

An introduction to statistical inference

Francisco Rodríguez-Sánchez

http://bit.ly/frod_san

Why statistics?

To answer questions like...

- ▶ what's the probability that something occurs?

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- ▶ does X influence Y? How much?

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- ▶ what's the probability that something occurs?
- ▶ does X influence Y? How much?
- ▶ can we predict Y knowing X, Z... How well?

To ensure correct inferences

111	451	368	80	46	83	74	29	71	489	43
439	164	94	45	73	38	98	25	75	340	3
235	166	172	54	91	89	40	78	45	731	19
10	30	62	49	32	11	10	10	10	10	10
1.433	895	2.132	2.380	3.860	2.775	1.580	2.000	3.000	3.000	3.000
1.870	2.845	1.001	1.920	1.740	2.981	3.000	1.580	1.580	1.580	1.580
2.427	1	1.33	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
2.424	2.657	1	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
1.692	84	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
1.199	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198
2.032	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198
3	2.032	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198	1.198
35	290	92	285	164	224	224	224	224	224	224
74	243	430	277	175	334	334	334	334	334	334
84	301	249	175	334	334	334	334	334	334	334
17	3.868	2.455	6.303	6.303	6.303	6.303	6.303	6.303	6.303	6.303

DATA

Inference



Bolker et al 2009 TREE:

'311 out of 537 GLMM analyses (58%) used these tools inappropriately'

Figure 1:

To get answers to tough problems

For example...

How many seeds do trees produce?



Figure 2:

Inferring tree fecundity

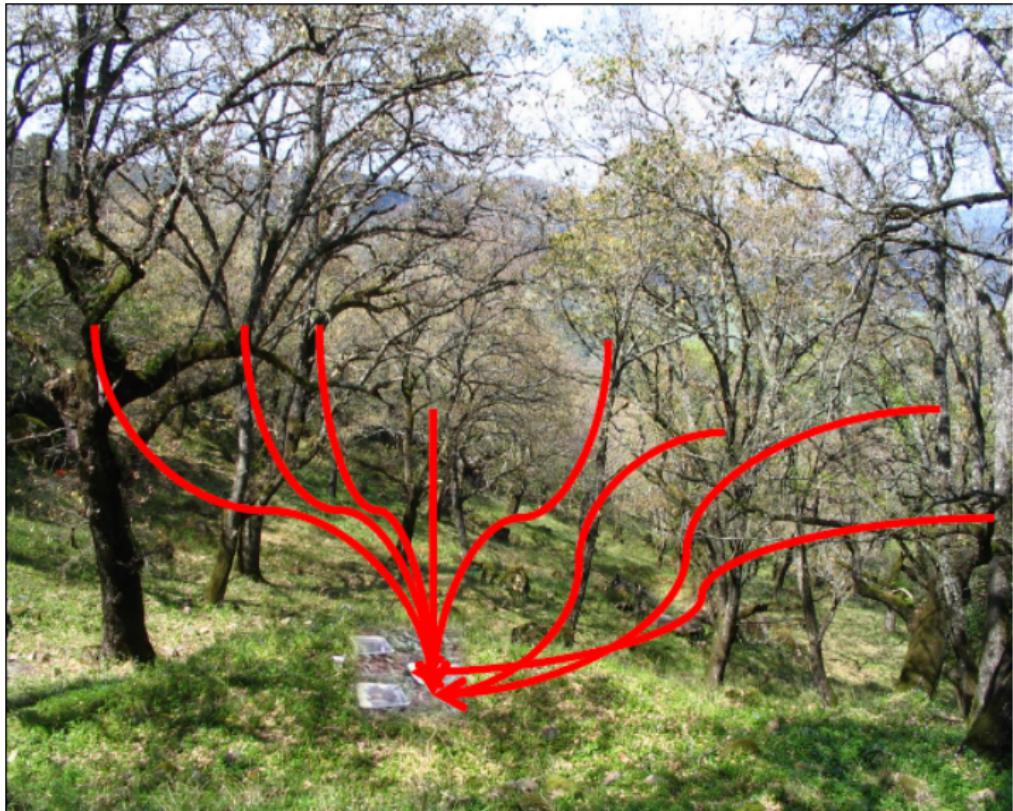


Figure 3:

Course goals

- ▶ **Understand** statistical inference

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- ▶ Avoid **misconceptions**

Course goals

- ▶ **Understand** statistical inference
- ▶ Avoid **misconceptions**
- ▶ Promote **good practices**

The purpose of models is not to fit data but to sharpen thinking

Sam Karlin

Topics

- ▶ Descriptive statistics

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- ▶ Graphics

Topics

- ▶ Descriptive statistics
- ▶ Graphics
- ▶ Sampling

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- ▶ Graphics
- ▶ Sampling
- ▶ Experimental design

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- ▶ Linear models & GLMs

Topics

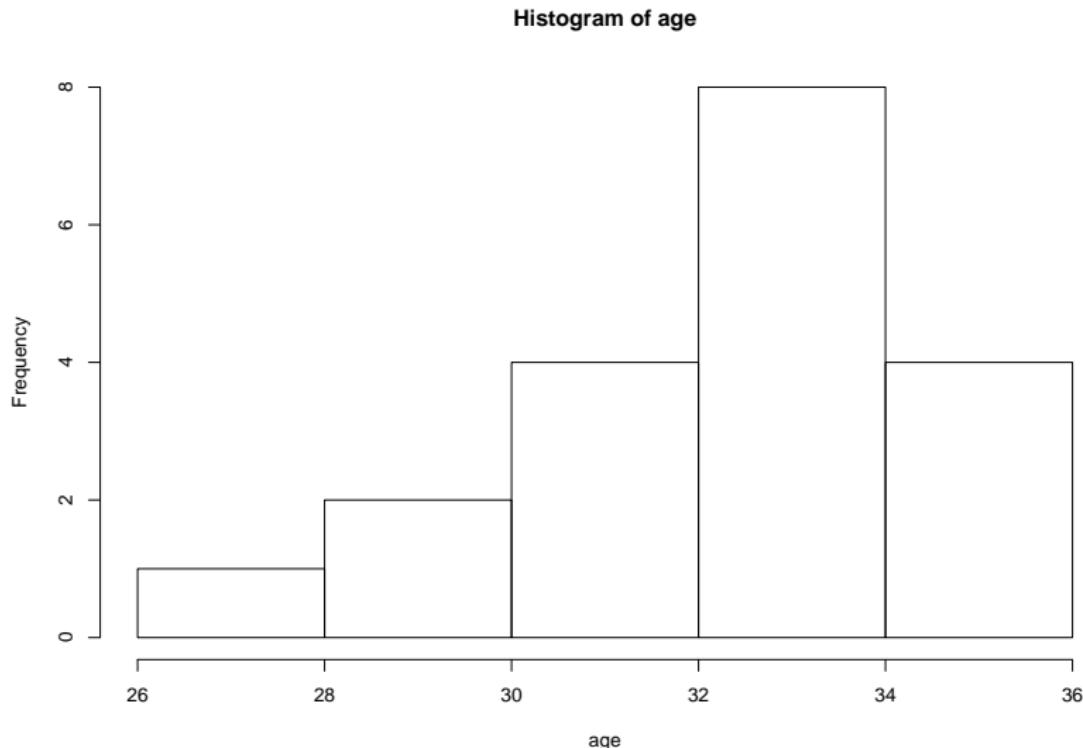
- ▶ Descriptive statistics
- ▶ Graphics
- ▶ Sampling
- ▶ Experimental design
- ▶ Hypothesis testing
- ▶ Bayesian inference
- ▶ Linear models & GLMs
- ▶ Model selection

Descriptive statistics

Guess my age

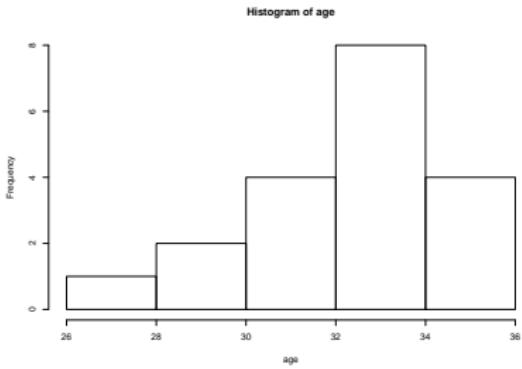
Graph your estimates

```
hist(age)
```



Summarise that distribution

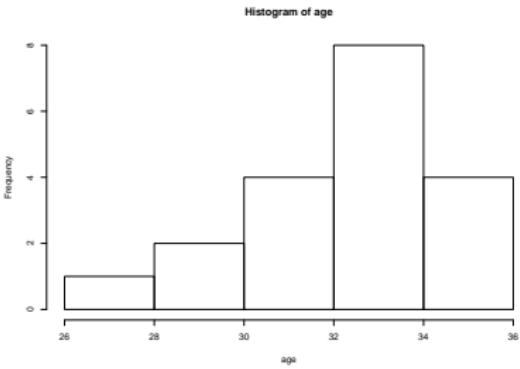
- ▶ Central tendency / location



Summarise that distribution

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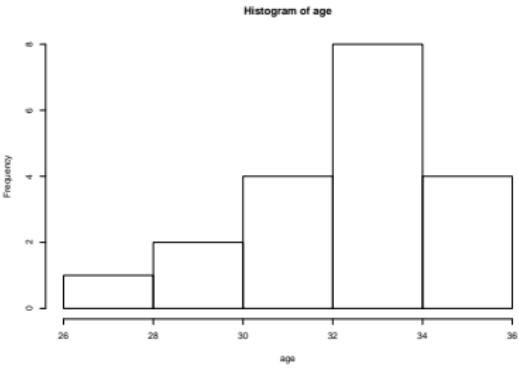
- ▶ mean



