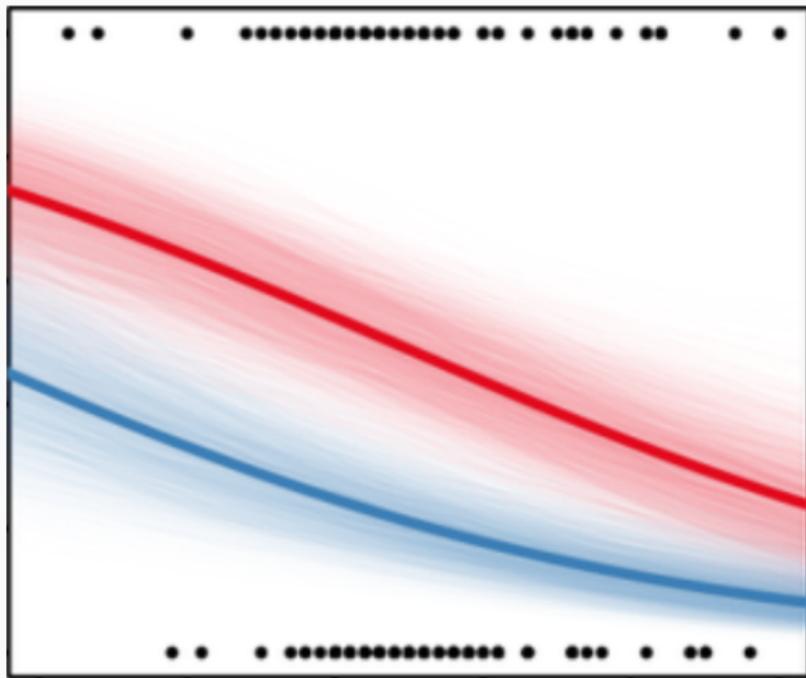


An introduction to statistical inference

Francisco Rodríguez Sánchez

@frod_san

<https://frodriguezsanchez.net>



Why statistics?

To answer questions like...

- what's the probability that something occurs?

To answer questions like...

- what's the probability that something occurs?
- does X influence Y? How much?

To answer questions like...

- what's the probability that something occurs?
- does X influence Y? How much?
- can we predict Y knowing X, Z... How well?

To ensure correct inferences

11	451	368	80	46	83	74	28	79	49	689	43
439	164	94	45	73	38	58	25	75	78	340	3
235	166	172	54	91	85	40	78	65	78	78	1
10	30	62	49	32	16	10	10	10	10	10	1
1.433	896	2.132	2.390	3.850	2.775	1.580	2.000	5.000	1.000	1.000	1
1.870	2.845	1.001	1.920	1.740	2.981	3.000	1.000	1.000	1.000	1.000	1
2.427	1.333	1.233	1.233	1.470	2.524	3.000	1.000	1.000	1.000	1.000	1
2.424	2.657	1.001	1.233	1.233	1.233	1.233	1.233	1.233	1.233	1.233	1
1.692	84	1.001	1.233	1.233	1.233	1.233	1.233	1.233	1.233	1.233	1
1.199	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1
2.032	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1
3	2.032	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1
35	290	92	285	163	224	234	234	234	234	234	234
74	243	430	277	175	234	234	234	234	234	234	234
84	249	301	47	3.858	2.459	6.303	9	9	9	9	9

Inference



Bolker et al 2009 TREE:

'311 out of 537 GLMM analyses (58%) used these tools
inappropriately'

To get answers to tough problems

How many seeds do trees produce?



Inferring tree fecundity



Course goals

- Understand statistical inference

Course goals

- Understand statistical inference
- Avoid misconceptions

Course goals

- Understand statistical inference
- Avoid misconceptions
- Promote good practices

*The purpose of models is not to fit data
but to sharpen thinking*

Sam Karlin

Topics

- Descriptive statistics

Topics

- Descriptive statistics
- Graphics

Topics

- Descriptive statistics
- Graphics
- Sampling

Topics

- Descriptive statistics
- Graphics
- Sampling
- Experimental design

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- Hypothesis testing

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Topics

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- Bayesian inference
- Linear models & GLMs
- Model selection

Descriptive statistics

Measure trunk diameter of 30 trees in your neighbourhood



Read data

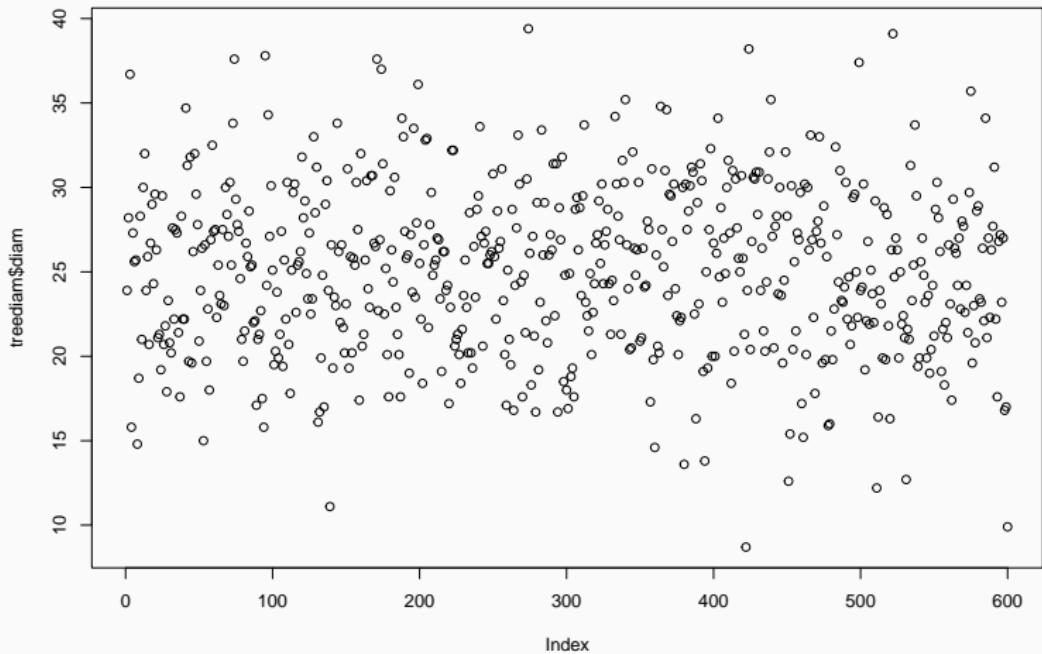
```
treediam <- read.csv("treediam.csv")
```

```
summary(treediam)
```

	site	tree	diam
Min.	: 1.00	Min. : 1.0	Min. : 8.70
1st Qu.	: 5.75	1st Qu.: 8.0	1st Qu.: 21.40
Median	:10.50	Median :15.5	Median :25.25
Mean	:10.50	Mean :15.5	Mean :25.04
3rd Qu.	:15.25	3rd Qu.:23.0	3rd Qu.: 28.40
Max.	:20.00	Max. :30.0	Max. :39.40

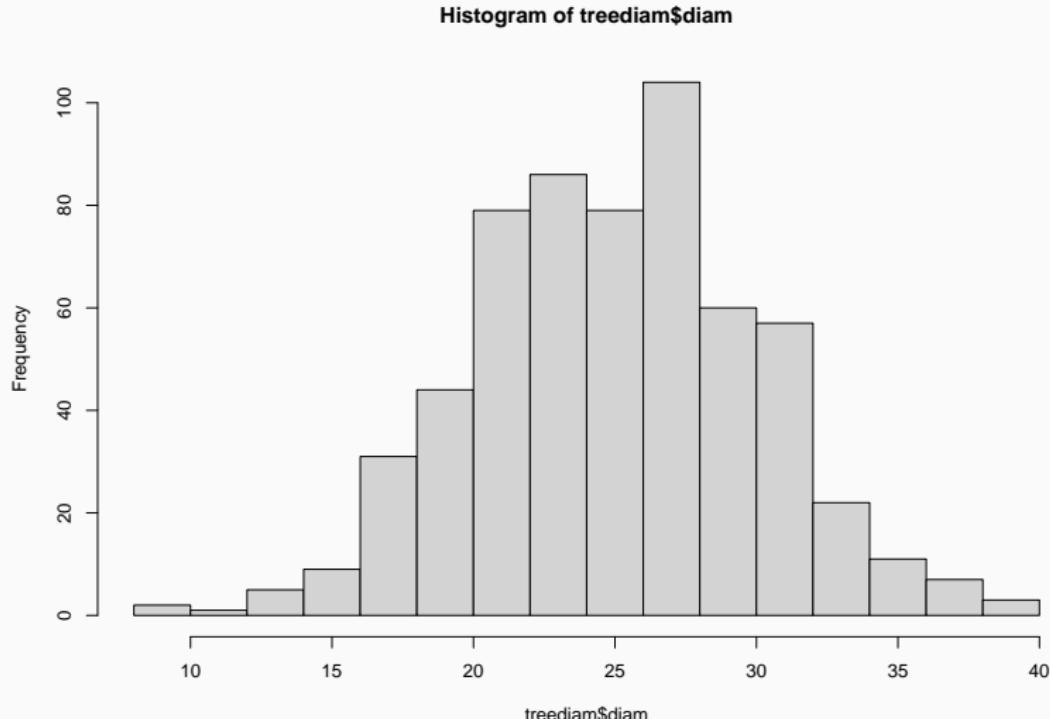
Visualisation of tree diameters

```
plot(treediam$diam)
```



Visualisation of tree diameters

```
hist(treediam$diam)
```



How well do these values
represent actual tree diameters
in your neighbourhood?

<https://pollev.com/franciscorod726>

- At what height did you measure?

- At what height did you measure?
- Did you include bark?

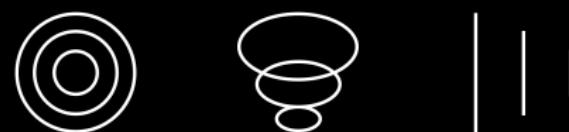
- At what height did you measure?
- Did you include bark?
- Did you measure with tape, caliper, by eye?

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- When did you measure: dawn, midday, night?

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- When did you measure: dawn, midday, night?
 - (trees may get thinner w/ high evapotranspiration)

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- Where did you measure?

- At what height did you measure?
- Did you include bark?
- Did you measure with tape, caliper, by eye?
- When did you measure: dawn, midday, night?
 - (trees may get thinner w/ high evapotranspiration)
- Where did you measure?
 - (differences among streets, species, etc)



TRUTH

TRUTH

TRUTH

Data are hardly ever objective.

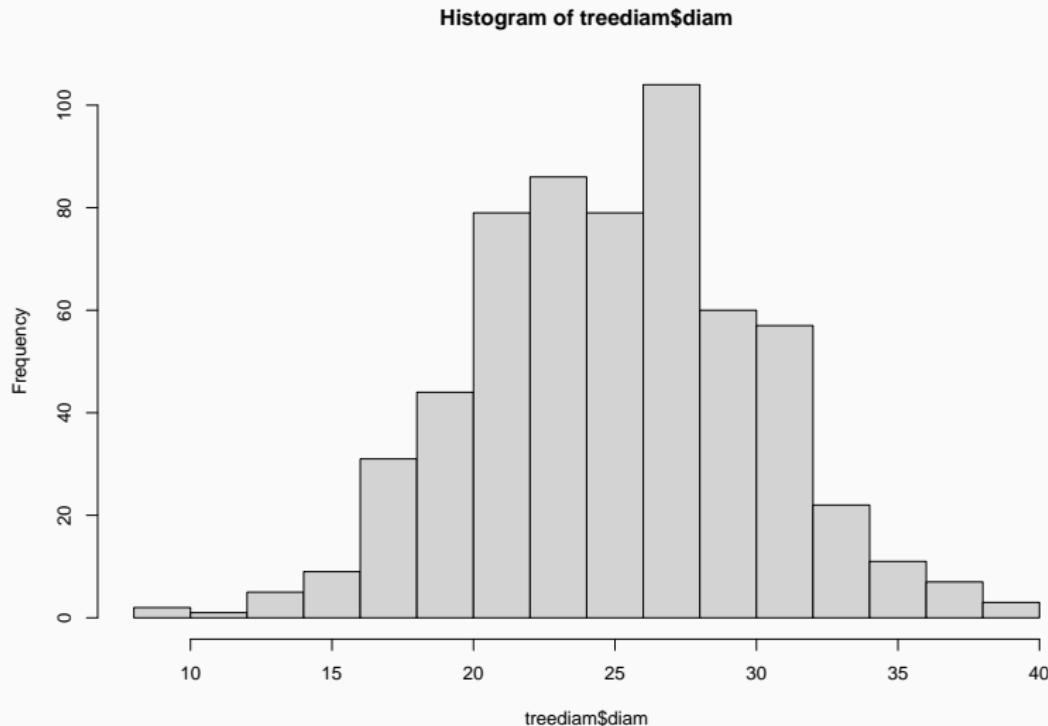
We decide **what to measure, when, where, and how**.

Always consider:

How well do data reflect what we are trying to measure?

Describing your data

How would you describe this distribution?



