Numerical Analysis: Homework #6

Due on February 27, 2015

Professor Mohler MWF 2:15

Rick Sullivan

Problem 1

Find the first three iterations obtained by the Power method applied to the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Use $x^{(0)} = (-1, 0, 1)^t$.

Solution

$$\begin{aligned} \vec{x} &= A\vec{x} \\ &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \begin{bmatrix} x_1 + x_2 + x_3 \\ x_1 + x_2 \\ x_1 + x_3 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \\ \vec{x^{(1)}} &= \vec{x}/||\vec{x}||_\infty \\ &= \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \vec{x} &= \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \\ \vec{x^{(2)}} &= \vec{x}/||\vec{x}||_\infty \\ &= \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \vec{x} &= \begin{bmatrix} -1 - 1 + 0 \\ -1 - 1 \\ -1 + 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} \\ \vec{x^{(3)}} &= \vec{x}/||\vec{x}||_\infty \\ &= \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} /2 \\ &= \begin{bmatrix} -1 \\ -1 \\ -1/2 \end{bmatrix} \end{aligned}$$

Problem 1 continued on next page...

Problem 2

Determine a singular value decomposition for the matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Solution