Numerical Analysis: Homework #7

Due on May 4, 2015

Professor Mohler MWF 2:15

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Problem 1

Consider the nonlinear system

$$\begin{aligned} 5x_1^2 - x_2^2 &= 0 \\ x_2 - (\sin(x_1) + \cos(x_2))/4 &= 0 \end{aligned}$$

a) Find a function $G(\vec{x})$ and a set D in $\mathbb{R}^2$ such that G has a unique fixed point in D.

b) Estimate the number of iterations required to approximate the exact solution within 10^{-5} in the $||\cdot||_\infty$ norm, given any initial guess in D.

Part a

$$\begin{aligned} G(\vec{x}) &= L(\vec{x}) \\ &= \begin{bmatrix} 5x_1^2 - x_2^2 + (2x_1)(x_1 - x_1) + (-2x_2)(x_2 - x_2) \\ x_2 - (\sin(x_1) + \cos(x_2))/4 + (-\cos(x_1)/4)(x_1 - x_1) + (1 + \sin(x_2))/4(x_2 - x_2) \end{bmatrix} \end{aligned}$$

Part b

Problem 2

Use two iterations of Newton's method with initial guess $vec0$ to approximate the solution to

$$\begin{aligned} 3x - \cos(yz) - 1/2 &= 0 \\ 4x^2 - 625y^2 + 2y - 1 &= 0 \\ e^{-xy} + 20z + \frac{10\pi - 3}{3} &= 0 \end{aligned}$$

Solution

$$J = \begin{bmatrix} 3 & -z\sin(yz) & -y\sin(yz) \\ 8x & 1250y + 2 & 0 \\ -ye^{-xy} & -xe^{-xy} & 20 \end{bmatrix}$$

Problem 2 continued on next page. . .

First iteration

$$J(\vec{0}) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$J(\vec{0})\vec{y} = -\vec{F}(\vec{0})$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 20 \end{bmatrix} \vec{y} = -\vec{F}(\vec{0})$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 20 \end{bmatrix} \vec{y} = -\begin{bmatrix} 3/2 \\ 1 \\ -\frac{10\pi}{3} \end{bmatrix}$$

$$\vec{y} = \begin{bmatrix} 9/2 \\ 2 \\ -\frac{200\pi}{3} \end{bmatrix}$$

$$\vec{x}^{(1)} = \vec{x}^{(0)} + \vec{y}$$

$$= \begin{bmatrix} 9/2 \\ 2 \\ -\frac{200\pi}{3} \end{bmatrix}$$

Second iteration

$$J(\vec{x}^{(1)}) = \begin{bmatrix} 3 & 181.38 & -1.73205 \\ 9 & 2502 & 0 \\ -.000247 & -.0005555 & 20 \end{bmatrix}$$

$$J(\vec{x}^{(1)})\vec{y} = -\vec{F}(\vec{x}^{(1)})$$

$$\begin{bmatrix} 3 & 181.38 & -1.73205 \\ 9 & 2502 & 0 \\ -.000247 & -.0005555 & 20 \end{bmatrix} \vec{y} = -\vec{F}(\vec{x}^{(1)})$$

$$\begin{bmatrix} 3 & 181.38 & -1.73205 \\ 9 & 2502 & 0 \\ -.000247 & -.0005555 & 20 \end{bmatrix} \vec{y} = -\begin{bmatrix} 3 \\ -2416 \\ 198.912 \end{bmatrix}$$

$$\vec{y} = \begin{bmatrix} 9/2 \\ 2 \\ -\frac{200\pi}{3} \end{bmatrix}$$

$$\vec{x}^{(1)} = \vec{x}^{(0)} + \vec{y}$$

$$= \begin{bmatrix} 9/2 \\ 2 \\ -\frac{200\pi}{3} \end{bmatrix}$$