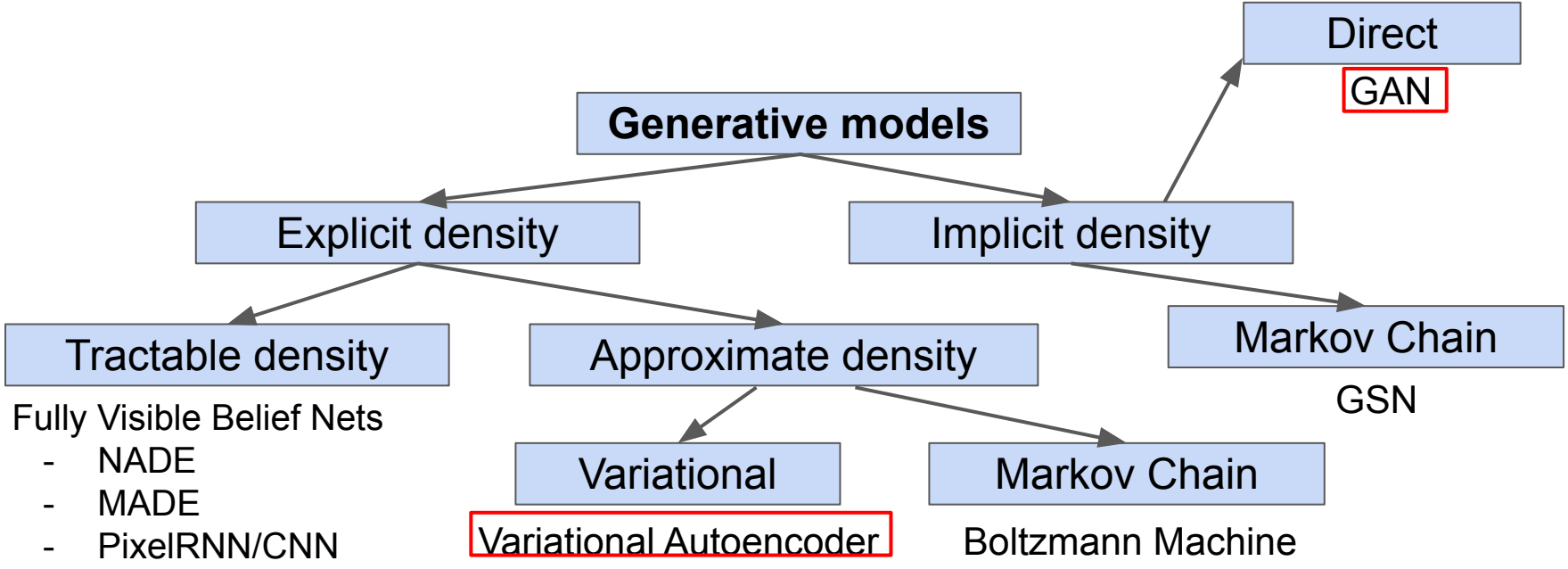
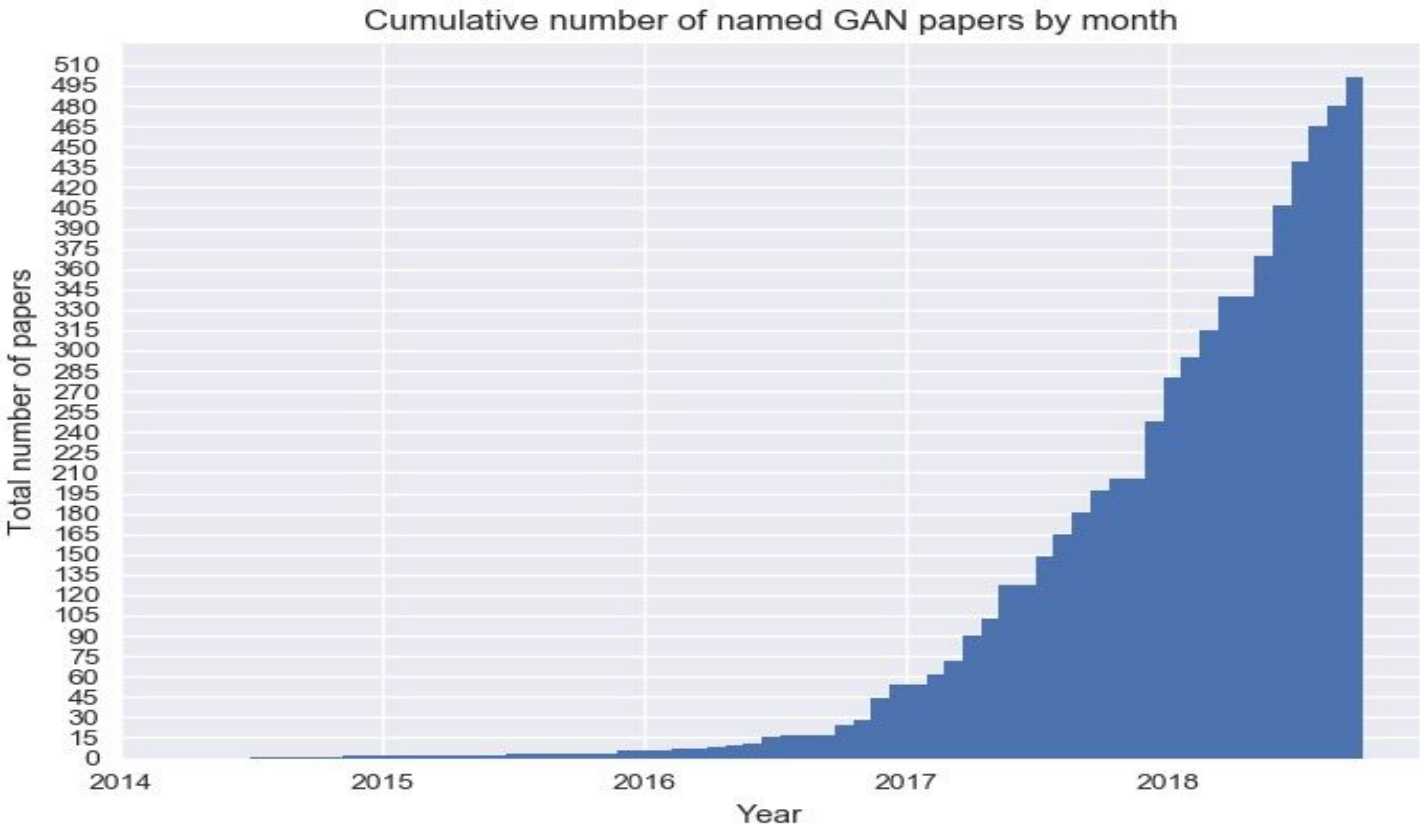
Deep generative methods

Stéphane Lathuilière
Assistant Professor
Télécom Paris

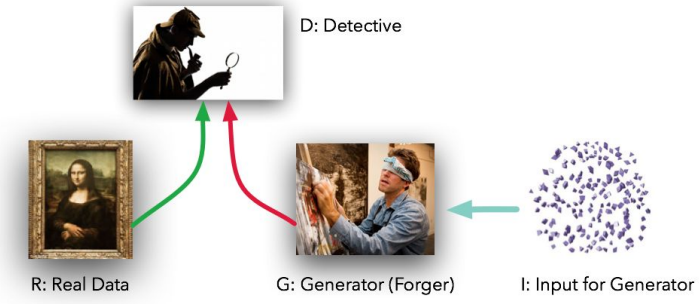
Taxonomy of Generative Models



GAN zoo



https://github.com/hindupuravinash/the-gan-zoo



Generative Adversarial Networks (GAN)

Generative Adversarial Networks

Problem: Want to sample from complex, high-dimensional training distribution. No direct way to do this!

Solution: Sample from a simple distribution, e.g. random noise. Learn transformation to training distribution.

Q: What can we use to represent this complex transformation?

Ian Goodfellow et al., “Generative Adversarial Nets”, NIPS 2014

Generative Adversarial Networks

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Solution: Sample from a simple distribution, e.g. random noise. Learn transformation to training distribution.

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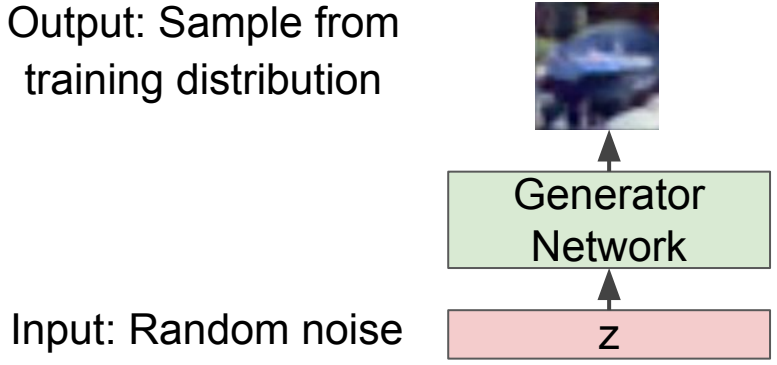
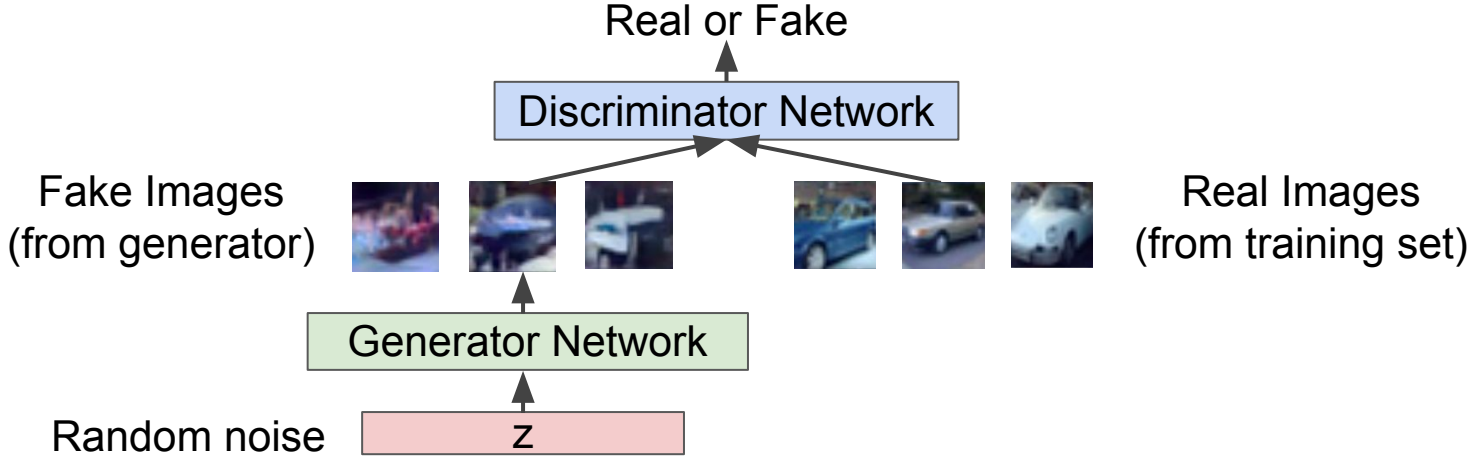
A: A neural network!



Training GANs: Two-player game

Generator network: try to fool the discriminator by generating real-looking images

Discriminator network: try to distinguish between real and fake images



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Train jointly in **minimax game**

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

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Discriminator output for real data x

Discriminator output for generated fake data G(z)

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Discriminator output for real data x

Discriminator output for generated fake data G(z)

- Discriminator (θ_d) wants to **maximize objective** such that $D(x)$ is close to 1 (real) and $D(G(z))$ is close to 0 (fake)
- Generator (θ_g) wants to **minimize objective** such that $D(G(z))$ is close to 1 (discriminator is fooled into thinking generated $G(z)$ is real)

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Alternate between:

1. **Gradient ascent** on discriminator

$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

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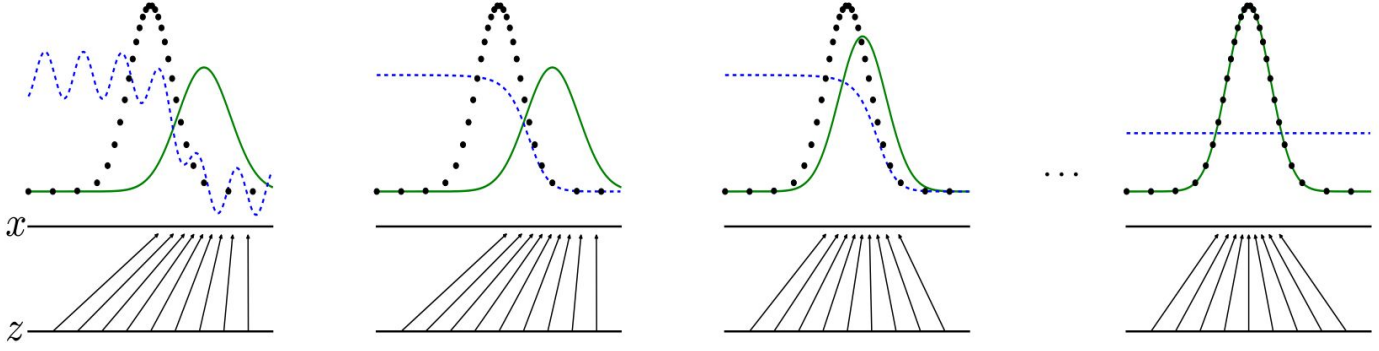

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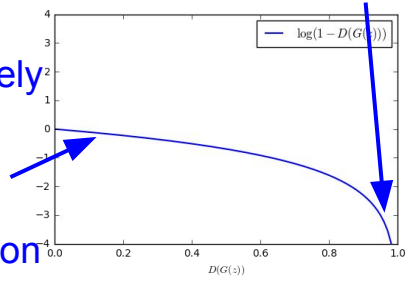
In practice, optimizing this generator objective does not work well!

Gradient signal dominated by region where sample is already good

When sample is likely fake, want to learn from it to improve generator. But gradient in this region is relatively flat!



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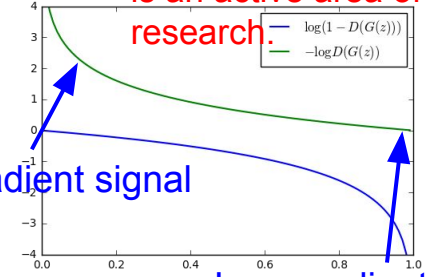
2. **Instead: Gradient ascent** on generator, different objective

$$\max_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(D_{\theta_d}(G_{\theta_g}(z)))$$

Instead of minimizing likelihood of discriminator being correct, now maximize likelihood of discriminator being wrong.
Same objective of fooling discriminator, but now higher gradient signal for bad samples => works much better! Standard in practice.

Aside: Jointly training two networks is challenging, can be unstable. Choosing objectives with better loss landscapes helps training, is an active area of research.



