CS4533 Lectures 9-10 Slides/Notes

Shading and Illumination (Notes, Ch 14)

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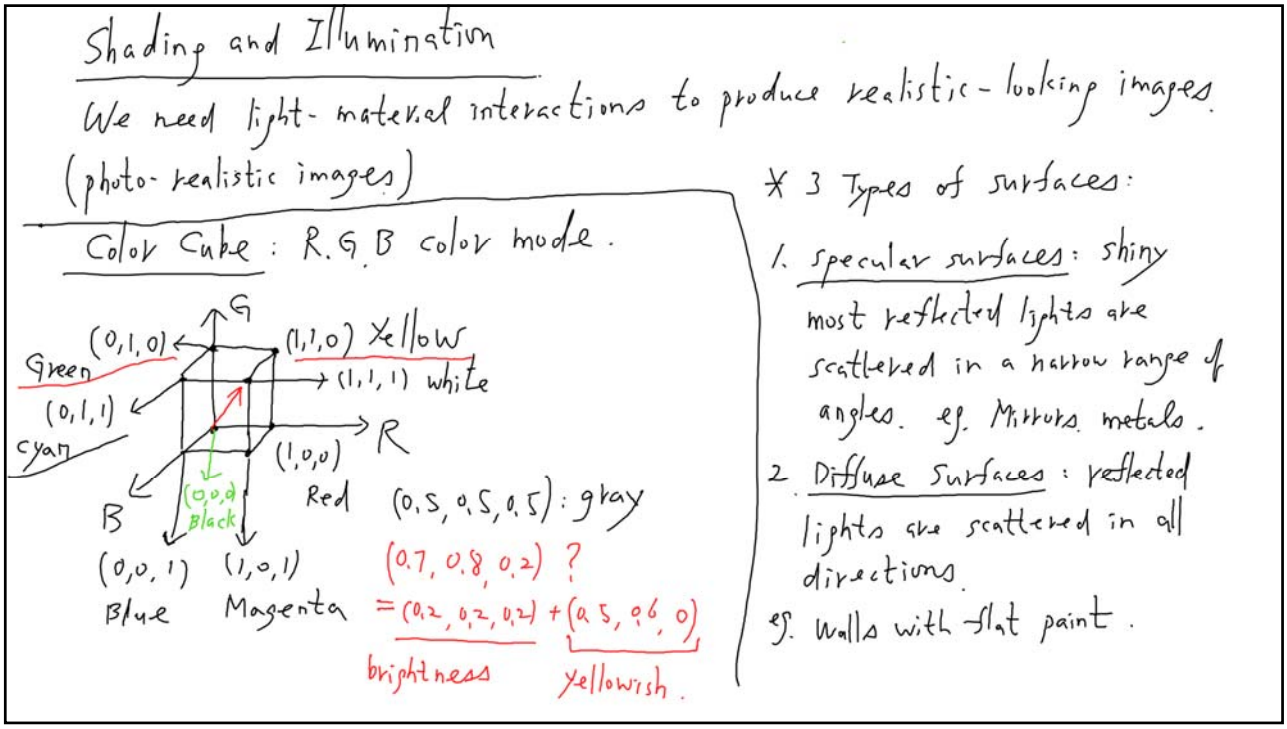
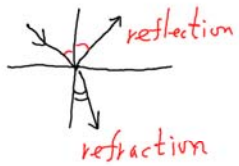


3. Translucent Surfaces : allow some lights to penetrate.



(refraction of lights)

eg. glass, water

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* 4 Types of Light Sources

- Describe a light source with a 3-component intensity function.

$$I = \begin{bmatrix} I_r \\ I_g \\ I_b \end{bmatrix}$$

I_r, I_g, I_b: intensity of independent red, green, blue components.

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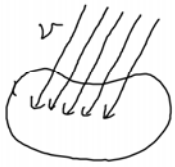
1. Ambient Light : uniform lighting (eg. classroom background lighting)

$$I_a = \begin{bmatrix} I_{ar} \\ I_{ag} \\ I_{ab} \end{bmatrix}$$

I_a is identical at every point in the scene.

2. Distant Light Source : light source is far away from the surface.



parallel light rays from the light source.

Specify the parallel ray vector V.

3. Point Source : emits light rays equally in all directions

Let P_0 be the location of the point source.

$I(P_0) = \begin{bmatrix} I_r(P_0) \\ I_g(P_0) \\ I_b(P_0) \end{bmatrix}$

Intensity at point P : $I(P, P_0) = \frac{1}{|P-P_0|^2} I(P_0)$

Intensity $\propto$ $\frac{\#\text{ rays}}{\text{area}}$

$\frac{k \text{ rays}}{4\pi d^2}$ (fixed k)

$\propto \frac{1}{d^2}$

Total: emitting k rays in all directions.

Total sphere area $= 4\pi d^2$.

* Define : $|P - P_0| = d$.

$I(P, P_0) = \frac{1}{d^2} I(P_0)$

* $\frac{1}{d^2}$ gives very high contrast. (bright or dark)

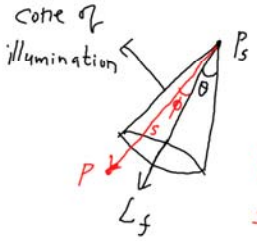
* In real world, sources are larger $\Rightarrow$ softer scenes.

* In real world, light sources are larger $\Rightarrow$ softer scenes.

$\Rightarrow$ usually we replace $\frac{1}{d^2}$ by $\frac{1}{a+bd+cd^2}$ where a, b, c are constants. We can adjust a, b, c to get more realistic effect.

($\frac{1}{d^2}$ is a special case with $a=b=0, c=1$)

4. Spotlights : The light has a narrow range of angles thru which the light is emitted.



P_s: spotlight position

L_f: focus direction

θ: cutoff angle $\in (0, 90^\circ]$

P: position of pt P being lit.

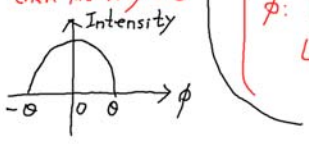
$S = P - P_s$: vector from P_s to P.