Summarise that distribution

► Central tendency / location

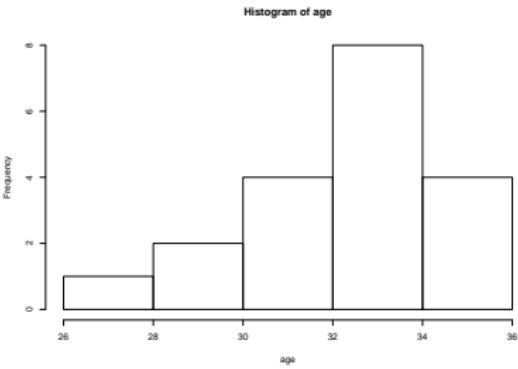
- ▶ mean
- ▶ median



Summarise that distribution

► Central tendency / location

- ▶ mean
- ▶ median
- ▶ mode

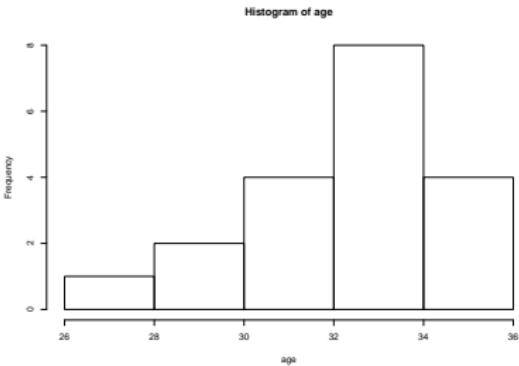


Summarise that distribution

- ▶ **Central tendency / location**

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- ▶ median
- ▶ mode

- ▶ **Variation / Spread**



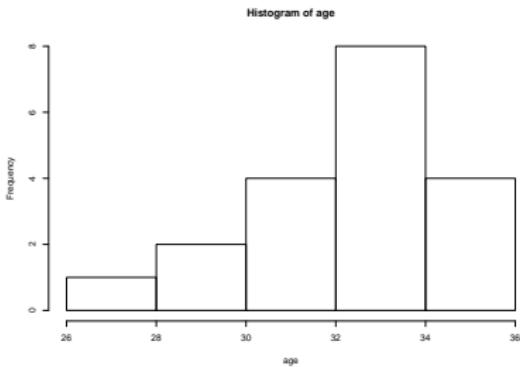
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- ▶ **Variation / Spread**

- ▶ min, max, range



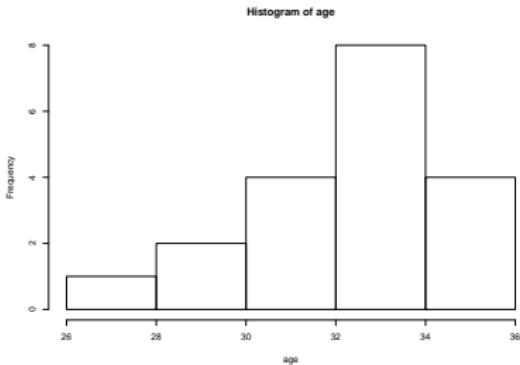
Summarise that distribution

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- ▶ min, max, range
- ▶ quantiles



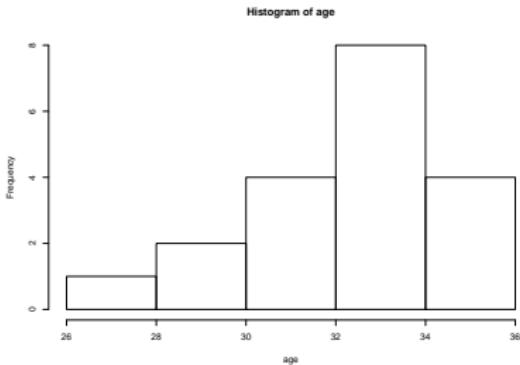
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- ▶ quantiles
- ▶ standard deviation



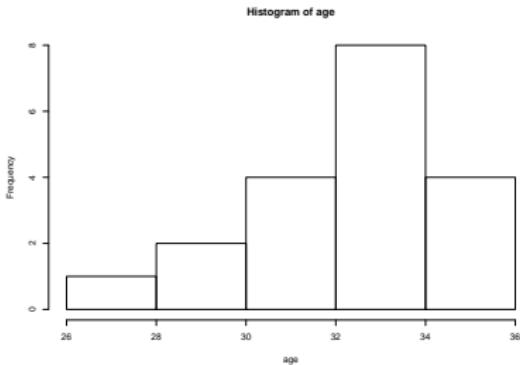
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- ▶ standard error



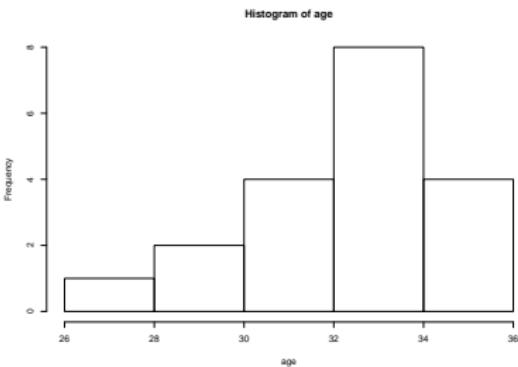
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- ▶ standard deviation
- ▶ standard error
- ▶ coefficient of variation



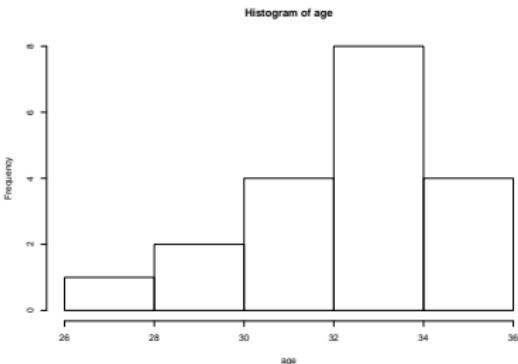
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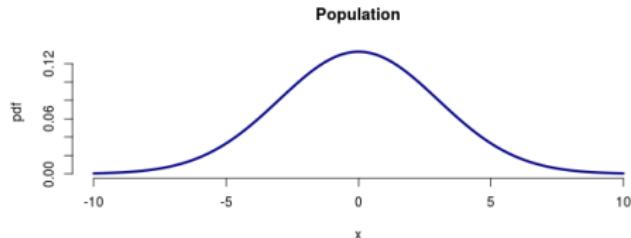
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- ▶ **Variation / Spread**

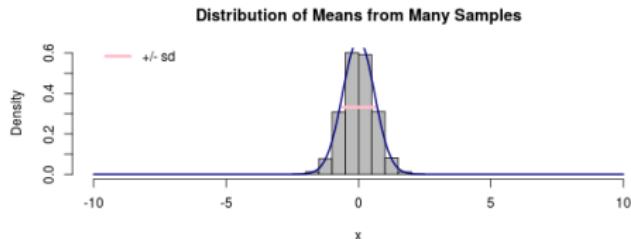
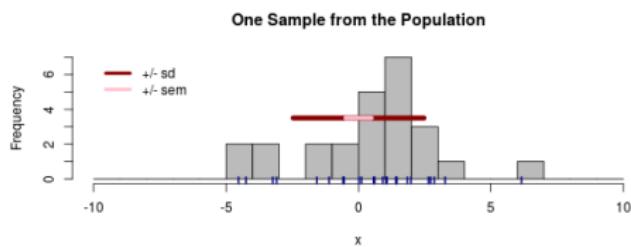
- ▶ min, max, range
- ▶ quantiles
- ▶ standard deviation
- ▶ standard error
- ▶ coefficient of variation
- ▶ confidence intervals



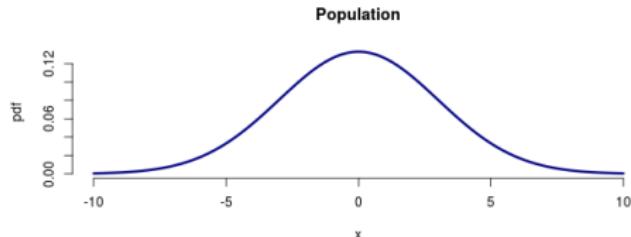
Relationship between SD and SEM



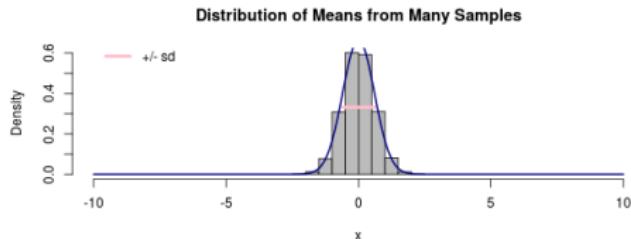
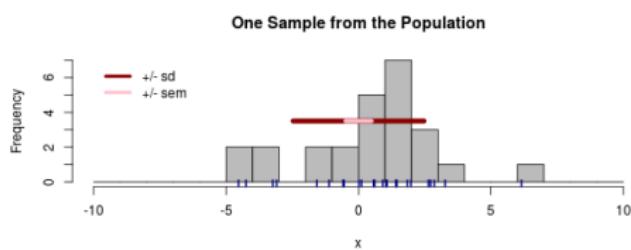
► SD quantifies scatter in population



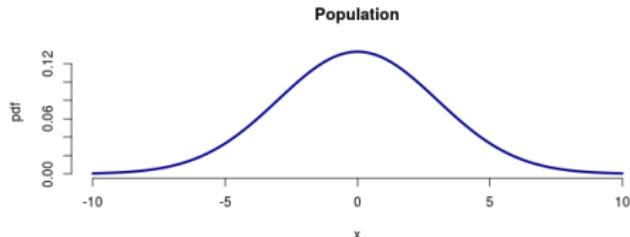
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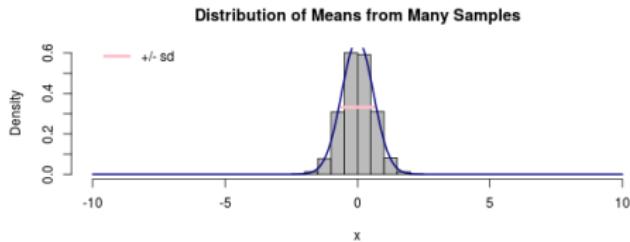
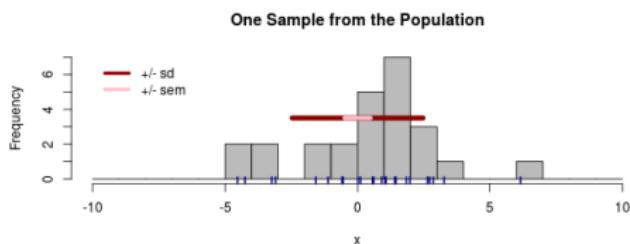
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- ▶ SEM quantifies uncertainty in parameter estimate (population mean)