High gradient signal

Low gradient signal

GAN Training Algorithm

- Update discriminator
  - Repeat for k steps:
    - Sample mini-batch of noise samples $z_1, \dots, z_m$ and mini-batch of real samples $x_1, \dots, x_m$
    - Update parameters of D by stochastic gradient ascent on

$$\frac{1}{m}\sum_{m}[\log D(x_m) + \log(1 - D(G(z_m)))]$$

- Update generator
  - Sample mini-batch of noise samples $z_1, \dots, z_m$
  - Update parameters of G by stochastic gradient ascent on

$$\frac{1}{m}\sum_{m}\log D(G(z_m))$$

GAN Results

MNIST digits

Toronto Face Dataset

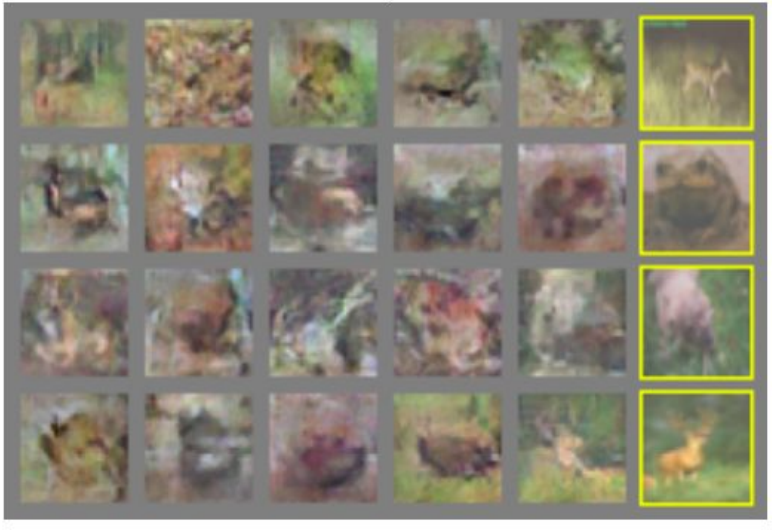
Nearest real image for sample to the left

I. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, Y. Bengio, Generative adversarial nets, NIPS 2014

GAN Results

CIFAR-10 (FC networks)

CIFAR-10 (conv networks)



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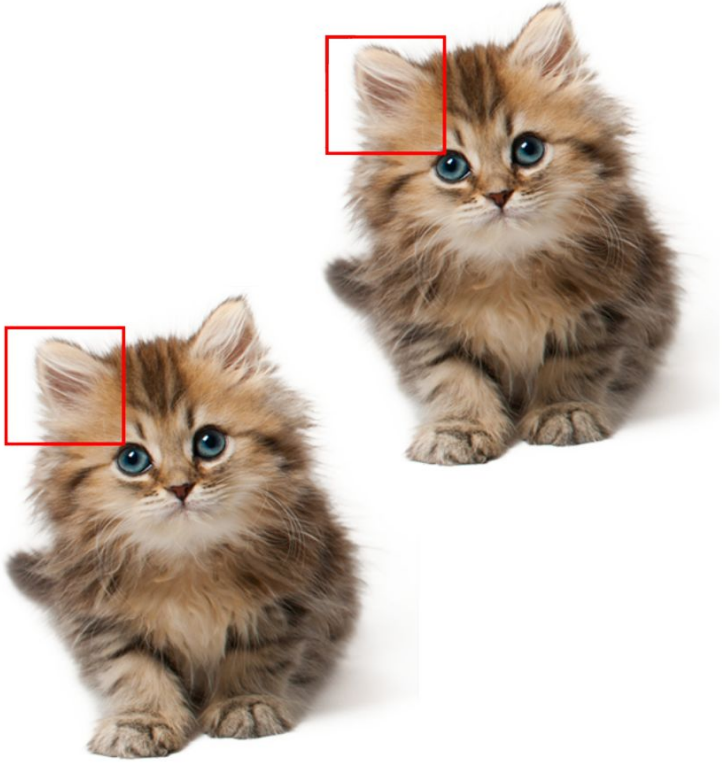
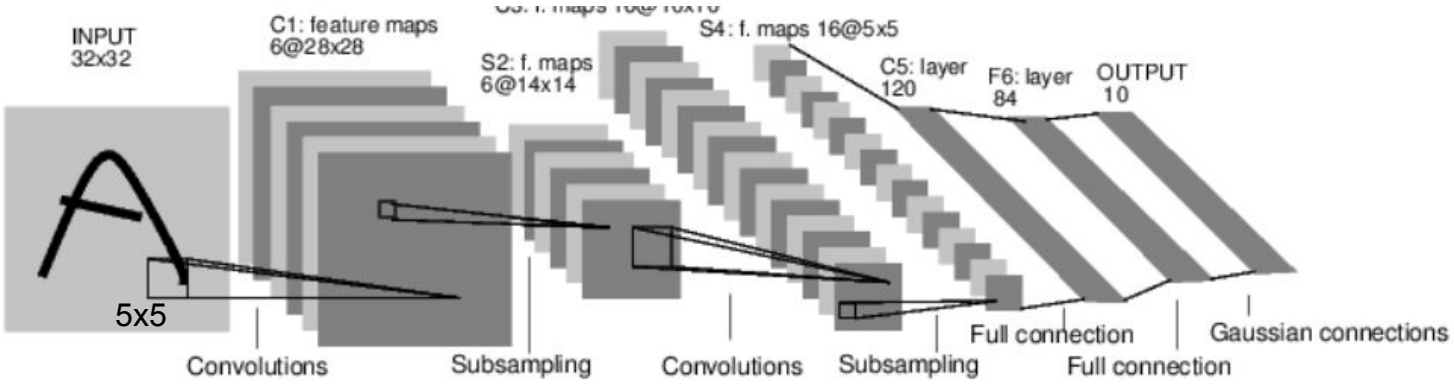
How to improve GAN?

Let’s go to basics!

Layers: convolution



Layers: convolution

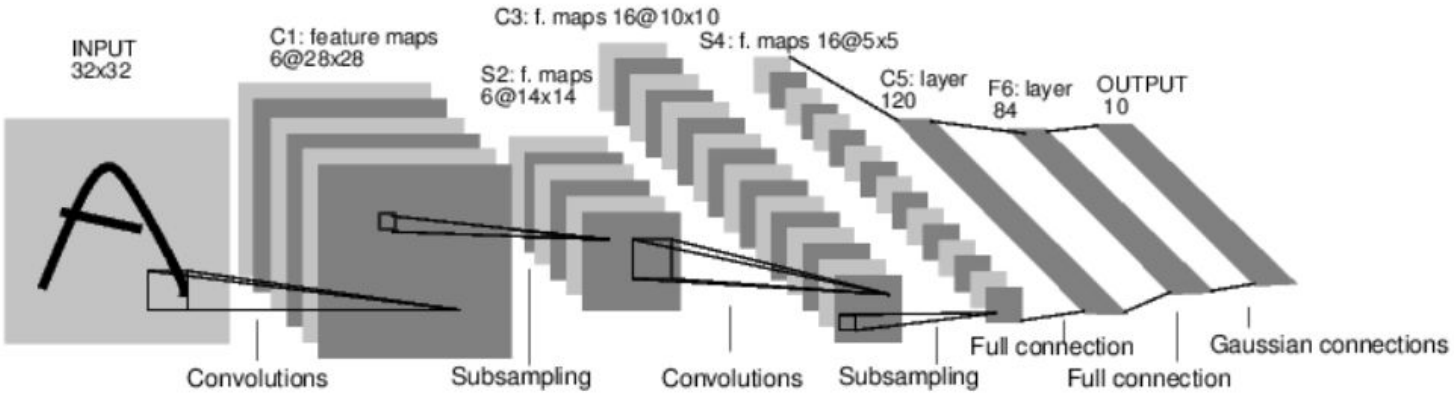


First layer:

- Without weight sharing:
- with weight sharing:

Y. LeCun, et al.: Gradient-Based Learning Applied to Document Recognition,1998

Layers: convolution



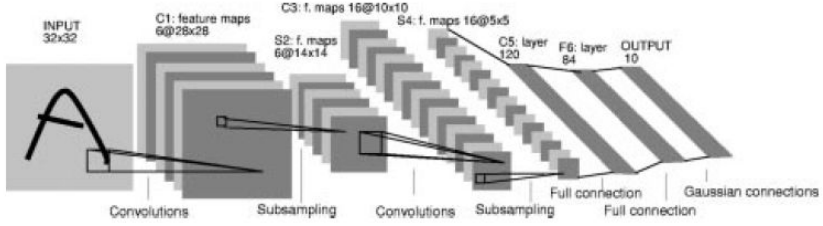
First layer:

- Without weight sharing: 5x5x6x28x28 = 117600
- with weight sharing: 5x5x6 = 150

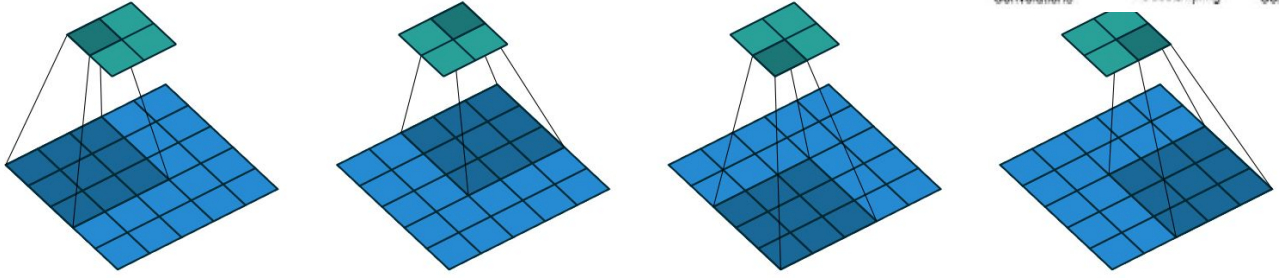
Y. LeCun, et al.: Gradient-Based Learning Applied to Document Recognition,1998

“De-convolution”

Architecture:



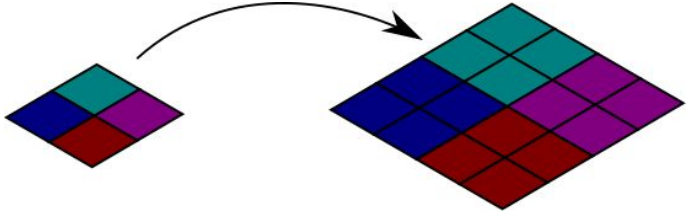
- Encoder: ConvNet architecture



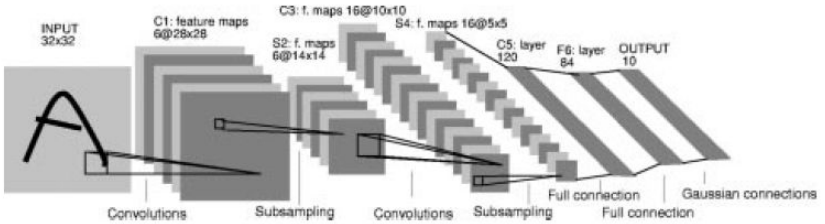
Convolving a 3 × 3 kernel over a 5 × 5 input using 2 × 2 strides

- Decoder: How to go from a vector to an image? How to “deconvolve”?

Up-sampling

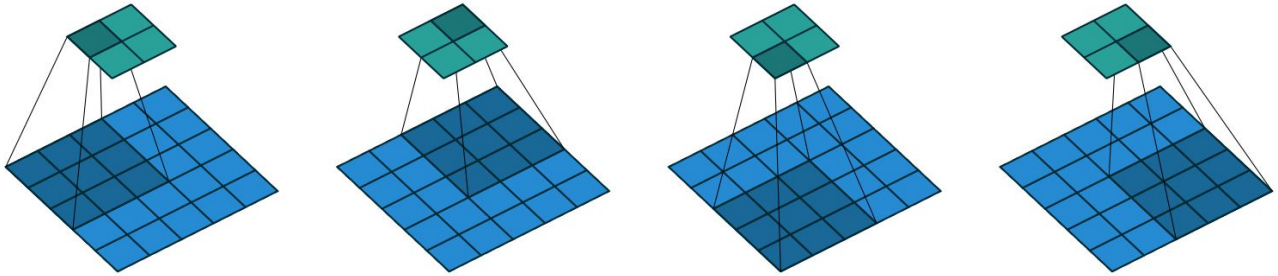


“De-convolution”



Architecture:

- Encoder: ConvNet architecture



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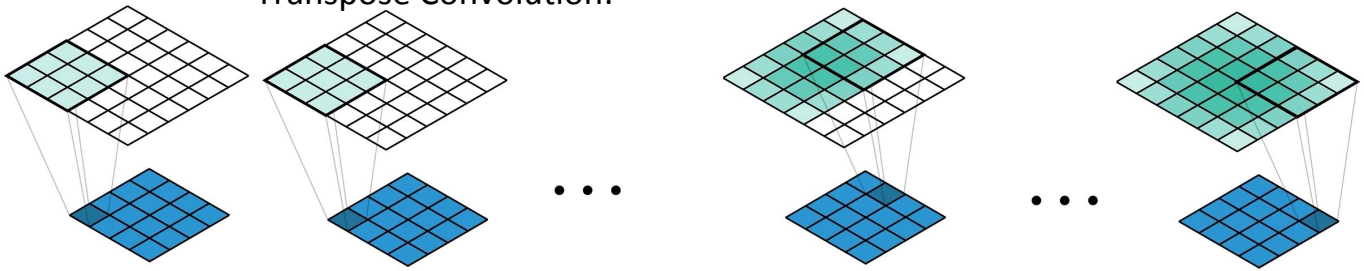
Transpose Convolution:



A guide to convolution arithmetic for deep learning ,Vincent Dumoulin, Francesco Visin

Gradient descent

- Optimization problem: $\min_\theta = L_{tot}(\theta)$

- Gradient Descent:

  Until convergence: $$\theta_{n+1} = \theta_n - \gamma \nabla L(\theta_n)$$

- Non convex problem -> Gradient descent does not work!

Gradient descent

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- Gradient Descent:

  Until convergence: $\theta_{n+1} = \theta_n - \gamma \nabla L(\theta_n)$

- Non convex problem -> Gradient descent does not work!
- Stochastic Gradient Descent:
  - We split the training in mini-batch: $L_{tot}(\theta) = \sum_b L_b(\theta)$

    Until convergence:

    For each batch b: $\theta_{n+1} = \theta_n - \gamma \nabla L_b(\theta_n)$

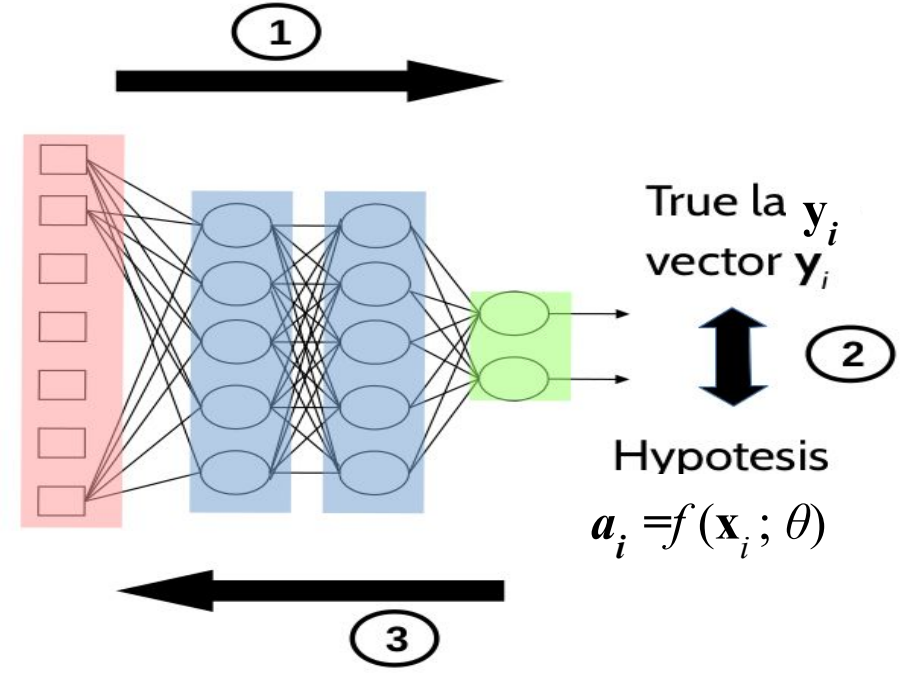
- How to compute the gradient efficiently?