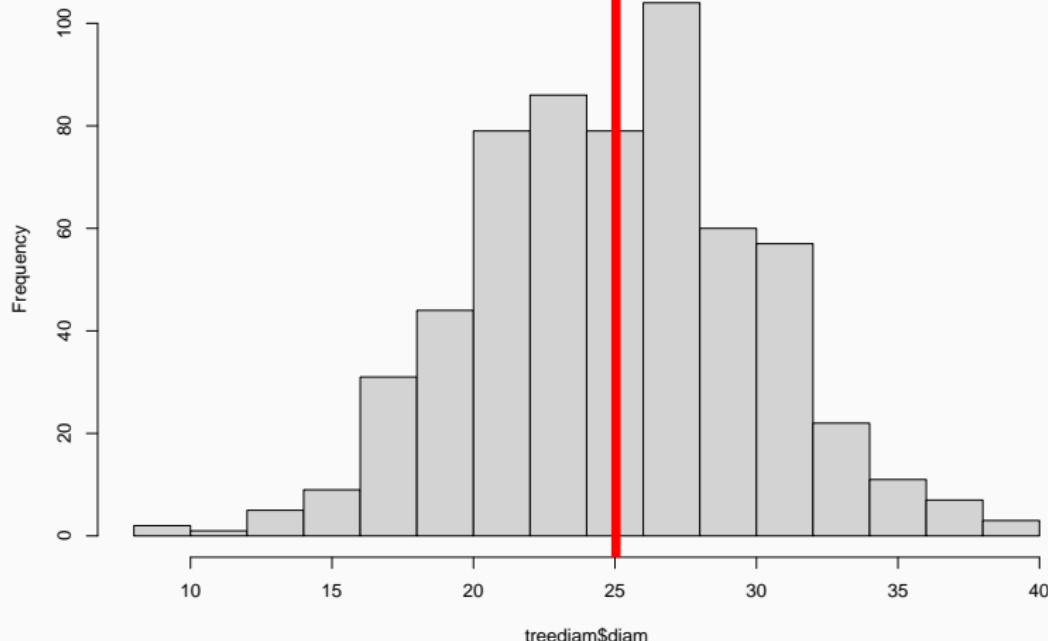
(Discuss with your partner)

Location / Central tendency

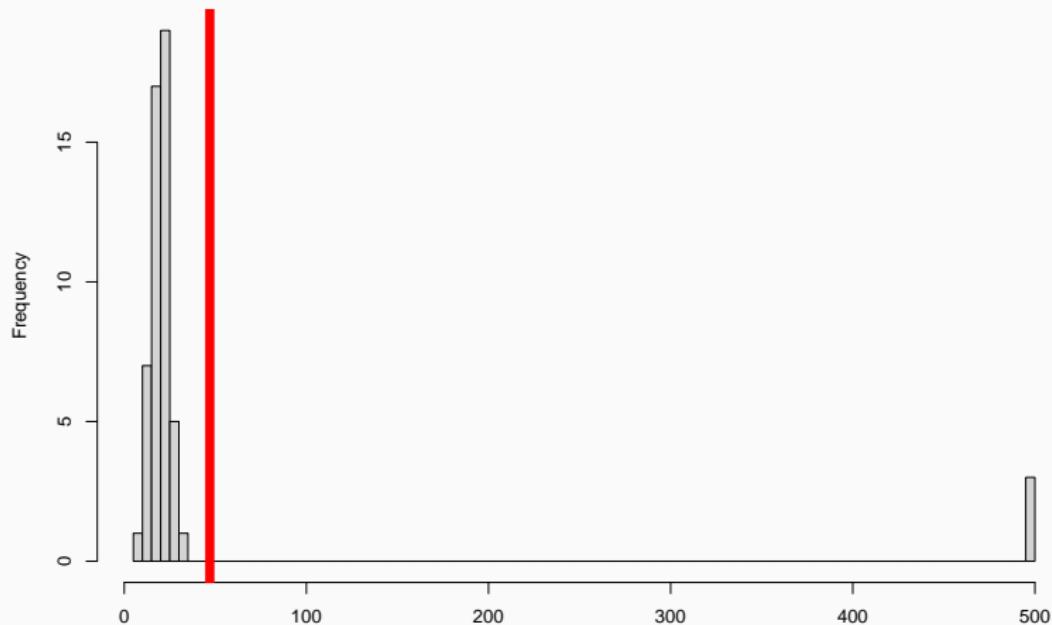
Mean / Average

$$\text{mean} = \frac{d_1 + d_2 + d_3}{n}$$

Histogram of treediam\$diam

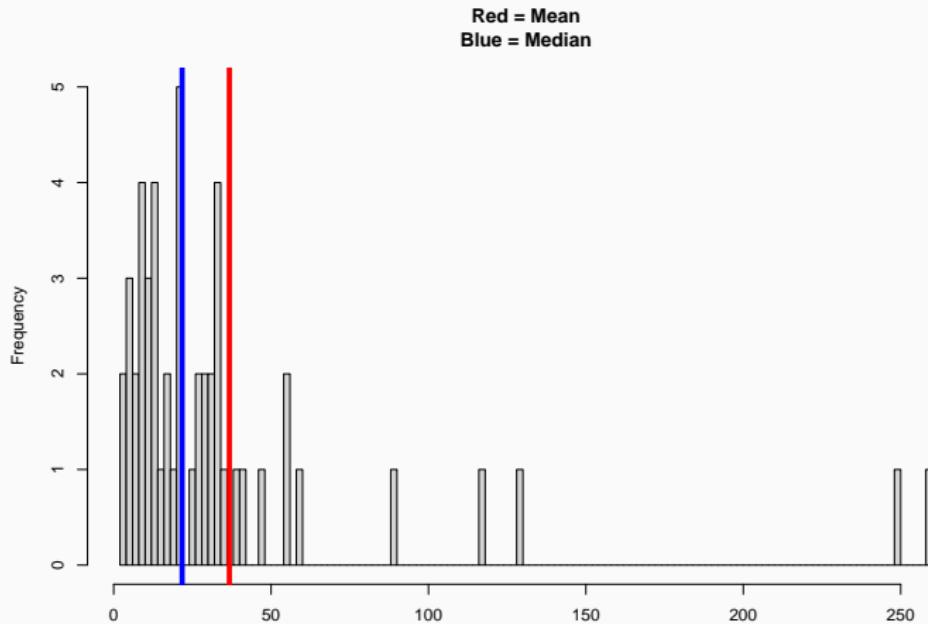


Mean is sensitive to skew/outliers



Median

50% percentile. Leaves half of the data values on each side



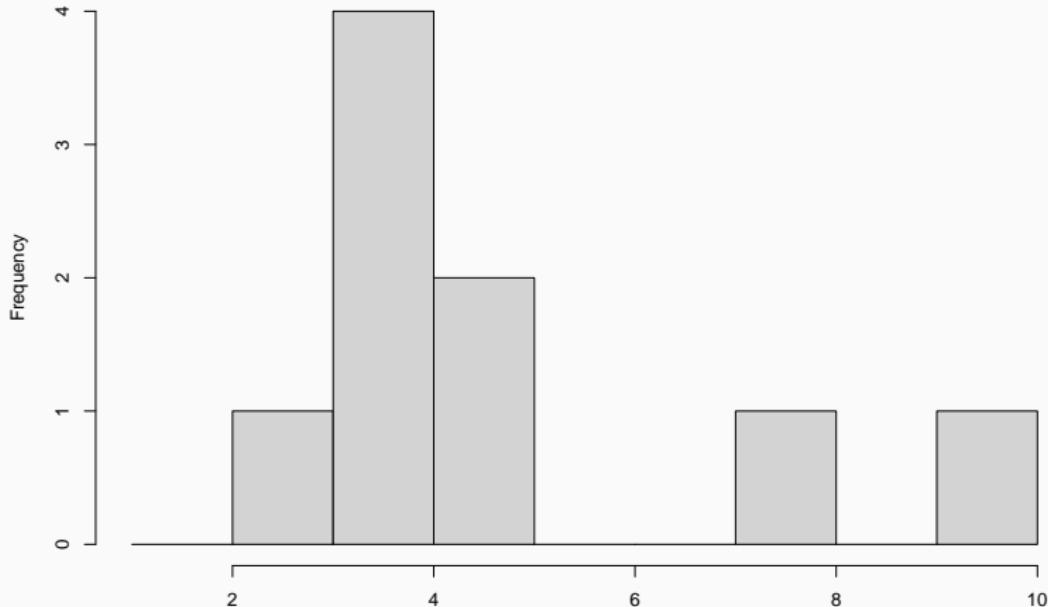
$$\text{Median of } c(2, 4, 6, 8, 10) = 6$$

$$\text{Median of } c(2, 4, 6, 8) = (4 + 6) / 2 = 5$$

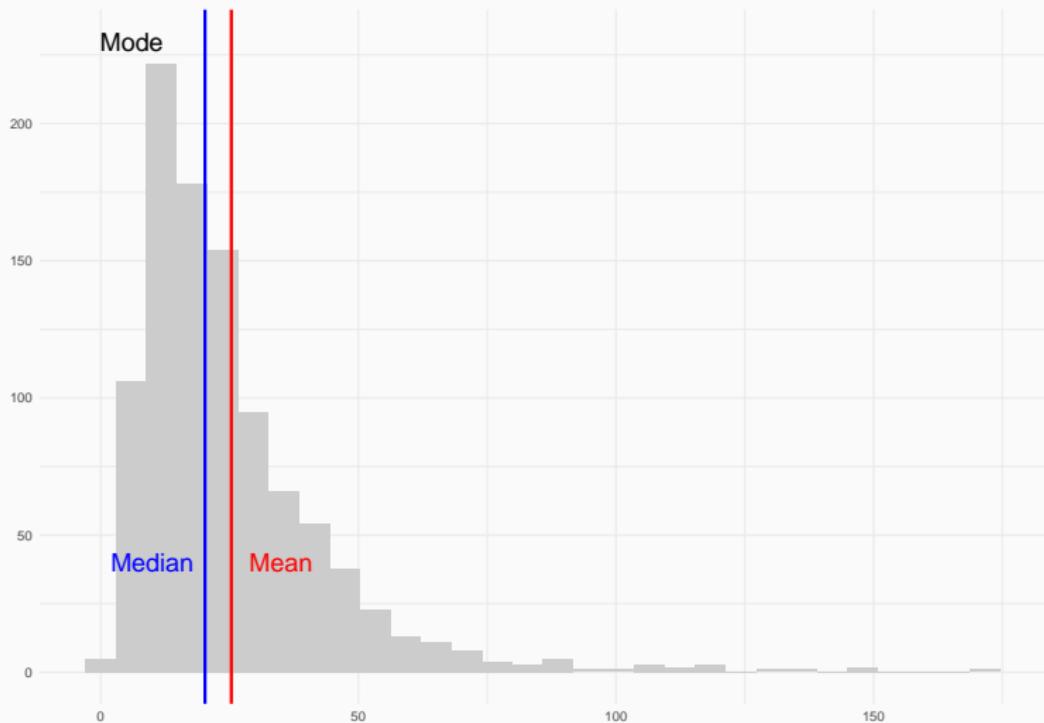
Mode

Most frequent value

Mode = 4

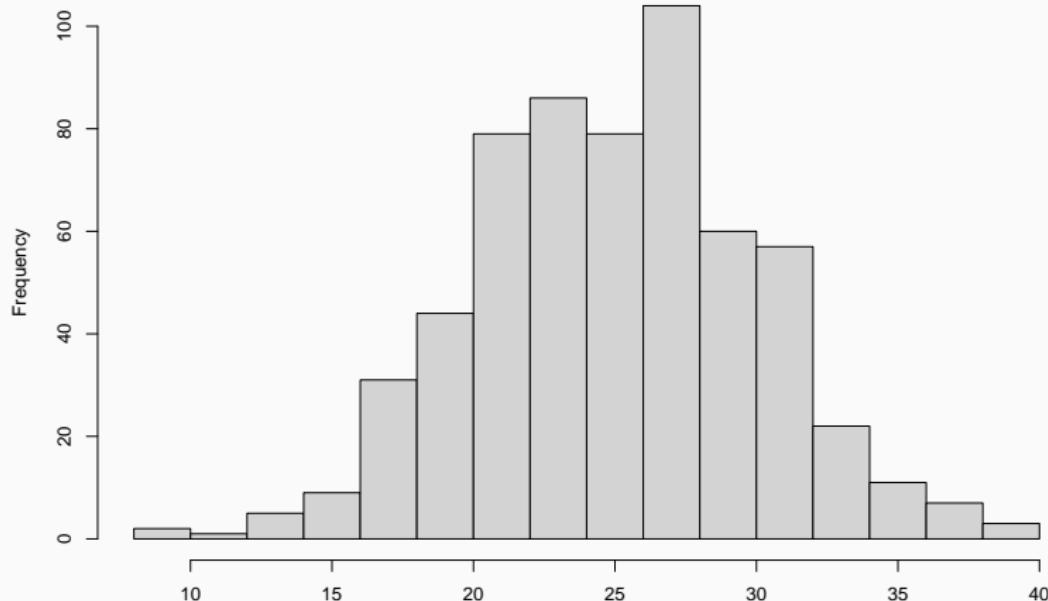


Describing the location / central tendency



Describing Variation / Spread

Minimum, Maximum, Range



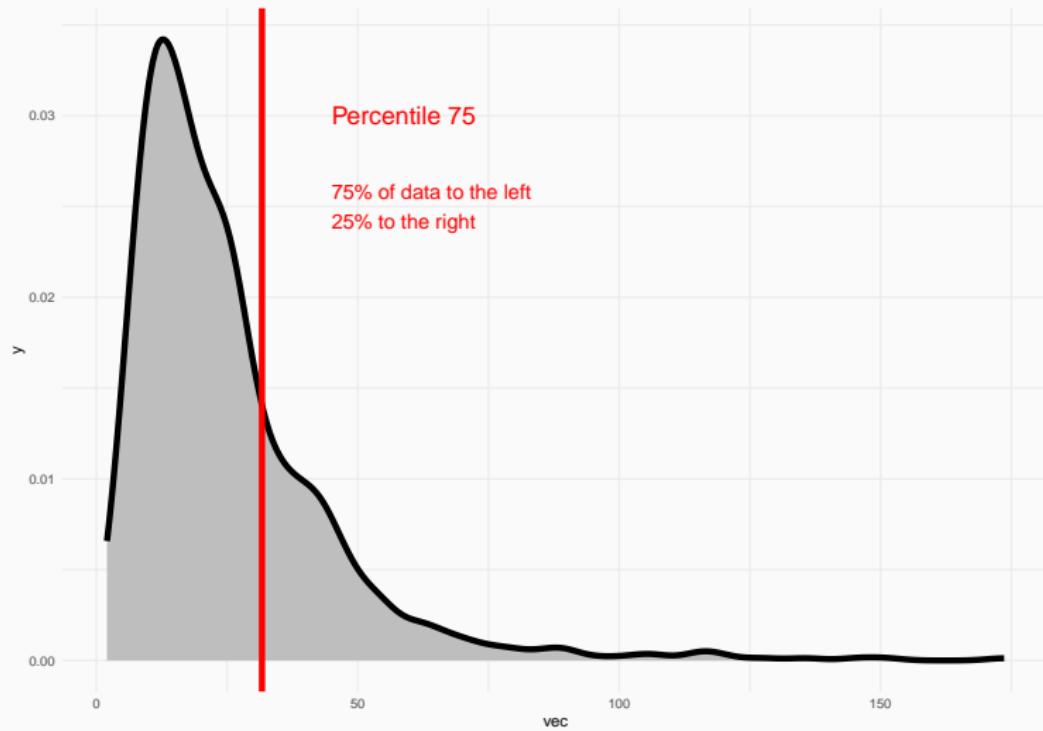
Minimum = 9.2

Maximum = 41.9

Range = 9.2, 41.9

Quantiles

Quartiles, Percentiles...