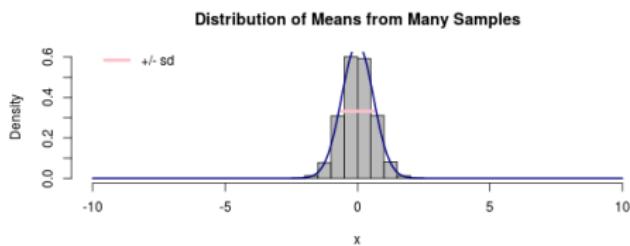
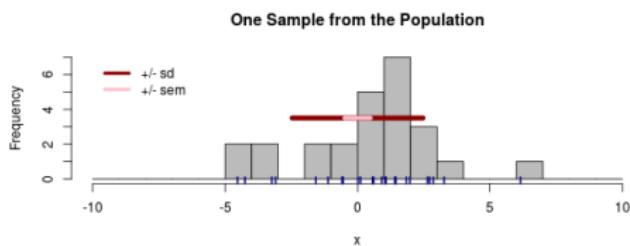
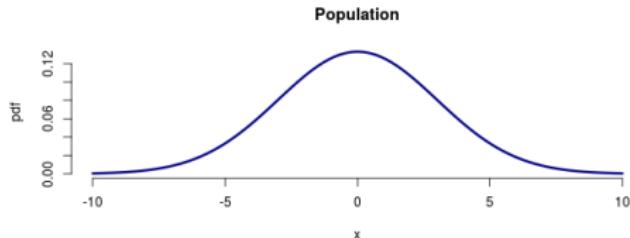
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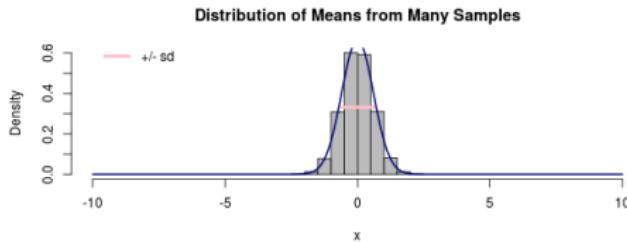
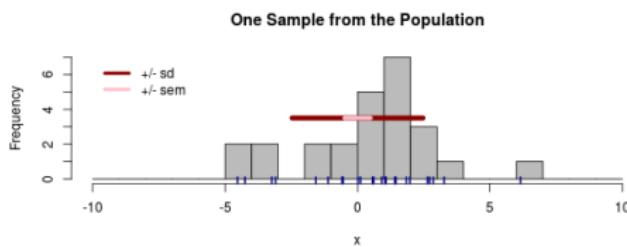
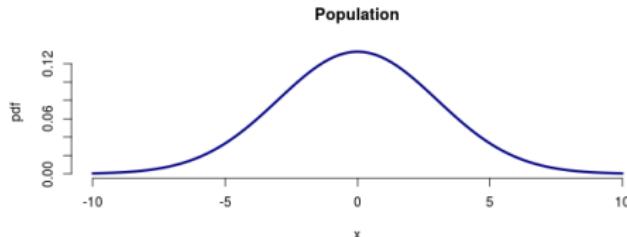


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- ▶ https://gallery.shinyapps.io/sampling_and_stderr/

In a Normal distribution

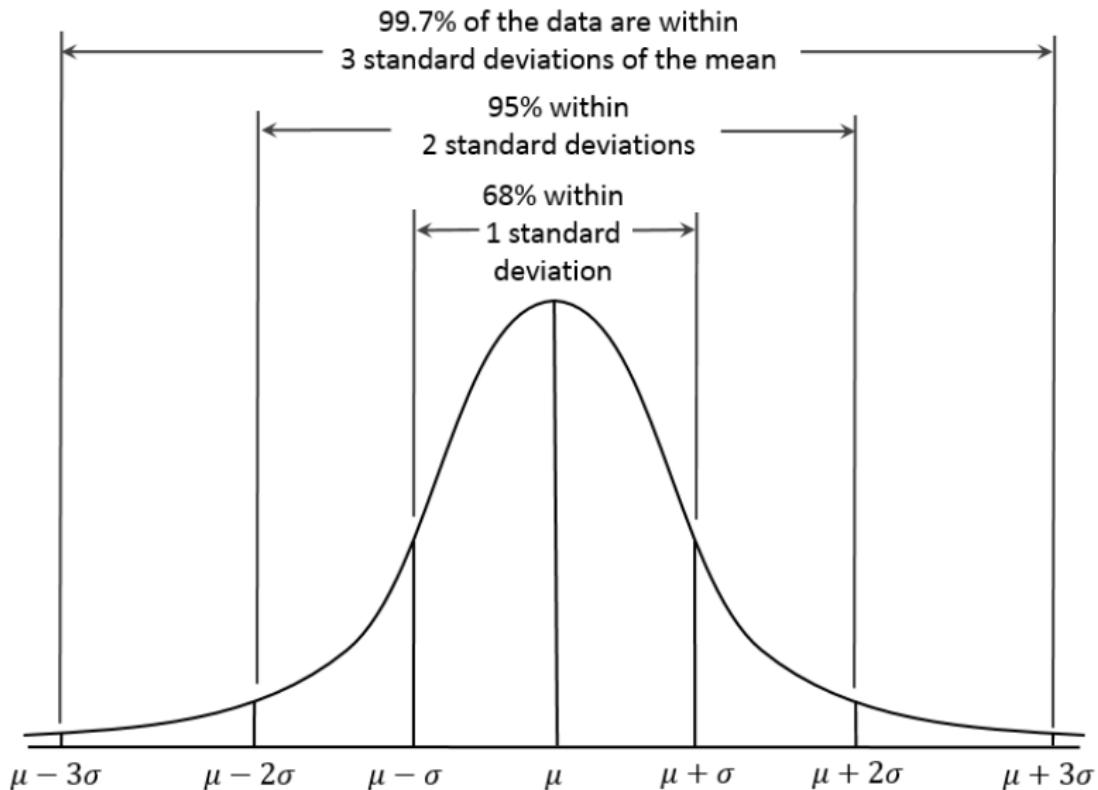
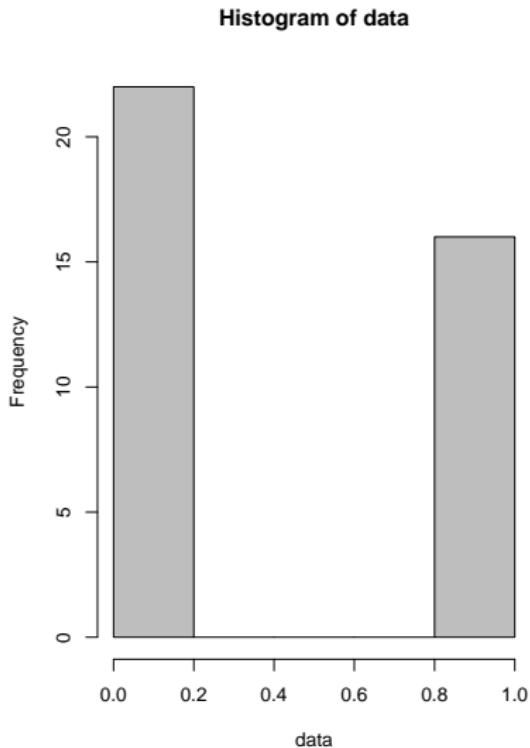
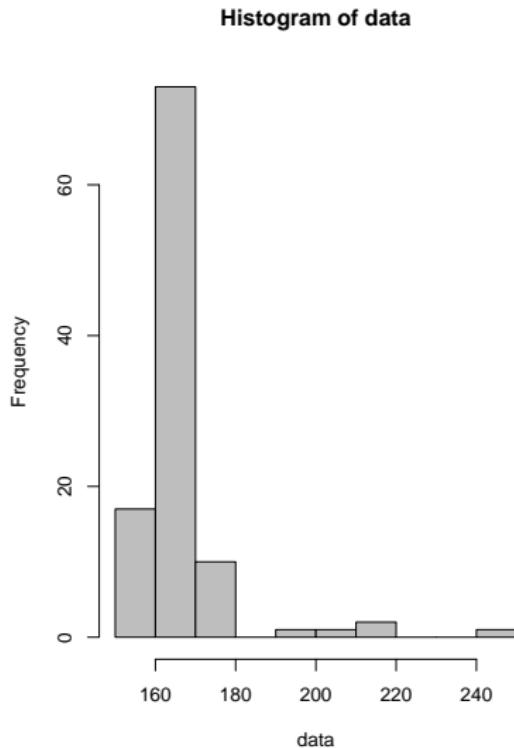


Figure 4:

What statistical descriptors are best? (and why)

<https://pollev.com/franciscorod726>



Sampling, confidence intervals, and Bayesian inference

Inference: from samples to population

We rarely measure the whole **population**, but take **samples** instead.



Figure 5:

What's the average height in this class?

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5. Do all CIs contain true mean height?

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- ▶ If we repeated the experiment, 95% of the CIs would contain the true value of X
- ▶ The probability that X is greater than 0 is at least 95%
- ▶ The probability that X equals 0 is smaller than 5%

<https://pollev.com/franciscorod726>

Understanding confidence intervals

- ▶ Summarise **uncertainty** in parameter estimates.

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- ▶ To read more: Morey et al (2015)

What happens if we increase sample size?

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- ▶ CI width *decreases* . . .

What happens if we increase sample size?

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- ▶ CI width *decreases* . . .
- ▶ but still 5% of CIs will NOT contain true mean!

Bayesian credible intervals

- ▶ Bayesian **credible** intervals do give the probability that true parameter value is contained within them.

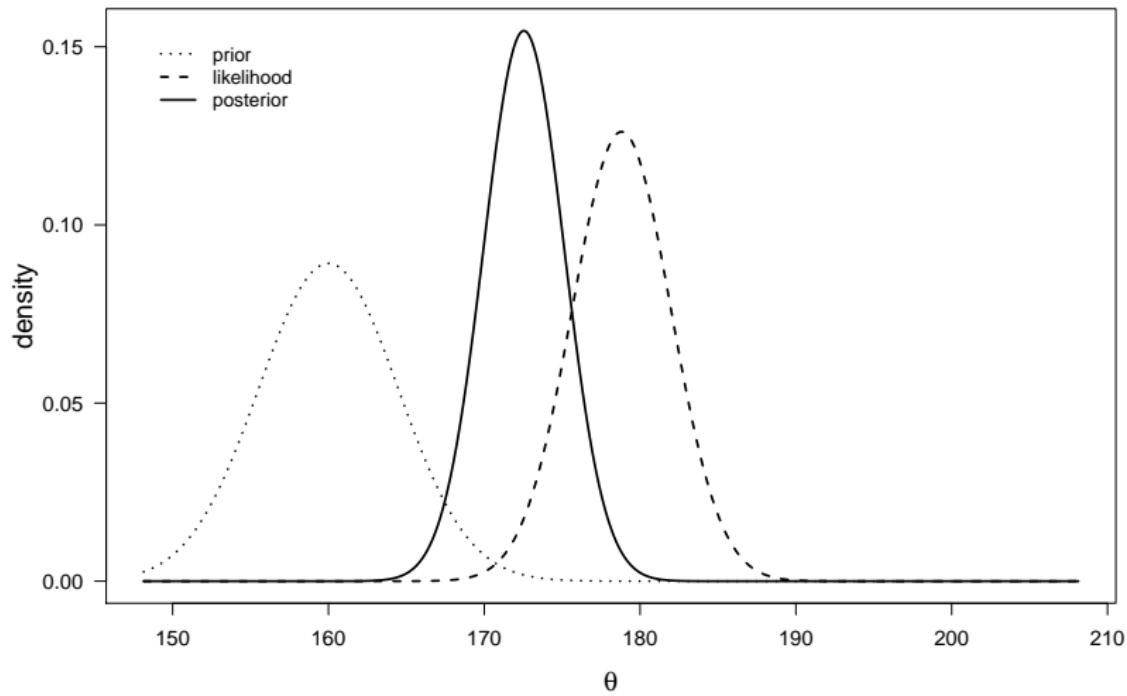
Bayesian credible intervals

- ▶ Bayesian **credible** intervals do give the probability that true parameter value is contained within them.
- ▶ Frequentist CIs and Bayesian credible intervals can be similar, but not always.