1986 – Backpropagation

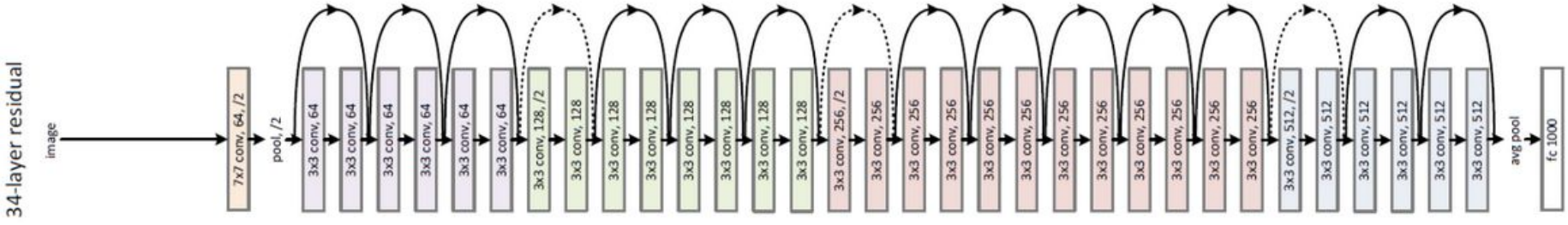
- Backpropagation revitalized the field
- Learning MLP for complicated functions can be solved
- Efficient algorithm which processes “large” training sets
- Allowed for complicated neural network architectures
- Today backpropagation is still at the core of neural network training

Backpropagation in a Nutshell

1. **Forward propagation**: sum inputs, produce activations, feed-forward
2. **Error estimation**.
3. **Back propagate** the error signal and used it to update weights

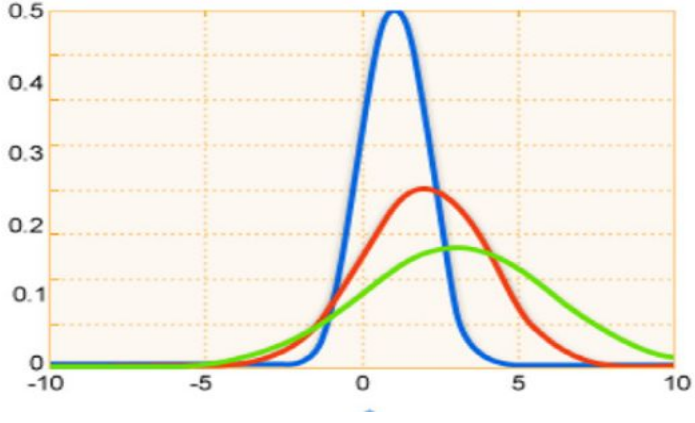


Layers: Batch-normalization



Parameter update:

$$\theta_{n+1} = \theta_n - \gamma \nabla L_b(\theta_n)$$



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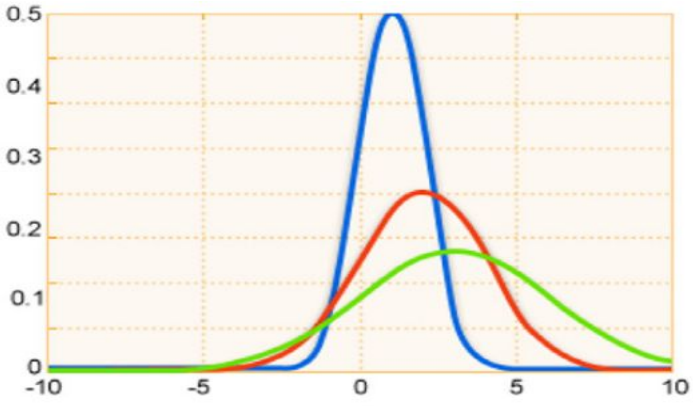

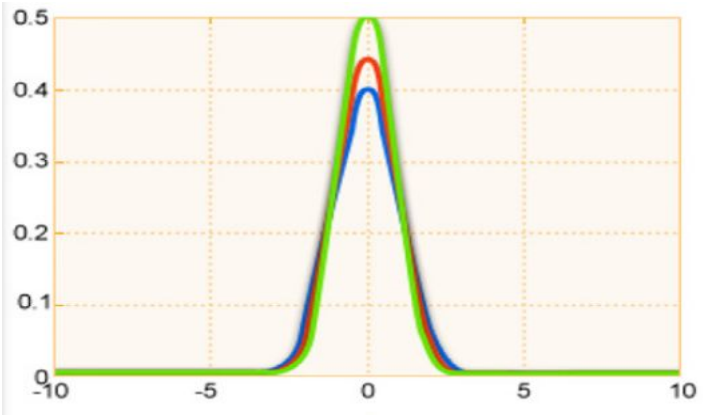

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β
Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift
Sergey Ioffe, Christian Szegedy

Layers: Batch-normalization

Advantages:

- Faster training
- Better optimization (lower final loss)
- Better generalization
- Specific usages in:
  - few-shot learning
  - domain adaptation
  - generation

Layers: Other normalizations

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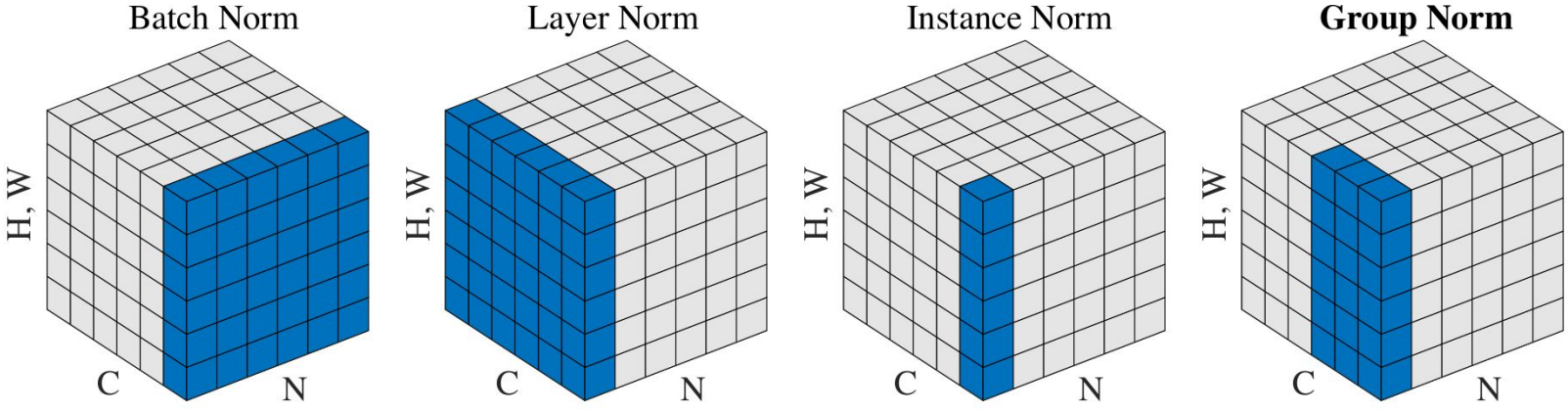
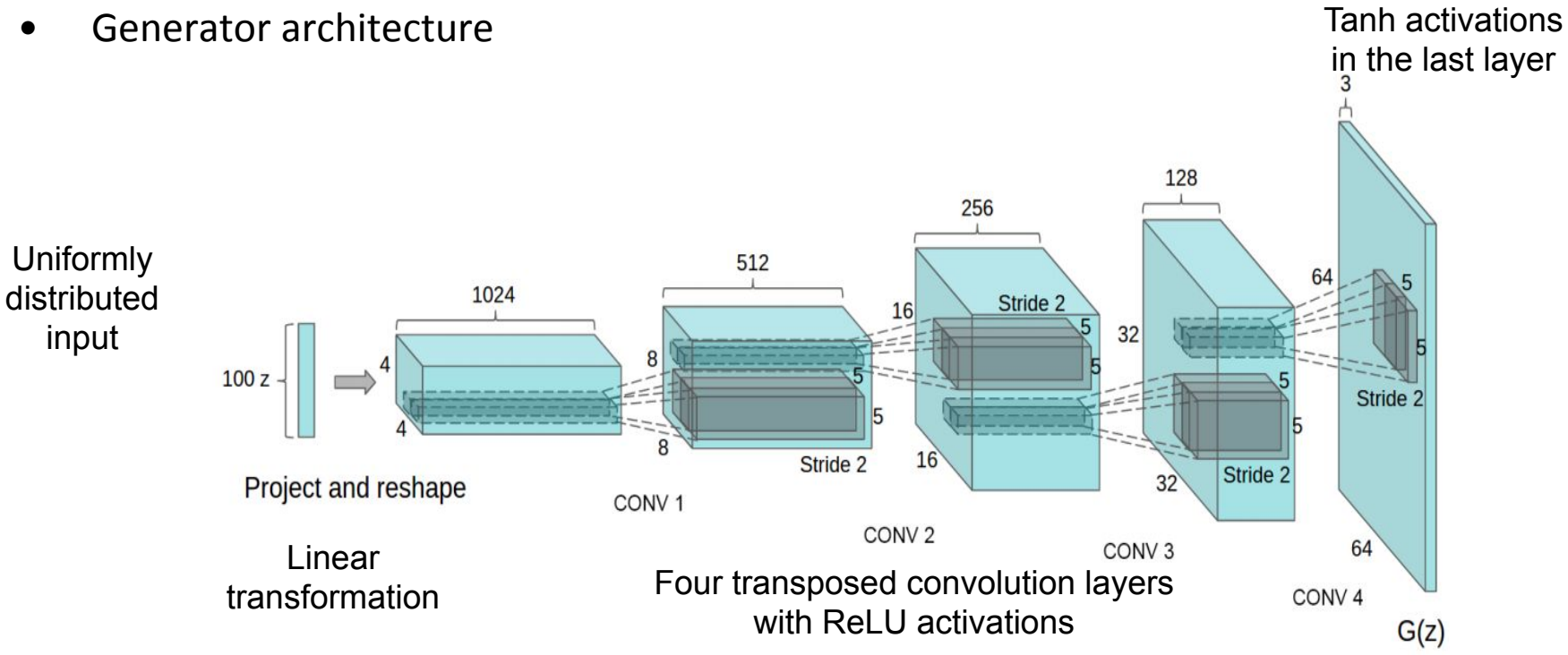


Group Normalization,Yuxin Wu, Kaiming He, ECCV 2018

DCGAN

- Propose principles for designing convolutional architectures for GANs, first model for large resolution image generation
- Generator architecture



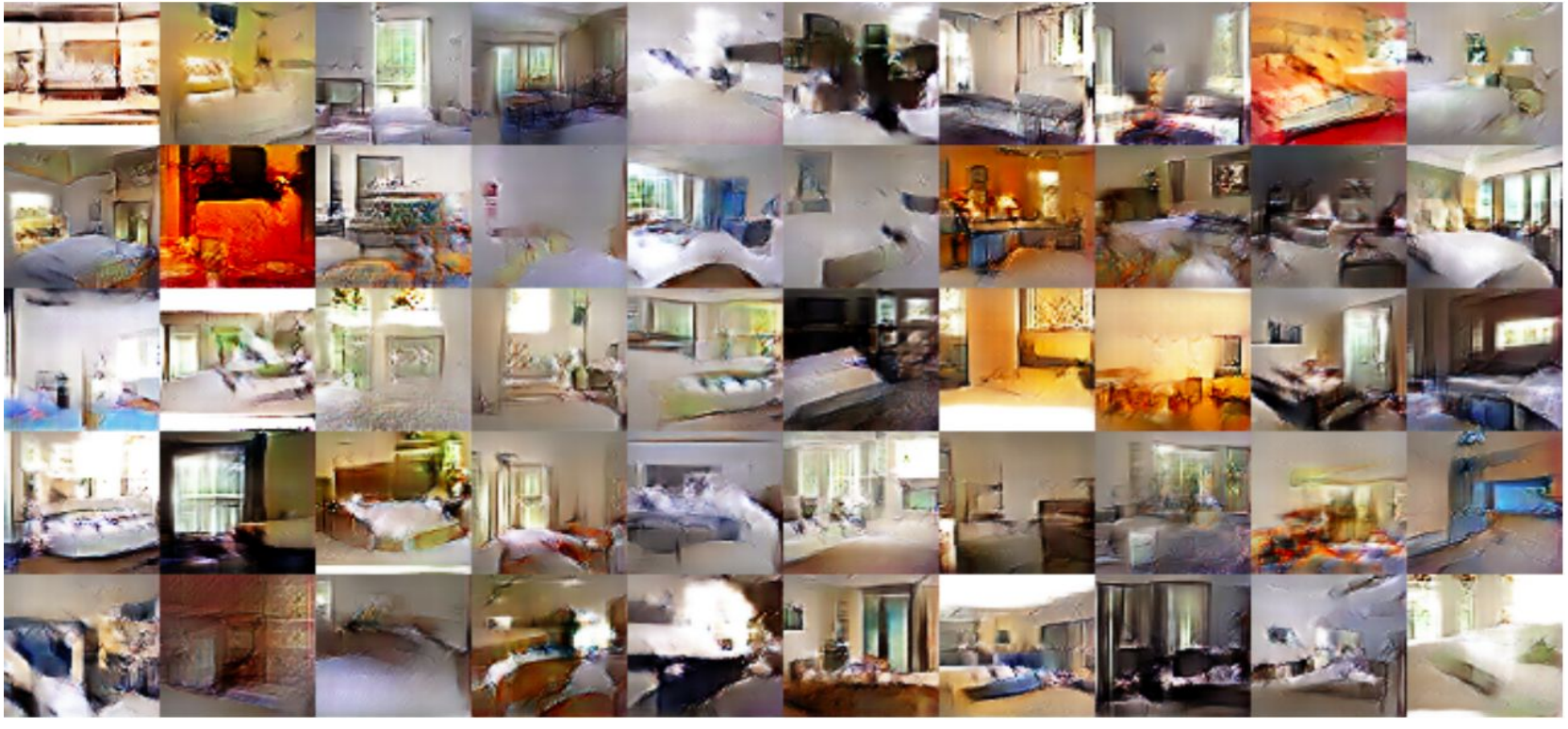
A. Radford, L. Metz, S. Chintala, Unsupervised representation learning with deep convolutional generative adversarial networks, ICLR 2016

DCGAN

- Discriminator architecture
  - No pooling, only strided convolutions
  - Use Leaky ReLU activations (sparse gradients cause problems for training)
  - Use only one FC layer before the softmax output
  - Use batch normalization after most layers
    (in the generator also)

