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TROTTING, PACING AND BOUNDING BY A QUADRUPED ROBOT

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Abstract—This paper explores the quadruped running gaits that use the legs in pairs: the trot (diagonal pairs), the pace (lateral pairs), and the bound (front and rear pairs). Rather than study these gaits in quadruped animals, we studied them in a quadruped robot. We found that each of the gaits that use the legs in pairs can be transformed into a common underlying gait, a *virtual biped gait*. Once transformed, a single set of control algorithms produce all three gaits, with modest parameter variations between them. The control algorithms manipulated rebound height, running speed, and body attitude, while a low-level mechanism coordinated the behavior of the legs in each pair. The approach was tested with laboratory experiments on a four-legged robot. Data are presented that show the details of the running motion for the three gaits and for transitions from one gait to another.

NOMENCLATURE

β	an angle that determines the turning moment about the yaw axis
$\dot{\gamma}_d$	desired rate of a hip actuator
$[\phi_P, \phi_R, \phi_Y]$	orientation of the body in the pitch, roll, and yaw axes
$[\tau_{P,i} \tau_{R,i} \tau_{Y,i}]$	torque exerted on the body about the pitch, roll, and yaw axes
$A_t B_t$	names for the virtul legs
f_x, f_y	forces measured at the hip actuators
i, j	indices indicating physical legs
$k_{\dot{\gamma}}$	hip actuator rate feedforward gain
k_f	actuator force feedback gain
k_p	actuator position feedback gain
$k_{p,P} k_{p,R}$	pitch and roll position feedback gains
k_v	actuator velocity feedback gain
$k_{v,P} k_{v,R}$	pitch and roll velocity feedback gain
$k_{\dot{x}}, k_{\dot{y}}$	foot placement feedback gain
k_1, k_2	feedback gains used to control turning
L	fore–aft hip spacing
L_1, L_2, L_3	hydraulic actuator setpoints used to specify thrust
$r_{s,i}$	length of the *i*th leg's air spring
t	time
T_s	duration of the stance phase
u_i	servovalve output signal for the *i*th actuator
u_x, u_y	servovalve output signals for the hip actuators
$w_i, w_{i,d}, \dot{w}_i$	position, desired position, and velocity of the *i*th actuator
$w_{t,i}$	length of the *i*th leg's hydraulic actuator
W	lateral hip spacing
$[x \;\; y \;\; z]$	position of the center of mass of the trunk in room coordinates
$[\dot{x}_d, \dot{y}_d]$	desired running speed
$[x_{f,d} y_{f,d}]$	desired position of the virtual foot with respect to the projection of the virtual hip
$x_{h,i,d}, y_{h,i,d}$	desired displacement of the *i*th foot with respect to the projection of the *i*th hip

INTRODUCTION

Running animals have control systems that stabilize the attitude and altitude of the body while propelling the system in the desired direction at the desired speed. The algorithms responsible for control and balance in running are difficult to study directly in animals because of the richness of the biomechanics and of the nervous system involved.

Rather than attempt to study the control of running in animals, we explored the control of running in a legged robot. Programs executing on a digital computer were used to sense and actuate the behavior of the four-legged running machine shown in Fig. 1. During each experiment the control computer acquired data from the machine's sensors, accepted commands from a human operator, performed calculations according to control algorithms, and issued commands to the machine's actuators. The quadruped machine exhibited behavior that we describe as running, including trotting, pacing, bounding, pronking, and transitions between these gaits.

A specific goal of this work was to see if control algorithms used previously for one- and two-legged running robots could be generalized for four-legged running. A more general goal was to understand quadruped running as a problem in dynamic balance and control, and to evaluate the effectiveness of various algorithms. A third goal was to explore the interplay between the mechanical elements of a system that moves and the information processing elements that guide and stabilize the movement: how might they both contribute to the motions needed for locomotion?

The following section describes the approach we took to controlling the behavior of a quadruped machine to make it trot, pace and bound. Subsequent sections describe the implementation that was used to test the control algorithms and present data collected from the machine during experiments. Generally, algorithms for one-legged hopping were found to generalize for four-legged running, with the addition of a low-level leg coordination mechanism.

APPROACH

We considered the three simplest quadruped running gaits, the trot, the pace and the bound. These gaits are simple in that the legs are used in pairs. In trotting the legs work in diagonal pairs: the left front and right rear legs (LF–RR), strike the ground at the same time, level the ground at the same time, and swing in phase with one another about their hips.* After a flight phase during which no feet touch the ground, the other diagonal pair (RF–LR) provides support. Pacing uses the legs in lateral pairs (LF–LR and RF–RR), and bounding uses the front legs as a pair (LF–RF) and the rear legs as a pair (LR–RR). We restrict attention to running, so all three gaits involve a strict alternation between support phases and flight phases. Figure 2 shows gait diagrams for the three quadruped running gaits that use the legs in pairs, the *pair gaits*. Good descriptions of the pair gaits have been available for over a hundred years (Marey, 1874; Muybridge, 1899).

The control task is to propel the body in the desired direction at the desired rate, to keep the body in a level posture, and to regulate the vertical rebounding motions of the body. In previous work, we studied these same problems in the context of one-legged hopping machines (Raibert, 1986a). The control for one-legged hopping was made simple by decomposing the algorithms into separate parts that regulated the body's hopping height, forward running speed, and attitude:

- *Hopping Height*—The control system used leg thrust during the stance phase to excite and modulate spring-mass oscillations of the springy leg and body.

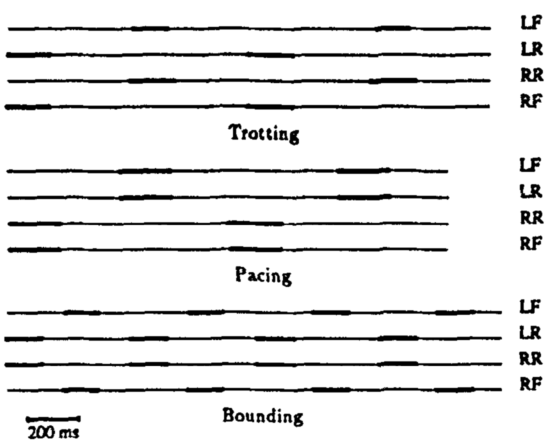


Fig. 2. Pair gaits. Gait diagrams showing the pattern of leg use for the trot, pace, and bound, as executed by the quadruped robot in experiments. Each of these gaits use the legs in pairs. The bars indicate periods of ground contact, as measured by load switches in the feet.

- *Forward Running Speed*—The control system positioned the feet during the flight phase to influence the accelerations of the body that would occur during the next stance phase. Symmetry was used to simplify the dynamics (Raibert, 1986b).
- *Body Attitude*—The control system kept the body level by exerting torques about the hip axes during the stance phase.

These algorithms for one-legged hopping were adapted for control of systems that run on two legs by recognizing that bipeds typically run with just one leg active at a time—to first order, the swing leg can be thought of as idle. The algorithms that specified leg thrust, leg placement, and hip torque for one-legged hopping were used to specify these same parameters for the active leg of the biped. Experiments with one- and two-legged systems showed that the three-part algorithms could provide balance and control for running in place, traversing simple paths, running fast (13 mph), switching gaits between hopping and running, climbing stairs, and performing simple gymnastic maneuvers (Raibert, 1986a; Hodgins *et al.*, 1986; Koechling and Raibert, 1988; Hodgins, 1989).

To generalize the one-leg control algorithms a step further, to the case of quadruped running, we invoke the concept of the *virtual leg*. The virtual leg was first introduced by Sutherland to simplify the design of a one-ton six-legged walking machine that carried a human driver (Sutherland and Ullner, 1984). Sutherland designed a hydraulic circuit that coupled the load-bearing behavior of two legs to act as though they were just one leg. This arrangement simplified control of the legs.

Generally, the virtual leg is a construct that allows several separate legs to be represented by fewer virtual legs. For instance, the virtual leg shown at the top right of Fig. 3 represents the two physical legs shown at the top left. So long as the two physical legs exert equal forces on the ground, equal torques at their hips, and their feet have equal horizontal displacements from their hips, then their behavior is precisely equivalent to the behavior of the virtual leg (Raibert *et al.*, 1986).

The value of this approach is that it lets us reduce the quadruped pair gaits to equivalent *virtual biped gaits*. Figure 3 shows the correspondences between each of the pair gaits and an equivalent virtual biped gait. The control techniques described earlier for bipeds can then be used to control the virtual biped gaits in quadrupeds. We have already shown that bipeds can be controlled as though they use just one leg at a time, and that the three-part hopping algorithms are effective. What remains is to provide a low-level mechanism that coordinates the behavior of the physical legs so that the virtual leg and the physical legs have mechanically equivalent effects on the behavior of the body. This approach allows us to transform trotting, pacing and bounding into a common underlying gait, the virtual biped gait.

* No distinction is made in this paper between hips and shoulders.

