

# CS4533 Lecture 11

## Slides/Notes

### **Shading and Illumination; Compositing (Notes, Ch 14, Notes)**

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**\* Continued on Shading and Illumination:**

- First we reviewed the “Overall Formula” of the Phong Reflection Model (shown in the next 2 slides) as presented last time, which is implemented in the sample program “Handout: rotate-cube-shading.cpp”.
- Discussed the sample program “Handout: rotate-cube-shading.cpp” (complete sample program has been posted at

<http://cse.poly.edu/cs653/Rotate-Cube-Shading.tar.gz>;

PDF file listing of the major files has been posted at

“NYU Classes -> Resources -> PDF Listing of Sample Code Major Files -> Handout-rotate-cube-shading.cpp.pdf”)

- Some screenshots of the sample program PDF with annotations are then shown next.
- Watched demo videos at “NYU Classes -> Media Gallery”:  
“Demo-Rotate-Cube-Shading” and “Demo-HW3-Parts-c-d”.

**\* Then we discussed a new topic: Composition Techniques.**

Overall formula:

$$I = \underbrace{K_a \cdot L_a}_{\text{global ambient light}} + \sum_{\text{light } i} (\text{Attenuation})_i \cdot [K_a \cdot L_a + K_d \cdot L_d \cdot \max\{\ell \cdot n, 0\} + \text{if } \ell \cdot n \geq 0 \cdot K_s \cdot L_s \cdot (\max\{\ell \cdot n, 0\})^\alpha]_i$$

Note: component-wise multiplications:  
 $K_a \cdot L_a, K_d \cdot L_d, K_s \cdot L_s$   
 They are attenuated differently

(1) If light  $i$  is a distant (directional) light, then

①  $(\text{Attenuation})_i = 1$

② vector  $\ell$  (from pt  $p$  to light source  $i$ ) =  $-(\text{distant light direction } L)$   $\ell = -L$

(2) If light  $i$  is a point source, then

①  $(\text{Attenuation})_i = \frac{1}{a + b \cdot d + c \cdot d^2}$

where  $d$  = distance from pt  $p$  to the light source  
 $a, b, c$ : constant, linear, quadratic attenuations

is  $\ell = -L$  (Also, use this  $\ell$  to compute  $h = \text{normalize}(\ell + v)$ )

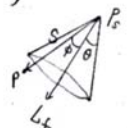
replaced with  $(n \cdot h)$

(3) If light  $i$  is a spotlight then

①  $(\text{Attenuation})_i = \frac{1}{a + b \cdot d + c \cdot d^2} \cdot (\text{spotlight-attenuation})_i$

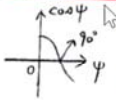
$a, b, c, d$  are as in (2) point source.

② (spotlight-attenuation) $_i$  = ?