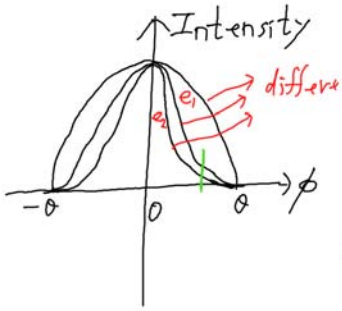
Intensity is attenuated by $(\cos\phi)^e$ $(e>1)$ when $\phi \le \theta$

e: spotlight exponent. describes how quickly the intensity drops off

ϕ: if $|L_f| = |S| = 1$, $L_f \cdot S = \cos\phi$.

If $\phi > \theta$ then intesity = 0





different e values. denoting different spotlight sources.

e_1 ? e_2 ($e_1 < e_2$ or $e_1 > e_2$?)

when $\phi \neq 0$ $\cos\phi < 1$. $e \uparrow$ $(\cos\phi)^e \downarrow$

$\Rightarrow e_2 > e_1$ $(e > 1)$

(eg. $\cos\phi = \frac{1}{2}$. $(\frac{1}{2})^1 = \frac{1}{2}$ $(\frac{1}{2})^2 = \frac{1}{4} < \frac{1}{2}$.)

Phong Reflection Model: describes material-light interactions.

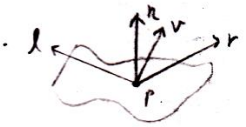
3 types of { material-light interactions / reflections } }
1. ambient reflection
2. diffuse =
3. specular =

3 components of the model (to be added together)

(Next class)

* **Phong Reflection Model** : 3 types of (material-light interactions) reflections: 1. ambient 2. diffuse 3. specular
(local ~~shading~~ lighting model)
(3 components of the model)

* **Vectors used by the Phong model** :



p: point on the surface we want to shade

n: surface normal at p

l: vector from p to the light source in question.

v: " p to the viewer

r: vector of a perfectly reflected ray from l
(computed from l and n)

* Each light source L has 3 terms for light-material interactions:
  - ambient color $L_a = \{0,0,0,0\}$ (R G B A)
  - diffuse color $L_d = \{0,0,0,0\}$
  - specular : $L_s = \{0,0,0,0\}$
* Surface material also has 3 terms
  - ambient reflection coefficient $k_a = \{0,0,0,0\}$ (R G B A)
  - diffuse : k_d
  - specular : k_s (discussed later)

* Same computation applied to each different point p on the surface.

(l, n, v, r are different for different points. (will simplify later))

3 reflection terms

1. **Ambient Reflection** : intensity of ambient light L_a is the same at every point on the surface

(ambient reflection coefficient) : $k_a = \{k_{ar}, k_{ag}, k_{ab}\}$ (surface property)

$k_{ac} \in [0,1] \quad \forall c = r, g, b$

$I_{ac} = k_{ac} \cdot L_{ac}$ for each $c = r, g, b$ i.e. for each color c, how much fraction of light intensity L_{ac} is reflected

ambient Light: $L_a = \{0,0,0,0\}$
material: $k_a = \{0,0,0,0 \neq 1.0\} \in [0,1]$

② $L_a = \{0,0,0,0\}$ (R G B A) ③ $I_a = k_a \cdot L_a$ (4 components each) ④ $\Rightarrow I_a = k_a \cdot L_a$ (Take (component-wise multiplication))

* $\because k_{ac} \in [0,1] \quad \forall c = r,g,b$. (k_{ar}, k_{ag}, k_{ab}) also makes a (color), extended with $(\alpha (=A) = 1.0)$

① $k_a = \{0,0,0,0\}$ (R G B A). we also call k_a the (ambient color) of the surface

2. **Diffuse Reflection** : rough surface, no prefered angle of reflection

diffuse Light: $L_d = \{0,0,0,0\}$
material: $k_d \in [0,1]$
$k_d = \{0,0,0,0 \neq 1.0\} \in [0,1]$

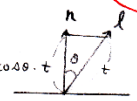
Perfectly diffuse surface: Lambertian surface

* Note: In $I_a = k_a L_a$, L_a can be a (global ambient light) as well

* Lambert's Law : we see only the vertical component of the incoming light

noon: brighter — afternoon: dimmer

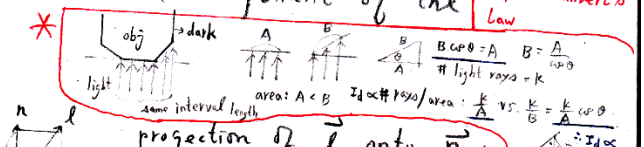
$I_d \propto \cos\theta$



projection of $\vec{l}$ onto $\vec{n}$: $t \cdot \cos\theta$

$\cos\theta = l \cdot n$ if $|l| = |n| = 1$

* Better way to explain Lambert's Law



$(\cos\theta) L_d = (l \cdot n) L_d$: vertical component of the light diffuse term L_d