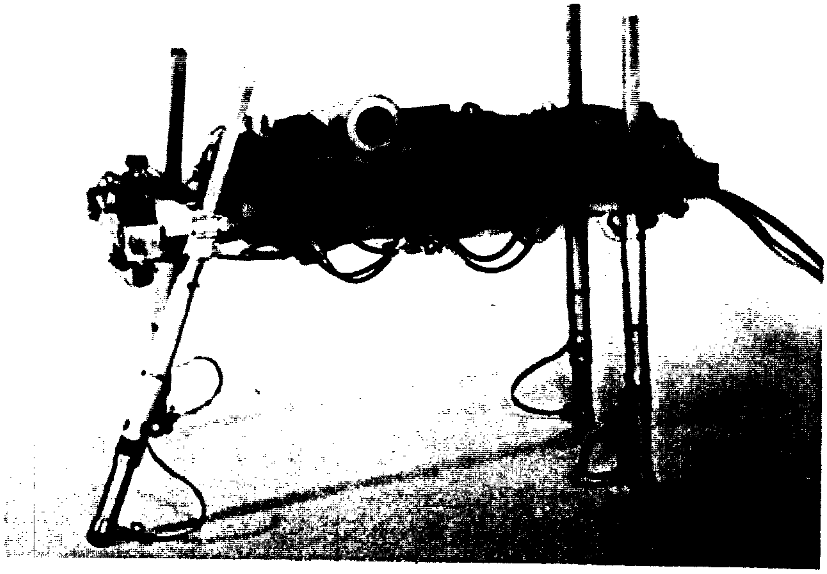


Fig. 1. Photograph of quadruped running machine used for experiments.

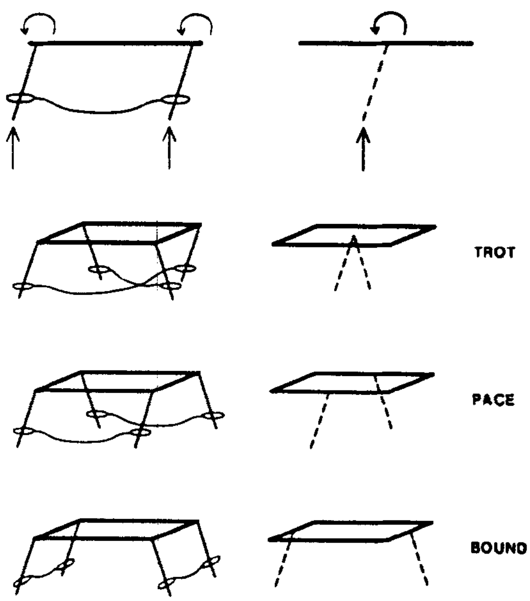


Fig. 3. Virtual legs. When two legs are coordinated to act in unison, they can be represented by a functionally equivalent *virtual leg*. The virtual leg and the original pair of physical legs both exert the same forces and moments on the body, so they both result in the same behavior. When each pair of legs is replaced by a virtual leg, the trot, the pace, and the bound are transformed into equivalent virtual biped gaits. One virtual leg is used for support at a time. Sutherland first introduced the concept of the virtual leg to simplify the design of a six-legged walking machine (Sutherland and Ullner, 1984).

In order to implement a control system based on the virtual leg concept, rules are needed for transforming the desired behavior of the virtual leg into the prescribed behavior of two physical legs.

- The legs should each exert axial thrust equal to half the axial thrust specified for the virtual leg.
- The hips should each exert a torque between the leg and body equal to half the hip torque specified for the virtual leg.
- The feet should strike the ground in unison and leave the ground in unison.
- The forward position of the feet with respect to their hips should equal the desired forward position of the virtual foot with respect to the virtual hip.

These rules eliminate degrees of freedom in the system, so they can be thought of as constraints or synergies. They allow the control system to map the desired system behavior into specific commands for each actuator of each leg.

We have outlined an approach to quadruped running that separates the problem into two parts. At one level, the control system guides and stabilizes motions of the body. This can be done the same way for the body of a four-legged system as for the body of one- or two-legged systems. At a lower level, the control system coordinates the behavior of individual legs to make them work together according to the rules of the virtual leg. The coordination specifies the relative placement of the feet, the relative thrust the legs deliver to the ground, and the relative hip torque exerted between the legs and the body. The virtual leg specifies how the individual legs should be controlled to make their net behavior equivalent to the desired behavior of the virtual leg.

ALGORITHMS FOR TROTTING, PACING AND BOUNDING

In order to evaluate the approach outlined in the last section, we did experiments with the machine shown in Figs 1 and 7. During experiments, a human operator communicated with the running machine through a control panel and joystick connected to the control computer. Experiments typically started out with the machine running in place. The machine ran forward when the operator specified a desired speed and direction of travel. The machine either traveled the length of the laboratory, with someone running behind carrying the umbilical, or it ran on a large treadmill. During each experiment the control computer recorded data from the sensors and from the internal variables of the control algorithms, and saved them for later analysis.

We designed the control system for these experiments according to the outline of the previous section. The control system used the three-part algorithms to specify desired behavior for each virtual leg and it used the rules of the virtual leg to coordinate the behavior of the physical legs.

Three-part locomotion algorithms

To control the pitch and roll attitude of the body during stance, the control system applied torques about the virtual hips, using linear servos:

$$u_x = -k_{p,P}\phi_P - k_{v,P}\dot{\phi}_P - k_{f,x} f_x - k_{\dot{\gamma}}\dot{\gamma}_d \qquad (1)$$

$$u_y = -k_{p,R}\phi_R - k_{v,R}\dot{\phi}_R - k_{f,y} f_y \qquad (2)$$

where

u_x, u_y	are the servovalve output signals for the hip actuators
ϕ_P, ϕ_R	are the pitch and roll angles of the body
$\dot{\gamma}_d$	is the desired rate of the actuator
f_x, f_y	are the forces measured in the hip actuators
$k_{p,P}, k_{v,P}, k_{p,R}, k_{v,R}, k_f, k_{\dot{\gamma}}$	are gains.

Hip actuator forces were included in the attitude control to help stabilize the under-damped modes caused by the lateral compliance of the foot pads.

To control the forward running speed, the control system positioned the foot of the virtual leg with respect to the center of mass of the body during each

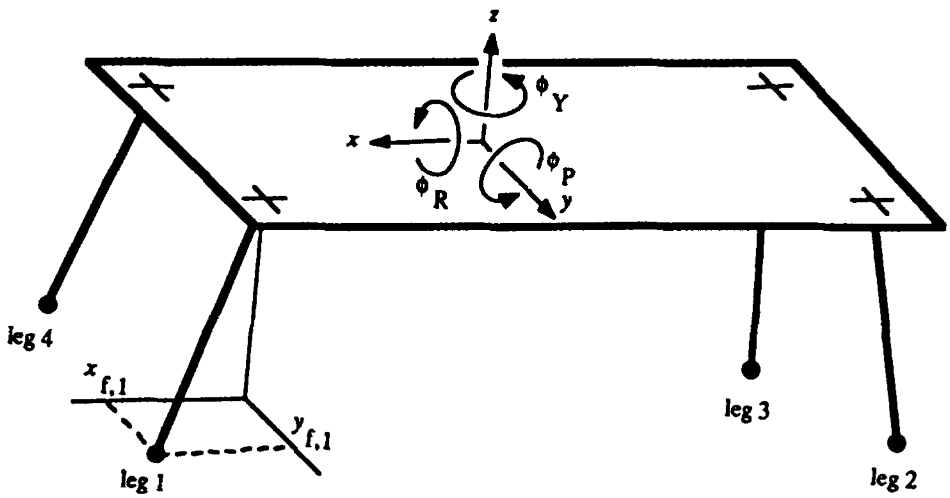


Fig. 4. Diagram of quadruped machine with variable used by control algorithms. Motion of the body is expressed in a Cartesian coordinate system fixed to the body at the center of the hips. Orientation of the body is expressed as rotations about the axes of this coordinate system. The position of the foot (x_f, y_f) with respect to the vertical projection of the hip is shown in the figure for leg 1. (The leg numbering convention is the same as for the pins of an integrated circuit.)

flight phase:

$$x_{f,d} = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) \tag{3}$$

$$y_{f,d} = \frac{\dot{y}T_s}{2} + k_{\dot{y}}(\dot{y} - \dot{y}_d) \tag{4}$$

where

$x_{f,d}, y_{f,d}$ is the desired displacement of the foot with respect to the projection of the center of mass
$\dot{x}, \dot{y}$ is the forward running speed
$\dot{x}_d, \dot{y}_d$ is the desired forward running speed
T_s is the duration of a support period
$k_{\dot{x}}, k_{\dot{y}}$ are gains.

The control system estimated the forward speed of the body, $(\dot{x}, \dot{y})$, using the assumption that the feet do not move with respect to the ground during the stance phase. Under this assumption, the backward motion of a foot with respect to the body is equal to the forward motion of the body with respect to the ground. Gyroscope and hip angle measurements were used together with kinematics to make this estimate. The control system assumed that the forward running speed did not change during flight. The control system measured the duration of each stance phase, T_s, and used the most recent value for control. A human operator used a two-axis joystick to specify the desired forward running speed $(\dot{x}_d, \dot{y}_d)$ during each experiment. See Raibert *et al.* (1986) for additional explanation and details of these control algorithms.

During fast forward running, the fore–aft hip actuators must move with substantial velocity if the body is to remain level with the foot stationary. An ideal torque control system would generate these high actuator rates as an outcome of body attitude control, without explicitly programming them. In the quadruped we obtained a more nearly ideal response by adding a term to the hip actuator output signal that was proportional to the desired actuator rate. The desired actuator rate was determined from the desired forward running speed, the measured pitch rate of the body, and the kinematics of the mechanism. The resulting term is the last term on the right-hand side of equation (1).

To control the vertical thrusting motion, the control system adjusted the hydraulic length of the virtual leg throughout the running cycle. Three parameters L_1, L_2, and L_3, $L_1 < L_2 < L_3$, were used to specify control of thrust throughout the locomotion cycle. When a virtual leg was in the swing phase, the desired hydraulic length was shortened to L_1 to keep the feet from touching the ground during the stance phase of the other virtual leg. When a virtual leg was preparing for landing or compressing under load of the body, the desired hydraulic length was set to the intermediate value L_2. During the second part of the stance phase the desired hydraulic length was increased to L_3 to provide a vertical thrust to the body. The operator specified L_1, L_2, and L_3 manually from the control panel.

