KS SUPPLY CHAIN FUNDAMENTALS

07 Linear Programming

SS 2025

PLM Institute of
Production and
Logistics Management

Learning Goals

- Describe what defines a linear program (LP).
- Solve a linear program with two variables using the graphical method.
- Solve simple linear programs with the help of Excel Solver.

- **include sustainability considerations into your decision making**

Literature

Stevenson, WJ: Operations Management. McGraw-Hill Education Ltd; latest edition - Chapter 19

Acknowledgements: icons are by fontawsome (latex) or by Pixelmeetup from www.flaticon.com

Linear Programming: a short history

- George B. Dantzig realized that a lot of problems in industry, business, and military could be described through linear equations and inequalities.
- This insight lead to the development of **linear programming** and the field of **Operations Research** (sometimes referred to as “Prescriptive Analytics”).
- In 1947 Dantzig developed the **Simplex method** for the solution of linear programs (LPs) which is still the standard method in all commercial software (eg Excel Solver, Google OR Tools) for LPs.
- *Dantzig also coined the statement saying that computers could only develop intelligence if a model was written and good algorithms were implemented.*

Many (almost all successful) companies have dedicated OR/Analytics departments, eg. Apple, Deutsche Bahn, Austrian Airlines, ... With the advancement of the digital transformation, data has become more and more important. No company will be able to stay competitive without the use of a data scientist/business analyst.

Linear Programming

A **linear Program** consists of

- Parameters (data)
- Decision variables
- Objective function
- Constraints

Relations between variables have to be linear.

Assumption:
two variables x_1 and x_2:

- $x_1 + x_2$ is linear
- $x_1 - x_2$ is linear
- $x_1 x_2$ is non-linear
- x_1 / x_2 is non-linear
- x_1^2 is non-linear

Example: YoungStars ☆

The small design company YoungStars was founded in 2018 by graduates of the JKU in Linz. The company produces two different types of products: deck chairs and patio tables. Both of these products are put together by using pre-built pieces made from wood from sustainable sources and painted in a variety of colors. The deck chair also requires some fabric to be attached. The fabric is made out of recycled PET bottles. The assembly area has a capacity of five hours per day and each product requires one hour to be put together. The painting area has a capacity of 12 hours per day. Each deck chair requires two hours to be painted and each patio table requires 3 hours. Attaching the fabric to the chair takes one hour and the capacity of that area is four hours.

YoungStars wants to maximize their profit and knows that with every deck chair they make a profit of 2000 euros and every table adds 1000 euros.

We assume that each produced deck chair and patio table will be sold to a customer.

How many deck chairs and patio tables should be produced by YoungStars every day?

On a side note: Is fabric made from PET bottles really sustainable?

The plastic bottle dilemma

- use of PET (polyethylene terephthalate) bottles reduces GHG by 75% compared to producing it from petroleum
- however, bottles are lost as “raw material” for new plastic bottles
- “The goal should be to establish a closed-loop circular economy for textiles’.’

https://thecircularlaboratory.com/recycled-polyester-and-the-plastic-bottle-dilemma

Back to the problem: how can we present it in a more structured way?

	deck chair	table	cap.
Assembly	1	1	5 h
Paint	2	3	12 h
Cloth	1	-	4 h
Profit	2	1	(t EUR)

$$\max 2x_1 + x_2 = z$$

subject to:

$$x_1 + x_2 \leq 5$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1 \leq 4$$
$$x_1, x_2 \geq 0$$

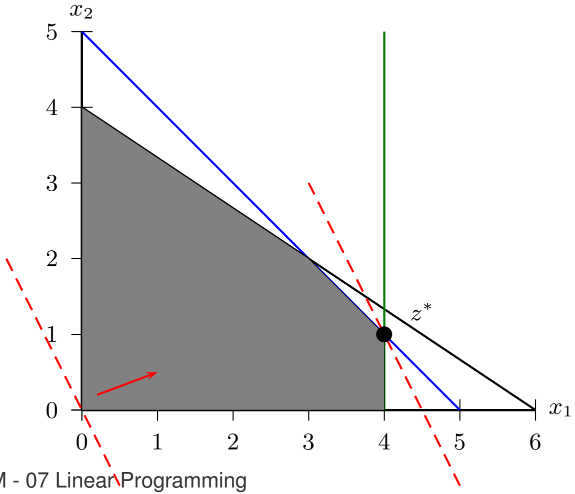
Graphical solution

Optimal solution: $x_1 = 4$, $x_2 = 1$, $z^* = 9$

$$\max 2x_1 + x_2 = z$$

subject to:

$$x_1 + x_2 \leq 5$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1 \leq 4$$
$$x_1, x_2 \geq 0$$



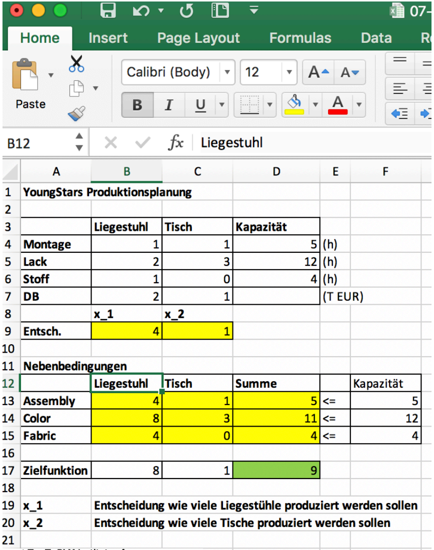
Graphic solution: how to

1. **Plot the constraints**
   for example: $2x_1 + 3x_2 \leq 12$
   Calculate the intersections with the x- and y-axis:
   $x_2 = 0 \rightarrow x_1 = 12/2 = 6$
   $x_1 = 0 \rightarrow x_2 = 12/3 = 4$
2. **Plot the feasible region**
3. **Plot the objective function**
   $2x_1 + x_2 = 0$
   $2x_1 = -x_2 \rightarrow x_2 = -2x_1$
   The slope of the line is: -2
4. **Determine the optimal solution**
   Intersection of lines: $x_1 = 4$ und $x_1 + x_2 = 5$
   $4 + x_2 = 5 \rightarrow x_2 = 1$