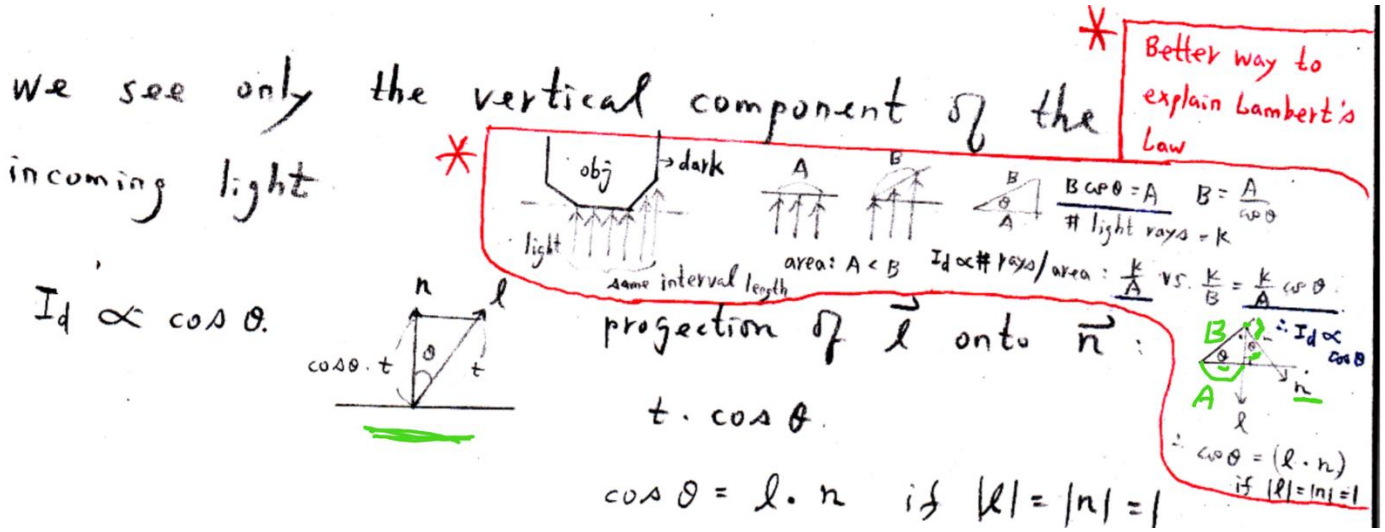
$k_d = \{0,0,0,0\}$ (R G B A) : (diffuse reflection coefficient) (Similar to k_a) each component $\in [0,1]$

$\Rightarrow I_d = k_d (l \cdot n) L_d$

($k_d \cdot L_d$: component-wise multiplication)

→ k_d is also called the (diffuse color) of the surface

Elaboration:



* If $l \cdot n < 0$ then no contribution. ie replace $(l \cdot n)$ with $\max\{(l \cdot n), 0\}$

* If a point source, need to account a distance attenuation

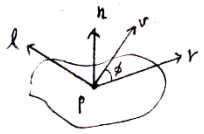
d: distance from the light source to the surface point p

$$\Rightarrow I_d = \frac{1}{a + b\,d + c\,d^2} K_d \cdot L_d \cdot \max\{(l \cdot n), 0\}$$

3. Specular Reflection: shiny, have highlight

(specular Light: $L_s = \{0,0,0,0\}$ material: ~~$K_s \in [0,1]$~~ $K_s = \{0,0,0,0\} \in [0,1]$)

eg. A red ball under white light will have a white highlight



r: direction of perfect reflection.

v: $\perp$ to the viewer

ϕ: angle between r & v

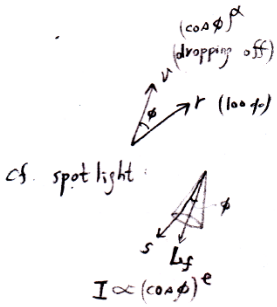
Approximation model: amount of specular reflection light seen by the viewer depends on ϕ:

$$I_s = K_s (\cos\phi)^\alpha L_s \quad (K_s \cdot L_s: \text{component-wise multiplication})$$

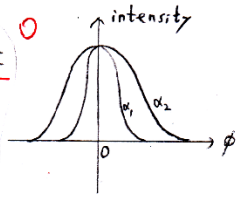
(K_s: specular reflection coefficient) $\{0,0,0,0\}$ each component $\in [0,1]$

(L_s: specular term of light)

K_s is also called the (specular color) of surface



α: shininess coefficient (material property)



$\phi \neq 0 \Rightarrow \cos\phi < 1$

$\alpha \uparrow \; (\cos\phi)^\alpha \downarrow$ ($\alpha > 1$)

$\alpha_1 > \alpha_2$.

$\Rightarrow$ Larger α, the reflected light is concentrated in a narrower region.

$\alpha \to \infty$: mirror $\quad \alpha \in [100, 500]$: metallic surface $\quad \alpha < 100$: broad highlights.

$\cos\phi = r \cdot v$ if $|r| = |v| = 1$, also no contribution if $r \cdot v < 0 \Rightarrow [\max(r \cdot v, 0)]^\alpha$

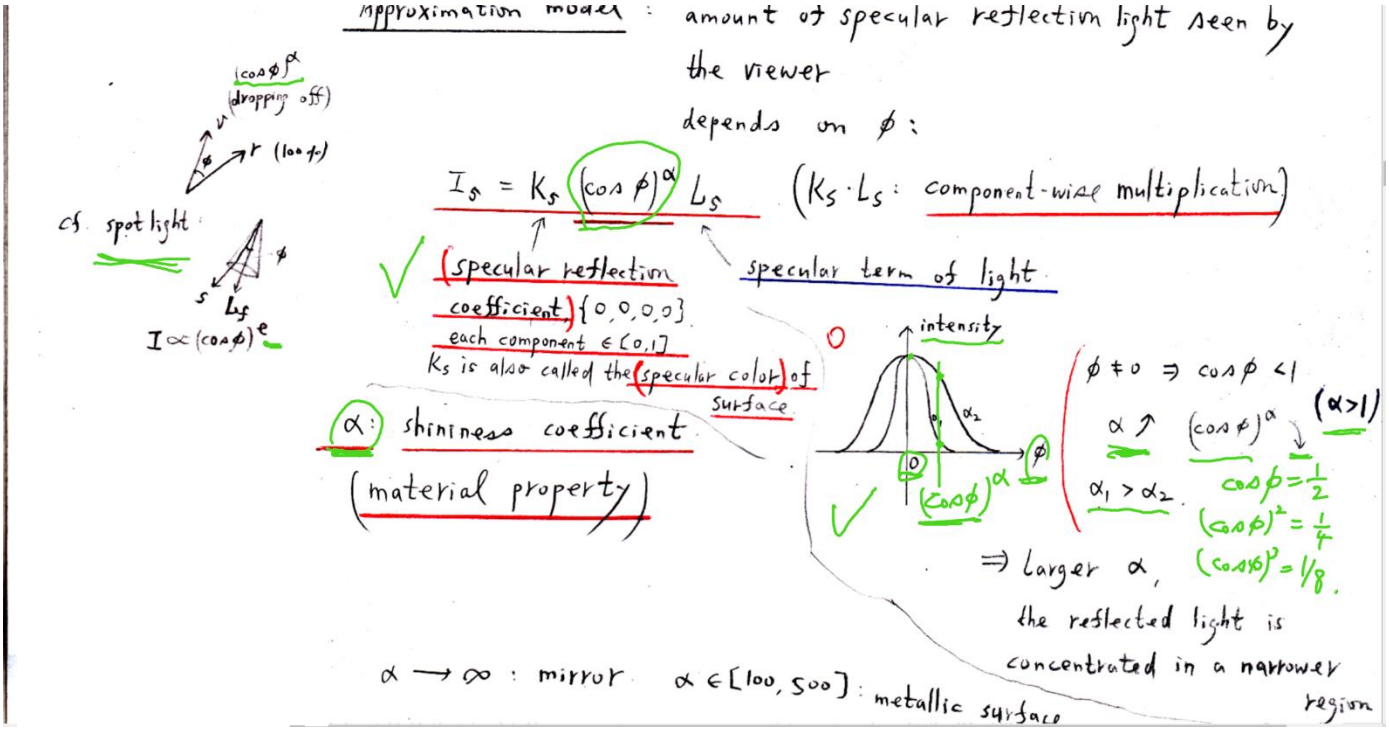
* If $l \cdot n < 0$ then no contribution.

