

Homework 1 (and learning L^AT_EX)

1.

$$M(1) = 3$$

$$M(2) = 3 + 4M(1) = 15$$

$$M(3) = 3 + 4M(2) = 63$$

$$M(n) = 3 + 4M(n-1)$$

$$\textit{Guess} : M(n) = 4^n - 1$$

$$\textit{Verify} : M(1) = 4^1 - 1$$

$$= 4 - 1$$

$$= 3 \checkmark$$

$$M(n+1) = 4^{n+1} - 1$$

$$= 4 * 4^n - 1$$

$$= 4(4^n - 1) + 4 - 1$$

$$= 4M(n) + 3 \checkmark$$

2.

$$C(1) = 2$$

$$C(n) = 4C(n/3) + \Theta(n^d), \quad n = 3, 9, 27, \dots$$

$$C(n) = aC(n/b) + \Theta(n^d)$$

$$a = 4, \quad b = 3$$

$$a) \quad d = 1 :$$

$$b^d = 3$$

$$a > b^d, \quad \text{so } C(n) \in \Theta(n)$$

$$b) \quad d = \log_3 4 :$$

$$b^d = 4$$

$$a = b^d, \quad \text{so } C(n) \in \Theta(n^{\log_3 4} \lg n)$$

$$c) \quad d = 2 :$$

$$b^d = 9$$

$$a < b^d, \quad \text{so } C(n) \in \Theta(n^{\log_3 4})$$

3.

```
template <class Item>
int partition(Item A[], int left, int right)
{
    Item pivot = 0;          //We want 0 to be our pivot
    int i = left;

    for (int j = left; j <= right; ++j)    //Include final element
        if (A[j] < pivot)
            swap(A[i++], A[j]);

    //Omit final swap

    return i;
}
```