$\theta$ : spotlight cut-off angle  $\theta \in [0, 90^\circ]$

(a) If  $\phi > \theta$  then contribution = 0



In the range  $[0, 90^\circ]$

$\phi > \theta \Leftrightarrow \cos \phi < \cos \theta$

$\Leftrightarrow (L_s \cdot S) < \cos \theta$

$\Leftrightarrow L_s \cdot (-\ell) < \cos \theta$

In particular,  $d = |P S|$

$\ell = \text{normalize}(P P_s)$

$= -s$

$\therefore s = -\ell$

If  $L_s \cdot (-\ell) < \cos \theta$  then (spotlight-attenuation) $_i = 0$

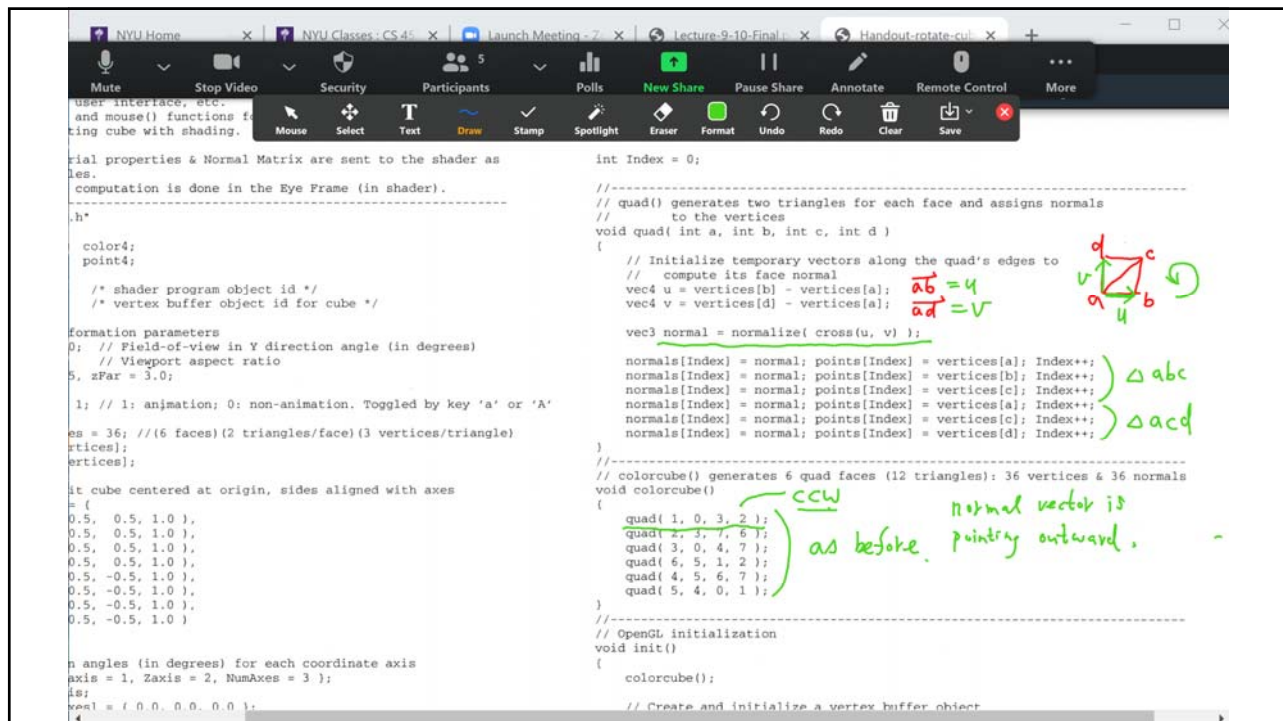
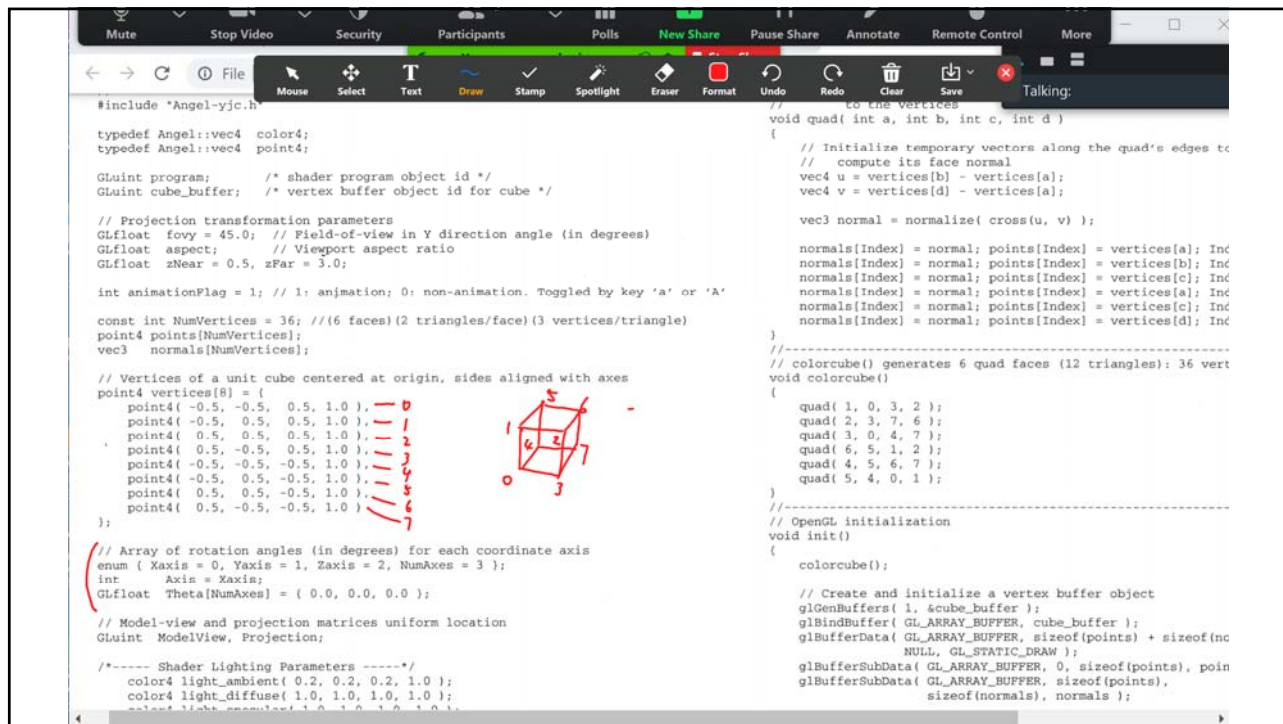
(b) Else