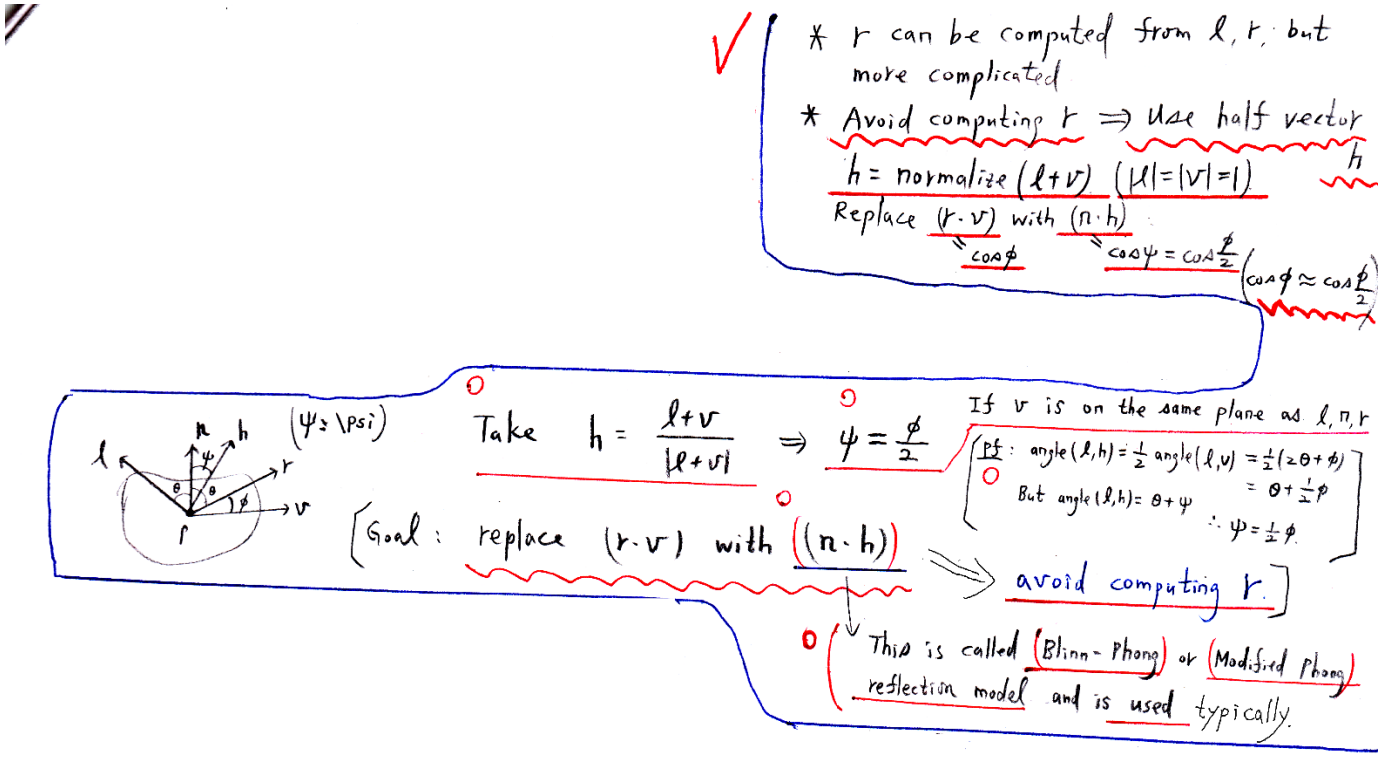
($\Rightarrow$ Define $[\text{if } l \cdot n \geq 0] = \begin{cases} 1 & \text{if } l \cdot n \geq 0 \\ 0 & \text{else} \end{cases}$)

$\Rightarrow \left(I_s = [\text{if } l \cdot n \geq 0] \cdot K_s \cdot L_s [\max(r \cdot v, 0)]^\alpha\right)$