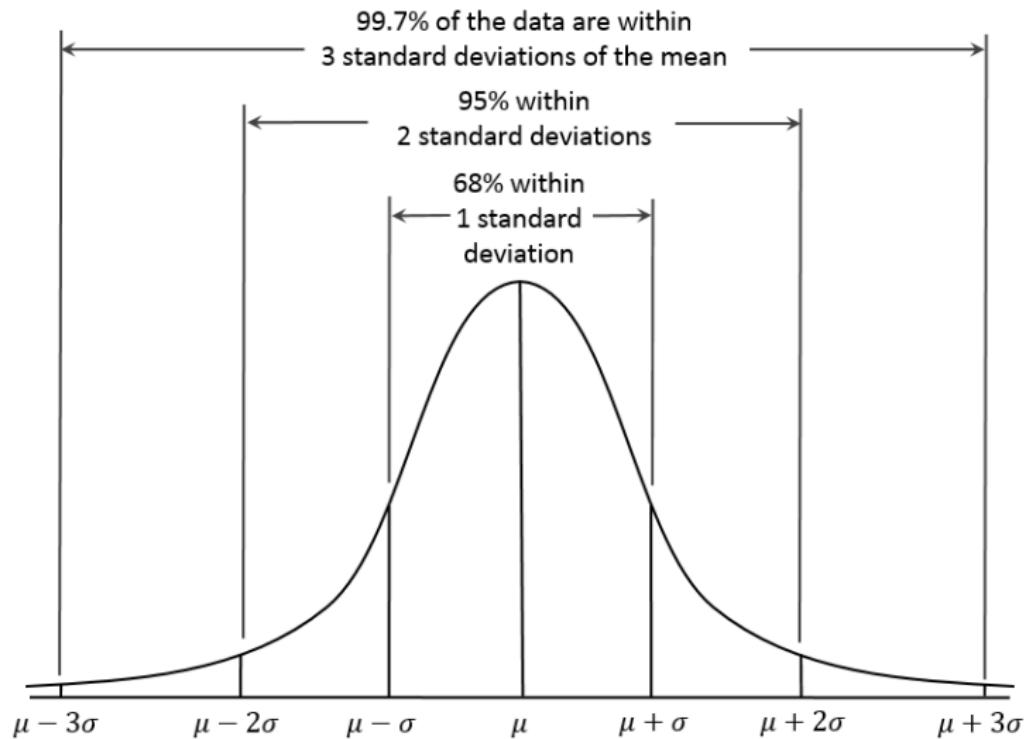


Standard Deviation

Average distance between data points and the mean

$$SD = \sqrt{\frac{\sum (x - \mu)^2}{n - 1}}$$

In a Normal distribution

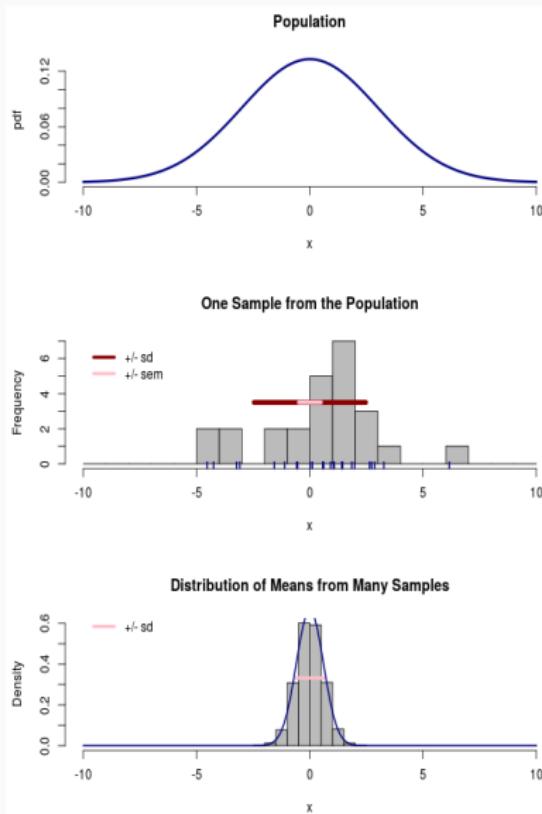


Standard Error of the Mean

$$SEM = \frac{SD}{\sqrt{n}}$$

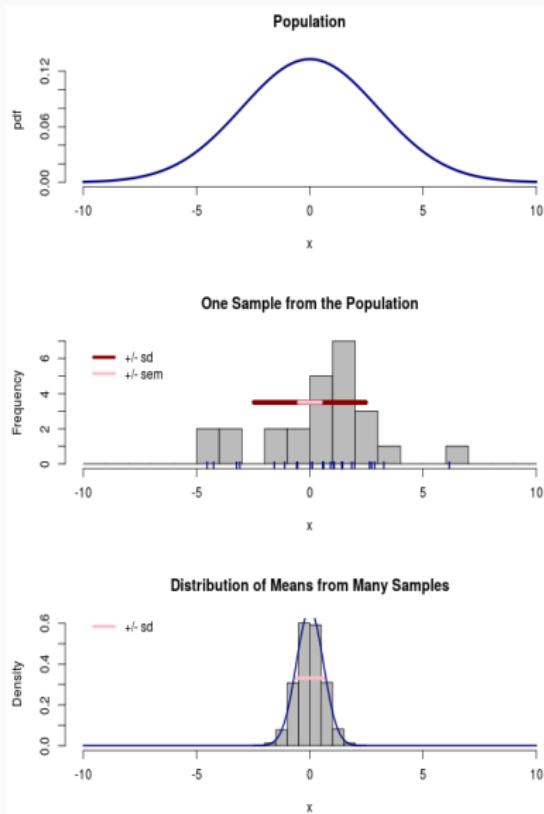
Estimates uncertainty (spread) of the parameter 'mean'

Relationship between SD and SEM



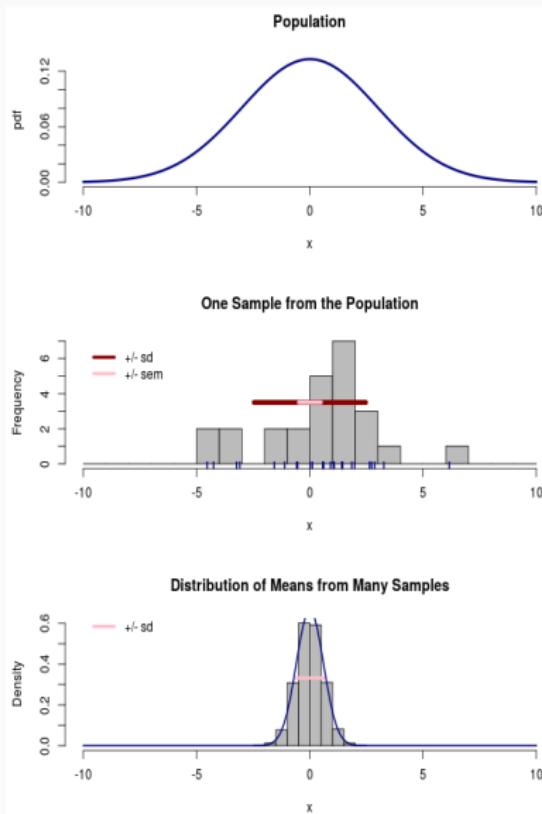
- https://gallery.shinyapps.io/sampling_and_stderr/

Relationship between SD and SEM



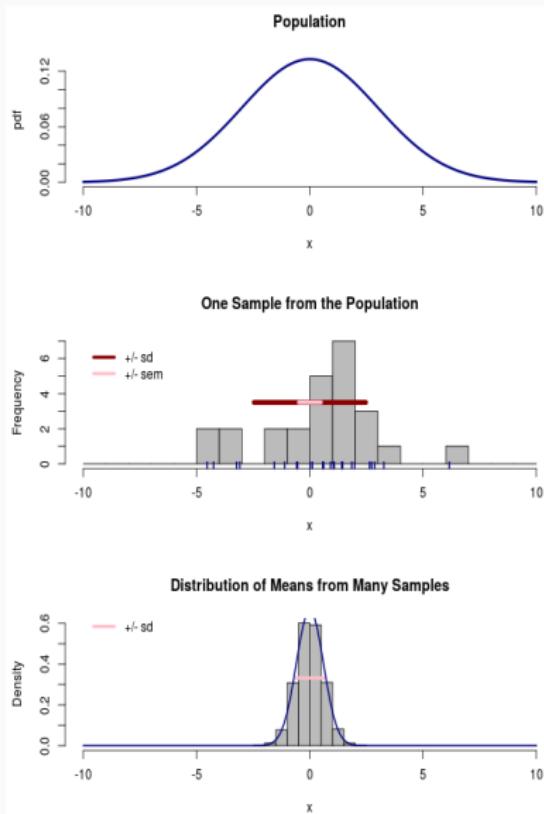
- https://gallery.shinyapps.io/sampling_and_stderr/
- SD quantifies scatter in population

Relationship between SD and SEM



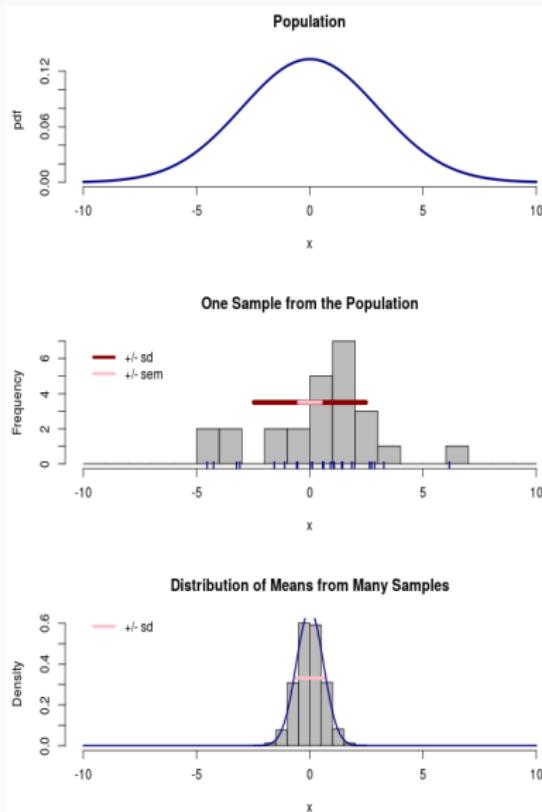
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Relationship between SD and SEM

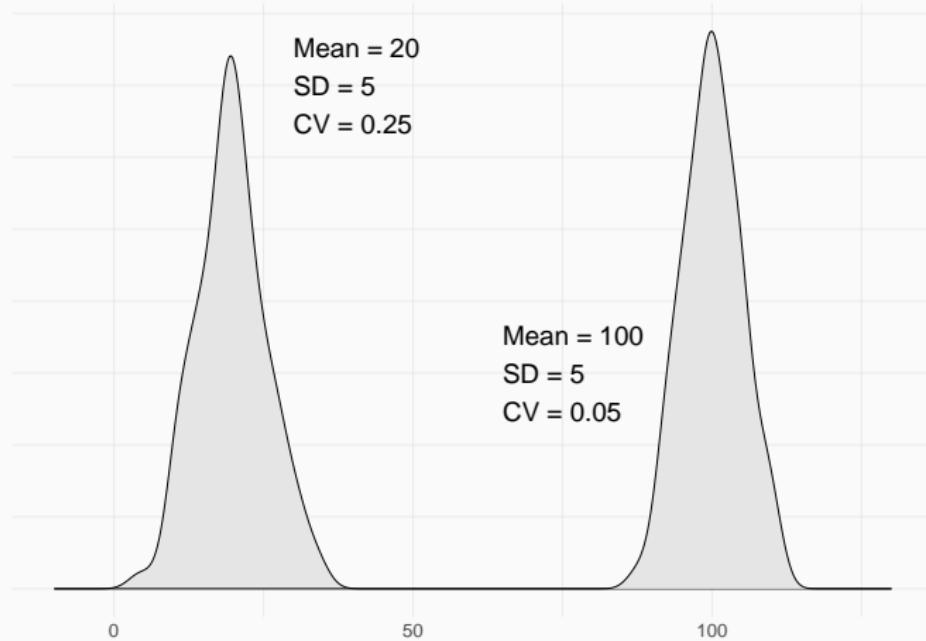


- https://gallery.shinyapps.io/sampling_and_stderr/
- SD quantifies scatter in population
- SEM quantifies uncertainty in parameter estimate (population mean)
- $\text{SEM} = \text{SD}/\sqrt{n}$
- SEM decreases with sample size (mean better known), SD does not.

