KS SUPPLY CHAIN FUNDAMENTALS

09a Inventory Management and Lot Sizing

SS 2025

JKU
PLM Institute of
Production and
Logistics Management

Learning Goals

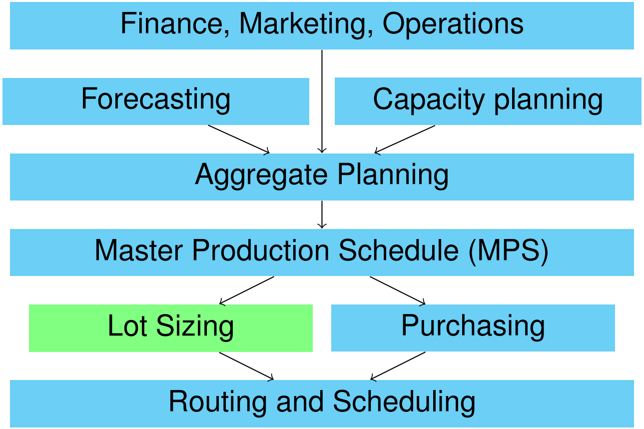
- Understand the relationship between Material Requirements Planning (MRP), lot sizing, and inventory management.
- Being able to apply the A-B-C approach.
- Understand the EOQ model.
- Being able to solve simple lot sizing problems by applying the Silver-Meal method.

Literature

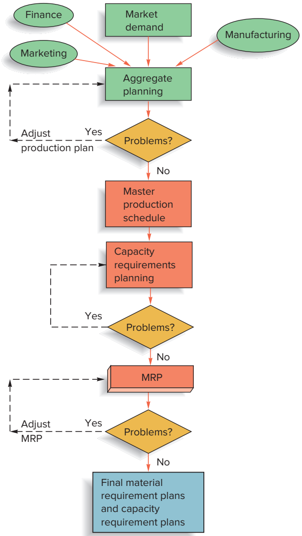
Stevenson, WJ: Operations Management. McGraw-Hill Education Ltd; latest edition - Chapters 12, 13, 14

Acknowledgements: icons are by fontawsome (latex) or by Pixelmeetup from www.flaticon.com

Classification



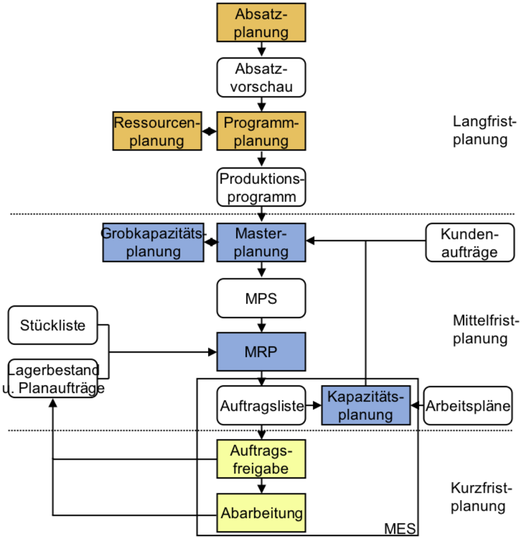
Manufacturing Resource Planning (MRP-II)*



*MRP (material requirements planning) is part of the MRP-II framework. Since they are abbreviated in the same way, manufacturing resource planning is abbreviated "MRP-II"

Stevenson (2017)

MRP-II (2)


Absatz-
planung
Absatz-
vorschau
Ressourcen-
planung
Programm-
planung
Produktions-
programm
Langfrist-
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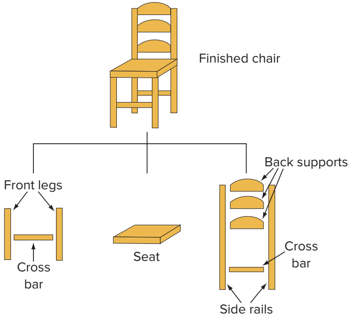
Grobkapazitäts-
planung
Master-
planung
Kunden-
aufträge
MPS
Stückliste
MRP
Mittelfrist-
planung
Lagerbestand
u. Planaufträge
Auftragsliste
Kapazitäts-
planung
Arbeitspläne
Auftrags-
freigabe
Abarbeitung
MES
Kurzfrist-
planung

MRP (Material Requirements Planning)

- ... follows after Master Production Scheduling (MPS).
- ... is a methodology generally used in production planning for assembled products (eg smartphones, cars, kitchen tables) to translate the requirements of the master production schedule (MPS) for the end items into time-phased requirements for subassemblies, component parts, and raw materials.
- In the context of MRP, **lot sizing** plays an important role.