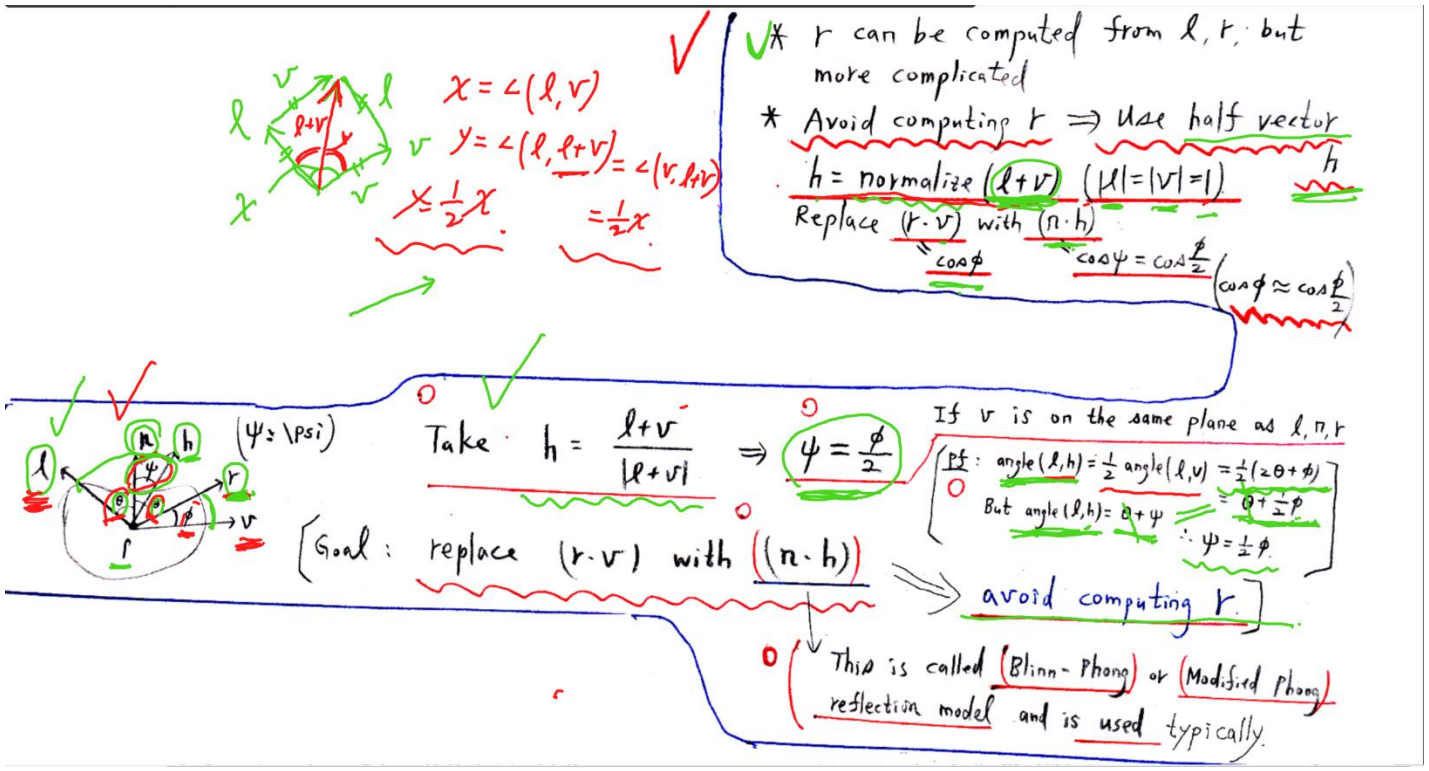
* Need to add distance attenuation if point source

Elaboration:





Elaboration:



Note: component-wise multiplications:
$K_a \cdot L_a$, $K_d \cdot L_d$, $K_s \cdot L_s$
They are attenuated differently

Overall formula:

$$I = K_a \cdot L_{a\_global} + \sum_{\text{light } i} (\text{Attenuation})_i \cdot \Big[K_a \cdot L_a + K_d \cdot L_d \cdot \max\{(\ell \cdot n), 0\} + [\text{if } \ell \cdot n \geq 0] \cdot K_s \cdot L_s \cdot \big(\max\{(r \cdot v), 0\}\big)^{\alpha} \Big]_i$$

(global ambient light)

replaced with $(n \cdot h)$

(1) If light i is a distant (directional) light, then

① $(\text{Attenuation})_i = 1$

② vector ℓ (from pt p to light source i) $= -$(distant light direction L)

L $\quad$ $\ell = -L$

ie $\ell = -L$ (Also, use this ℓ to compute $h = \text{normalize}(\ell + v)$)

(2) If light i is a point source, then

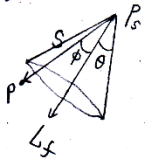
① $(\text{Attenuation})_i = \dfrac{1}{a + bd + cd^2}$

where d = distance from pt p to the light source position.
a, b, c: constant, linear, quadratic attenuations

(3) If light i is a spotlight then

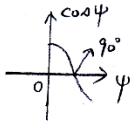
① $(\text{Attenuation})_i = \dfrac{1}{a + bd + cd^2} \cdot (\text{spotlight\_attenuation})_i$ $\quad$ a, b, c, d are as in (2) point source.

② $(\text{spotlight\_attenuation})_i = ?$



θ: spotlight cut-off angle $\quad \theta \in [0, 90°]$

(a) If $\phi > \theta$ then contribution $= 0$



In the range $[0, 90°]$

$\phi > \theta \iff \cos\phi < \cos\theta$

$\iff (L_f \cdot S) < \cos\theta$

$\iff L_f \cdot (-\ell) < \cos\theta$

In particular $d = |\overrightarrow{P_s P}|$

$\ell = \text{normalize}(\overrightarrow{P P_s})$

$= -S$

$\therefore S = -\ell$

If $L_f \cdot (-\ell) < \cos\theta$ then $(\text{spotlight\_attenuation})_i = 0$

(Let $L_f = \text{normalize}(L_f)$)

