UNIT

II. ELEMENTARY COMBINATIONS

Sum Rule :-

Basics of counting :

Sum Rule :- if an event can occur in 'M' ways and another event can occur in N ways. M (or) N can happen but two events can not happen simultaneously than one of the two events can occur in M + N ways.

Ex :- If there are 9 boys and 10 girls in a class. In how many ways 1 student can be selected as class represented

sol :- $M = 9$ $N = 10$ $M + N = 19$ ways

2). Suppose E is a event of selecting a prime number less than 10 and f is a event selecting a even number less than 10

sol :- $E = 2, 3, 5, 7$ $4 + 4 - 1$

$f = 2, 4, 6, 8$ $= 7$ ~~ways~~.

2 is repeating in both two case

3) In how many ways we can draw a ~~car~~ heart or speads from an ordinary deck of playing cards.

(ii) A hearts (or) ~~speads~~ ace.

iii) An ace (or) a king

iv) A card number 2 to 10 through

v) A number card (or) a king

i) sol:- $13+13 = 26$

ii) $13+4 = 17-1 = 16$

iii) $4+4 = 8$

iv) 36

v) 40

2 to 10

9 - hearts

9 - spods

9 -

9 -

4) How many ways can we get a sum of 4 (or) 8 two distinguishable dies are rolled.

sol:- 4 - (1,3)(3,1)(2,2)

8 - (4,4)(3,5),(5,3), (6,2)(2,6).

= 8 ways

* Product Rule :- If an event occur in M ways and second event occurs in N ways and if the number of ways the second event occur does not depend upon the first event occurs than the two events can occurs simultaneously in M.N ways

Ex 1) A book shelf 5 different math books, 6 diff computer books, 10 diff statistics in how many ways can we selected the books.

$6 \times 5 \times 10 = 300$ ways.

2) If 2 distinguish dics are rolled how many ways?

$6 \times 6 = 36$

3) suppose that two license plates of a certain state require 3 english letters followed by 4 digits.

(i) how many digits diff plates can be manufactured if repetition of digits are allowed

(ii) How many plates are possible if only the letters are repeated.

(iii) How many are possible of no repetition are repeated

(iv) How many are possible of no repetation are allowed

sol!- (i) $26^3 \cdot 10^4$ (ii) $26^3 \, {}^{10}P_4$ $\times 9 \times 8 \times 7$ (iii) $26^3 \cdot 10^4$
$26 \times 25 \times 24 \cdot 10^4$

(iv) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

* permutations without repetations :-

combination def :- A combination of n objects taken 'R' at a time is an anorder selection of 'R' objects.

permutation def :- A permutation of n objects take 'R' at a time is an ordered select (or) arrangments of 'R' objects.

Let $p(n, r)$ denote the no. of 'r' permutations of 'n' elements without repetations

$$p(n, r) = n(n-1) \cdots (n-r+1)$$

$$= \frac{n!}{(n-r)!}$$

(1) Q. How many different string of length '4' can be performed using the letter of the word "~~flower~~ 'FLOWER'.

sol:- $6P_4$

(2) Q. A soft ball team has 10 players. how many bat are possiable if everyone gets to bat.

sol:- $10P_{10} \Rightarrow 10!$

(3Q) In how many ways are there to distribute 10 different books among ~~50~~ 15 peoples. if no per is to recieve more than 1 book.