Implementation of virtual legs

In order to make the legs work together in pairs, the control system coordinated positioning of the feet, synchronized ground contact, and equalized axial leg thrust. Because of symmetry in the geometry of the quadruped machine, the desired position of the virtual foot with respect to the virtual hip could be used as the desired position of the physical feet with respect to the physical hips:

$$x_{h,i,d} = x_{h,j,d} = x_{f,d} \quad (5)$$

$$y_{h,i,d} = y_{h,j,d} = y_{f,d} \quad (6)$$

where

$x_{h,i,d}$, $y_{h,i,d}$ is the desired displacement of the ith foot with respect to the projection of the ith hip
$x_{f,d}$, $y_{f,d}$ is the desired displacement of the virtual foot with respect to the projection of the virtual hip
i, j are indices of two physical legs that form one virtual leg.

Once the desired foot displacements were known, transformations based on the kinematics of the legs, hips, and actuators were used to find actuator lengths that positioned the feet as desired. These transformations took into account the pitch and roll orientations of the body and the lengths of the legs.

To synchronize the instant of ground contact for the two legs forming a virtual leg, the control system servoed the leg lengths during flight, so that both feet had the same altitude above the ground. This adjustment affected only the difference in leg lengths, while L_2 determined the average leg length. Pitch and roll measurements made from onboard gyroscopes and a kinematic calculation were required to perform these adjustments.

To equalize the axial forces the legs delivered to the ground during stance, the control system differentially servoed the lengths of the leg hydraulic actuators:

$$w_{t,i,d} = w_{t,i} + \frac{r_{s,i} - r_{s,j}}{2} \quad (7)$$

where

$w_{t,i}$ is the hydraulic length of the ith leg
$w_{t,i,d}$ is the desired hydraulic length of the ith leg
$r_{s,i}$ is the air spring length of the ith leg.

This differential adjustment forced the air springs to assume equal lengths and therefore to apply equal axial force. Once again, values for L_2 and L_3 determined the average length of the hydraulic actuators during the first and second parts of the stance phase. The average hydraulic length at touchdown was determined by L_2, and the desired lengths of both hydraulic actuators were increased by $L_3 - L_2$ at the state transition from compression to thrust.

Yaw control

The algorithms used to control the quadruped, as stated so far, do not control the yaw orientation of the body. A torque about the yaw axis was generated by manipulating the position of the feet at touchdown. Foot position was selected so the axial load on the two legs in contact with the ground exerted a couple on the body about the yaw axis. The feet were positioned on a circle centered at the vertical projection of the center of mass. The forward speed calculation specified the average foot position with respect to the projection of the center of mass, but this leaves two degrees of freedom unspecified: the distance between the feet that form the virtual leg and the yaw orientation of the line passing through the feet. The distance between the feet is irrelevant here, but the control system manipulated the orientation of the line connecting the feet to generate yaw torque on the system.

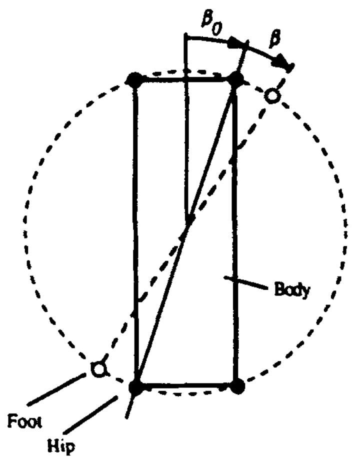


Fig. 5. Control of turning about the yaw axis. The diagram shows the quadruped viewed from above, indicating how placement of the feet can be used to generate a torque about the yaw axis. The torque is proportional to the angle between the line through the hips and the line through the feet, projected vertically. The foot placement shown in the figure would cause the machine to turn counter-clockwise. The filled circles indicate the location of the hips. The open circles indicate the placement of the feet. β is the angle that determines the turning moment. β_0 is the constant angle between the x-axis and the hips.

If the system were to run in place and each foot were placed directly under its hip, no torque would be exerted about the yaw axis. In this case the orientation of the line connecting the feet, viewed from above, would be the same as the orientation of the line connecting the hips (see Fig. 5). If the feet were positioned so the line connecting the feet was rotated about the center of mass, then the axial thrust of each leg would contribute to a moment acting about the yaw axis of the system. The moment would be proportional to the amount of leg rotation, β in the figure. The control system regulated the yaw orientation of the quadruped by manipulating β, without disturbing the average position of the feet.

To control yaw in the experiments, foot placement was used to generate a yaw moment. The operator specified the desired yaw rate with a lever on the control panel. The angle between the line connecting the hips and the line connecting the feet, β, was manipulated as follows:

$$\beta = k_1 \dot{\phi}_{Y,d} + k_2 \dot{\phi}_Y \quad (8)$$

where

ϕ_Y is the yaw angle of the body with respect to room coordinates

$\dot{\phi}_{Y,d}$ is the desired yaw angle rate of the body

k_1, k_2 are gains.

The yaw rate that appears in (8) is not the instantaneous yaw rate, but an average taken for an entire stride. Use of the average yaw rate over the stride permits there to be variations in yaw rate within the stride, without interfering with the control of the machine's facing direction.

Augmenting (3) and (4) above to include turning we have:

$$x_{f,d,i} = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) + D\cos(\beta + \beta_{0,i}) \quad (9)$$

$$y_{f,d,i} = \frac{\dot{y}T_s}{2} + k_{\dot{y}}(\dot{y} - \dot{y}_d) + D\sin(\beta + \beta_{0,i}) \quad (10)$$

where

i indicates the physical leg

$\beta_{0,i}$ is $\arctan(W/L)$ for $i = 1, 3$ and $-\arctan(W/L)$ for $i = 2, 4$

D $= \sqrt{(W^2/2 + L^2/2)}$

L, W are the length and width of the body measured by the hip separations.

Once these calculations for controlling vertical bouncing, forward speed, and turning were performed using equations (3)–(10), kinematic transformations were used to calculate set-points for each actuator. Equations (3)–(10) can be thought of as providing control at the *locomotion level*, whereas the actuators control at the *joint servo level*. Twelve linear servos acted on the hydraulic actuators to position the hips and leg lengths as required by the locomotion algorithms

$$u_i = -k_p(w_i - w_{i,d}) - k_v(\dot{w}_i) \quad (11)$$

where

u_i is the servovalve output signal for the ith actuator

$w_i, w_{i,d}, \dot{w}_i$ are the position, desired position, and velocity of the ith actuator

k_p, k_v are position and velocity gains.

Sequencing

In addition to providing control functions for the body and legs, quadruped locomotion required a mechanism to sequence the use of the legs. The sequencing mechanism selected which leg would next provide support so that it could move to a forward position for landing, and it assigned the thrust and attitude control functions to the leg currently providing support. The sequencing mechanism shortened the idle legs to keep them out of the way of the ground until they once again become active.

The control system used a finite state machine throughout the running cycle to perform this sequencing task, and to synchronize the actions of the control algorithms to the behavior of the machine. It kept track of the legs and assigned the three control functions to the appropriate virtual leg at the appropriate time. The finite state machine traversed ten states during each stride. Each state prescribed a set of sensor conditions that triggered transition into the state, and a set of control actions to be taken during the state. The state transitions synchronized the various control functions—vertical thrust, attitude control, and foot placement—to the ongoing behavior of the running machine. Figure 6 and Table 2 give the details of the state machine as implemented.

Differences in algorithms for trotting, pacing, and bounding

To a large degree, the control program used to produce trotting is the same as the program used to produce pacing and bounding. Of course, the identity of the legs forming each pair varied among the gaits. In addition, several parameters were adjusted individually to producing each gait.

The *track* parameter, the nominal horizontal separation of the feet, was normally set equal to the

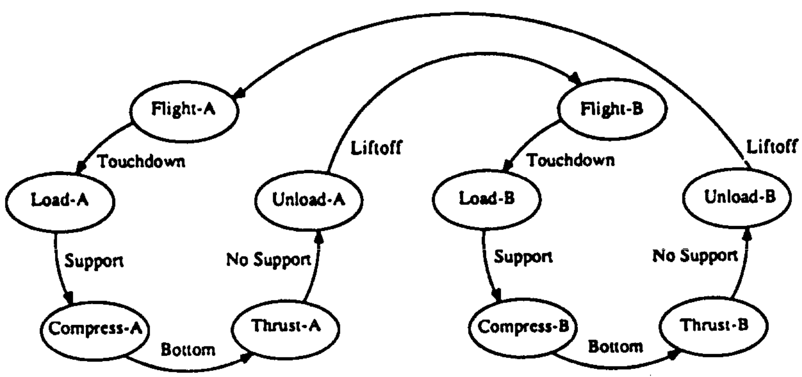


Fig. 6. Diagram of finite state machine used to synchronize the control programs to the behavior of the quadruped. Virtual leg B_i swings forward while virtual leg A_i provides support, and vice versa. State transitions are determined by events related to the support leg. See Table 2 for details.

