \cos xy(y) + \sin xy(2x)\right] - \left[x(-\sin xy)(y) + \cos xy(1)\right]$$

$$= x^2y^2 \cos xy + 2xy \sin xy + xy \sin xy - \cos xy$$

$$= \cos xy(x^2y^2-1) + 3xy \sin xy.$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

$\Rightarrow$ Eq (1) is not exact.

consider $Mx - Ny = \left[xy^2 \sin xy + y\cos xy\right]x - \left[x^2y \sin xy - x\cos xy\right]y$

$$= x^2y^2 \sin xy + xy\cos xy - x^2y^2 \sin xy + xy \cos xy$$

$$= 2xy \cos xy \neq 0$$

$$I.F \Rightarrow \frac{1}{Mx - Ny} = \frac{1}{2xy\cos xy}$$

Multiply eq (1) by I.F.

$$\Rightarrow \frac{1}{2xy\cos xy}\left[xy^2\sin xy + y\cos xy\right]dx + \frac{1}{2xy\cos xy}\left[x^2y\sin xy - x\cos xy\right]dy = 0$$

$$\left[\frac{y}{2}\tan xy + \frac{1}{2x}\right]dx + \left[\frac{x}{2}\tan xy - \frac{1}{2y}\right]dy = 0 \quad \text{(2)}$$

Compare eq (2) with $M_1 dx + N_1 dy = 0$

$$M_1 = \frac{y}{2}\tan xy + \frac{1}{2x} \qquad N_1 = \frac{x}{2}\tan xy - \frac{1}{2y}.$$

$$\frac{\partial M_1}{\partial y} = \frac{1}{2}\left[y(\sec^2 xy) + \tan xy\right]$$

$$= \frac{1}{2}\tan xy + \frac{xy}{2}\sec^2 xy$$

$$\frac{\partial N_1}{\partial x} = \frac{1}{2}\left[x(\sec^2 xy)(y) + \tan xy\right]$$

$$= \frac{1}{2}\tan xy + \frac{xy\sec^2 xy}{2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ Eq (2) is exact.

G.S $\Rightarrow \int M_1\,dx + \int N_1\,dy = c.$

$$\int\left[\frac{y}{2}\tan xy + \frac{1}{2x}\right]dx + \int\left[\frac{-1}{2y}\right]dy = c.$$

$$\frac{\cancel{y}}{2}\log\frac{\sec xy}{\cancel{y}} + \frac{1}{2}\log x - \frac{1}{2}\log y = \log c.$$

$$\cancel{\log}\left[\frac{(\sec xy\cdot(x)^{1/2}}{(y)^{1/2}}\right] = \cancel{\log} c$$

$$\sqrt{\frac{\sec xy(x)}{y}} = c$$

iv) $2xy\,dy - (x^2+y^2+1)\,dx = 0$

Sol:

$\cancel{x^2+y^2+1-2x}$

$(x^2+y^2+1)\,dx - 2xy\,dy = 0$ —— (1)

compare (1) with $M\,dx + N\,dy = 0$

$M = x^2+y^2+1$ $\qquad\qquad$ $N = -2xy$

$\frac{\partial M}{\partial y} = 2y$ $\qquad\qquad$ $\frac{\partial N}{\partial x} = -2y.$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ Eq. (1) is not exact.

consider $\frac{1}{N}\left[\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right]=-\frac{1}{2xy}\left[2y+2y\right]$

$$=\frac{-1}{\cancel{2}xy}(\overset{2}{\cancel{4}}y) = \frac{-2}{x} = f(x)$$

$$I\cdot F = e^{\int f(x)\,dx} = e^{\int\left(\frac{-2}{x}\right)dx} = e^{-2\int\frac{1}{x}dx} = e^{-2\log x}$$

$$= \frac{1}{x^2}$$

Multiply eq ① with I.F

$$\frac{1}{x^2}\left[x^2+y^2+1\right]dx - \frac{1}{x^2}\left[2xy\right]dy = 0$$

$$\left[1+\frac{y^2}{x^2}+\frac{1}{x^2}\right]dx - \left[\frac{2y}{x}\right]dy = 0 \quad \text{—— ②}$$

Compare eq ② with $M_1dx + N_1dy = 0$

$M_1 = 1+\frac{y^2}{x^2}+\frac{1}{x^2}$, $N_1 = -\frac{2y}{x}$

$\frac{\partial M_1}{\partial y} = \frac{2y}{x^2}$ $\qquad \frac{\partial N_1}{\partial x} = \frac{2y}{x^2}$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ Eq ② is exact.