sol:- $15P_{10}$

(4) Find the no. of different ways in which 4 boys and 6 girls may be arranged in a row. so that no ~~to~~ 2. boys shall be together

sol: B G B G B G B G B G B G B

$7P_4 \times 6P_6$

$\Rightarrow {}^7P_4 \times {}^6P_6$

5. In how many ways 7 women and 3 men be arranged in a row. If the 3 men must always stand next to each other.

$\boxed{- - -}$ - - - - - - -

$\Rightarrow 8!\,3!$

6. How many arrangments of the elements introduced "INTRODUCE" can we form.

(i) vowels are together — AEIOU · NTRDC

(ii) all vowels do not come together

Sol:- 9! ways

(i) $\boxed{4}\ \boxed{5} \Rightarrow {}^6P_6 \times {}^4P_4$

(ii) $9! - 6! \times 4! \Rightarrow$

$\cancel{362880} - \cancel{17280} = \cancel{345600}$

7. It is required to set 5 men and 4 women in a row so that women occupy even places. How many such arrangment are possible $\Rightarrow 4! \times 5!$

8. In how many way 6 men & 6 women be setted in a row.

(i) any person set next to another $\Rightarrow \cancel{6!\,6!}\cdot 2$ $12!$

(ii) men & women must occupy alternate seats

$6!\,6! \times 2$

9) In how many ways can 3 men & 3 women be sitt. at a round table.

(i) no restriction is impost $(n-1)! \Rightarrow (6-1)!$

(ii) two particular women must no sits together

mwm wmw (or) wmw mwm $(\cancel{n-1})\ 2 \cdot 3!\,3!$ ways

Combination without repitations

$$C(n, r) = \frac{p(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$

1) In how many ways can a hand of 5 cards be selected from a deck of 52 cards

sol:- $52C_5$

2) In how many wa can ~~item~~ team of 11 players be selected from 16 players.

$16C_{11} = 16C_5$

3) In how many ways a comitee of 5 members can be formed for 6 men and 5 women. such that

(i) It includes 2 women

(ii) It includes atleast 2 women

(iii) It includes atmost 2 women

(i) $6C_3 \cdot 5C_2$

(ii) $6C_3 \cdot 5C_2 + 6C_2 \cdot 5C_3 + 6C_1 \cdot 5C_4 + 6C_0\, 5C_5$

(iii) $6C_5 \cdot 5C_0 + 6C_4 5C_1 + 6_3 5C_2$

4) How many comitties of 6 (or) more can be choosen from 9 people

$\Rightarrow$ $9C_6 + 9C_7 + 9C_8 + 9C_9$

5) How many 5 card hands consists only hearts

$13C_5$

6) How many 5 cards hands consists of cards from a single suit

$13C_5$

permutations with constrained prep repitations :

$$P(n, q_1, q_2, q_3, \dots q_t) = \frac{n!}{q_1! q_2! q_3! \dots q_t!}$$

find the no. of permutations of the letter SUCCESS

$$= \frac{7!}{3!2!}$$

1) How many 9 letters words can be formed by using letter of the level DIFFICULT

$$= \frac{9!}{2!2!} = \frac{9!}{(2!)^2}$$

2) Find the no. of permutations of the letter of the level.

MASSASAGOA $\frac{10!}{4!3!}$

3) In How many ways of this all is so 11 as are together MSSSGUA $= \frac{7!}{4!3!}$

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7) The No. of arrangements of the letter in the word

TALLAHASSEE

$$\frac{11!}{\cancel{4}\,3!\,2!\,2!\,2!}$$

begins with s

$$\frac{3\times 9!}{4!} \quad \Bigg| \quad 3\times\frac{9!}{4!\,2!}$$

combinations with unlimited repetations :-

→ $V(n,r)$ = The no. of r combinations of distinct objects with unlimited repetations.

* The no. of binary numbers with $(n-1)$ one's and r ~~(or)~~ 0's.

→ $C(n-1+r, r) = C(n-1+r, n-1)$.

→ $(n+r-1)!\,/\,(r!\,(n-1)!)$

* The no. of ways of distributing are similar balls into 'n' no. boxes

* The no. of non-negative integral solutions to

of $(x_1+x_2+x_3, \ldots + x_n) = r$.

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1) In how many ways can we distribute 10 identical marbles among 6 distinct container.