Coefficient of Variation

Facilitates comparing spread of distributions with different means

$$CV = \frac{SD}{mean}$$

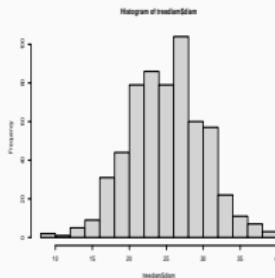


Summarise a distribution

Central tendency / location

- mean (average)

Variation / Spread

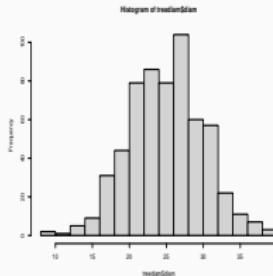


Summarise a distribution

Central tendency / location

- mean (average)
- median (50% percentile)

Variation / Spread

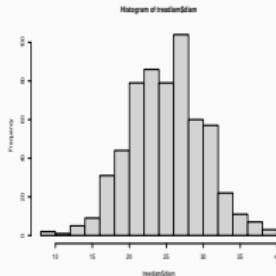


Summarise a distribution

Central tendency / location

- mean (average)
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- mode (most frequent value)

Variation / Spread



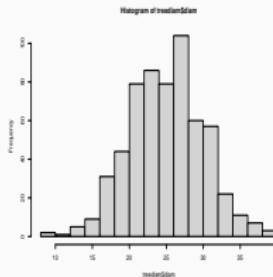
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Variation / Spread

- min, max, range



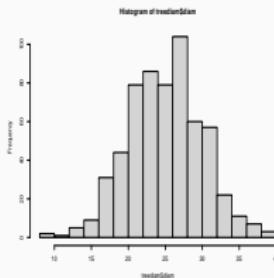
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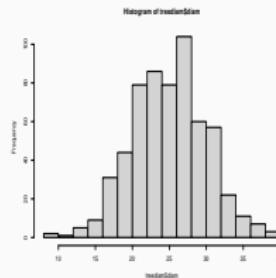
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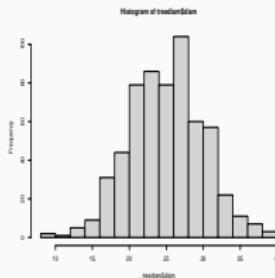
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- standard error of the mean



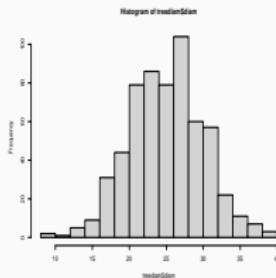
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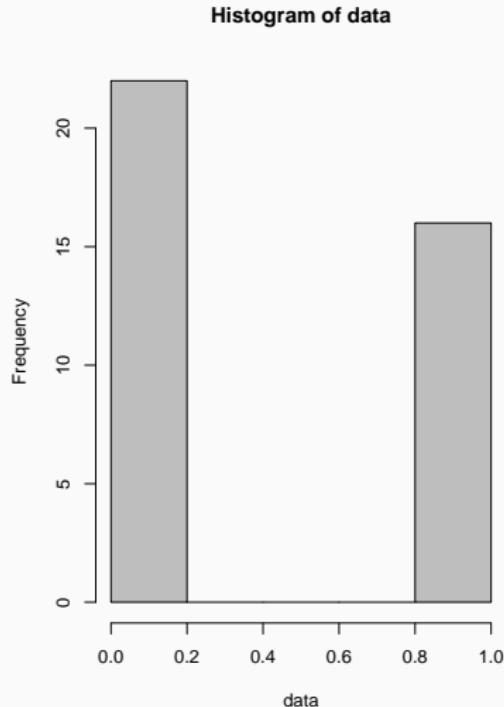
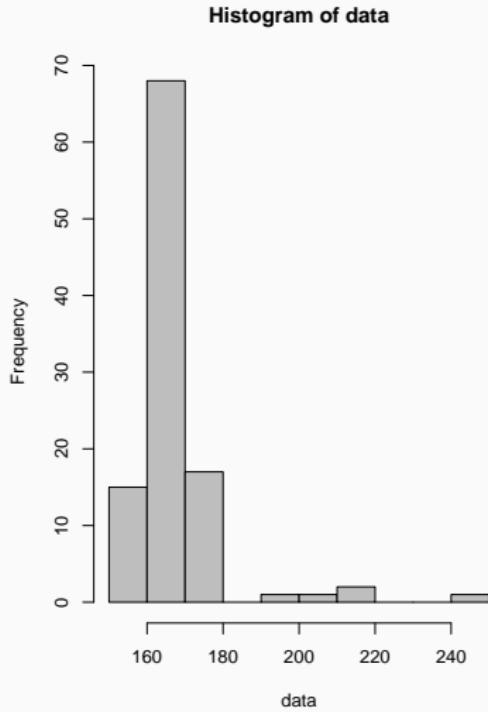
Variation / Spread

- min, max, range
- quantiles (quartiles, percentiles...)
- standard deviation
- standard error of the mean
- coefficient of variation



What statistical descriptors are best? (and why)

<https://pollev.com/franciscorod726>



Sampling, confidence intervals, likelihood and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples**.

Then we make inferences from sample to population.



If we sample 30 trees in our neighbourhood...

Can we extrapolate results to

- whole neighbourhood?
- whole city?
- whole country?
- the world?

What's the **suitable population** to make inferences given this sample?

Understanding confidence intervals

- Summarise **uncertainty** in parameter estimates.

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- Like person who tells truth 95% of the time, but we can't tell if a particular statement is true.
- It's a frequentist, long-run property.
- To read more: [Morey et al \(2015\)](#)

What happens if we increase sample size?

<https://rpsychologist.com/d3/CI/>

- CI width *decreases*...

What happens if we increase sample size?

<https://rpsychologist.com/d3/CI/>

- CI width *decreases*...
- but still 5% of CIs will NOT contain true mean!

If the 95% CI of X is (120, 150)...

- There is a 95% probability that X lies between 120 and 150

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If the 95% CI of X is (120, 150)...

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If the 95% CI of X is (120, 150)...

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- If we repeated the experiment, 95% of the CIs would contain the true value of X
- The probability that X is greater than 0 is at least 95%
- The probability that X equals 0 is smaller than 5%

<https://pollev.com/franciscorod726>

Bayesian credible intervals

- Bayesian **credible** intervals do give the probability that true parameter value is contained within them.

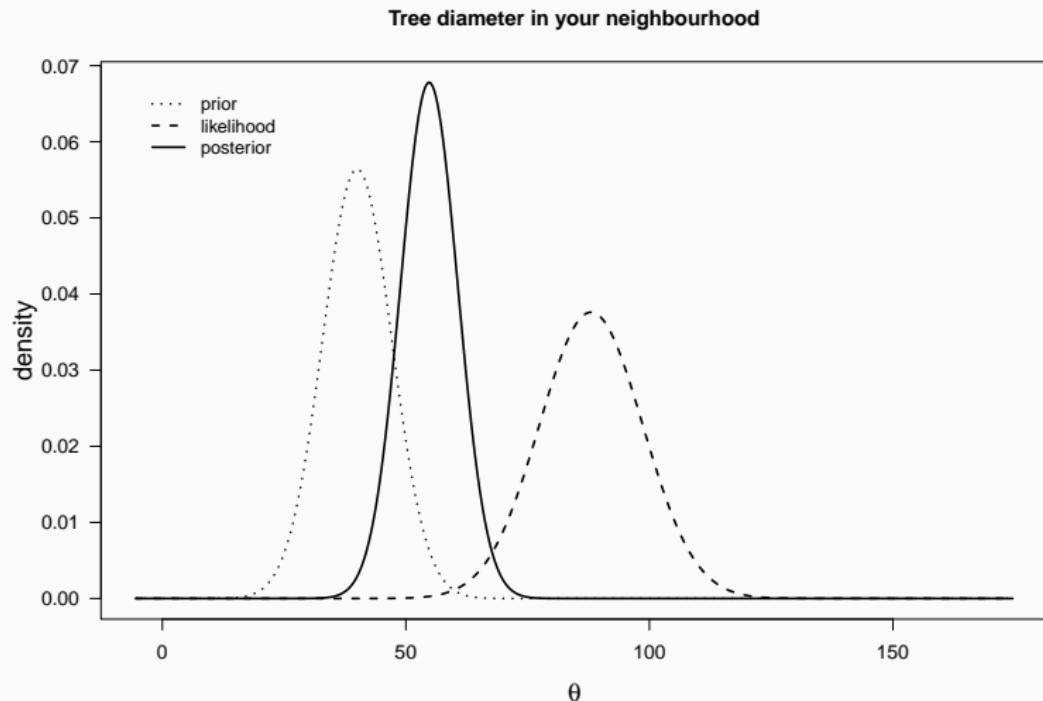
Bayesian credible intervals

- Bayesian **credible** intervals do give the probability that true parameter value is contained within them.
- Frequentist CIs and Bayesian credible intervals can be similar, but not always.

Bayesian inference: prior, posterior, and likelihood

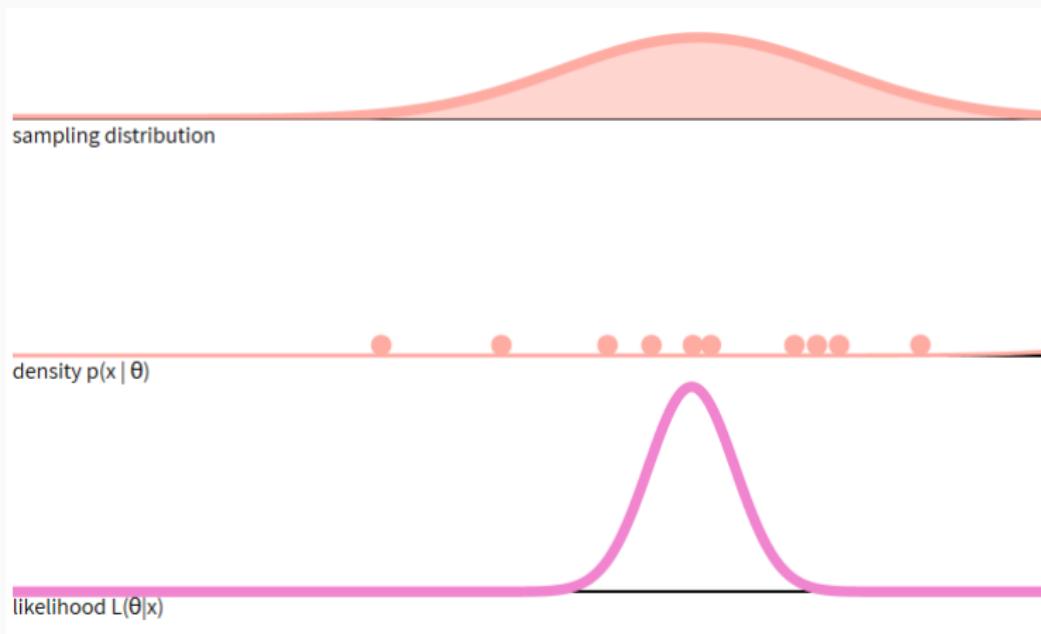
$$P(\text{Unknown} | \text{Data}) \propto P(\text{Data} | \text{Unknown}) \times P(\text{Unknown})$$

Posterior \propto Likelihood \times Prior

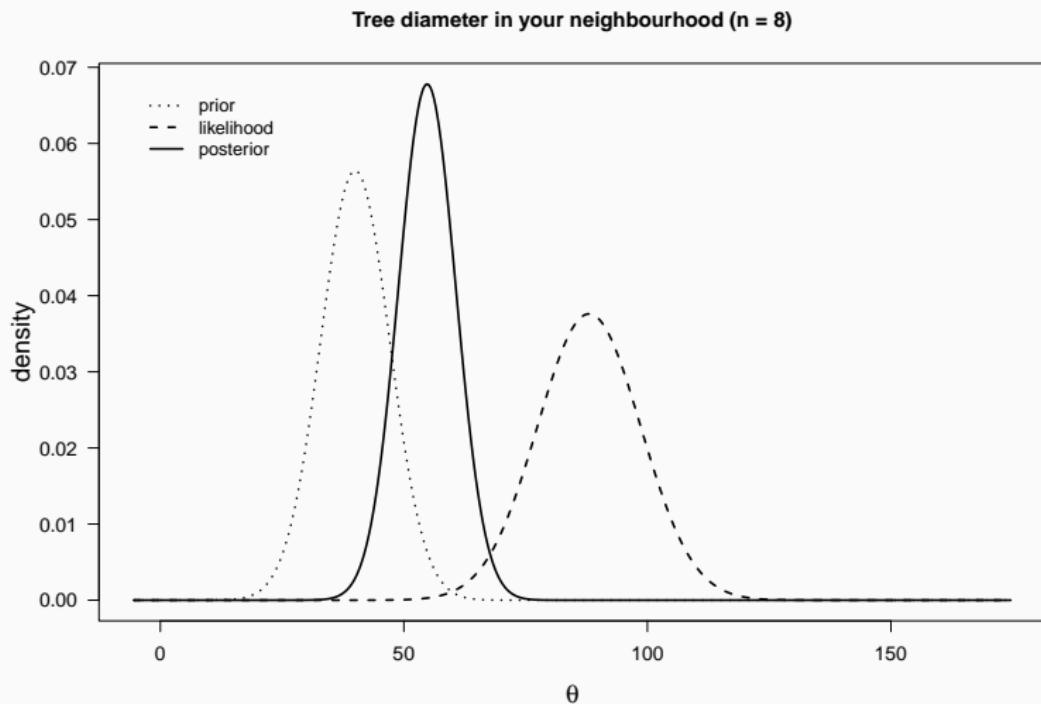


What is the likelihood?

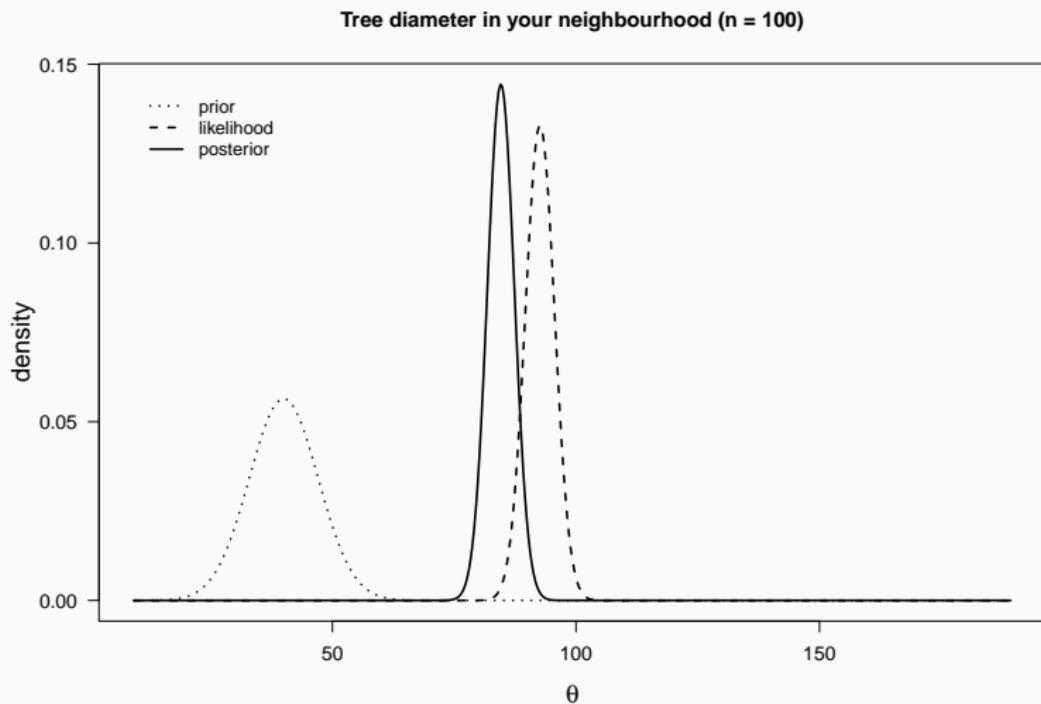
$$L(\theta|x) = P(x|\theta)$$



Bayesian inference: prior and likelihood produce posterior



With increasing sample size, likelihood dominates prior



More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference

More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference
- Bayesian Demo

More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference
- Bayesian Demo
- Bayesian inference for a population mean