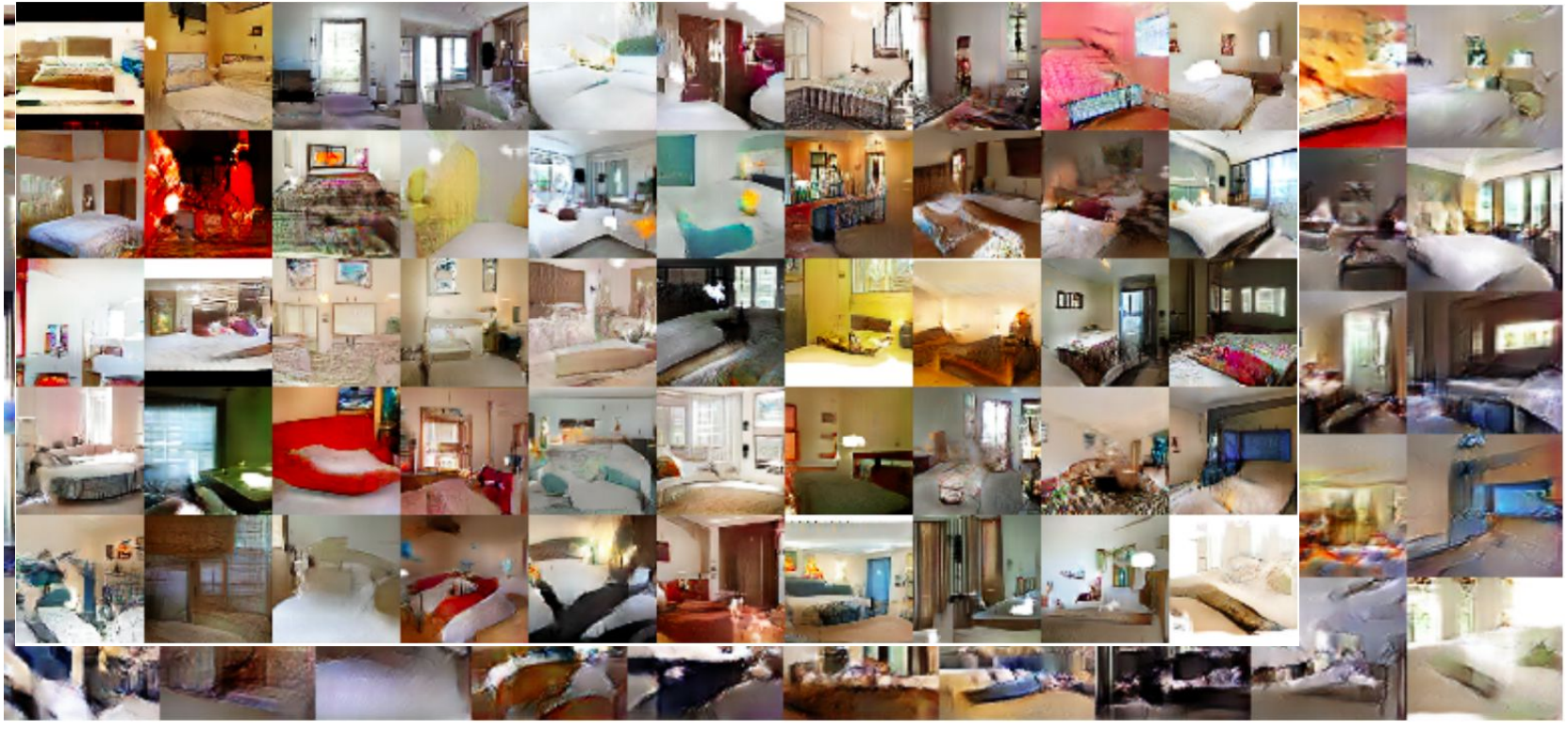
DCGAN results

Generated bedrooms after one epoch



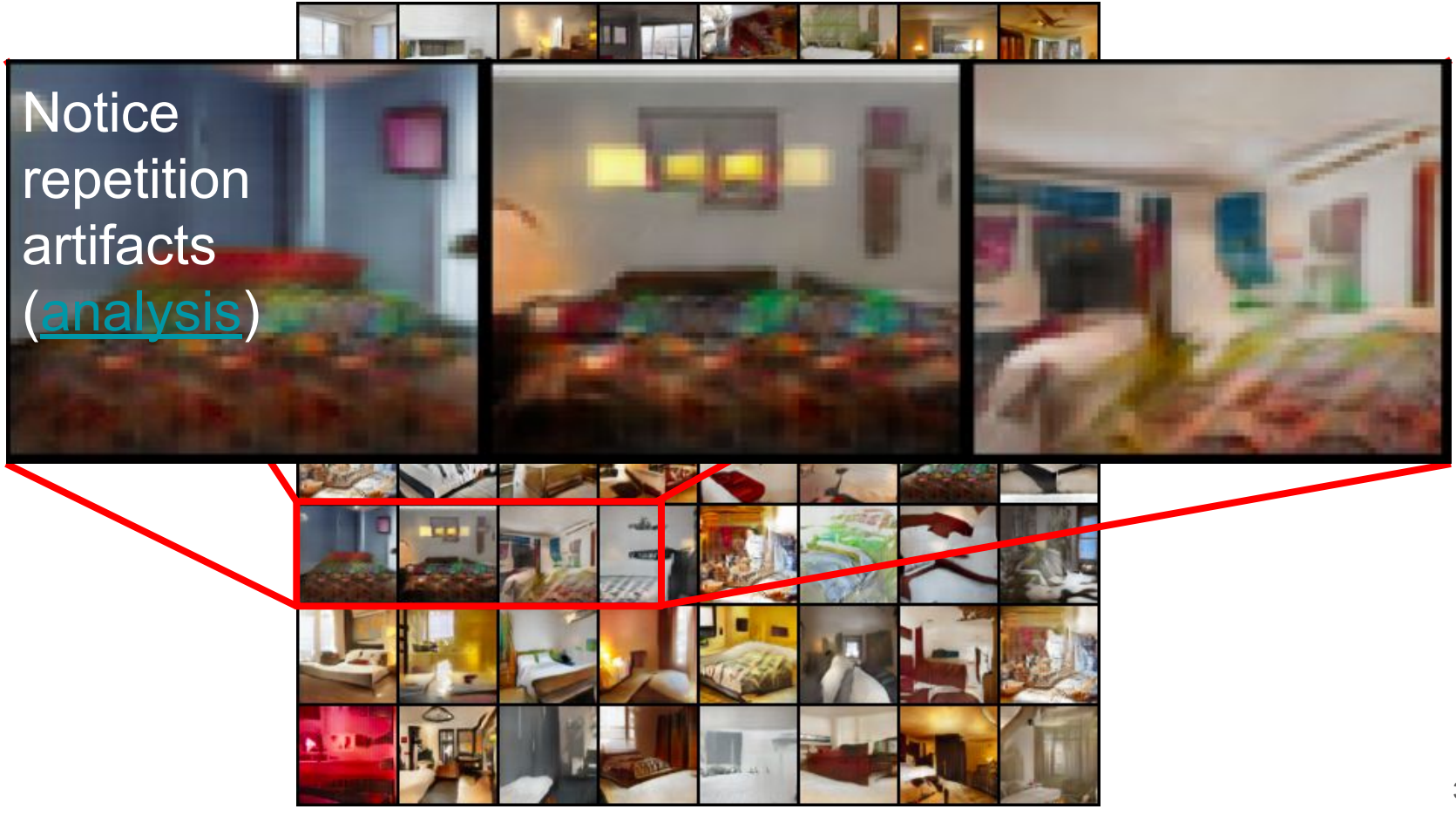
DCGAN results

Generated bedrooms after five epochs

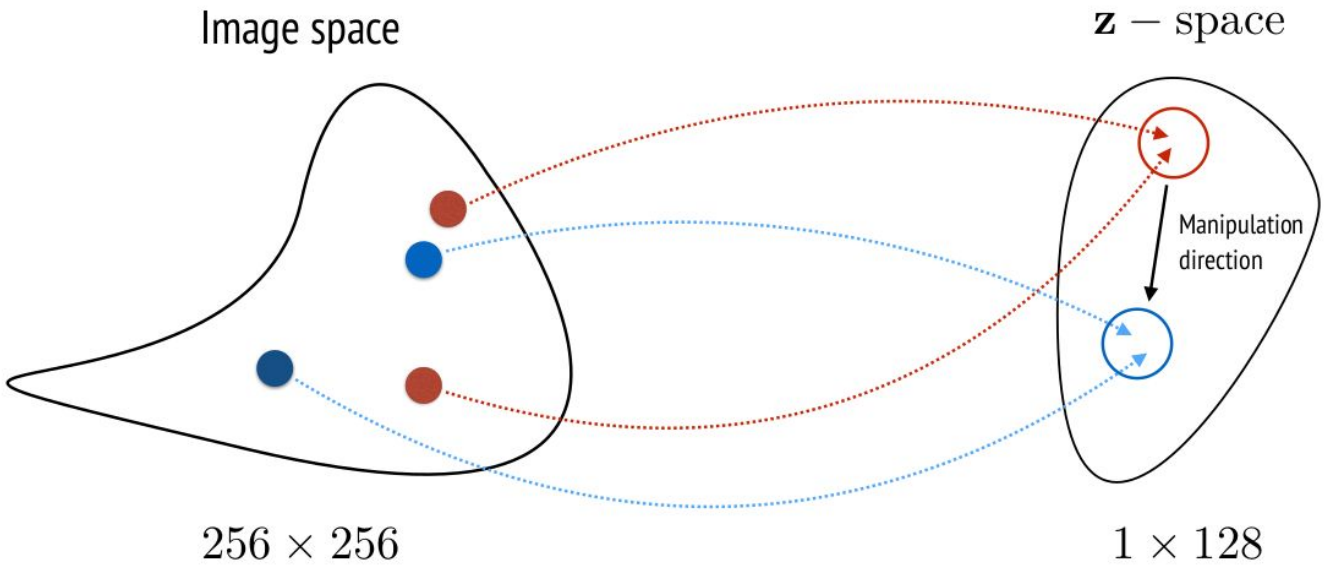


DCGAN results

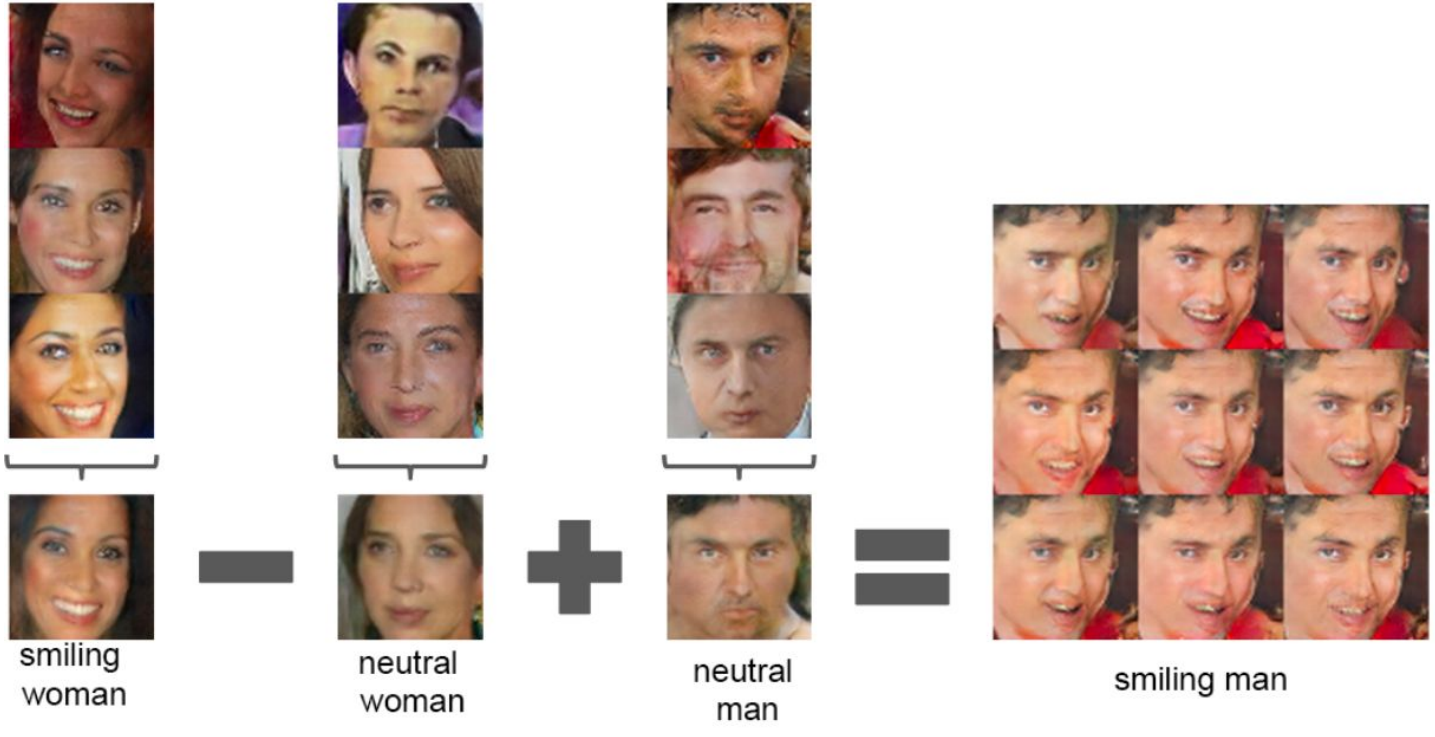
Generated bedrooms from reference implementation



DCGAN results

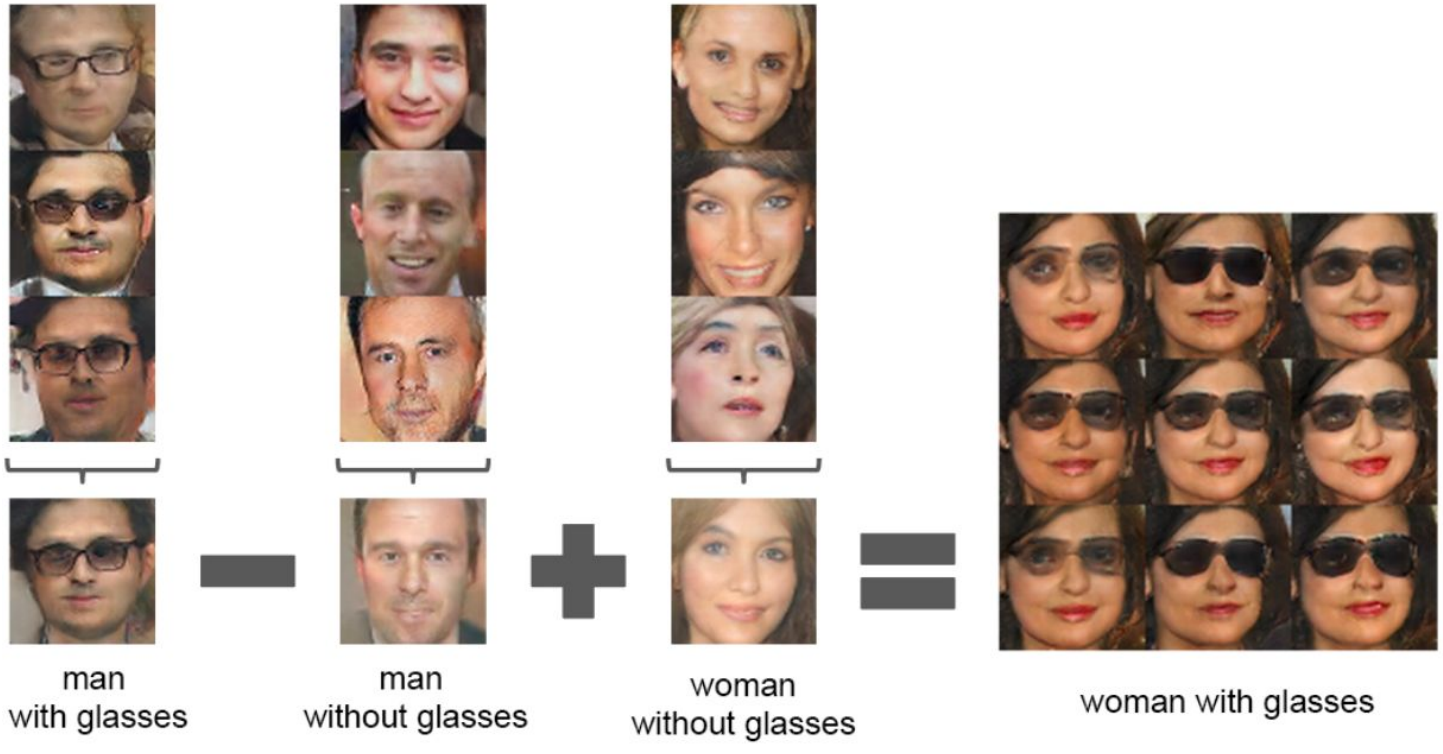


DCGAN results



For each column, the Z vectors of samples are averaged. Arithmetic was performed on the mean vectors creating a new vector Y . The center sample on the right hand side is produce by feeding Y as input to the generator. To demonstrate the interpolation capabilities of the generator, uniform noise sampled with scale +-0.25 was added to Y to produce the 8 other samples

DCGAN results



First model to bring the concept of **disentanglement**.

