

Assignment 3:

Projective Geometry

Computer Vision
National Taiwan University

Fall 2019

Part 1: Estimating Homography



Recap of Homography

- Matrix form:

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

- Equations:

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Recap of Homography

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

- Degree of freedom

- There are 9 numbers in H . Are there 9 DoF?
- No. Note that we can multiply all h_{ij} by nonzero k without changing the equations:

$$v_x = \frac{kh_{11}u_x + kh_{12}u_y + kh_{13}}{kh_{31}u_x + kh_{32}u_y + kh_{33}}$$

$$v_y = \frac{kh_{21}u_x + kh_{22}u_y + kh_{23}}{kh_{31}u_x + kh_{32}u_y + kh_{33}}$$



$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Enforcing 8 DoF

- **Solution 1:** set $h_{33} = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

- **Solution 2:** impose unit vector constraint

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Subject to

$$h_{11}^2 + \dots + h_{33}^2 = 1$$

Solution 1

- Set $h_{33} = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$
$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

- Multiply by denominator

$$(h_{31}u_x + h_{32}u_y + 1)v_x = h_{11}u_x + h_{12}u_y + h_{13}$$

$$(h_{31}u_x + h_{32}u_y + 1)v_y = h_{21}u_x + h_{22}u_y + h_{23}$$

- Rearrange

$$h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x = v_x$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y = v_y$$

Solution 1 (cont.)

- Solve linear system

$$\begin{array}{l}
 \text{Point 1} \\
 \text{Point 2} \\
 \text{Point 3} \\
 \text{Point 4} \\
 \text{Additional points}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix}
 u_{x,1} & u_{y,1} & 1 & 0 & 0 & 0 & -u_{x,1}v_{x,1} & -u_{y,1}v_{x,1} \\
 0 & 0 & 0 & u_{x,1} & u_{y,1} & 1 & -u_{x,1}v_{y,1} & -u_{y,1}v_{y,1} \\
 u_{x,2} & u_{y,2} & 1 & 0 & 0 & 0 & -u_{x,2}v_{x,2} & -u_{y,2}v_{x,2} \\
 0 & 0 & 0 & u_{x,2} & u_{y,2} & 1 & -u_{x,2}v_{y,2} & -u_{y,2}v_{y,2} \\
 u_{x,3} & u_{y,3} & 1 & 0 & 0 & 0 & -u_{x,3}v_{x,3} & -u_{y,3}v_{x,3} \\
 0 & 0 & 0 & u_{x,3} & u_{y,3} & 1 & -u_{x,3}v_{y,3} & -u_{y,3}v_{y,3} \\
 u_{x,4} & u_{y,4} & 1 & 0 & 0 & 0 & -u_{x,4}v_{x,4} & -u_{y,4}v_{x,4} \\
 0 & 0 & 0 & u_{x,4} & u_{y,4} & 1 & -u_{x,4}v_{y,4} & -u_{y,4}v_{y,4}
 \end{bmatrix} \\
 \vdots
 \end{array}
 \begin{array}{c}
 2N \times 8 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{c}
 \begin{bmatrix}
 h_{11} \\
 h_{12} \\
 h_{13} \\
 h_{21} \\
 h_{22} \\
 h_{23} \\
 h_{31} \\
 h_{32}
 \end{bmatrix} \\
 = \\
 \begin{bmatrix}
 v_{x,1} \\
 v_{y,1} \\
 v_{x,2} \\
 v_{y,2} \\
 v_{x,3} \\
 v_{y,3} \\
 v_{x,4} \\
 v_{y,4}
 \end{bmatrix} \\
 \vdots
 \end{array}
 \begin{array}{c}
 8 \times 1 \\
 2N \times 1
 \end{array}$$

Solution 1 (cont.)