G.S $\Rightarrow \int M_1 dx + \int N_1 dy = c$

$$\int \left[1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right] dx + \int (0)\, dy = c$$

$\cancel{x + y^2}$

$$x - \frac{y^2}{x} - \frac{1}{x} = c$$

$$\boxed{x^2 - y^2 - 1 = xc}$$

v) $2xy\,dx + (y^2 - x^2)\,dy = 0$ —— (1)

Sol Compare eq (1) with $M\,dx + N\,dy = 0$

$M = 2xy$ $\quad N = y^2 - x^2$

$\frac{\partial M}{\partial y} = 2x$ $\quad \frac{\partial N}{\partial x} = -2x$.

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\Rightarrow$ Eq (1) is not exact

consider, $\frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] = \frac{1}{2xy}\left[-2x - 2x\right]$

$$= \frac{-\cancel{4}^{2}\cancel{x}}{\cancel{2}\cancel{x}y} = \frac{-2}{y} = g(y)$$

$$I.F = e^{\int g(y)\,dy} = e^{\int \frac{-2}{y} dy} = e^{-2\log y} = \frac{1}{y^2}$$

Multiply eq(1) by I.F.

$$\frac{1}{y^2}\left[2xy\right]dx + \frac{1}{y^2}\left[y^2 - x^2\right]dy = 0$$

$$\left[\frac{2x}{y}\right]dx + \left[1 - \frac{x^2}{y^2}\right]dy = 0 \quad \text{—— (2)}$$

Compare eq(2) with $M_1 dx + N_1 dy = 0$

$$M_1 = \frac{2x}{y} \qquad N_1 = 1 - \frac{x^2}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{-2x}{y^2} \qquad \frac{\partial N_1}{\partial x} = \frac{-2x}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ Eq (2) is exact.

G.S $\Rightarrow$ $\int M_1 dx + \int N_1 dy = c$

$$\int \left(\frac{2x}{y}\right)dx + \int 0\,dy = c$$

$$\frac{x^2}{y} = c$$

$$\boxed{x^2 = cy}$$

vi) $y(xy+e^x)\,dx - e^x\,dy = 0$ —— (1)

Sol: Compare eq (1) with $M\,dx + N\,dy = 0$

$M = y(xy+e^x)$ $\quad N = -e^x.$

$= xy^2 + ye^x$

$\frac{\partial N}{\partial x} = -e^x.$

$\frac{\partial M}{\partial y} = 2xy + e^x.$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\Rightarrow$ Eq (1) is ~~ex~~ not exact.

Consider $\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = -\frac{1}{e^x}\left[2xy + e^x + e^x\right]$

$= -\frac{2xy + 2e^x}{e^x} = \frac{-2xy}{e^x} - 2 \neq f(x)$

Consider $\frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] = \frac{1}{xy^2 + ye^x}\left[-e^x - 2xy - e^x\right]$

$= \frac{1}{xy^2 + ye^x}\left[-2xy - 2e^x\right]$

$= \frac{-2\cancel{(xy+e^x)}}{y\cancel{(xy+e^x)}} = \frac{-2}{y} = g(y)$

$$I.F = e^{\int g(y)\,dy} = e^{\int -\frac{2}{y}dy} = e^{-2\log y}$$

$$= \frac{1}{y^2}$$

Multiply eq (1) by I.F

$$\frac{1}{y^2}\left[xy^2 + ye^x\right]dx - \frac{1}{y^2}\left[e^x\right]dy = 0$$

$$\left[x + \frac{e^x}{y}\right]dx - \left[\frac{e^x}{y^2}\right]dy = 0 \quad \text{—— (2)}$$

Compare eq (2) with $M_1\,dx + N_1\,dy = 0$

$$M_1 = x + \frac{e^x}{y} \qquad N_1 = \frac{-e^x}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{-e^x}{y^2} \qquad \frac{\partial N_1}{\partial x} = \frac{-e^x}{y^2}$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ Eq (2) is exact.