More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference
- Bayesian Demo
- Bayesian inference for a population mean
- Normal

More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference
- Bayesian Demo
- Bayesian inference for a population mean
- Normal
- Binomial

More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference
- Bayesian Demo
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- Binomial
- Own data

More apps to introduce Bayesian inference

- Wagenmaker's first lesson in Bayesian inference
- Bayesian Demo
- Bayesian inference for a population mean
- Normal
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- Own data
- Bayesian t-test

Bayesian statistics in practice

- Integrate information (prior)

Bayesian statistics in practice

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- Prior regularises unlikely estimates from data

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Bayesian statistics in practice

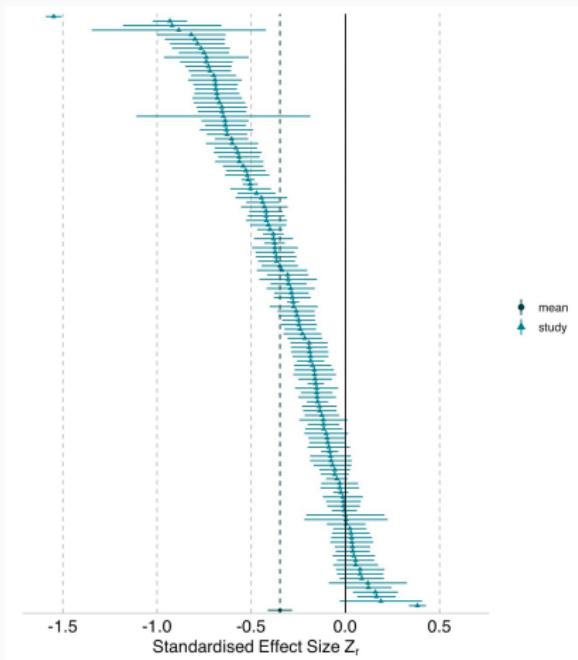
- Integrate information (prior)
- Prior regularises unlikely estimates from data
- Large dataset -> prior effect diminishes
- Uncertainty / Propagate errors

Uncertainty!

Even with same data, different teams reach different conclusions!

How does sibling competition affect nestling growth in blue tits?

Most teams found negative effect, but large variation in effect size. Some teams no or positive effect



73 teams testing the same hypothesis with the same data

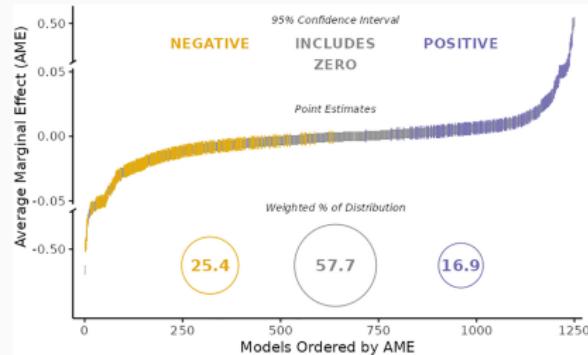
PNAS

RESEARCH ARTICLE

SOCIAL SCIENCES

OPEN ACCESS

Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty



"This reveals a **universe of uncertainty** that remains hidden when considering a single study in isolation"

"These results call for greater **epistemic humility** and **clarity** in reporting scientific findings"

Experimental design

How would you evaluate fertilizer effect?

Discuss with partner (5')



Experimental design principles

Replication

Replication!



Replication

- Replication is key: we need several samples.

Replication

- Replication is key: we need several samples.
- How many? As much as you can! See [Gelman & Carlin 2014](#).

H. Stern / A. Gelman

The most important aspect of a statistical analysis is not what you do with the data, it's what data you use

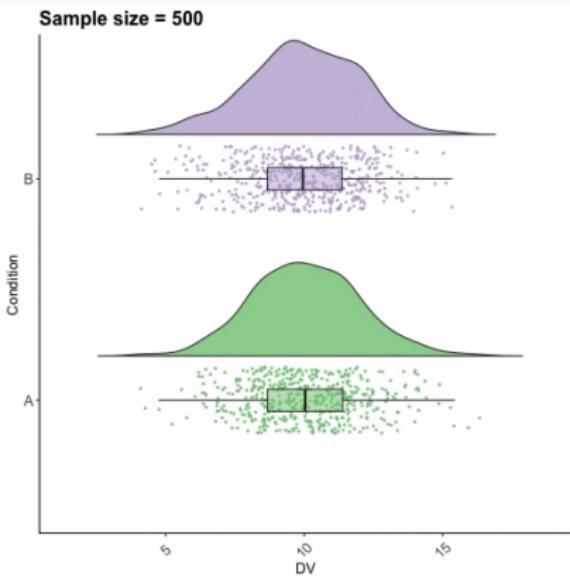
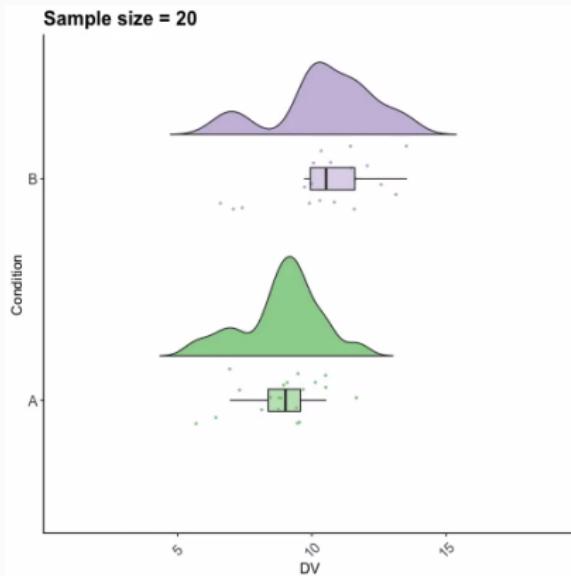
The importance of sample size

- Many studies have **too low sample sizes**.

The importance of sample size

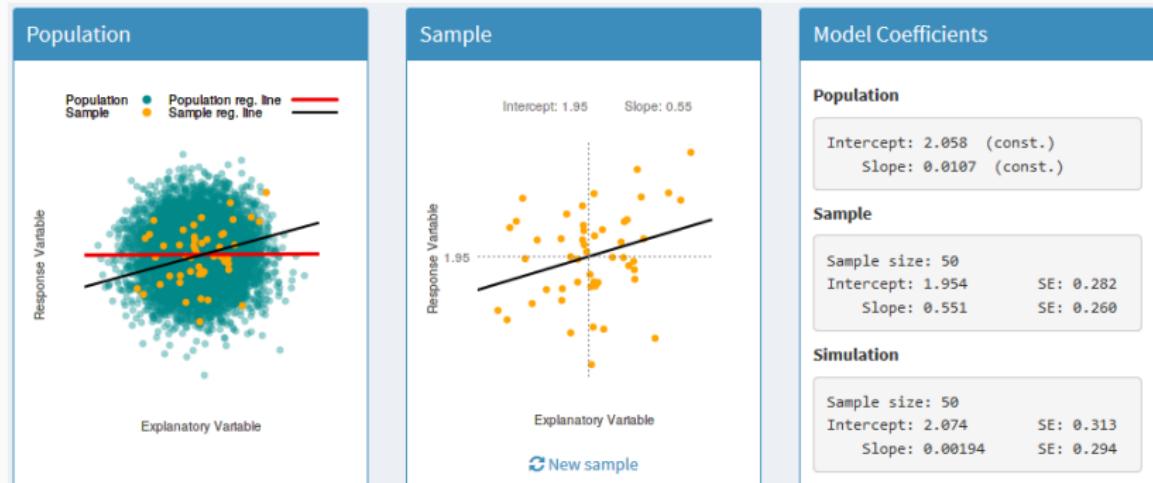
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https://twitter.com/ajstewart_lang/status/1020038488278945797

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<http://statisticalgate.com/regression-simulation/>

Low sample sizes may bias inferences

See [The evolution of correlations](#)

Stopping rules

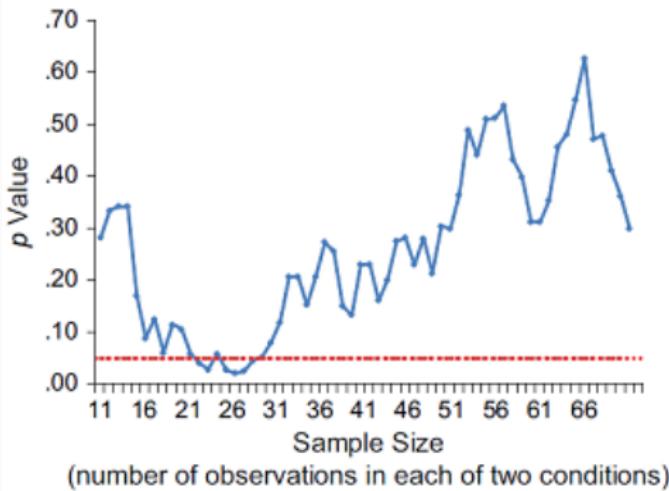


Fig. 2. Illustrative simulation of p values obtained by a researcher who continuously adds an observation to each of two conditions, conducting a t test after each addition. The dotted line highlights the conventional significance criterion of $p \leq .05$.

Sample size estimation

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Sample size estimation

Calculating sample size for Gaussian (Normal) response model:

- expected mean: 30
- expected sd: 10
- 10 parameters (predictors)
- expected R-squared: 0.2

```
library(pmsampsize)
pmsampsize(type = "c", parameters = 10, intercept = 30, sd = 10, rsquared = 0.2)
```

NB: Assuming 0.05 acceptable difference in apparent & adjusted R-squared

NB: Assuming MMOE <= 1.1 in estimation of intercept & residual standard deviation

SPP - Subjects per Predictor Parameter

	Samp_size	Shrinkage	Parameter	Rsq	SPP
Criteria 1	313	0.900	10 0.2	31.3	
Criteria 2	161	0.827	10 0.2	16.1	
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Final	313	0.900	10 0.2	31.3	

Minimum sample size required for new model development based on user inputs = 313

* 95% CI for intercept = (29.69, 30.31), for sample size n = 313

Sample size estimation

Calculating sample size for binary response model:

- expected prevalence: 0.1
- 20 parameters (predictors)
- expected R-squared: 0.2

```
library(pmsampsize)
pmsampsize(type = "b", parameters = 20, prevalence = 0.1, rsquared = 0.2)
```

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NB: Assuming 0.05 margin of error in estimation of intercept

NB: Events per Predictor Parameter (EPP) assumes prevalence = 0.1

	Samp_size	Shrinkage	Parameter	Rsq	Max_Rsq	EPP
Criteria 1	796	0.900	20	0.2	0.48	3.98
Criteria 2	738	0.893	20	0.2	0.48	3.69
Criteria 3	139	0.900	20	0.2	0.48	0.70
Final	796	0.900	20	0.2	0.48	3.98

Minimum sample size required for new model development based on user inputs = 796,
with 80 events (assuming an outcome prevalence = 0.1) and an EPP = 3.98

Randomization

Randomization



Randomization

- Haphazard \neq Random

Randomization

- Haphazard \neq Random
- Stratify: randomize within groups (e.g. species, soil types)

Controls

Have controls

- Untreated individuals, plots... (assigned randomly, of course).

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- Measure **before & after** treatment.
- Consider **blind designs** to avoid observer bias.

Experimental design principles

1. Replication

Experimental design principles

1. Replication
2. Randomization

Experimental design principles

1. Replication
2. Randomization
3. Controls

To read more

- Ruxton & Colegrave. Experimental Design for the Life Sciences.
OUP

The importance of sample size & good study design

The most important aspect of a statistical analysis is not what you do with the data, it's what data you use