- What might be wrong with solution 1?
- If h_{33} is actually 0, we can not get the right answer

Solution 2

- A more general solution by confining $h_{11}^2 + \dots + h_{33}^2 = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

- Multiply by denominator

$$(h_{31}u_x + h_{32}u_y + h_{33})v_x = h_{11}u_x + h_{12}u_y + h_{13}$$

$$(h_{31}u_x + h_{32}u_y + h_{33})v_y = h_{21}u_x + h_{22}u_y + h_{23}$$

- Rearrange

$$h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x - h_{33}v_x = 0$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y - h_{33}v_y = 0$$

Solution 2

- Similarly, we have a linear system like this:

$$\begin{matrix} 2N \times 9 & 9 \times 1 & & 2N \times 1 \\ & & & \\ & & & \end{matrix}$$
$$\mathbf{A} \mathbf{h} = \mathbf{b}$$

- Here, \mathbf{b} is all zero, so above equation is a [homogeneous system](#)
- Solve:
 - $Ah = 0$
 - SVD of $A = U\Sigma V^T$
 - Let h be the last column of V .

Input Data

- Canvas:



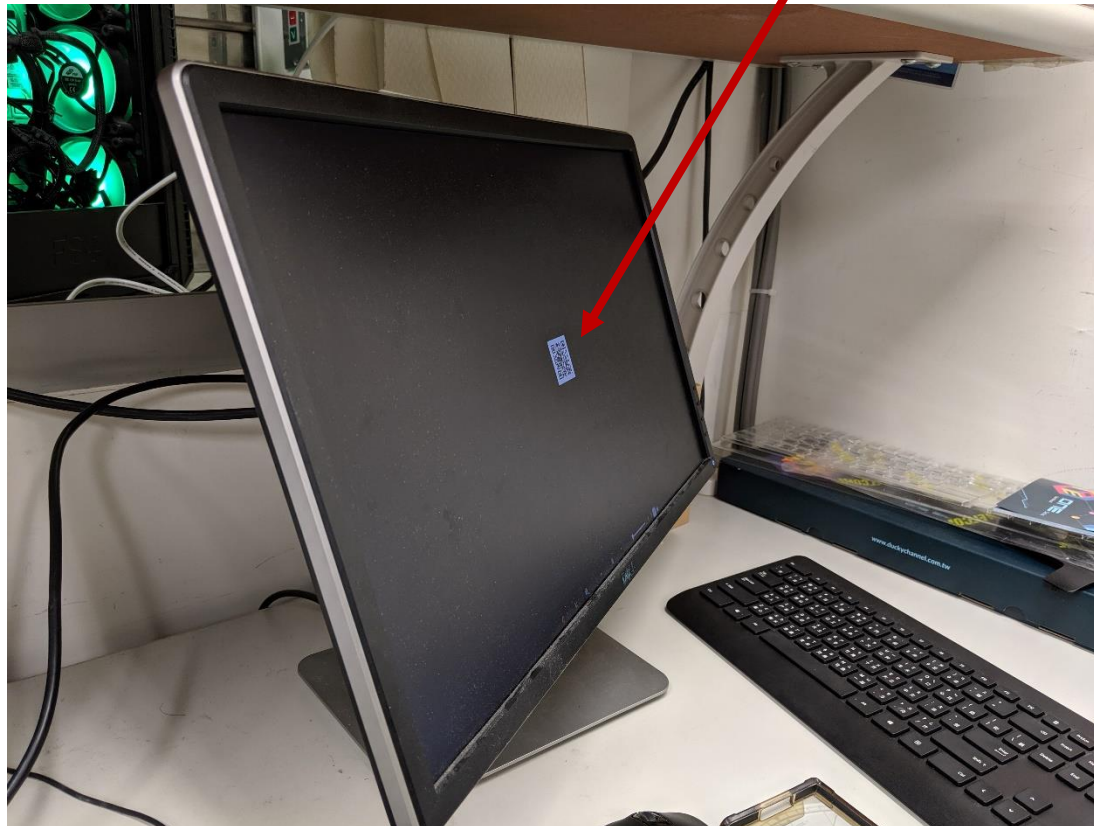
Input Data

- Material:



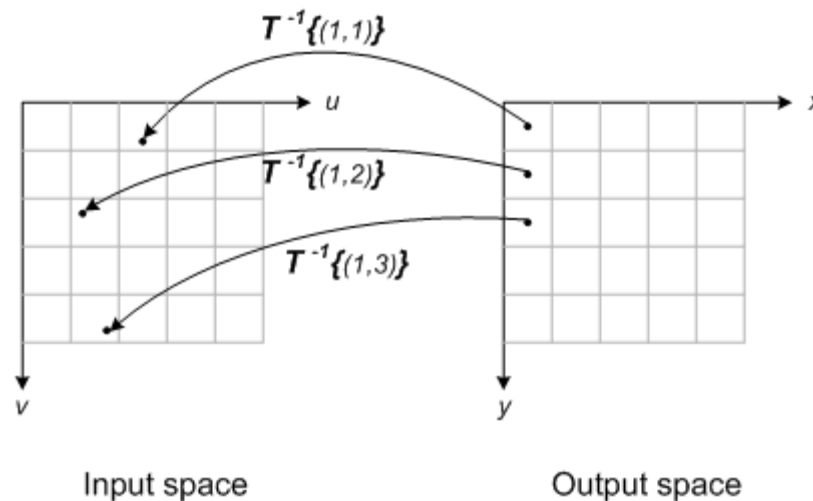
Part 2: Unwarp the Screen

Make the QR code frontal parallel



Backward Warping

- Why?
 - Prevent holes in output space
- Pixel value at sub-pixel location like (30.21, 22.74)?
 - Bilinear interpolation
 - Nearest neighbor