G.S $\Rightarrow \int M_1\,dx + \int N_1\,dy = c$

$$\int\left(x + \frac{e^x}{y}\right)dx + \int (0)\,dy = c$$

$$\boxed{\frac{x^2}{2} + \frac{e^x}{y} = c}$$

3) i) $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

Sol: $\cancel{(x+1)\,dy - y\,dx}$

$(x+1)\frac{dy}{dx} = \left[y - e^{3x}(x+1)^2\right] dx$

$\left[y - e^{3x}(x+1)^2\right] dx = x$

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3 i) $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

Sol: Dividing on b/s by $(x+1)$

$$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$$

$$\frac{dy}{dx} - \frac{1}{x+1}\,y = e^{3x}(x+1) \longrightarrow (1)$$

Compare eq (1) with $\frac{dy}{dx} + Py = Q$

$$P = \frac{-1}{x+1} \qquad Q = e^{3x}(x+1)$$

$$I.F \Rightarrow e^{\int P\,dx} = e^{\int\left[\frac{-1}{x+1}\right]dx} = e^{-\log(x+1)} = \frac{1}{x+1}$$

$$G.S \Rightarrow y(I.F) = \int (I.F)\,Q\,dx + c$$

$$y\left[\frac{1}{x+1}\right] = \int \left(\frac{1}{\cancel{x+1}}\right) e^{3x}\cancel{(x+1)}\,dx + c$$

$$\frac{y}{x+1} = \frac{e^{3x}}{3} + c \Rightarrow y = (x+1)\left[\frac{e^{3x}}{3} + c\right]$$

ii) $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$ —— (1)

Sol: Compare eq (1) with $\frac{dy}{dx} + Py = Q$

$P = \frac{1}{x\log x}$ $\quad Q = \frac{\sin 2x}{\log x}$

$I.F \Rightarrow e^{\int P\,dx} = e^{\int \left[\frac{1}{x\log x}\right] dx}$

$= e^{\int \frac{1/x}{\log x} dx} = e^{\log(\log x)} = \log x$

$G.S \Rightarrow y(I.F) = \int (I.F)\,Q\,dx + c$

$y(\log x) = \int (\cancel{\log x}) \left[\frac{\sin 2x}{\cancel{\log x}}\right] dx + c$

$y\log x = \int \sin 2x\,dx + c$

$y\log x = \frac{-\cos 2x}{2} + c$

$y = \frac{1}{\log x}\left[\frac{-\cos 2x}{2} + c\right]$

iii) $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Sol: $\cancel{\frac{dx}{dy}} = \cancel{\frac{\tan^{-1}y - x}{1+y^2}}$ $\quad (1+y^2)\frac{dx}{dy} = \tan^{-1}y - x$

$\cancel{\frac{dx}{dy}} +$ $\quad$ divide with $(1+y^2)$

$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$

$$\frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{\tan^{-1}y}{1+y^2} \quad \text{—— (1)}$$

compare with $\frac{dx}{dy} + Px = Q$

$$P = \left(\frac{1}{1+y^2}\right) \qquad Q = \frac{\tan^{-1}y}{1+y^2}$$

$$I.F \Rightarrow e^{\int P\,dy} = e^{\int\left(\frac{1}{1+y^2}\right)dy} = e^{\tan^{-1}y}.$$

G.S $\Rightarrow$ $x(I.F) = \int (I.F)Q\,dy + c$

$$x\left[e^{\tan^{-1}y}\right] = \int \left(e^{\tan^{-1}y}\right)\left[\frac{\tan^{-1}y}{1+y^2}\right]dy + c$$

consider $\int e^{\tan^{-1}y}\left[\frac{\tan^{-1}y}{1+y^2}\right]dy$

let $\tan^{-1}y = t$

$\frac{1}{1+y^2}\,dy = dt.$

$$\cancel{xe^t}\; x\,e^{\tan^{-1}y} = e^t \cdot t\,dt$$

$$= t\int e^t\,dt - \int\left(\frac{d}{dt}(t)\cdot\int e^t\,dt\right)dt + c$$

$$= te^t - \int e^t\,dt + c$$

$$\cancel{= e^t \int t\,dt}$$

$$= te^t - e^t + c$$

$$= e^{t}(t-1)+C$$

$$\boxed{\therefore x e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y-1)+c}$$

iv) $\frac{dy}{dx}+2y=e^{x}+x$, $y(0)=1$ —— ①

Sol Compare eq ① with $\frac{dy}{dx}+Py=Q$

$P=2$, $Q=e^{x}+x$.