H. Stern / A. Gelman

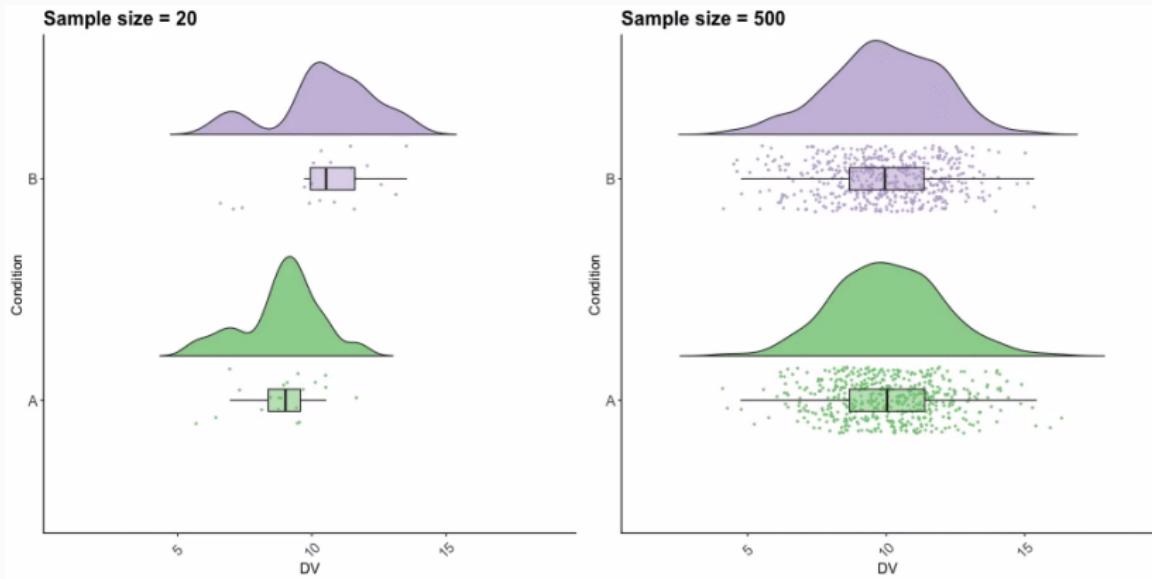
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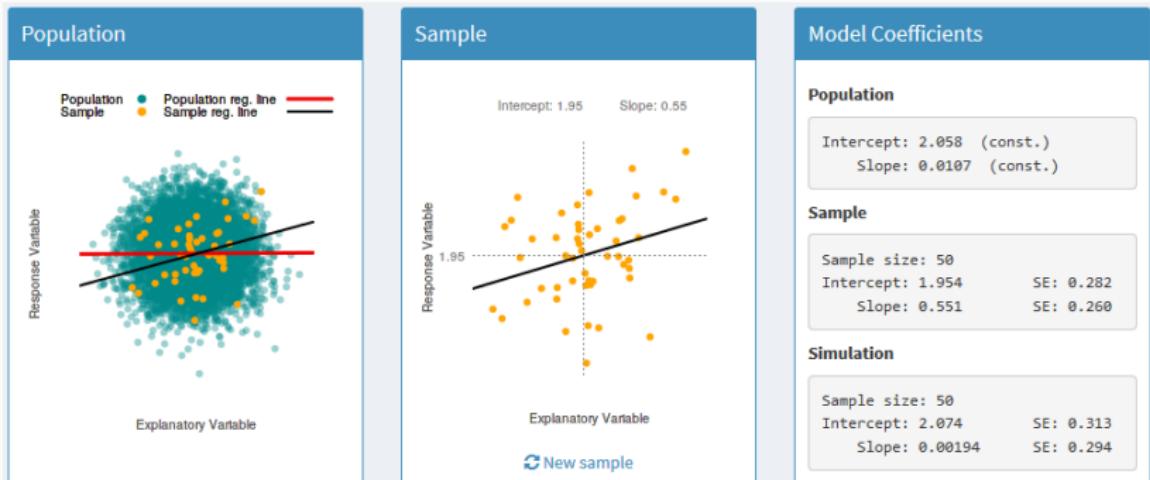
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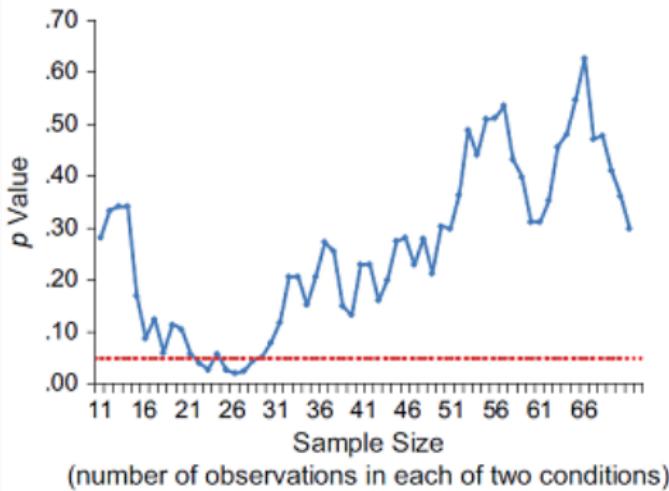


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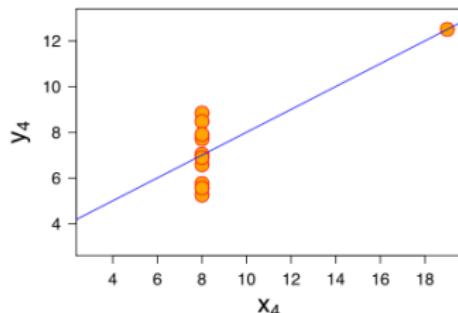
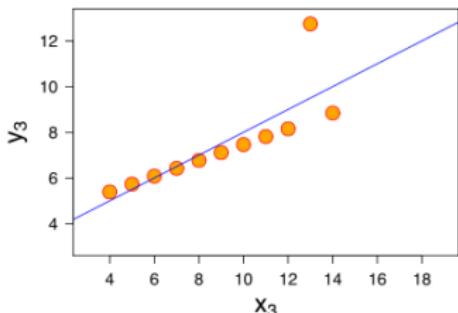
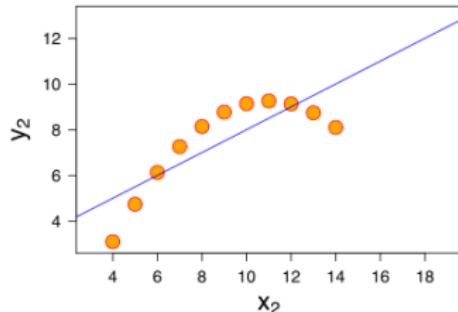
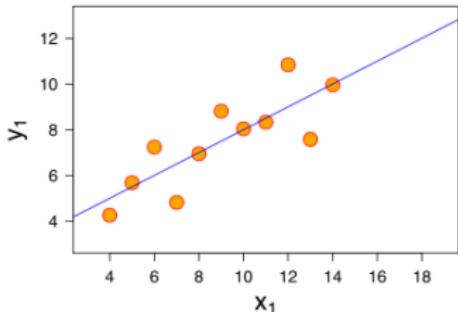
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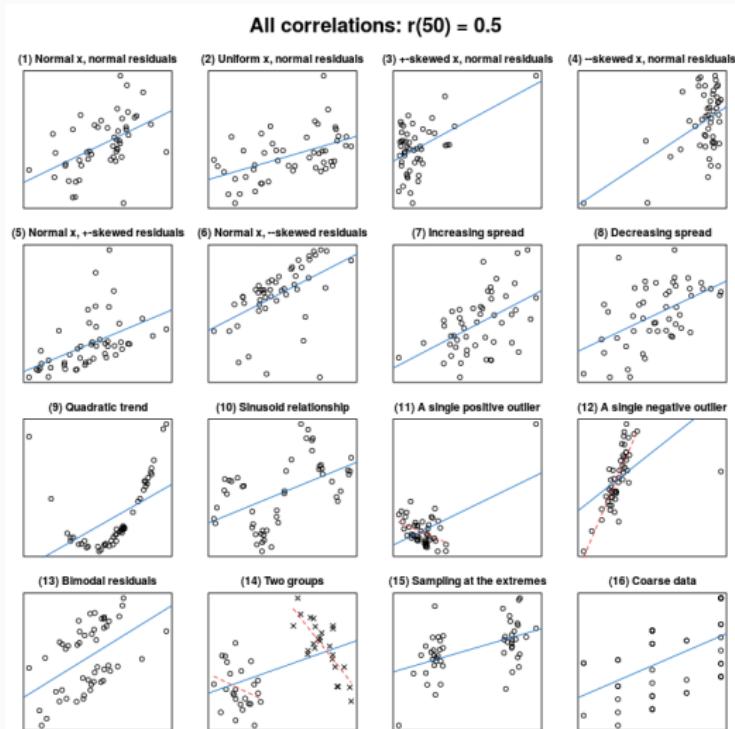
The importance of visualisation

Visualisation of data & models is
key

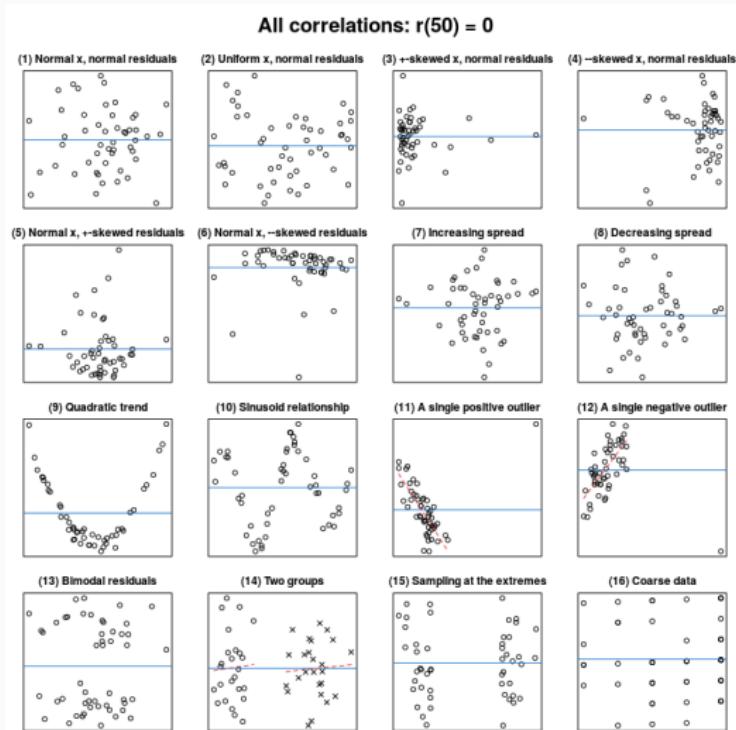
Always plot data and models



Don't use statistics blindly: Visualise

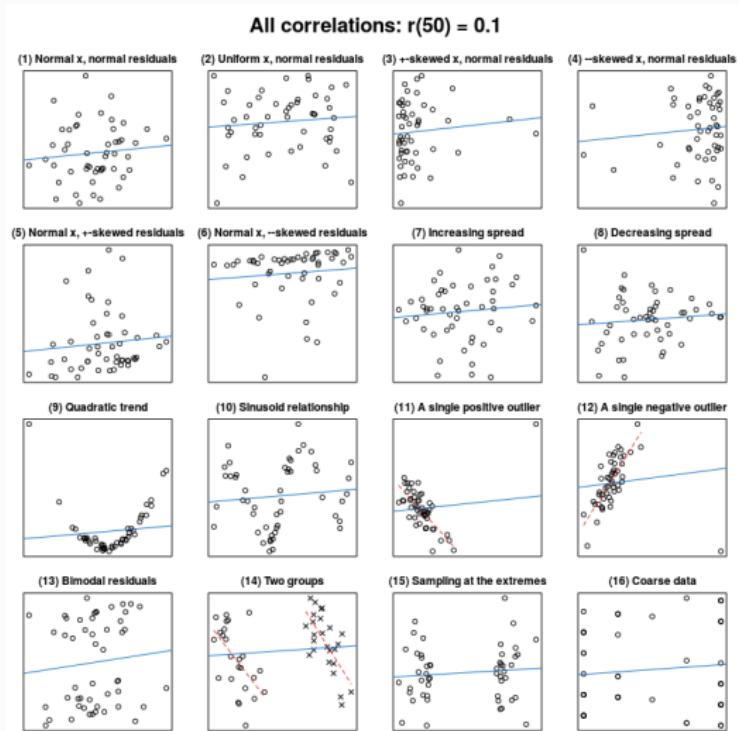


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<https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like>

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Plot. Check models. Plot. Check assumptions. Plot.

Lavine 2014 Ecology

Inference from observational studies

News: Hamburgers increase risk of heart attack

- In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.

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- <https://pollev.com/franciscorod726>

Bigger flowers increase reproductive success

- We found that plants with big flowers produced 30% more seeds...

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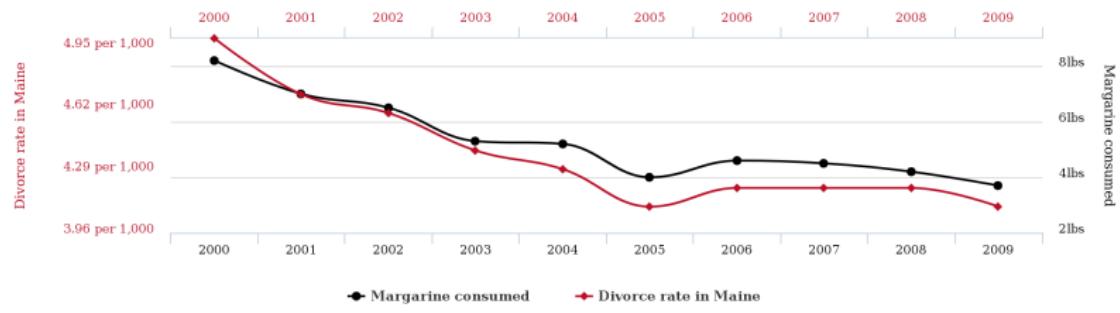
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Correlation vs Causation

Divorce rate in Maine correlates with Per capita consumption of margarine



<http://tylervigen.com/spurious-correlations>

Hypothesis testing

NHST concepts

Null and alternative hypotheses

- Tell me...

Null and alternative hypotheses

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- **Null hypothesis:** there is no difference between groups.

Null and alternative hypotheses

- Tell me...
- **Null hypothesis:** there is no difference between groups.
- **Alternative hypothesis:** groups are different.

In biology, everything is somewhat different

Are there any differences? A non-sensical question in ecology

Alejandro Martínez-Abraín

IMEDEA (CSIC-UIB), C/Miquel Marquès 21, 07190 Esporles, Majorca, Spain

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ABSTRACT

One of the main questions that ecologists pose in their investigations includes the analysis of differences in some trait between two or more populations. I argue here that asking whether there are differences or not between populations is biologically irrelevant, since no two living things are ever equal. On the contrary the appropriate question to pose is how large differences are between populations. That is, we urge a shift in interest from statistical significance to biological relevance for proper knowledge accumulation. I empha-

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 - Null hypothesis is true
 - No uncontrolled sources of bias (measurement or programming error, p-hacking, etc)

A very small p-value does NOT mean H₀ is automatically false

- A famous experiment found neutrinos faster than light

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- p-value < 10^{-7} -> reject null hypothesis of equal speed

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- In reality, measurement error (loose cable)

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- **Low P-value:** data unlikely if every model assumption (including H_0) were true.

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- See [Greenland et al 2016](#)

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The image shows a screenshot of a website. At the top, there is a dark red header bar with a 'MENU' dropdown, the 'nature' logo, and a 'Subs' button. Below the header, the word 'EDITORIAL' is followed by a date '20 MARCH 2019'. The main title of the article is 'It's time to talk about ditching statistical significance'.

<https://doi.org/10.1038/d41586-019-00857-9>

Are these two groups different?

```
t.test(group.A, group.B)
```

Welch Two Sample t-test

data: group.A and group.B

t = -1.6845, df = 6.3414, p-value = 0.1404

alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:

-32.85733 5.85733

sample estimates:

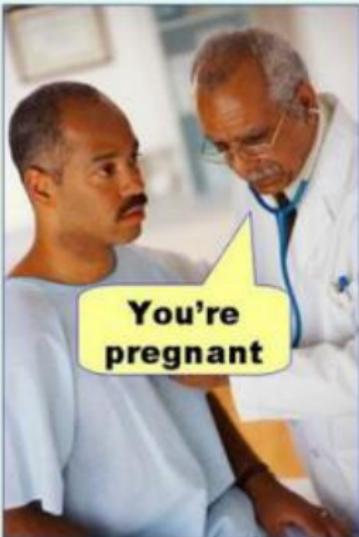
mean of x mean of y

164.8 178.3

<https://pollev.com/franciscorod726>

Rejecting hypotheses: two types of error

Type I error
(false positive)



Type II error
(false negative)



Rejecting hypotheses: two types of error

Statistics: Hypothesis Test	Null Hypothesis is True	Null Hypothesis is False
Reject Null Hypothesis	Type I Error	Correct
Fail to Reject Null Hypothesis	Correct	Type II Error

POWER: Probability of detecting true difference (rejecting H₀ when it's false).