Sol:-

$n = 6$ $\Rightarrow c(n-1+r, r)$

$r = 10$ $c(6-1+10, 10)$

$c(15, 10)$

$$\frac{(n+r-1)!}{r!\,(n-1)!} \Rightarrow \frac{15!}{10!\,(6-1)!} \Rightarrow \frac{15!}{10!\,5!}$$

2) In how many ways different outcomes are possible by 10 similar coins..

Sol:-

$r = 10$

$n = 2$ (H and T)

$c(2-1+10, 10)$

$c(11, 10)$

$$\frac{(n+r-1)!}{r!(n-1)!} \Rightarrow \frac{11!}{10!\,(2-1)!} \Rightarrow \frac{11!}{10!\,1!}$$

3) how many ways can 20 similar books ~~to~~ be placed ~~or~~ in 5 different shelfs.

$n = 5$ $c(5-1+20, 20)$

$r = 20$ $c(24, 20)$

$$\frac{(n+r-1)!}{r!(n-1)!} \Rightarrow \frac{24!}{20!\,4!}$$

4) The no. of non-negative integral solutions to

$x_1 + x_2 + x_3 + x_4 + x_5 = 50$

$r = 50$ $c(\overset{n}{54}, \overset{r}{50})$

$n = 5$

$$n \Rightarrow \frac{54!}{50!\,4!}$$

6) find the no. of integer solutions of $x_1+x_2+x_3+x_4+x_5 = 30$ where $x_1 \geq 2$, $x_2 \geq 3$, $x_3 \geq 4$, $x_4 \geq 2$, $x_5 \geq 0$.

**

Sol:- Sub above values in given eqn.

$$x_1+x_2+x_3+x_4+x_5 = 30$$

$$\underbrace{2+3+4+2+0}_{11} = 30$$
$$-11$$
$$19$$

$r = 19$

$n = 5$ $\Rightarrow$ $c(5-1+19, 19)$

$c(23, 19)$

$$= \frac{28!}{19!\,4!}$$

7) How many integral solutions are there to $x_1+x_2+x_3+x_4+x_5 = 20$ where $x_i \geq 2$

Sol:- $x_1+x_2+x_3+x_4+x_5 = 20$

$$\underbrace{2+2+2+2+2}_{10} = 20$$
$$-10$$

$r = 10$

$n = 5$ $\quad c(14, 10)$

$$= \frac{14!}{10!\,4!}$$

8) estimate the no. of non-negative integral solutions to the inequality $x_1+x_2+x_3+x_4+x_5 \leq 19$

for $r=0$, $n=5$ $\quad c(5-1+0, 0)$

$c(4, 0)$ $\quad \frac{4!}{0!\,4!} = 1$

* let $v(n,r)$ denote by the no. of permutations of 'n' objects with unlimited repetitions

$v(n,r) = 'n^r'$. each of r positions can be filled in n ways & so by product rule

$v(n,r) = n^r$

Q

(1) There are 25 True or false questions on an examination. how many different ways can student do the examination if he (or) she are choose to leave the answer blank.

Sol:-

$r = 25$

$n = 3$ $\quad n^r \Rightarrow 3^{25}$

(2) The results of 50 football games (win, loose, tie) are to be predicted. How many different forcast can contain exactly 28 correct results.

Sol:-

$50 - 28$

$r = 22$ $\quad n^r \Rightarrow 2^{22}$

$n = 2$

pigeonhole principle :-

Sum of the most important complicated results in modern theory floor from a very simple preposition.

→ If 'n' pigeon holes shelter 'n+1' (or) more pigeons, atleast one pigeon hole shelters atleast two pigeons.

→ To Apply a pigeonhole principle we must divide which objects will play the roles of the pigeon and which objects will play the roles of pigeon holes.