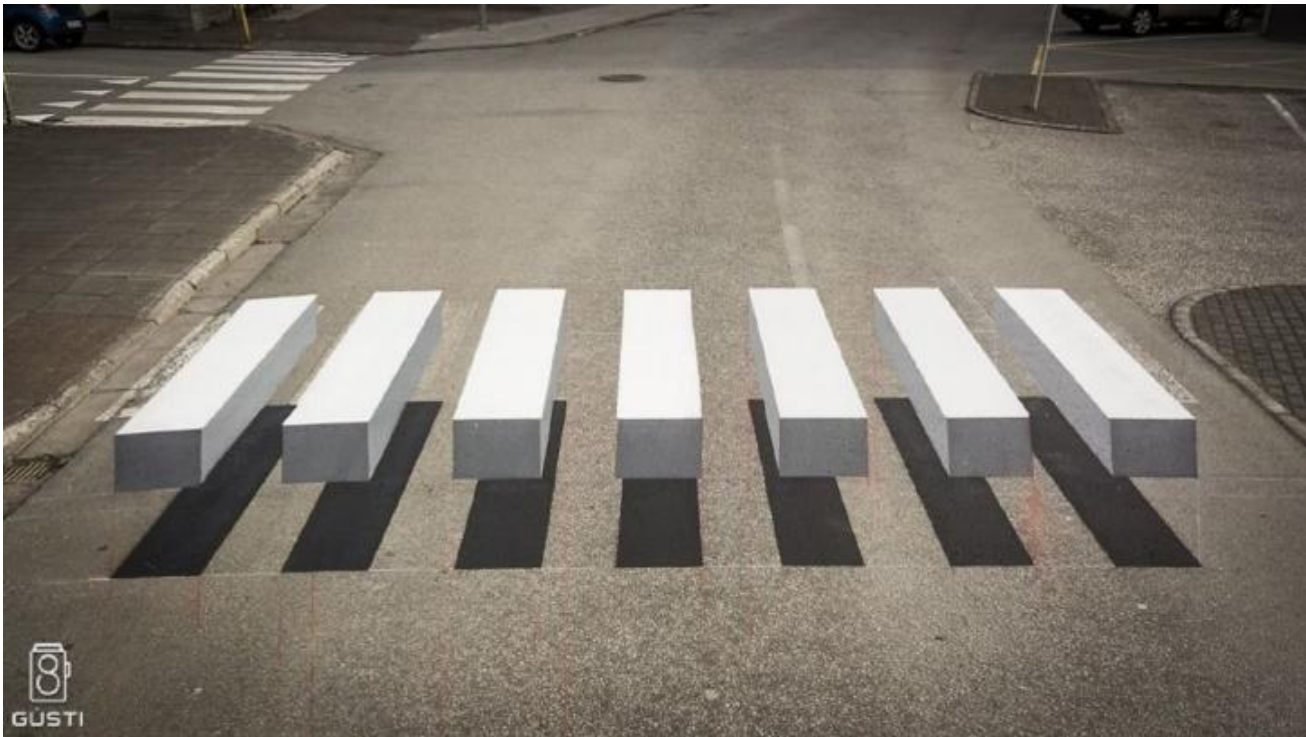
Part 3: Unwarp the 3D Illusion

- 3D illusion art



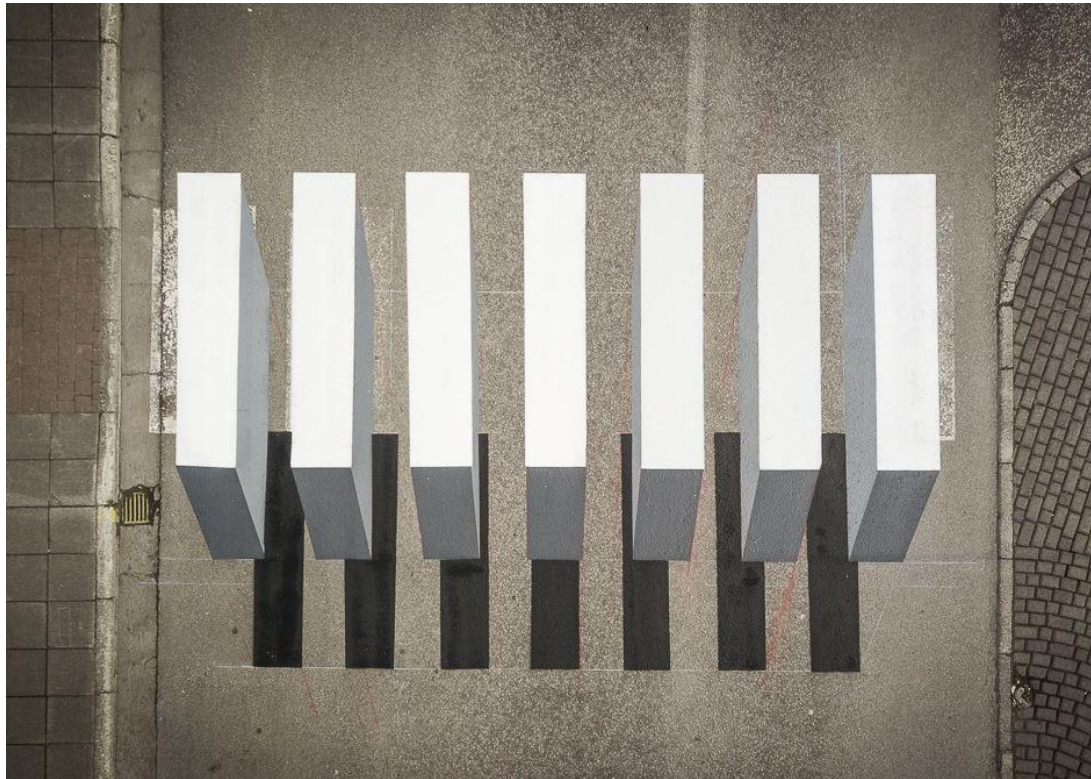
Part 3: Unwarp the 3D Illusion

- Input:



Part 3: Unwarp the 3D Illusion

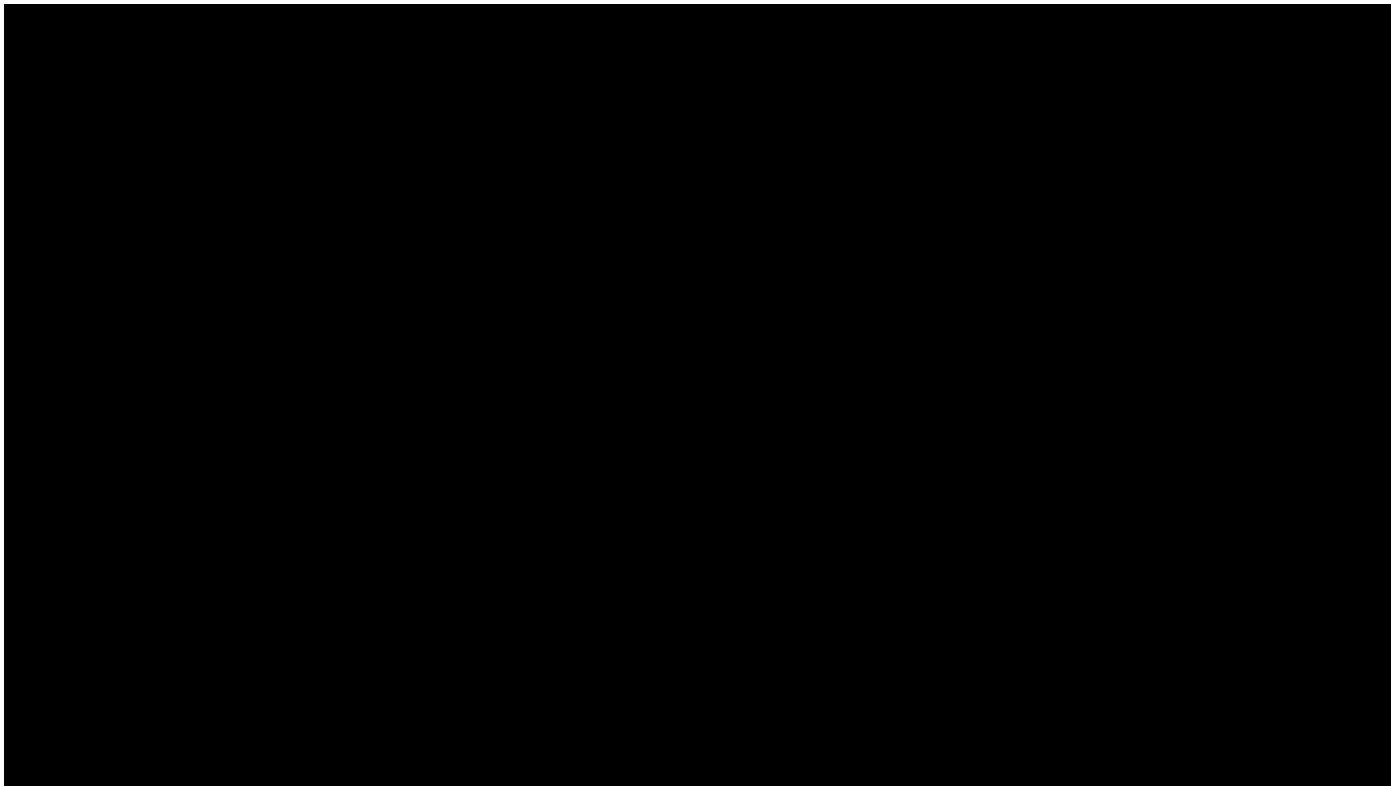
- Ground-truth top view:



Can you unwarp the input image to match the ground-truth top view?