Example: IKEA

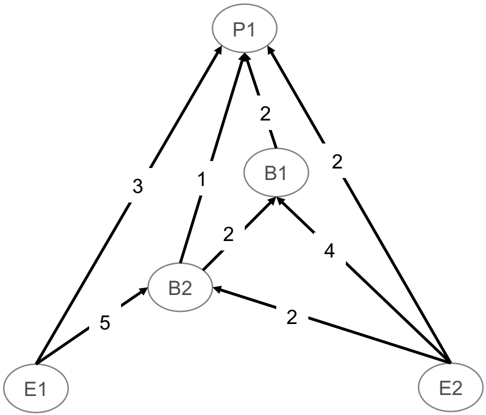
- Shopping at IKEA: You purchase a product consisting of one or multiple packages including building instructions.
- The building instructions list
  - all necessary parts (to be found inside the packages).
  - how to assemble the parts.
  - which parts are required for subassembly.
  - in which order the subassembly and final assembly have to be made (as well as its duration)



- **Operations management perspective:**
- Bill-of-materials (BOM)
- Material Requirements Planning (MRP) (when, how, how much, of what)
- If demand for a part is generated because it is needed to manufacture another part, we call this **dependent demand**, otherwise we call it **primary demand**.

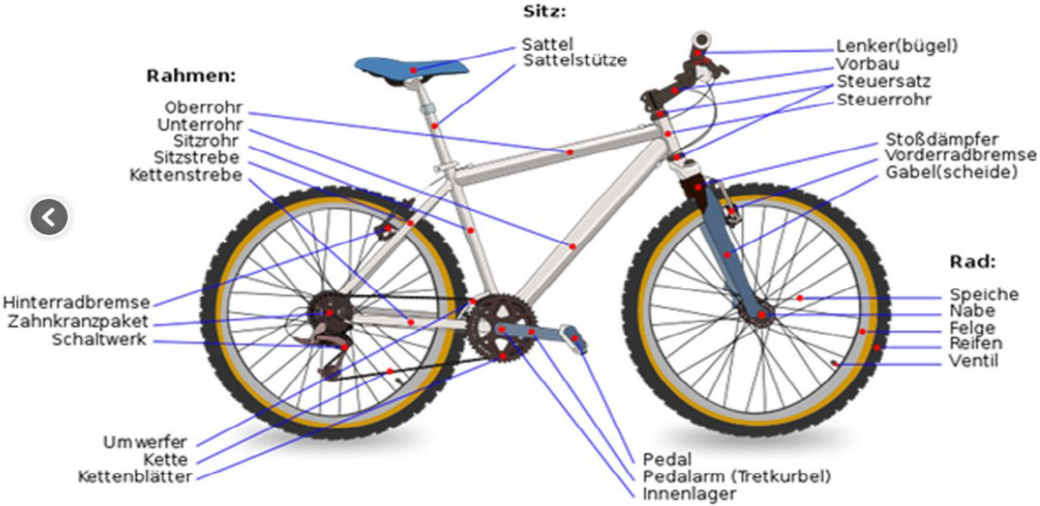
Gozinto graph

Term coined by Vasznoyi. He cited the fictitious Italian *Zepartzat Gozinto* - “the part that goes into”



P1: final product

Bike assembly



Lot size/Order quantity

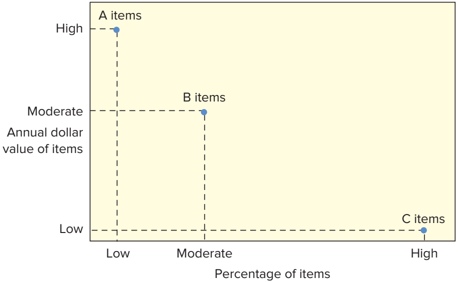
- The lot size (order quantity) that is to be **produced (ordered)** is an important figure in **MRP (inventory management).**
- There exists a trade-off between holding costs and setup costs (fixed costs per order or lot).