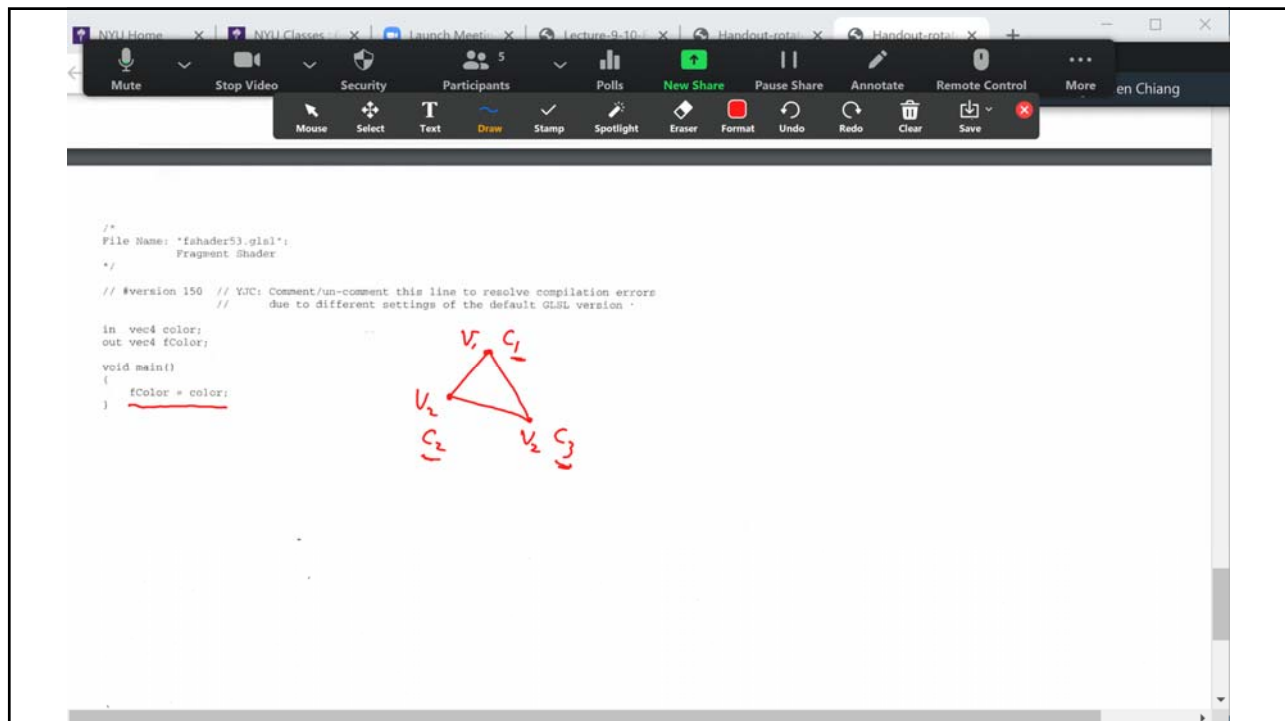
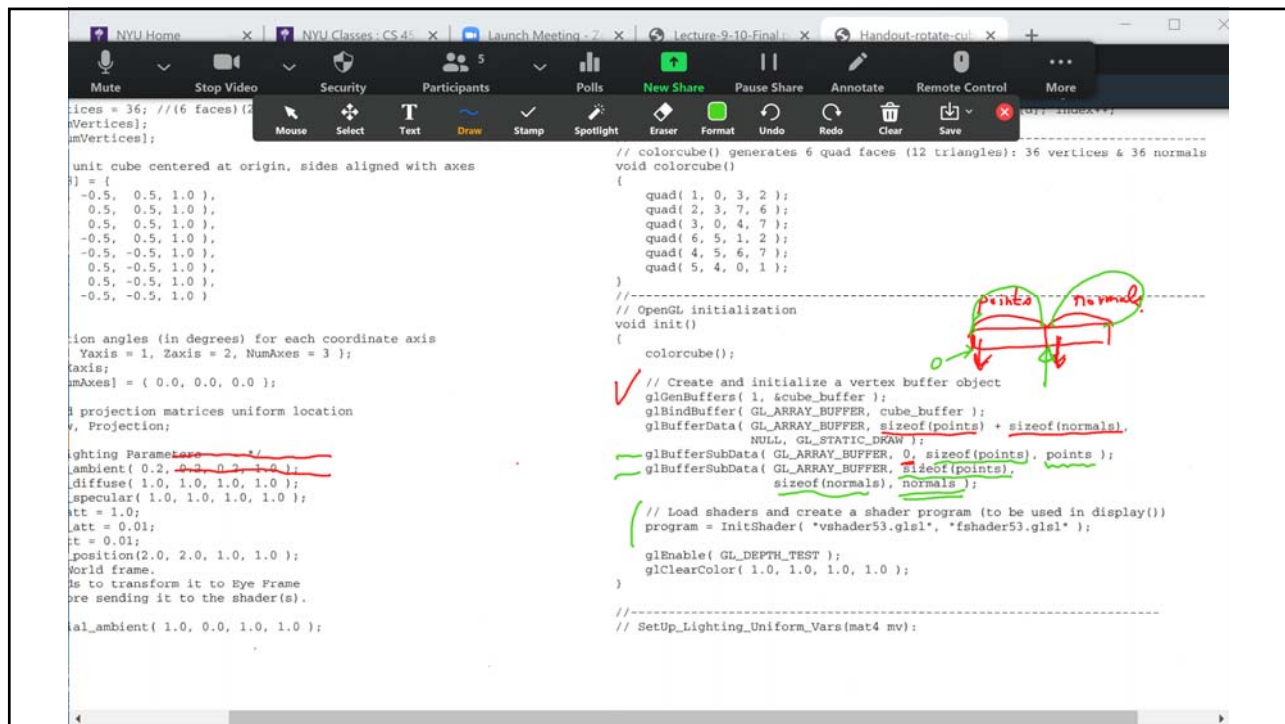
(spotlight-attenuation) $_i = (\cos \phi)^e$