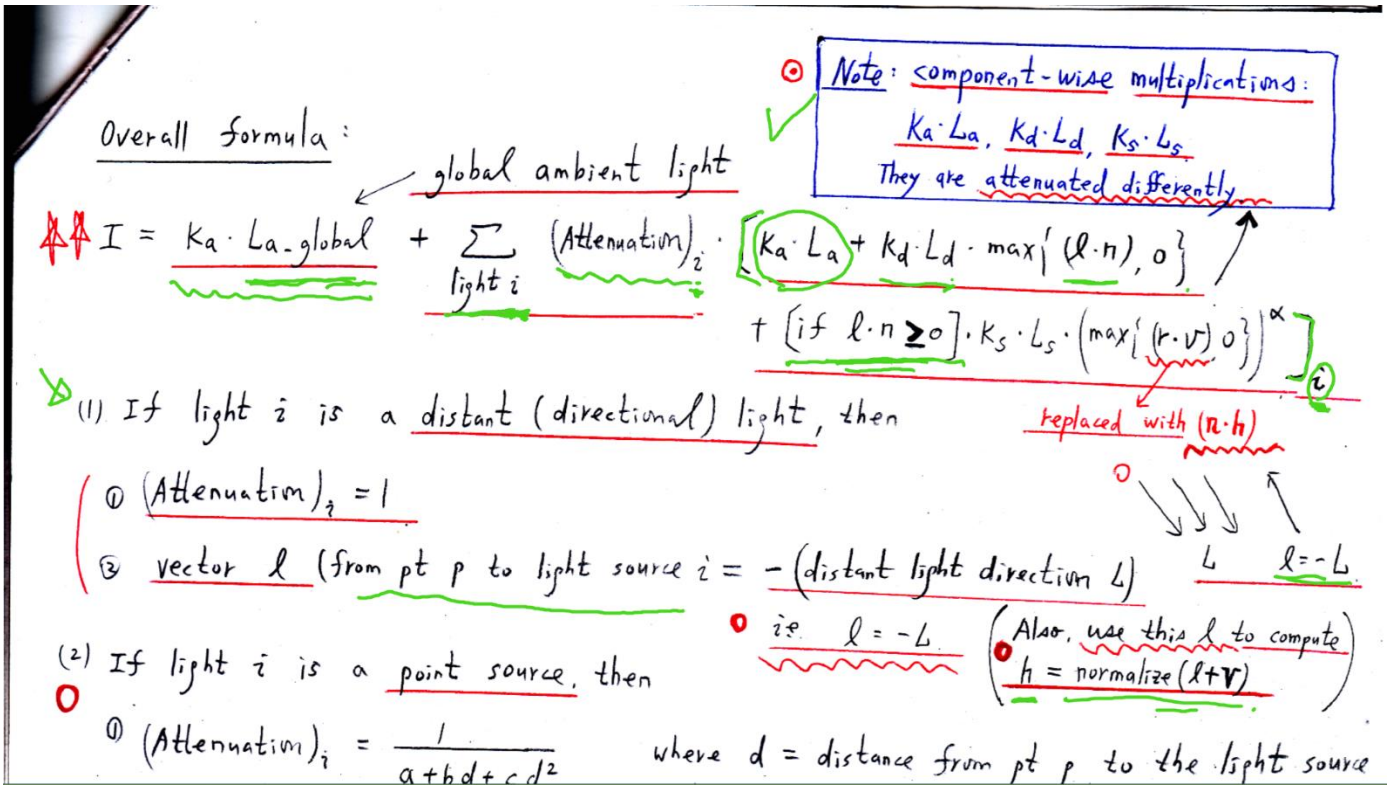
(b) Else

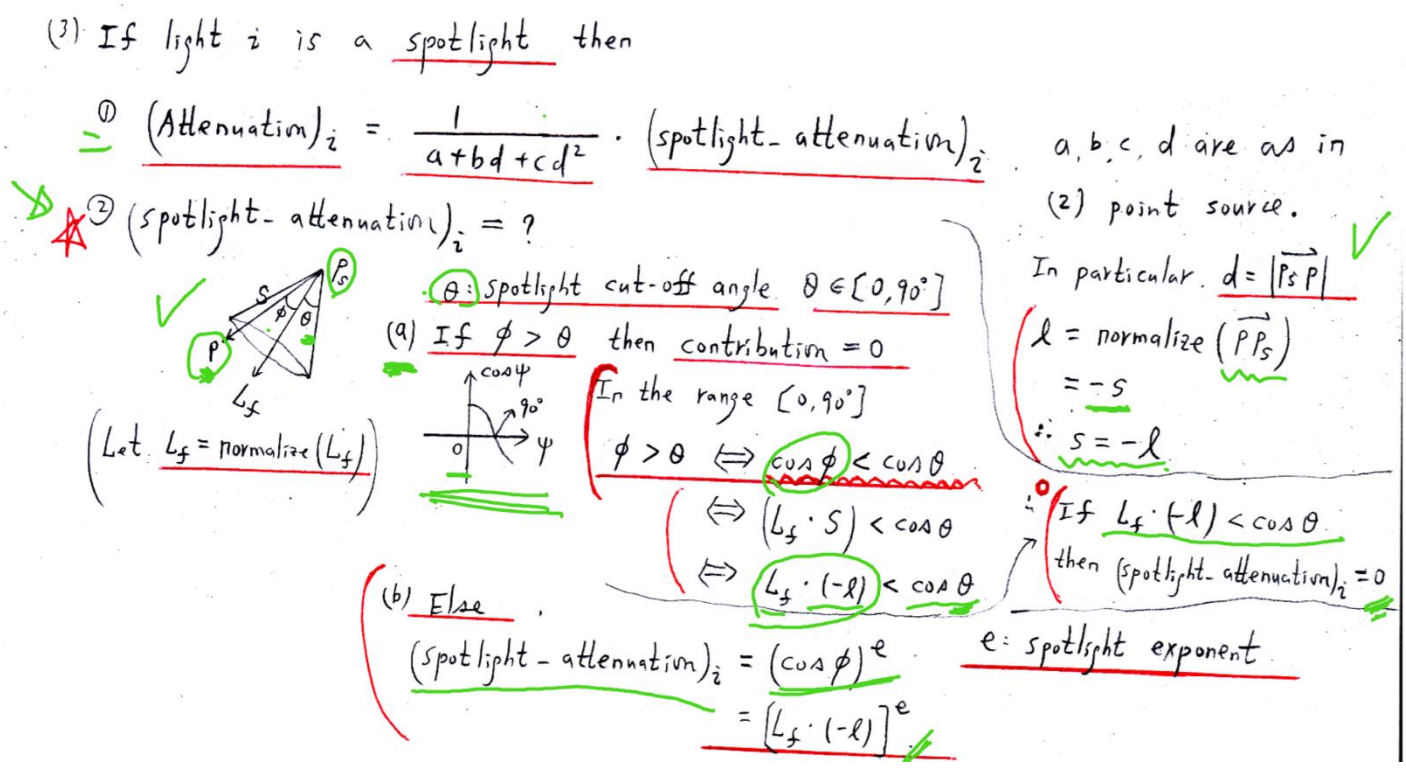
$(\text{spotlight\_attenuation})_i = (\cos\phi)^e$ $\quad$ e: spotlight exponent