Graphical solution

- Only possible with two variables.
- Realistic problems consider several thousands of variables.
- Realistic problems have to be solved using tailored software tools.
- Tool that we will use here: Excel Solver.
- Only works for comparatively small problems. More complex problems with even more decision variables require specialized software (eg IBM ILOG CPLEX or Gurobi $\rightarrow$ Major Operations, Transport and Supply Chain Management.)

Solving LP models using Excel Solver (1)

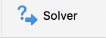


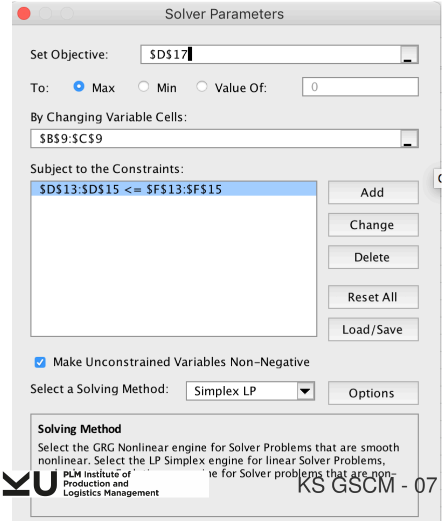
	A	B	C	D	E	F
1	**YoungStars Produktionsplanung**					
2						
3		**Liegestuhl**	**Tisch**	**Kapazität**		
4	**Montage**	1	1	5	(h)	
5	**Lack**	2	3	12	(h)	
6	**Stoff**	1	0	4	(h)	
7	**DB**	2	1		(T EUR)	
8		**x_1**	**x_2**			
9	**Entsch.**	4	1			
10						
11	**Nebenbedingungen**					
12		**Liegestuhl**	**Tisch**	**Summe**		Kapazität
13	**Assembly**	4	1	5	<=	5
14	**Color**	8	3	11	<=	12
15	**Fabric**	4	0	4	<=	4
16						
17	**Zielfunktion**	8	1	9		
18						
19	**x_1**	**Entscheidung wie viele Liegestühle produziert werden sollen**				
20	**x_2**	**Entscheidung wie viele Tische produziert werden sollen**				
21						

Excel formulas:
B13: =B4*B$9
C13: =C4*C$9
D13: =B13+C13 $(x_1 + x_2)$

Implement additional constraints and the objective function in the same way.

Solving LP models using Excel solver (2)

Calling the Excel-Solver add-in: Data $\rightarrow$ Solver 



Objective function: \$D\$17

Variable cells: \$B\$9:\$C\$9 (decision variables)

Constraints:
\$D\$13:\$D\$15 $\leq$ \$F\$13:\$F\$15
(Constraints can also be entered one by one)

Method: Simplex LP (for linear programs)

Example

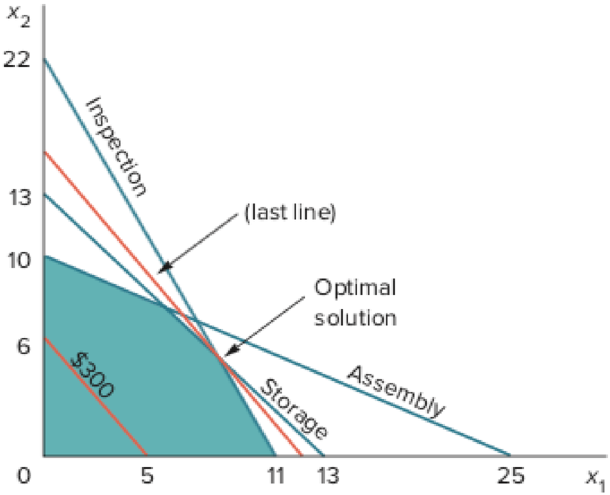
A company that assembles computers and computer equipment is about to start production of two new types of microcomputers. Each type will require assembly time, inspection time, and storage space. The amounts of each of these resources that can be devoted to the production of two new types of the microcomputers is limited. The manager of the firm would like to determine the quantity of each microcomputer to produce in order to maximize the profit generated by sales of these microcomputers. After meetings with the departments for design and production, the following information was obtained:

	Typ 1	**Typ 2**
Profit per unit	\$60	\$50
Assembly time per unit	4 hours	10 hours
Inspection time per unit	2 hours	1 hour
Storage space per unit	3 m^3	3 m^3

The manager also has acquired information on the availability of company resources. These amounts are 100 hours for assembly, 22 hours for inspection, and 39 m^3 of storage space.

The manager met with the firm's marketing manager and learned that demand for the microcomputers was such that whatever combination of these two types of microcomputer is produced, all of the output can be sold.

Example - Solution



Summary and outlook

- Many problems along the supply chain can be formulated as linear programs (eg production planning, location planning, routing)
- **Minimizing GHG emissions can be formulated as an objective function**
- LPs with two variables can be solved graphically.
- LPs with up to 200 variables can be solved using Excel Solver.