$e$ : spotlight exponent

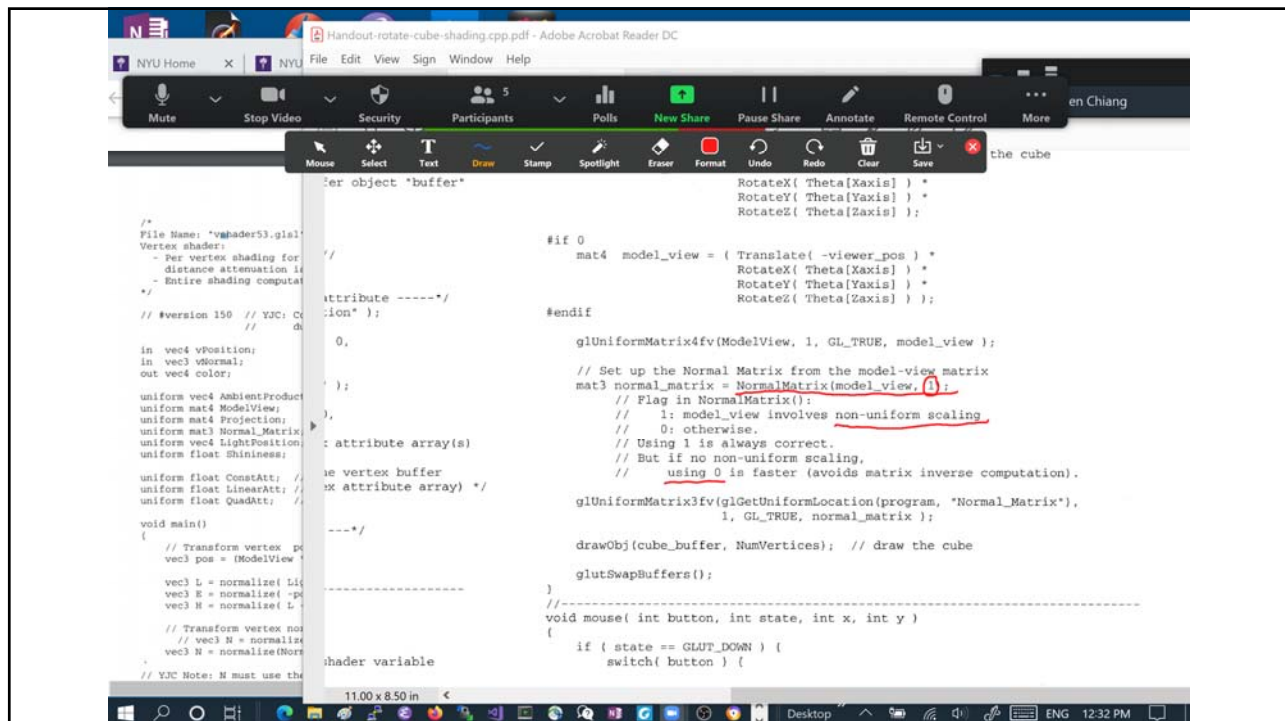
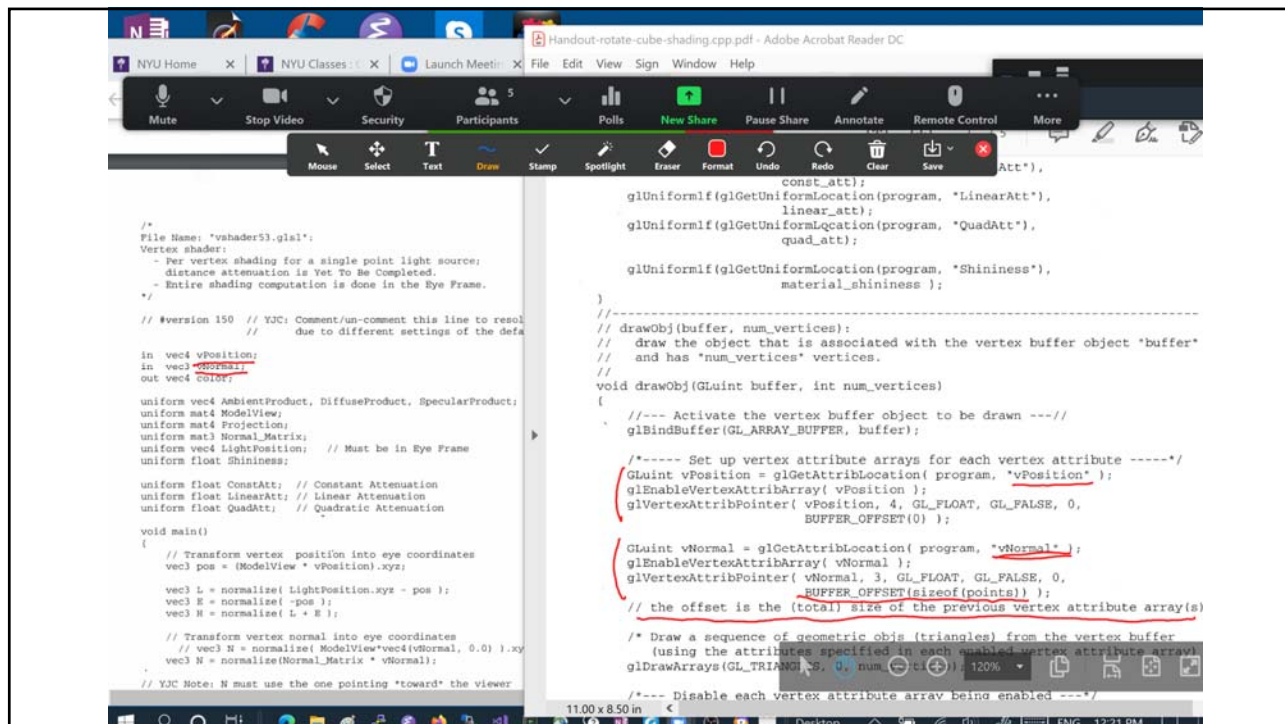
$= [L_s \cdot (-\ell)]^e$

