Graph Theory: Homework #7

Due on February 18, 2015

Professor McGinley MWF 9:15

Rick Sullivan

Problem 1

Determine all r, s such that $K_{r,s}$ is planar.

Solution

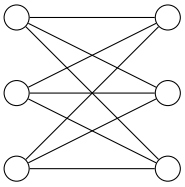
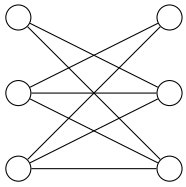
$K_{r,s}$ is planar whenever either r or s is less than or equal to 2.

Problem 2

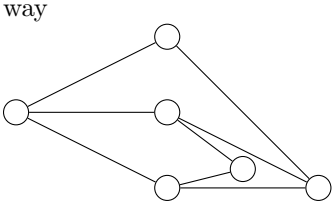
Show that the graph obtained by deleting one edge of $K_{3,3}$ is planar.

Solution

Proof. Removing any edge from $K_{3,3}$ produces an isomorphism of the same graph.

$K_{3,3}$:  Remove any one edge: 

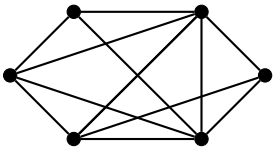
Moving the top and bottom vertices in the left set, we can embed the graph in the plane in the following way



□

Problem 3

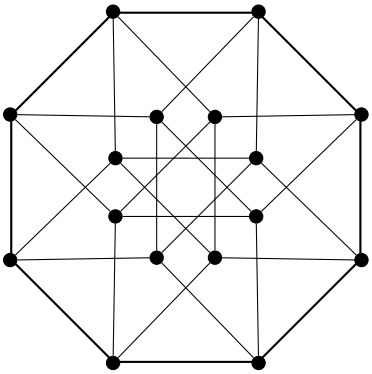
Determine (using the algorithm given in class) if the graph below is planar. If so, give its planar embedding.



Problem 4

Is the graph below planar? Use one of the theorems we talked about in class, not the algorithm.

Problem 4 continued on next page...



Solution

Proof. According to a corollary of Euler's formula, for a graph to be planar, $e(G) \leq 3n(G) - 6$.

$$\begin{aligned} n(G) &= 16 \\ e(G) &= 32 \\ e(G) &\leq 3n(G) - 6 \\ 32 &\leq 48 - 6 \\ 32 &\leq 42 \end{aligned}$$

For graphs with no triangular faces, it also must be true that $e(G) \leq 2n(G) - 4$. This graph has no triangular faces, so

$$\begin{aligned} e(G) &\leq 2n(G) - 4 \\ 32 &\leq 32 - 4 \\ 32 &\leq 28 \end{aligned}$$

This is not true, so this graph is not planar. □