Bayesian inference: prior, posterior, and likelihood

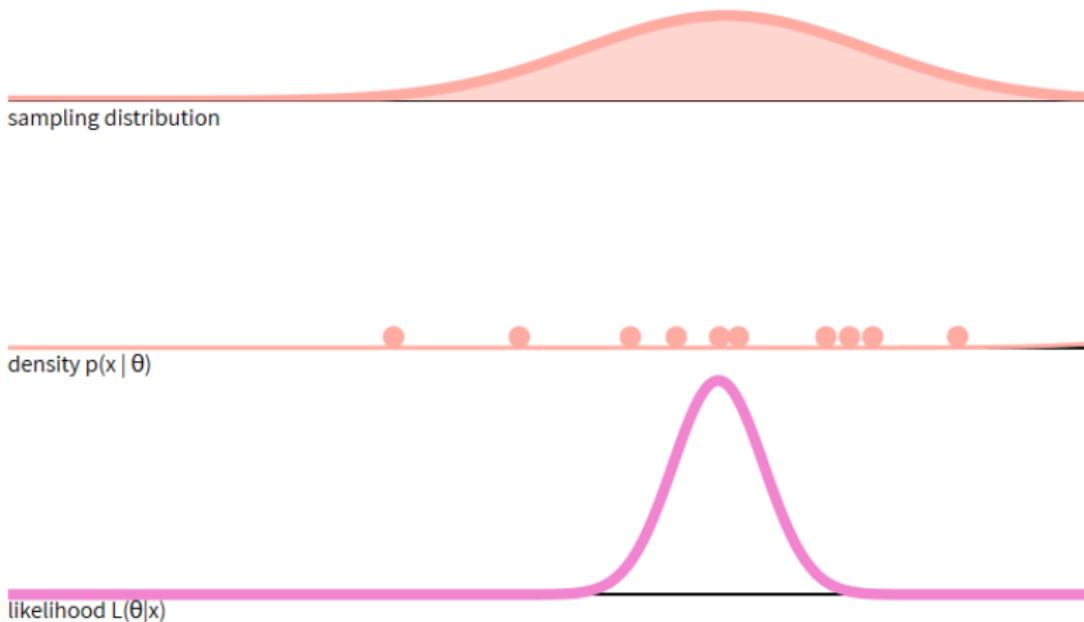
$$P(H|D) \propto P(D|H) \times P(H)$$

$$\text{Posterior} \propto \text{Likelihood} \cdot \text{Prior}$$



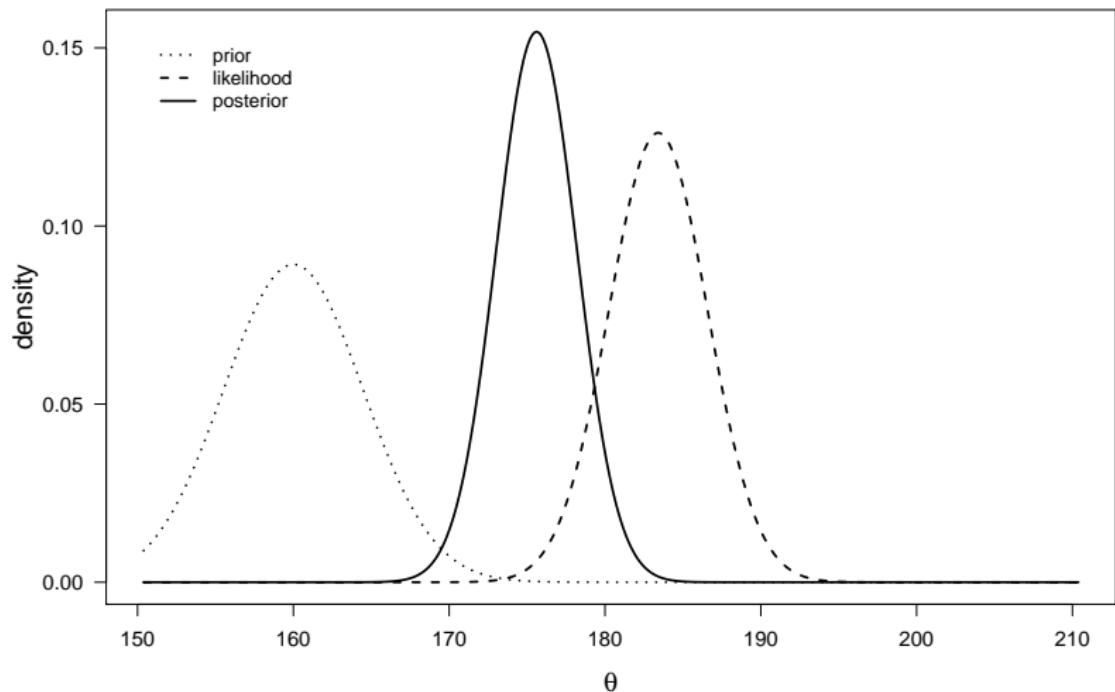
What is the likelihood?

$$L(\theta|x) = P(x|\theta)$$



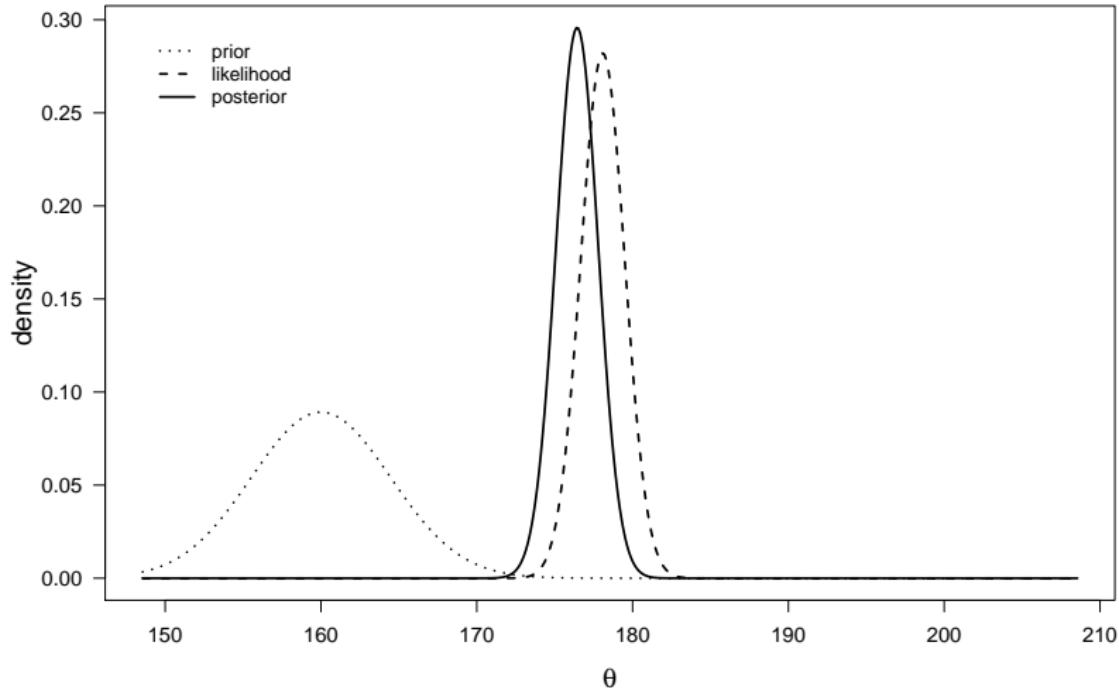
<https://seeing-theory.brown.edu/bayesian-inference/index.html>

Bayesian inference: prior and likelihood produce posterior



\$posterior.mean

With increasing sample size, likelihood dominates prior



\$posterior.mean

More apps to introduce Bayesian inference

- ▶ Normal

More apps to introduce Bayesian inference

- ▶ Normal
- ▶ Binomial

More apps to introduce Bayesian inference

- ▶ Normal
- ▶ Binomial
- ▶ Own data

More apps to introduce Bayesian inference

- ▶ Normal
- ▶ Binomial
- ▶ Own data
- ▶ Bayesian t-test

Experimental Design

How would you evaluate fertilizer effect?

Discuss with partner (5')



Figure 6:

Experimental design principles

Replication

Replication!



Figure 7:

Replication

- ▶ Replication is key: we need several samples.

Replication

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- ▶ How many? As much as you can! See Gelman & Carlin 2014.

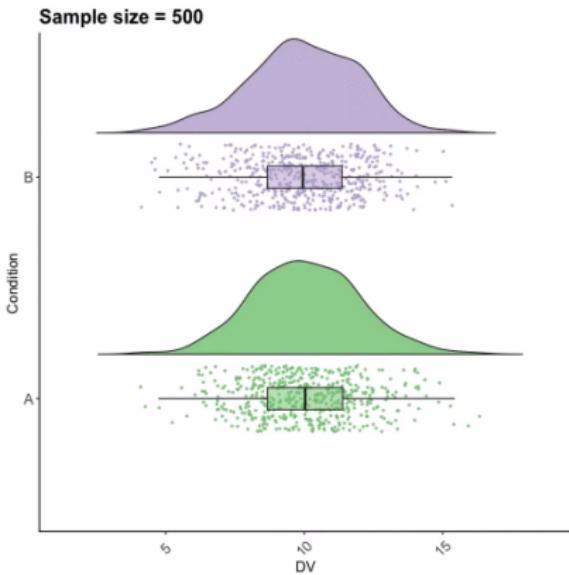
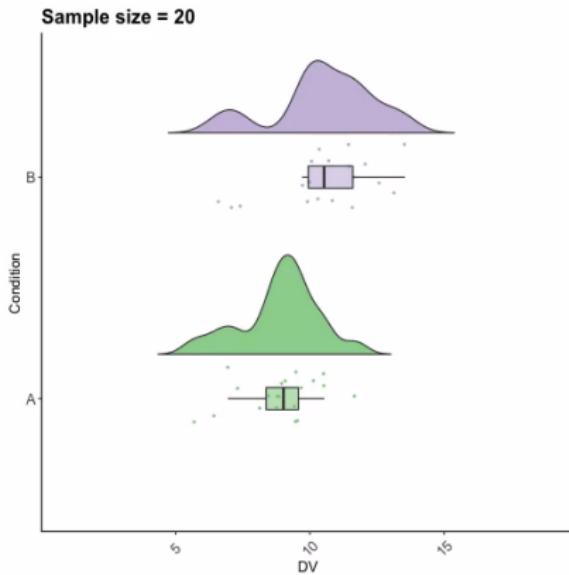
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- ▶ Replication is key: we need several samples.
- ▶ How many? As much as you can! See Gelman & Carlin 2014.
- ▶ Traditionally, ecology studies have had too low sample sizes.
- ▶ Low sample sizes miss subtle effects, but also prone to bias.