$I.F \Rightarrow e^{\int P dx} = e^{\int 2 dx} = e^{2x}$.

$G.S \Rightarrow y(I.F) = \int (I.F) Q \, dx + c$.

$$y(e^{2x}) = \int (e^{2x})(e^{x}+x)\, dx + c$$

$$ye^{2x} = \int e^{3x} dx + \int x e^{2x} dx + c$$

$$= \frac{e^{3x}}{3} + x\int e^{2x}dx - \int (x)' \int e^{2x} dx + c$$

$$ye^{2x} = \frac{e^{3x}}{3} + \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

when $x=0$, $y=1$

$$1 \times e^{0} = \frac{e^{0}}{3} + 0.\frac{e^{0}}{2} - \frac{e^{0}}{4} + c$$

$$1 = \frac{1}{3} - \frac{1}{4} + c$$

$$C = 1 - \frac{1}{3} + \frac{1}{4}$$

$$C = \frac{12-4+3}{12} = 11/12$$

$$ye^{2x} = \frac{e^{3x}}{3} + \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + \frac{11}{12}$$

$$\boxed{y = \frac{e^{x}}{3} + \frac{x}{2} - \frac{1}{4} + \frac{11}{12}e^{-2x}}$$

v) $(1+x^2)dy + 2xy\,dx = \cot x\,dx$

Sol: divide by 'dx'

$$(1+x^2)\frac{dy}{dx} + 2xy = \cot x \quad \text{---} (1)$$

~~compare with~~

divide by $(1+x^2)$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cot x}{1+x^2} \quad \text{---} (2)$$

Compare eq (2) with $\frac{dy}{dx} + Py = Q$.

$$P = \frac{2x}{1+x^2} \qquad Q = \frac{\cot x}{1+x^2}$$

$$I.F \Rightarrow e^{\int P\,dx} = e^{\int\left(\frac{2x}{1+x^2}\right)dx} = e^{\log(1+x^2)} = 1+x^2$$

G.S $\Rightarrow$ $y(I.F) = \int (I.F)Q\,dx + c$.

$$\Rightarrow y\left[\frac{1}{1+x^2}\right] = \int\left[\frac{1}{1+x^2}\right]\left[\frac{\cot x}{1+x^2}\right]dx + c$$

$$\frac{y}{1+x^2} = \int \frac{\cot x}{(1+x^2)^2}\,dx + c$$

$$\frac{y}{1+x^2}$$

$$\Rightarrow y(1+x^2) = \int (\cancel{1+x^2})\left[\frac{\cot x}{\cancel{1+x^2}}\right]dx + c$$

$$\Rightarrow y(1+x^2) = \int \cot x\,dx + c$$

$$y(1+x^2) = \log|\sin x| + c$$

$$y = \frac{1}{1+x^2}\left[\log|\sin x| + c\right]$$

4) i) $x\frac{dy}{dx} + y = x^3y^6$

Sol: divide by 'x'

$$\frac{dy}{dx} + \frac{y}{x} = x^2y^6. \quad \text{---} (1)$$

which is a bernouli equation

Divide (1) by 'y^6'

$$y^{-6}\frac{dy}{dx} + \left(\frac{1}{x}\right)\frac{y}{y^6} = x^2$$

G.S $\Rightarrow$ $y(I.F) = \int (I.F)\, Q\, dx + c$

$$y\left(e^{\log x}\right) = \int \left(e^{\log x}\right)\left[x^2 y^6\right] dx + c$$

$$y e^{\log x} =$$

$$y^{-6}\frac{dy}{dx} + y^{-5}\left(\frac{1}{x}\right) = x^2 \quad \text{---} \ (2)$$

let $y^{-5} = t$

$$-5y^{-6}\frac{dy}{dx} = \frac{dt}{dx}$$

$$y^{-6}\frac{dy}{dx} = -\frac{1}{5}\frac{dt}{dx} \quad \text{---} \ (3)$$

Substitute (3) in (2)

$$-\frac{1}{5}\frac{dt}{dx} + t\left(\frac{1}{x}\right) = x^2$$

$$\frac{dt}{dx} - \left(\frac{5}{x}\right)t = -5x^2 \quad \text{---} \ (4)$$

Eq (4) represents a linear eqn in t & x.