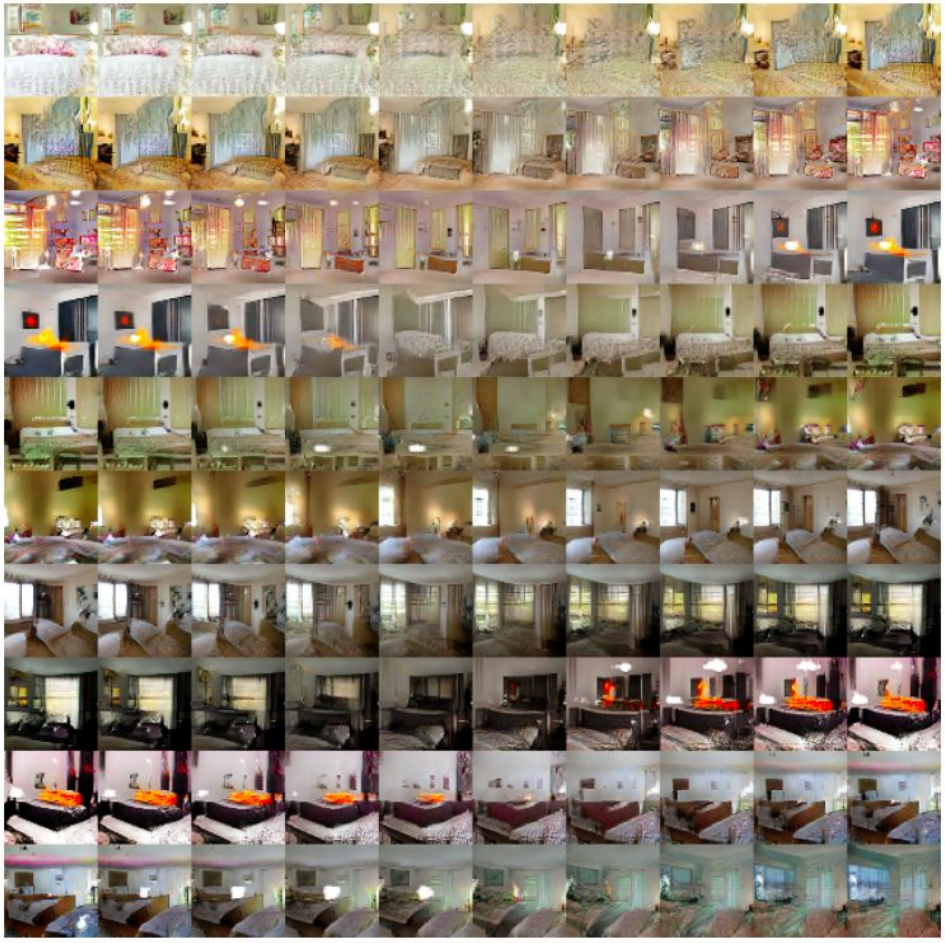
DCGAN results

- 

A ”turn” vector was created from four averaged samples of faces looking left vs looking right. By adding interpolations along this axis to random samples we were able to reliably transform their pose.

DCGAN results

Interpolation between different points in the z space



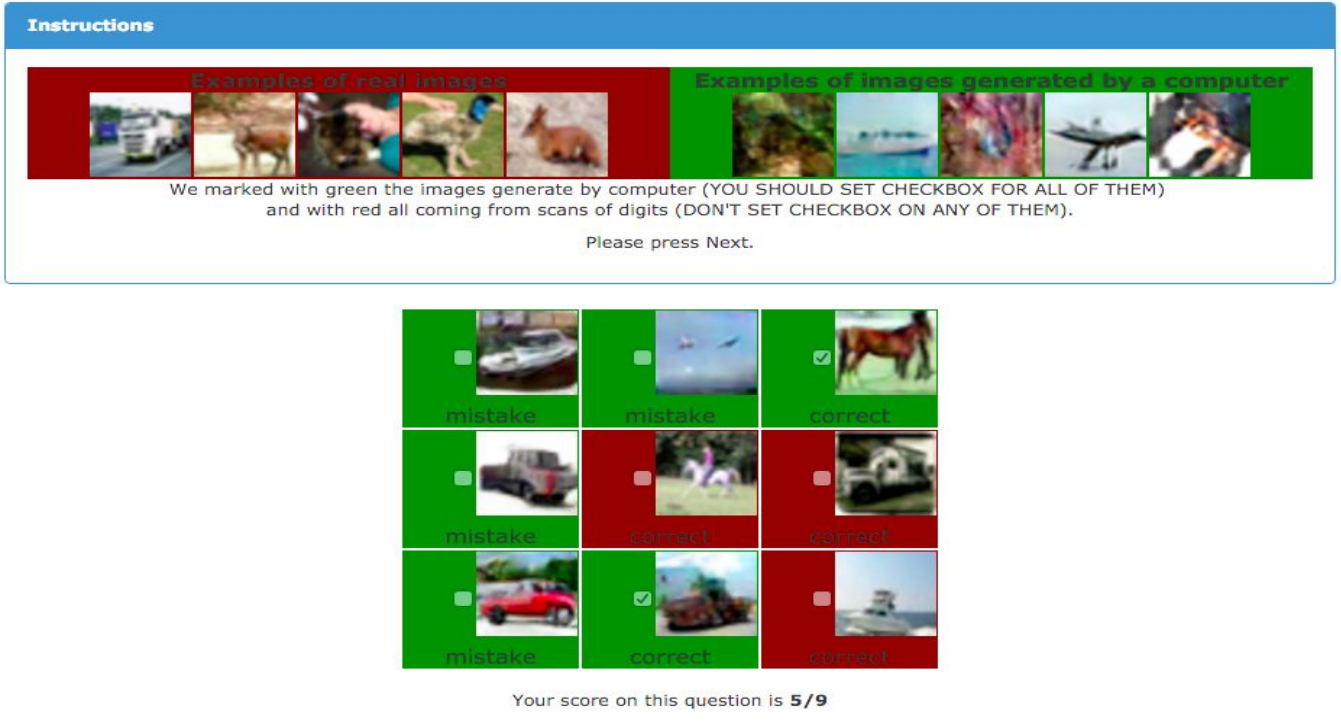
Evaluate GAN performance

- Showing pictures of samples is not enough, especially for simpler datasets like MNIST, CIFAR, faces, bedrooms, etc.
- We cannot directly compute the likelihoods of high-dimensional samples (real or generated) or compare their distributions
- Many GAN approaches claim mainly to improve stability, which is hard to evaluate
- For discussion, see Ian Goodfellow Twitter thread

Theis, L., Oord, A. V. D., & Bethge, M. (2016). A note on the evaluation of generative models. ICLR

Evaluate GAN performance

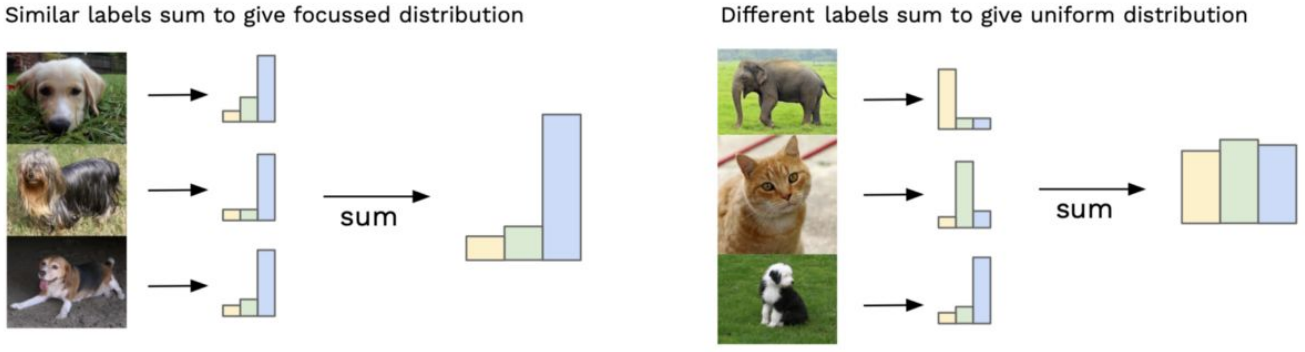
- Human performance



T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Inception Score

- Key idea: generators should produce images with a **variety** of **recognizable** object classes
- Two criteria:
  - A human, looking at a separate image, would be able to confidently determine what's in there (**saliency**).
  - A human, looking at a set of various images, would say that



T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Inception Score

- Key idea: generators should produce images with a **variety** of **recognizable** object classes
- Defined as:

$$\mathrm{IS}(G) = \exp\left(\mathbb{E}_{\mathbf{x}\sim p_g} D_{KL}(p(y|\mathbf{x}) \parallel p(y)) \right),$$

where $p(y|x)$ is the posterior label distribution returned by an image classifier (*e.g.*, InceptionNet) for sample x

- If the image contains a recognizable object, entropy of $p(y|x)$ should be low
- If generator generates images of diverse objects, the marginal distribution $p(y)$ should have high entropy
- Disadvantage: a GAN that simply memorizes the training data (overfitting) or outputs a single image per class (mode dropping) could still score well

T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Fréchet Inception Distance

- Key idea: fit simple distributions to statistics of feature activations for real and generated data; estimate divergence parametrically
  - Pass generated samples through a network (InceptionNet), compute activations for a chosen layer
  - Estimate multivariate mean and covariance of activations, compute Fréchet distance to those of real data

$$\mathrm{FID}(x, g) = ||\mu_x - \mu_g||_2^2 + \mathrm{Tr}(\Sigma_x + \Sigma_g - 2(\Sigma_x \Sigma_g)^{\frac{1}{2}}),$$

- Advantages: correlated with visual quality of samples and human judgment
- Disadvantage: cannot detect overfitting

M. Heusel, H. Ramsauer, T. Unterthiner, B. Nessler, S. Hochreiter, GANs trained by a two time-scale update rule converge to a local Nash equilibrium, NIPS 2017

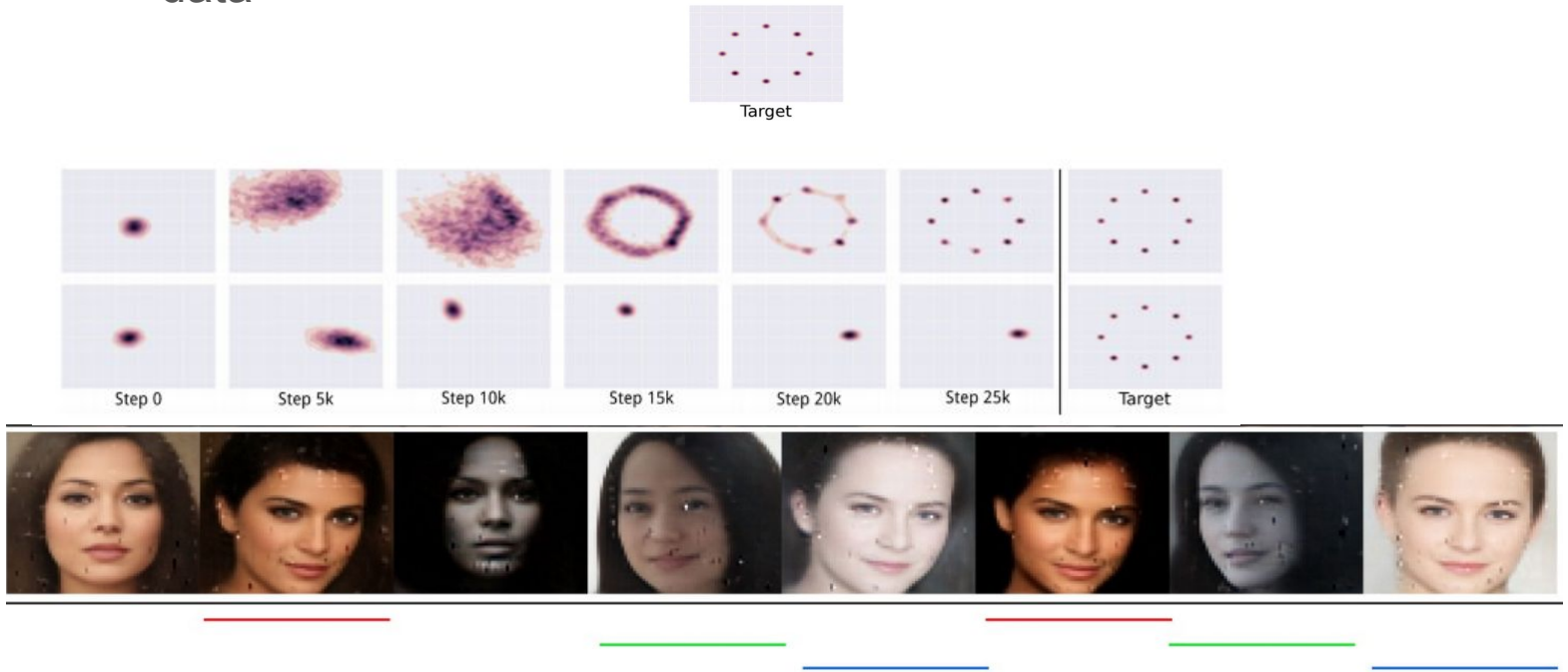
Issues with GAN training

- Stability
  - Parameters can oscillate or diverge, generator loss does not correlate with sample quality
  - Behavior very sensitive to hyper-parameter selection
  - Proposed several improvements, mostly changing the loss function