Combining (a), (b): (spotlight-attenuation) $_i = [\text{if } L_s \cdot (-\ell) \geq \cos \theta] \cdot [L_s \cdot (-\ell)]^e$









## \* Normal Matrix

\* Typically we perform shading computation in the (eye frame) (i.e. the right-handed eye frame where the eye/camera is at the origin looking at the -z direction. This is the frame obtained by applying LookAt() to the world frame).

Let  $\vec{T}$  be the tangent at pt  $p$  being shaded.

$\vec{n}$  normal

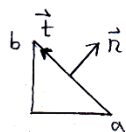
$\vec{n}, \vec{T}$  are in the model frame

$M$  the model-view matrix

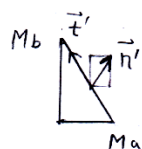
i.e.  $Mp$  puts  $p$  in the eye frame  
in  $x, y, z$ -dimensions are different

(1) Suppose  $M$  involves (non-uniform scaling) (i.e. scaling factors

e.g.  $S(1, 2)$   
in 2D:



$S(1, 2)$



$$\vec{T}' = M\vec{b} - M\vec{a} = M(\vec{b} - \vec{a})$$

$= M\vec{T}$  is the tangent after transformation

i.e. We can still apply  $M$  to  $\vec{T}$  to obtain the new tangent  $\vec{T}'$  correctly.

But applying  $M$  to  $\vec{n}$  does NOT give the correct normal vector.

(since  $\vec{n}'$  is NOT perpendicular to  $\vec{T}'$ )

(2) Deriving the correct matrix for normal vector: the (normal matrix)

$$\text{Let } \vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$$

$$\vec{n} \cdot \vec{T} = 0$$

The dot product can be expressed as matrix multiplication:

$$\vec{n} \cdot \vec{T} = \begin{bmatrix} \vec{n} \end{bmatrix}^T \begin{bmatrix} \vec{T} \end{bmatrix} = (\vec{n})^T \vec{T} \quad (*) \quad (\vec{n})^T: \text{transpose of } \vec{n} = \begin{bmatrix} \end{bmatrix}$$

From (\*), we have

$$0 = (\vec{n})^T \vec{T} = (\vec{n})^T M^{-1} (M\vec{T})$$

$$\text{But } M = \begin{bmatrix} l & T \\ 0 & 1 \end{bmatrix}$$

and the 4th component of  $\vec{T}$  is 0  $\Rightarrow$  We can ignore the 4th column of  $M$ , i.e.  $\begin{bmatrix} T_x \\ T_y \\ T_z \\ 1 \end{bmatrix}$  and can be ignored

