Graph Theory: Homework #3

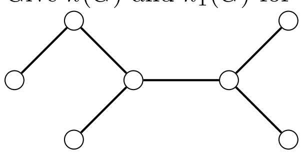
Due on January 21, 2015

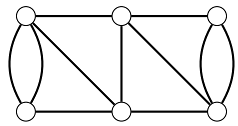
Professor McGinley MWF 9:15

Rick Sullivan

Problem 1

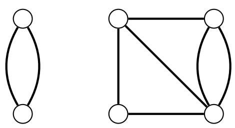
Give $k(G)$ and $k_1(G)$ for the following graphs:



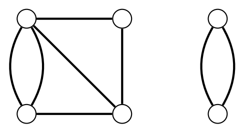


Solution
The first graph has $k(G) = 1$ and $k_1(G) = 1$. These can be found by removing any vertex with degree ¿ 1, or removing any edge.

The second graph has $k(G) = 2$ and $k_1(G) = 3$. $k(G)$ requires removing the two central vertices, while $k_1(G)$ requires removing either set of three edges like so:



or

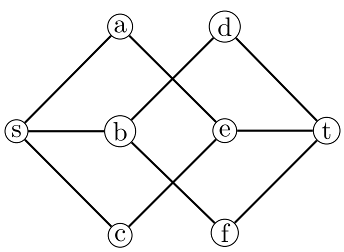


Problem 2

For the graph below:

a) Give three edges whose removal separates s from t. Is this the minimum number of such edges?

b) What is the maximum number of edge-disjoint paths from s to t and why?

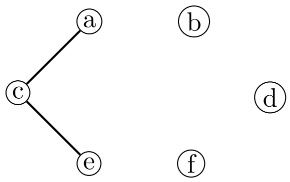


Part a
The edge set $s - a, s - b, s - c$ separates s from t when removed. This is not the minimum number of such edges; the set $s - b, e - t$ does the same.
Part b
The maximum number of edge-disjoint paths from s to t is 2: every path must take either edge $s - b$ or $e - t$. This is confirmed by the second answer of part a. Because the graph can be separated by removing those two edges, one of them must be traversed to travel from one subgraph to the other.

Problem 3

Find the shortest path from a to the other vertices.

Problem 3 continued on next page. . .



Solution

Problem 4

Find the articulation points (using the algorithm!)
Solution

Problem 5

Use induction on p to show that if G is a connected graph of order p, then the size of G is at least $p - 1$.

Solution

Problem 6

Find the smallest 3-regular graph simple graph having $k(G) = 1$.

Solution

Problem 7

Prove or disprove: If G is 2-connected, then for an $arbitrary u - v$ path P, there is $some other u - v$ path edge-disjoint from P.

Solution