Graph Theory: Homework #6

Due on February 11, 2015

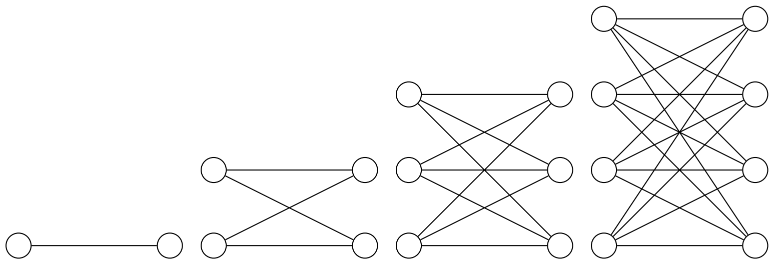
Professor McGinley MWF 9:15

Rick Sullivan

Problem 1

For which values of r is $K_{r,r}$ Hamiltonian?

Solution

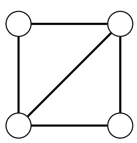


$K_{r,r}$ is Hamiltonian for all $r > 1$. Clearly there is no cycle on $K_{1,1}$. There exists a single Hamiltonian cycle on $K_{2,2}$ for each vertex. When we add 1 to r from a Hamiltonian complete bipartite graph, we can then include the two new vertices in the cycle because they are connected to every vertex in on the other side. Each successive graph is then also Hamiltonian.

Problem 2

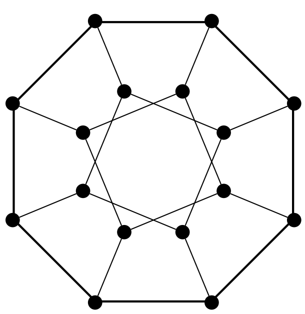
Give an example of a graph with more than two vertices that is Hamiltonian but not Eulerian.

Solution



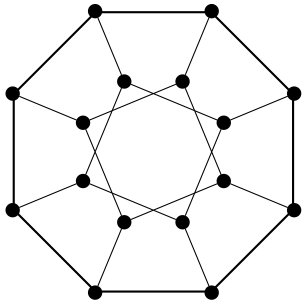
Problem 3

Decide if the graph below is Hamiltonian. Explain your answer.

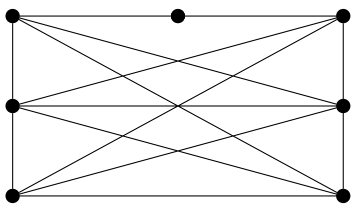


Solution

Yes, the graph is Hamiltonian with the following Hamiltonian cycle:

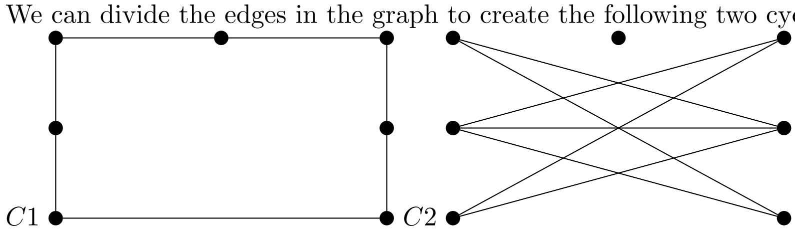


Problem 4

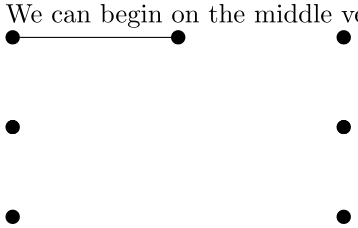


Solution

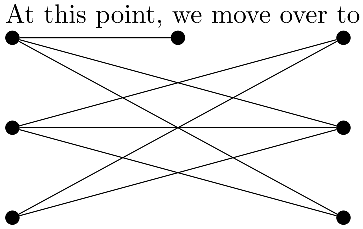
We can divide the edges in the graph to create the following two cycles



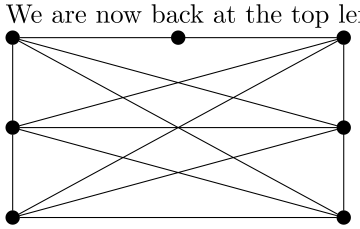
We can begin on the middle vertex, moving along $C1$ until we reach a shared vertex.



At this point, we move over to $C2$, tracing the entire cycle because there are no new cycles.



We are now back at the top left vertex, and complete tracing $C1$.



Problem 5

Show that if G is not 2-connected, G is not Hamiltonian.

Solution

If G is not 2-connected, there exists some vertex v in G of degree 1 or 0. In order for a graph to be Hamiltonian, there must exist a cycle that includes each vertex in $V(G)$.

Case 1: If v has degree 0, G is not connected, so no cycle connects all vertices.

Case 2: If v has degree 1, there is only one path to v, over that single edge. Therefore, there is no cycle containing v, so no cycle connects all vertices in G.