Low sample sizes very sensitive to random noise



https://twitter.com/ajstewart_lang/status/1020038488278945797

Sample size is very important

See *The evolution of correlations*

Stopping rules:

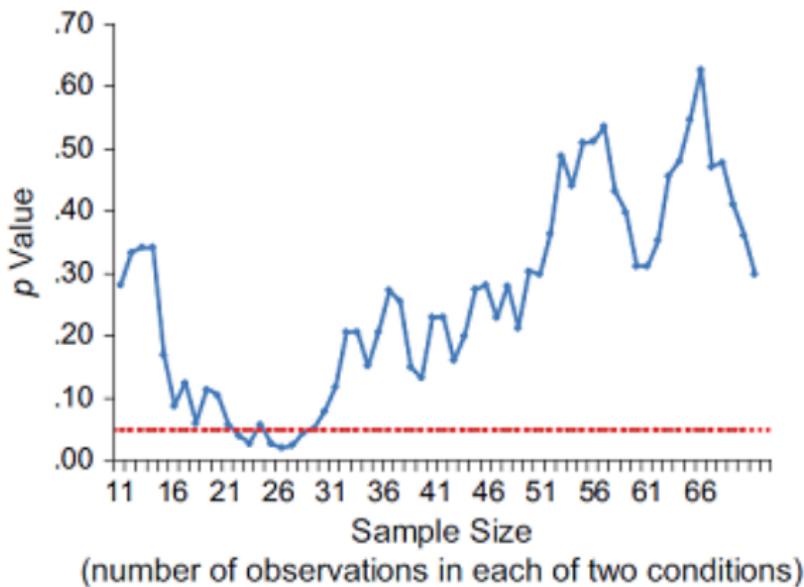


Fig. 2. Illustrative simulation of p values obtained by a researcher who continuously adds an observation to each of two conditions, conducting a t test after each addition. The dotted line highlights the conventional significance criterion of $p \leq .05$.

Sample size estimation

- ▶ Plan model **before** data collection.

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- ▶ Do simulations. Power/Sample size analyses (e.g.).

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- ▶ Do simulations. Power/Sample size analyses (e.g.).
- ▶ Plan to have at least **10-30 observations per predictor**.
- ▶ Complex models (w/ many predictors, interactions etc) require **high** sample sizes.

Randomization

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Figure 8:

Randomization

- ▶ Haphazard \neq Random

Randomization

- ▶ Haphazard \neq Random
- ▶ Stratify: randomize within groups (e.g. species, soil types)

Controls

Have controls

- ▶ Untreated individuals, plots... (assigned randomly, of course).

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- ▶ Must differ only in treatment (i.e. homogeneous environment).

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- ▶ Must differ only in treatment (i.e. homogeneous environment).
- ▶ Measure before & after treatment.

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- ▶ Must differ only in treatment (i.e. homogeneous environment).
- ▶ Measure before & after treatment.
- ▶ Consider blind designs to avoid observer bias.

Experimental design principles

1. Replication

Experimental design principles

1. Replication
2. Randomization

Experimental design principles

1. Replication
2. Randomization
3. Controls

To read more

- ▶ Ruxton & Colegrave. Experimental Design for the Life Sciences. OUP

Hypothesis testing

Does height differ between local and foreign students?

- Local people heights:

171 168 182 164 160

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
160	164	168	169	171	182

177 172 195 174 173 182 163 169 170 180

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
163.0	170.5	173.5	175.5	179.2	195.0

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177 172 195 174 173 182 163 169 170 180

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
163.0	170.5	173.5	175.5	179.2	195.0

- ▶ We know what happens in **our samples**, but want to extrapolate to the whole **population**.

If we sample students' heights in this class. . .

- ▶ Can we extrapolate results to

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- ▶ Can we extrapolate results to
 - ▶ this class?

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 - ▶ this class?
 - ▶ this university?
 - ▶ this city?

If we sample students' heights in this class. . .

- ▶ Can we extrapolate results to
 - ▶ this class?
 - ▶ this university?
 - ▶ this city?
 - ▶ the world?

If we sample students' heights in this class. . .

- ▶ Can we extrapolate results to
 - ▶ this class?
 - ▶ this university?
 - ▶ this city?
 - ▶ the world?
- ▶ What's the **suitable population** to make inferences given this sample?

NHST concepts

Null and alternative hypotheses

- ▶ Tell me...

Null and alternative hypotheses

- ▶ Tell me...
- ▶ **Null hypothesis:** there is no difference between groups.

Null and alternative hypotheses

- ▶ Tell me...
- ▶ **Null hypothesis:** there is no difference between groups.
- ▶ **Alternative hypothesis:** groups are different.

In ecology, everything is somewhat different

Are there any differences? A non-sensical question in ecology

Alejandro Martínez-Abraín

IMEDEA (CSIC-UIB), C/Miquel Marquès 21, 07190 Esporles, Majorca, Spain

ARTICLE INFO

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ABSTRACT

One of the main questions that ecologists pose in their investigations includes the analysis of differences in some trait between two or more populations. I argue here that asking whether there are differences or not between populations is biologically irrelevant, since no two living things are ever equal. On the contrary the appropriate question to pose is how large differences are between populations. That is, we urge a shift in interest from statistical significance to biological relevance for proper knowledge accumulation. I empha-

What is the p-value?

<https://pollev.com/franciscorod726>

P value

- ▶ Very complicated concept: even statisticians fail to describe it well.

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P value

- ▶ Very complicated concept: even statisticians fail to describe it well.
- ▶ Probability of observing data as or more extreme than these *if H₀ was true*.
- ▶ Low P-value: data unlikely if H₀ was true.
- ▶ Large P-value: data not unusual if H₀ was true.

Are differences *significant*?

- If $p < 0.05$, we **reject** H_0 .

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- ▶ This is **very widespread, but incorrect** practice.

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- ▶ This is **very widespread, but incorrect** practice.
- ▶ P-value is continuous. We must **avoid binary decisions** based on **arbitrary thresholds**.

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- ▶ **CAUTION:**
- ▶ This is **very widespread, but incorrect** practice.
- ▶ P-value is continuous. We must **avoid binary decisions** based on **arbitrary thresholds**.
- ▶ More on this later.

Let's do the test

```
t.test(h.sevi, h.out)
```

Welch Two Sample t-test

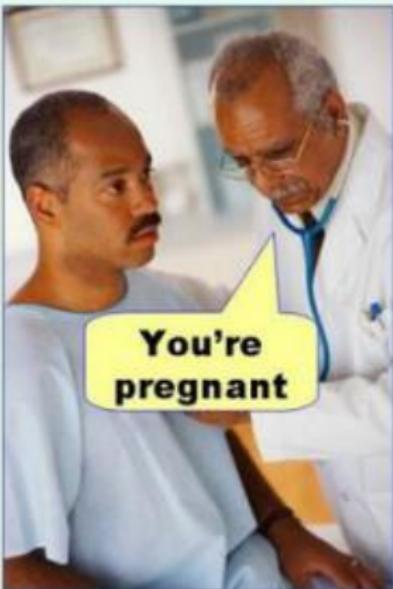
```
data: h.sevi and h.out
t = -0.20558, df = 6.346, p-value = 0.8436
alternative hypothesis: true difference in means is not equal to
95 percent confidence interval:
-14.0216 11.8216
sample estimates:
mean of x mean of y
172.2      173.3
```

Are heights different then?

Rejecting hypotheses: two types of error

Type I error

(false positive)



Type II error

(false negative)



Figure 9:

Rejecting hypotheses: two types of error

Statistics: Hypothesis Test	Null Hypothesis is True	Null Hypothesis is False
Reject Null Hypothesis	Type I Error	Correct
Fail to Reject Null Hypothesis	Correct	Type II Error

Figure 10:

Understanding NHST

<http://rpsychologist.com/d3/NHST/>

Example: biased coin

```
[1] 0 1 1 1 0 1 1 0 1 0
```

```
1-sample proportions test with continuity correction
```

```
data: sum(coin) out of ntrials, null probability 0.5  
X-squared = 0.1, df = 1, p-value = 0.7518
```

```
alternative hypothesis: true p is not equal to 0.5
```

```
95 percent confidence interval:
```

```
0.2736697 0.8630694
```

```
sample estimates:
```

```
p  
0.6
```

Correlation between variables

<http://rpsychologist.com/d3/correlation/>

Common pitfalls and good practice

A must read

Eur J Epidemiol (2016) 31:337–350
DOI 10.1007/s10654-016-0149-3



ESSAY

Statistical tests, *P* values, confidence intervals, and power: a guide to misinterpretations

Sander Greenland¹ · Stephen J. Senn² · Kenneth J. Rothman³ · John B. Carlin⁴ ·
Charles Poole⁵ · Steven N. Goodman⁶ · Douglas G. Altman⁷

<https://doi.org/10.1007/s10654-016-0149-3>

Good read

esa

ECOSPHERE

Applied statistics in ecology:
common pitfalls and simple solutions

E. ASHLEY STEEL,^{1,†} MAUREEN C. KENNEDY,² PATRICK G. CUNNINGHAM,³ AND JOHN S. STANOVICK⁴

Figure 11:

<https://doi.org/10.1890/ES13-00160.1>

Also <http://www.statisticsonewrong.com/>

Good read



Twenty tips for
interpreting
scientific claims

<https://doi.org/10.1038/503335a>

Visualisation of data and models is key

First things first

- ▶ Always

First things first

- ▶ Always
- ▶ Always

First things first

- ▶ Always
- ▶ Always
- ▶ Always

Plot data and models

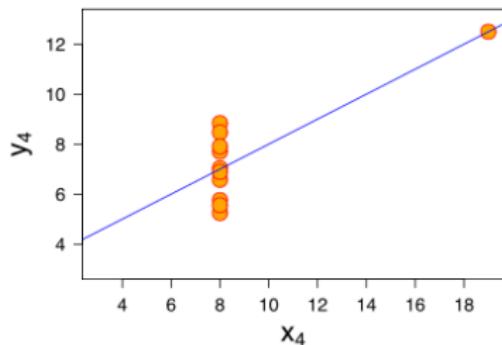
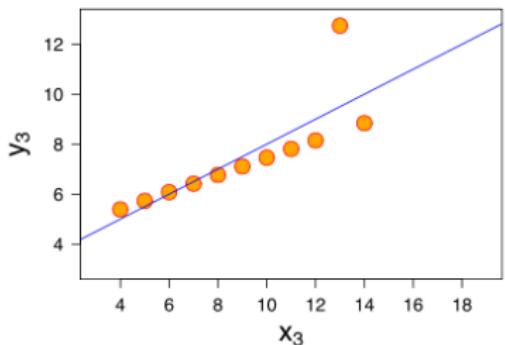
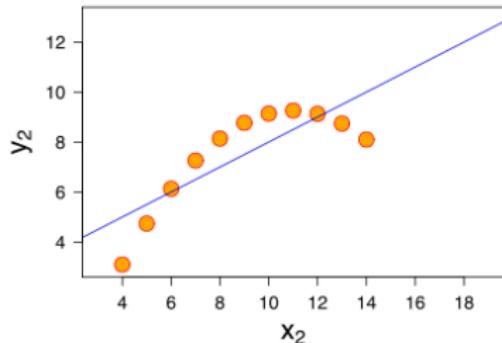
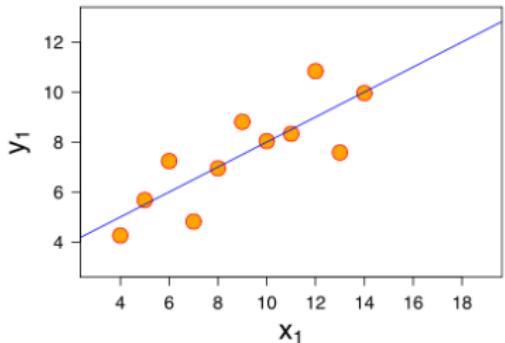
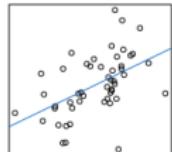


Figure 12:

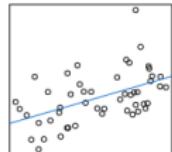
Don't use statistics blindly: Visualise

All correlations: $r(50) = 0.5$

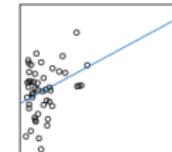
(1) Normal x, normal residuals



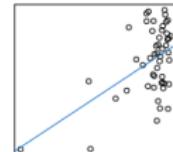
(2) Uniform x, normal residuals



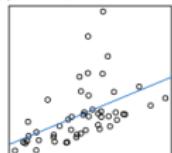
(3) +-skewed x, normal residuals



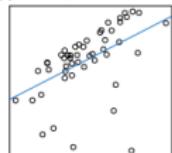
(4) --skewed x, normal residuals



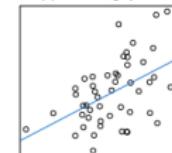
(5) Normal x, +-skewed residuals



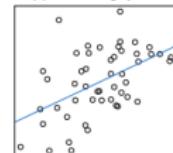
(6) Normal x, --skewed residuals



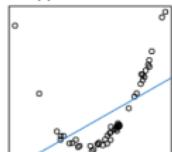
(7) Increasing spread



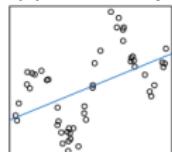
(8) Decreasing spread



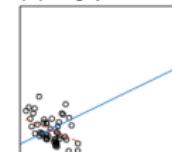
(9) Quadratic trend



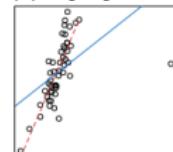
(10) Sinusoid relationship



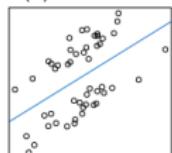
(11) A single positive outlier



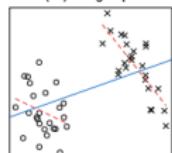
(12) A single negative outlier



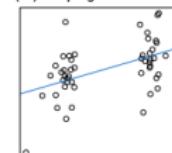
(13) Bimodal residuals



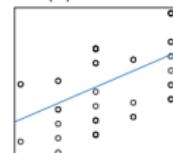
(14) Two groups



(15) Sampling at the extremes



(16) Coarse data

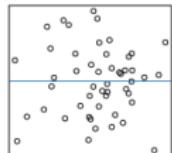


<https://janhove.github.io/teaching/2016/11/21/what-correlations-look-like>

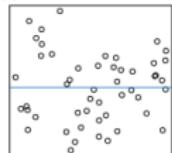
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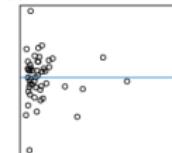
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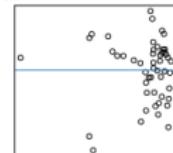
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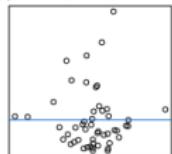
(3) +-skewed x, normal residuals



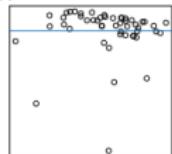
(4) --skewed x, normal residuals



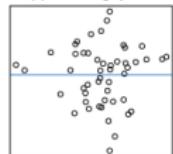
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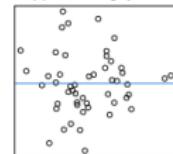
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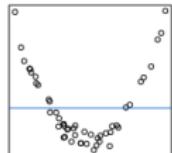
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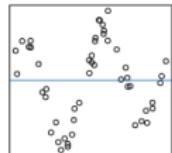
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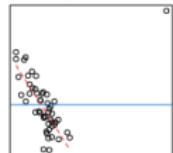
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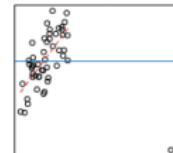
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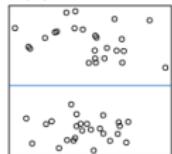
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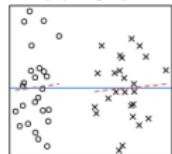
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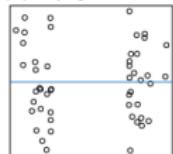
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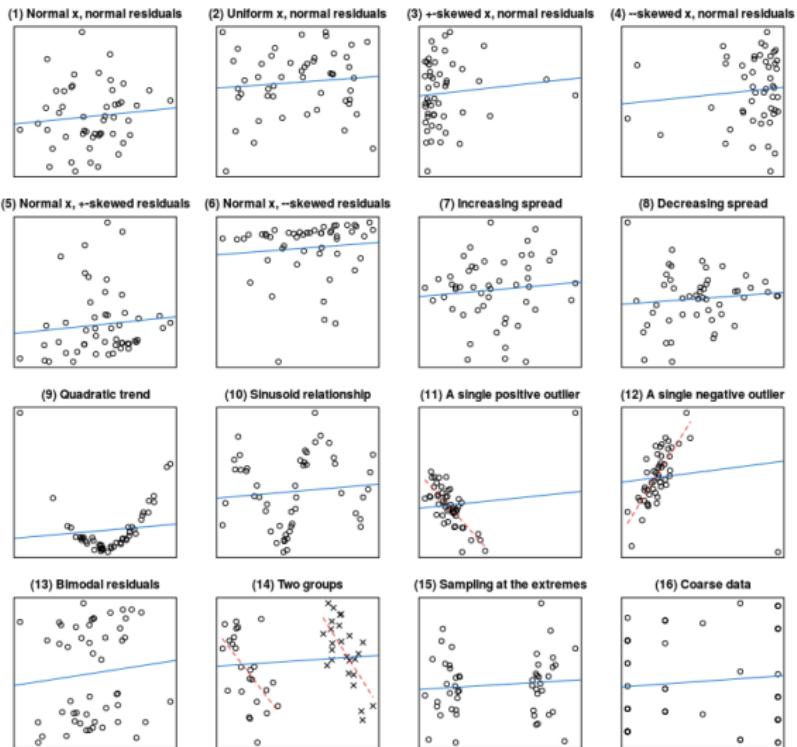
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//janhove.github.io/teaching/2016/11/21/what-correlations-look-like

Plot. Check models. Plot. Check assumptions. Plot.

Lavine 2014 *Ecology*

Inference from observational studies

News: Hamburgers increase risk of heart attack

- ▶ In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.

News: Hamburgers increase risk of heart attack

- ▶ In a sample of 10,000 people, it was found that people eating >2 hamburgers a week had 20% higher probability of heart attack.
- ▶ **Do hamburgers increase heart attacks?**

News: Hamburgers increase risk of heart attack

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- ▶ <https://pollev.com/franciscorod726>

Bigger flowers increase reproductive success

- ▶ We found that plants with big flowers produced 30% more seeds...