Classification of Goods : A-B-C approach

The **A-B-C approach** classifies inventory items according to some measure of importance, usually the annual euro value (it can also be used to, e.g., classify customers).

- **A items** Very important, (10-20% of items, 50-60% of total value)
- **B items** Moderately important
- **C items** Least important (50-60% of items, only 10-15% of total value)
  Example: screws. However, screws are not unimportant, as the assembly cannot be performed without them.

A-B-C Approach: Algorithm



Stevenson (2017)

1. For each item, multiply the yearly amount with the price per unit = “annual dollar value”
2. Sort parts by the calculated value in descending order
3. Classification into A, B, and C items

A-B-C Approach: Example (1)

Item Number	Annual Demand	×	Unit Cost	=	Annual Dollar Value
1	25		360		9.000
2	10		70		700
3	24		500		12.000
4	15		100		1500
5	7		70		490
6	10		1000		10.000
7	2		210		420
8	10		4000		40.000
9	80		10		800
10	5		200		1.000
Total					**75.910**

A-B-C Approach: Example (2)

Item Number	Annual Dollar Value	Classification	% of Items	cum. Percentage	Category
8	40.000	52.69%	10%	10%	A
3	12.000	15.81%	10%	20%	B
6	10.000	13.17%	10%	30%	B
1	9.000	11.86%	10%	40%	B
4	1.500	1.98%	10%	50%	C
10	1.000	1.32%	10%	60%	C
9	800	1.05%	10%	70%	C
2	700	0.92%	10%	80%	C
5	490	0.65%	10%	90%	C
7	420	0.55%	10%	100%	C

Economic Order Quantity (EOQ)

The EOQ formula can be used to identify a fixed order size that minimizes the total annual costs (inventory holding cost, setup costs, purchasing costs)

Assumptions of the basic model:

1. Only one product is involved.
2. Annual demand is known.
3. Demand is constant over the year.
4. Lead time is known and constant.
5. Each order is received in a single delivery.
6. There are no quantity discounts.

EOQ: Cost function and optimal order quantity

Q	Order quantity in units
H	Holding (carrying) cost per unit per year
S	Setup costs
D	Annual demand
c	Cost per unit

Cost function (TC)

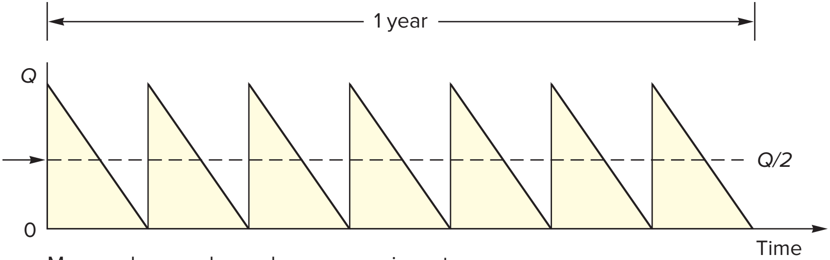
TC	=	setup cost	+	holding cost	+	(cost per unit)
TC	=	$\frac{D}{Q}S$	+	$H\frac{Q}{2}$	+	(Dc)

Optimal order quantity

$$Q^* = \sqrt{\frac{2DS}{H}}$$

EOQ: Explanation of the cost function (1)

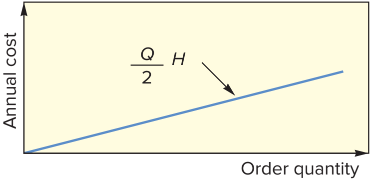
Average inventory level:

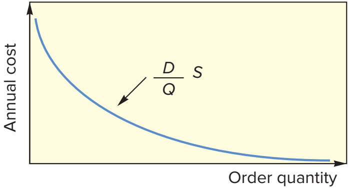


Many orders produce a low average inventory.