Is this coin biased?

```
[1] 0 1 1 0 0 0 1 0 0 0
```

```
1-sample proportions test with continuity correction
```

```
data: sum(coin) out of ntrials, null probability 0.5
X-squared = 0.9, df = 1, p-value = 0.3428
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
0.08094782 0.64632928
sample estimates:
p
0.3
```

<https://pollev.com/franciscorod726>

Understanding NHST

<http://rpsychologist.com/d3/NHST/>

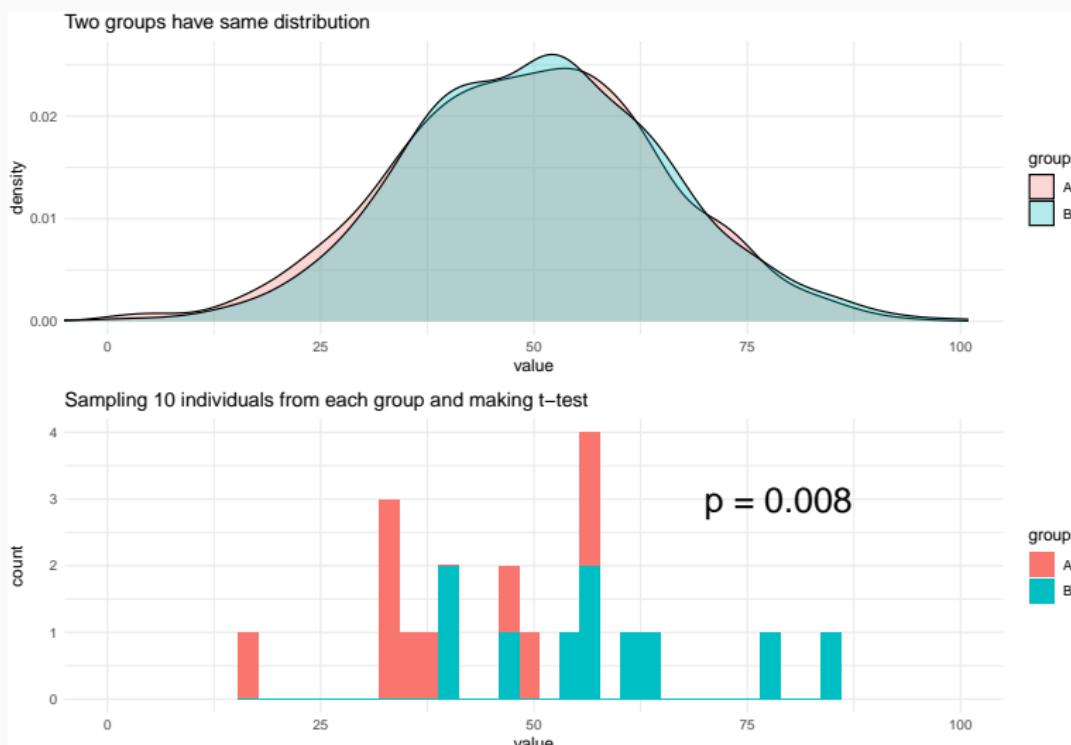
[http://daniellakens.blogspot.com/2017/12/
understanding-common-misconceptions.html](http://daniellakens.blogspot.com/2017/12/understanding-common-misconceptions.html)

NHST and p-values: common pitfalls

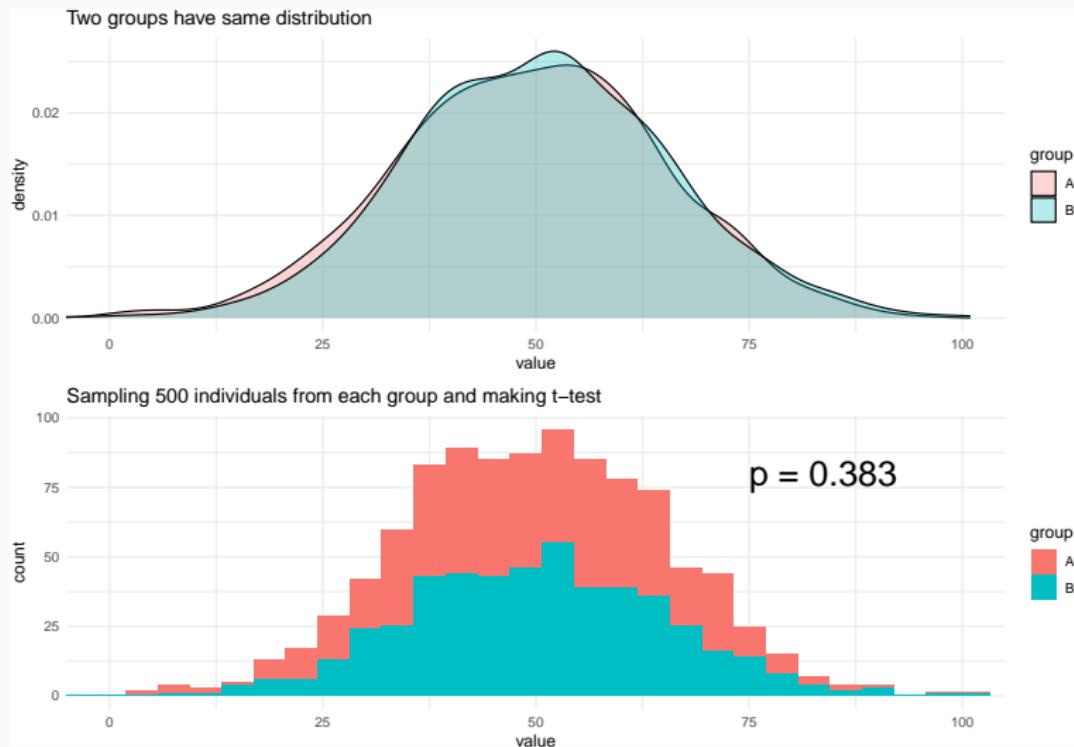
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A significant p-value does not mean we found a true difference

Particularly with low sample sizes



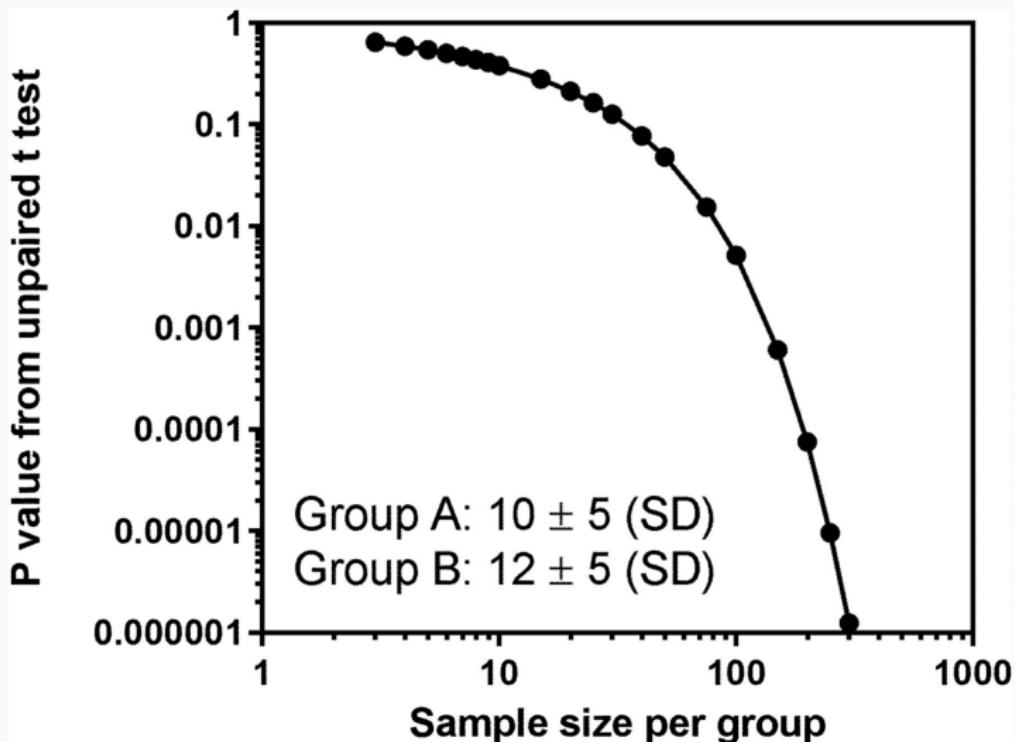
If sample size was larger...



With low sample size (power),
significant p-values
are most likely overestimates

Loken & Gelman 2014, Vasisth et al. 2018

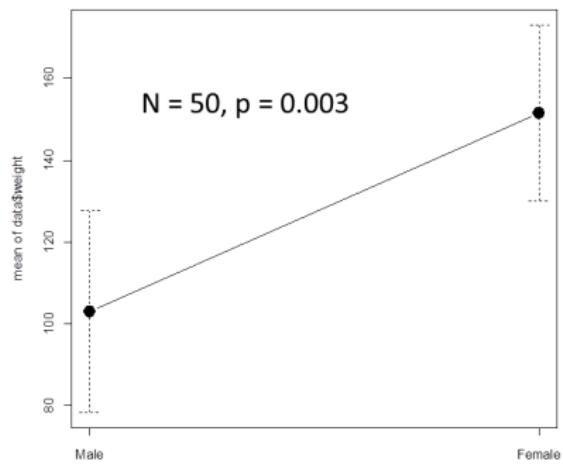
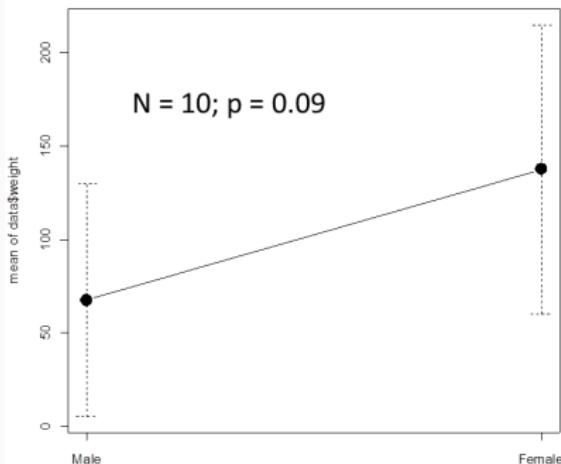
P-value depends on sample size



P-value depends on sample size

Same real difference is detected as significant or not depending on sample size

Real difference = 40 g



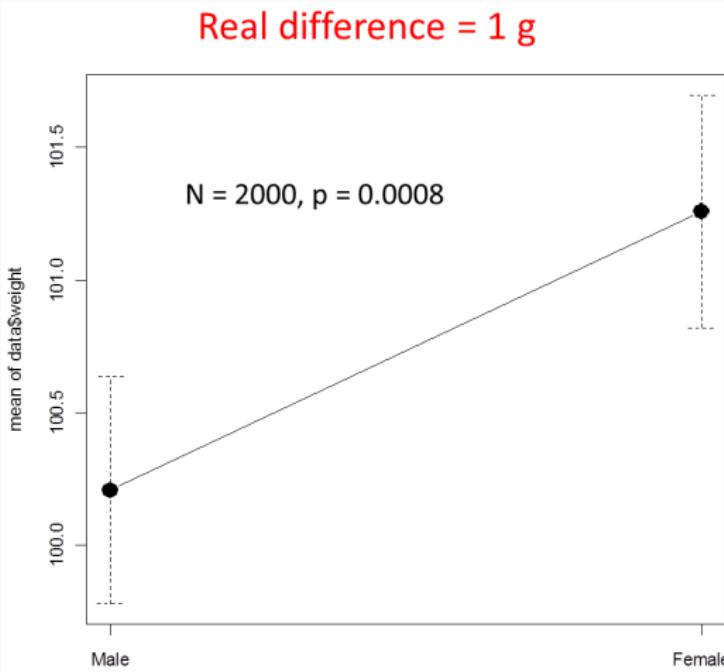
Statistically significant

\neq

biologically important

Statistically significant != biologically important

With big sample size, we can find highly significant but biologically unimportant differences.