$P = -\dfrac{5}{x}$ $\qquad$ $Q = -5x^2$

$$I.F \Rightarrow e^{\int P\,dx} = e^{\int -\frac{5}{x}\,dx} = e^{-5\log x} = \frac{1}{x^5}$$

G.S $\Rightarrow$ $t(I.F) = \int (I.F)\, Q\, dx + c$

$$t\left(\frac{1}{x^5}\right) = \int \left(\frac{1}{x^5}\right)(-5x^2)\,dx + c$$

$$\frac{t}{x^5} = -5\int x^{-3}\,dx + c$$

$$\frac{t}{x^5} = -5 \cdot \frac{x^{-2}}{-2} + c$$

$$\frac{t}{x^5} = \frac{5}{2x^2} + c$$

$$t = x^5\left[\frac{5}{2x^2} + c\right]$$

$$t = \frac{5x^3}{2} + cx^5.$$

$$y^{-5} = \frac{5x^3}{2} + cx^5.$$

$$\frac{1}{y^5} = \frac{5x^3}{2} + cx^5.$$

ii) $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y.$ —— (1)

Sol: Divide (1) by $\cos^2 y$

$$\frac{1}{\cos^2 y}\frac{dy}{dx} + \frac{x(2\sin y\cos y)}{\cos^2 y} = x^3$$

$\sec^2 y \cdot \frac{dy}{dx} + 2x \cdot \tan y = x^3$ —— (2)

let $\tan y = t$

$\sec^2 y \frac{dy}{dx} \quad \frac{dt}{dx}$ —— (3)

Sub (3) in (2)

$\frac{dt}{dx} \cancel{+ 2x \tan y = x^3}$

$\frac{dt}{dx} + 2xt = x^3$ —— (4)

Eq (4) represents a linear eqn in 't' & 'x'

$P = 2x$. $\quad Q = x^3$

$I \cdot F \Rightarrow e^{\int P\,dx} = e^{\int 2x\,dx} = e^{x^2}$.

G.S $\Rightarrow t(I \cdot F) = \int Q(I \cdot F)\,dx + c$

$\tan y (e^{x^2}) = \int x^3 (e^{x^2})\,dx + c$

let $x^2 = p$

$2x = \frac{dp}{dx}$

$x\,dx = \frac{1}{2} dp$.

$\tan y (e^{x^2}) = \int x^2 \cdot x \cdot (e^{x^2})\,dx + c$.

$$\tan y\,(e^{p}) = \int \frac{pe^{p}}{2}\,dp + c$$

$$= \frac{1}{2}\int pe^{p}\,dp + c$$

$$= \frac{1}{2}\left[p\int e^{p}\,dp - \int \frac{d}{dp}(p)\int e^{p}\,dp\right] + c$$

$$\tan y\, e^{p} = \frac{1}{2}\left[pe^{p} - e^{p}\right] + c$$

$$\tan y\,(e^{x^2}) = \frac{e^{p}}{2}(p-1) + c$$

$$\tan y\,(\cancel{e^{x^2}}) = \frac{\cancel{e^{x^2}}}{2}(p-1) + c$$

$$\boxed{\tan y = \frac{1}{2}(x^2-1) + c}$$

iii) $(1-x^2)\frac{dy}{dx} + xy = y^3 \sin^{-1}x$

Sol: Divide by '$1-x^2$'.

$$\frac{dy}{dx} + \left(\frac{x}{1-x^2}\right)y = y^3 \cdot \frac{\sin^{-1}x}{1-x^2} \quad \text{——} \; (1)$$