Table 1. Physical parameters of quadruped running machine

Parameter	Metric units	English units
Overall length	1.05 m	41.2 in
Overall height	0.95 m	37.5 in
Overall width	0.35 m	13.8 in
Overall mass	38 kg	85 lbm
Hip height (max)	0.668 m	26.31 in
Hip spacing (x)*	0.776 m	30.56 in
Hip spacing (y)	0.239 m	9.40 in
Leg sweep angle (x)	± 0.565 rad	$\pm 32.4^\circ$
Leg sweep angle (y)	± 0.384 rad	$\pm 22.0^\circ$
Leg stroke (hydraulic)	0.229 m	9.0 in
Leg stroke (spring)	0.102 m	4.0 in
Body mass	25.2 kg	55.4 lbm
Body moment of inertia (x)	0.257 kg m^2	880 lb in^2
Body moment of inertia (y)	1.60 kg m^2	5470 lb in^2
Body moment of inertia (z)	1.86 kg m^2	6340 lb in^2
Leg mass, total each	1.40 kg	3.08 lbm
Leg mass, unsprung	0.286 kg	0.63 lbm
Leg moment of inertia (about hip)	0.14 kg m^2	480 lb in^2
Leg spring stiffness @20 psi (fully extended)	2100 N m^{-1}	12 lbf in^{-1}
Hip torque, @3000 psi (x)	166 N m	1474 in lbf
Hip torque, @3000 psi (y)	116 N m	1030 in lbf
Leg thrust, @3000 psi	1147 N	258 lbf

*x—fore and aft, y—sideways, z—up and down.

Table 2. Finite state sequence used to synchronize the control algorithms to the behavior of the quadruped machine. The state shown in the left column is entered when the event listed in the center column occurs. States advance sequentially during normal running. A_t refers to the virtual leg made up of physical legs LF and RR for trotting, LF and LR for pacing, and LF and RF for bounding. B_t refers to the virtual leg made up of physical legs RF and LR for trotting, RF and RR for pacing, and LR and RR for bounding. During states 1–5, A_t is the support leg and B_t is the swing leg. During states 6–10, B_t is the support leg and A_t is the swing leg.

State	Trigger event	Action
1 Loading A_t	A_t touches ground	Equalize axial force A_t Zero hip torque A_t Shorten B_t Don't move hip B_t
2 Compression A_t	A_t air springs shortened	Equalize axial force A_t Erect body with hip A_t Shorten B_t Position B_t for landing
3 Thrust A_t	A_t air springs lengthening	Extend A_t, equalizing force Erect body with hip A_t Keep B_t short Position B_t for landing
4 Unloading A_t	A_t air springs near full length	Shorten A_t, equalizing force Zero hip torques A_t Keep B_t short Position B_t for landing
5 Flight A_t	A_t not touching ground	Shorten A_t Don't move hip A_t Lengthen B_t for landing Position B_t for landing

States 6–10 repeat states 1–5, with A_t and B_t reversed.

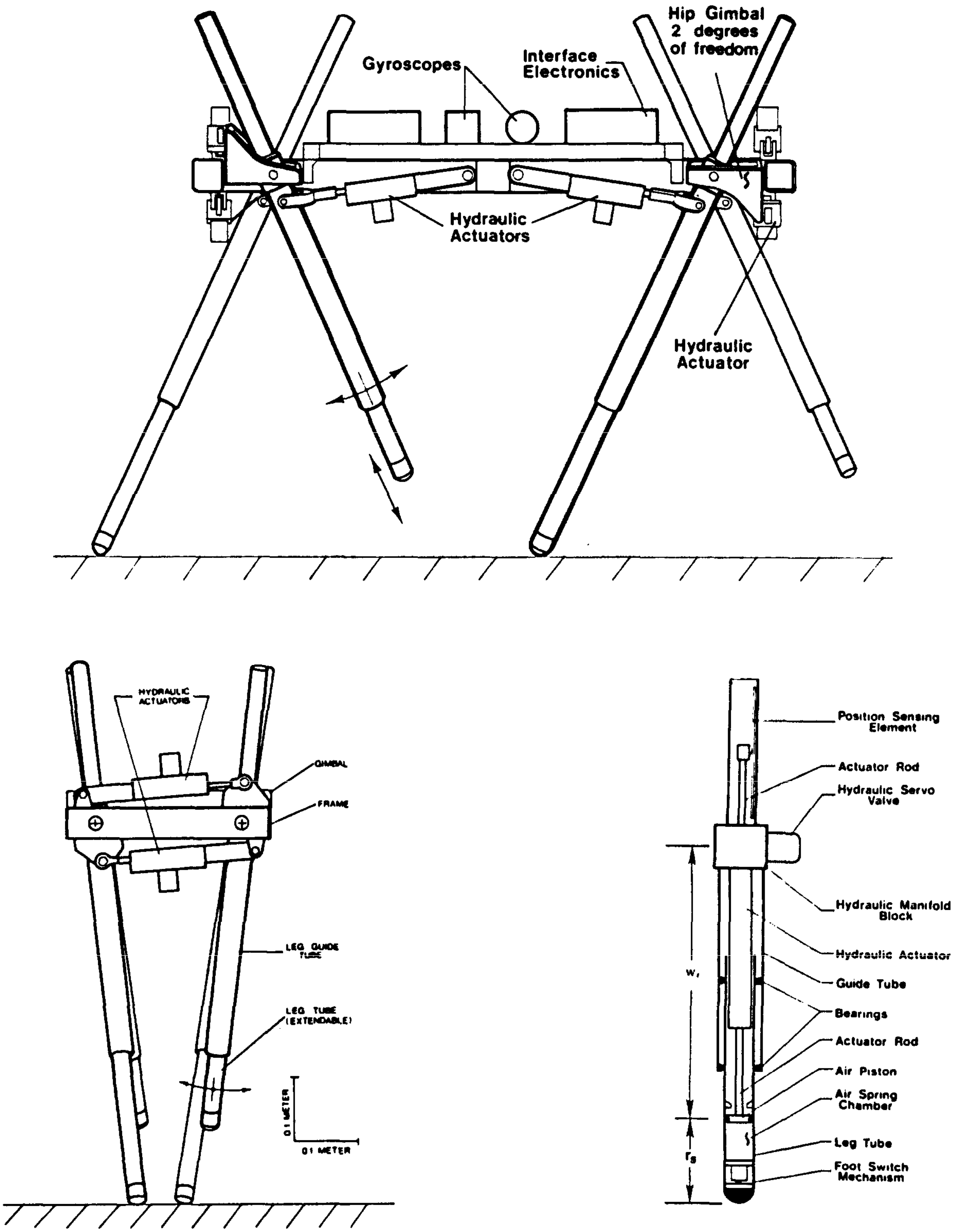


Fig. 7. Diagram of quadruped running machine used for experiments. The body is an aluminum frame, on which are mounted legs, hip actuators, gyroscopes, and computer interface electronics. Each hip has two low friction hydraulic actuators that position the leg fore and aft, and sideways. Sensors measure the position, velocity, and force of the hydraulic hip actuators, hydraulic leg length, overall leg length, leg spring length, contact between the feet and the floor, and the pitch, roll, and yaw orientations of the body. An umbilical cable connects the machine to hydraulic, pneumatic, and electrical power supplies, and to the control computer (DEC VAX/785), all of which are located nearby in the laboratory. The arrangement of spring in series with position source in each leg was motivated by simple muscle models. Physical parameters of the quadruped machine are given in Table 1.

hip spacing (0.23 m) for trotting and bounding. For pacing, the track parameter was reduced to 0.09 m. This value of the track parameter brought the feet closer to the midline, which reduced the roll motion of the body during the stance phase.

During the stance phase, the legs lengthened from L_2 to L_3 to drive the body upward. The amount the legs lengthened was about the same for trotting and pacing, but was larger by about 0.03 m for bounding. This difference was required because the thrust delivered by the legs caused the body to pitch during bounding, and the loading on the legs was reduced. To provide the same vertical acceleration required greater leg extension.

The most important variation in how the gaits were implemented concerns pitch control during bounding. We found that bounding did not require active control of the pitch attitude. The observed pitch oscillation was passively stabilized by the mechanical system. In previous work we found that passively stable pitch oscillations occurred in computer simulation of a planar model with two separated legs (Murphy and Raibert, 1985). In the bounding experiments reported here, the hip actuators were used to position the legs during the flight phase and to servo the hips to zero force during stance. They were not programmed to respond to errors in the pitch attitude of the body. Earlier implementations of bounding used the pitch control algorithms that were used for trotting and pacing, but data from those runs have not been included in this paper. Stability about the roll axis was controlled actively for all three pair gaits.

RESULTS

Data recorded during trotting experiments are shown in Figs 8 and 9. In trotting, the legs are used for support in diagonal pairs. The synchronization of foot impacts was controlled to within 12 ms in most cases, and equalization of pressure in the leg spring was controlled to a few psi, with transients reaching 70 psi briefly. The vertical bouncing motion of the body was regular and smooth.

Regulation of forward running speed was not perfect, as shown in Fig. 9. Only a rough relationship existed between the desired and actual running speeds. The errors in speed were due to known limitations of the velocity control algorithm. Hodgins (1989) describes improvements to the velocity control algorithm that reduce forward speed control errors to about ± 0.1 m s^{-1}, when tested with a planar biped running machine. One type of error in forward speed occurs during accelerations. The data in Fig. 9 show a marked increase in vertical bouncing at the point where forward speed decreases, at about $t = 15$ s. Some of the kinetic energy due to forward motion was converted into vertical motion during the deceleration. The effect is similar to that of a pole vaulter who converts forward speed into altitude.