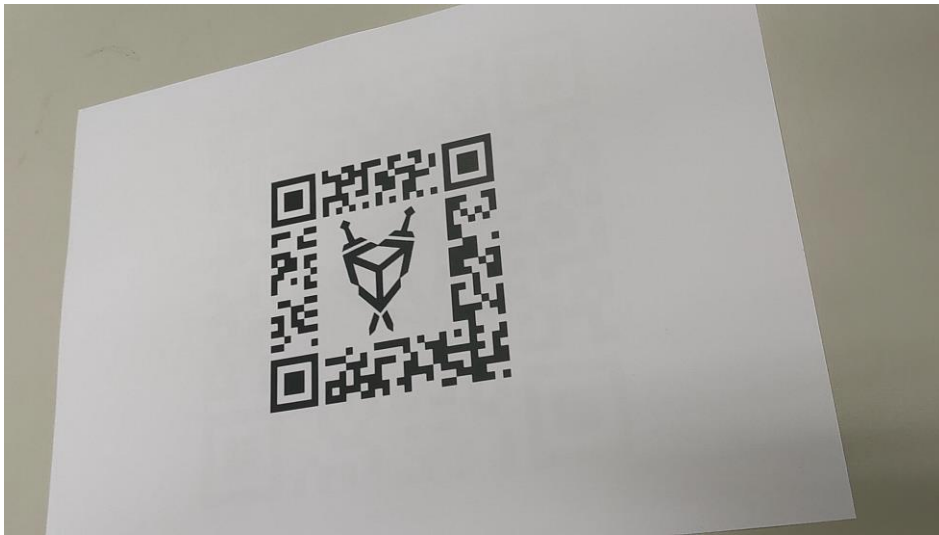
Part 4: Simple AR

- The simplest AR technic: Marker-based AR



Input Data

- A [video](#) and a template:



Input Data

- And a handsome guy:
 - We have cropped it into square for you.
 - You are also permitted to use your own image.



Assignment Description

- Part 1(4%)
 - Implement solution 1 or 2 for estimating homography.
 - Map 5 images of different people to the target surfaces (given in main.py). You can use whatever images you like. Include these images in your submission.
 - Include the function `solve_homography(u, v)` in your report.
- Part 2(3%)
 - Choose the unwarp region yourself.
 - The output image should contain the detectable QR code.
 - Include `the QR code` and the `decoded link` in your report.

Assignment Description

- Part 3(3%)
 - Unwarp the image to the **top view**.
 - Can you get the parallel bars from the top view?
 - If not, why? Discuss in your report.
- Part4(5%)
 - Find the pose between the video frames and the template.
 - Hints: feature matching, RANSAC, preprocess image and frame, etc.
 - This part is judged by the stability and **run time**.
 - The one whose execution time is more than **15 minutes** will get punishment.
 - 15 ~ 30 minutes: minus 1 points
 - 30 ~ 60 minutes: minus 2 points
 - Over 60 minutes: minus 3 points

Assignment Description

- For part1 to part3, we offer a template code:
 - You **cannot change** ***solve_homography*** and ***transform***
 - We will run `python3 main.py`
- For part4, we offer a template code for read video:
 - In this part, we **don't constrain the method** you use.
 - In the template code, the things you **cannot change** are “***template_path***” & “***video_path***”.
 - We will run `python3 part4.py <path>`
 - <path>: path to ar_marker.mp4
 - E.g., `python3 part4.py ./input/ar_marker.mp4`
- Notice:
 - If you are going to use opencv's feature tool, you should install “opencv-contrib” additionally.
 - As far as we know, version 3.4.2 can be used directly without any post-processing(we'll review your code with this version too).

Submission

- Code: main.py & part4.py(Python 3.5+)
- Input images for part 1 and part 4
 - Keep them in ./input
- Output images
 - part1.png, part2.png, part3.png
- Output video:
 - ar_video.mp4
- A PDF report, containing
 - Your student ID, name
 - Your answers to each part
 - Algorithm to the simple AR
 - Environment setting (package version)

Submission

- Compress all above files in a zip file named **StudentID.zip**
 - e.g. R07654321.zip
 - “./input” should be included
 - The unzipped file should be a folder named **StudentID** first.
 - If you want to upload a new version, add “_vx” at the end of original name, e.g. R07645321_v2.zip
- Submit to **sftp**
 - IP and port:
 - IP: 140.112.48.127
 - Port: 11000
 - User name: cv2019
 - Password: 9102vc
 - Platform suggestion:
 - MobaXterm
 - FileZilla
- Deadline: **12/3 11:00 pm**