Which is a Bernouli's Eqn

Divide eq (1) with 'y^3'

$$\frac{1}{y^3}\frac{dy}{dx} + \left(\frac{x}{1-x^2}\right)y^{-2} = \frac{\sin^{-1}x}{1-x^2} \quad \text{——} \; (2)$$

let $y^{-2} = t$

$$\cancel{-2y\frac{dy}{dx} = \frac{dt}{dx}}$$

$$-2y^{-3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{-2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dt}{dx} \quad \text{——} \ (3)$$

sub (3) in (2)

$$-\frac{1}{2}\frac{dt}{dx} + \left(\frac{x}{1-x^2}\right)t = \frac{\sin^{-1}x}{1-x^2}$$

Multiply with '-2'.

$$\frac{dt}{dx} - \frac{2x}{(1-x^2)}t = \frac{-2\sin^{-1}x}{1-x^2} \quad \text{——} \ (4)$$

It is a L.E in 't' & 'x'

$P = \frac{-2x}{1-x^2}$ $\qquad Q = \frac{-2\sin^{-1}x}{(1-x^2)}$

$$I.F \Rightarrow e^{\int P\,dx} = e^{\int \frac{-2x}{1-x^2}dx} = e^{\log|1-x^2|}$$

$$= 1-x^2$$

G.S $\Rightarrow t\,(I.F) = \int (I.F)\,Q\,dx + c$

$$t(1-x^2) = \int (1-x^2)\left[\frac{-2\sin^{-1}x}{1-x^2}\right] dx + c$$

$$t(1-x^2) = -2\int \sin^{-1}x\, dx + c$$

$$= -2\int \sin^{-1}x \cdot 1\, dx + c$$

$$= -2\left[\sin^{-1}x\int dx - \int \frac{d}{dx}(\sin^{-1}x)\int 1\, dx\right] + c$$

$$= -2\left[x\sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}} \cdot x\, dx\right] + c$$

$$= -2\left[x\sin^{-1}x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\, dx\right] + c$$

$$= -2\left[x\sin^{-1}x + \frac{1}{2}\int (-2x)(1-x^2)^{-1/2}\, dx\right] + c$$

$$f'(x) \quad f^n(x)$$

$$= -2\left[x\sin^{-1}x + \frac{1}{2}\frac{[1-x^2]^{-1/2+1}}{-\frac{1}{2}+1}\right] + c$$

$$t[1-x^2] = -2\left[x\sin^{-1}x + \sqrt{[1-x^2]}\right] + c$$

$$y^{-2} = \frac{-2}{[1-x^2]}\left[x\sin^{-1}x + \sqrt{1-x^2}\right] + \frac{c}{[1-x^2]}$$

iv) $e^x\frac{dy}{dx} = 2xy^2 + ye^x$ — (4)

Sol: Divide by 'e^x' on b/s

$$\frac{dy}{dx} - y = \frac{2xy^2}{e^x} \quad \text{— (1)}$$

~~let~~. Which is a Bernoulli's eqn.

Divide eqn (1) with 'y^2'.

$$y^{-2}\frac{dy}{dx} - y^{-1} = \frac{2x}{e^x} \quad \text{——} (2)$$

let $y^{-1} = t$

$$-1y^{-2} \cdot \frac{dy}{dx} = \frac{dt}{dx}$$

$$y^{-2}\frac{dy}{dx} = -\frac{dt}{dx} \quad \text{——} (3)$$

Sub (3) in (2)

$$-\frac{dt}{dx} - t = \frac{2x}{e^x}.$$

$$\frac{dt}{dx} + t = -\frac{2x}{e^x} \quad \text{——} (4)$$

compare with S.F.

$P = 1 \qquad Q = -\frac{2x}{e^x}.$

$$I.F \Rightarrow e^{\int P dx} = e^{\int 1 dx} = e^x$$

$G.S \Rightarrow t(I.F) = \int (I.F) Q\, dx + c$

$t(e^x) = \int (\cancel{e^x}) \left[\frac{-2x}{\cancel{e^x}}\right] dx + c \Rightarrow t(e^x) = \int -2x\, dx + c$

$te^x = -x^2 dx + c$

$\frac{e^x}{y} = -x^2 dx + c \Rightarrow \frac{e^x}{y} = -x^2 + c$