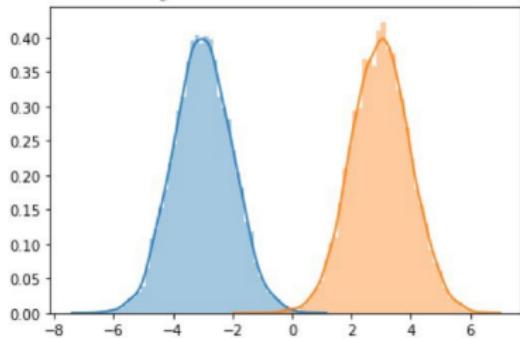
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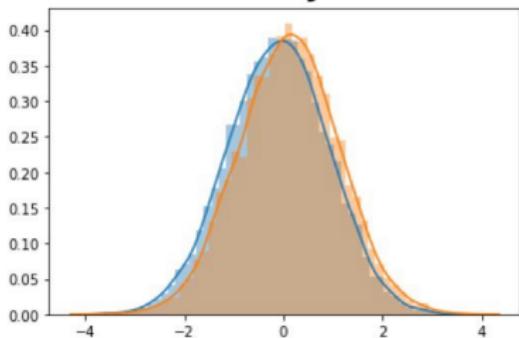
zara weinberg
@weinberz

friendly reminder about $p < 0.0001$:

What you think it means:



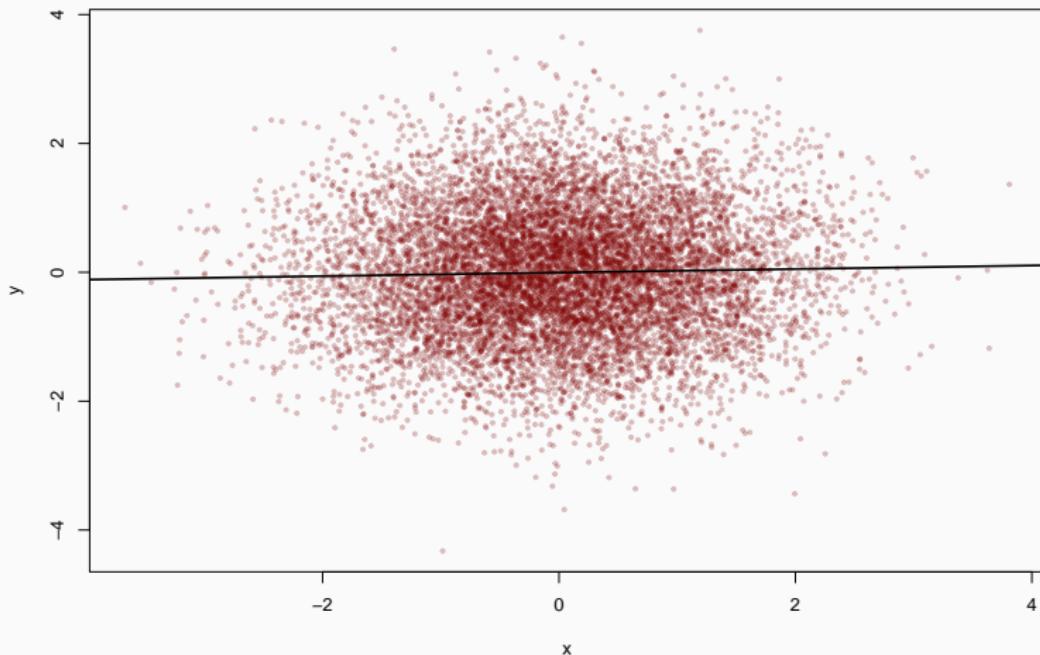
What it actually means:



<https://twitter.com/weinberz/status/1422405165236178947?s=20>

Statistically significant != biologically important

p = 0.005



Statistically significant != biologically important

- Statistically significant = unlikely to be zero

Statistically significant != biologically important

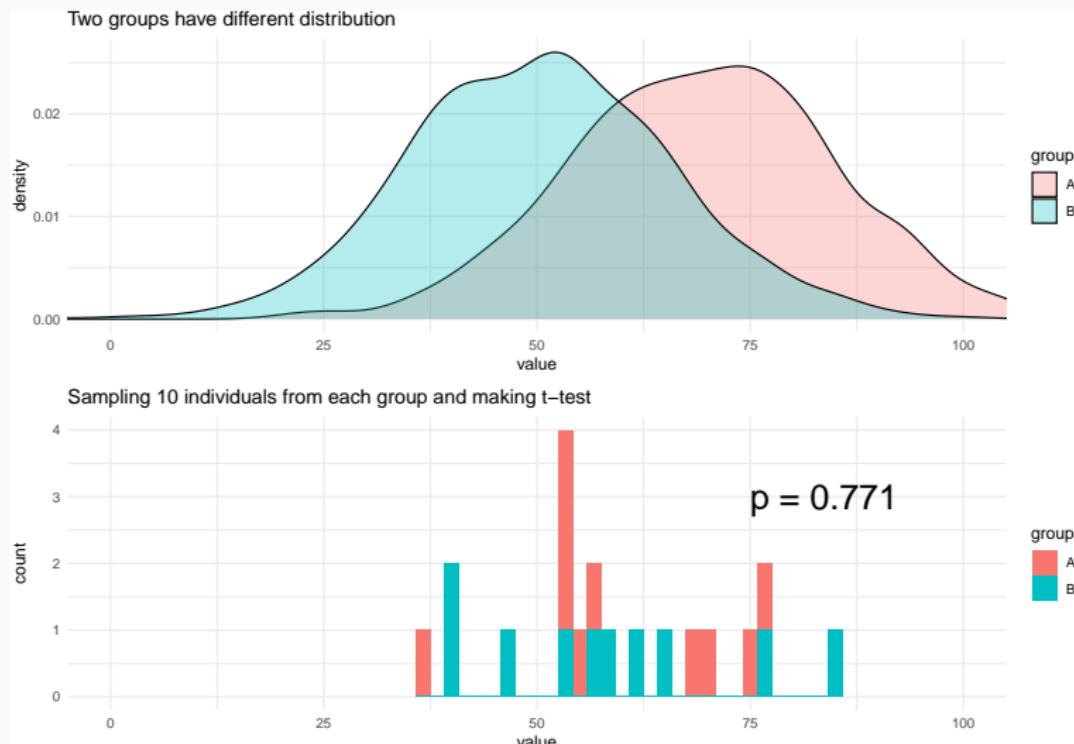
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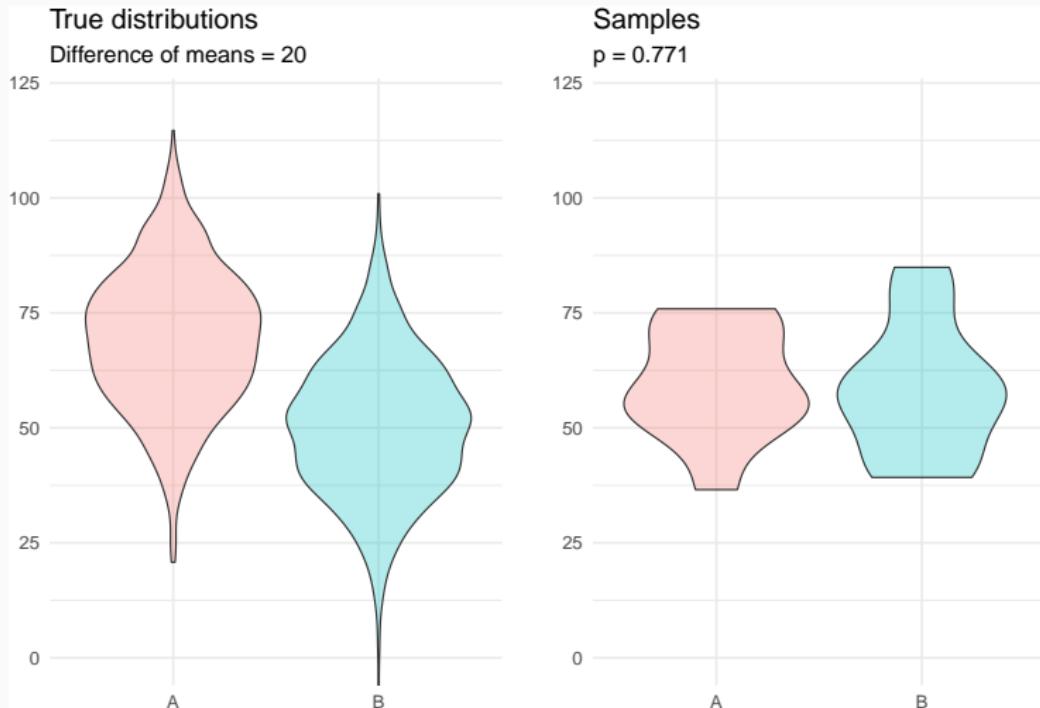
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- Beyond significant/not significant, look at **effect sizes and their uncertainty.**

‘Not significant’
does NOT mean
‘there is no effect’

'Not significant' does NOT mean 'they are equal'



'Not significant' does NOT mean 'there is no effect'



Failure to reject H_0 $\neq H_0$ is true

Absence of evidence \neq Evidence of absence

p-value > 0.05?

- “We were **unable** to find evidence against the hypothesis that $A = B$ with the current sample size” ([Harrell](#))

p-value > 0.05?

- “We were **unable** to find evidence against the hypothesis that A = B with the current sample size” ([Harrell](#))
- “Differences between groups were **not statistically clear**” ([Dushoff et al](#))

Is it safe to allow right turn with red lights?

- Right turn not allowed: 308 accidents



[https://www.statisticsonewrong.com/power.html#
the-wrong-turn-on-red](https://www.statisticsonewrong.com/power.html#the-wrong-turn-on-red)

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- Misinterpretation of underpowered study cost lives



<https://www.statisticsonewrong.com/power.html#the-wrong-turn-on-red>

0.05 is an arbitrary threshold

**The Difference Between “Significant” and “Not Significant” is not
Itself Statistically Significant**

Andrew GELMAN and Hal STERN

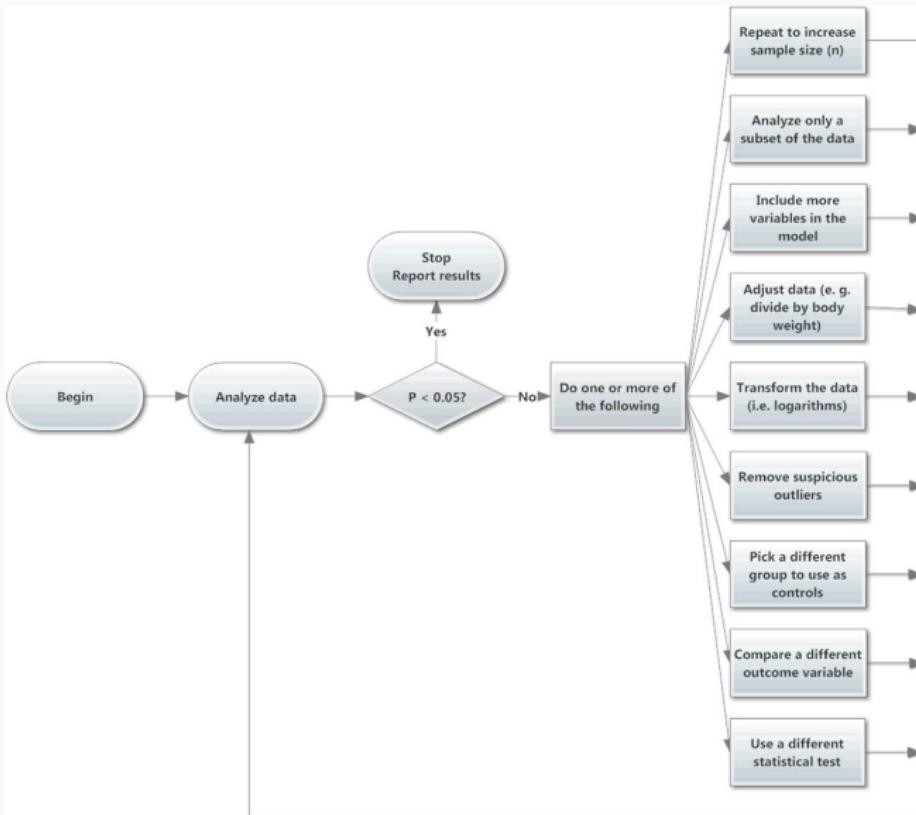
<http://dx.doi.org/10.1198/000313006X152649>

Multiple hypothesis testing



<https://xkcd.com/882>

How to make your results significant: *p*-hacking



How to make your results significant: *p-hacking*

1. Test multiple variables, then report the ones that are significant.

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4. Test different conditions (e.g. different levels of a factor) and report the ones you like.
 - To read more: [Simmons et al 2011](#)

p-hacking: try it yourself

<https://www.shinyapps.org/apps/p-hacker/>

<https://shiny.psy.lmu.de/felix/ShinyPHack/>

How to make your results significant: *p-hacking*

<https://www.youtube.com/watch?v=ZaNtz76dNSI>

ASA statement on p-values

- P-values do not measure the **probability of hypothesis** being true, or the probability that the data were produced by **random chance** alone.

<https://doi.org/10.1080/00031305.2016.1154108>

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- By itself, a p-value does NOT provide a good **measure of evidence** regarding a model or hypothesis.

<https://doi.org/10.1080/00031305.2016.1154108>

Good practice

A must read

Eur J Epidemiol (2016) 31:337–350
DOI 10.1007/s10654-016-0149-3



CrossMark

ESSAY

Statistical tests, *P* values, confidence intervals, and power: a guide to misinterpretations

Sander Greenland¹ · Stephen J. Senn² · Kenneth J. Rothman³ · John B. Carlin⁴ ·
Charles Poole⁵ · Steven N. Goodman⁶ · Douglas G. Altman⁷

<https://doi.org/10.1007/s10654-016-0149-3>

Good read

esa

ECOSPHERE

Applied statistics in ecology:
common pitfalls and simple solutions

E. ASHLEY STEEL,^{1,†} MAUREEN C. KENNEDY,² PATRICK G. CUNNINGHAM,³ AND JOHN S. STANOVICK⁴

<https://doi.org/10.1890/ES13-00160.1>

Also <http://www.statisticsdonewrong.com/>

Good read



Twenty tips for interpreting scientific claims

The New Statistics

Aim for estimation of effects and their uncertainty (SE, CI...)



General Article

The New Statistics: Why and How

Geoff Cumming

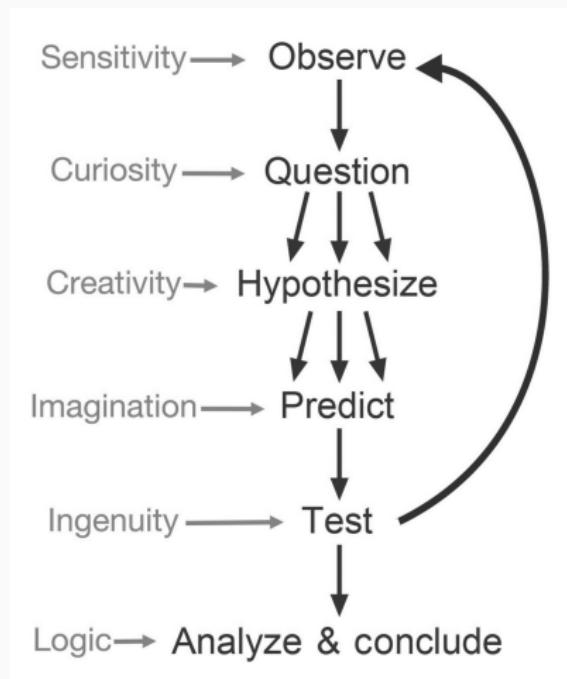
La Trobe University

Psychological Science
2014, Vol. 25(1) 7–29
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DOI: 10.1177/0956797613504966
pss.sagepub.com



<http://dx.doi.org/10.1177/0956797613504966>

Instead of falsifying null model, compare meaningful models



<https://doi.org/10.1242/jeb.104976>

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- Beyond Power Calculations: Assessing Type S (Sign) and Type M (Magnitude) Errors