Tutorial on GAN: summary https://arxiv.org/pdf/1701.00160.pdf
https://medium.com/@jonathan_hui/gan-why-it-is-so-hard-to-train-generative-advisory-networks-819a86b3750b

Issues with GAN training

- Mode collapse
  - Generator ends up modeling only a small subset of the training data



https://medium.com/@jonathan_hui/gan-why-it-is-so-hard-to-train-generative-advisory-networks-819a86b3750b

Training Tricks

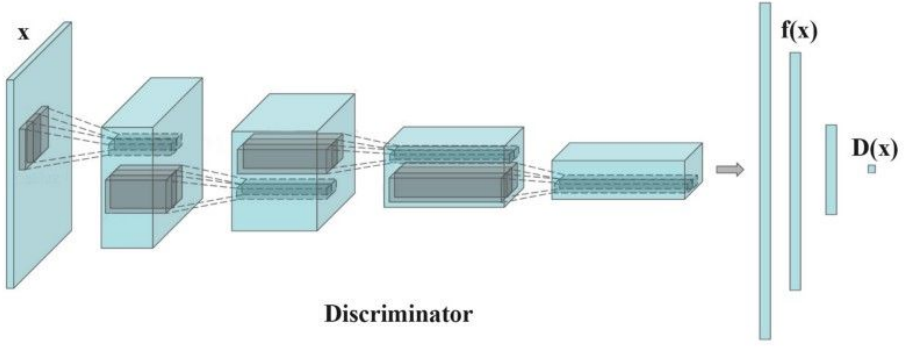
- Feature matching
- Use virtual batch norm
- Mini-batch discrimination (avoid mode collapsing by looking at more samples)
- Use label informations in the discriminator (C+1 categories)
- Label smoothing
- Historical averaging (introducing an additional cost function)

T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Training Tricks

- Feature matching
- Issue:
  - The generator tries to find the best image to fool the discriminator.
  - The “best” image keeps changing when both networks counteract their opponent.
  - The model does not converge and mode collapses.
- Add a loss for feature matching (expands the goal from beating the opponent to matching features in real images)

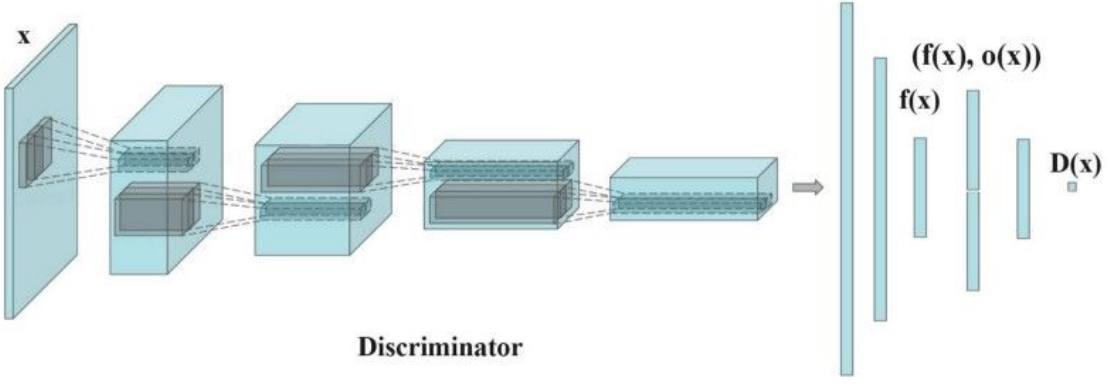
$$||\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\mathbf{f}(\boldsymbol{x}) - \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}\mathbf{f}(G(\boldsymbol{z}))||_2^2$$



T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Training Tricks

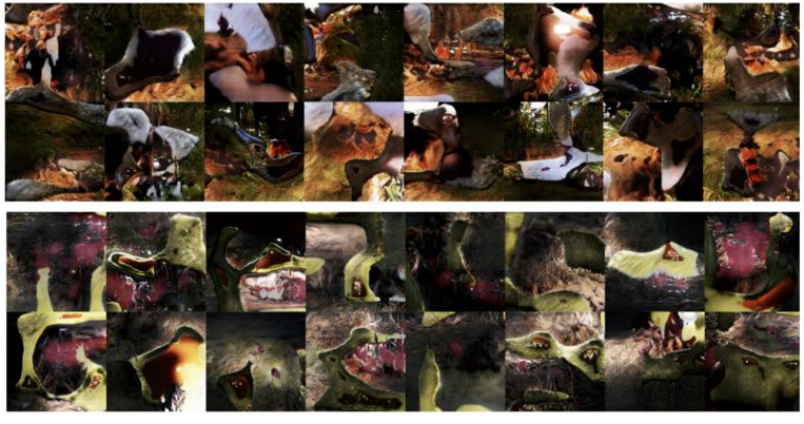
- Mini-batch discrimination
- Compute the similarity of the image ***x*** with images in the same batch. Append the similarity ***o(x)*** in one of the dense layers in the discriminator to classify whether this image is real or generated.
- If the mode starts to collapse, the similarity of generated images increases. The discriminator can use ***o(x)*** to detect generated images and penalize the generator if mode is collapsing.



T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Training Tricks

- Virtual batch norm
- Issue: Due to batch norm the generated images are not independent
- Idea: samples a reference batch before the training to compute the normalization parameters (μ and σ) and combine a reference batch with the current batch to compute the normalization parameters.



T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

Training Tricks

- Feature matching
- Use virtual batch norm
- Mini-batch discrimination (avoid mode collapsing by looking at more samples)
- Use label informations in the discriminator (C+1 categories)
- Label smoothing
- Historical averaging (introducing an additional cost function)

$$||\boldsymbol{\theta} - \frac{1}{t}\sum_{i=1}^{t}\boldsymbol{\theta}[i]||^2$$

T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

GAN Results

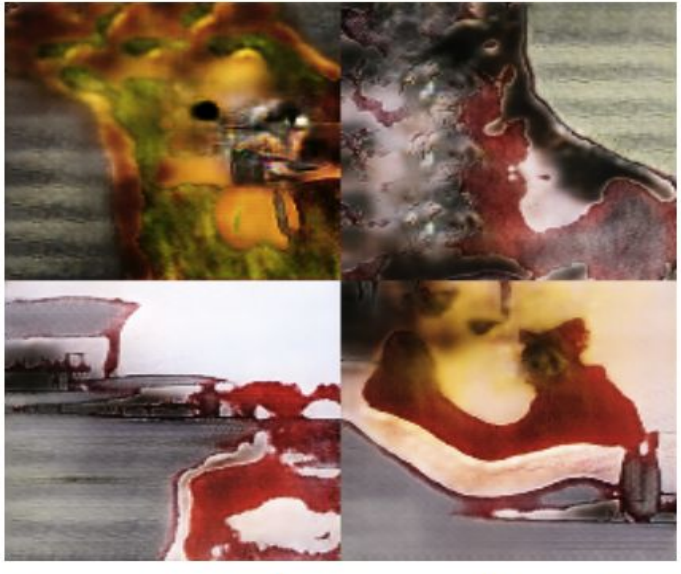
Original CIFAR-10 vs. Generated CIFAR-10 samples

T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

GAN Results

- ImageNet Results (128x128 pixels)

DCGAN

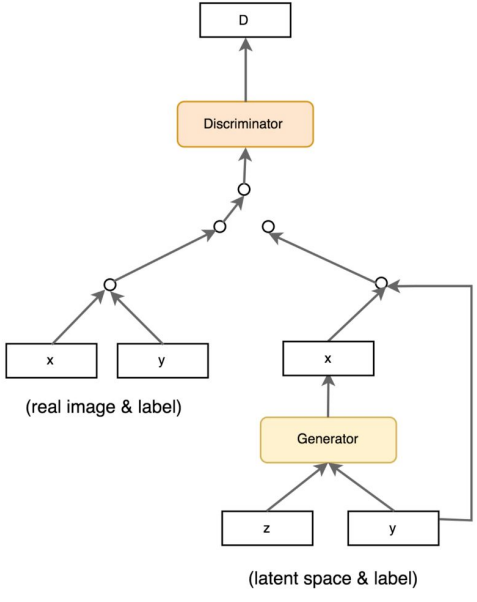


Improved DCGAN

T. Salimans, I. Goodfellow, W. Zaremba, V. Cheung, A. Radford, X. Chen, Improved techniques for training GANs, NIPS 2016

CGAN & InfoGAN

- Conditional GAN (CGAN)
  - Same loss as the original GAN
  - Use label info (1-hot encoding) both in generator and discriminator
  - Label as extension to the latent space



$$\min_G \max_D V(D,G) = \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x}|\boldsymbol{y})] + \mathbb{E}_{\boldsymbol{z}\sim p_z(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z}|\boldsymbol{y})))].$$

CGAN & InfoGAN

- Info GAN (label not given, treated as latent factor):
- Information Maximizing Generative Adversarial Nets

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

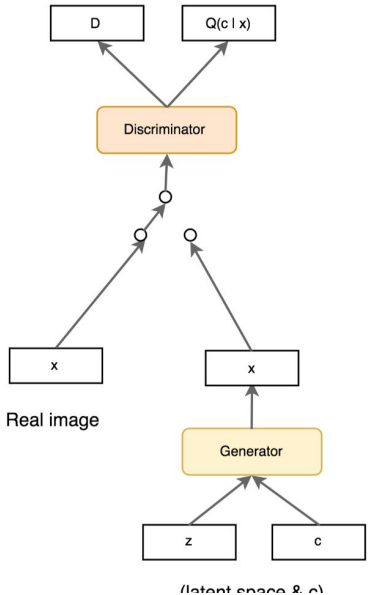
- X and Y are independent, then I(X; Y) = 0
- X is completely determined by Y, then H(Y|X)=0, I(X; Y) = H(Y)

Then I is maximized

Chen, X., Duan, Y., Houthooft, R., Schulman, J., Sutskever, I., & Abbeel, P. (2016). Infogan: Interpretable representation learning by information maximizing generative adversarial nets. NIPS.

CGAN & InfoGAN

- Info GAN (label not given, treated as latent factor):
  - Additional output for the discriminator $Q(c|x)$
  - Mutual information $I(c;x)$ measure how much we know c if we know x. It should be high.
  - The info in the latent code c should not be lost in the generation.



$$\min_G \max_D V_{infoGAN}(D, G) = V_{GAN}(D, G) - \lambda I(c; G(z, c))$$

$$V_{GAN}(D, G) \equiv \mathbb{E}_{x \sim p_{data}} log D(x) + \mathbb{E}_{z \sim p_z(z)} log(1 - D(G(z, c)))$$

$$\begin{aligned} I(c; G(z, c)) &= H(c) - H(c|G(z, c)) \\ &= \mathbb{E}_{x \sim G(z,c)}[\mathbb{E}_{c \sim P(c|x)}[log P(c|x)]] + H(c) \\ &\geq \mathbb{E}_{x \sim G(z,c)}[\mathbb{E}_{c \sim P(c|x)}[log Q(c|x)]] + H(c) \end{aligned}$$

Chen, X., Duan, Y., Houthooft, R., Schulman, J., Sutskever, I., & Abbeel, P. (2016). Infogan: Interpretable representation learning by information maximizing generative adversarial nets. NIPS.

InfoGAN Results

(a) Varying c_1 on InfoGAN (Digit type)

(b) Varying c_1 on regular GAN (No clear meaning)

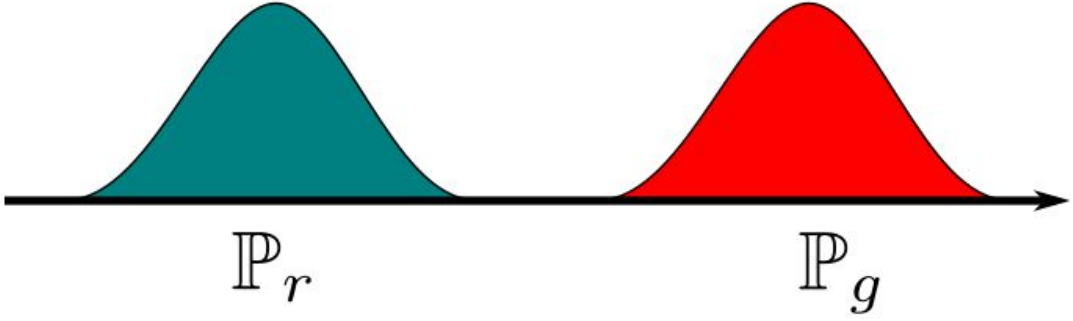
Chen, X., Duan, Y., Houthooft, R., Schulman, J., Sutskever, I., & Abbeel, P. (2016). Infogan: Interpretable representation learning by information maximizing generative adversarial nets. NIPS.

Wasserstein GAN: towards better losses

The *Kullback-Leibler* (KL) divergence

$$KL(\mathbb{P}_r \| \mathbb{P}_g) = \int \log \left(\frac{P_r(x)}{P_g(x)} \right) P_r(x) d\mu(x)$$

Problem when the distributions have $P_g(x)=0$ and $P_r(x)>0$:



$$KL(\mathbb{P}_r || \mathbb{P}_g) = +\infty$$

Wasserstein GAN, Martin Arjovsky, Soumith Chintala, Léon Bottou

Wasserstein GAN: towards better losses

The *Jensen-Shannon* (JS) divergence

$$JS(\mathbb{P}_r, \mathbb{P}_g) = KL(\mathbb{P}_r \| \mathbb{P}_m) + KL(\mathbb{P}_g \| \mathbb{P}_m)$$

where $\mathbb{P}_m$ is the mixture $(\mathbb{P}_r + \mathbb{P}_g)/2$.

Minimizing the GAN objective function with an optimal discriminator is equivalent to minimizing the JS-divergence

Wasserstein GAN, Martin Arjovsky, Soumith Chintala, Léon Bottou

Wasserstein GAN: towards better losses

The *Jensen-Shannon* (JS) divergence

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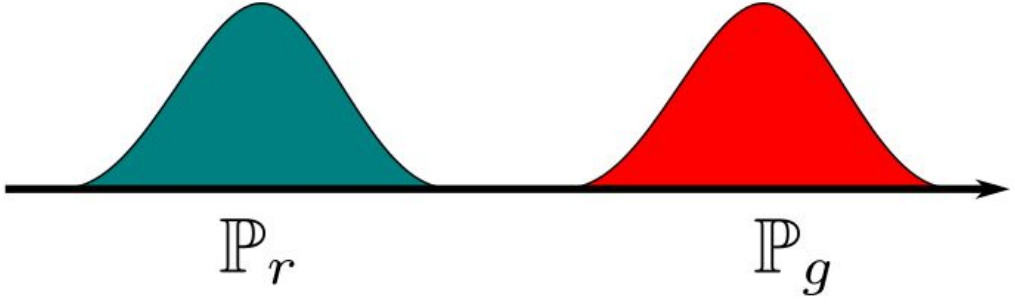


$$JS(\mathbb{P}_r, \mathbb{P}_g) = 2log(2)$$

Wasserstein GAN, Martin Arjovsky, Soumith Chintala, Léon Bottou

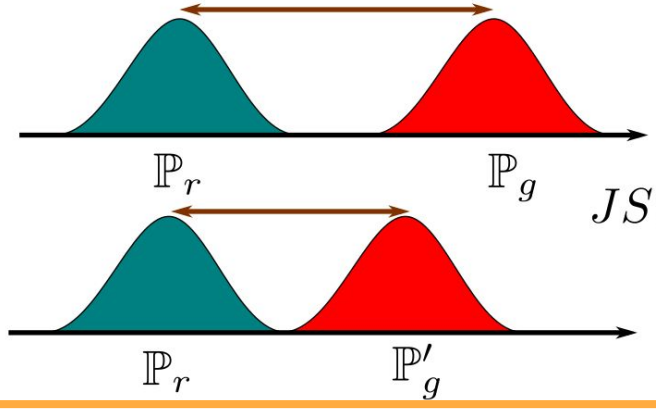
Wasserstein GAN: towards better losses

The *Jensen-Shannon* (JS) divergence

$$JS(\mathbb{P}_r, \mathbb{P}_g) = KL(\mathbb{P}_r \| \mathbb{P}_m) + KL(\mathbb{P}_g \| \mathbb{P}_m)$$

where $\mathbb{P}_m$ is the mixture $(\mathbb{P}_r + \mathbb{P}_g)/2$.