$\Rightarrow$  Then the 4th row of the remaining columns are 0

$\therefore$  In (\*) we can use  $l$  to replace  $M$ :

$$((\vec{n})^T l^{-1}) (l \vec{T}) = 0$$

$\vec{x}^T$  where  $\vec{x}$  is the transformed normal, in the form of (\*):

$$\therefore (\vec{x})^T = (\vec{n})^T l^{-1} \Rightarrow \vec{x} = ((\vec{n})^T l^{-1})^T = (l^{-1})^T (\vec{n})$$

cf. In (\*):  $(\vec{n})^T \vec{T} = 0$

Here:  $(\vec{x})^T \vec{T} = 0$

$\Rightarrow$  Desired normal  $\vec{x}$  is obtained by  $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$  where the 3x3 matrix  $N$  (normal matrix) is  $(l^{-1})^T$

Simplification:

- (3) If  $M$  only involves translations, rotations, uniform scaling, and LookAt() then: translations have no effect on  $l$  LookAt() has translation and rotation  $\Rightarrow l$  only involves rotations and uniform scaling
- But uniform scaling has no effect after we normalize the transformed normal vector

$\Rightarrow l \equiv R$ . But  $R^{-1} = R^t$

i  $(l^{-1})^t \equiv (R^{-1})^t = (R^t)^t = R \equiv l$

ie ① We can use  $(l)$  to replace  $(l^{-1})^t$

ie ② We can use the model-view matrix  $M$  ( $4 \times 4$ ) to apply to normal  $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$

①  $\equiv$  ②

(scaling factors in  $x$ ,  $y$ ,  $z$ -dim are all the same)

4 components

# Screenshot for Elaboration:

in 2D.

1. We can still apply  $M$  to  $\vec{t}$  to obtain the new tangent  $\vec{t}'$  correctly. But applying  $M$  to  $\vec{n}$  does NOT give the correct normal vector. (since  $\vec{n}'$  is NOT perpendicular to  $\vec{t}'$ )

(2) Deriving the correct matrix for normal vector: the (normal matrix)

Let  $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$   $\vec{n} \cdot \vec{t} = 0$  The dot product can be expressed as matrix multiplication:

$$\vec{n} \cdot \vec{t} = \begin{bmatrix} n_x & n_y & n_z & 0 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \\ t_w \end{bmatrix} = 0$$

From (\*) we have  $0 = (\vec{n})^t \vec{t} = (\vec{n})^t M^{-1} \begin{bmatrix} t_x \\ t_y \\ t_z \\ t_w \end{bmatrix}$  But  $M = \begin{bmatrix} l & T \\ 0 & 1 \end{bmatrix}$  and the 4th component of  $\vec{t}$  is 0  $\Rightarrow$  We can ignore the 4th column of  $M$  and can be ignored  $\Rightarrow$  Then the 4th row of the remaining columns are 0

$\therefore$  In (\*) we can use  $l$  to replace  $M$ :

$$(\vec{n})^t l^{-1} \vec{t} = 0$$

$(\vec{x})^t$  where  $\vec{x}$  is the transformed normal, in the form of (\*):

$$\vec{x} = (\vec{n})^t l^{-1} \Rightarrow \vec{x} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} l^{-1}$$

$\Rightarrow$  Desired normal  $\vec{x}$  is obtained by  $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$  where the  $3 \times 3$  matrix  $N$  (normal matrix) is  $(l^{-1})^t$

eye  
grave



## New Topic: Compositing Techniques

Recall:

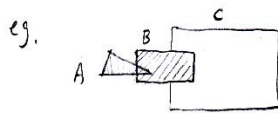
\* Fragments : generated by the rasterization of geometric primitives (polygons, etc.)  
Each fragment corresponds to a single pixel

\* Compositing Techniques : compositing,  $\alpha$ -blending

\* How do we model transparent objects? — the alpha channel

RGBA (RGB $\alpha$ ) color : (r, g, b,  $\alpha$ )  
 $\alpha$  → opacity :  $\begin{matrix} 1 & \text{opaque} \\ \updownarrow & \\ 0 & \text{transparent} \end{matrix}$   
 (transparency =  $1 - \alpha$ )

\*  $\alpha$  value controls how the RGB values are written to the frame buffer.