During forward trotting the inclination of the body about the pitch axis, ϕ_P, deviated from the desired value by up to 8°. The magnitude and sign of this error were generally related to the forward running speed. These errors in pitch orientation are caused by parasitic damping in the actuators, which increases with

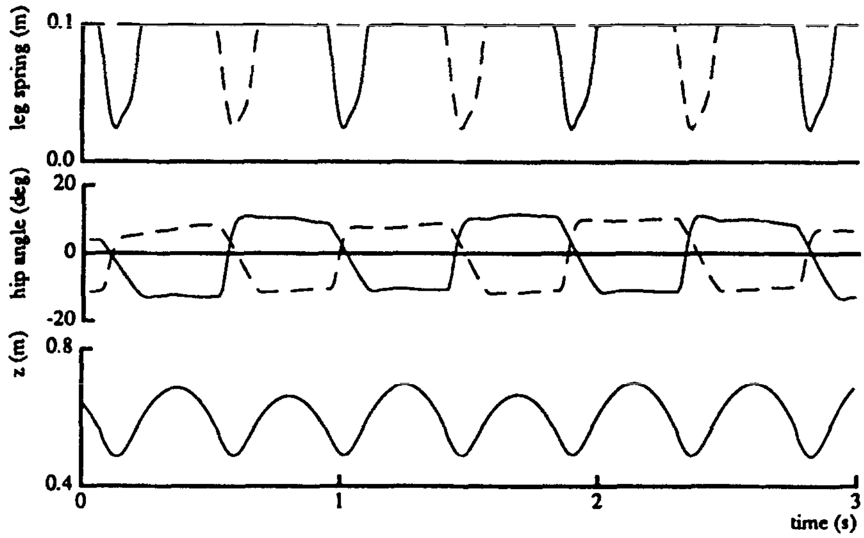


Fig. 8. The vertical bouncing motion. These data show the bouncing motion that underlies quadruped running. The data were recorded during trotting, for the left front and rear legs (LF shown solid, LR shown dashed). The top curve shows the compression of the air springs. The middle curve shows the hip joint motions for the same two legs. The bottom curve shows the altitude of the body above the floor, as estimated by the control system from internal joint sensors, data from the gyroscopes, and a rigid body model. (Data file Q86.343.5.)

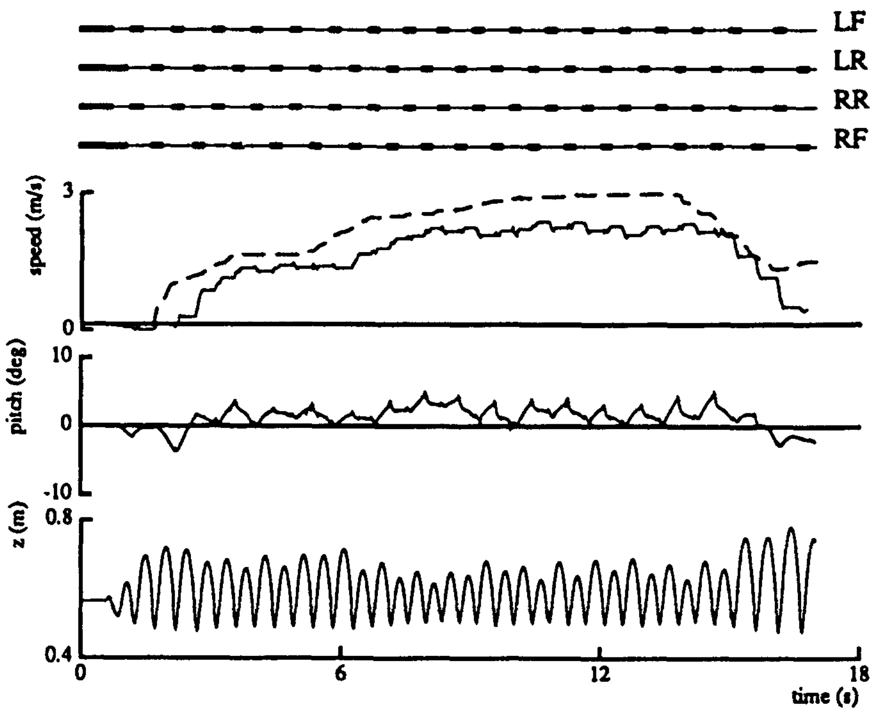


Fig. 9. Forward running. The machine stood on four legs until an operator initiated running (in this case trotting) by pressing a button. The operator used a joystick to specify the desired running speed (shown dashed) and direction. There was about a 0.3 m s^{-1} steady state discrepancy between the desired and measured forward running speed. The body tipped in the direction of running, as shown by the plot of pitch angle. Positive pitch indicates nose down. When the desired forward running speed was reduced at about 15 s, some of the energy due to forward motion was converted into vertical motion, as shown by the plot of z. (Data file Q86.343.5.)

running speed. The control system kept error about the roll axis within $\pm 5°$ in these experiments. The stability of the system about the roll axis was generally improved by adding weights to the body that increased its moment of inertia about the roll axis.

Data recorded during pacing are shown in Fig. 10. The behavior was similar to trotting, except for an oscillation about the roll axis of the body of about $\pm 5°$, and a small amount of lateral motion. Roll and lateral motions were both expected because the center of mass was not located over the virtual hips, as it was for trotting. During pacing the lateral position of the feet were biased so as to bring them near the center line. This was done by adding a *track* parameter to equation (6), which specified a fixed lateral offset in the desired placement of the feet. For the data shown in the figure, the nominal lateral foot separation was set to 0.1 m with the lateral hip separation fixed at 0.239 m. The machine paced with both larger and smaller lateral foot separations. We were not successful in making the machine pace with the feet on the center line because of leg and foot collisions.

Data for bounding are shown in Fig. 11. Bounding was characterized by large oscillations about the body pitch axis. These oscillations can be predicted from the large forward and rearward displacements of the virtual hips from the center of mass. The data show a pitch oscillation of $\pm 18°$, with little body rotation about the roll or yaw axes. Vertical displacement of the center of mass was about 0.05 m in bounding, as compared to about 0.2 m for trotting and pacing. Angular motion of the body was the primary factor that lifted and placed the legs in bounding, while vertical motion of the body was the primary factor in trotting and pacing.

Table 3 summarizes data for the quadruped machine trotting and bounding at top speed. The data were obtained during several seconds of steady running at the fastest speed for each gait. The results are expressed in terms of parameters normally used to characterize the behavior of running animals (Alexander *et al.*, 1977; McMahon *et al.*, 1987). The table also gives values for the parameter that would be expected of an ungulate, if it were the size and mass of the quadruped machine, and in the case of relative stride length, the speed of the quadruped. These values were obtained by evaluating allometric equations that summarize the observed behavior of ten species of African ungulates running at top speed (Alexander *et al.*, 1977) for values of the independent variables that correspond to the quadruped machine.

The allometric equations for ungulates predict values for stride frequency, relative stride length, and relative step length that are close to those observed for

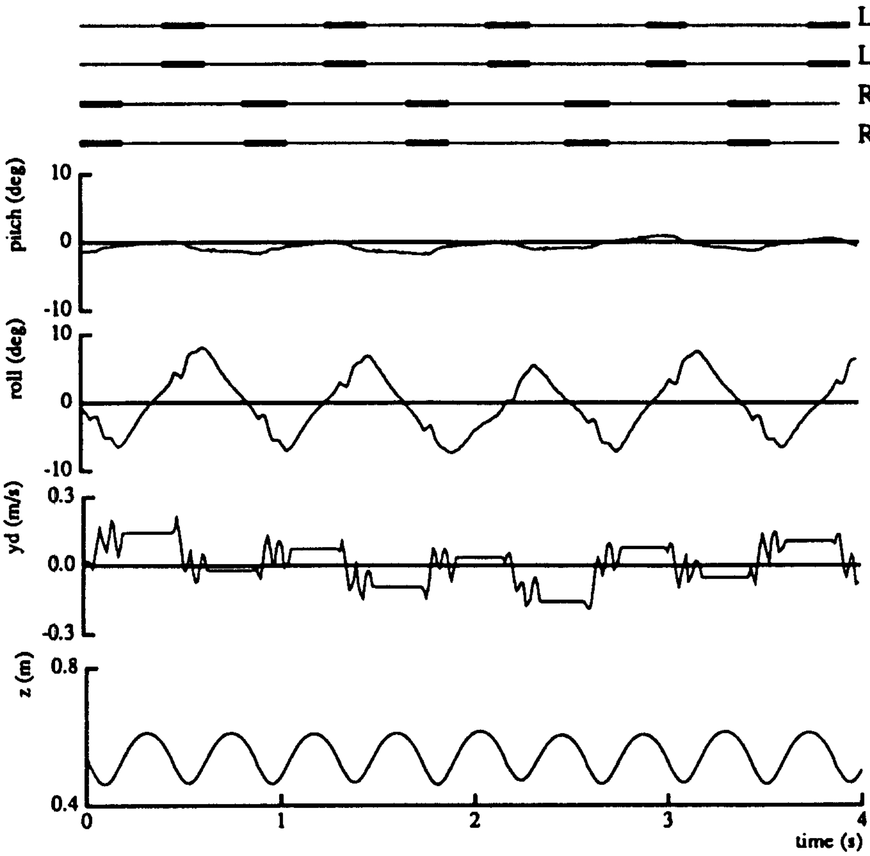


Fig. 10. Pacing. Data recorded as the quadruped paced in place. The roll oscillation and lateral translation were characteristic of pacing. Neither trotting nor pacing involve much pitching of the body. For pacing, the nominal lateral spacing of the feet, the track, was set to 0.09 m. This value is less than the hip spacing, which is 0.239 m. (Data file Q87.142.4.)