→ A generalisation of the pigeonhole principle is as follows

1) If 'K' pigeons are assigned to 'n' pigeonholes then one of the pigeon holes must contain atleast ~~one~~.

$$\left\lfloor \frac{K-1}{n} \right\rfloor + 1$$

Suppose there are 26 students and 7 cars to transport them. Then atleast one car must have 4 (or) more passanger

$\cancel{K} = \cancel{7}$ $\quad n = 7$

$\cancel{n} = 26$ $\quad K = 26$

$$\left\lfloor \frac{26-1}{n} \right\rfloor + 1 \Rightarrow \left\lfloor \frac{25}{7} \right\rfloor + 1$$

$$\Rightarrow 3.\cancel{5} + 1 \Rightarrow 4$$

2) If 401 letters where delivered to 50 apartments then prove that sum apartments recieved atleast 9 letters

$k = 401$

$n = 50$

$\left\lfloor \frac{401-1}{50} \right\rfloor + 1$

$\Rightarrow 8 + 1 = 9$

3) S.T in a group of 61 people atleast 6 people were born in the same month.

$k = 61$

$n = 12$

$\left\lfloor \frac{61-1}{12} \right\rfloor + 1$

$\left\lfloor \frac{60}{12} \right\rfloor + 1 \Rightarrow 6$

4) given 37 +ve integers then there must be atleast 4 of them that have the save remainder when divided by 12

$k = 37$

$n = 12$

$\left\lfloor \frac{37-1}{12} \right\rfloor + 1$

$3 + 1 = 4$

* The principle of Inclusion and exclusion :-

The sum rule by which we can count The no. of elements in the union of disjoint sets. however with the sets are not disjoints we must define the statement of sum rule to a rule commonly called the principle of inclusion and exclusion

" it is also called sieve rule "

The principle of inclusion, exclusion for two sets.
if a & b are subset of the universal set 'U' then
$|A \cup B| = |A| + |B| - |A \cap B|$.

problems

1) A sample of 80 people will be reveled that 25 like cinema and 60 like T.v programs. find the no. of people who like both programs.

$A = 25$, $B = 60$ $\quad A \cup B = 80$.

$|A \cup B| = |A| + |B| - |A \cap B|$.

$80 = 25 + 60 - \cancel{180}|A \cap B|$

$= \cancel{5}\ 75 - |A \cap B|$

$|A \cap B| = 5$

2) A certain computer centers employee's 100 computer programers of these 47 can programing Fortran language. 35 an pascal and 23 can programing both languages. how many can program in neither of these two languages

$A \cup B = 100$

$A = 47$, $B = 35$ $\quad |A \cap B| = 23$

$100 = 47 + 35 - 23$

$= 82 - 23$

$= 59$

3) Consider a set of integers from (1 to 250) find how many ways of these number are diviviable by

3 (or) 5 (or) ~~10~~ 7. also indicate how many are divisi[ble]
by 8 (or) 5 (or) 7. also indicate how many are divisi[ble]
by 3 (or) 7 but not by 5 and also indicate 3 (or)
[a]re divisible

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

A = 3

B = 5

C = 7

$$= \frac{250}{3} + \frac{250}{5} + \frac{250}{7} - \frac{250}{15} - \frac{250}{21} - \frac{250}{35} + \frac{250}{105}$$

$$= 83 + 50 + 35 - 16 - 11 - 7 + 2$$

$$= 168 - 170 - 34$$

$$|A \cup B \cup C| = 136 \;\text{//} \; - (B)$$

$$\overset{B}{(5)} = 136 - \frac{250}{5} \Rightarrow 136 - 50$$

$$= 86 \;\text{//}$$

(iii)

$$(A) = 8 \Rightarrow 136 - A \Rightarrow 136 -$$

$$\Rightarrow A + B - A \cap B$$

$$\Rightarrow 83 + 50 - 16$$

8 (or) 5

$$= \cancel{110} \;\text{//}\; 117 \;\text{//}$$

$$A + B - |A \cap C| \Rightarrow 83 + 50 - 11$$

$$= 133 - 11 \Rightarrow 1$$