$= [L_f \cdot (-\ell)]^e$

(Combining (a), (b): $(\text{spotlight\_attenuation})_i = [\text{if } L_f \cdot (-\ell) \geq \cos\theta] \cdot [L_f \cdot (-\ell)]^e$)

Elaboration:



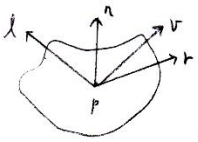


Polygonal shading : Phong reflection model : applied to every point on the surface
⇒ expensive

modeling curved surfaces in graphics : break curved surfaces into small flat polygons
⇒ simplify the shading calculation

* 3 ways of shading polygons :

1. Flat shading :



(r is defined from l, n)

l, n, v vary from point to point

① surface is a flat polygon ⇒ n is constant

② Assume a distant viewer ⇒ v =

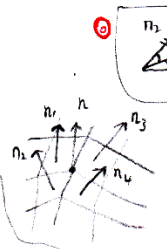
③ " distant light ⇒ l =

⇒ shading calculation is carried out once for each polygon

* short coming : small shading differences in adajent polygons will be seen with amplification by human eyes : overshooting the intensity on one side (HW demo) undershooting " the other
of an edge

2. Interpolative and Gouraud shading :

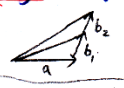
① average to get vertex normals :



$\because |n_1| = |n_2| = 1$
$\therefore n_1+n_2$ evenly divides the angle between n_1 & n_2 $\Rightarrow n_1+n_2$ (averaged the directions of n_1 & n_2.

$$\vec{n} = \frac{n_1+n_2+n_3+n_4}{|n_1+n_2+n_3+n_4|}$$

Note : Each n_i must be (unit length) so that each is (weighted equally) in the sum

eg. 

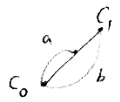
$a+b_1 \neq a+b_2$

average over the normals of the polygons sharing the vertex

② compute shade at vertices. (using Phong reflection model)

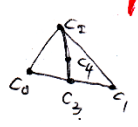
③ interpolate using vertex shade to get the shade for each point

* bilinear interpolation :



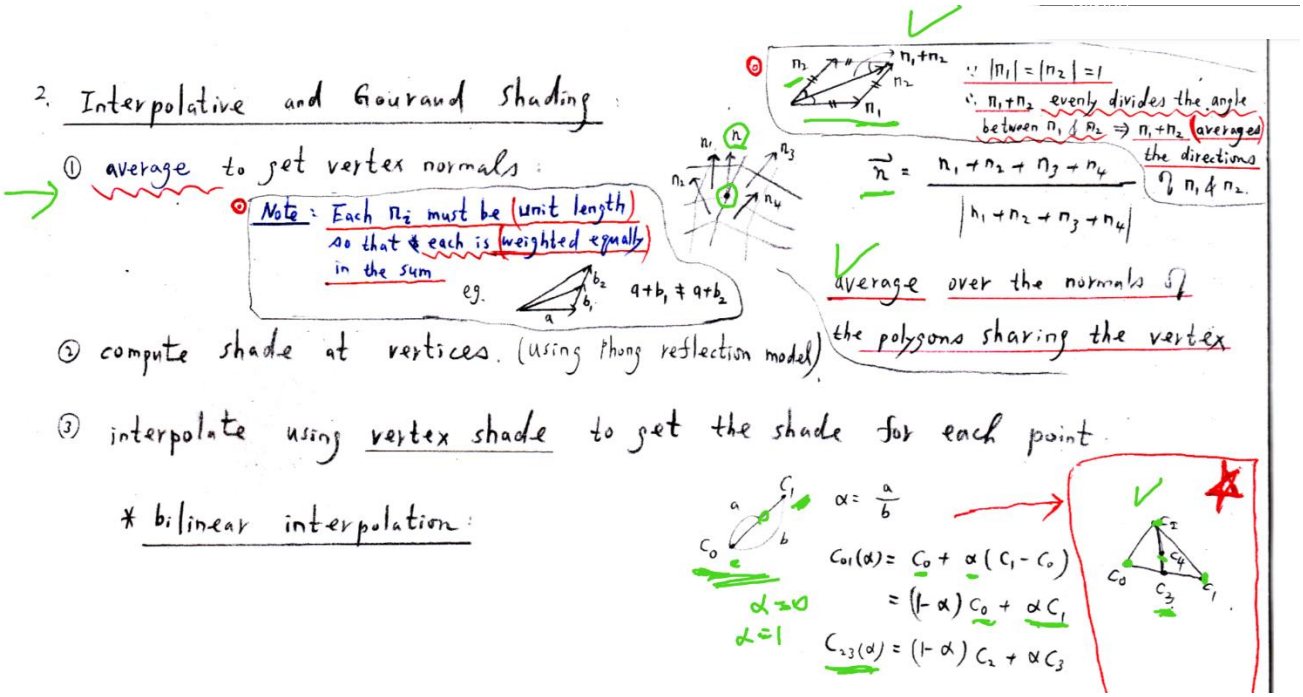
$$\alpha = \frac{a}{b}$$

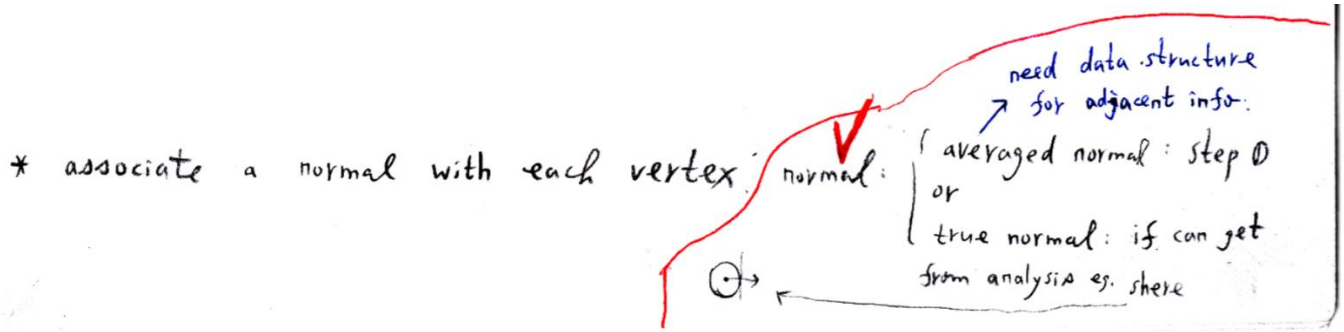
$$C_{01}(\alpha) = C_0 + \alpha(C_1 - C_0)$$
$$= (1-\alpha)C_0 + \alpha C_1$$
$$C_{23}(\alpha) = (1-\alpha)C_2 + \alpha C_3$$