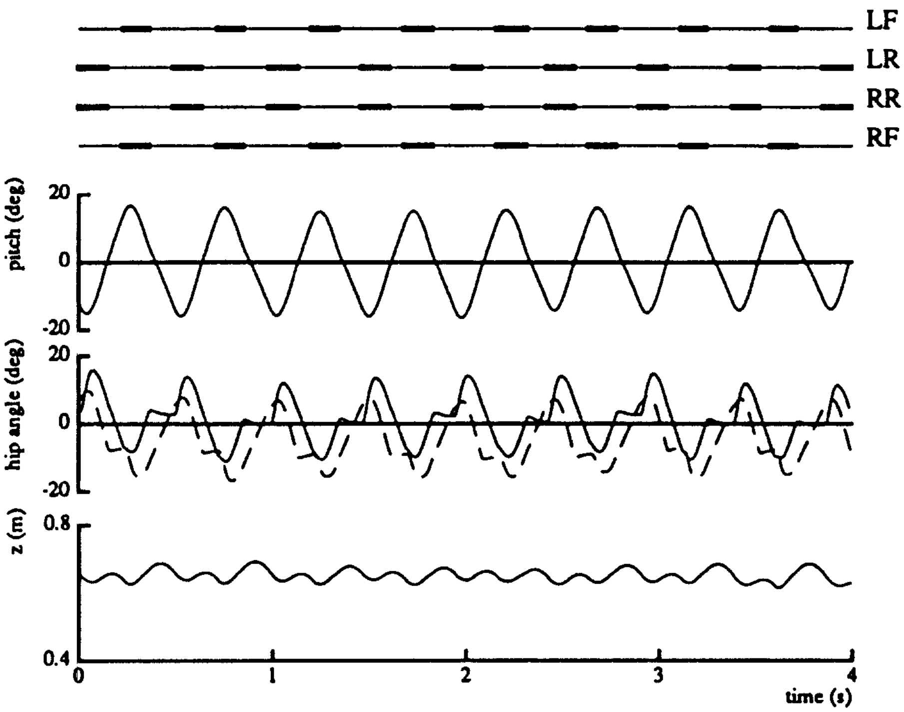


Fig. 11. Bounding. Bounding was characterized by large pitching motions of the body. Vertical motion of the center of mass was less than for trotting and pacing, even with greater leg thrust. For bounding, the nominal longitudinal spacing of the feet, the base, was set to 0.776 m, equal to the longitudinal hip spacing. (Data file Q87.167.1.)

Table 3. Summary data for quadruped trotting and bounding at top speed. Measurements were made by calculating values for each parameter for each stride, and averaging the results for several seconds of steady running. Resting leg length (0.56 m) and body length (0.78 m) were used to normalize the data to obtain dimensionless parameters. Data listed under 'Animals' were obtained by evaluating allometric equations given by Alexander *et al.* (1977, Table I) with quadruped machine parameters substituted for the independent variables: mass 38 kg, nominal body length 0.78 m, nominal leg length $l_0 = 0.56$ m, Froude number 1.53. The Groucho number, a dimensionless parameter that indicates the vertical stiffness of the system and the springiness of the running motion, was calculated using methods described by McMahon *et al.* (1987). (Trotting data Q86.343.5, bounding data Q88.167.2.)

Parameter	Trotting	Bounding	Animals
Mean stride frequency, s^{-1}	1.2	2.4	2.8
Duty factor	0.19	0.28	0.18
Stride length, m	1.87	1.20	
Relative stride length, *leg lengths*	3.4	2.2	2.12
Step length, m	0.35	0.34	
Relative step length, *leg lengths*	0.62	0.60	0.71
Maximum speed, $m\,s^{-1}$	2.2	2.9	
Maximum speed, *body lengths* s^{-1}	2.9	3.5	12.8
Froude number, $\dot{x}^2/(gl_0)$	0.88	1.53	
Groucho number, $\omega_0\dot{z}/g$	2.6	1.29	

the quadruped machine when bounding. The equations predict a duty factor that is about 50% smaller than observed in the machine when bounding, but close to the trotting value. The machine runs much more slowly than is predicted by the equations.† The Groucho number calculated for bounding is within the range normally observed for running animals (McMahon, 1987). The Groucho number observed for trotting is larger than found in most animals, but within the range of animal jumpers.

Two separate state machines for bounding and pronking

The state machine shown in Fig. 6 was used to generate trotting, pacing, and bounding. For bounding we also experimented with a variation of the control system that used a separate state machine for each virtual leg, as shown in Fig. 12. In this implementation the legs were paired as before, but the behavior of the front virtual leg (LF–RF) was tracked and controlled by one state machine, and the behavior of the rear virtual leg (LR–RR) was tracked and controlled by an entirely separate state machine. No explicit action was taken to synchronize the front and rear virtual legs.

Using separate state machines for the front and rear virtual legs, the system was observed to stablize in either of two gaits, pronking or bounding. In pronking, the system hopped on all four legs at once, with the pitch angle of the body nearly level and zero phase lag between the behavior of the front and rear virtual legs. The left half of Fig. 13 shows the quadruped pronking. In pronking the system rejected phase disturbances by returning itself to synchronous use of all four legs. There was nothing explicit in the control system to provide this synchronization.‡

Using separate state machines, it was also possible for the quadruped machine to stabilize in a bounding gait, with 180° phase between the behavior of the front and rear virtual legs. Data recorded during bounding using separate state machines are plotted in Fig. 14. They are difficult to distinguish from the other bounding data. In this experiment the machine started bounding in place, accelerated up to 3.0 $m\,s^{-1}$, then stopped as it approached the end of the running area. When the quadruped bounded in place with separate state machines, the phase relationship between front and rear legs was stable at 180°. When the quadruped traveled forward, the phase shifted to reduce the duration of extended flight phases (the ones occurring after the rear legs provide support) and increased the duration of the gathered flight phases (the ones occurring after the front legs provide support). This phenomenon can be seen in Fig. 14. The phase shift observed during bounding was caused by the same actuator damping problem that causes the system to pitch nose down during trotting.

† Whereas top running speed for animals is most likely limited by structural strength and energetics, the quadruped machine is currently limited by the ability to control motions of the body about the yaw axis when traveling faster than about 3 $m\,s^{-1}$. Koechling made a planar biped robot run at 5.5 $m\,s^{-1}$ using the same leg mechanism and actuators used by the quadruped (Koechling and Raibert 1988), suggesting that the quadruped has adequate strength and actuation for faster running.

‡ Control about the roll axis during pronking was marginally stable, which made extensive testing of this gait difficult. Roll stability was poor because the combined moment of inertia of the four legs about the roll axis was large compared to the moment of inertia of the body. In this case, the algorithm that stabilized lateral translation (equation 6) disturbed the roll posture of the body.

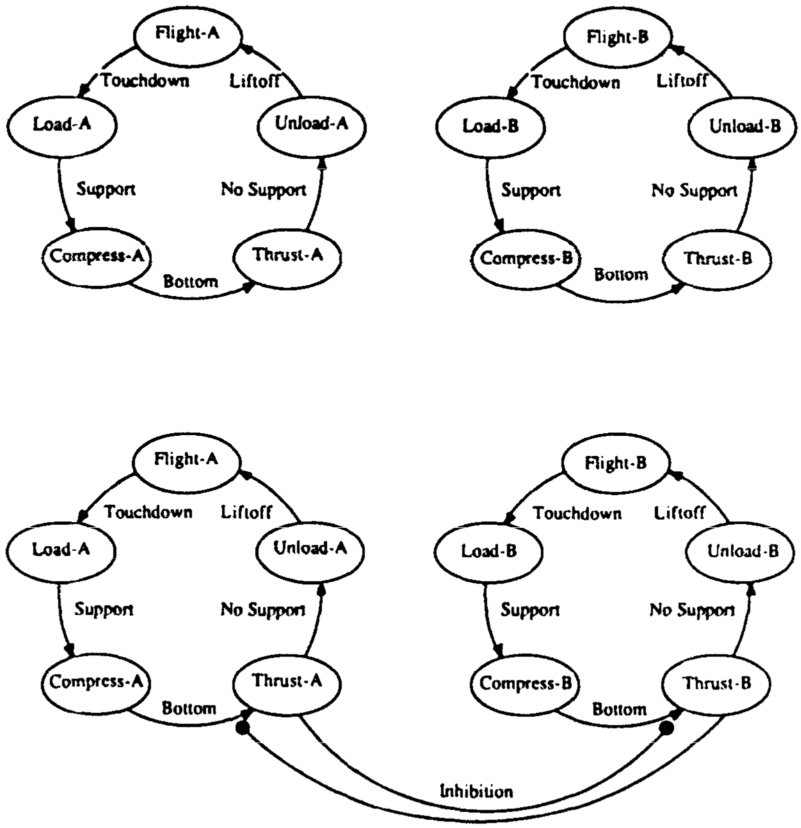


Fig. 12. Independent state machines assigned to the virtual legs for bounding. Top: one state machine tracks the behavior of the front virtual leg while a second state machine tracks the behavior of the rear virtual leg. The phase relationship between the behavior of the two virtual legs is not specified by the state machine, but results from mechanical and dynamic coupling in the system. Bottom: when inhibition is enabled, it prevents both virtual legs from thrusting at the same time. It prevents one state machine from entering the thrust state if the other state machine is already in the thrust state. The unequal thrust caused by inhibition induces pitching of the body, and thereby desynchronizes the two state machines.

To initiate bounding with separate state machines, an *inhibition* function was implemented, as shown in Fig. 12. Inhibition was used to artificially desynchronize the front and rear state machines by permitting only one of them to enter the thrust state at a time. With just one virtual leg thrusting at a time, a large pitch moment and pitching motion of the body was generated. This served to introduce a phase difference between the front and rear virtual legs, which eventually stabilized at 180°. The transition from pronk to bound using inhibition is shown in Fig. 14.

Gait transitions

In previous work we demonstrated that a planar biped could switch between an alternating gait and a hopping gait (Hodgins *et al.*, 1986). The approach was to execute the switch, or gait transition, during the flight phase, when the two gaits are nearly indistinguishable.