Problem:



$$JS(\mathbb{P}_r, \mathbb{P}_g) = JS(\mathbb{P}_r, \mathbb{P}'_g)$$

The *Earth-Mover* (EM) distance or Wasserstein-1

$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y)\sim\gamma} \left[\|x - y\| \right]$$

Wasserstein GAN: towards better losses

The *Earth-Mover* (EM) distance or Wasserstein-1

$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y)\sim\gamma}\left[\, \|x - y\| \,\right]$$

We can show:

$$W(\mathbb{P}_r, \mathbb{P}_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x\sim\mathbb{P}_r}[f(x)] - \mathbb{E}_{x\sim\mathbb{P}_\theta}[f(x)]$$

where the supremum is over all the 1-Lipschitz functions $f : \mathcal{X} \to \mathbb{R}$.

Wasserstein GAN, Martin Arjovsky, Soumith Chintala, Léon Bottou

Cédric Villani.Optimal Transport: Old and New. Grundlehren der mathema-tischen Wissenschaften

Wasserstein GAN: towards better losses

$$W(\mathbb{P}_r, \mathbb{P}_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x \sim \mathbb{P}_r}[f(x)] - \mathbb{E}_{x \sim \mathbb{P}_\theta}[f(x)]$$

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

Require: : α, the learning rate. c, the clipping parameter. m, the batch size. n_{critic}, the number of iterations of the critic per generator iteration.

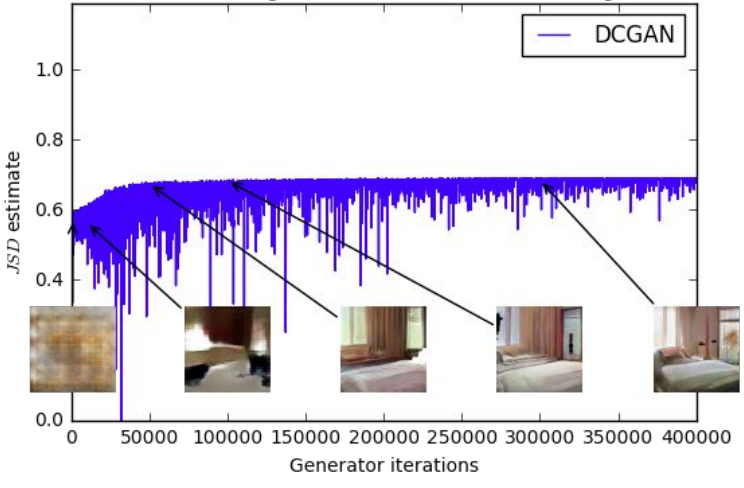
Require: : w_0, initial critic parameters. θ_0, initial generator's parameters.

while θ has not converged **do**
  for $t = 0, ..., n_{\text{critic}}$ **do**
    Sample $\{x^{(i)}\}_{i=1}^{m} \sim \mathbb{P}_r$ a batch from the real data.
    Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
    $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^{m} f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))\right]$
    $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$
    $w \leftarrow \text{clip}(w, -c, c)$
  end for
  Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
  $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))$
  $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$
end while

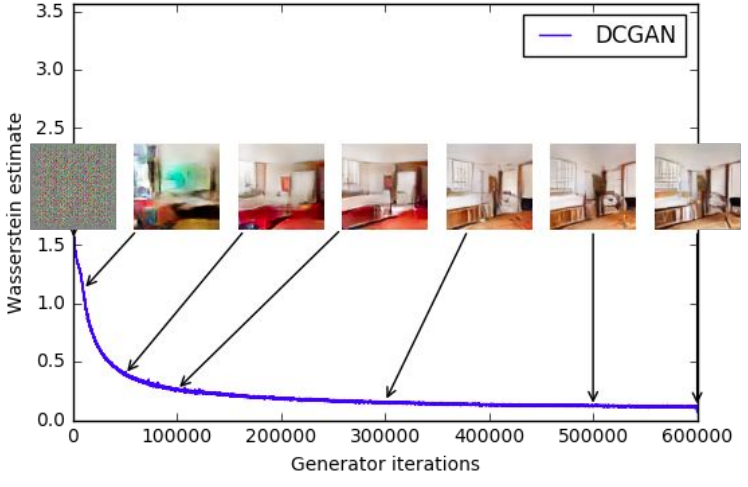
Wasserstein GAN

- Objective function value is more meaningfully related to quality of generator output

Original GAN divergence



WGAN divergence



Wasserstein GAN: Improved Training (WGAN_GP)

$$W(\mathbb{P}_r, \mathbb{P}_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x \sim \mathbb{P}_r}[f(x)] - \mathbb{E}_{x \sim \mathbb{P}_\theta}[f(x)]$$

Weight clipping: $w \leftarrow \text{clip}(w, -c, c)$

Gradient penalty (WGAN-GP):

$$L = \underbrace{\mathop{\mathbb{E}}_{\tilde{\boldsymbol{x}} \sim \mathbb{P}_g}[D(\tilde{\boldsymbol{x}})] - \mathop{\mathbb{E}}_{\boldsymbol{x} \sim \mathbb{P}_r}[D(\boldsymbol{x})]}_{\text{Original critic loss}} + \underbrace{\lambda \mathop{\mathbb{E}}_{\hat{\boldsymbol{x}} \sim \mathbb{P}_{\hat{\boldsymbol{x}}}}\left[(\|\nabla_{\hat{\boldsymbol{x}}} D(\hat{\boldsymbol{x}})\|_2 - 1)^2\right]}_{\text{Our gradient penalty}}.$$

Improved Training of Wasserstein GANs, Gulrajani et.al. Nips 2017

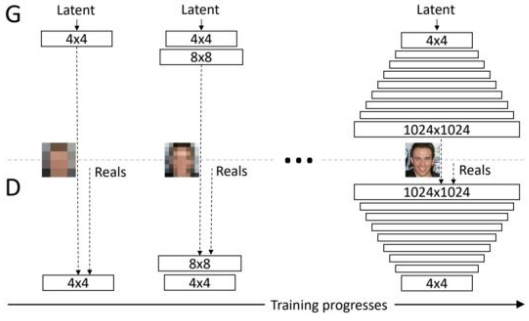
Lost functions

Name	Value Function
GAN	$L_D^{GAN} = E[\log(D(x))] + E[\log(1 - D(G(z)))]$ $L_G^{GAN} = E[\log(D(G(z)))]$
LSGAN	$L_D^{LSGAN} = E[(D(x) - 1)^2] + E[D(G(z))^2]$ $L_G^{LSGAN} = E[(D(G(z)) - 1)^2]$
WGAN	$L_D^{WGAN} = E[D(x)] - E[D(G(z))]$ $L_G^{WGAN} = E[D(G(z))]$ $W_D \leftarrow clip\_by\_value(W_D, -0.01, 0.01)$
WGAN_GP	$L_D^{WGAN\_GP} = L_D^{WGAN} + \lambda E[(\lvert\nabla D(\alpha x - (1 - \alpha G(z)))\rvert - 1)^2]$ $L_G^{WGAN\_GP} = L_G^{WGAN}$
DRAGAN	$L_D^{DRAGAN} = L_D^{GAN} + \lambda E[(\lvert\nabla D(\alpha x - (1 - \alpha x_p))\rvert - 1)^2]$ $L_G^{DRAGAN} = L_G^{GAN}$
CGAN	$L_D^{CGAN} = E[\log(D(x, c))] + E[\log(1 - D(G(z), c))]$ $L_G^{CGAN} = E[\log(D(G(z), c))]$
infoGAN	$L_{D,Q}^{infoGAN} = L_D^{GAN} - \lambda L_I(c, c')$ $L_G^{infoGAN} = L_G^{GAN} - \lambda L_I(c, c')$
ACGAN	$L_{D,Q}^{ACGAN} = L_D^{GAN} + E[P(class = c\lvert x)] + E[P(class = c\lvert G(z))]$ $L_G^{ACGAN} = L_G^{GAN} + E[P(class = c\lvert G(z))]$
EBGAN	$L_D^{EBGAN} = D_{AE}(x) + \max(0, m - D_{AE}(G(z)))$ $L_G^{EBGAN} = D_{AE}(G(z)) + \lambda \cdot PT$
BEGAN	$L_D^{BEGAN} = D_{AE}(x) - k_t D_{AE}(G(z))$ $L_G^{BEGAN} = D_{AE}(G(z))$ $k_{t+1} = k_t + \lambda(\gamma D_{AE}(x) - D_{AE}(G(z)))$

https://towardsdatascience.com/gan-ways-to-improve-gan-performance-acf37f9f59b

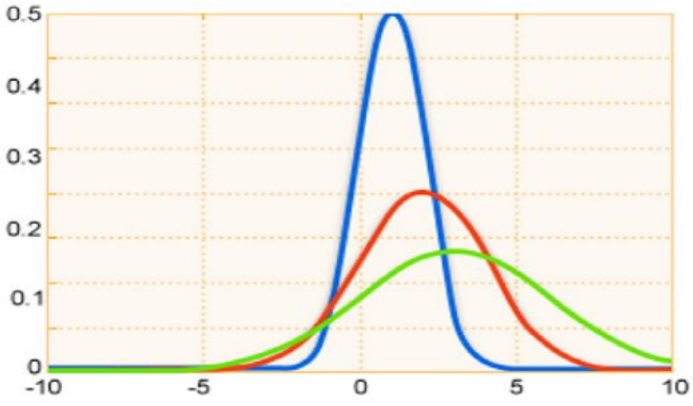
High Resolution

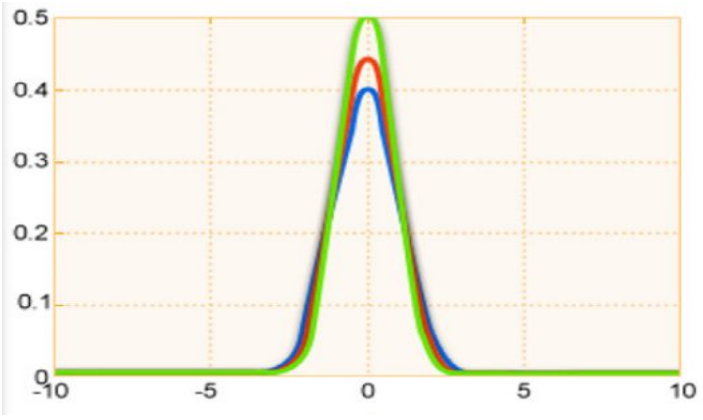
Progressive GAN (2018)



BigGAN (2018)

Inference-normalization





Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β
Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

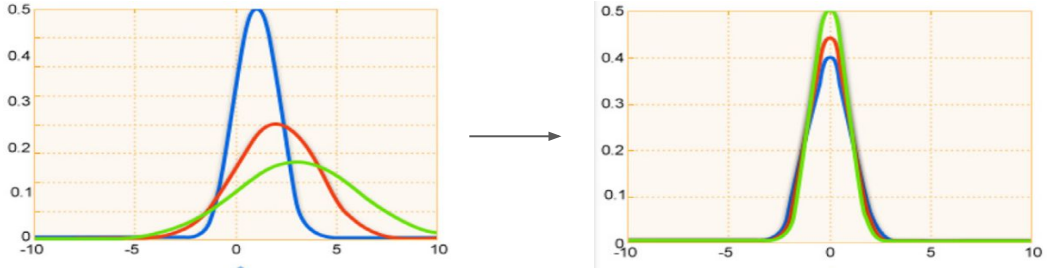
$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

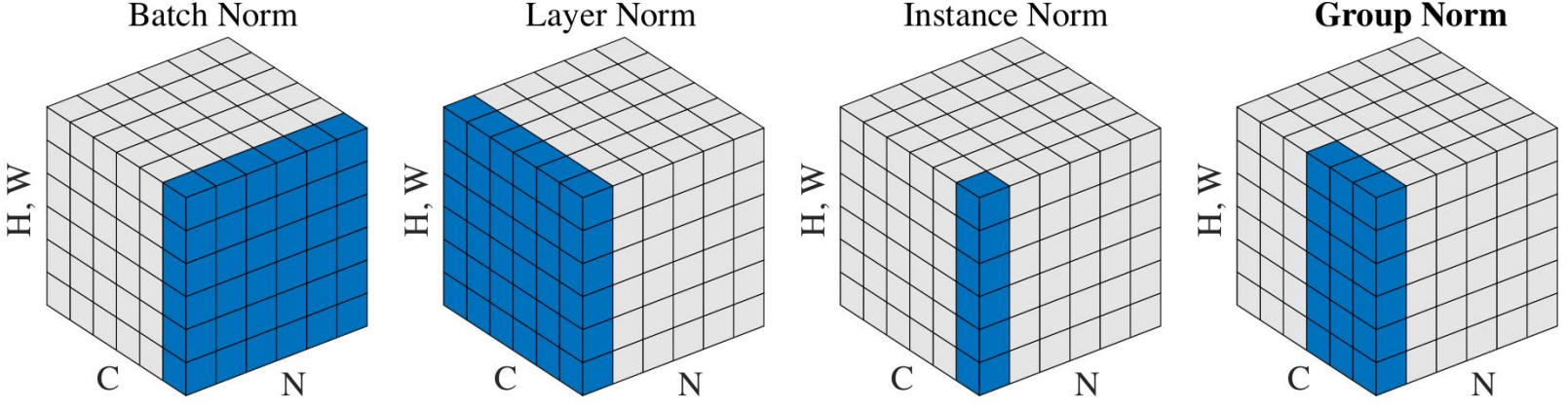
Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift
Sergey Ioffe, Christian Szegedy

Inference-normalization

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i$$

$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2$$



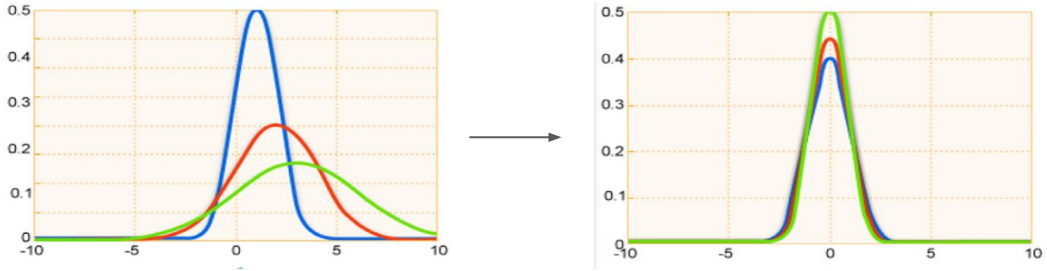


Group Normalization,Yuxin Wu, Kaiming He, ECCV 2018

Why inference-normalization?