Bigger flowers increase reproductive success

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Bigger flowers increase reproductive success

- ▶ We found that plants with big flowers produced 30% more seeds...
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Correlation vs Causation

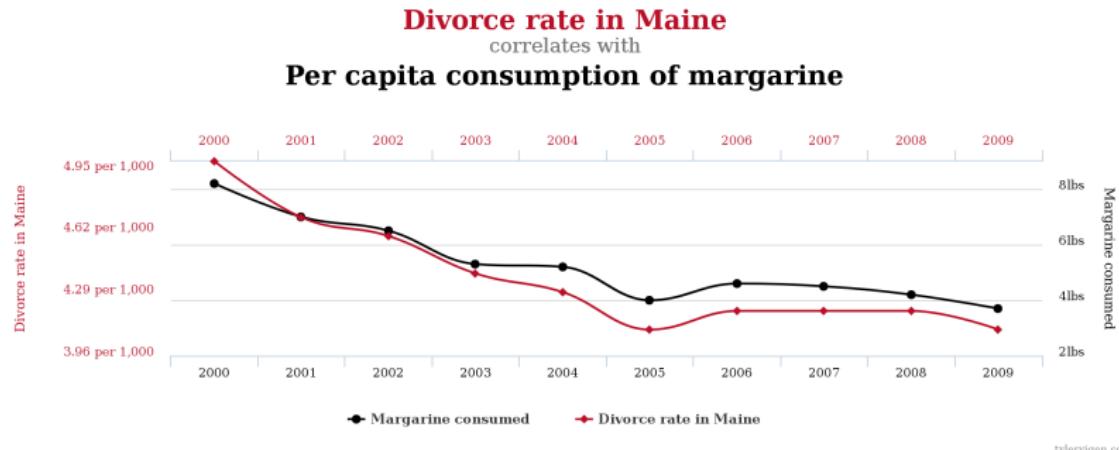


Figure 13:

<http://tylervigen.com/spurious-correlations>

Learning statistics through xkcd



NHST and p-values

In ecology, everything is somewhat different

Are there any differences? A non-sensical question in ecology

Alejandro Martínez-Abraín

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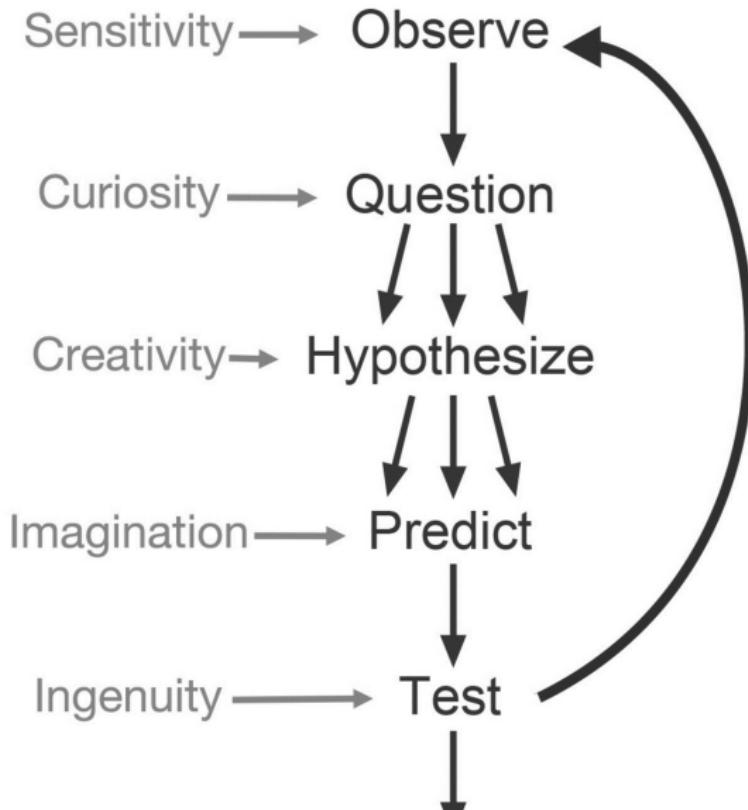
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Keywords:

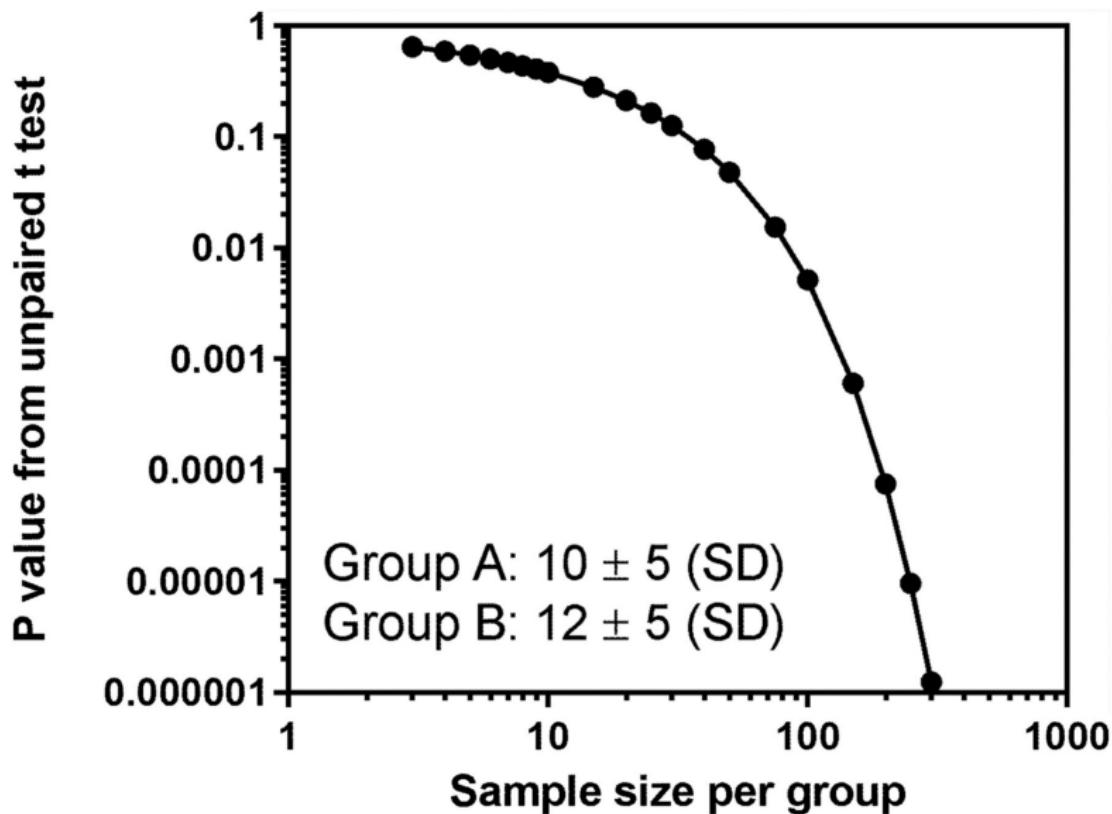
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One of the main questions that ecologists pose in their investigations includes the analysis of differences in some trait between two or more populations. I argue here that asking whether there are differences or not between populations is biologically irrelevant, since no two living things are ever equal. On the contrary the appropriate question to pose is how large differences are between populations. That is, we urge a shift in interest from statistical significance to biological relevance for proper knowledge accumulation. I empha-

Instead of falsifying a null model, estimate effects and compare meaningful models



P-value depends on sample size



P-value depends on sample size

- Same real difference is detected as significant or not depending on sample size:

Real difference = 40 g

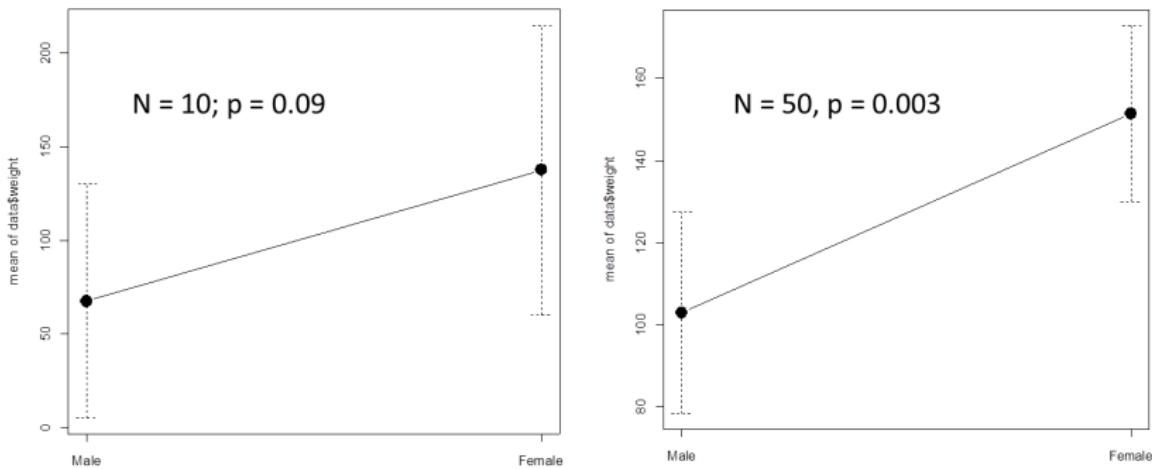
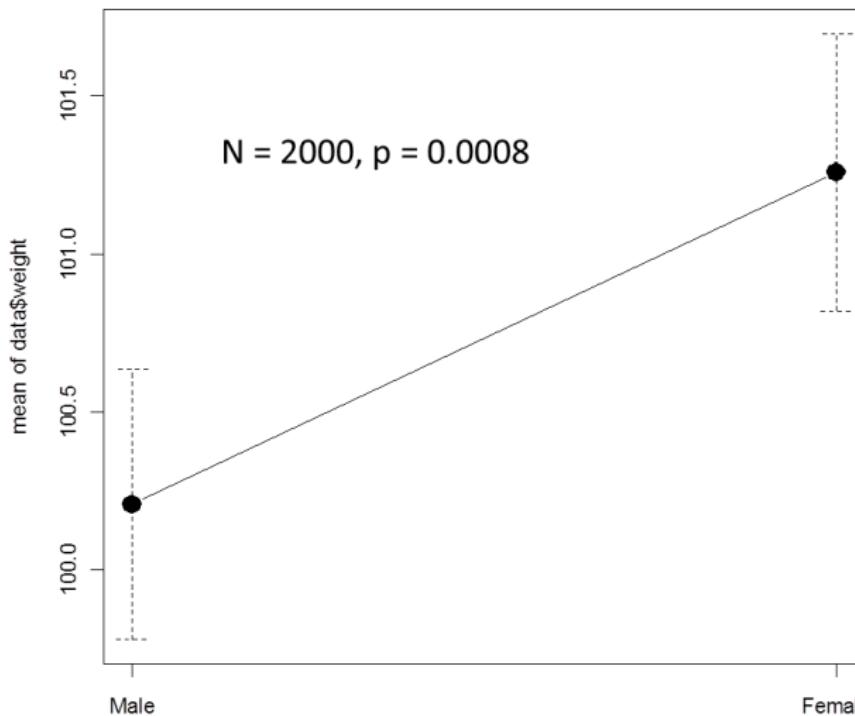


Figure 14:

Statistically significant != biologically important

- With big sample size, we can find **highly significant but biologically unimportant** differences.

Real difference = 1 g



Statistically significant \neq biologically important

- ▶ Statistically significant = unlikely to be zero

Statistically significant != biologically important

- ▶ Statistically significant = unlikely to be zero
- ▶ Good read: *significantly misleading*

Statistically significant != biologically important

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- ▶ My suggestion: avoid significant/not significant (and maybe p-values too)

Statistically significant != biologically important

- ▶ Statistically significant = unlikely to be zero
- ▶ Good read: *significantly misleading*
- ▶ My suggestion: avoid significant/not significant (and maybe p-values too)
- ▶ Beyond significance, look at *effect sizes*.

