Numerical Analysis: Homework #7

Due on May 4, 2015

Professor Mohler MWF 2:15

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Problem 1

Consider the nonlinear system

$$\begin{aligned} 5x_1^2 - x_2^2 = 0 \\ x_2 - (\sin(x_1) + \cos(x_2))/4 = 0 \end{aligned}$$

a) Find a function $G(\vec{x})$ and a set D in $\mathbb{R}^2$ such that G has a unique fixed point in D.

b) Estimate the number of iterations required to approximate the exact solution within 10^{-5} in the $||\cdot||_\infty$ norm, given any initial guess in D.

Part a

$$\begin{aligned} G(\vec{x}) &= L(\vec{x}) \\ &= \begin{bmatrix} 5x_1^2 - x_2^2 + (2x_1)(x_1 - x_1) + (-2x_2)(x_2 - x_2) \\ x_2 - (\sin(x_1) + \cos(x_2))/4 + (-\cos(x_1)/4)(x_1 - x_1) + (1 + \sin(x_2))/4(x_2 - x_2) \end{bmatrix} \end{aligned}$$

Part b

Problem 2

Use two iterations of Newton's method with initial guess $vec0$ to approximate the solution to

$$\begin{aligned} 3x - \cos(yz) - 1/2 = 0 \\ 4x^2 - 625y^2 + 2y - 1 = 0 \\ e^{-xy} + 20z + \frac{10\pi - 3}{3} = 0 \end{aligned}$$