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i$$

$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2$$



(a) Content image.

(c) Low contrast content image.

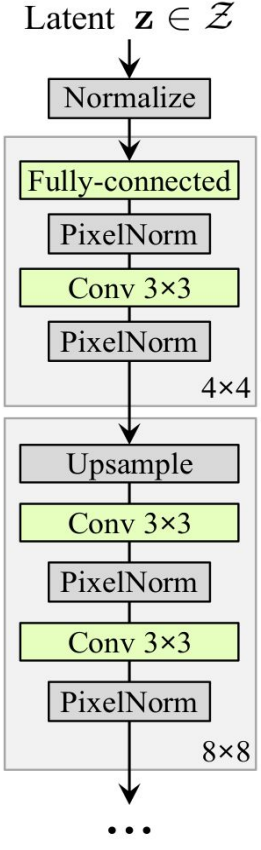
$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv$$

The two images are identical after instance normalization. We obtain style invariant representations.

Instance Normalization: The Missing Ingredient for Fast Stylization,Dmitry Ulyanov, Andrea Vedaldi, Victor Lempitsky

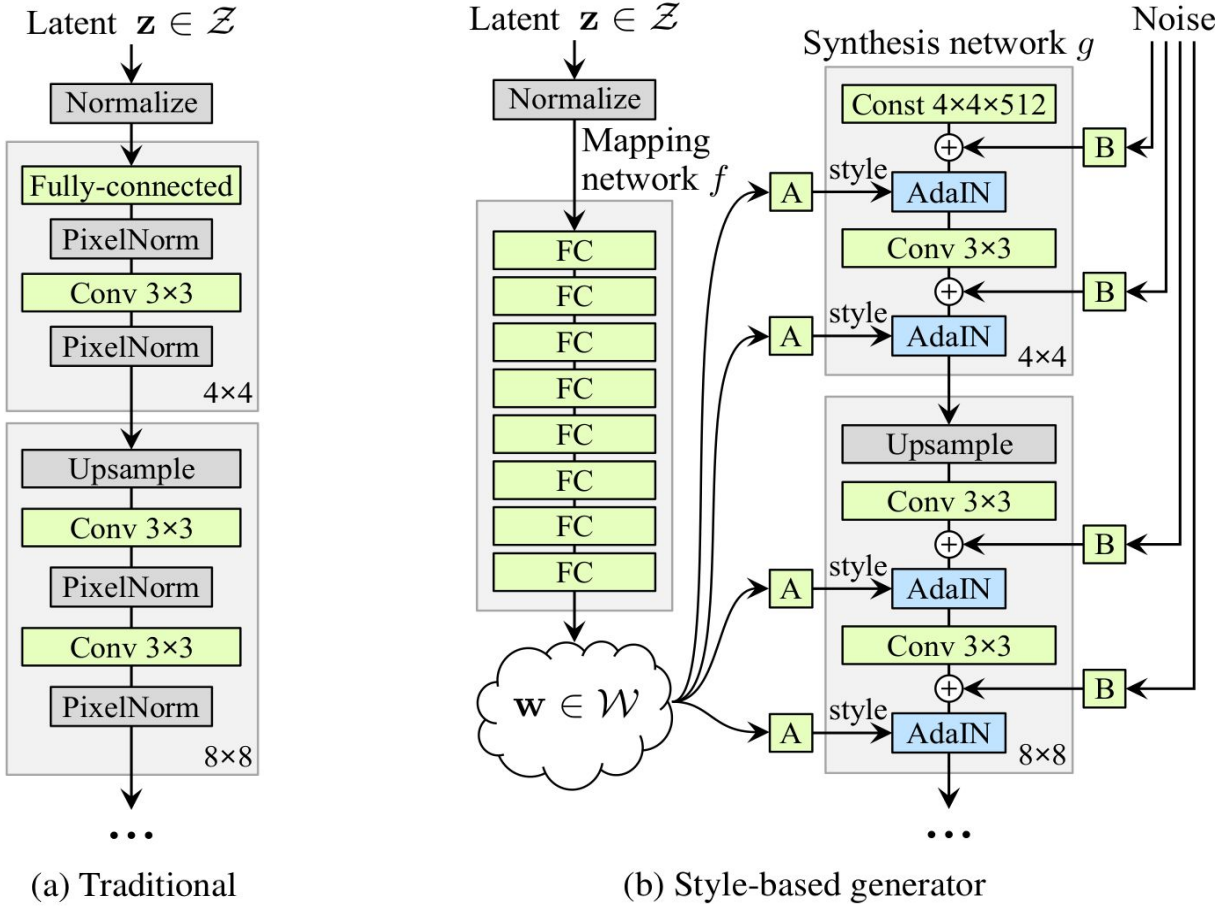
Style-Based GAN



(a) Traditional

A Style-Based Generator Architecture for Generative Adversarial Networks, Karras et.al. CVPR 2019

Style-Based GAN

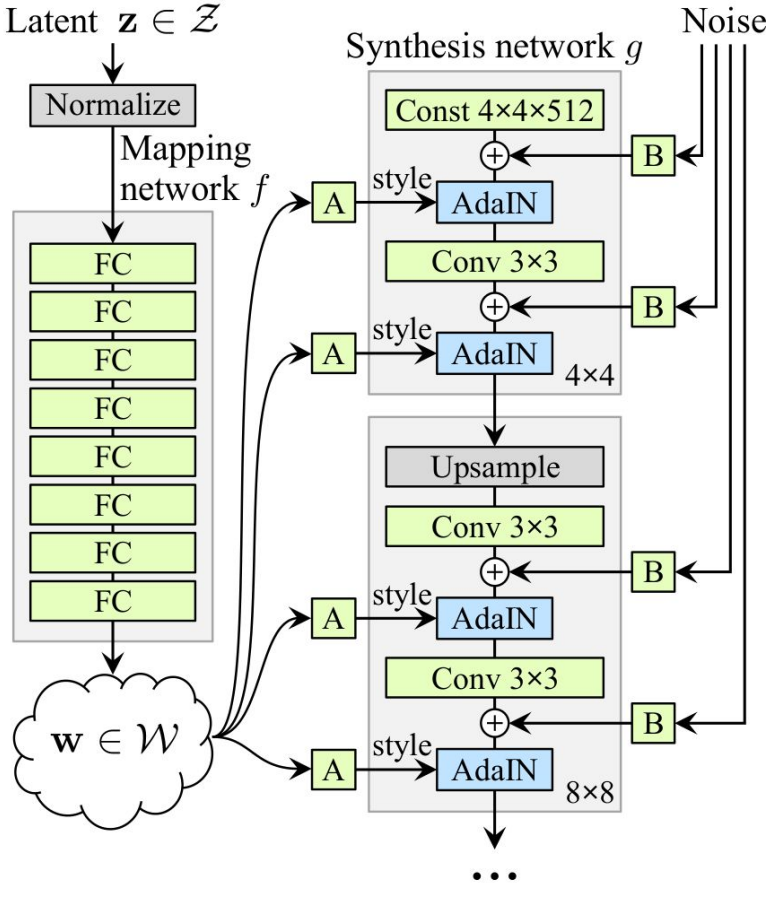


(a) Traditional

(b) Style-based generator

A Style-Based Generator Architecture for Generative Adversarial Networks, Karras et.al. CVPR 2019

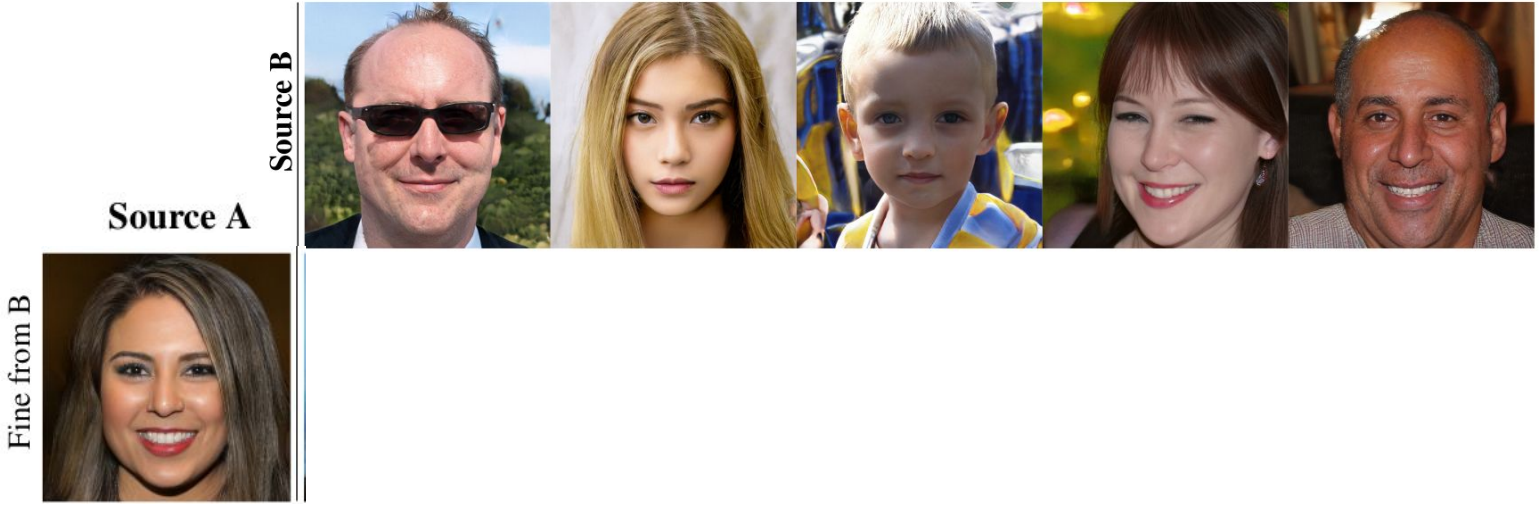
Style-Based GAN



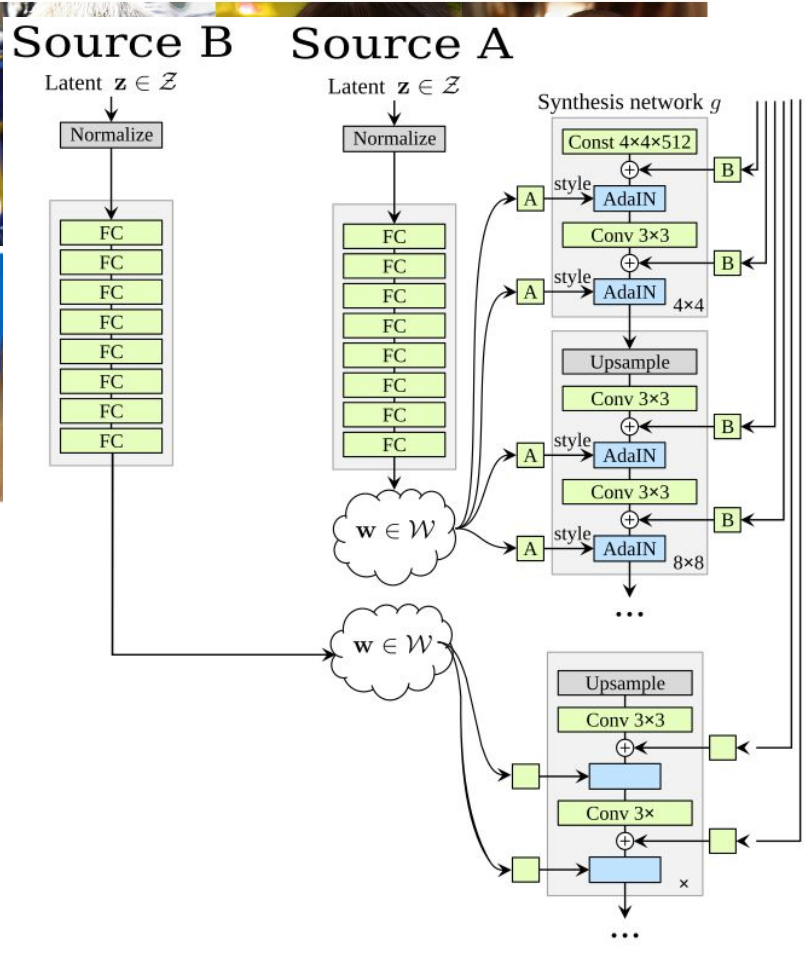
(b) Style-based generator

$$\text{AdaIN}(\mathbf{x}_i, \mathbf{y}) = \mathbf{y}_{s,i} \frac{\mathbf{x}_i - \mu(\mathbf{x}_i)}{\sigma(\mathbf{x}_i)} + \mathbf{y}_{b,i},$$

Style-Based GAN



Style-Based GAN



Style-Based GAN

Style-Based GAN

Style-Based GAN

Training GANs: Recap

Training GANs: Recap

Minimax objective function:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

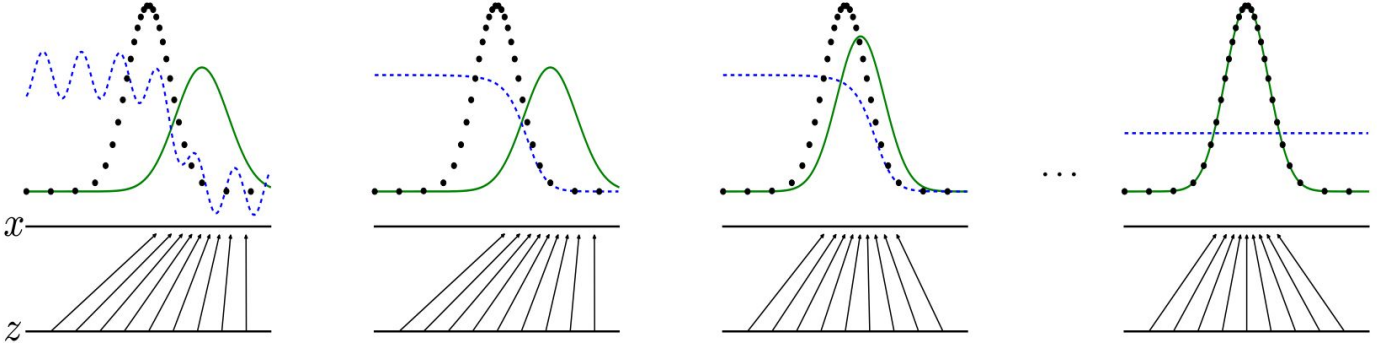
Alternate between:

1. **Gradient ascent** on discriminator

$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{data}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

2. **Gradient descent** on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$



Improving GANs: Recap

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Improving GANs: Recap

- Use convolutions
- Use BN
- Many optimization tricks
- Change the loss (e.g. Wasserstein loss)
- Improve the architecture using inference normalization