The quadruped presents a richer set of gait transition possibilities, as well as more complicated transitions. For instance, there are six different transitions possible among trotting, pacing, and bounding. During the flight phase these gaits differ with respect to the characteristic body motion. In trotting, the body is level during the flight phase. In pacing, the pitch angle of the body is level, but the roll angle of the body oscillates. In bounding the roll angle of the body is level, but the pitch angle undergoes oscillations of nearly $\pm 20°$.

One approach to achieving quadruped gait transitions designates a transition step, during which the control system generates the moment required to adjust the attitude motion of the body. For instance, a transition from trotting to bounding would introduce a pitching moment during the transition step, by differentially thrusting with the front and rear support legs. A transition from pacing to trotting would require a moment that eliminated body roll.

So far, we have used this approach on the pitch axis but not the roll axis. Roll axis oscillations are small enough to ignore. We have implemented gait transitions from trotting to pacing, trotting to bounding, and pacing to trotting. Data from two of these transitions are shown in Fig. 15. Transitions between trotting and pacing were typically quite smooth, with

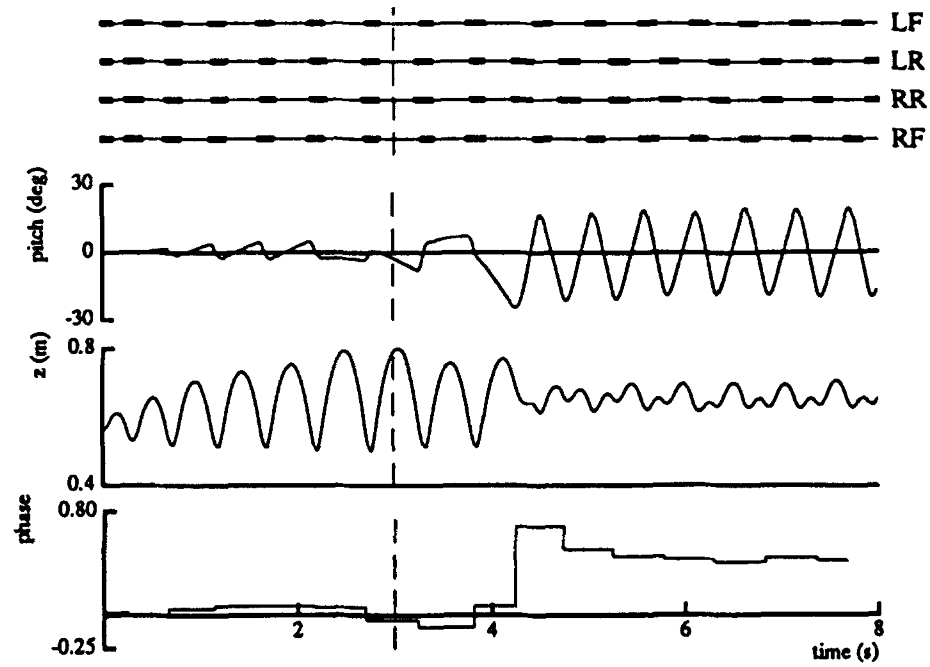


Fig. 13. Pronk and bound. The machine started out pronking. During each stance phase, the pitch angle of the body returned to nearly level, which synchronized the front and rear virtual legs and their state machines. The machine began to bound when the operator enabled inhibition after 3 s, indicated by vertical dashed line. Inhibition acted during two steps: from $t = 3.31$ to 3.40 s the front virtual leg was prevented from thrusting, and from $t = 3.84$ to 3.93 s the rear virtual leg was prevented from thrusting. Phase shifted at the pronk to bound transition. Phase was calculated as the time difference between the front and rear virtual legs striking the ground, normalized by the period of a stride for the front virtual leg. (Data file Q89.188.9.)

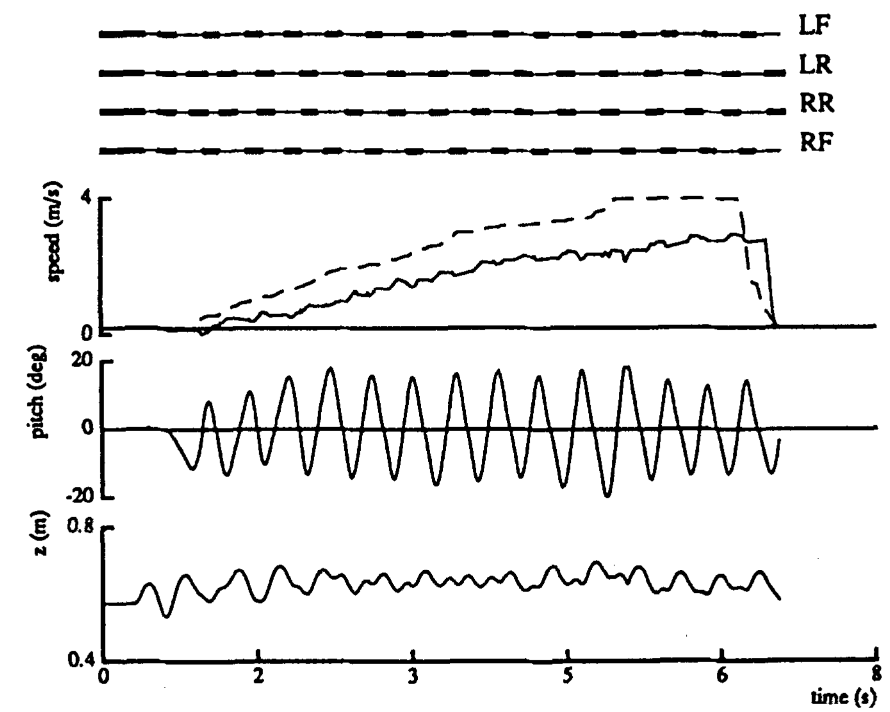
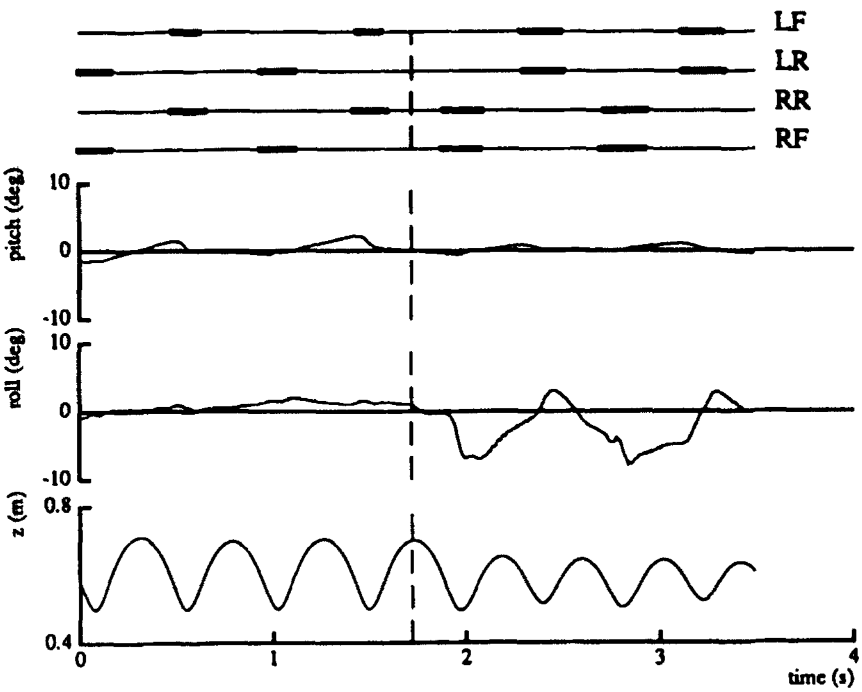
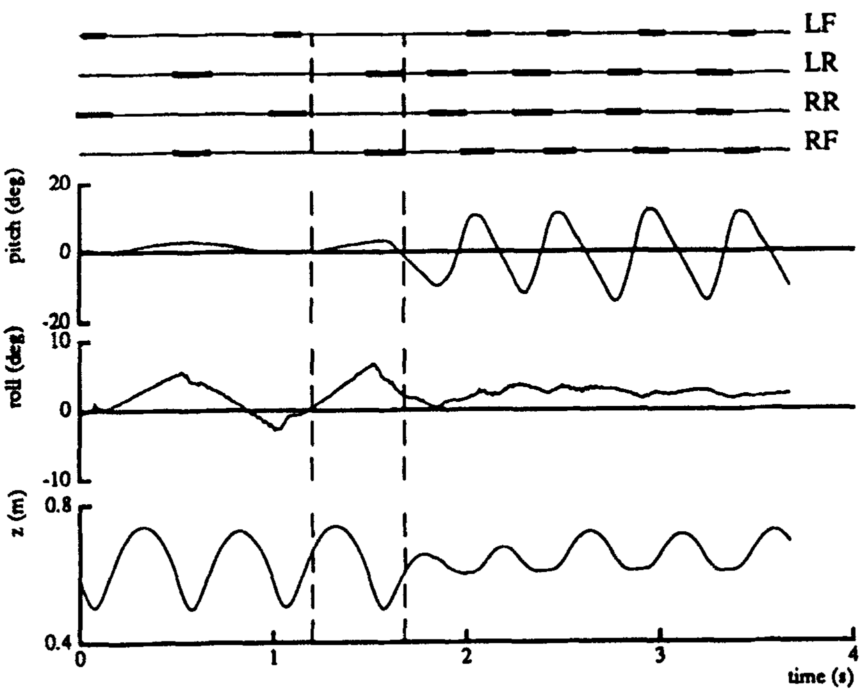


Fig. 14. Bounding with independent state machines for front and rear virtual legs. The machine started bounding in place, accelerated up to $3\ \mathrm{m\,s^{-1}}$, and stopped abruptly just before reaching the end of the laboratory. Once again, there is a substantial discrepancy between the desired forward running speed (shown dashed) and the measured running speed. As the machine increased speed, the phase relationship between front and rear ground impacts shifted away from 180°. (Data file Q87.196.4.)