'Not significant' does NOT mean 'there is no effect'

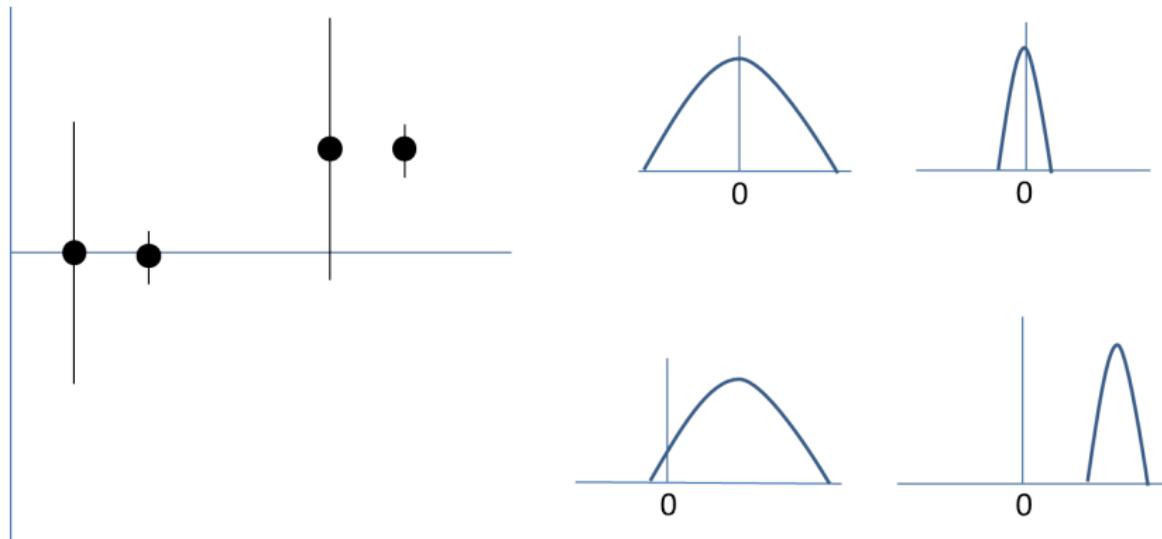


Figure 16:

- ▶ Absence of evidence \neq Evidence of absence

Failure to reject H_0 $\neq H_0$ is true

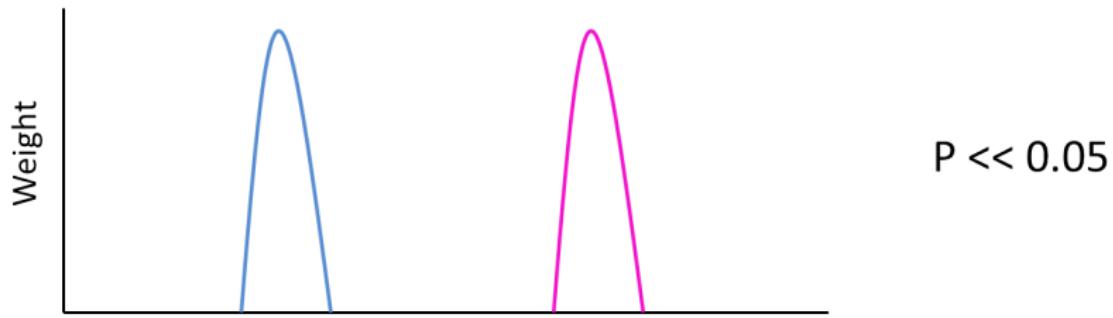
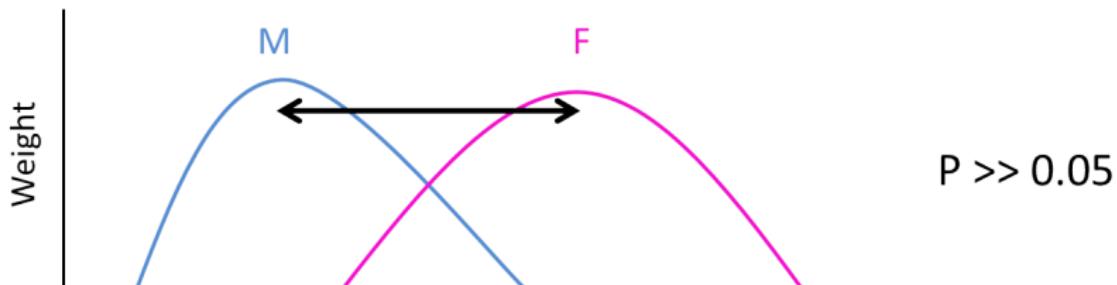


Figure 17:

p-value > 0.05?

"We were unable to find evidence against the hypothesis that A = B with the current sample size"

Is it safe to allow right turn with red lights?

- ▶ Right turn not allowed: 308 accidents

<https://www.statisticsdonewrong.com/power.html#the-wrong-turn-on-red>

Is it safe to allow right turn with red lights?

- ▶ Right turn not allowed: 308 accidents
- ▶ Right turn allowed: 337 accidents

<https://www.statisticsonthewrong.com/power.html#the-wrong-turn-on-red>

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- ▶ No *significant* difference, hence safe

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Is it safe to allow right turn with red lights?

- ▶ Right turn not allowed: 308 accidents
- ▶ Right turn allowed: 337 accidents
- ▶ No *significant* difference, hence safe
- ▶ Misinterpretation of underpowered study cost lives

[https:](https://www.statisticsdonewrong.com/power.html#the-wrong-turn-on-red)

//www.statisticsdonewrong.com/power.html#the-wrong-turn-on-red

0.05 is an arbitrary threshold

**The Difference Between “Significant” and “Not Significant” is not
Itself Statistically Significant**

Andrew GELMAN and Hal STERN

Figure 18:

<http://dx.doi.org/10.1198/000313006X152649>

Multiple hypothesis testing

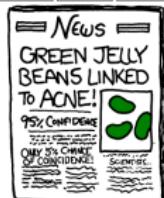
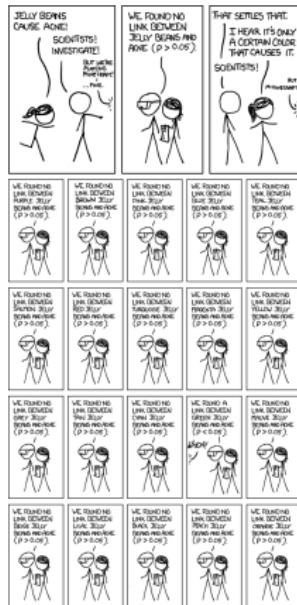
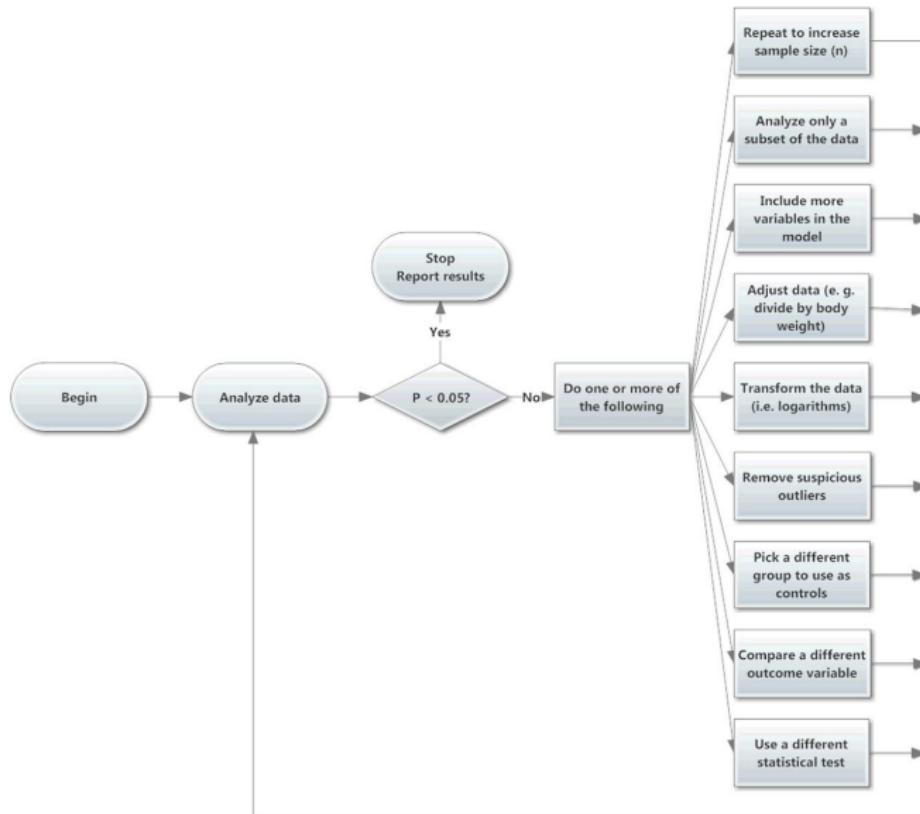


Figure 19:

How to make your results significant: *p-hacking*



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1. Test multiple variables, then report the ones that are significant.

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How to make your results significant: *p-hacking*

1. Test multiple variables, then report the ones that are significant.
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 3. Add covariates until effects are significant.
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- To read more: Simmons et al 2011

How to make your results significant: *p-hacking*

<https://www.youtube.com/watch?v=ZaNtz76dNSI>

ASA statement on p-values

- ▶ P-values do not measure the **probability of hypothesis** being true, or the probability that the data were produced by **random chance** alone.

<https://doi.org/10.1080/00031305.2016.1154108>

ASA statement on p-values

- ▶ P-values do not measure the **probability of hypothesis** being true, or the probability that the data were produced by **random chance** alone.
- ▶ Scientific conclusions or policy decisions should NOT be based only on **whether a p-value passes a specific threshold**.

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- ▶ Scientific conclusions or policy decisions should NOT be based only on **whether a p-value passes a specific threshold**.
- ▶ P-value, or statistical significance, does not measure the **size of an effect** or the **importance** of a result.
- ▶ By itself, a p-value does NOT provide a good **measure of evidence** regarding a model or hypothesis.

<https://doi.org/10.1080/00031305.2016.1154108>

The New Statistics

Aim for estimation of effects and their uncertainty (SE, CI...)



General Article

The New Statistics: Why and How

Geoff Cumming

La Trobe University

Psychological Science
2014, Vol. 25(1) 7–29
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DOI: 10.1177/0956797613504966
pss.sagepub.com



Figure 20:

<http://dx.doi.org/10.1177/0956797613504966>

How many types of errors?

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- ▶ **Type S (Sign):** estimating effect in opposite direction.
- ▶ **Type M (Magnitude):** Misestimating magnitude of the effect (under or overestimating).
- ▶ Beyond Power Calculations: Assessing Type S (Sign) and Type M (Magnitude) Errors

END



Figure 21:

Source code and materials: <https://github.com/Pakillo/stats-intro>