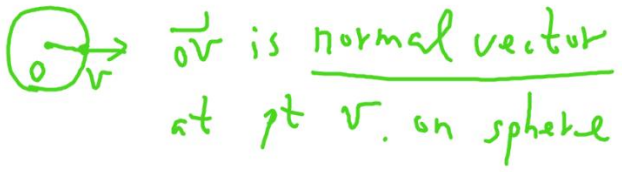


* associate a normal with each vertex : normal :
- averaged normal : step ① (need data structure for adjacent info.)
- or
- true normal : if can get from analysis eg. sphere

Elaboration:







3 Phong shading :

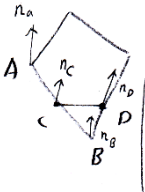
⓪ average to get vertex normals (as in step ① of Gouraud shading)

① interpolate to get normals at all points ✱

Note: flat polygon.
⓪ originally this polygon has its normal $\vec{n}$
Step ① may give each point in the interior a normal different from $\vec{n}$
Fine. the purpose is to smooth out the normal changes

Each vertex has a different normal by ⓪
⇒ want to have a smooth transition for normals

✱ use bi linear interpolation



$$n_{AB}(\alpha) = (1-\alpha)\, n_A + \alpha\, n_B$$

$$n_{CD}(\beta) = (1-\beta)\, n_C + \beta\, n_D$$

⊙ Note: by property of linear interpolation. min, max only occurs at endpoints
∴ max/min shading only occurs at vertices in Gouraud shading
⇒ Gouraud : has no highlight in interior of a polygon.
But Phong shading can.

② compute the shade at each point using its normal

(independent at each point)

computation

✱ Smoother than Gouraud shading

✱ Now is supported by programmable GPU Done in fragment shader

* Normal Matrix

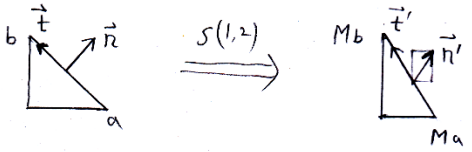
* Typically we perform shading computation in the (eye frame) (ie. the right-handed eye frame where the eye/camera is at the origin looking at the $-z$ direction. This is the frame obtained by applying LookAt( ) to the world frame)

Let $\vec{t}$ be the tangent at pt p being shaded.
$\vec{n}$ " normal "

$\vec{n}, \vec{t}$ are in the model frame
M the model-view matrix
ie Mp puts p in the eye frame

(1) Suppose M involves (non-uniform scaling) (ie scaling factors in x-, y-, z- dimensions are different)

eg. $S(1,2)$ in 2D.



$\vec{t'} = Mb - Ma = M(b-a)$
$= M\vec{t}$ is the tangent after transformation

ie. We can still apply M to $\vec{t}$ to obtain the new tangent $\vec{t'}$ correctly.
But applying M to $\vec{n}$ does NOT give the correct normal vector. (since $\vec{n'}$ is NOT perpendicular to $\vec{t'}$)

(2) Deriving the correct matrix for normal vector: the (normal matrix)

Let $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$ $\quad \vec{n} \cdot \vec{t} = 0$

The dot product can be expressed as matrix multiplication:

$\vec{n} \cdot \vec{t} = \begin{bmatrix} & & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = (\vec{n})^t \vec{t}$ ——— (*)
$= 0$

$(\vec{n})^t$: transpose of $\vec{n} = \begin{bmatrix} \end{bmatrix}$

From (*) we have $0 = (\vec{n})^t \vec{t} = \left((\vec{n})^t M^{-1}\right)\left(M\vec{t}\right)$ (♣)
$= I$

But $M = \begin{bmatrix} \ell & T \\ 0 & 1 \end{bmatrix}$ and the 4-th component of $\vec{t}$ is 0 ⇒ We can ignore the 4th column of M, ie $\begin{bmatrix} T \\ 1 \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \\ 1 \end{bmatrix}$

⇒ Then the 4th row of the remaining columns are 0 and can be ignored

∴ In (♣) we can use ℓ to replace M:

$\left((\vec{n})^t \ell^{-1}\right)\left(\ell \vec{t}\right) = 0$
$= (\vec{x})^t$ where $\vec{x}$ is the transformed normal, $\quad$ = transformed tangent $\vec{t'}$

in the form of (*):

∴ $(\vec{x})^t = (\vec{n})^t \ell^{-1} \Rightarrow \vec{x} = \left[(\vec{n})^t \ell^{-1}\right]^t = (\ell^{-1})^t (\vec{n})$

cf: In (*): $(\vec{n})^t \vec{t} = 0$
Here: $(\vec{x})^t \vec{t'} = 0$.

⇒ Desired normal $\vec{x}$ is obtained by $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ where the 3×3 matrix N (normal matrix) is $(\ell^{-1})^t$

Simplification:

(3) If M only involves translations, rotations, uniform scaling (scaling factors in X-, Y-, Z-dim are all the same), and LookAt( )

then: translations have no effect on ℓ
LookAt( ) has translation and rotation
$\Rightarrow$ ℓ only involves rotations and uniform scaling

But uniform scaling has no effect after we normalize the transformed normal vector

$\Rightarrow \ell \equiv R$. But $R^{-1} = R^t$

$\therefore (\ell^{-1})^t \equiv (R^{-1})^t = (R^t)^t = R \equiv \ell$

ie ① We can use (ℓ) to replace $(\ell^{-1})^t$

(ie ② We can use the model-view matrix M (4x4) to apply to normal $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$ (4 components)

① ≡ ②