Trot to Pace



Trot to Bound

Fig. 15. Gait transition: Top: data recorded during a transition from trotting to pacing. The transition occurred during a flight phase, as indicated by the vertical dashed line. There was little disruption of the running motion. Forward running speed was essentially zero. Bottom: the transition from trotting to bounding took several steps to stabilize. The vertical dashed lines bracket the transition step, which induced pitching of the body. (Data file Q87.335.3 and Q88.2.3.)

little disruption of the motion. For transitions to bounding, several steps were frequently required before the pitching motion of the body stabilized.

To initiate a gait transition, the operator pressed a button indicating that the machine should change gait beginning during the next flight phase. The operator continued to specify the desired forward and lateral running speeds, but the motions required during the transition were generated by the control system. All gait transitions were done while the quadruped ran in place or traveled at low speed. We have not written programs that attempt transitions from bounding to trotting or bounding to pacing. Such transitions will require algorithms for leg thrust that bring the pitch motion to zero rate and to an approximately level body angle. We also have not experimented with high speed gait transitions.

DISCUSSION

Displacements of the virtual legs

In trotting, the points half-way between the physical legs of each pair, the virtual hips, were both located under the center of mass of the body. Therefore, when each foot was positioned with respect to its hip, the virtual foot was positioned with respect to the center of mass, as specified by the one-leg algorithms.

In pacing and bounding, however, the center of the hips are displaced from the center of mass. In pacing, the virtual hips are displaced laterally from the center of mass by half the body width, or 0.119 m. In bounding, the virtual hips are displaced longitudinally from the center of mass by half the body length, or 0.388 m. As a result of these displacements, the control for bounding and pacing did not place the feet as required by the one-leg control algorithms: the placement errors alternated in sign on each step. These errors were responsible for the characteristic roll motions observed in pacing, and the characteristic pitching motion observed in bounding. Because the displacements alternate in sign on each step, there was a symmetry to the resulting accelerations that balanced out over an entire stride. Symmetries of this sort are described by Raibert (1986b).

It is possible to eliminate these displacements by specifying fixed offsets for each virtual leg. *Track* and *base* parameters can be used for this purpose. For instance, the displacement was reduced to 0.05 m for pacing. Despite the theoretical possibility of eliminating these displacements entirely, there are practical limitations. One limitation comes from the range of motion of the legs. In bounding, the front legs can not reach far enough backward to be placed under the center of mass, nor can the rear legs reach far enough forward. A second limitation concerns foot collisions. If the feet were placed on the centerline in either pacing or bounding, it would be difficult to keep the legs from colliding. Of course, there are animals that do not suffer from either of these limitations.

An alternative method for controlling yaw

There is an alternative method for controlling yaw and turning to the one that was implemented. It is possible to manipulate the pitch and roll hip torques exerted during the stance phase to achieve a corrective torque about the yaw axis. The sum of the pitch and roll torques exerted on the body by the hips was determined by the virtual leg calculation. However, the difference in hip pitch and roll torques is free to be used to control yaw.

If the hips were separated W laterally, L longitudinally, and the legs were length r, then the yaw torque would be

$$\tau_Y = \frac{L}{2r}(\tau_{R,i} - \tau_{R,j}) + \frac{W}{2r}(\tau_{P,i} - \tau_{P,j}) \qquad (12)$$

where $\tau_{R,i}$ is the roll hip torque exerted by leg i, and $\tau_{P,i}$ is the pitch hip torque exerted by leg i. Pure yaw torque would be obtained without internal forces in the closed chain when

$$\tau_{P,i} = -\tau_{P,j} \qquad (13)$$

$$\tau_{R,i} = -\tau_{R,j} \qquad (14)$$

$$W\tau_R = L\tau_P. \qquad (15)$$

This manipulation would leave the sum of the pitch hip torques available for controlling the body pitch angle, and the sum of the roll hip torques available for controlling the body roll angle. This method could be used in conjunction with the previous method of influencing yaw described earlier, with this one exerting torques during stance and the other positioning the feet during flight. This method has not been implemented.

Force-equalizing virtual legs

A consequence of coordinating the legs of a pair so that they exert equal forces on the ground is the loss of passive stability that a pair of legs might otherwise provide. An ordinary table resists tipping when unevenly loaded because the legs near the load generate more supporting force than the legs that are far from the load. If a table had force-equalizing legs, then an uneven load would cause the legs near the load to shorten, the legs remote from the load to lengthen, and the surface to tip. This force-equalized behavior should be expected, since it is precisely the behavior of a table with just one leg located in the middle.

The experiments reported in this paper showed that an approach which discards passive stability of the legs is workable. It leaves us with a design philosophy problem: on the one hand, the force-equalizing virtual leg permitted relatively sophisticated behavior with a simple implementation. On the other hand, a well-engineered control system should take advantage of the intrinsic mechanical stability of the mechanism. That approach would ultimately lead to the most efficient system, both in terms of energy and control.

Despite these limitations, it is entirely possible that four-legged animals use force equalization when they trot, pace, or bound. One might find out by measuring the axial forces that develop in the legs of running quadrupeds, perhaps using sets of force platforms. The experiment would disturb one of the feet during stance by shifting the support surface upward or downward. If force equalization were in effect, the difference in axial leg force would not be affected by the manipulation. Exact force equalization is unlikely to be found, because the distribution of mass in animals' bodies is skewed by the asymmetric placement of their heads and the unequal lengths of the fore and hind legs. One might find an asymmetry in force equalization comparable to the skewness of body mass distribution.

Gait selection

This paper presents data for three quadruped gaits and describes rudimentary techniques for switching between gaits. It is silent, however, on the question of how to choose which gait to use. In animals, energetic cost seems to be an important factor in selecting a gait. Animals change gait as they change speed, apparently to minimize the cost of transportation (Hoyt and Taylor, 1981). The geometry of animals may also enter into gait selection. At low running speeds, for instance, long-legged animals use a pace rather than a trot, presumably to avoid interference between the front and rear legs on each side (Hildebrand, 1960). Other factors, such as the range of leg motion and leg stiffness may also be important. Despite these potential factors, the experiments reported here do not suggest good criteria for selecting one gait over another.

This paper refers to the pair gaits as 'simple' gaits. They are simple because of the regular alternation between flight phases and double support phases, and because all the legs move the same way. Less simple quadruped gaits are the canter and gallop, which mix flight phases with single, double, and triple support phases. The key problem for the less simple gaits is how more than one leg can work together to rebound the body, without making them either synchronous or entirely disjoint in their motion, and without giving up the strain energy absorbed by the legs.

One way to approach these less simple gaits might be to generalize the virtual leg concept to apply to the behavior of physical legs that act in sequence, but overlap in time. One might separate each support phase into the subintervals during which a fixed number of legs provides support. Then the entire support phase might be represented by a sequence of virtual support phases. For a rotary gallop the sequence of phases would be (1) right rear, (2) right rear and left rear, (3) left rear, (4) left rear and left front, (5) left front, (6) left front and right front, and (7) right front. Again, the key difficulty is to find a mechanism that can mediate the smooth exchange of support from one leg to another, without disrupting the bouncing motion of the body.

Relevance to animals

There is no reason to assume that the mechanisms and algorithms studied in the context of legged robots are the same as those used by animals. However, experience implementing an artificial system that performs a task like the one an animal performs can provide a better appreciation of the task, and perhaps insights into strengths and weaknesses of various approaches. Specific control algorithms can also be used as initial hypotheses that are detailed, concrete, and testable in animals.

One could argue that robots have certain advantages over animals, when it comes to being the subject of motor behavior experiments. One important advantage is the experimenter's knowledge of the intended function of all components in the system. The control system architecture, control method, and implementation details are all known when the behavioral data are examined. This knowledge provides an important tool in interpreting observed behavior. Another advantage to studying robots comes from the experimenter's freedom to simplify and instrument the system. One can design a system with just enough complexity to be interesting, but no more. It is usually feasible to make direct measurements of important variables.

Robots also have disadvantages. The primary disadvantage is that the quality and richness of their behavior does not compare well to that of animals. Also, it is possible to study a form of animal behavior when little is known about it, whereas robots are useful only when knowledge is already quite advanced. Bootstrapping is required before there is any behavior to study at all, and then it is usually impoverished when compared to animal behavior. Finally, for those interested in the specific details of how an animal performs a task, rather than in the nature of the task performed, the study of robots may provide only indirect clues.

SUMMARY

We describe control algorithms for quadruped trotting, pacing, and bounding. The high-level part of the control performs three tasks: it regulates the vertical bouncing motion, stabilizes the forward running speed, and keeps the body level. The high-level algorithms are like those used previously to control one-legged hopping machines. The low-level part of the control system coordinates the behavior of legs by manipulating the relative placement of the feet on the ground, the relative forces the legs of a pair exert on the ground, and the net hip torque the legs exert on the body.

Experiments with a four-legged running machine verify the general approach outlined in the paper. The control system used the one-legged algorithms, a finite state machine, and virtual legs to make it run with trotting, pacing and bounding gaits.

Gait transitions from trotting to pacing and from pacing to trotting were accomplished at low forward speed by switching from one gait to the other during the flight phase. Transitions from trotting to bounding were accomplished by introducing an adjustment step, during which differential thrust of the fore and hind legs gave the body a pitch moment.

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