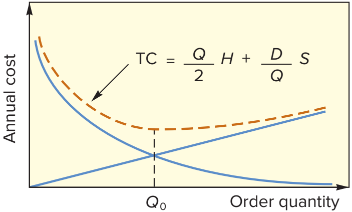
EOQ: Explanation of the cost function (2)

Holding costs and setup costs





Total costs (TC)



EOQ: Example

A local distributor of a national tire company wants to know their optimal order quantity:
D = 9600 tires
H = 16 EUR per tire and year
S = 75 EUR per order

$$Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \cdot 9600 \cdot 75}{16}} = 300 \text{ tires}$$

Number of orders per year: D/Q = 9600/300 = 32 orders

Time inbetween 2 orders: Q/D = 300/9600 = 1/32 years. Assuming 288 work days in a year, the time is 9 work days.

Total costs = $16 \cdot 300/2 + 75 \cdot 9600/300 = 4800$

Silver-Meal Heuristic: heuristic lot sizing

"Heuristic" means that the method usually computes a good solution following the defined rules but it does not necessarily produce the optimal (or best possible) solution.

Assumption: Demand is deterministic, but varying over time.

Idea: Summarize into one lot while average costs decrease.

Algorithm:

1. Set $t = 1$ (first period), set $j = 1$ (number of periods covered)
2. Compute $C_t(1)$ (cost of production in period t for period t)
3. **do**
   3.1 $j = j + 1$
   3.2 Calculate average cost in period t for j periods $C_t(j)$

   until $C_t(j) > C_t(j-1)$ or end of planning horizon
4. **if** $t + j - 2 < T$ **then**
   4.1 set lot $y_t = D_t + .. + D_{t+j-2}$
   4.2 set $t = t + j - 1$, set $j = 1$ and continue with step (2)
5. set $y_t = D_t + .. + D_T$

Silver-Meal Heuristic: heuristic lot sizing

$C(j)$ average setup and holding costs per period over j periods
$(D_1, \ldots, D_n)$ Demand for the next n periods

in period 1 for period 1: $C(1) = S$
in period 1 for periods 1,2: $C(2) = (S + HD_2)/2$
in period 1 for periods 1,2,3: $C(3) = (S + HD_2 + 2HD_3)/3$
In general: $C(j) = (S + HD_2 + 2HD_3 + \ldots + (j-1)HD_j)/j$

As soon as $C(j) > C(j-1)$, STOP and fix lot $y_1 = D_1 + D_2 + \ldots + D_{j-1}$, restart the process in period j.

Silver-Meal Heuristic - Example (1)

Computer Case Production Demand $D = (18, 30, 42, 5, 20)$; $n = 5$, Holding costs $H = 2$ EUR; Setup costs $S = 80$ EUR

Start in period 1

$C(1) = 80$

$C(2) = [80 + 2 \cdot 30]/2 = 70$

$C(3) = [80 + (2 \cdot 30) + (2 \cdot 2 \cdot 42)]/3 = 102,67$ STOP, as $C(3) > C(2)$, set

$y_1 = D_1 + D_2 = 18 + 30 = 48$

Start in period 3

$C(1) = 80$

$C(2) = [80 + (2 \cdot 5)]/2 = 45$

$C(3) = [80 + (2 \cdot 5) + (2 \cdot 2 \cdot 20)]/3 = 56.67$ STOP, as $C(3) > C(2)$; set

$y_3 = D_3 + D_4 = 42 + 5 = 47$

Silver-Meal Heuristic - Example (2)

Period 5 is the last period; $y_5 = D_5 = 20$

Lots $y = (48, 0, 47, 0, 20)$

Total costs $80 + (2 \cdot 30) + 80 + (2 \cdot 5) + 80 = 310$

Table:

Period t	1	2	3	4	5
Demand D_t	18	30	42	5	20
C(1) C(2) C(3) C(4)	80 70 **102,67 > 70**		80 45 **56,67 > 45**		80
Lot y_t	48	0	47	0	20

Summary

- In the context of MRP-II (manufacturing resource planning, also known as “Produktionsplanung und -steuerung (PPS)” in German) a master production schedule is created which is based on aggregate planning. It is followed by material requirements planning (MRP). This step includes lot sizing decisions (demands for raw material, components, etc. maybe ordered or produced in larger batches depending on setup and inventory holding costs)
- The A-B-C approach is used to distinguish between “important” and less important products.
- The EOQ (economic order quantity) formula is used to calculate the optimal order quantity of a product with constant demand.
- The Silver-Meal Heuristic can be used to support lot sizing decisions for items with deterministic but varying demand over time. Since it is a heuristic, the solution may not be optimal.