Numerical Analysis: Homework #6

Due on February 27, 2015

Professor Mohler MWF 2:15

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Problem 1

Find the first three iterations obtained by the Power method applied to the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Use $x^{(0)} = (-1, 0, 1)^t$.

Solution

$$\begin{aligned}
\vec{x} &= A\vec{x} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\
&= \begin{bmatrix} x_1 + x_2 + x_3 \\ x_1 + x_2 \\ x_1 + x_3 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \\
\vec{x^{(1)}} &= \vec{x}/||\vec{x}||_\infty \\
&= \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\vec{x} &= \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \\
\vec{x^{(2)}} &= \vec{x}/||\vec{x}||_\infty \\
&= \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\vec{x} &= \begin{bmatrix} -1 - 1 + 0 \\ -1 - 1 \\ -1 + 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} \\
\vec{x^{(3)}} &= \vec{x}/||\vec{x}||_\infty \\
&= \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} /2 \\
&= \begin{bmatrix} -1 \\ -1 \\ -1/2 \end{bmatrix}
\end{aligned}$$

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Problem 2

Determine a singular value decomposition for the matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Solution

Using the python script

```python
import numpy as np
a = [[0,1,1],[0,1,0],[1,1,0],[0,1,0],[1,0,1]]
u,s,v = np.linalg.svd(a)
print('U:%s' % u)
print('S:%s' % s)
print('V:%s' % v)

# Approximations
u_2 = [[x] for x in u[:,0]]
s_2 = s[0]
v_2 = v[0]
print('Approximation reconstruction:\n%s' % (np.multiply(np.multiply(u_2, s_2)
    , v_2)))
```

we get the following output:

```
U:[[ -5.47722558e-01  -2.77555756e-16  -7.07106781e-01  -1.32058463e-01
    -4.27271064e-01]
 [ -3.65148372e-01  -4.08248290e-01   1.52655666e-16  -5.43516408e-01
    6.36073827e-01]
 [ -5.47722558e-01   1.38777878e-16   7.07106781e-01  -1.32058463e-01
    -4.27271064e-01]
 [ -3.65148372e-01  -4.08248290e-01   1.66533454e-16   8.07633333e-01
    2.18468301e-01]
 [ -3.65148372e-01   8.16496581e-01  -2.22044605e-16   1.32058463e-01
    4.27271064e-01]]
S:[ 2.23606798  1.41421356  1.        ]
V:[[ -4.08248290e-01  -8.16496581e-01  -4.08248290e-01]
 [  5.77350269e-01  -5.77350269e-01   5.77350269e-01]
 [  7.07106781e-01   3.88578059e-16  -7.07106781e-01]]
Approximation reconstruction:
[[ 0.5         1.          0.5       ]
 [ 0.33333333  0.66666667  0.33333333]
 [ 0.5         1.          0.5       ]
```

Problem 2 continued on next page. . .

```
 [ 0.33333333  0.66666667  0.33333333]
 [ 0.33333333  0.66666667  0.33333333]]
```

To create a 5×1, 1×1, and a 1×3 matrix whose product is approximately A, we can just take elements in the manner we discussed in class from our solution provided by the script. This will give us the matrices

$$\begin{bmatrix} -0.547722558 \\ -0.365148372 \\ -0.547722558 \\ -0.365148372 \\ -0.365148372 \end{bmatrix}, \begin{bmatrix} 2.23505798 \end{bmatrix}, \begin{bmatrix} -0.408248290 & -0.816496581 & -0.408248290 \end{bmatrix}$$

The product of these matrices is the matrix

$$\begin{bmatrix} 0.5 & 1 & 0.5 \\ 0.33333 & 0.66667 & 0.33333 \\ 0.5 & 1 & 0.5 \\ 0.33333 & 0.66667 & 0.33333 \\ 0.33333 & 0.66667 & 0.33333 \end{bmatrix}$$