B is opaque, blocking C in <sup>the</sup> overlapped portion.

A is transparent, the portion overlapped with B is blended with the color of B (blending the colors of A & B)

\* Many fragments, each coming from a different object, may correspond

to the same pixel  $\Rightarrow$  each such fragment contributes <sup>to</sup> the color of the pixel.

the final color of the pixel is obtained by blending the fragment colors.

The corresponding objects are blended or composited together.

\* When a polygon is processed, pixel-size fragments are computed.

The fragments are assigned colors based on the shading model used.

Regard the fragment as the (source pixel)

the frame-buffer pixel as the (destination pixel)

Previously : z-buffer, opaque : source pixel is closer to viewer  $\Rightarrow$  source pixel (replaces) the destination pixel.

destination pixel :  $\Rightarrow$  source pixel is (blocked) no action.

Now : blend the source and destination pixels in various ways

color of source pixel:  $s = [s_r \ s_g \ s_b \ s_a]$  source blending factor  $b = [b_r \ b_g \ b_b \ b_a]$

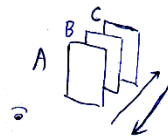
destination:  $d = [d_r \ d_g \ d_b \ d_a]$  destination:  $c = [c_r \ c_g \ c_b \ c_a]$

$$d' \leftarrow bs + cd$$

compositing: replace  $d$  with  $d' = [b_r s_r + c_r d_r \ b_g s_g + c_g d_g \ b_b s_b + c_b d_b \ b_a s_a + c_a d_a]$

the resulting  $r, g, b, a$  values are clamped to  $[0.0, 1.0]$   $\begin{cases} \geq 1 \Rightarrow 1.0 \\ \leq 0 \Rightarrow 0.0 \end{cases}$

## Depth Cueing and Fog



"Over" operation:

back-to-front.

$$\begin{cases} C_d' = \alpha_s C_s + (1 - \alpha_s) C_d \\ \alpha_d' = \alpha_s + (1 - \alpha_s) \alpha_d \end{cases}$$

transparency: the fraction that the "behind color" survives

\* Depth Cueing: create illusion of depth by drawing objects farther from the viewer dimmer

\* Fog Effect: extend depth cueing.

See Handout for full details

$(A \text{ over } (B \text{ over } C))$ : back to front  
 $((A \text{ over } B) \text{ over } C)$ : front to back.

create the illusion of partially translucent space (fog) between the object and the viewer, by blending in a (distance-dependent color) as each fragment is processed

$f$ : fog factor, given by the fog equation  $f(z)$ , ( $f = f(z)$ )

$C_s$ : fragment color

$C_f$ : fog color

$z$ : distance between a (fragment) being rendered and the (viewer) given in the (eye coordinates)

(\*\* Note: The Handout for the "Over" operation is posted at NYU Classes:

"Resources -> Handouts -> CS4533\_Over-Op-Associativity.pdf")

Resulting color:  $C_{s'} = f C_s + (1-f) C_f$  — (\*)

fog-mode

fog equation  $f(z) = f$

linear fog

$$f = \frac{\text{end} - z}{\text{end} - \text{start}}$$

linear, depth-cueing effect.

exponential fog

$$f = e^{-(\text{density} \cdot z)}$$

exponential

exponential square fog

$$f = e^{-(\text{density} \cdot z)^2}$$

Gaussian

} fog effect

\*  $f$  specified is clamped to  $[0,1]$  and then used in (\*) to compute  $C_{s'}$ .

o From fog equation:  
 $z \uparrow \Rightarrow f \downarrow$  ( $f$  is clamped to  $[0,1]$ )  
Plugging  $f$  into (\*)  
 $\Rightarrow C_s$  has more weight  
 $\Rightarrow$  when object is farther ( $z \uparrow$ )  
we see more of the fog color ( $C_f$ )